

P1. (10 points) Bob needs to use a 3-bit up-counter. However, he only has a 4-bit synchronous down-counter and several NOT gates. He is NOT allowed to modify the internal structure of the down-counter. How can he construct the 3-bit up-counter using only the devices he has?

P2. (30 points) Design a modulo-6 counter, which counts in the sequence 0, 1, 2, 3, 4, 5, 0, 1,... The counter counts the clock pulses if its enable input  $w$  is equal to 1, otherwise it does not increment its count.

- Use D flip-flops in your circuit and include the state diagram, state-assigned table, next-state expressions and output expressions all clearly labeled. Let  $y_2$ ,  $y_1$ , and  $y_0$  be the current state values.
- Repeat Problem 1 using JK flip-flops. Work through the entire design process rather than simply converting the D flip-flops to JK flip-flops. Use the same state diagram.
- Repeat Problem 1 using T flip-flops. Work through the entire design process rather than simply converting the D flip-flops to T flip-flops. Use the same state diagram.

P3. (10 points) Derive the circuits that implement the state tables shown below. Compare the costs of these circuits.

Present state	Next state				Output $z$
	$DN = 00$	01	10	11	
S1	S1	S3	S2	—	0
S2	S2	S4	S5	—	0
S3	S3	S6	S7	—	0
S4	S1	—	—	—	1
S5	S3	—	—	—	1
S6	S6	S8	S9	—	0
S7	S1	—	—	—	1
S8	S1	—	—	—	1
S9	S3	—	—	—	1

Present state	Next state				Output $z$
	$DN = 00$	01	10	11	
S1	S1	S3	S2	—	0
S2	S2	S4	S5	—	0
S3	S3	S2	S4	—	0
S4	S1	—	—	—	1
S5	S3	—	—	—	1

P4. (10 points) Consider the state machine specified by the following state transition table.

Current		Input $I$	Next	
$X$	$Y$		$X$	$Y$
0	0	0	1	1
0	0	1	0	1
0	1	0	0	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0

- (a) Draw the state transition diagram of the machine.  
(b) Write two next-state expressions for X and Y that will implement the transitions of the state machine. Make your expressions as simple as possible, and use XOR gates and NOT gates only.  
(c) Implement the state machine using D flip-flops, XOR gates, and NOT gates.  
(d) Suppose the machine is initially in 00 (i.e.,  $X=0$  and  $Y=0$ ). Indicate for each input sequence below, the state the machine is in after the last digit has been read in. Assume the digits are read in from left to right.
- a. 11111                      b. 1001110000101100                      c. 436 1s followed by 325 0s

P5. (10 points) Deduce the purpose of a state machine with the next state logic and output logic given below. Let X, Y, and Z be the values of the three D flip-flops,  $X_{next}$ ,  $Y_{next}$ , and  $Z_{next}$  be the next state expressions, and ABC be the three-bit output of the circuit.

- (a) Construct the state assigned table from the expression.  
(b) Draw the state diagram from the table.  
(c) Explain in one sentence the functionality of the circuit.

$$X_{next} = X'YZ + XY' + XZ'$$

$$Y_{next} = Y'Z + YZ'$$

$$Z_{next} = Z'$$

$$A = X'$$

$$B = Y$$

$$C = Y + Z$$

P6. (10 points) A state machine has one input P in addition to the clock input and one output Q. The value of Q is 1 if the total number of 1's in the sequence of input P is either a multiple of 2 or a multiple of 3. Otherwise, the output value Q is 0. Draw a state transition diagram for the state machine using as few states as possible.

P7. (10 points) Derive a minimal state table for an FSM that acts as a three-bit parity generator. For every three bits that are observed on the input w during three consecutive clock cycles, the FSM generates the parity bit  $p = 1$  if and only if the number of 1s in the three-bit sequence is odd.

P8. (10 points) A state machine has one input P in addition to the clock input and one output Q. The value of Q is 1 if the total number of 1's and the total number of 0's in the sequence of input P are both even. Otherwise, the output value Q is 0. Draw a state transition diagram for the state machine using as few states as possible.