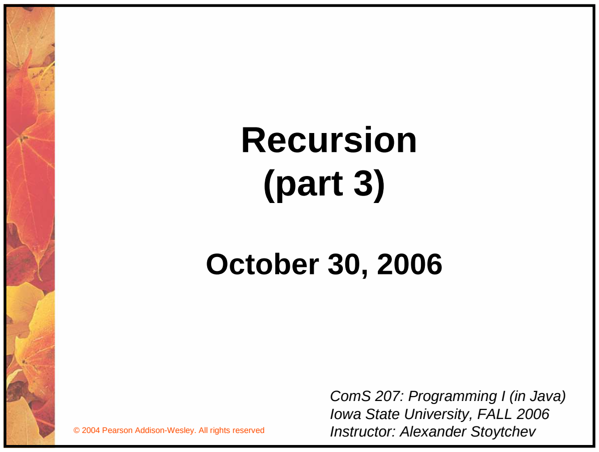
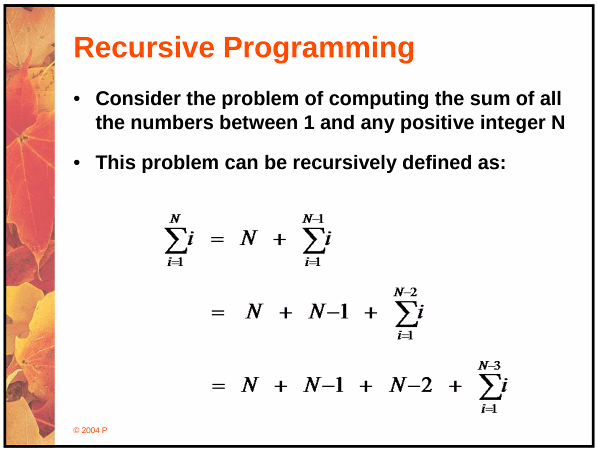


Recursion
(part 3)

October 30, 2006

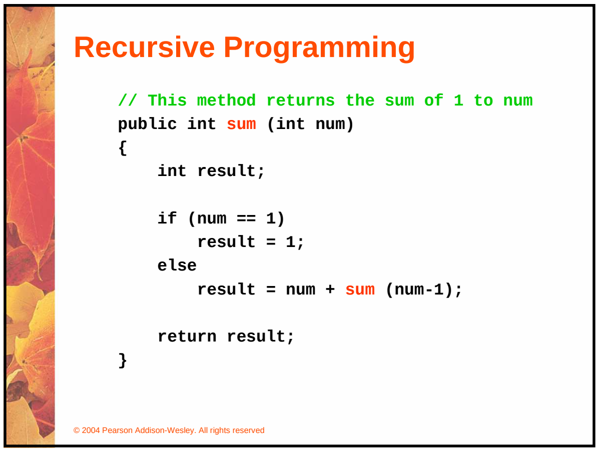
ComS 207: Programming I (in Java)
Iowa State University, FALL 2006
Instructor: Alexander Stoytchev



Recursive Programming

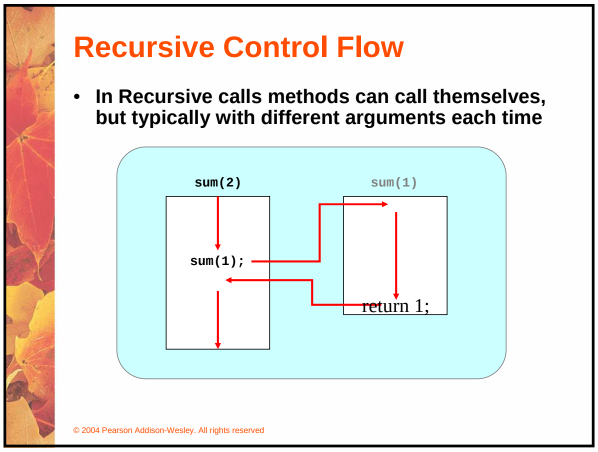
- Consider the problem of computing the sum of all the numbers between 1 and any positive integer N
- This problem can be recursively defined as:

$$\begin{aligned}\sum_{i=1}^{N} i &= N + \sum_{i=1}^{N-1} i \\ &= N + N-1 + \sum_{i=1}^{N-2} i \\ &= N + N-1 + N-2 + \sum_{i=1}^{N-3} i\end{aligned}$$



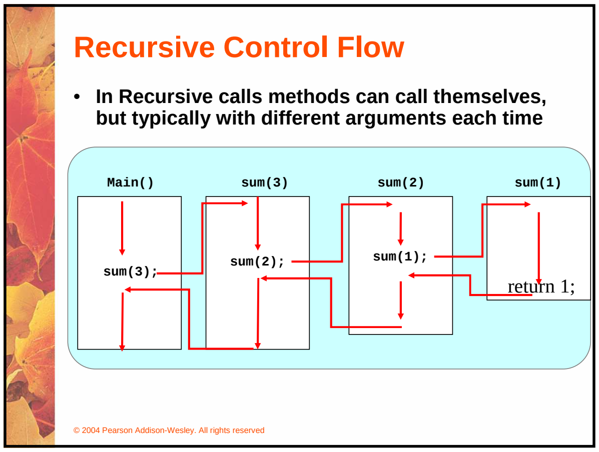
Recursive Programming

```
// This method returns the sum of 1 to num
public int sum (int num)
{
    int result;

    if (num == 1)
        result = 1;
    else
        result = num + sum (num-1);

    return result;
}
```



Recursive Control Flow

- In Recursive calls methods can call themselves, but typically with different arguments each time



Recursive Control Flow

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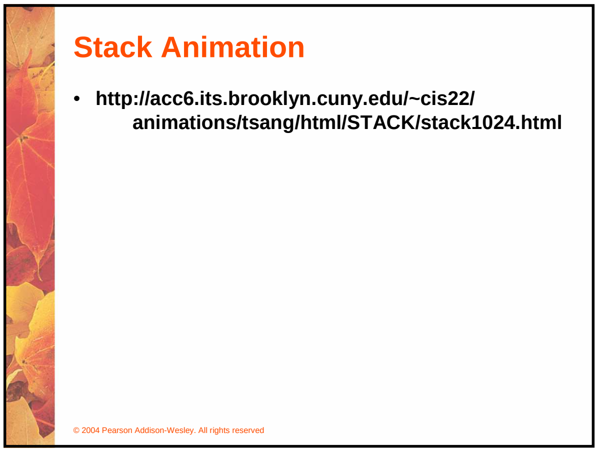
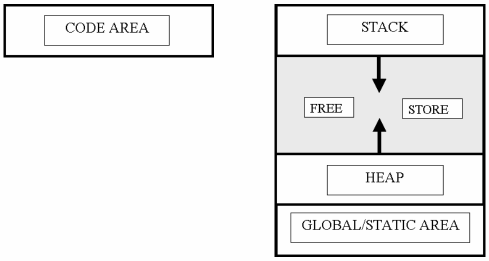


Stack Animation

- http://acc6.its.brooklyn.cuny.edu/~cis22/animations/tsang/html/STACK/stack1024.html

Memory Organization



CODE AREA

STACK

FREE

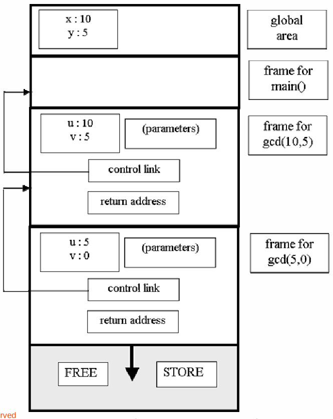
STORE

HEAP

GLOBAL/STATIC AREA

[http://www.dickinson.edu/~ziantzl/cs500_fall2002/Lec10/handouts_files/]

The stack during a recursive call to gcd



x : 10
y : 5

global area

frame for main()

u : 10
v : 5

(parameters)

control link

return address

frame for gcd(10,5)

u : 5
v : 0

(parameters)

control link

return address

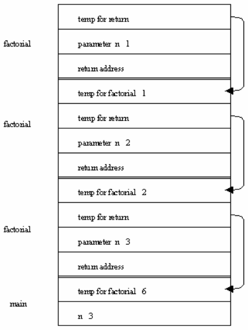
frame for gcd(5,0)

FREE

STORE

[http://www.dickinson.edu/~ziantzl/cs500_fall2002/Lec10/handouts_files/image003.gif]

The stack during a recursive call to factorial



factorial

temp for return

parameter n 1

return address

temp for factorial 1

factorial

temp for return

parameter n 2

return address

temp for factorial 2

factorial

temp for return

parameter n 3

return address

temp for factorial 6

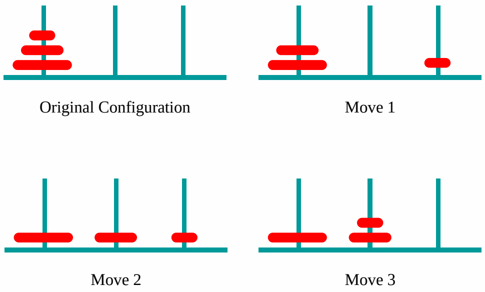
main

n 3

Towers of Hanoi

- **The *Towers of Hanoi* is a puzzle made up of three vertical pegs and several disks that slide on the pegs**
- **The disks are of varying size, initially placed on one peg with the largest disk on the bottom with increasingly smaller ones on top**
- **The goal is to move all of the disks from one peg to another under the following rules:**
  - **We can move only one disk at a time**
  - **We cannot move a larger disk on top of a smaller one**

Towers of Hanoi



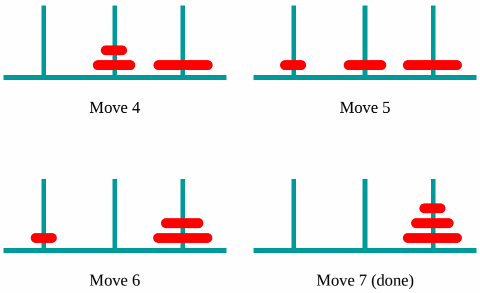
Original Configuration

Move 1

Move 2

Move 3

Towers of Hanoi



Move 4

Move 5

Move 6

Move 7 (done)

Animation of the Towers of Hanoi

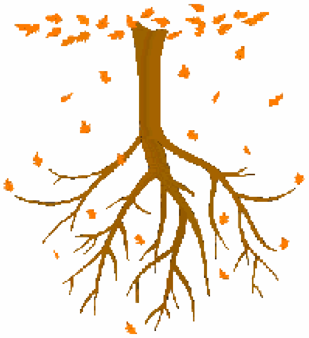
**http://www.cs.concordia.ca/~twang/
WangApr01/RootWang.html**

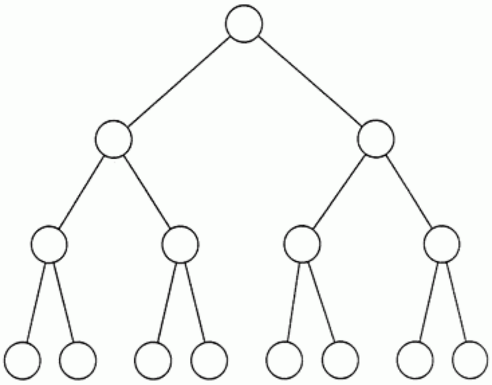
Mystery Recursion on HW8

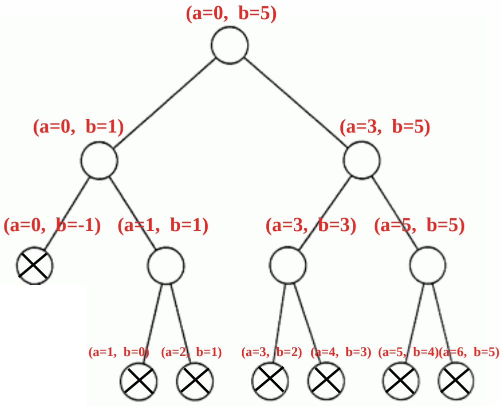
```
 public static void mystery1(int a, int b)
{
        if (a <= b) {
            int m = (a + b) / 2;
            System.out.print(m + " ");
            mystery1(a, m-1);
            mystery1(m+1, b);
        }
}

public static void main(String[] args) {
        mystery1(0, 5);
        System.out.println();
}
```

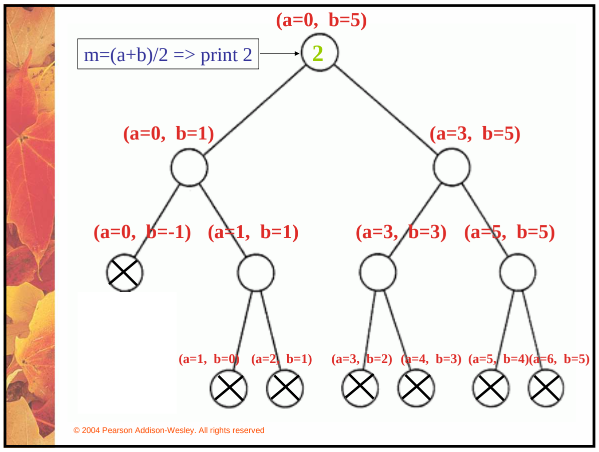
Think of recursion as a tree …

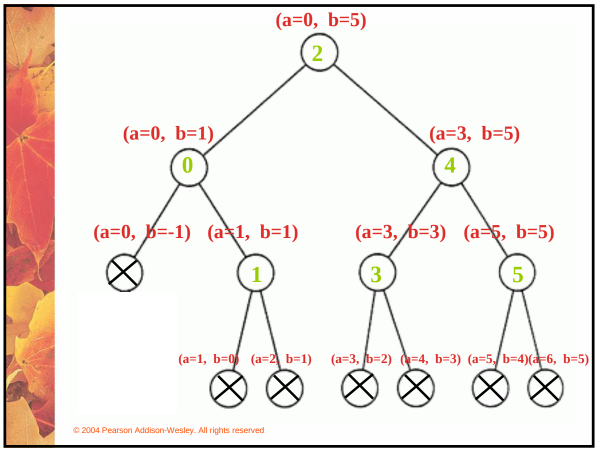
… an upside down tree

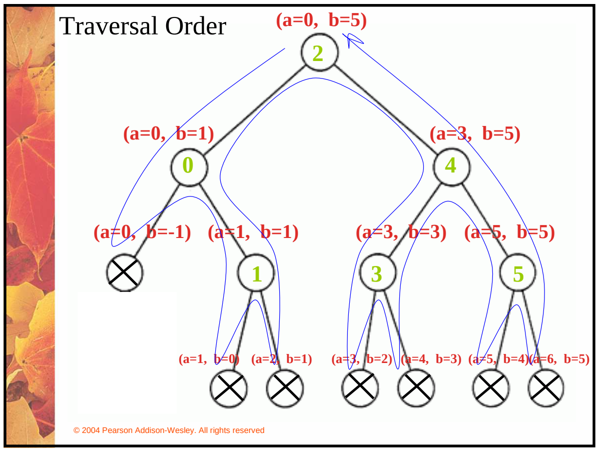


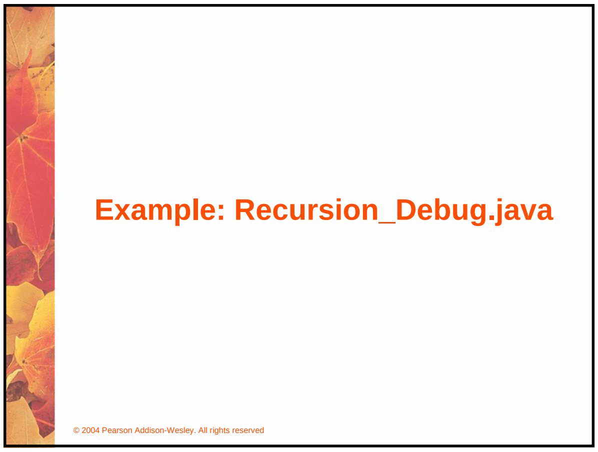


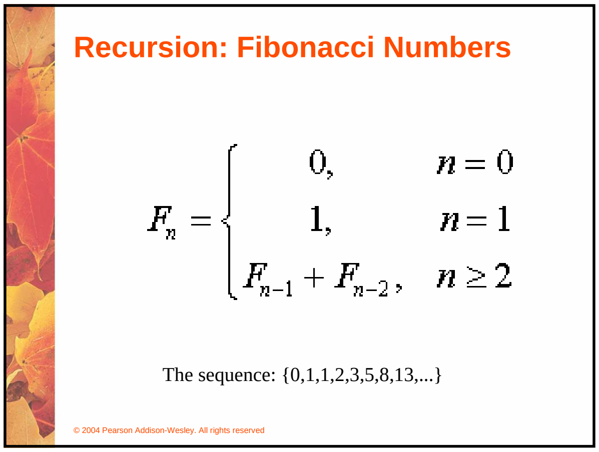






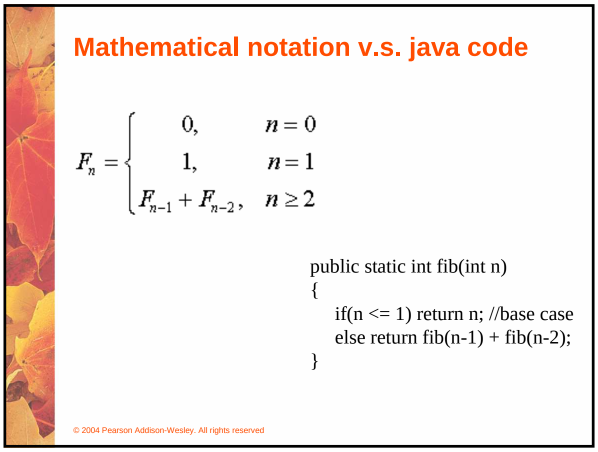

Traversal Order


Example: Recursion_Debug.java


Recursion: Fibonacci Numbers

$$F_n = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ F_{n-1} + F_{n-2}, & n \geq 2 \end{cases}$$

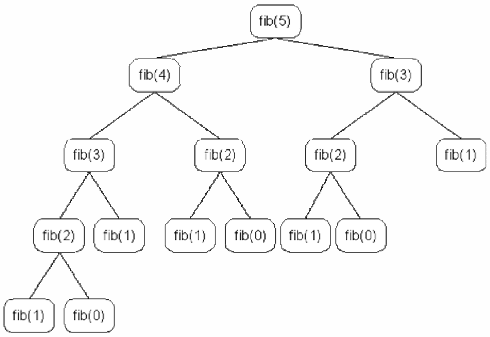
The sequence: {0,1,1,2,3,5,8,13,...}


Mathematical notation v.s. java code

$$F_n = \begin{cases} 0, & n = 0 \\ 1, & n = 1 \\ F_{n-1} + F_{n-2}, & n \geq 2 \end{cases}$$

```
public static int fib(int n)
{
    if(n <= 1) return n; //base case
    else return fib(n-1) + fib(n-2);
}
```

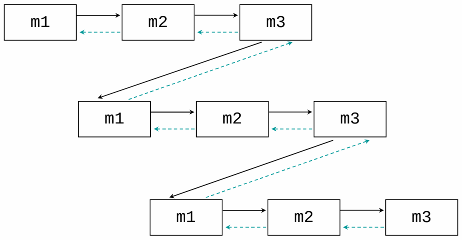
Execution Trace



Indirect Recursion

- A method invoking itself is considered to be *direct recursion*
- A method could invoke another method, which invokes another, etc., until eventually the original method is invoked again
- For example, method `m1` could invoke `m2`, which invokes `m3`, which in turn invokes `m1` again
- This is called *indirect recursion*, and requires all the same care as direct recursion
- It is often more difficult to trace and debug

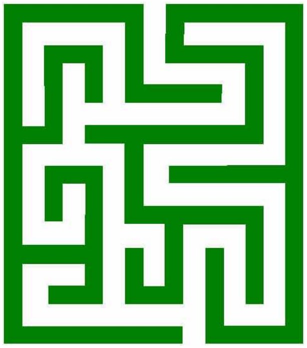
Indirect Recursion

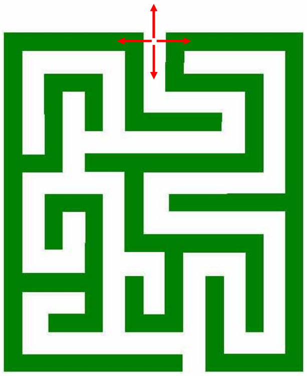


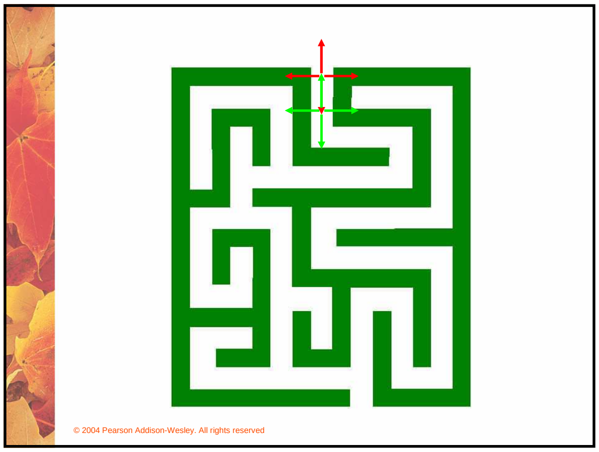
Maze Traversal

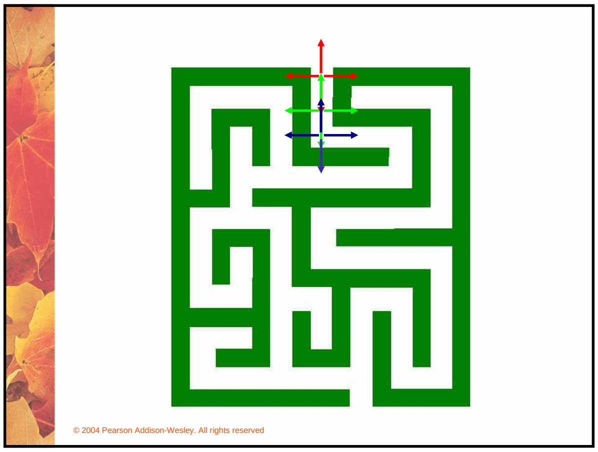
- We can use recursion to find a path through a maze
- From each location, we can search in each direction
- Recursion keeps track of the path through the maze
- The base case is an invalid move or reaching the final destination
- See `MazeSearch.java` (page 583)
- See `Maze.java` (page 584)

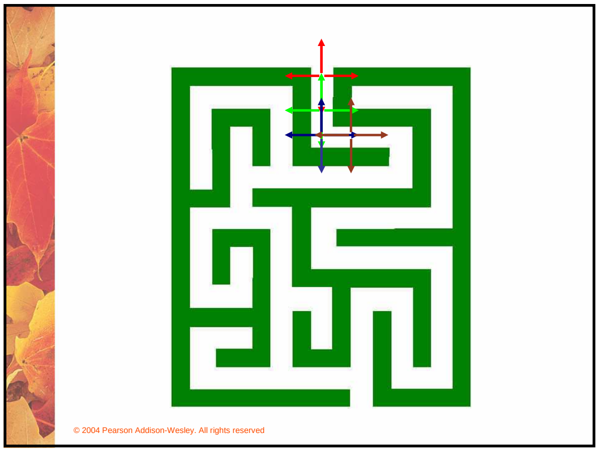
Traversing a maze

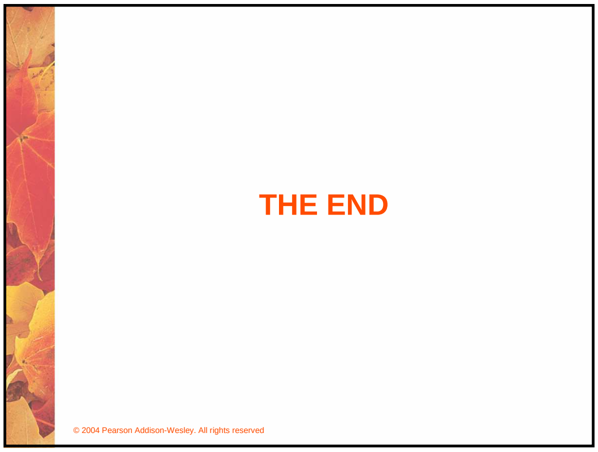












THE END