

Binary Number Representation and Arithmetic

Assigned Date: Seventh Week
Due Date: Oct. 12, 2015

P1. (6 points) How many bits are required to represent each of the following sets of integers to represent unsigned integers in binary?

- (a) The integers from 0 to 511 inclusively.
- (b) The integers from 0 to 2,047 inclusively.
- (c) The integers from 0 to 1,234,567 inclusively.

P2. (6 points) How large a value can be represented by each of the unsigned binary quantities?

- (a) A 8-bit quantity.
- (b) A 12-bit quantity.
- (c) A 20-bit quantity.

P3. (8 points) Convert each of the following binary numbers into decimal. Assume these quantities represent unsigned integers.

- (a) 1110
- (b) 10110
- (c) 1001111
- (d) 10000000

P4. (8 points) Convert each of the following decimal numbers into binary.

- (a) 10
- (b) 15
- (c) 201
- (d) 1023

P5. (4 points) Suppose a jogger wants to use her five fingers to count laps as she circles a track. Each finger can be in two different states to represent a binary digit. How many laps can she conveniently count? Briefly justify your answer.

P6. (6 points) How many ternary (base 3) digits are required to represent numbers in the following ranges?

- (a) The integers from 0 to 255 inclusively.
- (b) The integers from 0 to 4,095 inclusively.
- (c) The integers from 0 to 1,234,567 inclusively.

P7. (6 points) Convert each of the following binary numbers into hexadecimal.

- (a) 1011
- (b) 11111
- (c) 1011101

P8 (6 points) Convert each of the following hexadecimal numbers into binary.

- (a) D4
- (b) 8E3
- (c) ABCDEF

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P9. (6 points) Convert each of the following decimal numbers into hexadecimal.

- (a) 123
- (b) 210
- (c) 1023

P10. (6 points) Convert each of the following hexadecimal numbers into decimal.

- (a) 5E
- (b) B2
- (c) 3D8

P11. (12 points) An expedition to Mars found the ruins of a civilization. The explorers were able to translate the mathematical equations:

$$5x^2 - 50x + 125 = 0$$

with the solutions: $x = 5$ and $x = 8$.

The $x = 5$ solution seemed okay, but $x = 8$ was puzzling. The problem should be because Martians were using a non-decimal number system. Therefore, "50" is not fifty, but "50" in base b ($50_b = 5 \times b + 0 \times 1 = 5b$). The explorers reflected on the way in which Earth's number system developed. How many fingers would you say the Martians had? *Hint:* What should be the value of the base b such that both 5 and 8 are solutions of the equation?

P12. (6 points) What is the value represented by the bit string 110101 if:

- (a) it is in sign-and-magnitude representation?
- (b) it is in 1's complement representation?
- (c) it is in 2's complement representation?

P13. (6 points) Convert the following decimal numbers to binary using 6-bit 2's complement representation.

- a) -16_{10} b) 13_{10} c) -3_{10} d) -10_{10} e) 26_{10} f) -31_{10}

P14. (6 points) Answer the following:

- (a) What advantage does 2's complement have over 1's complement?
- (b) If you want to write the number 710 using 2's complement representation, what do you need to do?
- (c) If you want to write the number -710 using 2's complement representation, what do you need to do?

P15. (8 points) Assume the following numbers are represented as 4-bit words in 2's complement form. Perform the following operations and identify, in each case, whether or not an overflow occurs:

$$\begin{array}{r} \text{(a)} \quad 1111 \\ + \quad 0001 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(b)} \quad 1000 \\ + \quad 1110 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(c)} \quad 1111 \\ - \quad 0001 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(d)} \quad 1000 \\ - \quad 1101 \\ \hline \end{array}$$