Cpr E 281 HW05
ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Number Representation and Arithmetic Circuits
Assigned Date: Fifth Week
Due Date: Monday, Oct. 3, 2016

P1. (10 points) An expedition to Mars found the ruins of a civilization. The explorers were able to translate the mathematical equations:

$$5x^2 - 50x + 125 = 0$$

with the solutions: x=5 and x=8. The x=5 solution seemed okay, but x=8 was puzzling. The problem should be because Martians were using a non-decimal number system. Therefore, "50" is not fifty, but "50" in base b ($50b = 5\times b + 0\times 1 = 5b$). The explorers reflected on the way in which Earth's number system developed. How many fingers would you say the Martians had? (*Hint*: What should be the value of the base b such that both 5 and 8 are solutions of the equation?)

P2. (15 points) Complete the following table by converting the integers in decimal to **5-bit signed numbers** in binary. (1 point for each cell.)

	Decimal	Sign-and-Magnitude	1's Complement	2's Complement
Example	**-5**	**10101**	**11010**	**11011**
(a)	**-15**			
(b)	**-10**			
(c)	**-1**			
(d)	**0**			
(e)	**7**			

P3. (18 points) Perform the following conversions: (3 points each)

a) $(10011)_2$ in 5-bit sign-and-magnitude to 5-bit 1's complement
b) $(10011)_2$ in 5-bit sign-and-magnitude to 5-bit 2's complement
c) $(11000)_2$ in 5-bit 1's complement to 5-bit sign-and-magnitude
d) $(11000)_2$ in 5-bit 1's complement to 5-bit 2's complement
e) $(101110)_2$ in 6-bit 2's complement to 6-bit sign-and-magnitude
f) $(101110)_2$ in 6-bit 2's complement to 6-bit 1's complement

P4. (12 points) Negate the following **6-bit 2's complement** binary numbers: (3 points each)

a) $(001010)_2$
b) $(110011)_2$
c) $(100100)_2$
d) $(010001)_2$

P5. (10 points) Consider **4-bit 2's complement** representation for signed numbers.

a) (2 points) What is the largest integer in decimal that can be represented?
b) (2 points) What is the smallest integer in decimal that can be represented?
c) (6 points) For n-bit 2's complement representation, what are the largest and smallest integers in decimal that can be represented?

P6. (10 points) Perform the following additions of **5-bit unsigned numbers** in binary and identify if overflow occurs. Check your answers by converting the numbers to decimal.

a) (5 points) 01111 + 01010

```
    0 1 1 1 1
+   0 1 0 1 0
-------------
    □ □ □ □ □
```

b) (5 points) 11000 + 01101

```
    1 1 0 0 0
+   0 1 1 0 1
-------------
    □ □ □ □ □
```

P7. (15 points) Perform the following operations of **5-bit 2's complement numbers** in binary and identify if overflow occurs. Check your answers by converting the numbers to decimal.

a) (5 points) 01111 + 01010

```
    0 1 1 1 1
+   0 1 0 1 0
-------------
    □ □ □ □ □
```

b) (5 points) 11000 + 01101

```
    1 1 0 0 0
+   0 1 1 0 1
-------------
    □ □ □ □ □
```

c) (5 points) 01010 - 11101

```
    0 1 0 1 0
-   1 1 1 0 1
-------------
    □ □ □ □ □
```

P8. (10 points) Read Section 3.2 from the textbook. A full-adder (FA) can be constructed with two half-adders (HAs). From Figure 3.4 on page 129 one can infer that the carry-out function for a FA is given by:

$$c_{i+1} = x_i y_i + c_i (x_i \oplus y_i)$$

On the other hand, the textbook states on page 126 that the carry-out function for a FA is:

$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

Prove that these two functions are the same using Boolean algebra.