HCI/ComS 575X: Computational Perception
Homework 1 (part 1)
Out: Wed. Jan 14, 2009
Due: Wed. Jan 28, 2009
For extra credit submit by Fri. Jan 16 before 4pm.
(submit it to HCI/ComS 575X basket in VRAC front office.)

Note: Not all assignments are created equal :(

These problems are designed to test your background knowledge in linear algebra and statistics. Please show the intermediary steps of your calculations (not just the final answers).

1. **Calculate the following determinants.**

$$(a) \begin{vmatrix} -1 & 9 \\ 8 & -5 \end{vmatrix} \quad (b) \begin{vmatrix} 3 & -7 \\ 6 & 8 \end{vmatrix} \quad (c) \begin{vmatrix} 3 & 0 & 4 \\ 2 & 3 & 2 \\ 0 & 5 & -1 \end{vmatrix} \quad (d) \begin{vmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{vmatrix}$$

2. **Show that the value of the determinant does not depend on θ.**

$$\begin{vmatrix} sin\theta & -cos\theta & sin\theta - cos\theta \\ cos\theta & sin\theta & sin\theta + cos\theta \\ 0 & 0 & 1 \end{vmatrix}$$

3. **Calculate the inverse of the given matrix.**

$$(a) \begin{bmatrix} 5 & 10 \\ 4 & 7 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix} \quad (d) \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{bmatrix}$$

4. **Solve the following systems of equations. In (b) describe the set of all b for which $Ax = b$ has a solution.**

$$(a) \begin{array}{rcrcr} 5x_1 & + & 7x_2 & = & 3 \\ 2x_1 & + & 4x_2 & = & 1 \end{array} \qquad (b) \begin{array}{rcrcrcr} x_1 & - & 3x_2 & - & 4x_3 & = & b_1 \\ -3x_1 & + & 2x_2 & + & 6x_3 & = & b_2 \\ 5x_1 & - & x_2 & - & 8x_3 & = & b_3 \end{array}$$

5. **Let $u = (3, -1, -5)$, $v = (0, -1, -3)$, and $w = (6, -2, 3)$. Compute:**

$$(a)\ u \cdot w \qquad (b) \|u\|^2 \qquad (c)\ v \times w \qquad (d)\ dist(u, v) = \|u - v\| \qquad (e)\ proj_v u = \frac{u \cdot v}{\|v\|^2} v$$

6. **Let $u = (3, -1, -5)$, $v = (0, -1, -3)$, and $w = (6, -2, 3)$. Consider whether the following problems are possible. Compute AND verify the answer if the problem is possible. Otherwise give a SHORT explanation if the problem is not possible.**

   (a)Find a vector $t \in \Re^3$ that is perpendicular to both v and w.

   (b)Find a vector $t \in \Re^3$ that is perpendicular to u, v and w.

7. **Find an equation for the plane passing through the given points.**

$$(a) \quad P(3,7,4), Q(6,0,1), R(1,1,3) \qquad\qquad (b) \quad P(-1,3,-2), Q(4,5,4), R(-8,5,0)$$

8. **Find the characteristic polynomial and the eigenvalues of the following matrices.**

$$(a) \quad \begin{bmatrix} 7 & 4 \\ -3 & -1 \end{bmatrix} \quad (b) \quad \begin{bmatrix} 4 & 0 & 0 \\ 5 & 3 & 2 \\ -2 & 0 & 2 \end{bmatrix} \quad (c) \quad \begin{bmatrix} 4 & -7 & 0 & 2 \\ 0 & 3 & -4 & 6 \\ 0 & 0 & 3 & -8 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. **In the following problems, find the 3×3 matrices that produce the described composite 2D transformations, using homogenous coordinates.**

   (a)Translate by $(-2, 3)$, and then scale the x-coordinate by .8 and the y-coordinate by 1.2.

   (b)Reflect points through the x-axis, and then rotate $30°$ about the origin.

10. **Given the vector $v = [x, y]^T$, find a rotation matrix R which rotates the vector by $60°$ counter-clockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.**

11. **Find the equation $y = B_0 + B_1 x$ of the least-squares line that best fits the data points (2,1), (5,2), (7,3), (8,3).**

12. **Given the following experimental data points: find the mean, subtract the mean from all data points, construct the sample covariance matrix, and find the principal components of the data. (Hint: (19, 12) is one data point)**

$$\begin{bmatrix} 19 & 22 & 6 & 3 & 2 & 20 \\ 12 & 6 & 9 & 15 & 13 & 5 \end{bmatrix}$$