Space-Efficient Tracking of Persistent Items in a Massive Data Stream

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Abstract

Motivated by scenarios in network anomaly detection, we consider the problem of detecting persistent items in a data stream, which are items that occur "regularly" in the stream. In contrast with heavy-hitters, persistent items do not necessarily contribute significantly to the volume of a stream, and may escape detection by traditional volume-based anomaly detectors.

We first show that any online algorithm that tracks persistent items exactly must necessarily use a large workspace, and is infeasible to run on a traffic monitoring node. In light of this lower bound, we introduce an approximate formulation of the problem and present a small-space algorithm to approximately track persistent items over a large data stream. We experimented with three different datasets to see how the accuracy and memory footprint of the algorithm varies with the skewness of the dataset. Our algorithms performed best for the two datasets out of three which had highest skewness of persistence and lowest mean persistence. To our knowledge, this is the first systematic study of the problem of detecting persistent items in a data stream, and our work can help detect anomalies that are temporal, rather than volume based.

Index Terms

Data streams; persistence; sketches; hash-based filters

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1 Introduction

We consider the problem of tracking *persistent* items in a large data stream. This problem has particular relevance while mining various network streams, such as the traffic at a gateway router, connections to a web service, etc. Informally, a persistent item is one that occurs "regularly" in the stream.

More precisely, suppose that the time at the stream processor is partitioned into non-overlapping intervals called "timeslots". Consider a stream of elements of the form (d, t) where d is an item identifier, and t is a timeslot during which the item arrived. The t values are in an increasing order within the stream. Multiple items can arrive in the same timeslot, and the same item may arrive multiple times within a time slot. Suppose the total number of timeslots in the stream is n. The persistence of an item d is defined to be the number of distinct timeslots in which d was observed. The persistence of any item is an integer between 0 and n (inclusive). An item is said to be α-persistent, for some constant $0 < \alpha \leq 1$, if its persistence is at least αn. Given a user-defined α, the problem is to output the set of α-persistent items in the stream.

Persistent items exhibit a repeated and regular pattern of arrival, and are significant for many applications. Giroire *e*t al. [1] monitored traffic from end-hosts to detect communication across botnet channels. They observed that persistent destinations were likely to belong to one of two classes: (1) either they were malicious hosts associated with a botnet, or (2) they were frequently visited benign hosts. It was also observed that the latter set of hosts could be identified easily and assembled into a "whitelist" of known good destinations. They found that tracking persistent items in the network stream, followed by filtering out items contained in the whitelist, resulted in reliable identification of botnet traffic.

More broadly, persistent items are often associated with specific anomalies in the context of network streams: periodic connections to an online advertisement in a pay-per-click revenue model [2] is an indicator of click fraud [3], repeated (failed) connections observed in the stream is indicative of a failed or unreachable web service [4]; botnets periodically "phone home" to their bot controllers [1]; attackers regularly scan for open ports on which vulnerable applications are usually deployed [5]. While the narrative in this paper draws from applications in the network monitoring space, it appears that the problem of detecting persistent items in a data stream is broadly applicable in other data monitoring applications. For example, persistent use of gathering techniques such as telephone interception or satellite imaging might indicate an "Advanced Persistent Threat" (APT) [6] for a target group, e.g., a government.

The persistent items in a stream could be very different from the frequently occurring items (or "heavy-hitters") in a stream. An item is called a ϕ-heavy hitter if it contributes to at least a ϕ fraction of the entire volume of the stream. There is a large body of literature on heavy-hitter identification (including [7], [8], [9], [10], [11], [12]). A persistent item need not be a heavy hitter. For example, the item may appear only once in each time slot and may not contribute significantly to the stream volume. Such "stealthy" behavior was indeed observed in botnet traffic detection [1]; the highly persistent destinations which were not contained in the whitelist did not contribute in any meaningful way to the traffic volume. In fact, the traffic to these destinations was stealthy and very low volume, perhaps by design to evade detection by traditional volume-based detectors. Conversely, a heavy-hitter need not be a persistent item either – for example, an item may occur a number of times in the stream, but all its occurrences maybe within only a couple of timeslots. Such an item will have a low persistence. Clearly, the set of persistent items in a stream can be very different from the set of heavy-hitters in the stream; their intersection can very well be empty. There seems to be no easy reduction from the problem of tracking persistent items to the problem of tracking heavy-hitters. For example, one could attempt to devise a "filter" that eliminated duplicate occurrences of an item within a time slot, and then apply a traditional heavy-hitter algorithm on the resulting "filtered" stream. But this approach does not work in small space, because such a filter would itself take space proportional to the number of distinct items that appeared within the timeslot, and this number maybe very large, especially for the type of network traffic streams that we are interested in.

A closely related problem is the problem of identifying *heavy distinct hitters (HDHs)* in a data stream (Venkataraman *et al.*[13] and Bandi *et. al.* [14]). In the heavy distinct hitters problem, we are given a stream S' of (x, y) pairs, of length N. For a parameter $\beta, 0 < \beta \leq 1$, the set of β-HDHs in S' is defined as the set of all those values of x that have occurred with more than $N\beta$ distinct values of y. There is a reduction from the problem of tracking persistent items to that of identifying HDHs, as follows. Consider the identification of α-persistent items on a stream S of (d, t) pairs, of length N. Let n denote the total number of timeslots in S. Consider a stream S' of (x, y) pairs where for each element $(d, t) \in S$, there is an element $(x = d, y = t)$ in S'. Then, the $\frac{n\alpha}{N}$-HDHs in S' are the set of α-persistent items in S. There are two significant issues with using such a reduction for solving our problem using an algorithm (such as in [13], [14]) for HDH identification. (1)The first one is that for HDH identification, the threshold $\frac{n\alpha}{N}$ should be known beforehand. Though n, the number of timeslots is usually known before the stream is observed, the number of packets N is not known beforehand, so the prior algorithms for HDHs cannot be directly used. (2)Next, even if we were to modify the algorithms for HDHs to work with an "adaptive threshold", that can change as the number of elements increases (which seems non-trivial), there is special structure in the data in the persistent items identification problem that can be used here. In the heavy distinct hitter problem on a stream of (x, y) values, there is no relative ordering required on the y values, and the same (x, y) tuple can re-occur at arbitrary positions in the stream. But in the persistent items problem on a stream of (d, t) tuples, the t values must be in a non-decreasing order (since they represent the times of observation at the stream processor). An important consequence of this difference is that the algorithms for HDH identification ([13], [14]) need to use "distinct counters" (such as in [15], [16]) to count the number of distinct y values associated with each value of x. Hence, the space complexity of their algorithms is the number of counters maintained multiplied by the space taken by an (approximate) distinct counter. Approximate distinct counting is inherently expensive space-wise, since it has been shown [17] that maintaining distinct counters with a relative error of ϵ requires $\Omega(1/\epsilon^2)$ space. Our algorithm does not need to use approximate distinct counters, making it simpler, more efficient, and easier to implement.

Prior work in Giroire *et al.* [1] used the following method to track persistent items in a stream of network traffic. For each distinct item in the stream, their method maintained (1)The number of timeslots in which the item has appeared in the stream so far, and (2)Whether or not the item has appeared in the current timeslot. This allowed them to exactly compute the number of timeslots that each item has appeared in, and hence exactly track the set of persistent items. However, the space taken by this scheme is proportional to the number of distinct items in the stream. The stream could have a very large number of distinct items (for example, IP sources, or destinations), and the memory overhead may render this infeasible on a typical network monitor or a router. Thus the challenge is to track the persistent items in a stream using a small workspace, and minimal processing per element. Further, all tracking must be done online, and the system does not have the luxury of making multiple passes through the data.

1.1 Contributions

In this work, we present the first small-space approximation algorithm for tracking persistent items in a data stream, and an evaluation of the algorithm. Our contributions are as follows.

Space Lower Bound: We first consider the problem of exactly tracking all α-persistent items in a stream, for some user-defined $\alpha \in (0, 1]$. For this problem, we show that any algorithm that solves it must use $\Omega(m \log n\alpha)$ space, where m is the number of distinct items in the stream, and n is the total number of slots, *even when the number of persistent items is much smaller than* m.

Approximate Tracking of Persistent Items: In light of the above lower bound, we define an approximate version of the problem. We are given two parameters, α - the threshold for persistence, and $\epsilon < \alpha$, an approximation (or "uncertainty") parameter. The task is to report a set of items with the following properties: every item that is α-persistent is reported, and no item with persistence less than $(\alpha - \epsilon)$ is reported. We also formulate this problem for a "sliding window" of the most recently observed items of the stream.

Small Space Algorithm: For the above problem of approximate tracking of persistent items, we present a randomized algorithm that can approximately track the α-persistent items using space that is typically much smaller than the number of distinct items in the stream. The expected space complexity of the algorithm is $O\left(\frac{P}{\epsilon n}\right)$, where P is the sum of the persistence values of all items in the stream, and n is the total number of timeslots. The algorithm has a small probability of a false negative (i.e. an α-persistent item is missed). This probability can be made arbitrarily small, at the cost of additional space. Note that any algorithm will need space that is at least as large as the size of the output, i.e., the number of α-persistent items in the stream. The worst case scenario is when every item is α-persistent, forcing the algorithm to use space proportional to the number of distinct items! Fortunately, this situation does not seem to occur in practice and only a fraction of items are very persistent, and this helps our algorithm considerably. We also prove that if persistence of different items in a stream follow a power law distribution, then the space taken by our algorithm is $O\left(\frac{1}{\epsilon}\right)$.

Sliding Windows: In most network monitoring applications, the data set of interest is not the entire traffic stream, but only a window of the recent past. Formally, we define a window as follows:

Definition 1.1: A window S_ℓ^r consists of all stream elements (d_i, t_i) whose timeslots are in the range $[\ell, r]$, i.e. $S_\ell^r = \{(d_i, t_i) \in S | \ell \leq t_i \leq r\}$.
The size of window S_ℓ^r is defined as $(r - \ell + 1)$, i.e. the number of timeslots it encompasses. For instance, Giroire *et al* [1] used this sliding window model in their work on botnet traffic detection. Though the size of the data set has decreased when compared with the fixed window case, maintaining statistics over a sliding window is still a hard problem, since the data contained within a sliding window is often too large to be stored completely within the memory of the stream processor. This is a harder problem than the fixed window, since it has to deal with (old) elements falling off the window. We present an extension to our fixed window algorithm to handle the sliding window model. Interestingly, the expected space cost of our sliding window algorithm is within a factor of two of the space cost of the fixed window algorithm.

Experimental Evaluation: We evaluate our algorithm against three datasets: a large, real-world network traffic trace (which we call **HeaderTrace**) collected from an Internet backbone link, as well as two artifically created datasets, which we call **Synthetic1** and **Synthetic2** respectively, the latter having a skewness of persistence (17.17) which is three times that of the former (5.67). In other words, **Synthetic1** had a more *uniform* distribution of skewness than **Synthetic2** or **HeaderTrace**.

Our algorithm performed best on **HeaderTrace** and **Synthetic2**, and a little worse on **Synthetic1**. On **HeaderTrace**, our small-space algorithm uses up to 85% less space than the naive algorithm and typically incurs a false positive rate of less than 1% and a false negative rate of less than 4%. We also see that false positive rate never exceeds 3% for any parameter setting, while the false negative rate stays below 5% for all but the most aggressive thresholds for persistence. For **Synthetic2**, which had a skewness about twice that of **HeaderTrace**, the maximum False Positive Rate (FPR) was 2.2%, the typical False Negative Rate (FNR) being about 6%. The skewness of persistence for the **Synthetic1** dataset was about 60% of that of **HeaderTrace** (9.28). Although the maximum FNR for **Synthetic1** is 11.5% (the theoretical maximum FNR is 13%) and the maximum FPR is 15.6%, the typical FNR and FPR are both within 4%. The comparative performance on the three datasets shows that our algorithm in fact works better for datasets with high skewness of peristence and low mean persistence, which is very typical of real-life network traffic.

1.2 Roadmap

The rest of this paper is organized as follows. A precise statement of the problem is presented in Section 2, followed by a lower bound on the space cost of exactly tracking the persistent items in a stream. Our algorithms for the fixed and sliding windows models are presented in Section 3, followed by their analysis and correctness. Experimental results are described in Section 4. A detailed discussion of related work is presented in Section 5.

2 Problem Definition

Consider a world where time is divided into timeslots (or slots) that are numbered $1, 2, \ldots$. Let S be a stream of elements of the form $S = \langle (d_1, t_1), (d_2, t_2), \ldots \rangle$. Each element is a tuple (d_i, t_i), where d_i is an item identifier (IP address, hostname, etc), and t_i is the time slot during which the element arrived. It is assumed that the t_is are in non-decreasing order. All elements that have the same values of t_i are said to be in the same timeslot. Clearly, a timeslot consists of elements that form a contiguous subsequence of the observed stream.

The duration of a timeslot depends on the application on hand. In the botnet detection application [1], the duration of a timeslot was chosen to be between 1 hour and 24 hours, primarily because these were suspected to be the possible lengths of time between successive connections from the (infected) client to malicious destinations, for the botnets that they considered. Since then, there have been other botnet attacks that work on a much smaller timescale (see Section 4 for a discussion). In an eventual solution to botnet attack detection, we may need to consider running the algorithm simultaneously with different timeslot durations, to monitor multiple types of attacks.

For a given window S_ℓ^r, as defined in Definition 1.1, we define the persistence of an item in that window as follows:

Definition 2.1: The persistence of an item d over a window S_ℓ^r, denoted $p_d(\ell, r)$, is defined as the number of distinct slots in $\{\ell, \ell+1, \ldots, r\}$ that d appeared in.

$$p_d(\ell, r) = |\{t | ((d, t) \in S) \wedge (\ell \leq t \leq r)\}|$$

Definition 2.2: An item d is said to be α-persistent in window S_ℓ^r if $p_d(\ell, r) \geq \alpha(r - \ell + 1)$. In other words, d must have occurred in at least an α fraction of all slots within the window.

We state two versions of the problem, the first version for a fixed window, and the second version for a sliding window. In practice, the sliding window version is more useful.

2.1 Exact Tracking of Persistent Items

Problem 1: **Identifying Persistent Items Over a Fixed Window:** Devise a space-efficient algorithm that takes as input a prespecified window $W = S_1^n$ and a persistence threshold α, and at the end of observing the stream, returns the set of all items that are α-persistent. In other words, the algorithm will report every item that is α-persistent in W and will not report any item that is not α-persistent.

A straightforward algorithm for this problem would track every distinct item in the stream, and for each distinct item, count the number of slots (from 0 to $n-1$) during which the item appeared. For a single item, its persistence can be tracked in a constant number of bytes (assuming that the item identifier and slot number can be stored in constant space), by maintaining a counter for the number of timeslots the item has appeared in so far, in addition to one bit of state for whether or not the item has appeared in the current timeslot. The total space consumed by the naive algorithm is of the order of the number of distinct items in the stream. In general, this would be a large number and the space overhead may make it infeasible for this algorithm to be deployed within a network router.

Space Lower Bound for Exact Tracking: We now show that any algorithm that solves Problem 1 exactly must require $\Omega(m)$ space in the worst case, where m is the number of distinct items in the input. Importantly, $\Omega(m)$ space is needed even if the number of persistent items is much smaller than m.

Lemma 2.1: Any algorithm that can exactly solve Problem 1 must use $\Omega(m \log(n\alpha + 1))$ bits of space in the worst case, where m is the number of distinct items in the input.

Proof: Without loss of generality, suppose that the m distinct items that appear in the stream are labeled $1, 2, 3, \ldots, m$. Consider the state of the stream after observing $k = (n - \alpha n)$ timeslots. For i from 1 to m, let n_i denote the number of timeslots among $1, 2, \ldots, k$ during which item i has appeared. Consider the vector $u = \langle n_1, n_2, \ldots, n_m \rangle$. Consider the following set V of possible assignments to u, where each component in u is chosen from the range $0, 1, 2, \ldots, \alpha n$. The size of V is $(1 + n\alpha)^m$. We show that any algorithm that solves Problem 1 must be able to distinguish between two distinct vectors in V, and hence must have a different state of its memory for two input streams that result in different assignments to u.

We use proof by contradiction. Suppose the above was not true, and there were two input streams A and B which, at the end of k slots, resulted in vectors $v_A, v_B \in V$ respectively. Suppose $v_A \neq v_B$ but the states of the algorithm's memory were the same after observing the two inputs. Now, v_A and v_B must differ in at least one coordinate. Without loss of generality, suppose they differed in coordinate 1, so $n_1(A) \neq n_1(B)$, and without loss of generality suppose $n_1(A) < n_1(B)$. Consider the rest of the stream, from slot $n - n\alpha$ onwards. Suppose these slots had $n\alpha - n_1(B)$ slots in which item 1 occurred. Clearly, appending this stream to stream A results in a stream with n slots where the persistence of item 1 is $n_1(A) + (n\alpha - n_1(B)) = n\alpha - (n_1(B) - n_1(A)) < n\alpha$, and appending this same stream to stream B results in a stream with n slots where the persistence of item 1 is $n\alpha$. Thus, item 1 must be reported as α-persistent in the latter case, and not in the former case. But this is not possible, since the algorithm has the same memory state for both A and B, and sees the same substream henceforth, leading to a contradiction.

To distinguish between any two vectors in V, the algorithm needs at least $\log |V|$ bits of memory. Since the size of V is $(n\alpha + 1)^m$, the lower bound is $\Omega(m \log(n\alpha + 1))$ bits. $\square$

2.2 Approximate Tracking of Persistent Items

In light of the above lower bound on the space cost of exact tracking of persistent items, we define a relaxed version of the problem. Here, in addition to the persistence threshold α, the user provides two additional parameters, $\epsilon \in [0, 1]$, an "uncertainty parameter", and $\delta \in [0, 1]$, an error probability.

Problem 2: **Approximate Tracking of Persistent Items over a Fixed Window:** Given a fixed window $W = S_1^n$, persistence threshold α, approximation parameter ϵ, and error probability δ, devise a small space algorithm that returns a set of items with the following properties.

A. *If an item is persistent, it is reported with high probability.* For any item d, if $p_d(1, n) \geq \alpha n$, then d is reported as persistent with a probability at least $(1 - \delta)$.

B. *Items that are far from persistent are not reported.* If $p_d(1, n) < (\alpha - \epsilon) \cdot n$, then d is not reported.

Sliding Windows. The sliding window version of the problem requires that we continuously monitor the window of the n most recent timeslots in the stream.

Problem 3: **Approximately identifying Persistent Items over a Sliding Window:** The problem of approximately tracking persistent items over a sliding window is the same as the above Problem 2, except that the window of interest, W, is the set of the n most recent timeslots in the stream, and changes continuously with time.

The fixed window version is a special case of the sliding window, where the window is equal to the entire stream. The space lower bound for fixed window obviously applies to the sliding window version, hence it is also necessary to consider an approximate version of the problem for sliding windows, if we are to achieve a small space solution.

3 An Algorithm for Approximate Tracking of Persistent Items

We present algorithms for approximate tracking of persistent items in a stream. We first present the algorithm for tracking persistent items over a fixed window, followed by a proof of correctness and analysis of complexity. We then present the algorithm for sliding window.

3.1 Fixed Window

Intuition. The goal is to track the persistence of as few items in the stream as possible, and hence minimize the workspace used by the algorithm. Ideally, we track (and hence, use space for) only the α-persistent items in the stream, and not the rest. But this is impossible, since we do not know in advance which items are α-persistent.

The strategy is to set up a hash-based "filter". Each stream element is sent through this filter, and if it is selected by the filter, then the persistence of the corresponding item is tracked in future timeslots. The filter behaves in such a way that if the same item reappears in the same timeslot, then its chances of being selected by the filter are not enhanced, but if the same item reappears in different timeslots, then its chances of passing the filter get progressively better. For achieving the above, the filter for an item is selected to be dependent on the output of a hash function whose inputs are both the item identifier as well as the timeslot within which it appeared.

Let h denote a hash function that takes two inputs, and whose output is a random real number in the range $[0, 1]$. For item d arriving in slot t, the item passes through the filter if $h(d, t) < \tau$, for some pre-selected threshold τ. The value of τ is chosen to be small enough that an item with a small value of persistence is not likely to cross this filter; in particular, transient items which only occur in a constant number of timeslots will almost certainly not make it. Note that if the same item d reappears in the same timeslot t, then the hash output $h(d, t)$ is the same as before, hence the probability of the item passing the filter does not increase.

After an item has passed the filter, the persistence of this item in the remaining timeslots is tracked exactly, since this requires only a constant amount of additional space (per item). Finally, the persistence of an item is estimated as the number of slots that it has appeared in since it started being tracked (this is known exactly), plus an estimate of the number of slots it had to appear in before we started tracking it. An item is returned as α-persistent if its estimated persistence is greater than a threshold T (decided by the analysis). Note that there may be items which are being tracked because they passed the filter, but are not returned as α-persistent, since the estimate of their persistence did not exceed T.

The higher the threshold τ, the greater is the accuracy in our estimate of the persistence, but this comes at the cost of higher memory consumption since more items will now pass the filter. Setting the value of τ gives us a way to tradeoff accuracy versus space.

Formal Description. Let $D(S)$ denote the set of distinct items in the stream S, and suppose that the timeslots of interest are $1, 2, \ldots, n$. The stream processor tracks only a subset of $D(S)$, and maintains a data structure that we call a "sketch", which summarizes the stream elements seen so far. Let $\mathcal{S}$ denote the sketch data structure maintained by the algorithm.

$\mathcal{S}$ is a set of tuples of the form (d, n_d, t_d), where d is an item that has appeared in the stream, n_d is the number of slots in which d has appeared, since we started tracking it, and t_d is the most recent timeslot during which d has appeared. For each item d, if d is being tracked, then there is a tuple of the form $(d, \cdot, \cdot)$ belonging in $\mathcal{S}$; if d is not being tracked, then there is no such tuple in $\mathcal{S}$. For each item d, there can never be more than one tuple of the form $(d, \cdot, \cdot)$ in $\mathcal{S}$ at a time. We say $d \in \mathcal{S}$ to mean "there is a tuple $(d, \cdot, \cdot)$ belonging to $\mathcal{S}$". Similarly, we say $d \notin \mathcal{S}$.

The inputs to the algorithm are the persistence threshold α, the total number of slots n, approximation parameter ϵ, and error probability δ. The algorithm selects a hash function $h(d, t)$ where d is an item, and t is the timeslot number. It is assumed that $h(d, t)$ is a uniform random real number in $(0, 1)$, and that the outputs of h on different inputs are mutually independent; when presented with the same input (d, t), the hash function returns the same output. We note that it is possible to work with weaker assumptions of hash functions whose range is a finite set of integers, but we assume the current model for simplicity and ease of exposition.

Before any element arrives, Algorithm 1 Sketch-Initialize is invoked to initialize the data structures. When an element (d, t) arrives, Algorithm 2 is invoked to update the $\mathcal{S}$ data structure. When there is a query for persistent items in the stream, Algorithm 3 Detect-Persistent-Items is called to process the query and will return a list of all items deemed persistent.

```
Algorithm 1: Sketch-Initialize($m, n, \alpha, \epsilon, \delta$)
  Input: Size of domain $m$; Total number of slots $n$; persistence threshold $\alpha$; parameter
         $\epsilon$; error probability $\delta$
1 Initialize the hash function $h : ([1, m] \times [1, n]) \to (0, 1)$;
2 $\mathcal{S} \leftarrow \phi$; $\tau \leftarrow \frac{2}{\epsilon n}$; $T \leftarrow \alpha n - \frac{\epsilon n}{2}$
```

Algorithm 2: Sketch-Update(d, t)

```
Input: $d$ is an item; $t$ is the timeslot of arrival
if $d \in \mathcal{S}$ then
    if $t_d < t$ then
        /* $d$ appeared in a new slot */
        $n_d \leftarrow n_d + 1$; $t_d \leftarrow t$;
    end
else
    if $h(d, t) < \tau$ then
        /* Start tracking item $d$ from now onwards */
        $\mathcal{S} \leftarrow \mathcal{S} \cup (d, 1, t)$;
    end
end
```

Algorithm 3: Detect-Persistent-Items

```
foreach tuple $(d, n_d, t_d) \in \mathcal{S}$ do
    $\hat{p_d} \leftarrow n_d + \frac{1}{\tau}$
    if $\hat{p_d} \geq T$ then
        Report $d$ as a persistent item
    end
end
```

3.1.1 Analysis of the Fixed Window Algorithm

We present the proof of correctness and analysis of space complexity. Consider an item d, with persistence $p_d = p_d(1, n)$. For parameter q, $0 < q \leq 1$, let $G(q)$ denote the geometric random variable with parameter q, i.e., the number of Bernoulli trials until a success (including the trial when the success occurred), where the different trials are all independent, and the success probability is q in each trial.

For each item d that appeared in the stream, there are two possibilities: (1) either d is tracked by the algorithm from some timeslot t onwards, or (2) d is not tracked by the algorithm, because none of the tuples (d, t) were selected by the filter.

In each distinct slot where d appears, the probability of d being sampled into the sketch is τ. If $G(\tau) > p_d$, then this will lead to case (2) above, and d will fail to make it into the sketch $\mathcal{S}$. On the other hand, if $G(\tau) \leq p_d$, this will lead to case (1), and d will be inserted into the sketch at some timeslot in Algorithm 2, and the counter $n_d = p_d - G(\tau) + 1$.

Lemma 3.1: **False Negative:** If an item d has $p_d \geq \alpha n$, then the probability that this item will not be reported as α-persistent by Algorithm 3 is no more than e^{-2}.

Proof: From Algorithm 3, the item will not be reported if $\hat{p_d} < T$, i.e., $n_d + \frac{1}{\tau} < T$. Using $\tau = \frac{2}{\epsilon n}$ and $T = \alpha n - \frac{\epsilon n}{2}$, we get:

$$\begin{aligned}
\Pr[\text{False Negative}] &= \Pr[p_d - G(\tau) + 1 + \frac{1}{\tau} < T] \\
&= \Pr[G(\tau) > 1 + \frac{1}{\tau} + p_d - T] \\
&= \Pr[G(\tau) > 1 + \frac{1}{\tau} + \frac{\epsilon n}{2} + (p_d - \alpha n)] \\
&\leq \Pr[G(\tau) > \frac{2}{\tau}]
\end{aligned}$$

In the last step, we have used the fact $p_d \geq \alpha n$, and $\frac{1}{\tau} = \frac{\epsilon n}{2}$. Using the fact $\Pr[G(p) > t] = (1-p)^t$, we get

$$\Pr[\text{False Negative}] \leq (1-\tau)^{\frac{2}{\tau}} \leq e^{-2}$$

In the last step, we have used the inequality $1 - x \leq e^{-x}$. □

Lemma 3.2: **Items that are far from persistent are not reported:** If an item d has $p_d < (\alpha - \epsilon)n$, then d will not be reported by Algorithm 3 as an α-persistent item.

Proof: We prove this by contradiction. For such an item, the value of n_d at the end of observation is $n_d = p_d - G(\tau) + 1$. If that item is reported as α-persistent, then by Algorithm 3, we must have:

$$\begin{aligned}
n_d + \frac{1}{\tau} \geq T &\Rightarrow p_d - G(\tau) + 1 + \frac{1}{\tau} \geq \alpha n - \frac{1}{\tau} \\
&\Rightarrow G(\tau) \leq (p_d - \alpha n) + 1 + \frac{2}{\tau} \\
&\Rightarrow G(\tau) \leq p_d - (\alpha - \epsilon)n + 1 \\
&\Rightarrow G(\tau) \leq 0
\end{aligned}$$

But since $G(\tau)$ is the number of slots the item has to appear in until it gets into the sketch, it must be positive; so we reach a contradiction. □

Lemma 3.3: The expected space taken by the $\mathcal{S}$ is $O\left(\sum_{d \in D(S)} \min(1, 2\tau p_d)\right)$, where $D(S)$ is the set of all distinct items in stream S. We assume that storing a tuple (d, n_d, t_d)

takes a constant amount of space, provided $\tau \leq 1/2$.

Proof: The space taken by $\mathcal{S}$ is a random variable, since the decision of whether or not to allocate space to an item is a randomized decision. For item d, let random variable Z_d be defined as follows. $Z_d = 1$ if the algorithm tracks d, i.e $d \in \mathcal{S}$, and $Z_d = 0$ otherwise.

Let $Z = \sum_{d \in D(S)} Z_d$. If we assume that the space required for storing a single tuple $(d, \cdot, \cdot)$ in $\mathcal{S}$ is a constant number of bytes, say c, then the space used by $\mathcal{S}$ is cZ bytes. Now, for the random variable Z, by linearity of expectation, we get:

$$E[Z] = E\left[\sum_{d \in D(S)} Z_d\right] = \sum_{d \in D(S)} E[Z_d] = \sum_{d \in D(S)} \Pr[Z_d = 1] \tag{1}$$

$$\Pr[Z_d = 0] = (1 - \tau)^{p_d} \tag{2}$$

Using Taylor's expansion,

$$\begin{aligned} e^{-2\tau} &\leq 1 - 2\tau + 4\tau^2/2 \\ &\leq 1 - 2\tau + \tau = 1 - \tau \quad \text{(assuming } \tau \leq 1/2\text{)} \end{aligned}$$

Using the above in Equation 2, we get:

$$\Pr[Z_d = 0] \geq (e^{-2\tau})^{p_d} = e^{-2\tau p_d}$$

Thus,

$$\begin{aligned} \Pr[Z_d = 1] &= 1 - \Pr[Z_d = 0] \leq (1 - e^{-2\tau p_d}) \\ &\leq (1 - (1 - 2\tau p_d)) \\ &\quad \text{(using } e^{-x} > 1 - x\text{)} \\ &= 2\tau p_d \end{aligned}$$

Using the above in Equation 1, we get:

$$E[Z] \leq \sum_{d \in D(S)} 2\tau p_d \tag{3}$$

We can actually get a **tighter bound for the space**, as follows. For a highly persistent item item d with $p_d = n$, $2\tau p_d = \frac{4}{\epsilon}$, which is greater than 1. On the other hand, for a transient item with $p_d < \frac{\epsilon n}{4}$, $2\tau p_d < 1$. The way the sketch is designed for the fixed window case, an item can have no more than one tuple in the sketch. So, we can write Equation 3

$$E[Z] \leq \sum_{d \in D(S)} \min(1, 2\tau p_d) \tag{4}$$

□

Discussion: The expression for the space complexity shows that the expected space required for an item d is proportional to p_d/n. Note that p_d can range from 1 till n, but in a typical stream, the persistence of most items can be expected to be small, with only a few items having a large persistence. Thus, in the typical case, for example, with a Zipfian distribution of packet frequencies and persistence, the space taken by the sketch will be much smaller than the number of distinct items in the input.

Space Complexity for Specific Distributions. Let $P = \sum_{d \in D(S)} p_d$ denote the sum of the persistence values of all items in the stream. We now consider the case when the persistence values followed a Zipfian distribution. The persistence of the k^{th} most persistent item is $\rho_k = \frac{c}{k^\beta}$, for $k \in 1, 2, ..., |D(S)|$, for a normalization constant $c > 0$ and $\beta = 1 + \theta$ where $\theta > 0$ is a fixed constant. In such a case, we prove that $P = O(n)$, leading to a constant space complexity, independent of the number of distinct items in the input.

Lemma 3.4: If the persistence of items in $D(S)$ followed a Zipfian distribution with a parameter $\beta > 1$ that is a fixed constant, then the space complexity of the sketch is $O(\frac{1}{\epsilon})$.

Proof: Since the persistence of an item is bounded by n, $\rho_1 = c \leq n$. Let $\zeta(\cdot)$ be the Reimann Zeta function.

$$\sum_{d \in D(S)} p_d = \sum_{k \leq |D(S)|} \frac{c}{k^\beta} \leq c \sum_{k=1}^{\infty} \frac{1}{k^\beta} = c\zeta(\beta) \leq n\zeta(\beta)$$

Thus, from Lemma 3.3, we have

$$\begin{aligned} E[Z] &\leq \sum_{d \in D(S)} \min(1, 2\tau p_d) \\ &\leq \sum_{d \in D(S)} 2\tau p_d \\ &= \frac{4}{\epsilon n} \sum_{d \in D(S)} p_d \\ &\leq \frac{4}{\epsilon n} n\zeta(\beta) = \frac{4\zeta(\beta)}{\epsilon} \end{aligned}$$

By the Maclaurin-Cauchy test, we know for any fixed constant $\beta > 1$, the series represented by $\zeta(\beta)$ converges, which proves the lemma. □

For example, if $\beta = 1.5$, then $\zeta(1.5) = 2.6$. In this case, we get: $\sum_{d \in D(S)} p_d \leq 2.6n$, and thus, from Lemma 3.3, $E[Z] < \frac{(4)\cdot(2.6)}{\epsilon} < \frac{11}{\epsilon}$.

Theorem 3.1: The above algorithms 2 and 3 can be used in an algorithm for tracking persistent items in a fixed window with the following properties:

A. Each α-persistent item is reported with probability at least $1 - \delta$.

B. No item d such that $p_d < (\alpha - \epsilon)n$ is reported.

C. The space complexity of the algorithm is $O\left(\frac{P \log(1/\delta)}{\epsilon n}\right)$, where $P = \sum_{d \in D(S)} p_d$.

D. The expected processing time per stream element is $O(\log \frac{1}{\delta})$.

Proof: Algorithms 2 and 3 achieve most of the above properties. From Lemma 3.1, we get that the probability of a persistent item not being reported is no more than e^{-2}. The only task now is to bring down the probability of a false negative to δ.

To achieve this, we run $(1/2) \ln \frac{1}{\delta}$ instances of Algorithm 2 in parallel, and return the union of the items reported by all the instances. For an item that is persistent, it is not reported only if it is missed by every instance. The probability that this happens is no more than $\left(e^{-2}\right)^{(1/2) \ln \frac{1}{\delta}}$, which is δ. For an item d whose persistence is less than $(\alpha - \epsilon)n$, from Lemma 3.2, we see that the item is not returned by any instance, and hence will not be present in aggregated result, proving property B.

Property C follows from Lemma 3.3, adding a multiplying factor of $O(\log \frac{1}{\delta})$. For the time complexity (property D), we note that Algorithm 2 can be made to run in constant expected time if the sketch $\mathcal{S}$ is organized as a hash table with the item identifier as the key. $\square$

3.2 Sliding Windows

In this setting, we are interested only in the substream of elements that belong to the n most recent timeslots. If c is the current timeslot, then the window of interest is S_{c-n+1}^{c}. Note that n here does not represent the number of timeslots in the stream, but the number of timeslots in the window. We now present an algorithm solving Problem 3. The intuition for the sliding window algorithm is as follows.

Suppose we started a new fixed window data structure for each new timeslot. This would suffice, since any sliding window query in the future will be covered by one of these fixed window data structures. For now, suppose that $\mathcal{S}_t$ was the fixed window data structure that we start from time t onwards (this will serve the window S_t^{t+n-1}). At first glance, it seems like this would be too much space, since the cost would be n times the space for a single fixed window data structure.

The space can be reduced through the following observations: (1) when we start a fixed window data structure at a particular timeslot t, say, only a few of the items (approximately a τ fraction of the items) that arrive in timeslot t will be selected into this data structure; (2) for those items d that were not selected into $\mathcal{S}_t$ in timeslot t, the tuple for d in $\mathcal{S}_t$ can be shared with the tuple for d in $\mathcal{S}_{t+1}$; (3) further, when the current timeslot is t, we can afford to discard $\mathcal{S}_r$ for $r \le (t-n)$, since these data structures will never be used in a future query.

Thus, the sketch used by our algorithm at time c is effectively $\cup_{i=c-n+1}^{c} \mathcal{S}_i$, where $\mathcal{S}_i$ is the fixed window sketch starting at timeslot i. Through observation (2), we reduce the space by having a single tuple for d in $\mathcal{S}_i, \mathcal{S}_{i+1}, \ldots, \mathcal{S}_j$ such that j is the first timeslot in $i, i+1, \ldots, j$ where d was selected into the sketch.

The formal description of the algorithm for the sliding window model is presented in Algorithms 4, 5, 6 and 7. The sketch $\mathcal{S}$ is a set of tuples of the form $(d, t, n_{d,t}, t_{d,t})$, where d is an item identifier, t is the timeslot when this tuple was created, $n_{d,t}$ is the number of timeslots since t when d has reappeared, and $t_{d,t}$ is some state that we maintain to eliminate counting reoccurrences of d within the same timeslot. In the following discussion, we say "(d,t) belongs in the sketch", or "$(d,t) \in \mathcal{S}$", if there is a 4-tuple of the form $(d,t,\cdot,\cdot)$ in the sketch. In our sketch, for any item d and timeslot t, there can be at most one tuple of the form $(d,t,\cdot,\cdot)$.

Algorithm 4: Sliding-Window-Sketch-Initialize $(m, n, N, \alpha, \epsilon, \delta)$

```
Input: Size of domain $m$; window size $n$; maximum number of slots $N$; persistence
       threshold $\alpha$; parameter $\epsilon$; error probability $\delta$
1 Initialize the hash function $h : ([1,m] \times [1,N]) \to (0,1)$;
2 $\mathcal{S} \leftarrow \phi; \tau \leftarrow \frac{2}{\epsilon n}; T \leftarrow (\alpha - \frac{\epsilon}{2})n$
```

Algorithm 5: Sliding-Window-Sketch-Update(d, t)

```
   Input: $d$ is an item; $t$ is the timeslot of arrival
1  if $(d,t) \in \mathcal{S}$ then
2      return
3  end
   // Consider starting a new tuple, tracking $d$ from slot $t$
      onwards.
4  if $h(d,t) < \tau$ then
5      $\mathcal{S} \leftarrow \mathcal{S} \cup (d,t,1,t)$
6  end
7  foreach $t'$ such that $(d,t') \in \mathcal{S}$ do
8      Let $(d, t', n_{d,t'}, t_{d,t'})$ be the tuple corresponding to $(d,t')$
       // Incorporate $(d,t)$ into this tuple if not been done yet
9      if $t_{d,t'} < t$ then
           // $d$ has not been seen in slot $t$ by this tuple
10         $n_{d,t'} \leftarrow n_{d,t'} + 1; t_{d,t'} \leftarrow t$
11     end
12 end
```

During the initialization phase of the algorithm, $\mathcal{S}$ is initialized to empty, τ to $\frac{2}{\epsilon n}$, and T to $\alpha n - \frac{\epsilon n}{2}$. When we want to add an element (d,t) to the sketch, there are two possible cases. First, if there is an entry in the sketch of the form $(d,t,\cdot,\cdot)$, then this element can be safely ignored, since the same combination of item and timeslot has been observed earlier. Otherwise, if (d,t) hashes to an appropriately small value (less than τ), then a new entry is created for tracking d, starting from time t onwards, that will serve to answer queries on certain windows that include t within them. Simultaneously, (d,t) is used to update each of the tuples in $\mathcal{S}$ that track d. Whenever time advances, and the window slides forward from t to $t+1$, all entries $(d,t',\cdot,\cdot)$ in $\mathcal{S}$ such that $t' \leq (t-n)$ are discarded, because stream windows of current and future interest will not be served by this entry. Let $p_d^t = p_d(t-n+1,t)$ denote the persistence of d over the window $[t-n+1,t]$.

Algorithm 6: Actions taken when timeslot changes from $c-1$ to c

```
  // Discard old items
1 Discard items $(d,t,\cdot,\cdot) \in \mathcal{S}$ where $t \leq (c-n)$
```

Algorithm 7: Sliding-Window-Detect-Persistent-Items(c)

```
  Input: $c$ is the current timeslot. The window of interest is $[c-n+1,c]$.
1 Let $\mathcal{S}_{cur}$ be all tuples $(d,t',n_{d,t'},t_{d,t'})$ in $\mathcal{S}$ such that both the following conditions
  are true: (A) $t' \geq (c-n+1)$ and (B) There is no $t''$ such that $(d,t'') \in \mathcal{S}$ and
  $(c-n+1) \leq t'' < t'$.
2 foreach tuple $(d,\cdot,n_d,t_d) \in \mathcal{S}_{cur}$ do
3     $\hat{p_d^c} \leftarrow n_d + \frac{1}{\tau}$
4     if $\hat{p_d^c} \geq T$ then
5         Report $d$ as a persistent item in the window
6     end
7 end
```

3.2.1 Correctness and Complexity

For a pair (d,t) where d is an item identifier and t is a time slot, (d,t) is said to be stored in $\mathcal{S}$ at time c if there exists a tuple $(d,t,\cdot,\cdot)$ in $\mathcal{S}$ at time c.

Lemma 3.5: **Items that are far from persistent in the window are not reported:** At time c, if an item d has $p_d^c < (\alpha - \epsilon n)$, then d will not be reported as persistent in the window in Algorithm 7.

Proof: Consider such an item d, where $p_d^c < (\alpha - \epsilon n)$. We analyze the instances when d was processed by Algorithm 5. If d was never stored in the sketch from time $c-n+1$ onwards, then there will not exist a tuple $(d,t',\cdot,\cdot)$ in $\mathcal{S}$ at time c, and d will not be reported by Algorithm 7.

Suppose at time c, there existed a tuple (d, t', n_d, t_d) in $\mathcal{S}$, such that $t' \geq (c-n+1)$. This tuple was inserted into the sketch at time t'. From Algorithm 5, it can be seen that n_d is equal to the number of occurrences of d in timeslots $t', t'+1, t'+2, \ldots, c$. This number cannot be more than p_d^c, and hence $n_d \leq p_d^c < (\alpha - \epsilon)n$.

In Algorithm 7, for item d, it must be true that:

$$\hat{p_d^c} = n_d + \frac{1}{\tau} < (\alpha - \epsilon)n + \frac{\epsilon n}{2} = \alpha n - \frac{\epsilon n}{2} = T$$

Since $\hat{p_d^c} < T$, d will not be reported as persistent. □

Lemma 3.6: **Sliding Window False Negative:** At time c, if an item d has $p_d^c \geq \alpha n$, then the probability that this item will not be reported as α-persistent in the current window by Algorithm 7 is no more than e^{-2}.

Proof: Suppose that d was sampled into the sketch later than time $(c-n)$, i.e., there exists a tuple $(d, t, n_d, \cdot)$ such that $t > (c-n)$. In such a case, Algorithm 7 selects the tuple (d, t', n_d, t_d) such that (A) $t' > (c-n)$ and (B) there is no tuple $(d, t'', \cdot, \cdot)$ in $\mathcal{S}$ such that $t'' < t'$. In other words, t' is the earliest timeslot in $[c-n+1, c]$ when a sketch for d was initialized. Thus, it follows that from time $c-n+1$ onwards (inclusive), d was not selected into the sketch till time t'. The number of times that d needs to occur in slots $c-n+1, c-n+2, \ldots$ till it is sampled into $\mathcal{S}$ is $G(\tau)$ (the geometric random variable with parameter τ). The counter n_d keeps track of the number of times d occurred in different timeslots starting from slot t' (inclusive). Since d occurred in the window in a total of p_d^c distinct slots, $n_d = p_d^c - G(\tau) + 1$.

$$\Pr[\text{False Negative}] = \Pr[p_d^c - G(\tau) + 1 + \frac{1}{\tau} < T]$$

In the proof of Lemma 3.1, it is shown that the above probability is no more than e^{-2} if $p_d^c \geq \alpha n$, and the lemma follows. □

3.3 Space Complexity

The following result is useful for the space complexity, and follows directly from the definitions in Algorithms 5 and 6.

Fact 3.1: A tuple (d,t) is stored in $\mathcal{S}$ at time c if and only if both the following conditions are true:

A. $t > (c-n)$

B. $h(d,t) < \tau$

Lemma 3.7: **Space Complexity:** Let Z_c denote the number of tuples in $\mathcal{S}$ at time c, and D denote the set of all distinct items that appeared during timeslots $c-n+1$ till c. Then,

$$E[Z_c] = \frac{2}{\epsilon n}\sum_{d \in D} p_d^c$$

Proof: First, it can be verified that in Algorithm 5, if the same tuple (d,t) occurs multiple times, then the effect on the sketch is the same as if (d,t) occurred only once in the stream. Thus we can ignore repeated arrivals of the same tuple (d,t).

For each tuple (d,t) that arrived, let random variable $Z_{d,t}^c$ be defined as follows. $Z_{d,t}^c$ is 1 if tuple (d,t) is stored in $\mathcal{S}$ at time c. Let $D(S)$ denote the set of all distinct tuples (d,t) in the stream so far.

We have

$$Z_c = \sum_{(d,t)\in D(S)} Z_{d,t}^c$$

From Fact 3.1, we have that $Z_{d,t}^c = 0$ if $t \le (c-n)$. Thus, we can rewrite the above as:

$$Z_c = \sum_{\{(d,t)|t>(c-n)\}} Z_{d,t}^c \tag{5}$$

To compute the expectation of Z_c, we use linearity of expectation:

$$E[Z_c] = E\left[\sum_{\{(d,t)|t>(c-n)\}} Z_{d,t}^c\right] = \sum_{\{(d,t)|t>(c-n)\}} E\left[Z_{d,t}^c\right]$$

For a tuple (d,t) such that $t > (c-n)$, $Z_{d,t}^c$ is equal to 1 if it was sampled into the sketch at time t i.e., if $h(d,t) < \tau$. The probability of this event is $\tau = \frac{2}{\epsilon n}$. Let D denote the set of all distinct items that appeared in the stream during a timeslot i such that $(c-n) < i \leq c$.

$$E[Z_c] = \sum_{\{(d,t)|t>(c-n)\}} \tau = \sum_{d \in D} (p_d^c \cdot \tau) = \frac{2}{\epsilon n} \sum_{d \in D} p_d^c$$

$\square$

4 Evaluation

We evaluated our small space algorithm and contrasted its performance with that of a naive (exact) algorithm, by running the two on the following three (one real, two synthetic) datasets described below. The goal of our experiments is to show how the performance of our algorithm varies with the skewness of persistence of the items appearing in a stream.

Dataset design:

- **HeaderTrace:** This is a real-world traffic trace dataset. The trace used had 885 million packets collected during a 3-hour period from a large Internet backbone link (source: CAIDA [18]). The data consists of timestamped packet headers, with the source and destination addresses, in addition to other attributes. From this packet header trace, we extracted a sequence of (destination IP address, timestamp) pairs which forms the input data stream. We divided the entire trace into slots of 30 seconds (to obtain a trace of 360 slots). The sliding window length was set to 100 slots.
- **Synthetic1:** This is a synthetic dataset that comprised of 1,024,680,418 (timeslot, itemID) tuples. The item-identifiers were from the universe $\{1, \ldots, 4000000\}$, and the trace was simulated for a period of 30 days, the length of each timeslot being 15 minutes. Hence, there were $30 \cdot 24 \cdot 60/15 = 2880$ distinct timeslots. We split the universe of size 4000000 in 10 disjoint groups, and defined a list F of 10 fractions (the constraint being $\sum_{i=1}^{10} F_i = 1$) where F_i represented what fraction of the universe belongs to group i. We also kept a list P of 10 fractions, where P_i indicated the persistence of F_i over the whole trace of 2880 slots. The values of F_i's and P_i's are all listed in Table 1. As an example, for $i = 1$, $F_i = 0.01$ and $P_i = 0.95$, which implies

group 1 comprised of 1% of the items of the universe (i.e., 40000 items), each of which will occur in $2880 \cdot 0.95 = 2736$ slots on expectation. Note that, the way we assigned the values of F_i's and P_i's mimics the real-life fact that the distribution of persistence is very skewed, so more than 50% of the items in the universe occur in less than 3 slots (on expectation). In practice, we put an item in group i in slot $j \in \{1, \ldots, 2880\}$ with probability P_i. Also, before generating the actual tuples, we created a random permutation of the universe $\{1, \ldots, 4000000\}$ by FisherYates shuffle [19].

- **Synthetic2:** This is a synthetic dataset that comprised of 123,408,469 (timeslot, itemID) tuples. Like Synthetic1, for this also, the item-identifiers were from the universe $\{1, \ldots, 4000000\}$, and the trace was simulated for 2880 distinct timeslots. It differs from Synthetic1 in the values of F_i's and P_i's, and the difference is evident in Table 1. Note that, for Synthetic2, 86% of the items in universe have a persistence of 0.001 only, whereas for Synthetic1, 55% of the items in universe have that persistence. On the other hand, for Synthetic1, 1% of the items in universe have a persistence of 0.95, whereas for Synthetic2, 0.1% of the items have that high persistence. This explains why the skewness of Synthetic2 is about thrice that of Synthetic1, and the mean persistence of Synthetic2 is about $\frac{1}{9}^{th}$ of that of Synthetic1.

There is no obvious choice on what should be a suitable duration of the timeslot, since prior research has shown that the delay between successive botnet-related communications to the same destination can range from a few minutes to a few days. A duration of a few minutes is reasonable, since many botnets have multiple events occurring within this time frame. For example, Li *et al* [20] observed periodic botnet-related events about every half an hour. Rajab *et al* [21] reported that the average “staying time” for bots that they monitored was about 25 minutes, and 90% of them lasted less than 50 minutes. Over a 24-hour window, the BRAT project [22] reported probes by 8 fast-flux botnets which showed periodicity, the periods being in the range of 1-10 minutes. Porras *et al* [23] showed that for iKeeB, the iPhone-based botnet, a compromised iPhone runs a shell script once every 5 minutes. For the **HeaderTrace** dataset, we finally decided on a duration of 30 seconds so that our 3 hour trace led to a sufficient number of slots, and for the **Synthetic1** and

TABLE 1: Distribution of persistence for all datasets

Partition of universe	Synthetic1		Synthetic2		HeaderTrace
	F_i	P_i	F_i	P_i	
$i = 1$	0.01	0.95	0.001	0.95	
$i = 2$	0.02	0.75	0.002	0.75	
$i = 3$	0.03	0.55	0.003	0.55	
$i = 4$	0.04	0.35	0.004	0.35	
$i = 5$	0.05	0.25	0.005	0.25	
$i = 6$	0.06	0.15	0.006	0.15	
$i = 7$	0.07	0.1	0.007	0.1	
$i = 8$	0.08	0.05	0.01	0.05	
$i = 9$	0.09	0.01	0.1	0.01	
$i = 10$	0.55	0.001	0.862	0.001	
$\sum_{i=1}^{10} F_i$	1		1		
Size of universe	4,000,000		4,000,000		2,047,953
Exp. packets	1,024,704,000		123,402,240		
Actual packets	1,024,680,418		123,408,469		885,055,227
Mean persistence	0.089		0.01		0.0177
Third moment of persistence (m_3)	0.0105		0.0014		0.0043
Variance of persistence (m_2)	0.015		0.0019		0.006
Skewness of persistence ($m_3/{m_2}^{3/2}$)	5.67		17.17		9.28

Synthetic2 datasets, we chose the length to be 15 minutes. This helped us evaluate the scalability of our algorithm with increasing number of timeslots, and also to experiment with different slot-lengths. With the above setting of parameters, for the **HeaderTrace** and the synthetic datasets, we had reasonably large number of timeslots (360 and 2880 respectively) as well as a large number of packets per timeslot.

The algorithms were implemented in C++ using the STL extensions. For the hash functions in the small space algorithm (Algorithm 5), we used an endian-neutral implementation of the *Murmur Hash* algorithm [24], which is generally considered to generate high quality hash outputs.

We obtained the ground truth about the persistence of individual items (IP addresses for **HeaderTrace**) by running the naive algorithm over the input data streams. Note that, although for the synthetic datasets, we determined which item will have how much persistence, the actual data was generated by a probabilistic process, so we still needed to collect the *actual* persistence values of the items. In the process, for the **HeaderTrace** dataset, we discovered that a large fraction of the windows did not contain many persistent items. On such windows, our algorithm will run in a space-efficient manner, but we did not consider these windows since there would not be enough data for a fair comparison.

To simplify the presentation, on **HeaderTrace**, we focus on 11 specific "query" windows: these are $[1, 100], [26, 125], [51, 150], \ldots, [251, 350]$. On both the synthetic datasets, the query windows are $[1, 288], [289, 576], \ldots, [2593, 2880]$, so on these two, the time-duration of the query window was $288{\cdot}15/60 = 72$ hours. We use window $[a, b]$ to denote the window of all timeslots starting from a till b (both endpoints included).

On **HeaderTrace**, we found that the cumulative distribution of the persistence values in the dataset was highly skewed, for every query window that we tried. We present the CDF of persistence for three out of the 11 query windows: $[1, 100], [101, 200]$ and $[201, 300]$ in Figure 1, but all the 11 query windows showed similar pattern. For example, in the $[101, 200]$ window, more than 50% IP addresses occur in 1 slot only, and 95% of the IP addresses occur in 20 or less slots. This confirms the utility of an algorithm like ours, which requires less space when items have lower average persistence.

We made the distribution of **Synthetic1** less skewed (Figure 2) than that of **HeaderTrace** (skewnesses are respectively 5.67 and 9.28, as shown in Table 1), to construct **Synthetic1** as an adversarial input dataset, and found the results for **HeaderTrace** better than those for **Synthetic1**. In Figure 2, we present the CDF for only one query window as the distributions are identical across all windows, because of the way the dataset is generated. Then again, we constructed **Synthetic2** as the dataset with highest skew (17.17, as in Table 1) of all three, and thus it shows some improvement over **Synthetic1**, as we explain later.

Metrics: The following metrics were used. For parameter α, an item that is not α-persistent, as per the definition of α-persistence given in Definition 2.2, is called "transient". Note that this implies that an item with persistence between αn and $(\alpha - \epsilon)n$ is also treated as transient.

The **False Negative Rate (FNR)** is defined as the ratio of the number of α-persistent items that were not reported by the small space algorithm to the total number of α-persistent items.

The **False Positive Rate (FPR)** is defined as the ratio of the number of transient items that were reported as persistent by the algorithm to the total number of transient items.
The **Space Compression (SC)** is defined as the ratio of the number of tuples stored by the naive algorithm to the number of tuples stored by the small space algorithm.
The **Physical Space Compression (PSC)** is defined as the maximum Resident Set Size of the naive algorithm to that of the small-space algorithm. The Resident Set Size (RSS) is the part of the memory of a process that is held in RAM. We do not use the Virtual Set Size (VSZ) because that includes the swapped out memory for a process.

The notion of Space Compression (SC) is a logical one, and for the sliding window version of the problem (Problem 3), we were interested in the number of tuples of the form $(d, t, \cdot, \cdot)$, as referred to in Algorithms 5 to 7.

In the actual implementation, for each distinct item d, we maintained a sorted list (of variable size) of $(t', n_{d,t'})$ tuples, ordered by t', where $n_{d,t'}$ indicates in how many distinct slots d has appeared since its appearance in slot t'. The sorted list helped us to check by binary-search if an item d has occurred in a given slot t'. When d appears in a slot t' it has not appeared in before, the tuple $(t', n_{d,t'})$ is initialized only if $h(d, t') < \tau$. Note that $t_{d,t'}$ - the last timeslot d has appeared in since its appearance in t', does not depend on t', and hence we maintained a single copy of this variable for each item d.

Since the resident set is specific to a process run by the operating system, for computing the Physical Space Compression (PSC), for each combination of α and ϵ, we actually created a new process so that the resident set is created afresh. We used the `getrusage()` function of C++ to measure the maximum resident set size of the process in kilobytes. However, we are more interested in knowing by how much our algorithm reduces the memory usage compared to a naive algorithm, and how it varies with the tunable parameters of the algorithm. We expect the Physical Space Compression for (α, ϵ') to be higher than that for (α, ϵ) when $\epsilon' > \epsilon$ (because τ is lower for ϵ'), but we found that because of the way memory allocation algorithms work, if the algorithm runs first for (α, ϵ) and then for (α, ϵ') (using the same process), then, the memory allocated for (α, ϵ) is enough to accomodate the algorithm for (α, ϵ'), and the space-saving due to (α, ϵ') does not get reflected.

Note that both the numerator and the denominator of each metric depend on the query window $[c - n + 1, c]$ (n is the window length). To measure the ratios, we ran the small-space algorithm on the query windows defined previously and in each window, recorded all the items that were marked as persistent by the algorithm. The only source of randomness in each run is the output of the Murmur Hash function and we ran each simulation thrice using different seeds (we saw very minor variation in the results when different seeds were used.) Thus, for each parameter setting we had 11×3 data points for **HeaderTrace**, and 10×3 data points for both the synthetic datasets, and in each we recorded the false positives, the false negatives, and the number of tuples that were tracked. The ratios computed (by comparing to the naive algorithm) are then averaged across all the runs.

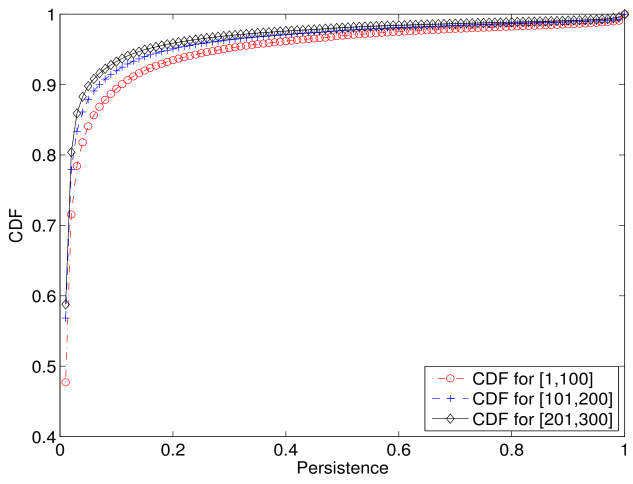


Fig. 1: CDF of persistence values from 3 windows for the **HeaderTrace** dataset

Observations along metrics:

For every value of α, the **False Negative Rate** (Figure 4a for **HeaderTrace**, Figure 5a for **Synthetic1** and Figure 6a for **Synthetic2**) increases as ϵ increases, which is expected. However, although Lemma 3.6 bounds the False Negative Rate to $\frac{1}{e^2} = 13\%$, the algorithm performed much better in practice - we found that even for $\alpha = 0.3$ and $\epsilon = 0.21$, the FNR

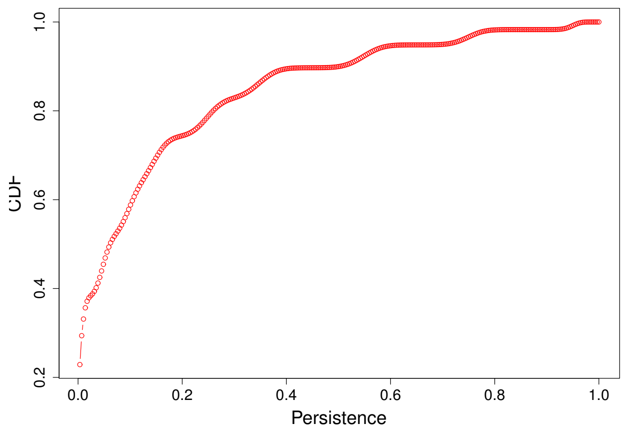


Fig. 2: CDF of persistence values from the [1,288] window for the **Synthetic1** dataset

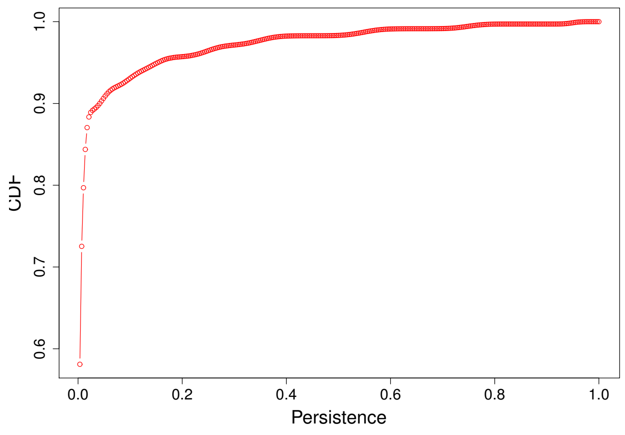
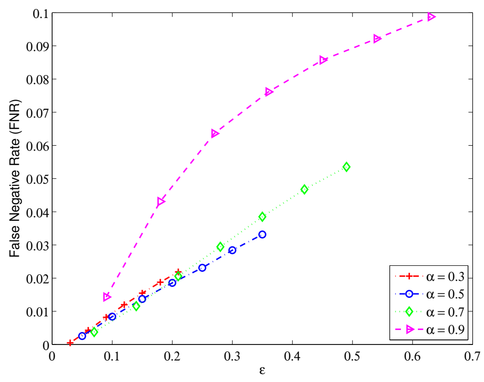
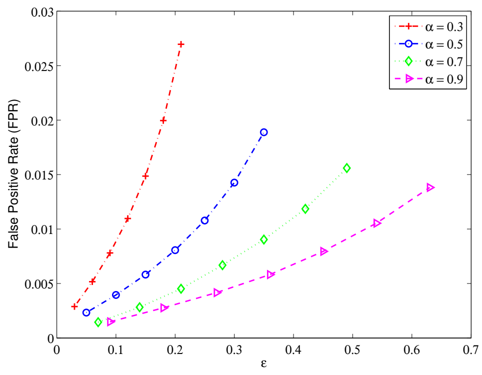


Fig. 3: CDF of persistence values from the [1,288] window for the **Synthetic2** dataset

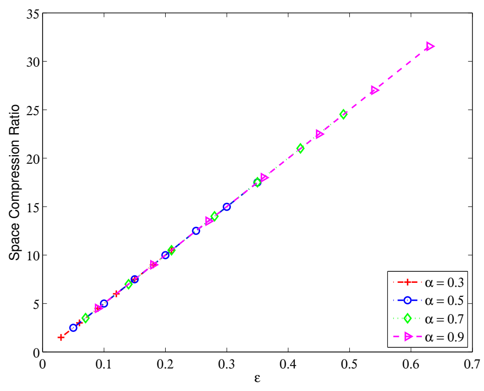
was as low as 2% for **HeaderTrace** and ∼3.5% for both the synthetic datasets. Note that $\frac{\epsilon}{\alpha}$ is a relative measure of error tolerance in α, which in this case is as high as 70%. The highest FNR we ever got was less than 10% for **HeaderTrace**, less than 12% for **Synthetic1** and 12.7% for **Synthetic2**. However, for all the three datasets, this was for the highest setting of α ($\alpha = 0.9$) - the number of false negatives for this were higher than for the other settings, for similar values of ϵ. One possible reason is that for $\alpha = 0.9$, an item that was 0.9-persistent had persistence very close to $0.9n$. Whereas, many of the items that



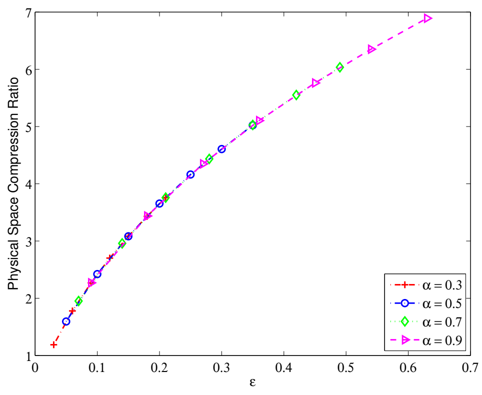
(a) Variation of FNR with α and ϵ



(b) Variation of FPR with α and ϵ



(c) Variation of SC with α and ϵ

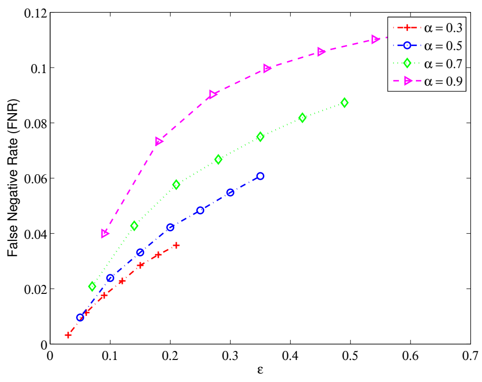


(d) Variation of PSC with α and ϵ

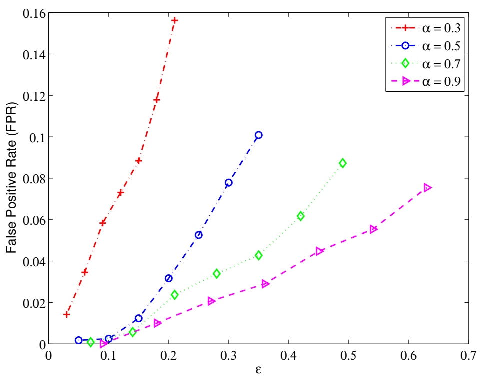
Fig. 4: Trade-off between accuracy and space for the small-space algorithm over sliding windows for the **HeaderTrace** dataset. Each point in each plot is an average from 33 data points - 3 runs over 11 query windows each. Note that the Y-axis is different for each plot. Also, for each value of α, the values of ϵ range from 0.1α to 0.7α.

were 0.3-persistent had persistence values that were much larger than $0.3n$. Items that have persistence values close to the threshold, but higher than it, have a greater chance of not being reported than items whose persistence values are far above the threshold. Hence, the false negative ratio for $\alpha = 0.9$ is a little higher.

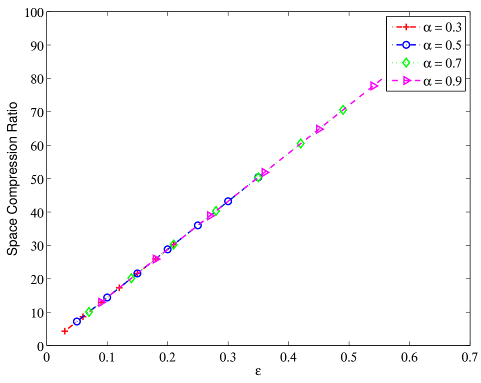
The **False Positive Rate**, similar to the False Negative Rate, shows (Figure 4b for **HeaderTrace**, Figure 5b for **Synthetic1** and Figure 6b for **Synthetic2**) an increasing trend as ϵ increases. The maximum FPR was 2.69% for **HeaderTrace** and 2.2% for **Synthetic2** (both for $\alpha = 0.3$ and $\epsilon = 0.21$). Moreover, all of Figures 4b, 5b and 6b show that for the same value of ϵ, the FPR is lower for higher values of α. The possible reason is that when α



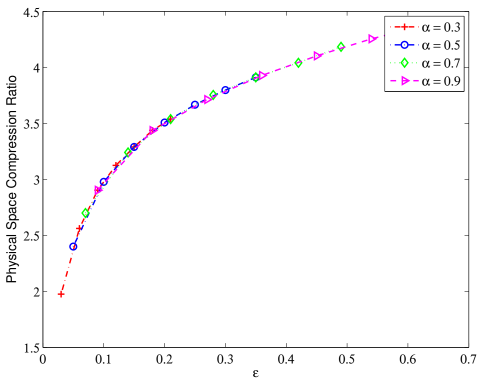
(a) Variation of FNR with α and ϵ



(b) Variation of FPR with α and ϵ



(c) Variation of SC with α and ϵ

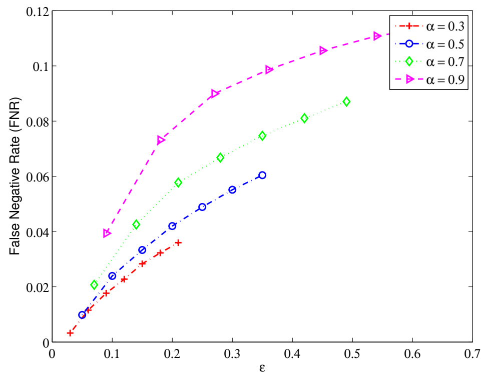


(d) Variation of PSC with α and ϵ

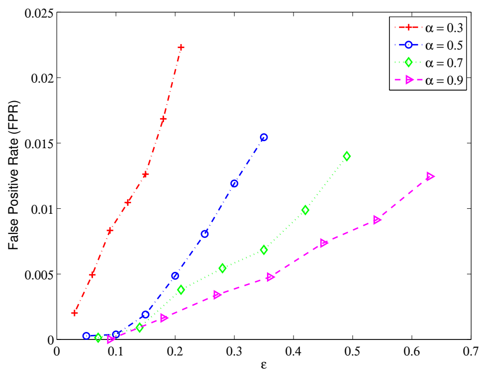
Fig. 5: Trade-off between accuracy and space for the small-space algorithm over sliding windows for the **Synthetic1** dataset. Each point in each plot is an average from 30 data points - 3 runs over 10 query windows each. The Y-axis is different for each plot. For each value of α, the values of ϵ range from 0.1α to 0.7α.

is very high (e.g. 0.9), most items have persistence much lower than αn (as is evident from the CDFs in Figures 1 and 2), hence are very unlikely to cross the threshold T in Algorithm 7.

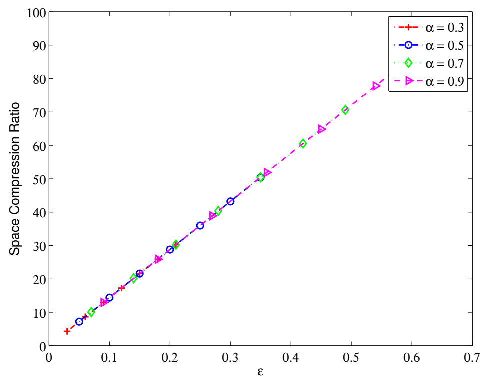
The (Logical) **Space Compression** increases *linearly* with ϵ (Figures 4c for **HeaderTrace**, 5c for **Synthetic1** and 6c for **Synthetic2**), and we found the Space Compression is close to $\frac{1}{\tau} = \frac{\epsilon n}{2}$, for all values of α and all three datasets. This is expected since the naive algorithm creates a new tuple for an item everytime it appears in a different slot - where the small-space algorithm creates a tuple with probability τ only. For $\alpha = 0.9$, with $\epsilon = 0.63$, the logical space compression was as high as 32 for **HeaderTrace**, and as high



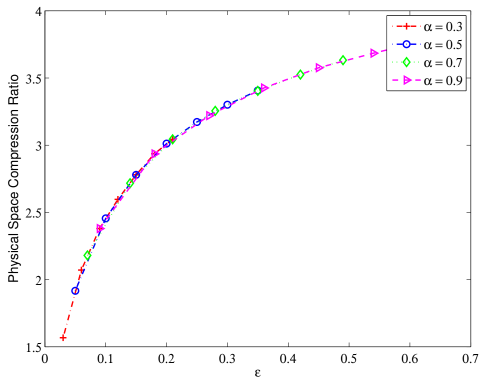
(a) Variation of FNR with α and ϵ



(b) Variation of FPR with α and ϵ



(c) Variation of SC with α and ϵ

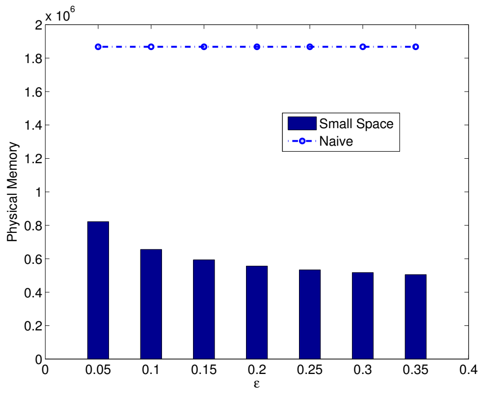


(d) Variation of PSC with α and ϵ

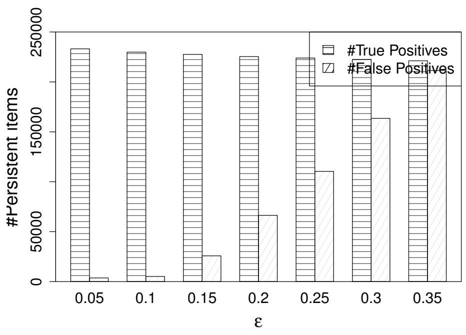
Fig. 6: Trade-off between accuracy and space for the small-space algorithm over sliding windows for the **Synthetic2** dataset. Other details are same as **Synthetic1**.

as 91 for **Synthetic1** and ∼100 for **Synthetic2**. The higher Space Compression for the synthetic datasets compared to **HeaderTrace** is justified by the larger value of the window length n (288 as opposed to 100). For higher values of α, we could achieve better Space Compression as the tolerance ϵ could be made higher while keeping the false positives and the false negatives small enough.

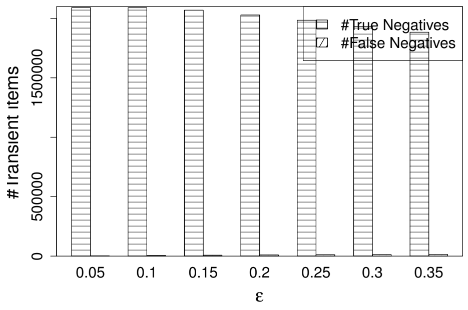
Like its logical counterpart, the **Physical Space Compression** also increases with ϵ (Figure 4d for **HeaderTrace**, 5d for **Synthetic1** and 6d for **Synthetic2**), and for each distinct value of α, the Physical Space Compression grows almost linearly with ϵ. For higher values of α, we could achieve better Space Compression as the tolerance ϵ could be made higher. While the size of the **HeaderTrace** dataset was 58 GB, the maximum resident set



(a) Variation of actual memory used with ϵ



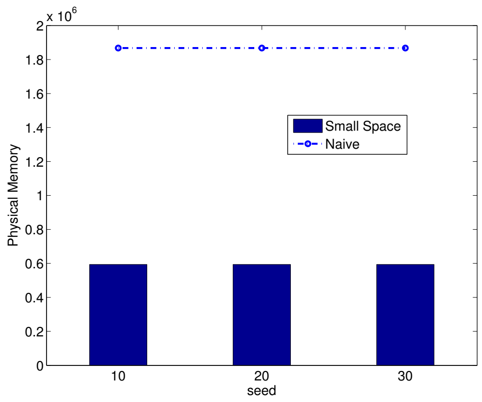
(b) Variation of number of true positives and false positives with ϵ



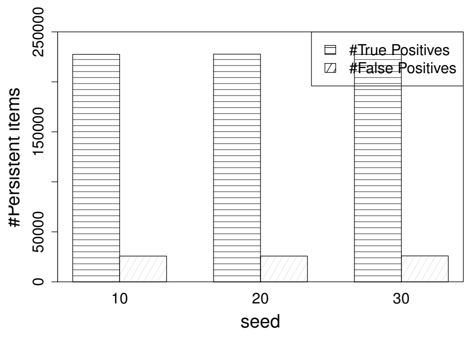
(c) Variation of number of true negatives and false negatives with ϵ

Fig. 7: The variation of the physical memory taken, the number of true positives, false positives, true negatives and false negatives with ϵ for the **Synthetic1** dataset. All the plots are for $\alpha = 0.5$ and the query window $[2593, 2880]$. So, each point in each plot is an average from 3 data points corresponding to the 3 different seed values $(10, 20, 30)$. Note that the horizontal line in the first plot represents the actual memory taken by the naive algorithm, and hence does not vary with ϵ. The Y-axis is different for each plot. The values of ϵ range from 0.1α to 0.7α.

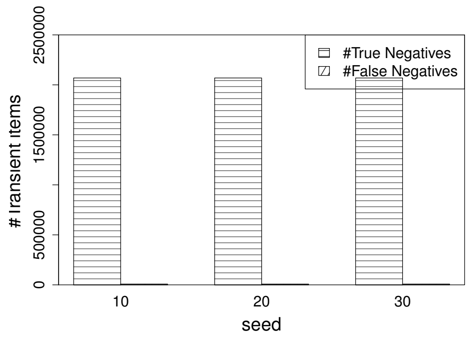
size of the naive algorithm went up to 3 GB (at the query window [251,350]), whereas for typical parameters like $\alpha = 0.5$ and $\epsilon = 0.35$, the small-space algorithm took less than $\frac{1}{5}^{th}$ (600 MB) memory (on average) compared to the naive algorithm. For **Synthetic1**, the dataset size was 12 GB, the maximum resident set size of the naive algorithm went up to 1.8 GB, whereas for $\alpha = 0.5$ and $\epsilon = 0.35$, the small-space algorithm took space between 350 and 500 MB. For **Synthetic2**, the dataset size was 1.5 GB, the maximum resident set size of the naive algorithm went up to 736 MB, whereas for $\alpha = 0.5$ and $\epsilon = 0.35$, the small-space algorithm took space between 140 and 200 MB.



(a) Variation of actual memory used with seed



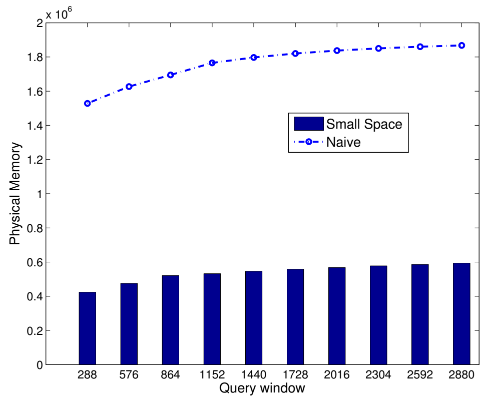
(b) Variation of number of true positives and false positives with seed



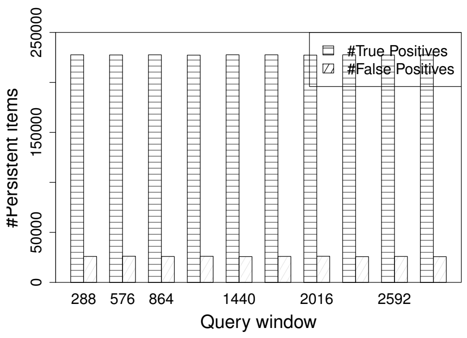
(c) Variation of number of true negatives and false negatives with seed

Fig. 8: The variation of the physical memory taken, the number of true positives, false positives, true negatives and false negatives with the seed of the random number generator for the **Synthetic1** dataset. All the plots are for $\alpha = 0.5, \epsilon = 0.15$ and the query window $[2593, 2880]$. Note that the horizontal line in the first plot represents the actual memory taken by the naive algorithm, and hence does not vary with the seed. The Y-axis is different for each plot. The values of the seed used are 10, 20 and 30.

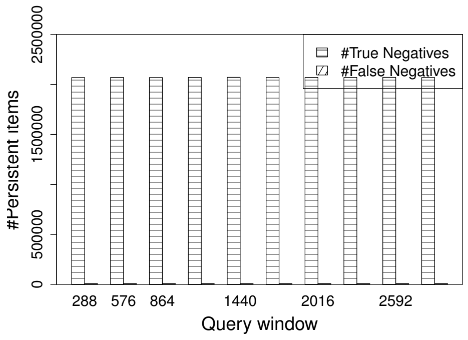
Variation with ϵ, seed and query window: Figures 7a through 9c take a closer look at some of the absolute numbers (actual memory used, number of true and false positives, number of true and false negatives) rather than ratios for the **Synthetic1** dataset, and show how they vary with ϵ, the seed value of the random number generator and the different query windows. Figure 7a shows how the physical memory (in KB) varies with ϵ for $\alpha = 0.5$ and the query window $[2593, 2880]$. Since the memory taken by the naive algorithm does not depend on α or ϵ, it is constant throughout at 1.86 GB, and the memory



(a) Variation of actual memory used with query window



(b) Variation of number of true positives and false positives with query window



(c) Variation of number of true negatives and false negatives with query window

Fig. 9: The variation of the physical memory taken, the number of true positives, false positives, true negatives and false negatives with the query window for the **Synthetic1** dataset. All the plots are for $\alpha = 0.5, \epsilon = 0.15$ and seed = 10. Note that the horizontal line in the first plot represents the actual memory taken by the naive algorithm, and shows slight increase with the progress of time (increasing query window number). The Y-axis is different for each plot. The query windows are $[1, 288], [289, 576], \ldots, [2593, 2880]$ and the values on the X-axis are the endpoints of the query windows.

taken by the small space algorithm falls from 800 MB to 500 MB as ϵ increases from 0.05 to 0.35.

In Figure 7b, the actual number of persistent items in the window $[2593, 2880]$ is constant at 235,353; and we can see that with increasing ϵ, the number of true positives reduces only a little, remaining very close to the number of actual persistent items throughout. The number of false positives is as low as ~3,600 when $\epsilon = 0.05$, and for typical values of ϵ (e.g., 0.15) that will probably be used in practice for a problem like this, the number of false positives is $\sim$25k.

In Figure 7c, the actual number of transient items in the window $[2593, 2880]$ is constant at $\sim$2.1m; and we can see that with increasing ϵ, the number of true negatives reduces only a little, remaining very close to the number of actual transient items throughout. The number of false negatives is as low as $\sim$2,200 when $\epsilon = 0.05$, and even when ϵ is as high as 0.35, the number of false negatives increases only to $\sim$14k. Note that, for many practical applications, it is important to keep the number/rate of false negatives much lower compared to the number/rate of false positives, and comparing Figure 7b and 7c shows that our algoithm meets that criterion.

Figure 8a shows how the physical memory (in KB) varies with the seed of the random number generator for $\alpha = 0.5$, $\epsilon = 0.15$ and the query window $[2593, 2880]$. Since the memory taken by the naive algorithm does not depend on the seed of the random number generator, it is constant throughout at 1.86 GB, and the memory taken by the small space algorithm also remains practically constant at $\sim$593MB, which justifies averaging the actual memory footprint over the 3 different seed values.

In Figure 8b, the actual number of persistent items in the window $[2593, 2880]$ is constant at 235,353; and we can see that with change in the seed, the number of true positives remains practically constant at $\sim$227k, and so does the number of false positives at $\sim$26k.

In Figure 8c, the actual number of transient items in the window $[2593, 2880]$ is constant at $\sim$2.1m; and we can see that with change in the seed, the number of true negatives remains practically constant at $\sim$2.07m, and so does the number of false negatives at $\sim$7.8k.

Figure 9a shows how the physical memory (in KB) varies with the query window for $\alpha = 0.5$ and $\epsilon = 0.15$, the seed of the random number generator being 10. Unlike Figure 7a or Figure 8a, the physical memory depends on the number of distinct items in the window, and although we generated the items uniformly across the slot range $[1, 2880]$, we see it increased gradually with increasing slot number. However, while the space taken by the naive algorithm varied from 1.5 GB to 1.86 GB, the space taken by the small-space algorithm varied from $\sim$400 MB to $\sim$600MB.

Figure 9b shows how the number of persistent items varies with the query window for $\alpha = 0.5$ and $\epsilon = 0.15$, the seed of the random number generator being 10. Since for the **Synthetic1** dataset, each persistent item was distributed uniformly across the slot range $[1, 2880]$, the actual number of persistent items across the different windows was almost constant at $\sim$235k; and we can see that with change in the query window, the number of true positives also remains practically constant at $\sim$227k, and so does the number of false positives at $\sim$26k.

Figure 9c shows how the number of transient items varies with the query window for $\alpha = 0.5$ and $\epsilon = 0.15$, the seed of the random number generator being 10. For the **Synthetic1** dataset, like the persistent items, each transient item was also distributed uniformly across the slot range $[1, 2880]$, hence the actual number of transient items across the different query windows is practically constant at $\sim$2.1m; and we can see that with change in the query window, the number of true negatives remains practically constant at $\sim$2.07m, and so does the number of false negatives at $\sim$7.8k.

The small variation in the actual memory used, the number of true and false positives and the number of true and false negatives by the small-space algorithm, as shown in Figures 8a through 9c justifies our averaging of these quantities across the different seed values and query windows.

Comparsion among three datasets: The FPR for **HeaderTrace** was much lower than that for **Synthetic1** at comparable points, e.g., at $\alpha = 0.3, \epsilon = 0.21$ and in comparable query windows, for **Synthetic1**, the FPR is 16%, whereas for **HeaderTrace**, the FPR is 2.7%. For **Synthetic1**, at an identical query window, there are $\sim$300k false positives out of $\sim$1931k transient items, and for **HeaderTrace**, there are $\sim$21k false positives out of $\sim$802k transient items. The lower FPR for **HeaderTrace** probably arises out of the fact that the CDF curve (Figure 1) grows more steeply than the CDF curve for **Synthetic1** (Figure 2), so the fraction of items whose persistence come anywhere close to 0.3 is much less. To demonstrate the difference among the three datasets, we present the 93^{rd} percentile value of persistence for each. For **HeaderTrace**, there are total $\sim$829k distinct items in the window [251,350], but ~770k of these items occur in 10 or less slots out of 100, i.e., the 93^{rd} percentile value of persistence is 10%. As a comparison, for **Synthetic1**, there are total ~2330k distinct items in the window [2593, 2880], but ~2167k of these items occur in 161 or less slots out of 288 (161/288 = 56%), i.e., the 93^{rd} percentile value of persistence is 56%. But then again, for **Synthetic2**, the FPR improves significantly over **Synthetic1** - for $\alpha = 0.3$ and $\epsilon = 0.21$, **Synthetic2** gives an FPR of 2.2% in pretty much all query windows - so the FPR becomes comparable to that of **HeaderTrace**. To compare with **Synthetic1**, there are ~1394k distinct items in the window [2593, 2880], but ~1296k of them occur in 28 or less slots out of 288, i.e., the 93^{rd} percentile value of persistence is 9.7% (see Figure 3). The reason for lower FPR for **Synthetic2** is similar to that for **HeaderTrace**. This shows that the FPR improves with skewness of the data, and our algorithm in fact performs well for datasets with realistic skewness.

The physical memory compression ratio is less for **Synthetic1** than for **HeaderTrace** for similar reasons - many transient items find room into the sketch for having persistence close to the threshold. Although the skew for **Synthetic2** is more than that of **Synthetic1**, it has similar values of logical and physical memory compressions since the proportion of items from the universe that have similar persistence values bear similar ratios to each other, e.g., for **Synthetic1**, 1% of the items have persistence 0.95 and 2% have persistence 0.75; whereas for **Synthetic2**, 0.1% of the items have persistence 0.95 and 0.2% have persistence 0.75 (first two rows of Table 1).

In Lemma 3.7, we showed that for a given value of ϵ and length of sliding window (n), the expected number of tuples in the sketch is proportional to the sum of the persistence values of all items appearing in the window ($\sum_{d \in D} p_d^c$). Hence, the physical memory taken should also vary with the sum of the persistence values. We present the following example to demonstrate this: in the $[1, 288]$ window, the number of distinct items for **Synthetic1** and **Synthetic2** are respectively ~2.33m and ~1.39m, and the sum of persistence values for **Synthetic1** and **Synthetic2** respectively are about 20.47×10^4 and 1.4×10^4. For $\alpha = 0.5$ and $\epsilon = 0.35$, the memory footprints by the small-space algorithm for this combination of parameters for **Synthetic1** and **Synthetic2** are respectively 353 MB and 142 MB, so

Synthetic1 takes about 2.5 times more memory than **Synthetic2**.

5 Related Work

A large body of literature on network anomaly detection has focused on detecting volume-based anomalies, i.e., tracking IPs which send or receive an unusally large volume of traffic over an interval of time. While volume-based anomaly detection is relevant for Denial-of-Service type attacks like SYN flood [25], UDP flood [26], Ping flood or P2P attacks, there are many "stealthy" attacks [27], which can bypass the radar by never sending traffic in large volume, yet remaining active over long windows in time, and probing the target network/host once in a while. For example, port scans [5] look for open ports on remote hosts that have applications with known vulnerabilities deployed on those ports; bots installed on compromised hosts in a botnet keep on communicating with the C&C server, etc. Our work differs from these in that persistent items may not result in large volumes of traffic and may escape detection by a volume-based system.

It is interesting to compare how algorithmic techniques for identifying heavy-hitters (or "frequent items") may work for the problem of identifying persistent items. Broadly, the techniques in the literature can be classified into "counter-based", "quantile algorithms", "sketches", or "random sampling-based" (see [28]). Counter-based techniques such as the Misra-Gries algorithm [7], and the "Space-Saving" algorithm [29] rely on maintaining per-item counters for counting the number of occurrences of each item that has been currently identified as being frequent; these counters are occasionally decremented to ensure that the space taken by the data structure is small. The difficulty in using this technique for our problem is that it is not easy to ensure that re-occurrences of the same item within a timeslot have no effect on the system state. For example, in the Misra-Gries algorithm, if there is a decrement of the counters between two occurrences of an item within the same timeslot, it seems hard to ensure that the second occurrence has no effect on the system state, especially given that the increment due to the first occurrence may have disappeared from the system (due to the decrement). The same argument is true for Lossy Counting too [8]. Quantile-based algorithms such as Greenwald and

Khanna, or [30], the q-digest [31] view the space of all items as being a bijection with the set of integers, and associate counts with different ranges in this space of all items. In the q-digest algorithm, there are no decrements to these counters, so one may use "distinct counters" such as those by Flajolet-Martin [16], or Gibbons and Tirthapura [15], or Kane, Nelson, and Woodruff [32], instead of regular counters. Such an approach based on maintaining distinct counters would not only be more complex than our approach, but also likely have a greater space complexity, since maintaining distinct counters with a relative error of ϵ requires $\Omega(1/\epsilon^2)$ space [17]. The sketch approach, such as count-sketch [33] or count-min sketch [12] also maintains multiple counters, each of which is the sum of many random variables. Replacing each such counter with a distinct counter leads to its own set of difficulties, one of which is the space complexity of distinct counting, explained above, and the other being the fact that each distinct counter is only approximate (exact distinct counting necessarily requires large space [34]), while the analyses in [33] and [12] rely on the different counters in the data structure being exact.

Finally, our algorithm is inspired by the random sampling approaches based on the "sample and count" scheme of Alon *et al.* [34], [35] and the "sticky sampling" algorithm of Manku and Motwani [8]. Both these algorithms use the following idea: "sample a random element in the stream, and track reoccurrences of this element exactly". In these works, the idea was applied to a different context than ours – sample and count was applied to track the size of a self-join in limited storage, and sticky sampling was used in the identification of heavy hitters using limited space. Our algorithm has the following technical differences when compared with the above works. The sampling of an item is done using a hash function that is based on the item identifier and the timeslot in which it arrived in. This hash-based sampling avoids giving greater sampling probability to an item if it occurs multiple times within the same timeslot. Further, reoccurrences are tracked in such a way that we do not overcount if the same item appears again in the same timeslot. In addition, we show how to handle sliding windows using nearly the same space, while the above works do not address the context of sliding windows. A distinguishing aspect of our work on sliding windows is that while the extension to sliding windows often requires asymptotically greater space than for the infinite window case (for example, see Arasu and Manku [36]), in our case the space complexity is asymptotically the same.

Persistence is exploited to detect botnet traffic in [1], using an algorithm that tracked the state of every distinct item that arrived within the sliding window. Hence the memory used is of the order of the number of distinct items times the window size, which is potentially very high. In contrast, our algorithm tracks persistent items using much smaller space, while giving up some accuracy.

There has been much work in estimating various properties of the frequency distribution of stream items, including the frequency moments of a stream [34], [37], [32], heavy-hitters [8], [9], [12], [38], and the entropy [39], [40], [41]. Unlike the set of persistent items, all the above properties depend only on the frequency distribution of items in the stream – they are unaffected by re-ordering of the stream elements, or by changing the times at which the elements arrive. In contrast, the set of persistent items in a stream is affected by the time and order in which elements arrive.

In a recent work on a temporal property of a stream, Chen *et al* [42] addressed the problem of tracking long-duration flows from network streams. They identified flows for which the difference of timestamps between the first and the last packet in the flow exceed some threshold d. A flow might continue for a long duration and yet the total number of bytes sent in the flow may not be high enough to be detected by the heavy-hitter algorithms; whereas some other flow of shorter duration might qualify as a heavy-hitter because it sends many more bytes. Clearly, a long-lived flow is not necessarily persistent.

6 Conclusion

We formulated the problem of detecting *persistent* items in a data stream. Our lower bound result shows that an exact algorithm for the problem, which reports *all* persistent items, would need a prohibitively high memory, and is therefore impractical. Subsequently, we presented an approximate formulation of the problem that explores a tradeoff between space and accuracy in identifying persistent items. Allocating more memory leads to more accurate answers and this allows operators to tune their systems appropriately depending on the amount of resources available.

By running simulations of both the naive (exact) and small space algorithms on a real as well as two synthetic traffic datasets with different skewness, we demonstrate that our algorithm works very well in practice: for the real trace, it uses up to 85% *less* space than the naive (exact) algorithm and incurs a false positive rate (and false negative rate) of less than 1% (and 4% respectively) for typical values of the parameters. We also see that false positive rate never exceeds 3% for any parameter setting, while the false negative rate stays below 5% for all but the most aggressive thresholds for persistence. For the synthetic trace with low skewness, the small-space algorithm uses up to 80% less space than the naive one, the false positive rate is less than 2% and the false negative rate is about 4% for typical parameter values (e.g., $\alpha = 0.5$ and $\epsilon = 0.15$). The maximum false positive rate is less than 3% for the real trace and the synthetic trace with higher skewness. The empirical false positive and false negative rates, for most parameters, are much better than the analytical bounds: and our experiment across the three different datasets shows that the false positive rate improves for data with higher skewness.

REFERENCES

[1] F. Giroire, J. Chandrashekar, N. Taft, E. Schooler, and D. Papagiannaki, "Exploiting temporal persistence to detect covert botnet channels," in *Proceedings of the 12th International Symposium on Recent Advances in Intrusion Detection (RAID)*, 2009, pp. 326–345.

[2] "Google AdWords," http://www.google.com/ads/adwords2/.

[3] L. Zhang and Y. Guan, "Detecting click fraud in pay-per-click streams of online advertising networks," in *Proceedings of the 28th IEEE International Conference on Distributed Computing Systems (ICDCS)*, 2008, pp. 77–84.

[4] S. Guha, J. Chandrashekar, N. Taft, and K. Papagiannaki, "How healthy are today's enterprise networks?" in *Proceedings of the 8th ACM SIGCOMM Conference on Internet Measurement (IMC)*, 2008, pp. 145–150.

[5] S. Staniford, J. A. Hoagland, and J. M. McAlerney, "Practical automated detection of stealthy portscans," in *Journal of Computer Security*, 2002, vol. 10, no. 1/2, pp. 105–136.

[6] "Advanced Persistent Threat," http://www.usenix.org/event/lisa09/tech/slides/daly.pdf.

[7] J. Misra and D. Gries, "Finding repeated elements," *Science of Computer Programming*, vol. 2, no. 2, pp. 143–152, 1982.

[8] G. S. Manku and R. Motwani, "Approximate frequency counts over data streams," in *Proceedings of 28th International Conference on Very Large Data Bases (VLDB)*, 2002, pp. 346–357.

[9] C. Estan and G. Varghese, "New directions in traffic measurement and accounting: Focusing on the elephants, ignoring the mice," *ACM Transactions on Computer Systems*, vol. 21, no. 3, pp. 270–313, 2003.

[10] M. Charikar, K. Chen, and M. Farach-Colton, "Finding frequent items in data streams," *Theoretical Computer Science*, vol. 312, no. 1, pp. 3–15, 2004.

[11] G. Cormode, F. Korn, S. Muthukrishnan, and D. Srivastava, "Diamond in the rough: Finding hierarchical heavy hitters in multi-dimensional data," in *Proceedings of the ACM SIGMOD International Conference on Management of Data (SIGMOD)*, 2004, pp. 155–166.

[12] G. Cormode and S. Muthukrishnan, "An improved data stream summary: the count-min sketch and its applications," *Journal of Algorithms*, vol. 55, no. 1, pp. 58–75, 2005.

[13] S. Venkataraman, D. X. Song, P. B. Gibbons, and A. Blum, "New streaming algorithms for fast detection of superspreaders," in *Proceedings of the Network and Distributed System Security Symposium (NDSS)*, 2005, pp. 149–166.

[14] N. Bandi, D. Agrawal, and A. E. Abbadi, "Fast algorithms for heavy distinct hitters using associative memories," in *Proceedings of the 27th IEEE International Conference on Distributed Computing Systems (ICDCS)*, 2007, pp. 6–14.

[15] P. Gibbons and S. Tirthapura, "Estimating simple functions on the union of data streams," in *Proceedings of the 13th ACM Symposium on Parallel Algorithms and Architectures (SPAA)*, 2001, pp. 281–291.

[16] P. Flajolet and G. N. Martin, "Probabilistic counting algorithms for database applications," *Journal of Computer and System Sciences*, vol. 31, pp. 182–209, 1985.

[17] P. Indyk and D. Woodruff, "Tight lower bounds for the distinct elements problem," in *Proceedings of the 44th IEEE Symp. on Foundations of Computer Science (FOCS)*, 2003, pp. 283–288.

[18] CAIDA, "OC48 traces dataset," https://data.caida.org/datasets/oc48/oc48-original/20020814/5min/.

[19] R. Durstenfeld, "Algorithm 235: Random permutation," *Communications of the ACM*, vol. 7, no. 7, p. 420, 1964.

[20] Z. Li, A. Goyal, Y. Chen, and V. Paxson, "Automating analysis of large-scale botnet probing events," in *Proceedings of the 2009 ACM Symposium on Information, Computer and Communications Security (ASIACCS)*, 2009, pp. 11–22.

[21] M. A. Rajab, J. Zarfoss, F. Monrose, and A. Terzis, "A multifaceted approach to understanding the botnet phenomenon," in *Proceedings of the 6th ACM SIGCOMM Conference on Internet Measurement (IMC)*, 2006, pp. 41–52.

[22] "Botnet Reporting and Termination," http://spamtrackers.eu/wiki/index.php/Botnet_Reporting.

[23] P. Porras, H. Saidi, and V. Yegneswaran, "An Analysis of the iKeeB (duh) iPhone botnet (worm)," http://mtc.sri.com/iPhone/.

[24] A. Appleby, "Murmurhash 2.0," http://sites.google.com/site/murmurhash/.

[25] "CERT advisory CA-1996-21 TCP SYN flooding and IP spoofing attacks," http://www.cert.org/advisories/CA-1996-21.html.

[26] "CERT advisory CA-1996-01 UDP port denial-of-service attack," http://www.cert.org/advisories/CA-1996-01.html.

[27] Y. Gao, Y. Zhao, R. Schweller, S. Venkataraman, Y. Chen, D. Song, and M.-Y. Kao, "Detecting stealthy attacks using online histograms," in *International Workshop on Quality of Service*, 2007.

[28] G. Cormode and M. Hadjieleftheriou, "Finding frequent items in data streams," *Proceedings of the VLDB Endowment (PVLDB)*, vol. 1, no. 2, pp. 1530–1541, 2008.

[29] A. Metwally, D. Agrawal, and A. E. Abbadi, "Efficient computation of frequent and top-k elements in data streams," in *Proceedings of the 10th International Conference on Database Theory (ICDT)*, 2005, pp. 398–412.

[30] M. Greenwald and S. Khanna, "Space efficient online computation of quantile summaries," in *Proceedings of the 20th ACM International Conference on Management of Data (SIGMOD)*, 2001, pp. 58–66.

[31] N. Shrivastava, C. Buragohain, D. Agrawal, and S. Suri, "Medians and beyond: new aggregation techniques for sensor networks," in *Proceedings of the 2nd International Conference on Embedded Networked Sensor Systems (SenSys)*, 2004, pp. 239–249.

[32] D. M. Kane, J. Nelson, and D. P. Woodruff, "An optimal algorithm for the distinct elements problem," in *Proceedings of the 29th ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems (PODS)*, 2010.

[33] M. Charikar, K. Chen, and M. Farach-Colton, "Finding frequent items in data streams," in *Proceedings of the 29th International Colloquium Automata, Languages and Programming (ICALP)*, 2002, pp. 693–703.

[34] N. Alon, Y. Matias, and M. Szegedy, "The space complexity of approximating the frequency moments," *Journal of Computer and System Sciences*, vol. 58, no. 1, pp. 137–147, 1999.

[35] N. Alon, P. B. Gibbons, Y. Matias, and M. Szegedy, "Tracking join and self-join sizes in limited storage," *Journal of Computer and System Sciences*, vol. 64, no. 3, pp. 719–747, 2002.

[36] A. Arasu and G. Manku, "Approximate counts and quantiles over sliding windows," in *Proceedings of the 23rd ACM SIGMOD-SIGACT-SIGART Symposium on Principles of Database Systems (PODS)*, 2004, pp. 286–296.

[37] D. P. Woodruff, "Optimal space lower bounds for all frequency moments," in *Proceedings of the 15th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA)*, 2004, pp. 167–175.

[38] L.-K. Lee and H. F. Ting, "A simpler and more efficient deterministic scheme for finding frequent items over sliding windows," in *Proceedings of the 25th ACM SIGACT-SIGMOD-SIGART Symposium on Principles of Database Systems (PODS)*, 2006, pp. 290–297.

[39] A. Lall, V. Sekar, M. Ogihara, J. Xu, and H. Zhang, "Data streaming algorithms for estimating entropy of network traffic," in *Proceedings of the Joint International Conference on Measurement and Modeling of Computer Systems (SIGMETRICS/Performance)*, 2006, pp. 145–156.

[40] A. Chakrabarti, K. D. Ba, and S. Muthukrishnan, "Estimating entropy and entropy norm on data streams," in *Proceedings of the 23rd Annual Symposium on Theoretical Aspects of Computer Science (STACS)*, 2006, pp. 196–205.

[41] G. Nychis, V. Sekar, D. G. Andersen, H. Kim, and H. Zhang, "An empirical evaluation of entropy-based traffic anomaly detection," in *Proceedings of the 8th ACM SIGCOMM Conference on Internet Measurement (IMC)*, 2008, pp. 151–156.

[42] A. Chen, Y. Jin, and J. Cao, "Tracking long duration flows in network traffic," in *Proceedings of the 29th IEEE International Conference on Computer Communications (INFOCOM)*, 2010, pp. 206–210.