

CprE 281: Digital Logic

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Minimization

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 21 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

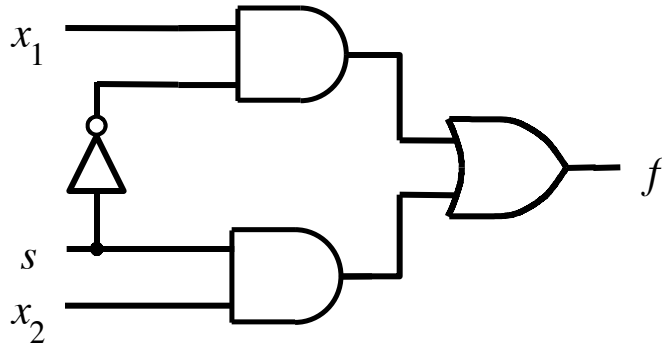
Quick Review

Example:
K-Map for the 2-1 Multiplexer

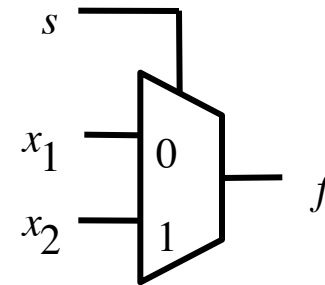
2-1 Multiplexer (Definition)

- **Has two inputs: x_1 and x_2**
- **Also has another input line s**
- **If $s=0$, then the output is equal to x_1**
- **If $s=1$, then the output is equal to x_2**

Circuit for 2-1 Multiplexer



(b) Circuit



(c) Graphical symbol

Truth Table for a 2-1 Multiplexer

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

[Figure 2.33a from the textbook]

Let's Draw the K-map

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

[Figure 2.33a from the textbook]

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

$s \backslash x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

$s \backslash x_1 x_2$	00	01	11	10
0	0	1	0	0
1	0	1	1	1

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

$s \backslash x_1 x_2$	00	01	11	10
0	0	1	0	0
1	0	1	1	1

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

$s \backslash x_1 x_2$	00	01	11	10
0	0	1	0	0
1	0	1	1	1

$$f(s, x_1, x_2) = \bar{x}_1 x_2 + s x_1$$

Let's Draw the K-map

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

		$x_1 x_2$			
	s	00	01	11	10
0	0	0	1	0	0
1	0	0	1	1	1

$f(s, x_1, x_2) = \bar{x}_1 x_2 + s x_1$

Something is wrong!

Compare this with the SOP derivation

Let's Derive the SOP form

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Where should we
put the negation signs?

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\overline{s} \ x_1 \ \overline{x_2}$
0 1 1	1	$\overline{s} \ x_1 \ x_2$
1 0 0	0	
1 0 1	1	$s \ \overline{x_1} \ x_2$
1 1 0	0	
1 1 1	1	$s \ x_1 \ x_2$

Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\bar{s} \ x_1 \ \bar{x}_2$
0 1 1	1	$\bar{s} \ x_1 \ x_2$
1 0 0	0	
1 0 1	1	$s \ \bar{x}_1 \ x_2$
1 1 0	0	
1 1 1	1	$s \ x_1 \ x_2$

$$f(s, x_1, x_2) = \bar{s} \ x_1 \ \bar{x}_2 + \bar{s} \ x_1 \ x_2 + s \ \bar{x}_1 \ x_2 + s \ x_1 \ x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

Let's Draw the K-map again

s x_1 x_2	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

[Figure 2.33a from the textbook]

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

		$x_1 x_2$			
		00	01	11	10
s	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

	s	$x_1 x_2$	00	01	11	10
0			m_0	m_2	m_6	m_4
1			m_1	m_3	m_7	m_5

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

		s	x_1				
	x_2			00	01	11	10
0	m_0	m_2	m_6	m_4			
1	m_1	m_3	m_7	m_5			

The order of the labeling matters.

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	0	1	0	0
	1	0	1	1	1

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

A Karnaugh map for the function $f(s, x_1, x_2)$. The map is a 2x4 grid with s and x_1 as columns and x_2 as rows. The columns are labeled 00, 01, 11, 10 and the rows are labeled 0, 1. The cells contain the function values: (0,00)=0, (0,01)=1, (0,11)=0, (0,10)=0, (1,00)=0, (1,01)=1, (1,11)=1, (1,10)=1. Two groups of 1s are circled in cyan: a vertical group of two cells at $x_1=01$ (rows $x_2=0$ and $x_2=1$), and a horizontal group of two cells at $x_2=1$ (columns $s=11$ and $s=10$). Vertical lines extend from the bottom of each group.

$x_2 \backslash s x_1$	00	01	11	10
0	0	1	0	0
1	0	1	1	1

Let's Draw the K-map again

	s	x_1	x_2	$f(s, x_1, x_2)$
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	1
m_3	0	1	1	1
m_4	1	0	0	0
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

$s \ x_1$	x_2	00	01	11	10
0	0	0	1	0	0
1	0	0	1	1	1

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

This is correct!

Two Different Ways to Draw the K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		x_2x_3			
		00	01	11	10
x_1	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

Another Way to Draw 3-variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		x_1	
		0	1
x_2x_3	00	m_0	m_4
	01	m_1	m_5
	11	m_3	m_7
	10	m_2	m_6

Gray Code

- **Sequence of binary codes**
- **Consecutive lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

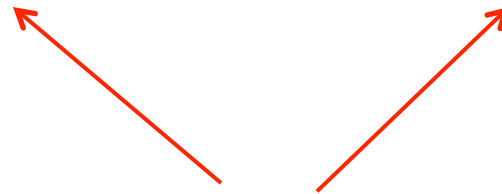
	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

x_2 \ $s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Adjacency Rules

x_2	$s x_1$	00	01	11	10
0		000	010	110	100
1		001	011	111	101



adjacent
columns

Gray Code & K-map

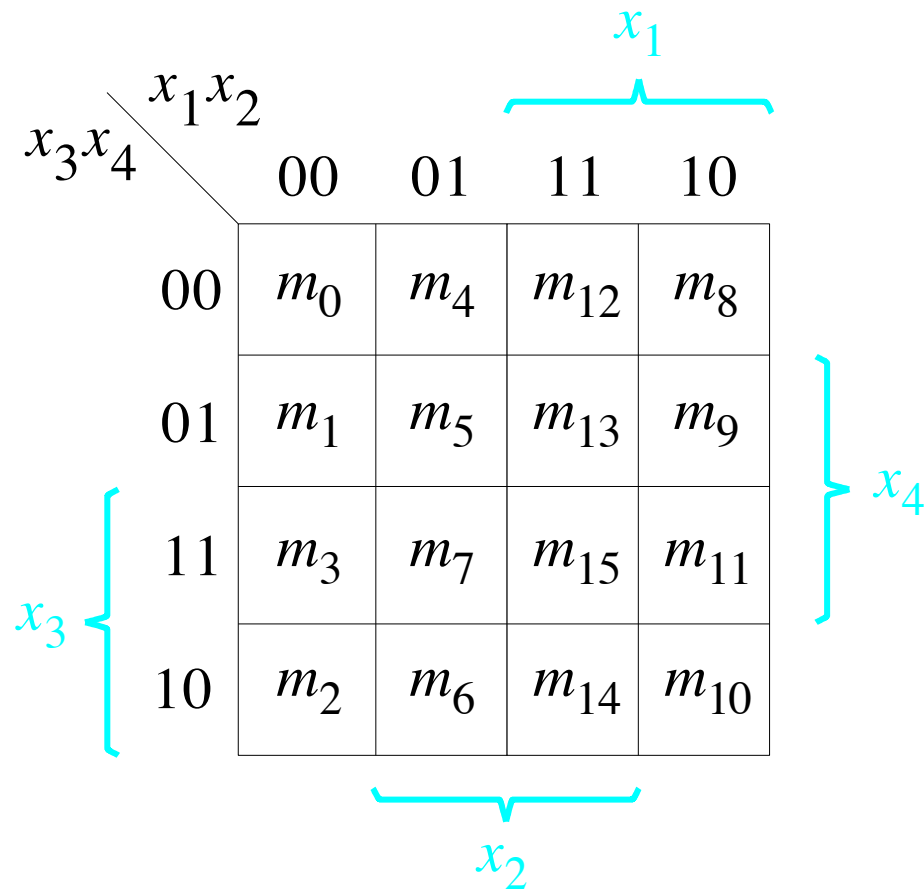
	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

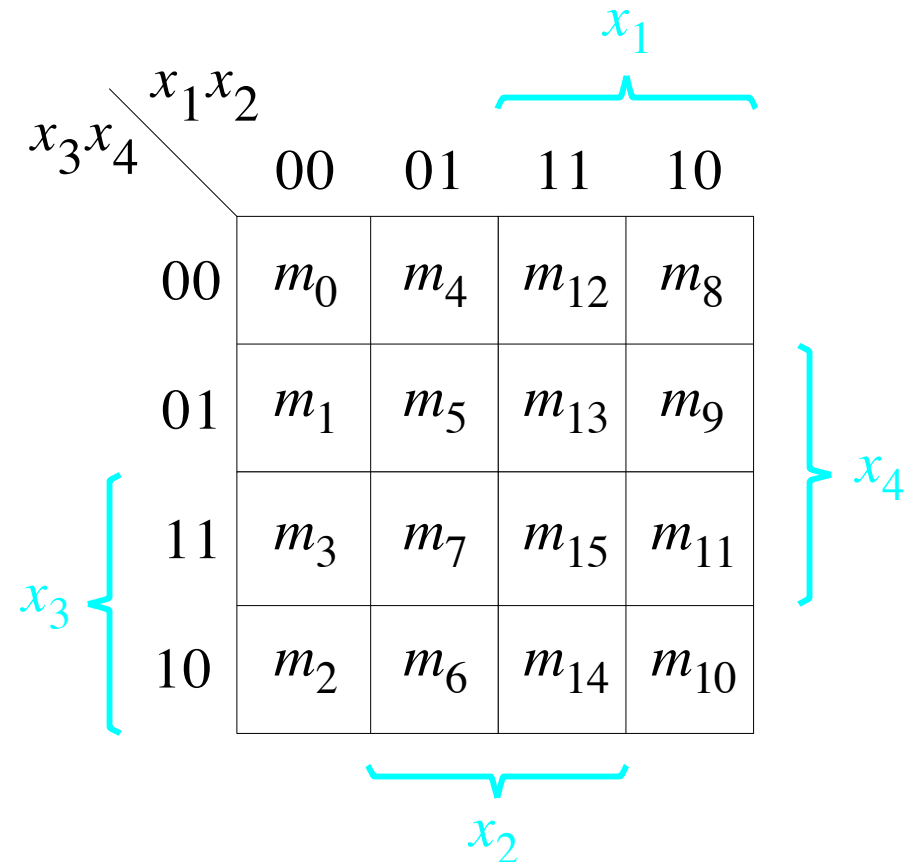
A four-variable Karnaugh map



[Figure 2.53 from the textbook]

A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns

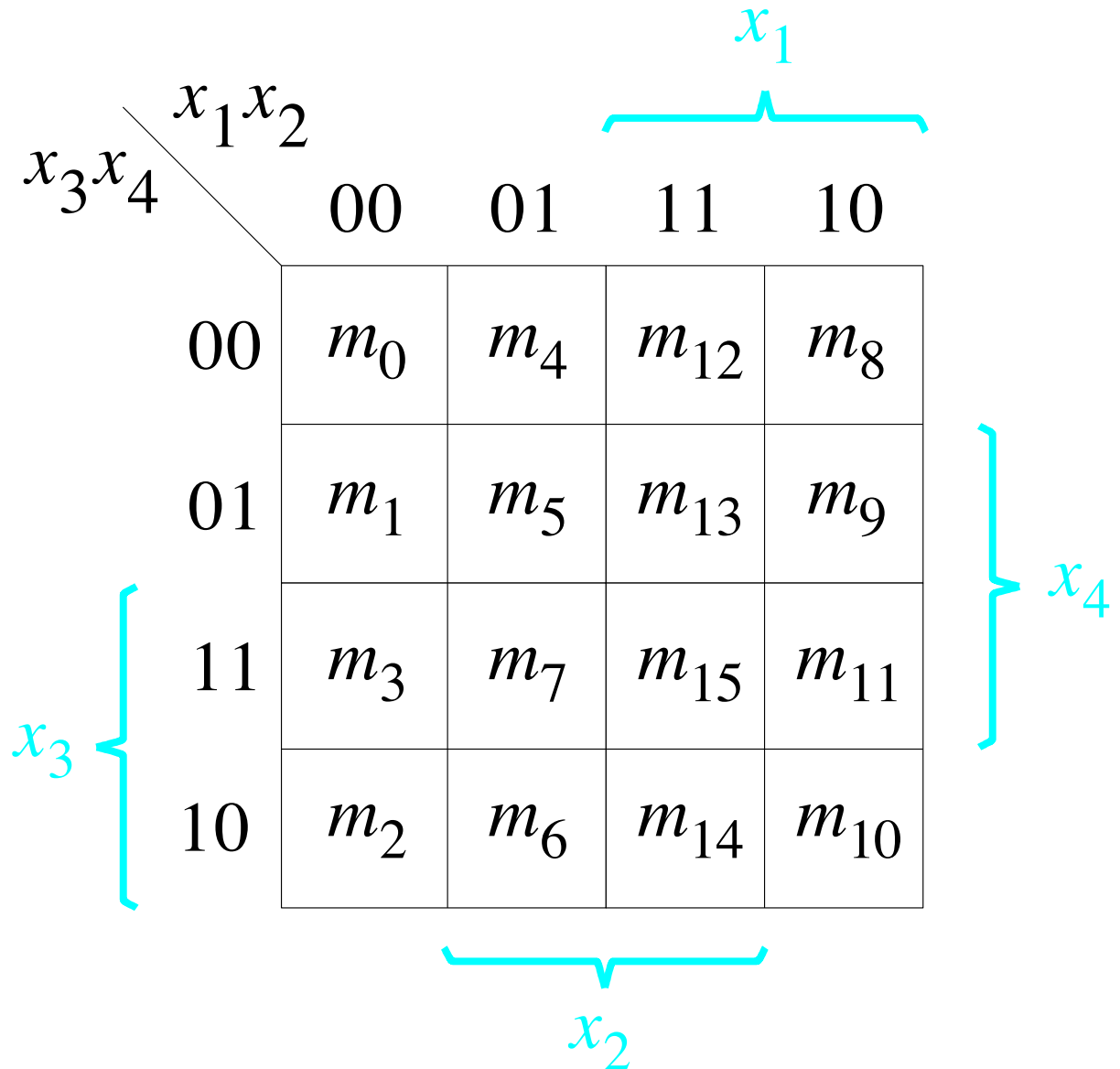
		x_1x_2			
		00	01	11	10
x_3x_4	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

adjacent
columns

adjacent
rows

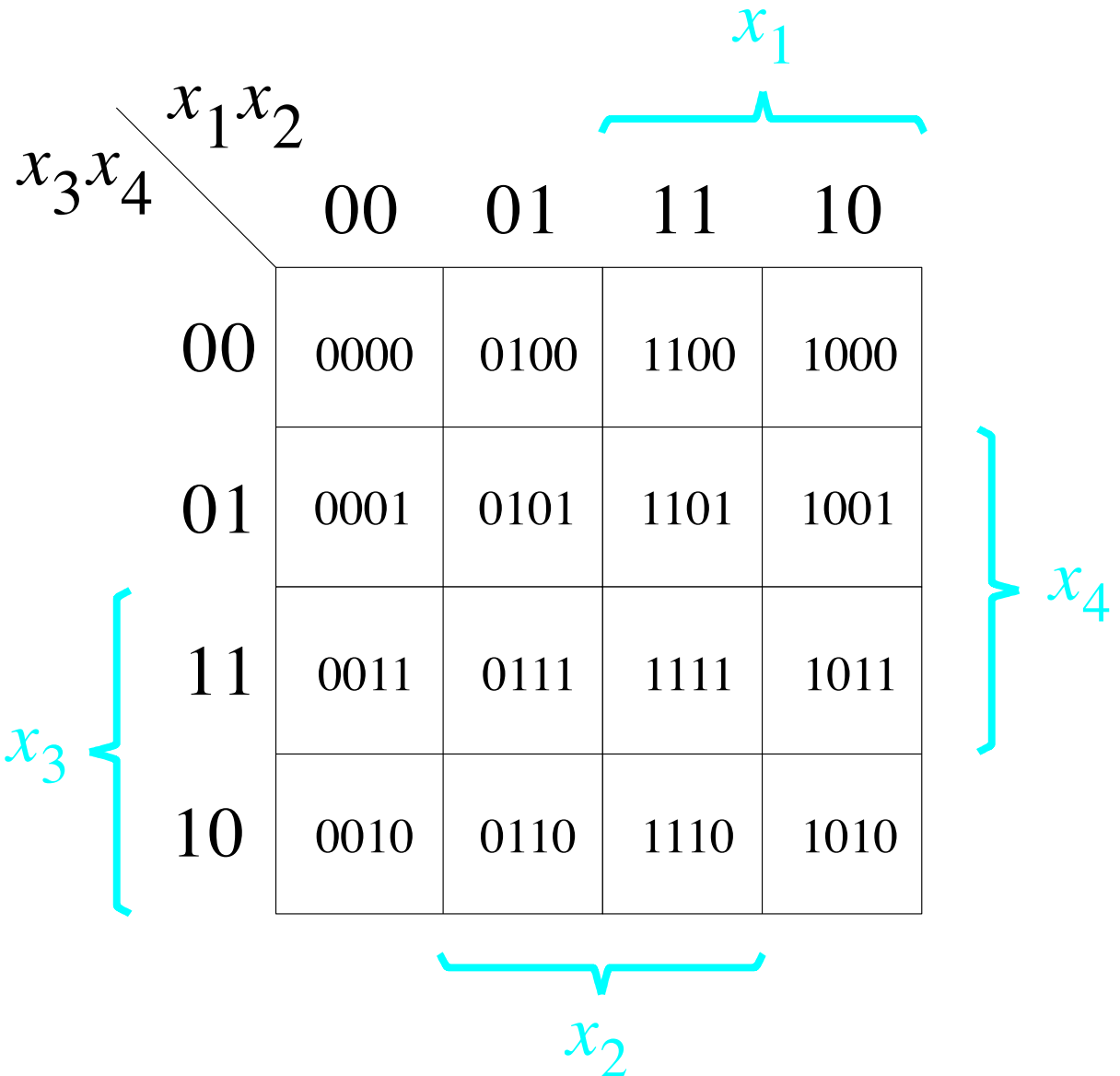
Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

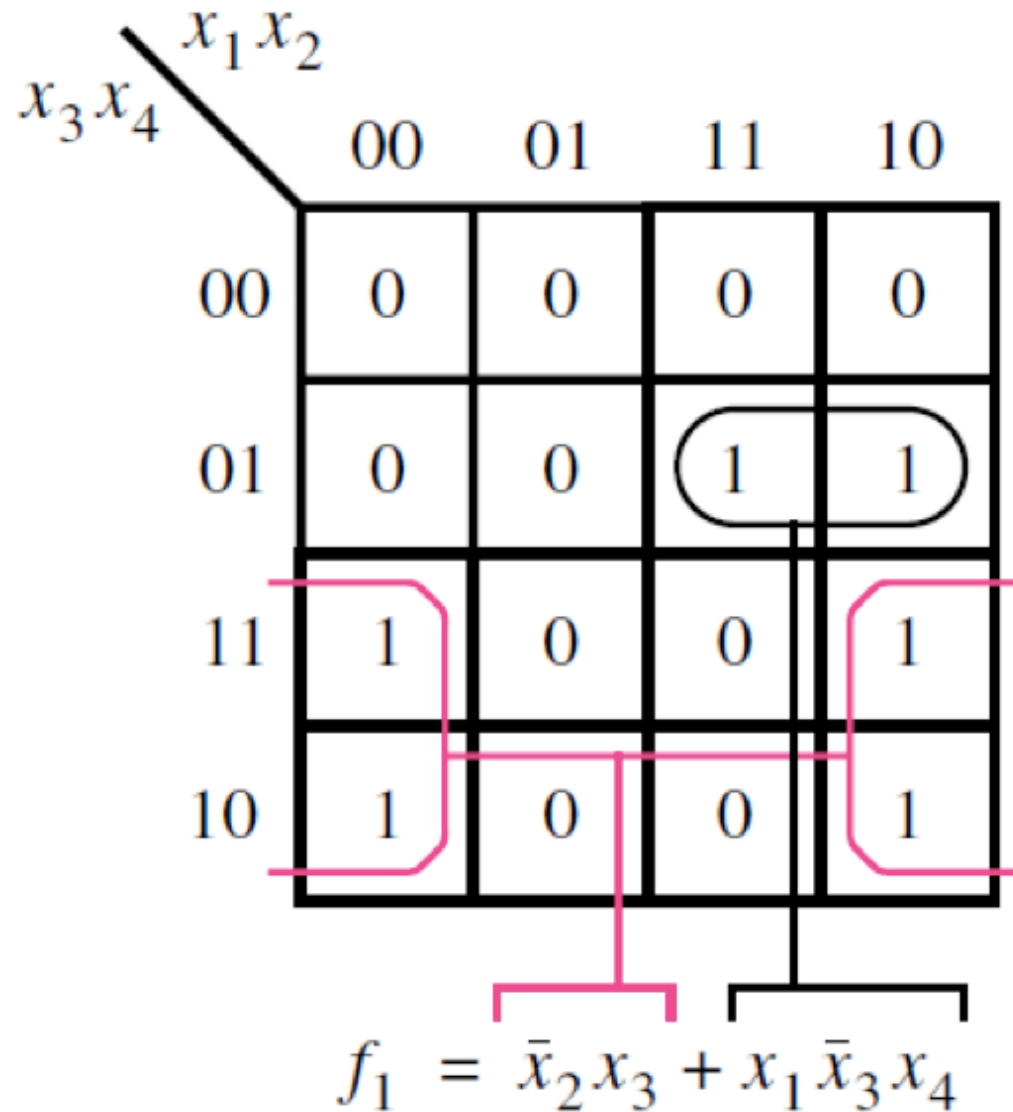


Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15

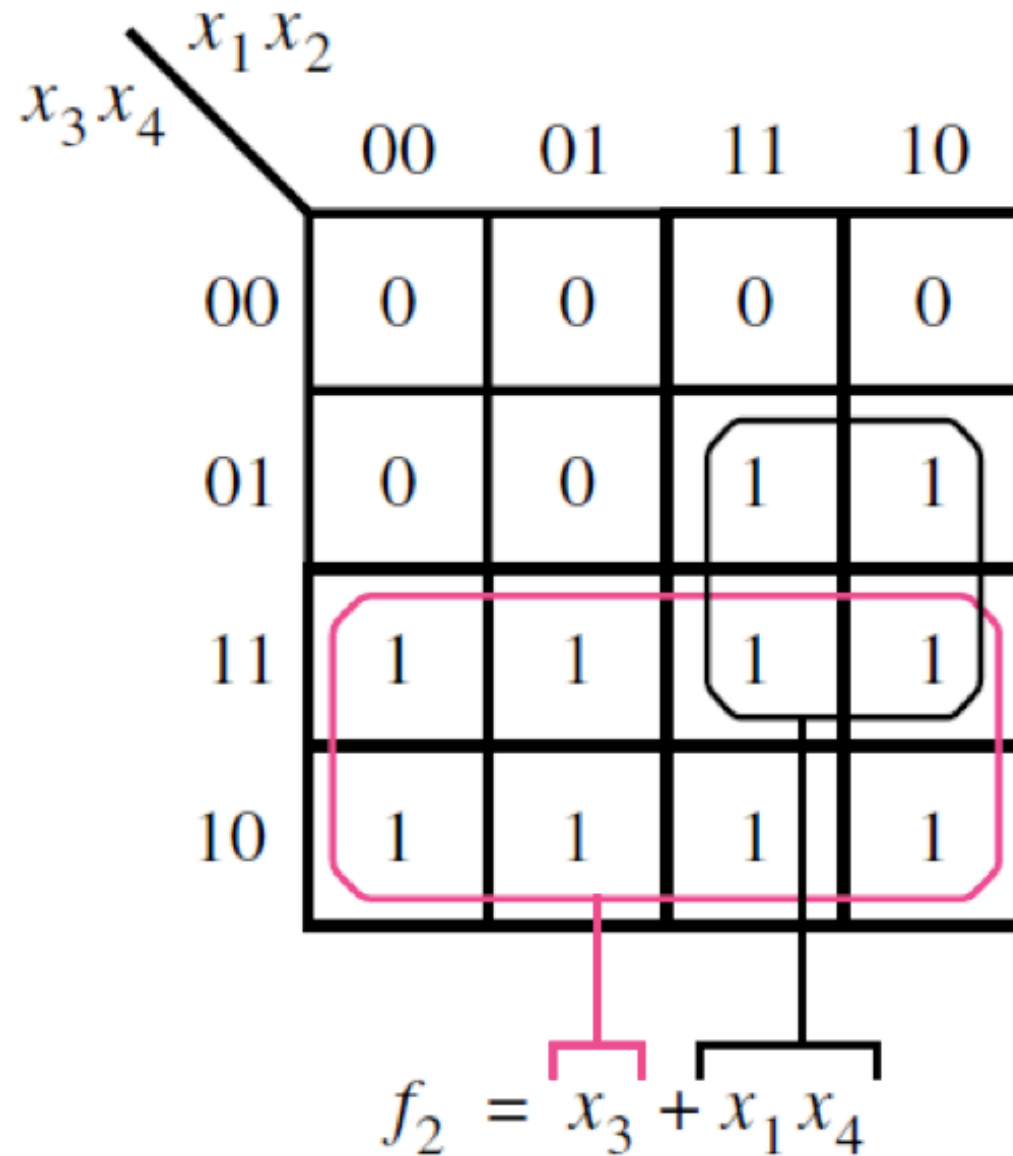


Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Strategy For Minimization

Grouping Rules

- **Group “1”s with rectangles**
- **Both sides a power of 2:**
 - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
 - **Try to use as few groups as possible to cover all “1”s.**
 - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don’t use a 2x1 even if that is enough).**

Terminology

Literal: a variable, complemented or uncomplemented

Some Examples:

- \bar{X}_1
- X_2

Terminology

- **Implicant: product term that indicates the input combinations for which function output is 1**

- **Example**

- \bar{x}_1 - indicates that $\bar{x}_1\bar{x}_2$ and \bar{x}_1x_2 yield output of 1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	0

Terminology

- **Prime Implicant**
 - Implicant that cannot be combined into another implicant with fewer literals
 - Some Examples

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	1	0

Not prime

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	1	0

Prime

Terminology

- **Essential Prime Implicant**
 - Prime implicant that includes a minterm not covered by any other prime implicant
 - Some Examples

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	0	0

The Karnaugh map shows a 2x4 grid of cells. The columns are labeled 00, 01, 11, and 10, and the rows are labeled 0 and 1. The cells contain the following values: (0,00)=0, (0,01)=1, (0,11)=1, (0,10)=1, (1,00)=1, (1,01)=1, (1,11)=0, (1,10)=0. There are four prime implicants highlighted: a blue square covering (0,01) and (1,01), a red square covering (0,11) and (1,01), a red square covering (0,11) and (0,10), and a red square covering (1,00) and (1,01). The minterm (0,11) is covered by two prime implicants, while the minterm (0,10) is only covered by one, making it an essential prime implicant.

Terminology

- **Cover**

- **Collection of implicants that account for all possible input valuations where output is 1**

- **Ex. $x_1' x_2 x_3 + x_1 x_2 x_3' + x_1 x_2' x_3'$**

- **Ex. $x_1' x_2 x_3 + x_1 x_3'$**

		$x_1 x_2$			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	1	0	0

Example

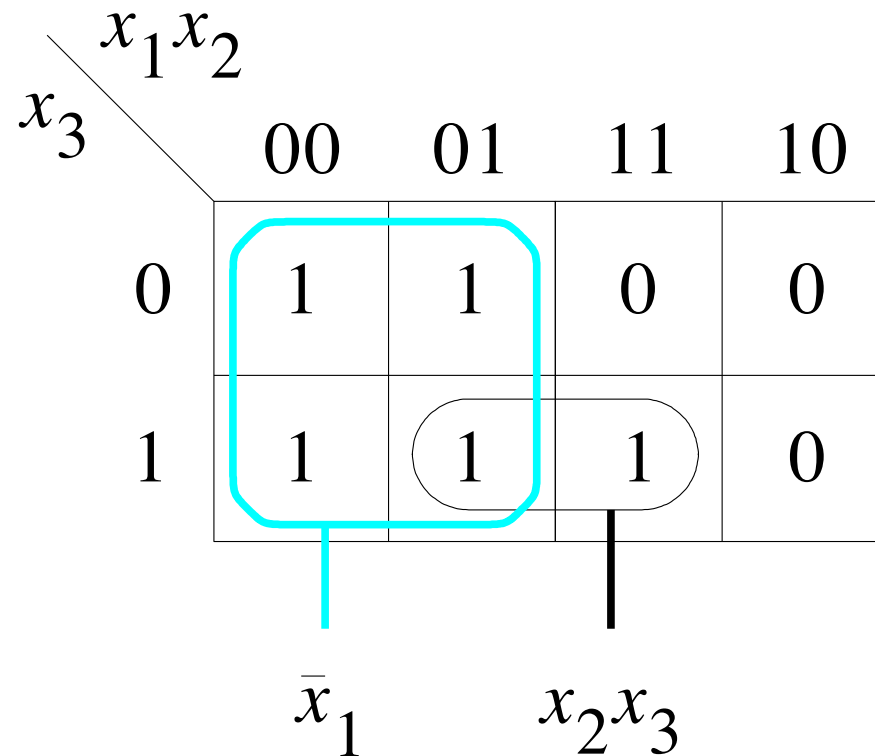
- Give the Number of
 - Implicants?
 - Prime Implicants?
 - Essential Prime Implicants?

$x_3 \backslash x_1 x_2$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Why concerned with minimization?

- **Simplified function**
- **Reduce the cost of the circuit**
 - **Cost: Gates + Inputs**
 - **Transistors**

Three-variable function $f(x_1, x_2, x_3) = \Sigma m(0, 1, 2, 3, 7)$

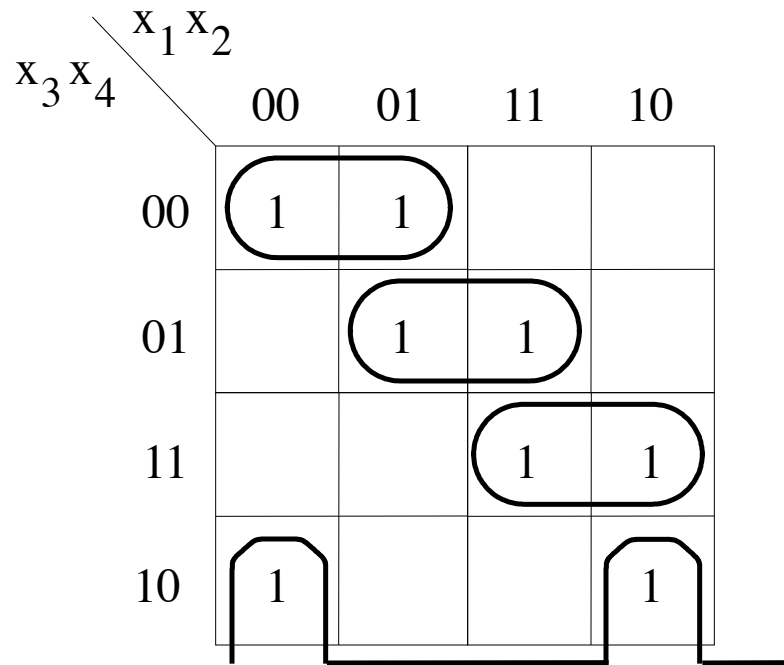


[Figure 2.56 from the textbook]

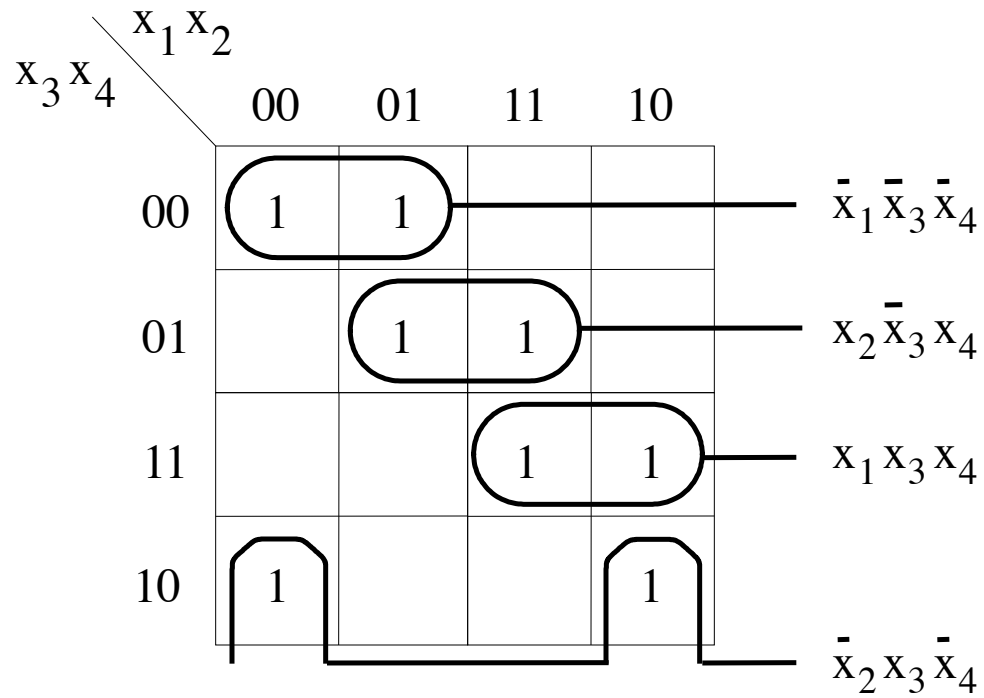
Example

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	1	1		
	01		1	1	
	11			1	1
	10	1			1

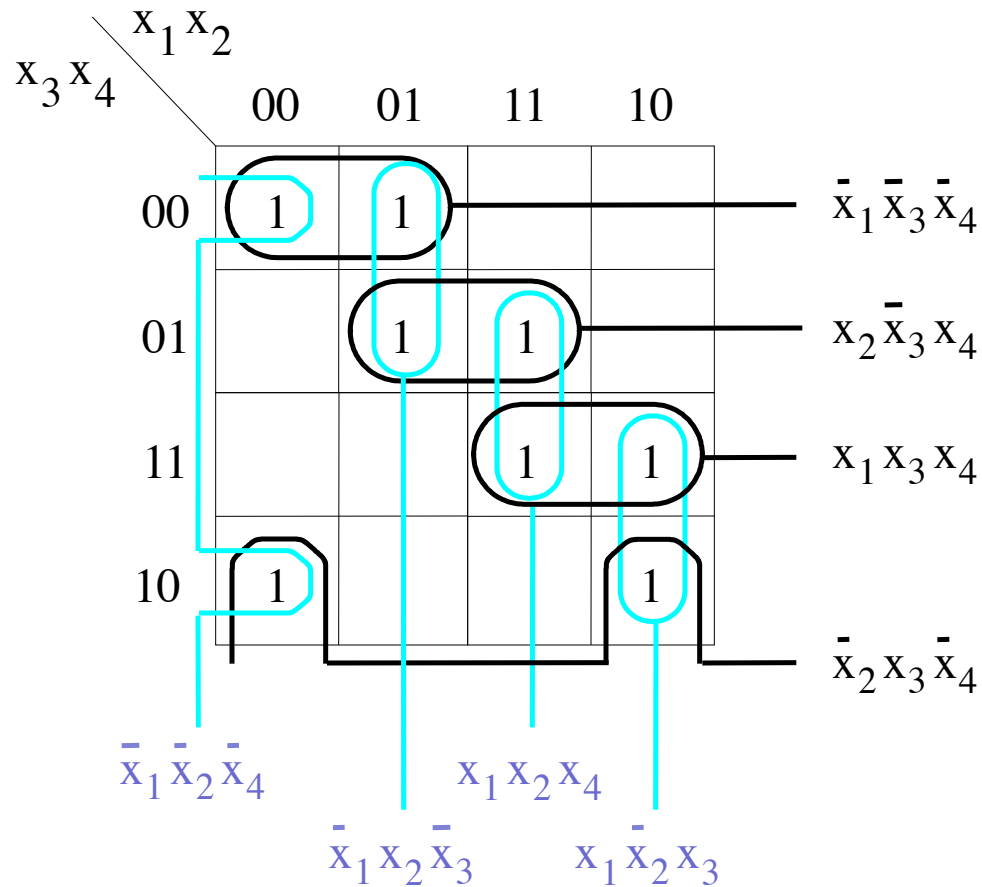
Example



Example

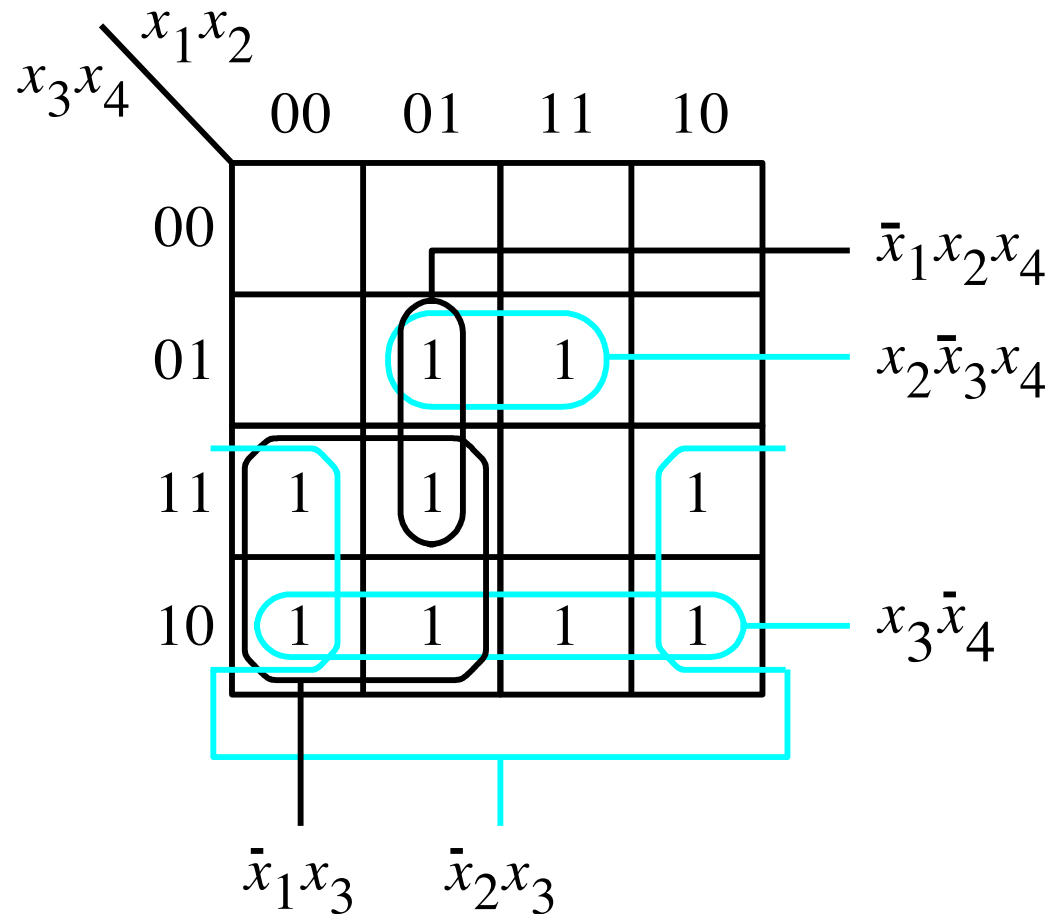


Example: Another Solution



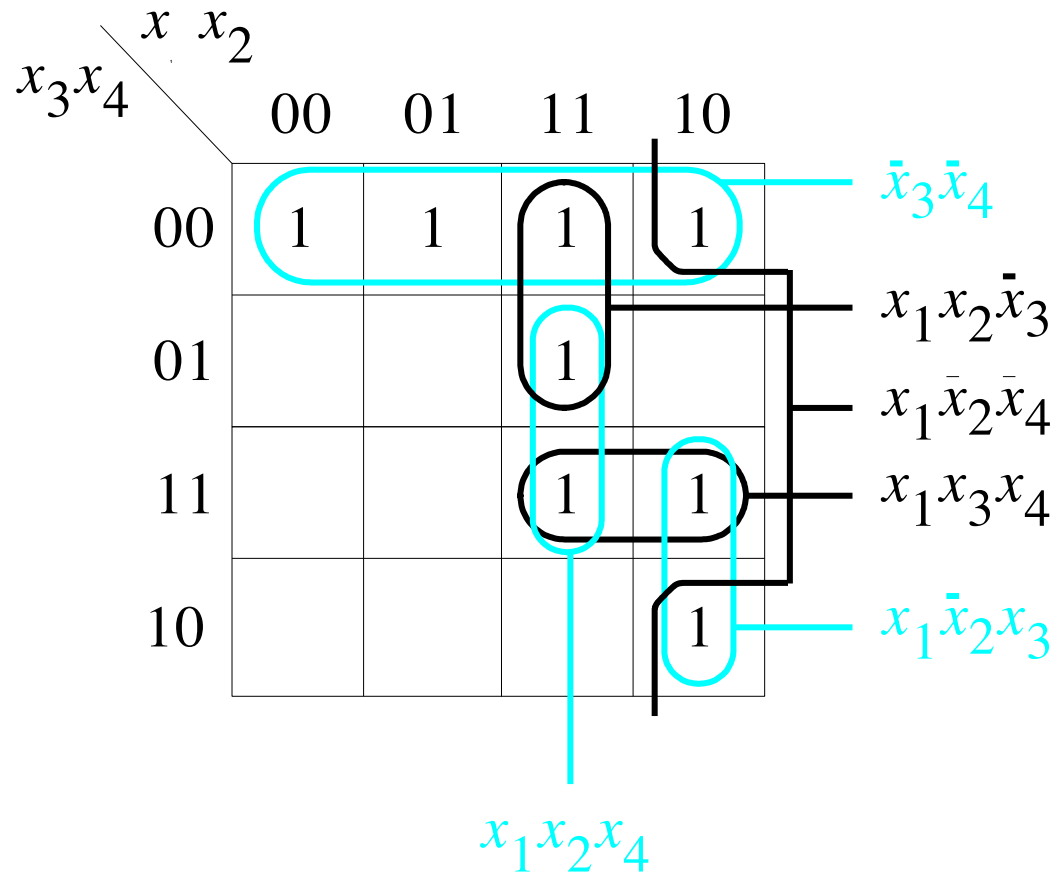
[Figure 2.59 from the textbook]

$$f(x_1, \dots, x_4) = \sum m(2, 3, 5, 6, 7, 10, 11, 13, 14)$$



[Figure 2.57 from the textbook]

$$f(x_1, \dots, x_4) = \sum m(0, 4, 8, 10, 11, 12, 13, 15)$$



[Figure 2.58 from the textbook]

Minimization of Product-of-Sums Forms

Do You Still Remember This Boolean Algebra Theorem?

14a. $x \cdot y + x \cdot \bar{y} = x$

14b. $(x + y) \cdot (x + \bar{y}) = x$

Combining

Let's prove 14.b

x	y	$(\mathbf{x} + \mathbf{y}) \cdot (\mathbf{x} + \overline{\mathbf{y}}) = \mathbf{x}$
0	0	
0	1	
1	0	
1	1	

Let's prove 14.b

x	y	$(\mathbf{x} + \mathbf{y}) \cdot (\mathbf{x} + \overline{\mathbf{y}}) = \mathbf{x}$
0	0	0
0	1	1
1	0	1
1	1	1

Let's prove 14.b

x	y	$(\mathbf{x} + \mathbf{y}) \cdot (\mathbf{x} + \overline{\mathbf{y}}) = \mathbf{x}$
0	0	0 1
0	1	1 0
1	0	1 1
1	1	1 1

Let's prove 14.b

x	y	$(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} + \overline{\mathbf{y}}) = \mathbf{x}$			
0	0	0	0	1	
0	1	1	0	0	
1	0	1	1	1	
1	1	1	1	1	

Let's prove 14.b

x	y	$(\mathbf{x} + \mathbf{y}) \bullet (\mathbf{x} + \overline{\mathbf{y}}) = \mathbf{x}$				
0	0	0	0	1	0	
0	1	1	0	0	0	
1	0	1	1	1	1	
1	1	1	1	1	1	

Let's prove 14.b

x	y	$(x + y) \cdot (x + \bar{y}) = x$				
0	0	0	0	1	0	
0	1	1	0	0	0	
1	0	1	1	1	1	
1	1	1	1	1	1	

They are equal.

Grouping Example

	x_1	0	1
x_2			
0		0	1
1		1	1

M_0

	x_1	0	1
x_2			
0		1	0
1		1	1

M_2

Grouping Example

	x_1	0	1
x_2			
0		0	1
1		1	1

M_0

*

	x_1	0	1
x_2			
0		1	0
1		1	1

M_2

=

	x_1	0	1
x_2			
0		0	0
1		1	1

$M_0 * M_2$

Grouping Example

	x_1	0	1
x_2	0	0	1
	1	1	1

M_0

*

	x_1	0	1
x_2	0	1	0
	1	1	1

M_2

=

	x_1	0	1
x_2	0	0	0
	1	1	1

$M_0 * M_2$

Grouping Example

	x_1	0	1
x_2	0	0	1
	1	1	1

M_0

*

	x_1	0	1
x_2	0	1	0
	1	1	1

M_2

=

	x_1	0	1
x_2	0	0	0
	1	1	1

$M_0 * M_2$

Grouping Example

	x_1	0	1
x_2	0	0	1
	1	1	1

M_0

$(x_1 + x_2)$

*

	x_1	0	1
x_2	0	1	0
	1	1	1

M_2

$(\bar{x}_1 + x_2)$

=

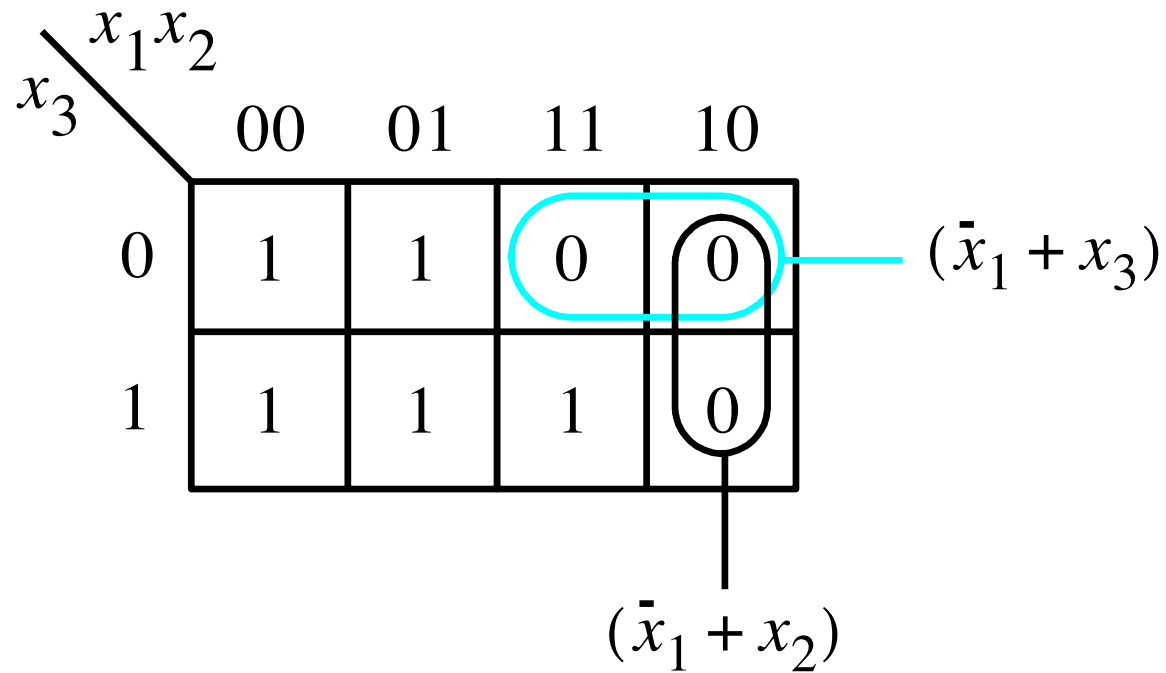
	x_1	0	1
x_2	0	0	0
	1	1	1

$M_0 * M_2$

x_2

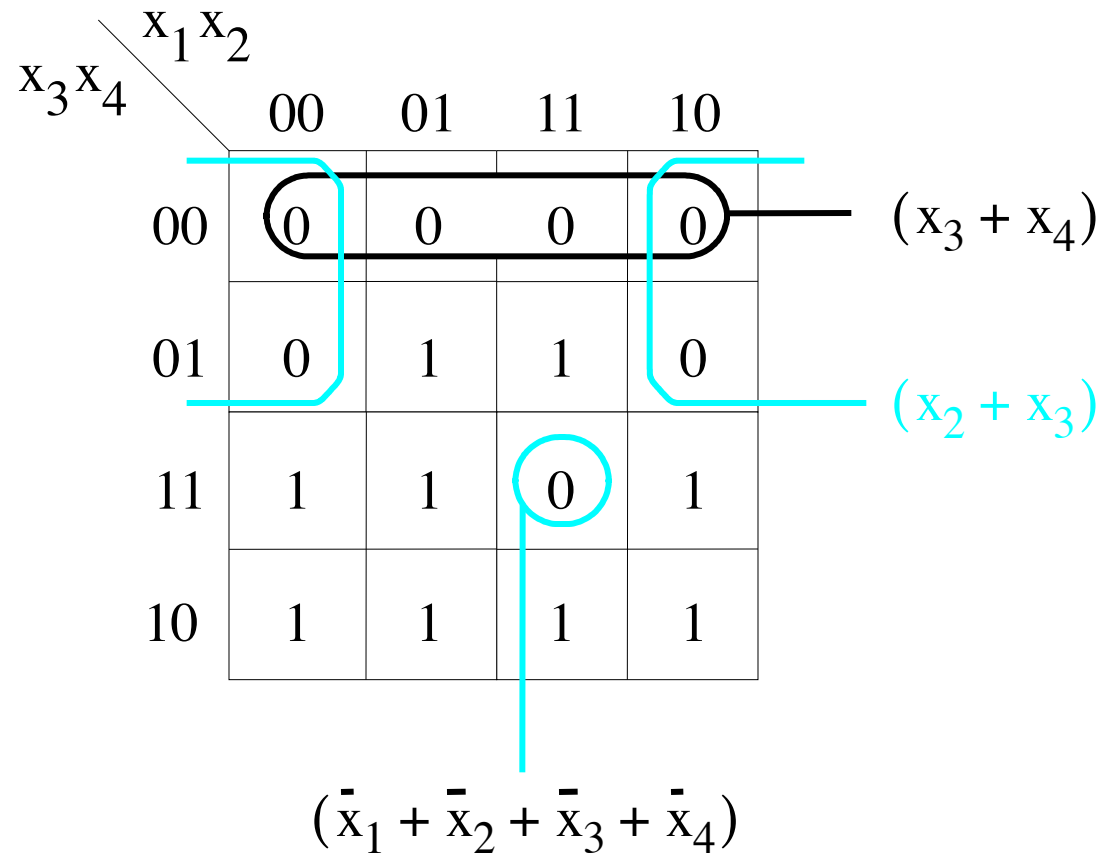
Property 14b (Combining)

POS minimization of $f(x_1, x_2, x_3) = \Pi M(4, 5, 6)$



[Figure 2.60 from the textbook]

POS minimization of $f(x_1, \dots, x_4) = \prod M(0, 1, 4, 8, 9, 12, 15)$



[Figure 2.61 from the textbook]

Questions?

THE END