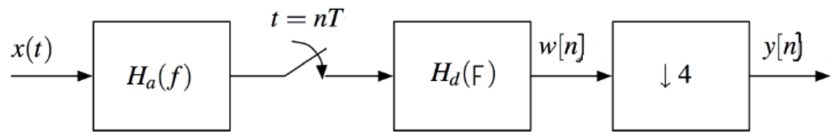
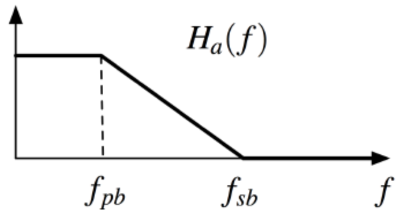
Problem 1 (Signal Processing)

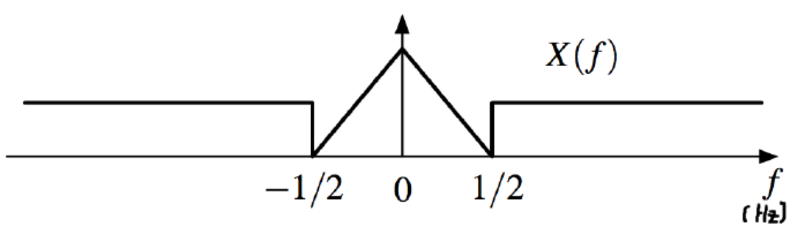
Consider the following system:



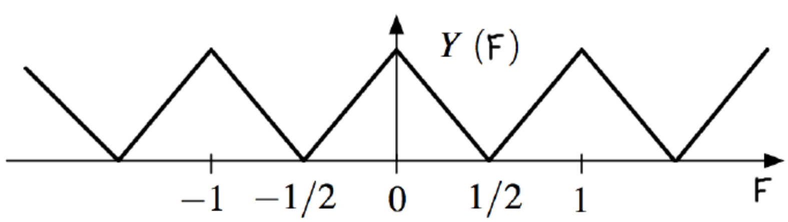
where the real-valued continuous-time signal $x(t)$ is first filtered by a non-ideal lowpass filter with a frequency response $H_a(f)$, shown below (f denotes the continuous-time frequency in Hz):



Then, the output of this filter is sampled and filtered by a discrete-time lowpass filter $H_d(F)$ (where $F = fT$ denotes the discrete-time frequency). For convenience, we assume that this discrete-time lowpass filter is *ideal*. Finally, the signal is decimated by a factor of 4. The analog input $x(t)$ contains a signal of interest in the frequency band $-1/2$ Hz $< f < 1/2$ Hz and an interfering signal in the bands $f < -1/2$ Hz and $f > 1/2$ Hz that we wish to suppress. The spectrum of the input signal is:



The desired output-signal spectrum is



We wish to find the set of parameters that make the continuous-time filter $H_a(f)$ as easy to build as possible, meaning we wish to have the **largest possible** transition band $f_{sb} - f_{pb}$.

(a) **(2 pts)** Plot the spectrum (i.e. discrete-time Fourier transform) $W(F)$ of the signal $w[n]$ before the decimator.

(b) **(2 pts)** Determine the impulse response of the ideal discrete-time filter.

(c) **(2 pts)** Determine the sampling rate $f_s = 1/T$ for the sampler.

(d) **(4 pts)** Determine the passband and stopband frequency edges f_{pb} and f_{sb} that correspond to the **widest possible** transition width for the continuous-time filter.

Problem 2(Signal Processing)

Let k be an unknown integer and h_{k-1}, h_k and h_{k+1} be unknown reals. The coefficients $h_i, i = k-1, k, k+1$ represent the non-zero taps of a filter denoted by $h[n]$. $H(e^{j\omega})$ represents the frequency response of $h[n]$. Let $y[n]$ denote the response of $h[n]$ to an input $x[n]$. You are given the following information about $h[n]$.

1. $e^{j\omega}H(e^{j\omega})$ is real and even.
2. If $x[n] = (-1)^n$ for all n. Then the signal $y[n] = 0$ for all n.
3. If $x[n] = (\frac{1}{4})^n u[n]$ where $u[n]$ is the unit step function, then $y[2] = 25/16$.

a) Find the values of k and h_{k-1}, h_k, h_{k+1}.

b) Determine $y[n]$ for the input $x[n]$ shown in the figure.

c) Provide magnitude and phase plots of the frequency response $H(e^{j\omega})$.