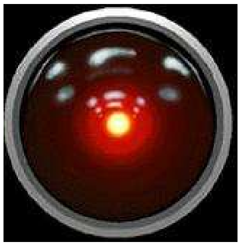


HCI/ComS 575X:
Computational Perception

Instructor: Alexander Stoytchev
http://www.cs.iastate.edu/~alex/classes/2007_Spring_575X/

Image Filtering

January 24, 2007

Reading Today's Lecture

- Jain, Kasturi, and Schunck (1995). Machine Vision, ``Chapter 4: Image Filtering,'' McGraw-Hill, pp. 112-139.

Reading for Next Time

- Burt and Adelson (1983). ``The Laplacian Pyramid as a Compact Image Code,'' IEEE Transactions on Communications, vol. 31(4), pp. 532-540.

- Posted on the reading web page
- (not WebCT)

Some Questions from Last Lecture

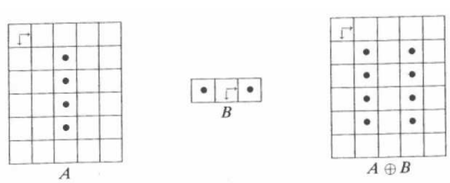
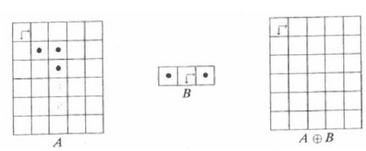


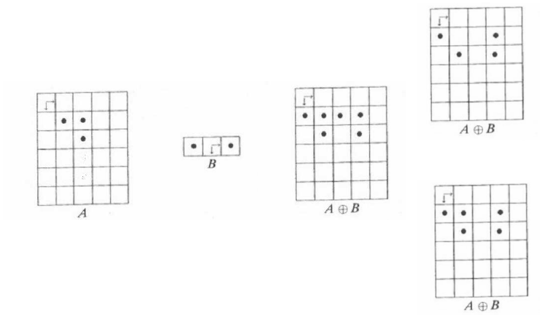
Figure 5.2 Set A dilated by a structuring element B that does not contain the origin. As a result, the dilated set is not even guaranteed to have a single point in common with A. However, there are always translations of $A \oplus B$ that can contain A.

[Haralick and Shapiro (1993). "Computer and Robot Vision," Ch. 5]

What would be the result?



Which one is correct?

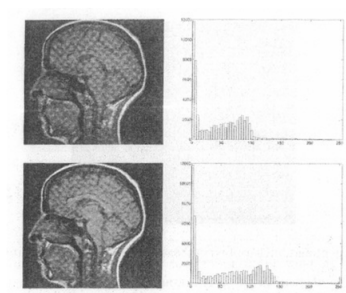


Let's verify this using matlab

Histogram Modification

- Scaling
- Equalization
- Normalization

Histogram Scaling



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

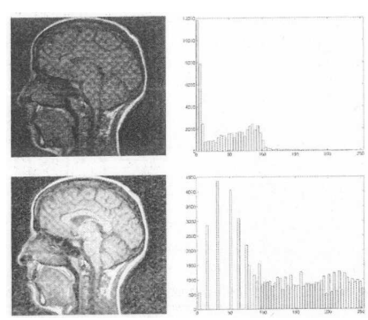
Histogram Scaling (Contrast Stretching)

the pixels in the range $[a, b]$ are expanded to fill the range $[z_1, z_k]$

$$
\begin{aligned}
z' &= \frac{z_k - z_1}{b - a}(z - a) + z_1 \\
&= \frac{z_k - z_1}{b - a} z + \frac{z_1 b - z_k a}{b - a}.
\end{aligned}
$$

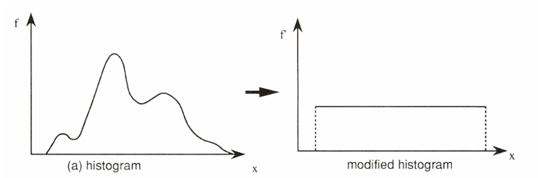
[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Histogram Equalization



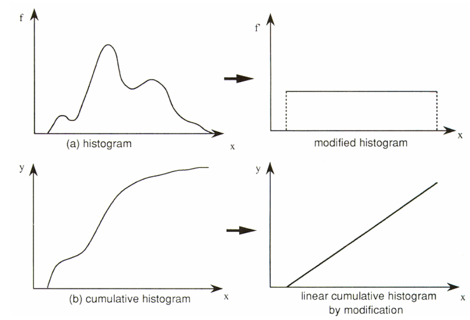
[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Histogram Equalization



[http://www.profc.udec.cl/~gabriel/tutoriales/rsnote/cp10/cp10-3.htm]

Histogram Equalization



[http://www.profc.udec.cl/~gabriel/tutoriales/rsnote/cp10/cp10-3.htm]

Histogram Equalization

The number of pixels At level z_i in the old histogram

The number of pixels At level z_1 in the new histogram

$$\sum_{i=1}^{k_1-1} p_i \leq q_1 < \sum_{i=1}^{k_1} p_i.$$

The pixels at levels $z_1, z_2, \ldots, z_{k_1-1}$ map to level z_1 in the new image.

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Histogram Equalization

$$\sum_{i=1}^{k_2-1} p_i \leq q_1 + q_2 < \sum_{i=1}^{k_2} p_i.$$

The next range of pixel values, $z_{k_1}, \ldots, z_{k_2-1}$, maps to level z_2.

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Histogram Normalization

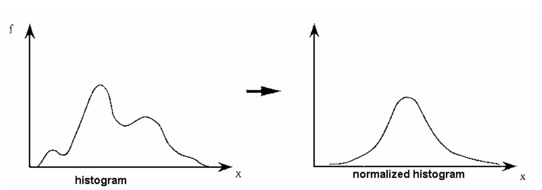
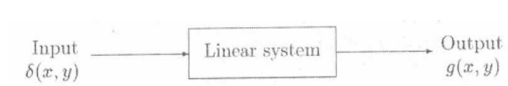


Figure 10.3.2 Normalized histogram

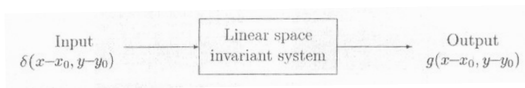
[http://www.profc.udec.cl/~gabriel/tutoriales/rsnote/cp10/cp10-3.htm]

Linear Systems



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

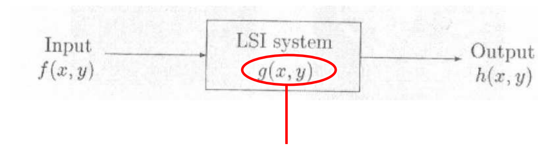
Linear Space Invariant System



A system whose response remains the same irrespective of the position of the input pulse is called a space invariant system.

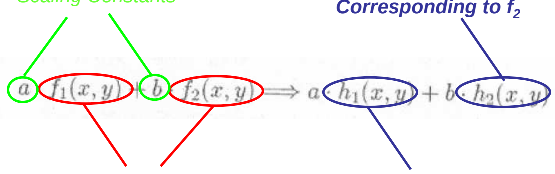
[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Linear Space Invariant System



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

This relation must hold



$$a\, f_1(x,y) + b\, f_2(x,y) \Longrightarrow a\, h_1(x,y) + b\, h_2(x,y)$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Convolution

For such a system the output h(x,y) is the convolution of f(x,y) with the impulse response g(x,y)

$$\begin{aligned} h(x,y) &= f(x,y) \star g(x,y) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\, g(x-x', y-y')\, dx'\, dy'. \end{aligned}$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

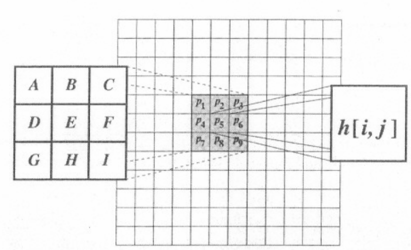
Convolution

$$\begin{aligned} h(x,y) &= f(x,y) \star g(x,y) \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')\, g(x-x', y-y')\, dx'\, dy'. \end{aligned}$$

$$\begin{aligned} h[i,j] &= f[i,j] \star g[i,j] \\ &= \sum_{k=1}^{n}\sum_{l=1}^{m} f[k,l]\, g[i-k, j-l]. \end{aligned}$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

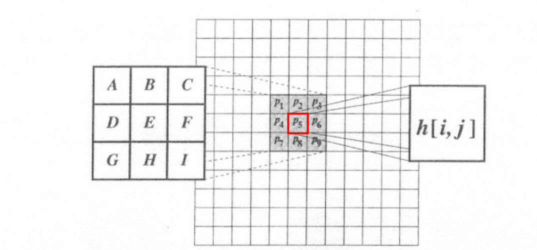
Example of 3x3 convolution mask



$$h[i,j] = A\,p_1 + B\,p_2 + C\,p_3 + D\,p_4 + E\,p_5 + F\,p_6 + G\,p_7 + H\,p_8 + I\,p_9$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Example of 3x3 convolution mask



$$h[i,j] = A\,p_1 + B\,p_2 + C\,p_3 + D\,p_4 + E\,p_5 + F\,p_6 + G\,p_7 + H\,p_8 + I\,p_9$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

In plain words

Convolution is essentially equivalent to computing a weighted sum of image pixels.

Convolution is a linear operation

$$g[i,j] \star \{a_1 h_1[i,j] + a_2 h_2[i,j]\} = a_1\{g[i,j] \star h_1[i,j]\} + a_2\{g[i,j] \star h_2[i,j]\}$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Types of Image Noise

- Salt and Pepper Noise
  - random occurrences of black and white pixels
- Impulse noise
  - Random occurrences of white pixels only
- Gaussian noise
  - Variations of intensity that are drawn from a Gaussian or normal distribution

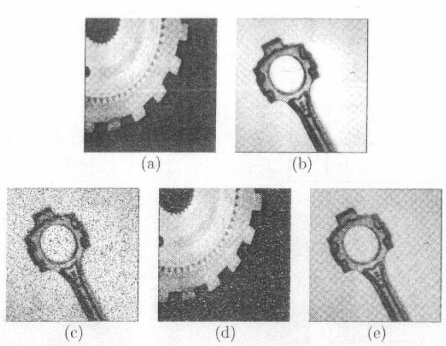


Figure 4.5: Examples of images corrupted by salt and pepper, impulse, and Gaussian noise. (a) & (b) Original images. (c) Salt and pepper noise. (d) Impulse noise. (e) Gaussian noise.

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Mean Filter

- Arbitrary neighborhood

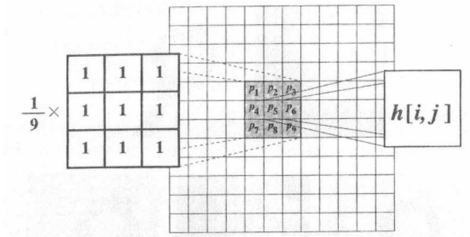
$$h[i,j] = \frac{1}{M} \sum_{(k,l)\in N} f[k,l]$$

- For a 3x3 neighborhood

$$h[i,j] = \frac{1}{9} \sum_{k=i-1}^{i+1} \sum_{k=j-1}^{j+1} f[k,l].$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

3x3 Mean Filter



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

3x3 Linear Smoothing Filter

In general, it is a good idea to have only a single peak in your smoothing filter:

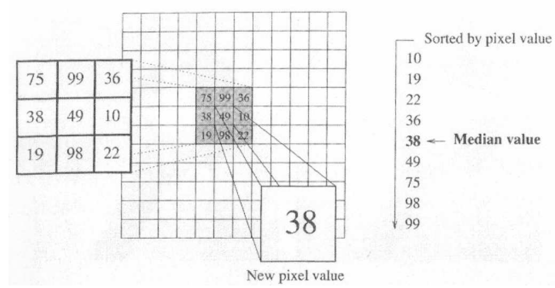
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Median Filter

- Sort the pixels into ascending order by their gray level values
- Select the value of the middle pixel as the new value for pixel [i, j]

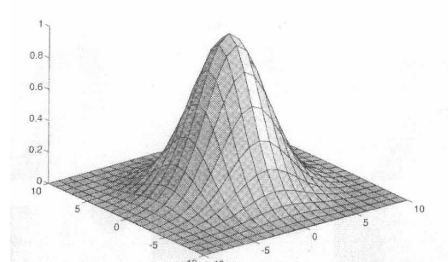
3x3 Median Filter



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Matlab Demo

Gaussian Smoothing



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

The Gaussian Function

- Zero mean 1D Gaussian

$$g(x) = e^{-\frac{x^2}{2\sigma^2}},$$

- Zero mean 2D Gaussian for image processing applications

$$g[i,j] = e^{-\frac{(i^2+j^2)}{2\sigma^2}},$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Gaussian Properties

- Rotationally symmetric in 2D
- Has a single peak
- The width of the filter and the degree of smoothing are determined by sigma
- Large Gaussian filters can be implemented very efficiently using small Gaussian filters

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Rotational Symmetry

- Original formula

$$g[i,j] = e^{-\frac{(i^2+j^2)}{2\sigma^2}}.$$

- Switch to polar coordinates

$$r^2 = i^2 + j^2$$

- Result (does not depend on θ)

$$g(r,\theta) = e^{-\frac{r^2}{2\sigma^2}},$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Gaussian Separability

$$\begin{aligned} g[i,j] \star f[i,j] &= \sum_{k=1}^{m}\sum_{l=1}^{n} g[k,l]\, f[i-k,j-l] \\ &= \sum_{k=1}^{m}\sum_{l=1}^{n} e^{-\frac{(k^2+l^2)}{2\sigma^2}} f[i-k,j-l] \\ &= \sum_{k=1}^{m} e^{-\frac{k^2}{2\sigma^2}} \left\{ \sum_{l=1}^{n} e^{-\frac{l^2}{2\sigma^2}} f[i-k,j-l] \right\}. \end{aligned}$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

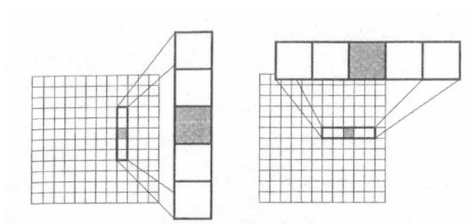
Gaussian Separability

$$\begin{aligned} g[i,j] \star f[i,j] &= \sum_{k=1}^{m}\sum_{l=1}^{n} g[k,l]\, f[i-k,j-l] \\ &= \sum_{k=1}^{m}\sum_{l=1}^{n} e^{-\frac{(k^2+l^2)}{2\sigma^2}} f[i-k,j-l] \\ &= \sum_{k=1}^{m} e^{-\frac{k^2}{2\sigma^2}} \left\{ \sum_{l=1}^{n} e^{-\frac{l^2}{2\sigma^2}} f[i-k,j-l] \right\}. \end{aligned}$$

The convolution of the input image f[i,j] with a vertical 1D Gaussian function

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Cascading Gaussians



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

The convolution of a Gaussian with itself yields a scaled Gaussian with larger sigma

$$
\begin{aligned}
g(x) \star g(x) &= \int_{-\infty}^{\infty} e^{-\frac{\xi^2}{2\sigma^2}} e^{-\frac{(x-\xi)^2}{2\sigma^2}}\, d\xi \\
&= \int_{-\infty}^{\infty} e^{-\frac{(\frac{x}{2}+\xi)^2}{2\sigma^2}} e^{-\frac{(\frac{x}{2}-\xi)^2}{2\sigma^2}}\, d\xi, \quad \xi \rightarrow \xi + \frac{x}{2} \\
&= \int_{-\infty}^{\infty} e^{-\frac{(2\xi^2+\frac{x^2}{2})}{2\sigma^2}}\, d\xi \\
&= e^{-\frac{x^2}{4\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{\xi^2}{\sigma^2}}\, d\xi \\
&= \sqrt{\pi}\sigma e^{-\frac{x^2}{2(\sqrt{2}\sigma)^2}}.
\end{aligned}
$$

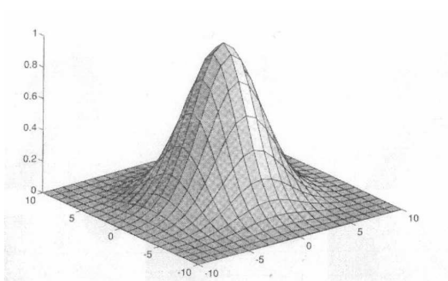
[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Properties

The product of the convolution of two Gaussian functions with a spread σ is a Gaussian function with a spread $\sqrt{2}\sigma$ scaled by the area of the Gaussian filter

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Designing Gaussian Filters



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Pascal's Triangle (Binomial Expansion)

$$(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n.$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

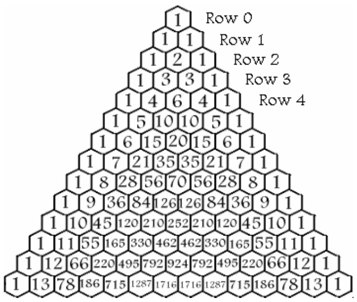
Example: 6 choose 3

For example, [6:3] = (6 * 5 * 4 * 3 * 2 * 1) / (3 * 2 * 1 * 3 * 2 * 1) = 20

Binomial Coefficients

(x+1)^0 = 1
(x+1)^1 = 1 + x
(x+1)^2 = 1 + 2x + x^2
(x+1)^3 = 1 + 3x + 3x^2 + x^3
(x+1)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4
(x+1)^5 = 1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5 .....

Pascal's Triangle



[http://ptri1.tripod.com/]

A Five Point Approximation

1	4	6	4	1

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Another Way: Compute the Weights

- Start with a discrete Gaussian

$$g[i,j] = c\,e^{-\frac{(i^2+j^2)}{2\sigma^2}}$$

- Normalize the weights

$$\frac{g[i,j]}{c} = e^{-\frac{(i^2+j^2)}{2\sigma^2}}$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Example: sigma^2=2, n=7

$[i,j]$	-3	-2	-1	0	1	2	3
-3	.011	.039	.082	.105	.082	.039	.011
-2	.039	.135	.287	.368	.287	.135	.039
-1	.082	.287	.606	.779	.606	.287	.082
0	.105	.368	.779	1.000	.779	.368	.105
1	.082	.287	.606	.779	.606	.287	.082
2	.039	.135	.287	.368	.287	.135	.039
3	.011	.039	.082	.105	.082	.039	.011

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

To keep them all integers

$$\frac{g[3,3]}{k} = e^{-\frac{(3^2+3^2)}{2(2)^2}} = 0.011 \implies k = \frac{g[3,3]}{0.011} = \frac{1.0}{0.011} = 91.$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Integer Weights

$[i,j]$	-3	-2	-1	0	1	2	3
-3	1	4	7	10	7	4	1
-2	4	12	26	33	26	12	4
-1	7	26	55	71	55	26	7
0	10	33	71	91	71	33	10
1	7	26	55	71	55	26	7
2	4	12	26	33	26	12	4
3	1	4	7	10	7	4	1

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Normalization constant

$$\sum_{i=-3}^{3} \sum_{j=-3}^{3} g[i,j] = 1115.$$

$$h[i,j] = \frac{1}{1115}(f[i,j] \star g[i,j])$$

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

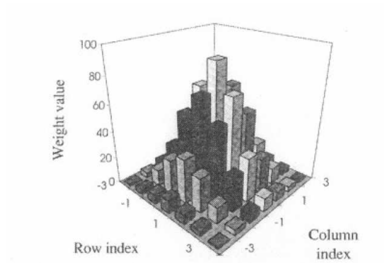
Discrete Gaussian Filters

7x7 Gaussian Mask

1	1	2	2	2	1	1
1	2	2	4	2	2	1
2	2	4	8	4	2	2
2	4	8	16	8	4	2
2	2	4	8	4	2	2
1	2	2	4	2	2	1
1	1	2	2	2	1	1

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

3D Plot of the 7x7 Gaussian



[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

15 x 15 Gaussian Mask

2	2	3	4	5	5	6	6	6	5	5	4	3	2	2
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
6	8	11	13	16	18	19	20	19	18	16	13	11	8	6
6	8	10	13	15	17	19	19	19	17	15	13	10	8	6
5	7	10	12	14	16	17	18	17	16	14	12	10	7	5
5	7	9	11	13	14	15	16	15	14	13	11	9	7	5
4	5	7	9	10	12	13	13	13	12	10	9	7	5	4
3	4	6	7	9	10	10	11	10	10	9	7	6	4	3
2	3	4	5	7	7	8	8	8	7	7	5	4	3	2
2	2	3	4	5	5	6	6	6	5	5	4	3	2	2

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

Properties of Discrete Gaussian Filters

- Step 1: smooth with n x n discrete Gaussian Filter
- Step 2: smooth the intermediary result from Step 1 with m x m discrete Gaussian Filter
- Step 1 + Step 2 are equivalent to smoothing the original with (n+m-1)x(n+m-1) discrete Gaussian Filter

[Jain, Kasturi, and Schunck (1995). Machine Vision, Ch. 4]

THE END