

**P1. (20 points)** Use a K-map to find the minimal sum-of-products (SOP) expression for the following four problems. Show the terms that are grouped in each K-map.

a) (5 points)

		BC			
		00	01	11	10
A	0	0	0	0	1
	1	1	0	0	1

b) (5 points)

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	0	1	1	0
	11	0	0	0	0
	10	1	0	0	1

c) (5 points)  $F(A, B, C) = \sum m(1, 2, 3, 5, 7)$

d) (5 points)  $F(A, B, C, D) = \sum m(1, 3, 4, 5, 6, 7, 9, 11, 13, 15)$

**P2. (15 points)** Use a K-map to find the minimal product-of-sums (POS) expression for the following three problems. Show the terms that are grouped in each K-map.

a) (5 points)

		BC			
		00	01	11	10
A	0	0	0	0	1
	1	1	0	0	1

b) (5 points)

		CD			
		00	01	11	10
AB	00	1	0	0	1
	01	0	1	1	0
	11	0	1	1	0
	10	1	0	0	1

c) (5 points)  $F(A, B, C, D) = \prod M(5, 7, 11, 13, 15)$

**P3. (15 points)** A four-variable function  $F(w,x,y,z)$  is called a *majority* function if  $F = 1$  when any three or all four of its input variables are equal to 1.

- (5 points) Draw the truth table for the majority function.
- (5 points) Use a K-map to derive the minimal SOP expressions for the majority function.
- (5 points) Use a K-map to derive the minimal POS expressions for the majority function.

**P4. (10 points)** Use a K-map to derive the minimal SOP expressions for the following Boolean function:

$$F(A,B,C,D) = ACD' + C'D + AB' + ABCD$$

**P5. (20 points)** Design a circuit that accepts a 4-bit number  $X = x_3x_2x_1x_0$  as input and generates a 1-bit output  $P$  that is equal to 1 if the input number is a prime. (0 and 1 are not prime; 2, 3, 5, etc., are prime.)

- (10 points) Write down the truth table for the output  $P$ .
- (10 points) Derive the simplest SOP expressions for the output  $P$ .

**P6. (20 points)** Design a circuit that accepts a 3-bit number  $X = x_2x_1x_0$  as input and generates a 6-bit number  $Y = y_5y_4y_3y_2y_1y_0$  as output, which is equal to the square of the input number (i.e.,  $Y = X^2$ ).

- (10 points) Write down the truth table for the six output lines  $y_5y_4y_3y_2y_1y_0$  that jointly represent the number  $Y$  in binary.
- (10 points) Derive the simplest SOP expressions for each bit of the output. That is, derive six expressions: one for  $y_5$ , another for  $y_4$ , and so on.