

Binary Number Representation and Arithmetic

Assigned Date: Seventh Week
Due Date: Mar. 3, 2014

P1. (6 points) How many bits are required to represent each of the following sets of integers to represent unsigned integers in binary?

- (a) The integers from 0 to 255 inclusively.
- (b) The integers from 0 to 4,095 inclusively.
- (c) The integers from 0 to 1,234,567 inclusively.

P2. (6 points) How large a value can be represented by each of the unsigned binary quantities?

- (a) A 6-bit quantity.
- (b) A 10-bit quantity.
- (c) A 16-bit quantity.

P3. (8 points) Convert each of the following binary numbers into decimal. Assume these quantities represent unsigned integers.

- (a) 1011
- (b) 10010
- (c) 1101010
- (d) 10000000

P4. (8 points) Convert each of the following decimal numbers into binary.

- (a) 6
- (b) 12
- (c) 100
- (d) 511

P5. (4 points) Suppose a jogger wants to use her ten fingers to count laps as she circles a track. Each finger can be in two different states to represent a binary digital. How many laps can she conveniently count? Briefly justify your answer.

P6. (6 points) How many trinary (base 3) digits are required to represent numbers in the following ranges?

- (a) The integers from 0 to 255 inclusively.
- (b) The integers from 0 to 4,095 inclusively.
- (c) The integers from 0 to 1,234,567 inclusively.

P7. (6 points) Convert each of the following binary numbers into hexadecimal.

- (a) 1011
- (b) 10010
- (c) 1101010

P8 (6 points) Convert each of the following hexadecimal numbers into binary.

- (a) 3A
- (b) 2B3
- (c) EDCBA

Binary Number Representation and Arithmetic

Assigned Date: Seventh Week
Due Date: Mar. 3, 2014

P9. (6 points) Convert each of the following decimal numbers into hexadecimal.

- (a) 111
- (b) 151
- (c) 511

P10. (6 points) Convert each of the following hexadecimal numbers into decimal.

- (a) 3A
- (b) 6B
- (c) 1F6

P11. (12 points) An expedition to Mars found the ruins of a civilization. The explorers were able to translate the mathematical equations:

$$5x^2 - 50x + 125 = 0$$

with the solutions: $x = 5$ and $x = 8$.

The $x = 5$ solution seemed okay, but $x = 8$ was puzzling. The problem should be because Martians were using a non-decimal number system. Therefore, "50" is not fifty, but "50" in base b ($50_b = 5 \times b + 0 \times 1 = 5b$). The explorers reflected on the way in which Earth's number system developed. How many fingers would you say the Martians had? *Hint:* What should be the value of the base b such that both 5 and 8 are solutions of the equation?

P12. (6 points) What is the value represented by the bit string 101101 if:

- (a) it is in sign-and-magnitude representation?
- (b) it is in 1's complement representation?
- (c) it is in 2's complement representation?

P13. (6 points) Negate the following binary numbers in 4-bit 2's complement representation:
(Remark: Negate means you find the negative of the number.)

- (a) 0001
- (b) 1010
- (c) 1001

P14. (6 points) Give the 4-bit 2's complement representation for the following decimal numbers:

- (a) -7
- (b) -1
- (c) 7

P15. (8 points) Assume the following numbers are represented as 4-bit words in 2's complement form. Perform the following operations and identify, in each case, whether or not an overflow occurs:

$$\begin{array}{r} (a) \quad 1111 \\ + \quad 0001 \\ \hline \end{array}$$

$$\begin{array}{r} (b) \quad 1000 \\ + \quad 1111 \\ \hline \end{array}$$

$$\begin{array}{r} (c) \quad 1111 \\ - \quad 0001 \\ \hline \end{array}$$

$$\begin{array}{r} (d) \quad 1000 \\ - \quad 1111 \\ \hline \end{array}$$