

Arithmetic Circuits and Combinational-Circuit Building Blocks

Assigned Date: Eighth Week
Due Date: Oct. 19, 2015

P1. (6 points) Negate the following binary numbers in 4-bit 2's complement representation:
(Remark: Negate means you find the negative of the number.)

- (a) 0001
- (b) 1100
- (c) 0111

P2. (20 points) Consider the addition of the two n-bit 2's complement numbers $X=x_{n-1}x_{n-2}\dots x_1x_0$ and $Y=y_{n-1}y_{n-2}\dots y_1y_0$. Suppose the sum is $s_{n-1}s_{n-2}\dots s_1s_0$ and the carry is $c_n c_{n-1}\dots c_2 c_1$.

- (a) If X is positive, Y is negative, and $c_{n-1}=0$, what should be the values of c_n and s_{n-1} ? Will overflow occur?
- (b) If X is negative, Y is negative, and $c_{n-1}=0$, what should be the values of c_n and s_{n-1} ? Will overflow occur?
- (c) Following the idea in part (a) and (b), please construct a truth table list the values of c_n and s_{n-1} for all combinations of the sign of X, the sign of Y, and the value of c_{n-1} . For each combination, please also state if overflow occurs or not.
- (d) Based on the truth table in part (c), prove that $\text{Overflow} = c_n \oplus c_{n-1}$.

P3. (6 points) Give the 4-bit 2's complement representation for the following decimal numbers:

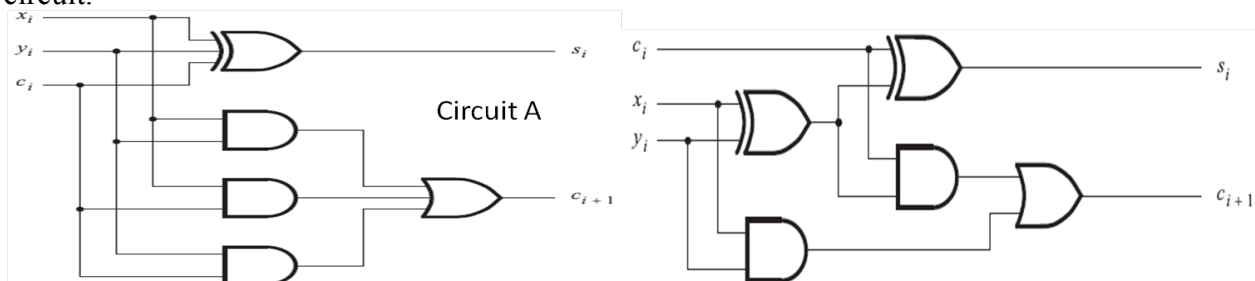
- (a) -6
- (b) -1
- (c) 6

P4. (10 points) Design a circuit to add 1 to a given n-bit number (i.e., design an increment-by-1 circuit) using n half-adders.

P5. (10 points) Using IEEE 754 SINGLE-PRECISION FLOATING-POINT FORMAT

- A) Represent the decimal number 13.5 in IEEE 754 single-precision floating-point format.
- B) What is the decimal value of the following IEEE 754 single-precision floating-point number? 10111111 00101000 00000000 00000000

P6. (10 points) Prove that the following two circuits are different representations of the full-adder circuit.



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- P7. (10 points) Consider constructing a $2^n \times 1$ multiplexer using only 2×1 multiplexers, with n being a positive integer.
- (a) How many 2×1 multiplexers would a $2^n \times 1$ multiplexer require? Give an answer in terms of n .
 - (b) Design an 8×1 multiplexer ($8=2^3$) using a minimal number of 2×1 multiplexers. Please label all signals clearly.
- P8. (18 points) Consider a function F with 4 bits of input A_3, A_2, A_1, A_0 such that the output of F is 1 if the unsigned binary number represented by $A_3A_2A_1A_0$ is integer divisible by 3 or 7 (i.e., 0, 3, 6, 7, 9, 12, 14 or 15). Otherwise, the output of F is 0.
- (a) Write the truth table for F .
 - (b) Implement F using a 16×1 MUX and nothing else.
 - (c) Implement F using an 8×1 MUX, some AND gates, some OR gates, and some NOT gates.
- P9. (10 points) Given a supply of 2-to-4 decoders, show how to get a 4-to-16 decoder circuit. Assume each of the 2-to-4 decoders has an ENABLE input (ENABLE = 1 enables the decoder), but you need not include an enable capability on the 4-to-16 decoder circuit.