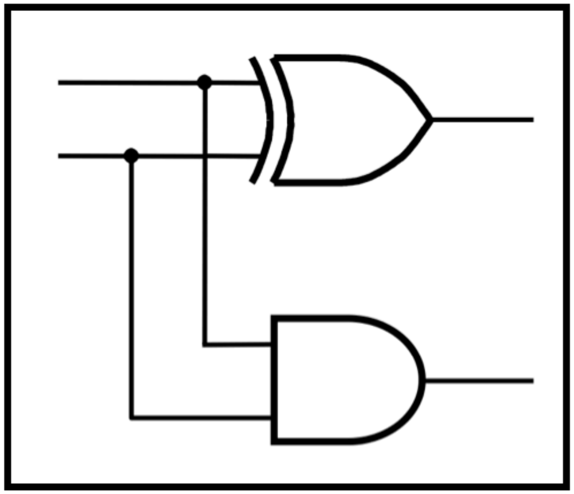


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Synthesis
Using AND, OR, and NOT Gates

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW2 is due on Wednesday Sep 9 @ 4pm

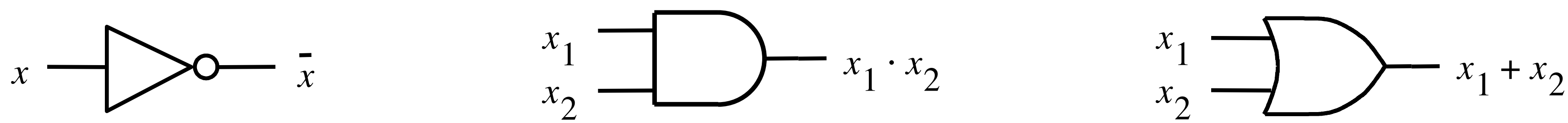
- Please write clearly on the first page (in block capital letters) the following three things:
  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter

Administrative Stuff

- Next week we will start with Lab2
- It will be graded!
- Print the answer sheet for that lab and do the prelab at home. Otherwise you'll lose 20% of your grade for that lab.

Quick Review

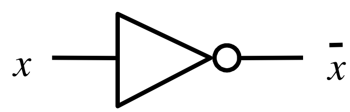
The Three Basic Logic Gates



NOT gate AND gate OR gate

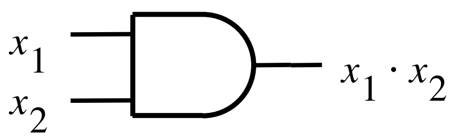
[Figure 2.8 from the textbook]

Truth Table for NOT



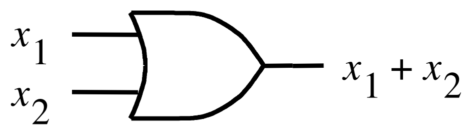
x	$\overline{x}$
0	1
1	0

Truth Table for AND



x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for OR



x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

Truth Tables for AND and OR

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1
		AND	OR

[Figure 2.6b from the textbook]

Operator Precedence

- In regular arithmetic and algebra multiplication takes precedence over addition

- This is also true in Boolean algebra

Operator Precedence
(three different ways to write the same)

$$x_1 \cdot x_2 + \overline{x}_1 \cdot \overline{x}_2$$

$$(x_1 \cdot x_2) + ((\overline{x}_1) \cdot (\overline{x}_2))$$

$$x_1 x_2 + \overline{x}_1 \overline{x}_2$$

DeMorgan's Theorem

15a. $\overline{x \bullet y} = \overline{x} + \overline{y}$

15b. $\overline{x + y} = \overline{x} \bullet \overline{y}$

Function Synthesis

Synthesize the Following Function

x_1	x_2	$f(x_1,x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

1) Split the function into 4 functions

x_1	x_2	$f_{00}(x_1,x_2)$	$f_{01}(x_1,x_2)$	$f_{10}(x_1,x_2)$	$f_{11}(x_1,x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

2) Write Expressions for all four

x_1	x_2	$f_{00}(x_1,x_2)$	$f_{01}(x_1,x_2)$	$f_{10}(x_1,x_2)$	$f_{11}(x_1,x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

x_1x_2 $\quad$ $\bar{x}_1x_2$ $\quad$ 0 $\quad$ $\bar{x}_1\bar{x}_2$

3) Then just add them together

x_1	x_2	$f_{00}(x_1,x_2)$	$f_{01}(x_1,x_2)$	$f_{10}(x_1,x_2)$	$f_{11}(x_1,x_2)$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	0	0
1	1	0	0	0	1

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 x_2 + 0 + \bar{x}_1 \bar{x}_2$$

A function to be synthesized

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

[Figure 2.19 from the textbook]

Let's look at it row by row.
How can we express the last row?

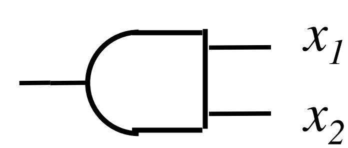
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	$\boxed{1}$

Let's look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	
1	0	0	
1	1	$\boxed{1}$	$x_1 x_2$

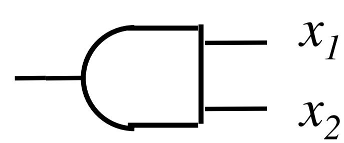
Let's look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



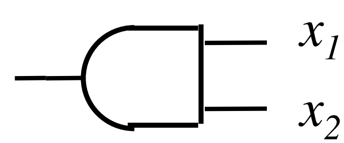
What about this row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



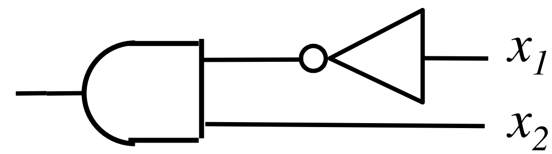
What about this row?

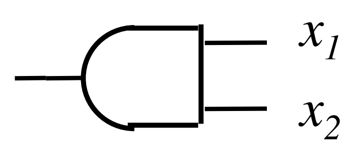
x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	$\bar{x}_1 x_2$
1	0	0	
1	1	1	



What about this row?

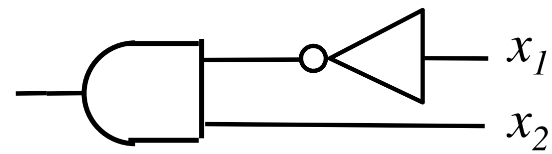
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

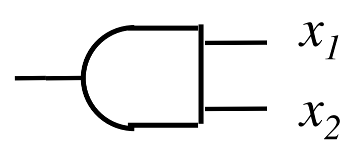




What about the first row?

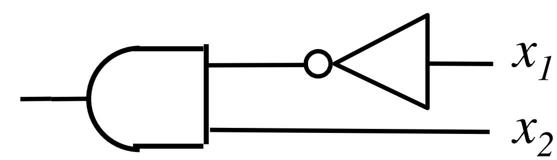
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

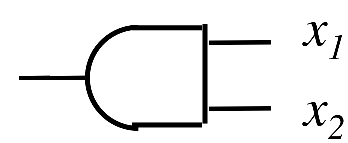




What about the first row?

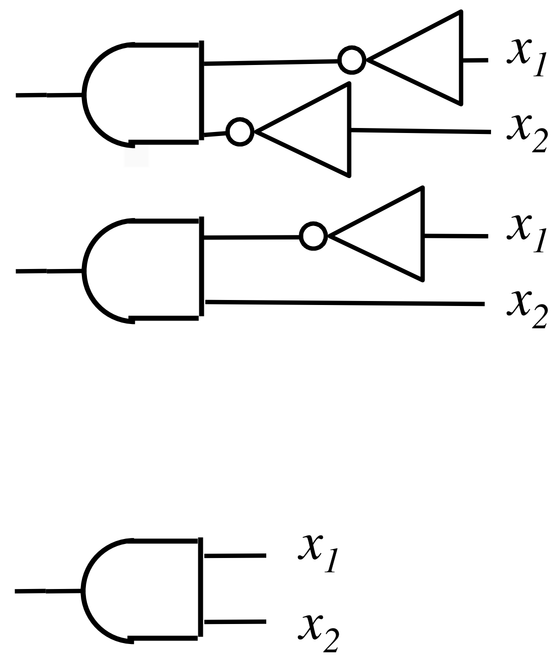
x_1	x_2	$f(x_1, x_2)$	
0	0	$\boxed{1}$	$\bar{x}_1\bar{x}_2$
0	1	1	
1	0	0	
1	1	1	





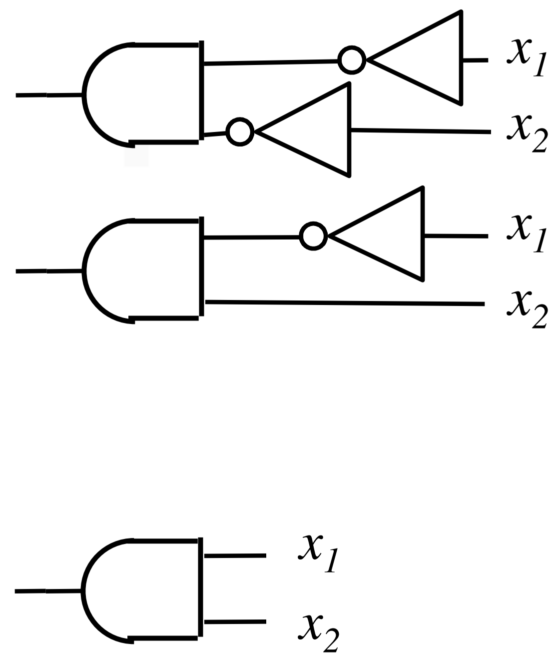
What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

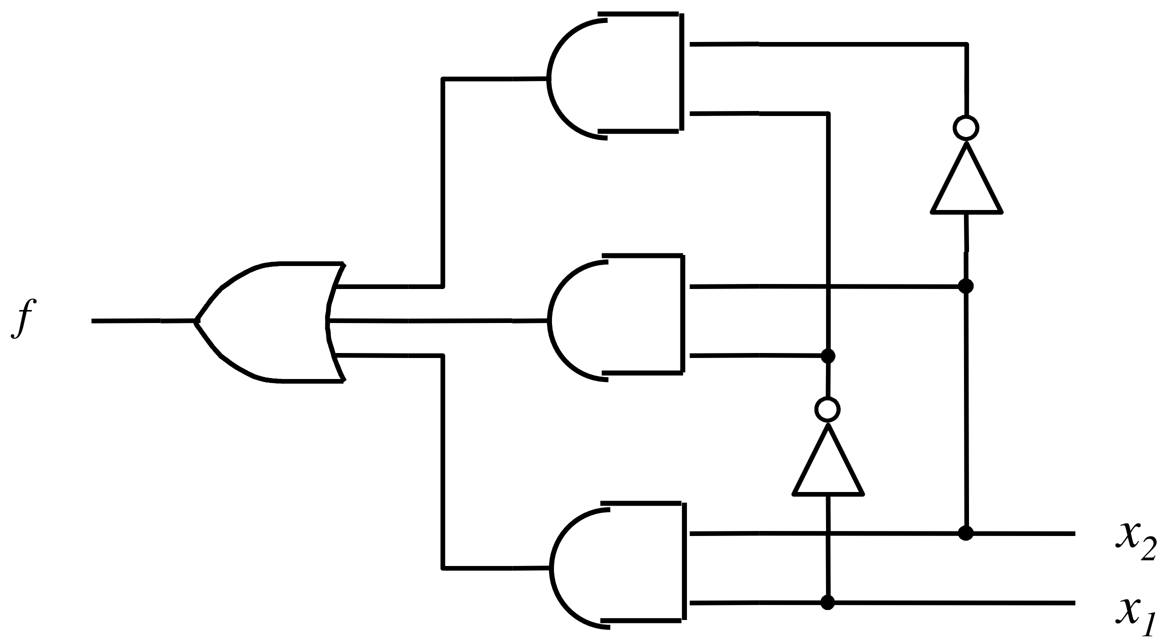


Finally, what about the zero?

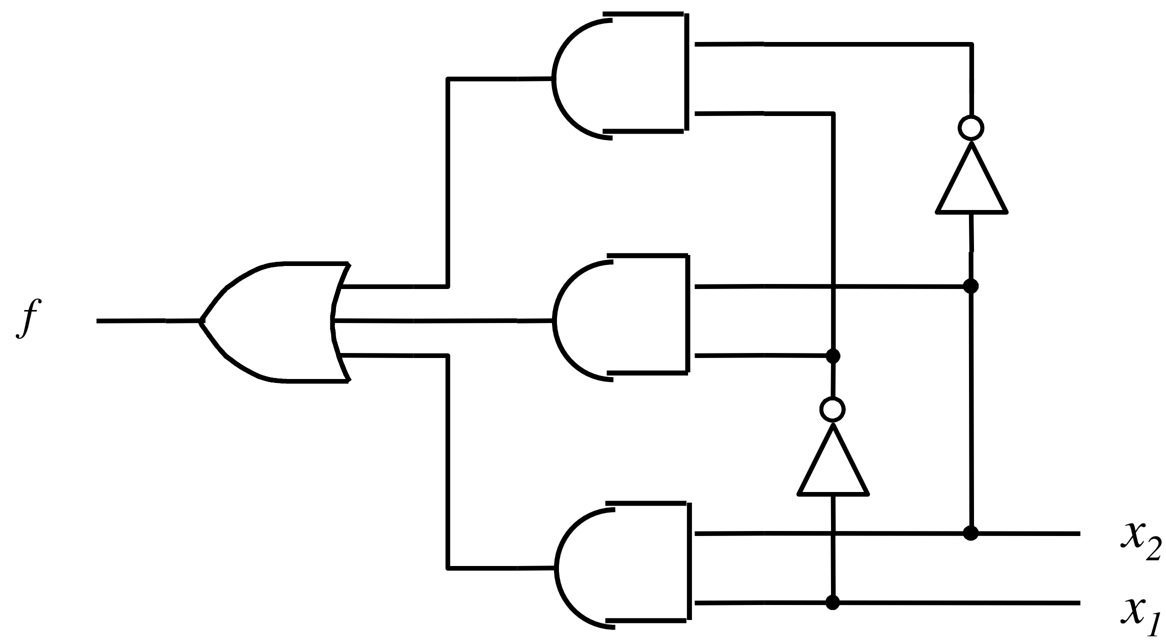
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



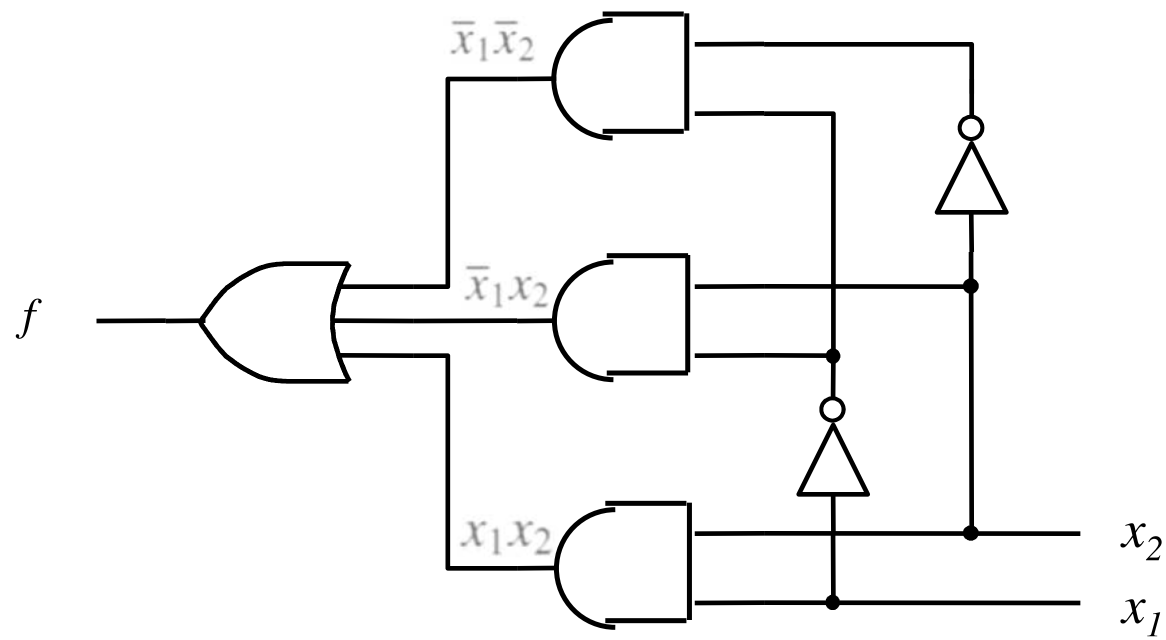
Putting it all together



Let's verify that this circuit implements correctly the target truth table

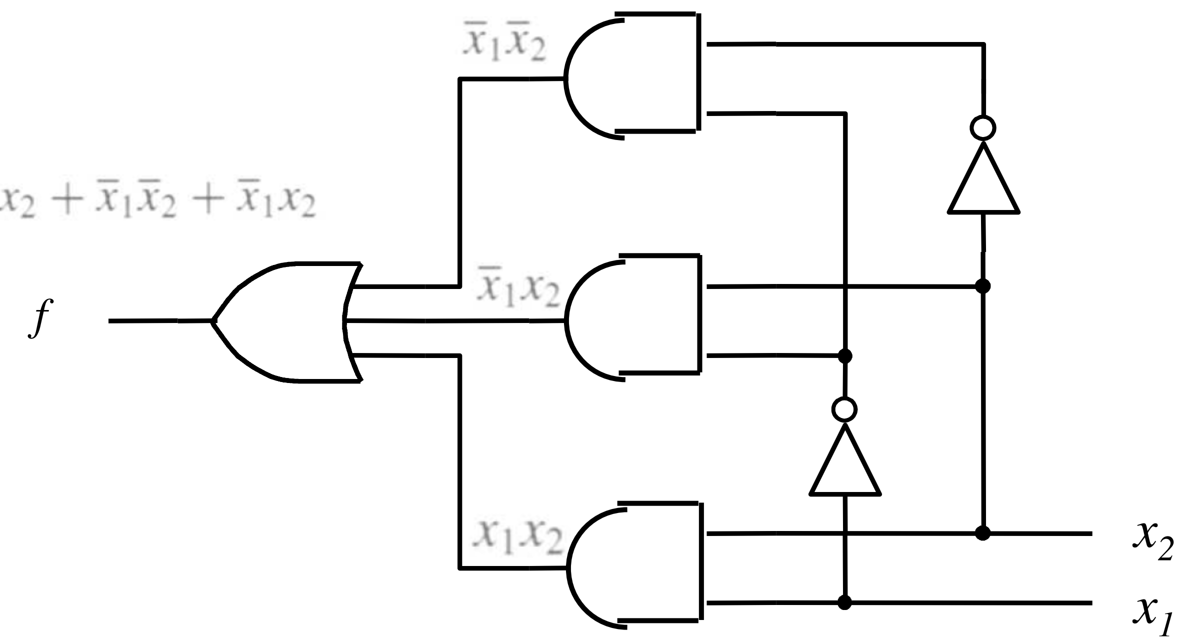


Putting it all together

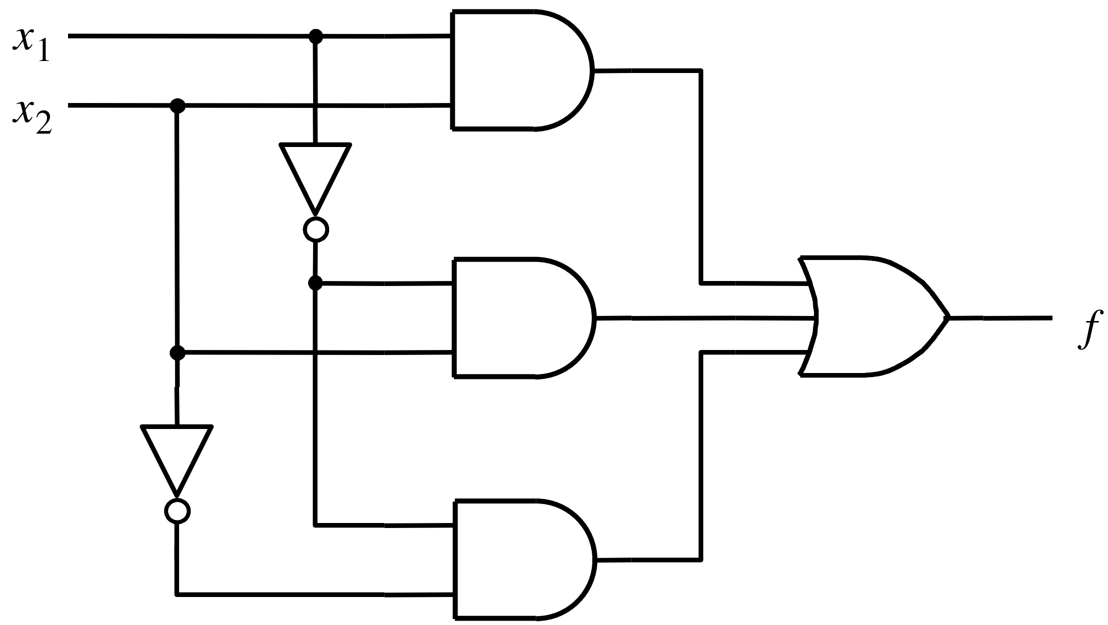


Putting it all together

$$f(x_1, x_2) = x_1x_2 + \overline{x}_1\overline{x}_2 + \overline{x}_1x_2$$



Canonical Sum-Of-Products (SOP)



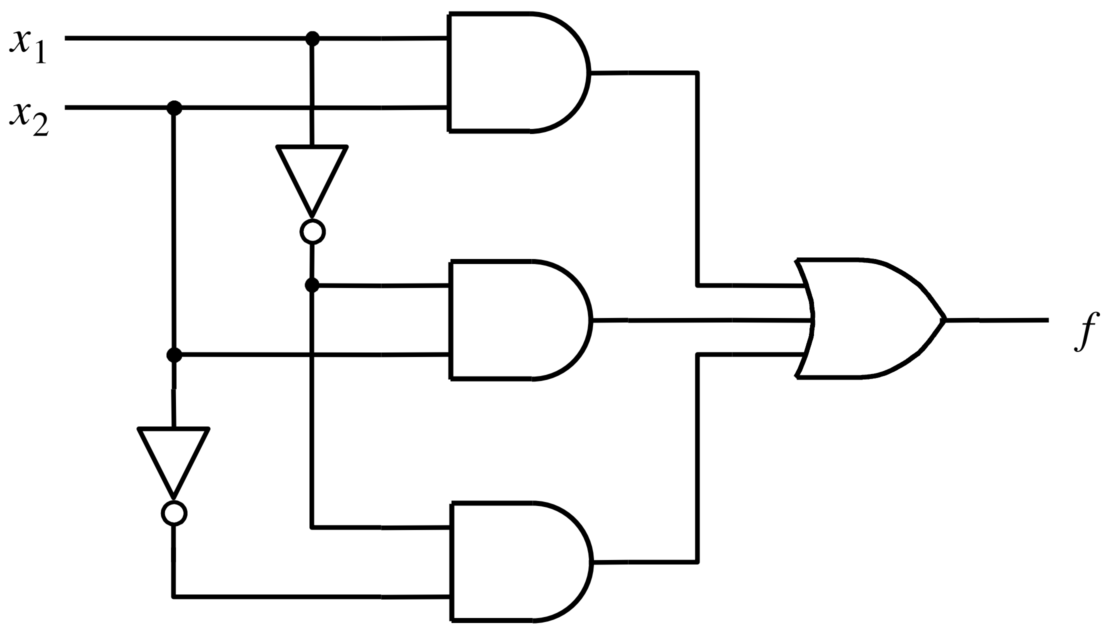
$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

[Figure 2.20a from the textbook]

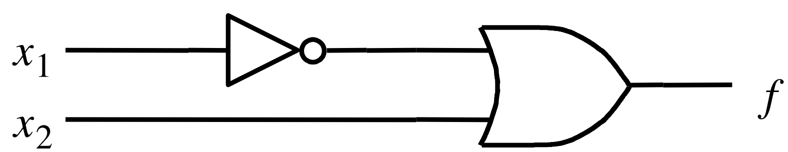
Summary of This Procedure

- **Get the truth table of the function**
- **Form a product term (AND gate) for each row of the table for which the function is 1**
- **Each product term contains all input variables**
- **In each row, if x_i=1 enter it at x_i, otherwise use $\overline{x_i}$**
- **Sum all of these products (OR gate) to get the function**

Two implementations for the same function



(a) Canonical sum-of-products



(b) Minimal-cost realization

[Figure 2.20 from the textbook]

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \boxed{\bar{x}_1x_2}$$

replicate this term

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

group
these terms

$$f(x_1, x_2) = \boxed{x_1x_2} + \boxed{\bar{x}_1\bar{x}_2} + \boxed{\bar{x}_1x_2} + \boxed{\bar{x}_1x_2}$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

These two terms are trivially equal to 1

$$f(x_1, x_2) = (\boxed{x_1 + \bar{x}_1}) x_2 + \bar{x}_1 (\boxed{\bar{x}_2 + x_2})$$

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

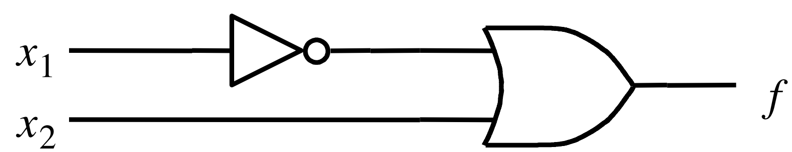
Drop the 1's

$$f(x_1, x_2) = \boxed{1} \cdot x_2 + \bar{x}_1 \cdot \boxed{1}$$

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

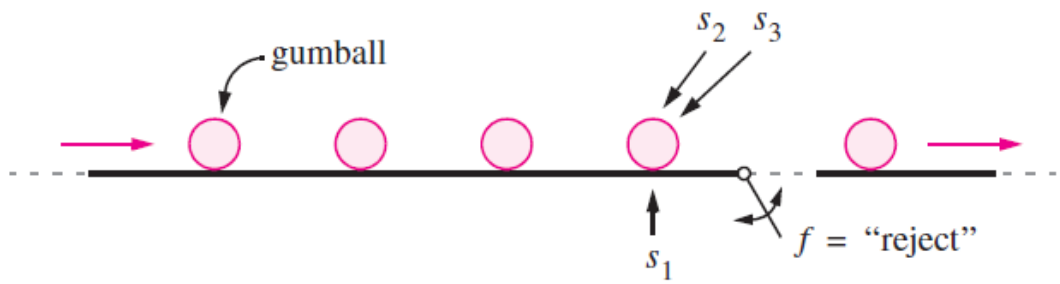
Minimal-cost realization

$$f(x_1, x_2) = x_2 + \bar{x}_1$$



[Figure 2.20b from the textbook]

Let's look at another problem



(a) Conveyor and sensors

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) Truth table

[Figure 2.21 from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 2.21b from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\overline{s}_1 \overline{s}_2 s_3$
0	1	0	0	
0	1	1	1	$\overline{s}_1 s_2 s_3$
1	0	0	0	
1	0	1	1	$s_1 \overline{s}_2 s_3$
1	1	0	1	$s_1 s_2 \overline{s}_3$
1	1	1	1	$s_1 s_2 s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1\bar{s}_2 s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1 s_2 s_3$
1	0	0	0	
1	0	1	1	$s_1\bar{s}_2 s_3$
1	1	0	1	$s_1 s_2\bar{s}_3$
1	1	1	1	$s_1 s_2 s_3$

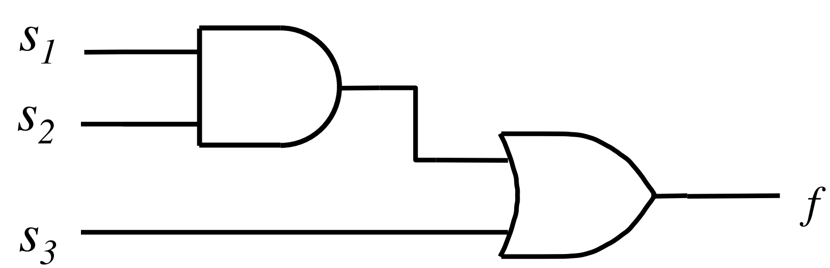
$$f = \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2\bar{s}_3 + s_1 s_2 s_3$$

Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3 \\
&= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1 s_3(\bar{s}_2 + s_2) + s_1 s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2 \\
&= s_3 + s_1 s_2
\end{aligned}
$$

Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3 \\
&= \bar{s}_1s_3(\bar{s}_2 + s_2) + s_1s_3(\bar{s}_2 + s_2) + s_1s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1s_3 + s_1s_3 + s_1s_2 \\
&= s_3 + s_1s_2
\end{aligned}
$$



Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

$$
\begin{aligned}
f &= m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1 \\
&= m_0 + m_1 + m_3 \\
&= \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + x_1 x_2
\end{aligned}
$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$
\begin{aligned}
\overline{f}(x_1, x_2) &= m_2 \\
&= x_1\overline{x}_2
\end{aligned}
$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$\begin{aligned}\overline{f}(x_1, x_2) &= m_2 \\ &= x_1\overline{x}_2\end{aligned}$$

$$\begin{aligned}\overline{\overline{f}} = f &= \overline{x_1\overline{x}_2} \\ &= \overline{x}_1 + x_2\end{aligned}$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$\begin{aligned}\overline{f}(x_1, x_2) &= m_2 \\ &= x_1\overline{x}_2\end{aligned}$$

$$\begin{aligned}\overline{\overline{f}} = f &= \overline{x_1\overline{x}_2} \\ &= \overline{x}_1 + x_2\end{aligned}$$

$$f = \overline{m}_2 = M_2$$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	$\boxed{1}$
2	0	1	0	0
3	0	1	1	0
4	1	0	0	$\boxed{1}$
5	1	0	1	$\boxed{1}$
6	1	1	0	$\boxed{1}$
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

$$\begin{aligned} f &= (\bar{x}_1 + x_1)\bar{x}_2x_3 + x_1(\bar{x}_2 + x_2)\bar{x}_3 \\ &= 1 \cdot \bar{x}_2x_3 + x_1 \cdot 1 \cdot \bar{x}_3 \\ &= \bar{x}_2x_3 + x_1\bar{x}_3 \end{aligned}$$

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	$\boxed{0}$
1	0	0	1	1
2	0	1	0	$\boxed{0}$
3	0	1	1	$\boxed{0}$
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	$\boxed{0}$

$$
\begin{aligned}
f &= \overline{m_0 + m_2 + m_3 + m_7} \\
&= \overline{m}_0 \cdot \overline{m}_2 \cdot \overline{m}_3 \cdot \overline{m}_7 \\
&= M_0 \cdot M_2 \cdot M_3 \cdot M_7 \\
&= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)
\end{aligned}
$$

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f = ((x_1 + x_3) + x_2)((x_1 + x_3) + \bar{x}_2)(x_1 + (\bar{x}_2 + \bar{x}_3))(\bar{x}_1 + (\bar{x}_2 + \bar{x}_3))$$

$$f = (x_1 + x_3)(\bar{x}_2 + \bar{x}_3)$$

Shorthand Notation

- **Sum-of-Products**

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

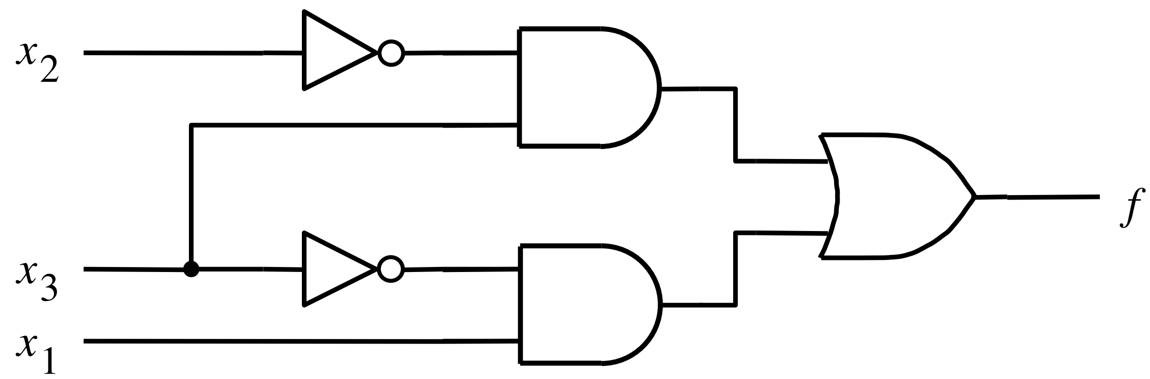
- **Product-of-sums**

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

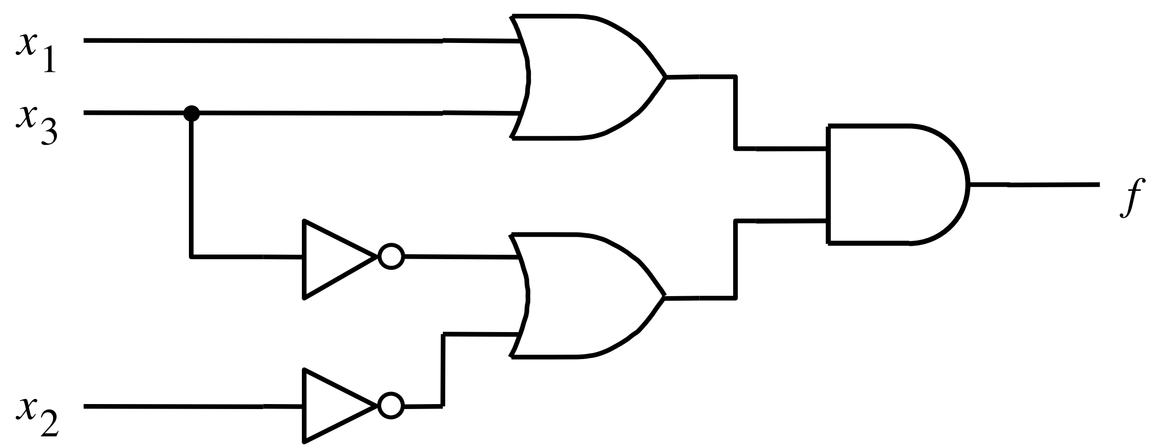
or

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

Two realizations of that function



(a) A minimal sum-of-products realization



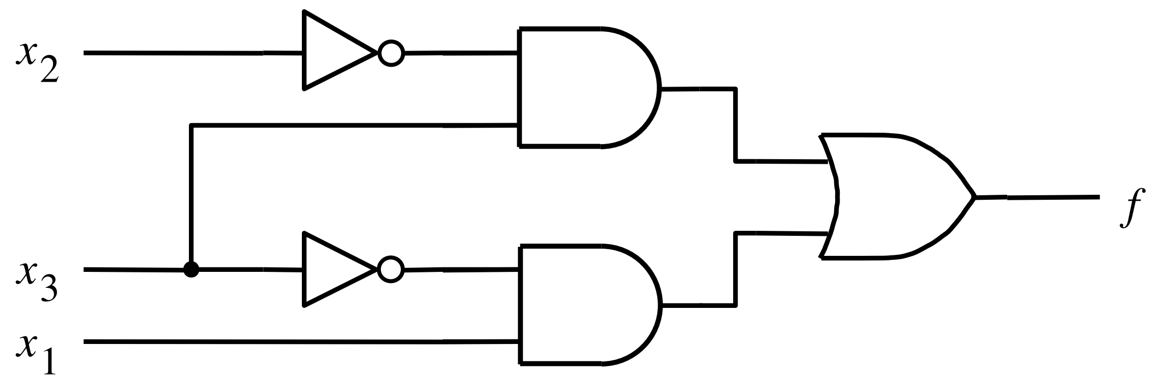
(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

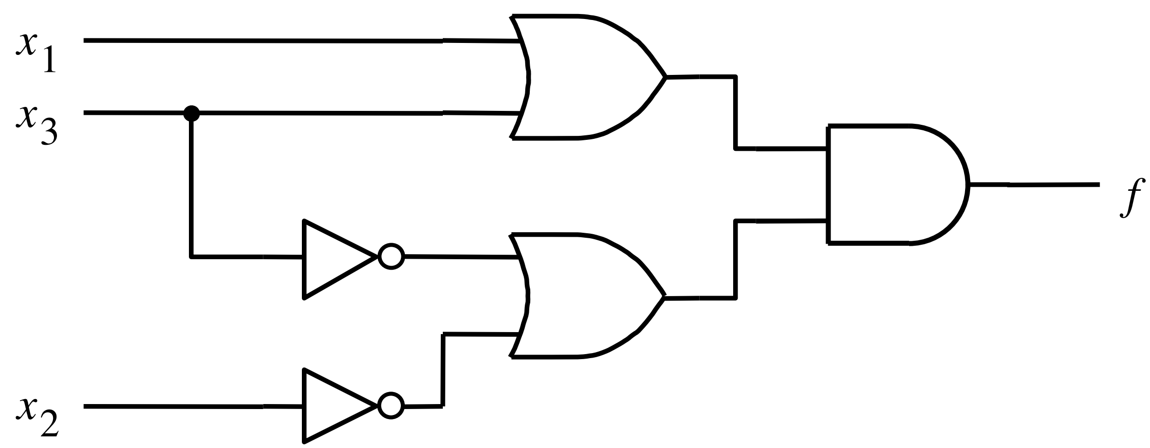
The Cost of a Circuit

- **Count all gates**
- **Count all inputs/wires to the gates**

What is the cost of each circuit?



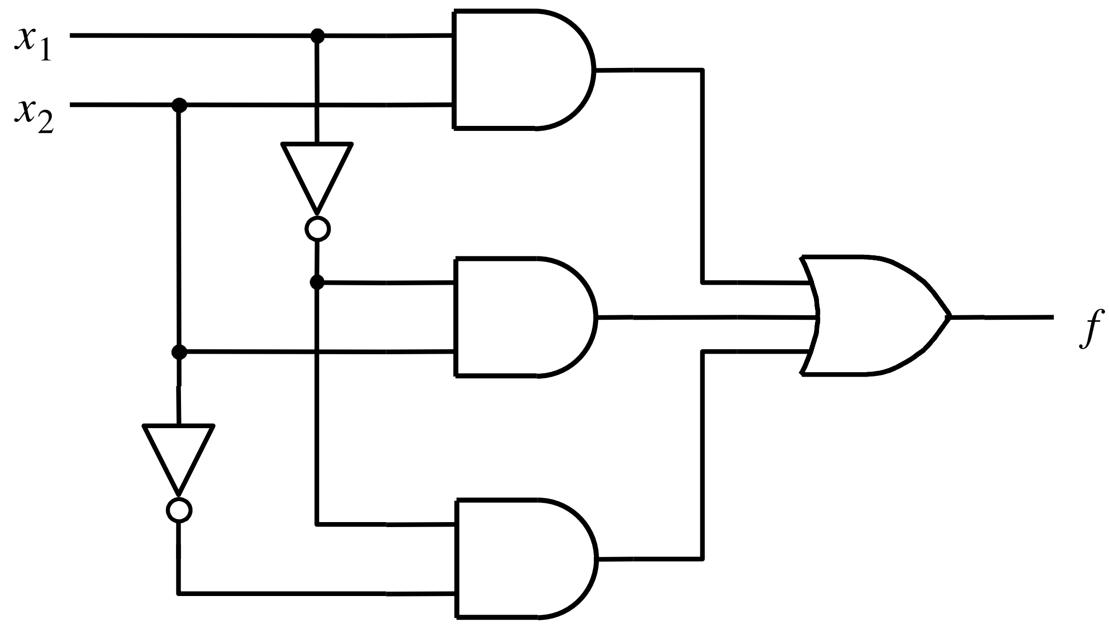
(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

What is the cost of this circuit?



Questions?

THE END