

# HCI/CprE/ComS 575: Computational Perception

## Homework 1 (part 1)

Out: Wed. Jan 13, 2010

Due: Wed. Jan 27, 2010

For extra credit submit by Fri. Jan 15 before 4pm.

(submit it to the secretary in the VRAC front office.)

Note: Not all assignments are created equal :(

These problems are designed to test your background knowledge in linear algebra and statistics. Please show the intermediary steps of your calculations (not just the final answers).

1. Calculate the following determinants.

$$(a) \begin{vmatrix} \sqrt{2} & \sqrt{6} \\ 4 & \sqrt{3} \end{vmatrix} \quad (b) \begin{vmatrix} a-3 & 5 \\ -3 & a-2 \end{vmatrix} \quad (c) \begin{vmatrix} -1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2 \end{vmatrix} \quad (d) \begin{vmatrix} c & -4 & 3 \\ 2 & 1 & c^2 \\ 4 & c-1 & 2 \end{vmatrix}$$

2. Show that the value of the determinant does not depend on  $\theta$ .

$$\begin{vmatrix} \sin\theta & \cos\theta & 0 \\ -\cos\theta & \sin\theta & 0 \\ \sin\theta - \cos\theta & \sin\theta + \cos\theta & 1 \end{vmatrix}$$

3. Calculate the inverse of the given matrix.

$$(a) \begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix} \quad (b) \begin{bmatrix} -3 & 6 \\ 4 & 5 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad (d) \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

4. Solve the following systems of equations.

$$(a) \begin{cases} x_1 + x_2 = 2 \\ 5x_1 + 6x_2 = 9 \end{cases} \quad (b) \begin{cases} x_1 + 3x_2 + x_3 = 4 \\ 2x_1 + 2x_2 + x_3 = -1 \\ 2x_1 + 3x_2 + x_3 = 3 \end{cases}$$

5. Let  $u = (3, 2, -1)$ ,  $v = (0, 2, -3)$ , and  $w = (2, 6, 7)$ . Compute:

$$(a) u \cdot v \quad (b) \|u\| \quad (c) v \times w \quad (d) u \times (v \times w) \quad (e) (u \times v) \times w$$

6. Find the orthogonal projection of  $u$  on  $a$  (i.e., find  $proj_a u = \frac{u \cdot a}{\|a\|^2} a$ )

(a)  $u = (6, 2), a = (3, -9)$

(b)  $u = (3, 1, -7), a = (1, 0, 5)$

7. Find the angle  $\theta$  between the two vectors  $u$  and  $v$ .

(a)  $u = (2, 3), v = (5, -7)$

(b)  $u = (1, 0, 0), v = (1, 1, 1)$

8. Find an equation for the plane passing through the given points.

(a)  $P(-4, -1, -1), Q(-2, 0, 1), R(-1, -2, -3)$       (b)  $P(5, 4, 3), Q(4, 3, 1), R(1, 5, 4)$

9. Find all values of  $\lambda$  for which the determinant of the matrix is equal to 0.

(a)  $\begin{vmatrix} \lambda - 2 & 1 \\ -5 & \lambda + 4 \end{vmatrix}$       (b)  $\begin{vmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{vmatrix}$

10. Find the eigenvalues of the following matrices.

(a)  $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$       (b)  $\begin{bmatrix} 3 & 0 & -5 \\ \frac{1}{5} & -1 & 0 \\ 1 & 1 & -2 \end{bmatrix}$

11. Given the vector  $v = [x, y]^T$  find a rotation matrix  $R$  which rotates the vector by  $60^\circ$  counter-clockwise. Give the values for the vector  $v' = [x', y']^T$  in terms of  $x$  and  $y$ , where  $v' = Rv$ .

12. Write down the analytical form of the 3D rotation matrices  $R_x$ ,  $R_y$ , and  $R_z$  which rotate a vector about the  $X$ ,  $Y$ , and  $Z$  axis, respectively.

13. By example, show that the 3D rotation matrices are not commutative.