

P1. (10 points) Use Venn diagrams to prove the following two equations:

$$(x_1 + x_2 + x_3) \cdot (x_1 + x_2 + \bar{x}_3) = x_1 + x_2$$
$$\overline{x + y} = \bar{x} \cdot \bar{y}$$

P2. (10 points) Use Boolean algebra to prove the first equation in P1.

P3. (10 points) Using a truth table to check the validity of the following:

$$(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2) = (x_1 + x_2)(x_2 + x_3)(\bar{x}_1 + \bar{x}_3)$$

P4. (10 points) Draw logic circuits for the following three expressions:

$$\bar{x}_1 x_3 + x_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 + x_1 \bar{x}_2$$
$$x_1 \bar{x}_3 + x_2 x_3 + \bar{x}_2 \bar{x}_3$$
$$(x_1 + x_3)(\bar{x}_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2)$$

P5. (20 points) Consider the logic function:

$$f(a,b,c) = a'b'c' + ab'c + a'bc + a'b'c + ab'c'$$

where a' is an alternative notation for \bar{a} , b' for \bar{b} , and c' for \bar{c} .

(a) (4 points) Draw the logic circuit for the function f given above.

(b) (4 points) Let the cost of a logic circuit be the total number of gates plus the total number of inputs to all gates in the circuit. (See pages 49-50 in the book for examples.) What is the cost of the circuit in (a)?

(c) (4 points) Simplify f using Boolean algebra as much as possible.

(d) (4 points) Draw the logic circuit for the simplified version of f in (c).

(e) (4 points) What is the cost of the circuit in (d)?

P6. (20 points) A function f has four inputs x , y , z , and w and one output such that the output is a 1 if and only if the number of 1s in the inputs is either one or three.

- (a) (4 points) Derive the truth table for f .
- (b) (4 points) Write the canonical sum-of-products expression for f . Do not use the shorthand notation.
- (c) (2 points) Write the canonical sum-of-products expression for f in shorthand notation.
- (d) (2 points) Write the canonical sum-of-products expression for \bar{f} in shorthand notation.
- (e) (4 points) Write the canonical product-of-sums expression for f . Do not use the shorthand notation.
- (f) (2 points) Write the canonical product-of-sums expression for f in shorthand notation.
- (g) (2 points) Write the canonical product-of-sums expression for \bar{f} in shorthand notation.

P7. (10 points) Use algebraic manipulation to show that for three input variables x_1 , x_2 , and x_3 the following is true:

$$\prod M(0, 1, 2, 3, 4, 5, 6) = x_1x_2x_3$$

Show the steps in your derivation.

P8. (10 points) Use algebraic manipulation to find the minimum sum-of-products expression for the following function:

$$f = x_1\bar{x}_2\bar{x}_3 + x_1x_2x_4 + x_1\bar{x}_2x_3\bar{x}_4$$