

CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Incompletely Specified Functions & Multiple-Output Circuits

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW4 is out**
- **It is due on Monday Sep 19 @ 4pm.**
- **Please write clearly on the first page (in block capital letters) the following three things:**
 - **Your First and Last Name**
 - **Your Student ID Number**
 - **Your Lab Section Letter**
- **Also, staple all of your pages together**

Administrative Stuff

- **Homework 5 is out, but it is not due for 2 more weeks.**
- **It is due on Oct 3.**

- **No HW due on Sep 26.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 23.**
- **Where: This classroom**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be open book and open notes (you can bring up to 3 pages of handwritten notes).**
- **Sample exams are posted on the class web page**

Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**

- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**

- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**

- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**

Quick Review

The Combining Theorems of Boolean Algebra

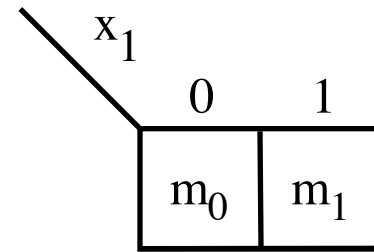
$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

One-Variable K-map

x_1	
0	m_0
1	m_1

(a) Truth table

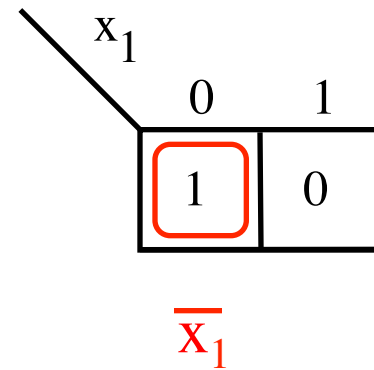


(b) Karnaugh map

One-Variable K-map

x_1	
0	1
1	0

(a) Truth table

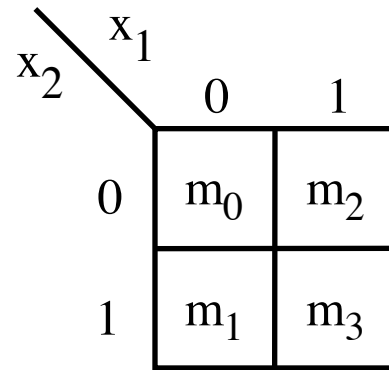


(b) Karnaugh map

Two-Variable K-map

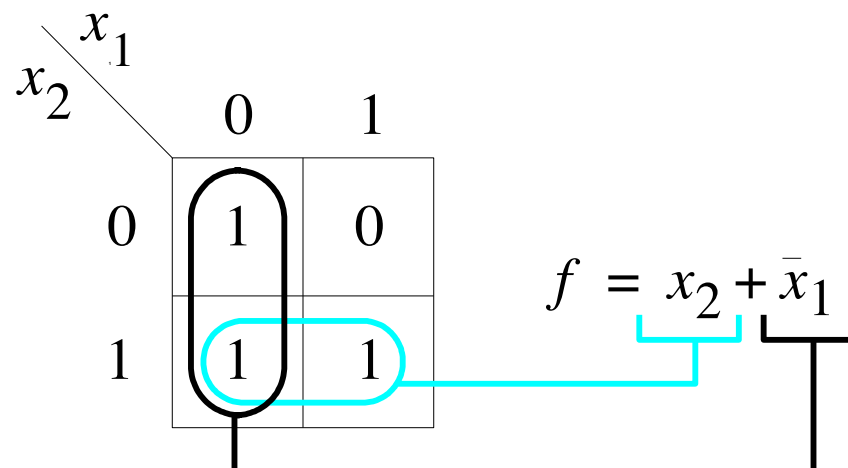
x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



(b) Karnaugh map

Two-Variable K-map



These are all valid groupings

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

Groupings: m_0, m_1 (vertical), m_2, m_3 (vertical)

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

Groupings: m_2, m_3 (vertical), m_0, m_1 (horizontal)

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

Groupings: m_0, m_2 (horizontal), m_1, m_3 (horizontal)

	x_1	0	1
x_2	0	m_0	m_2
	1	m_1	m_3

Groupings: m_1, m_3 (horizontal), m_0, m_2 (horizontal)

These are also valid

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

But try to use larger rectangles if possible.

These two are not valid

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

	x_1		
x_2		0	1
0		m_0	m_2
1		m_1	m_3

Three-Variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

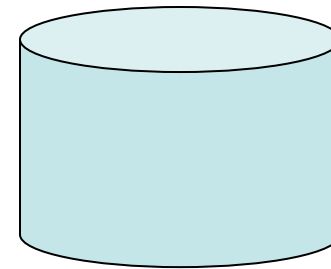
Notice the placement of

- **Variables**
- **Binary pair values**
- **Minterms**

Adjacency Rules

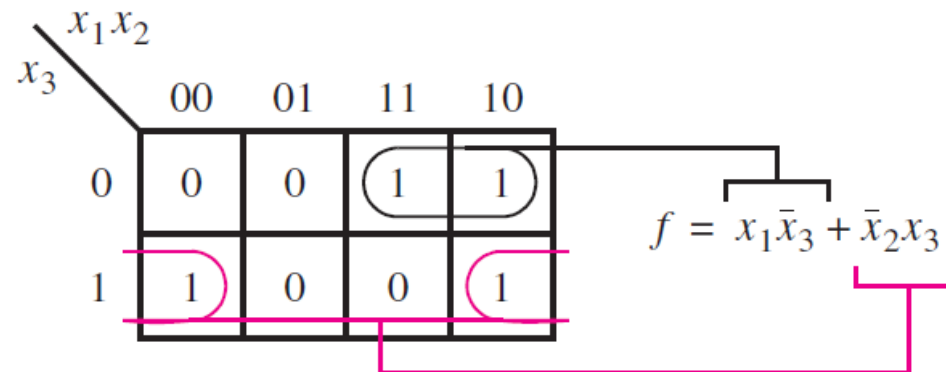
		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

adjacent
columns

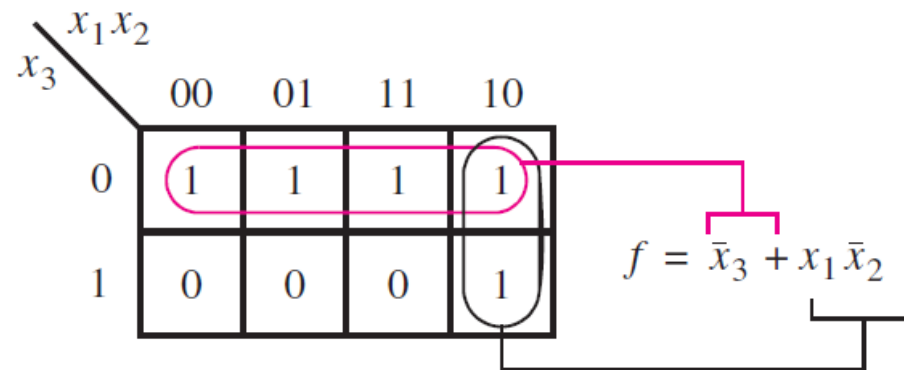


As if the K-map were
drawn on a cylinder

Three-Variable K-map



(a) The function of Figure 2.23



(b) The function of Figure 2.48

Two Different Ways to Draw the K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		x_2x_3			
		00	01	11	10
x_1	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

Another Way to Draw 3-variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		x_1	
		0	1
x_2x_3	00	m_0	m_4
	01	m_1	m_5
	11	m_3	m_7
	10	m_2	m_6

Gray Code

- **Sequence of binary codes**
- **Consecutive lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

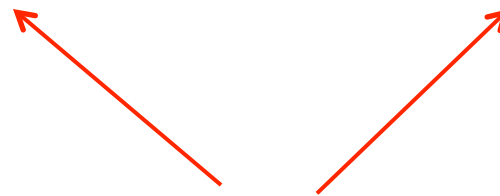
	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Adjacency Rules

$x_2 \backslash s x_1$	00	01	11	10
0	000	010	110	100
1	001	011	111	101



adjacent
columns

Gray Code & K-map

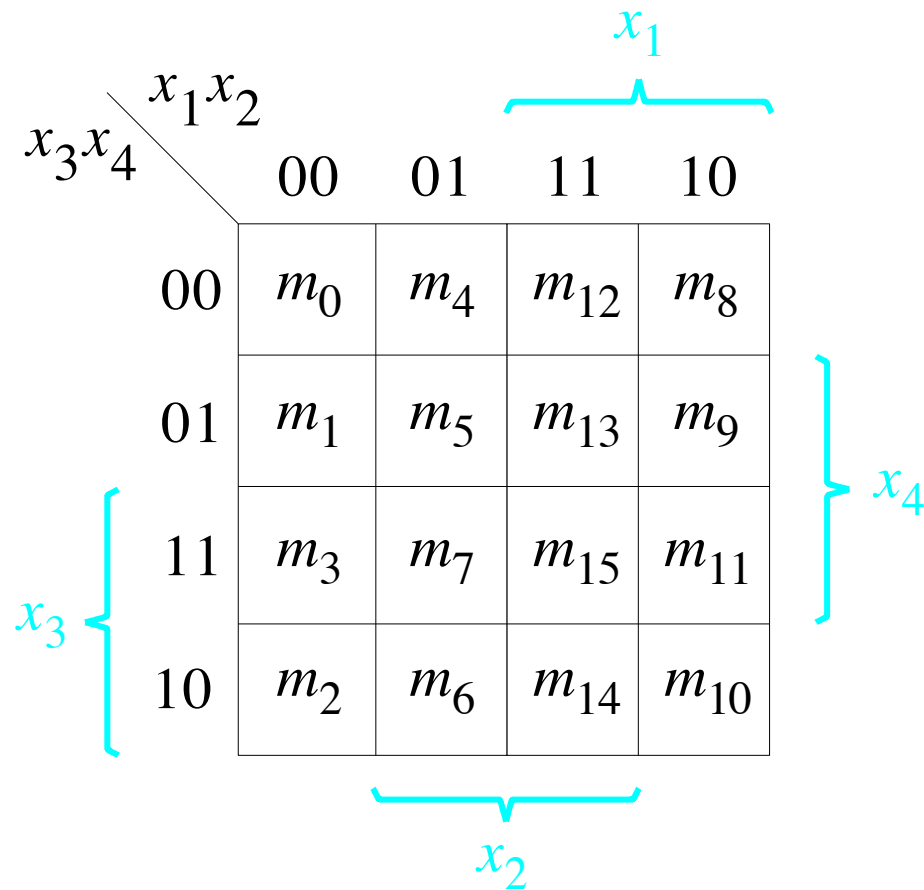
	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

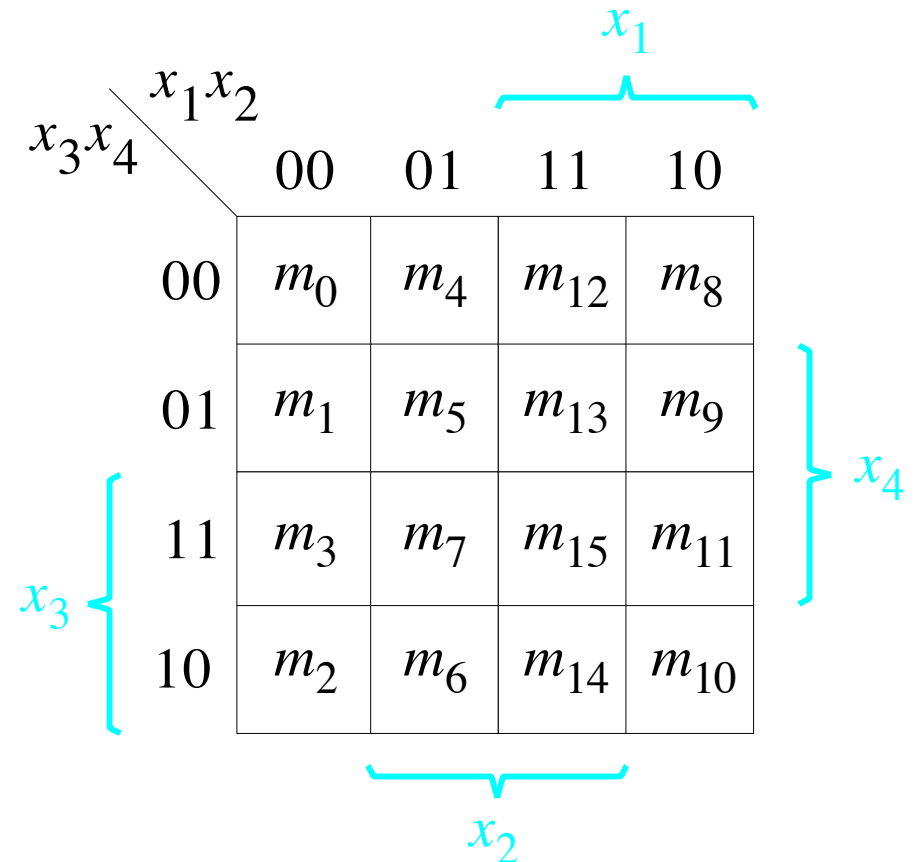
A four-variable Karnaugh map



[Figure 2.53 from the textbook]

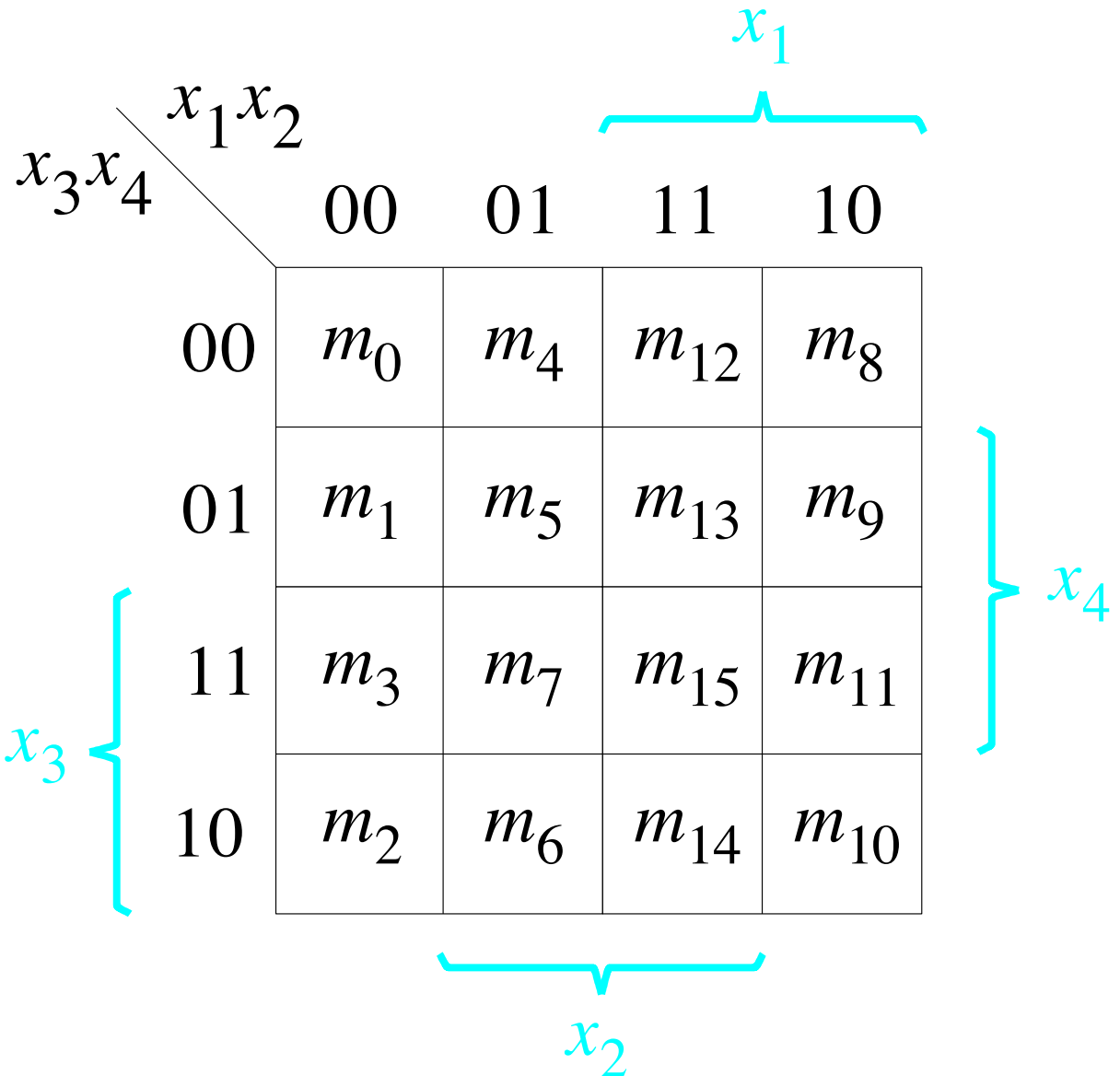
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



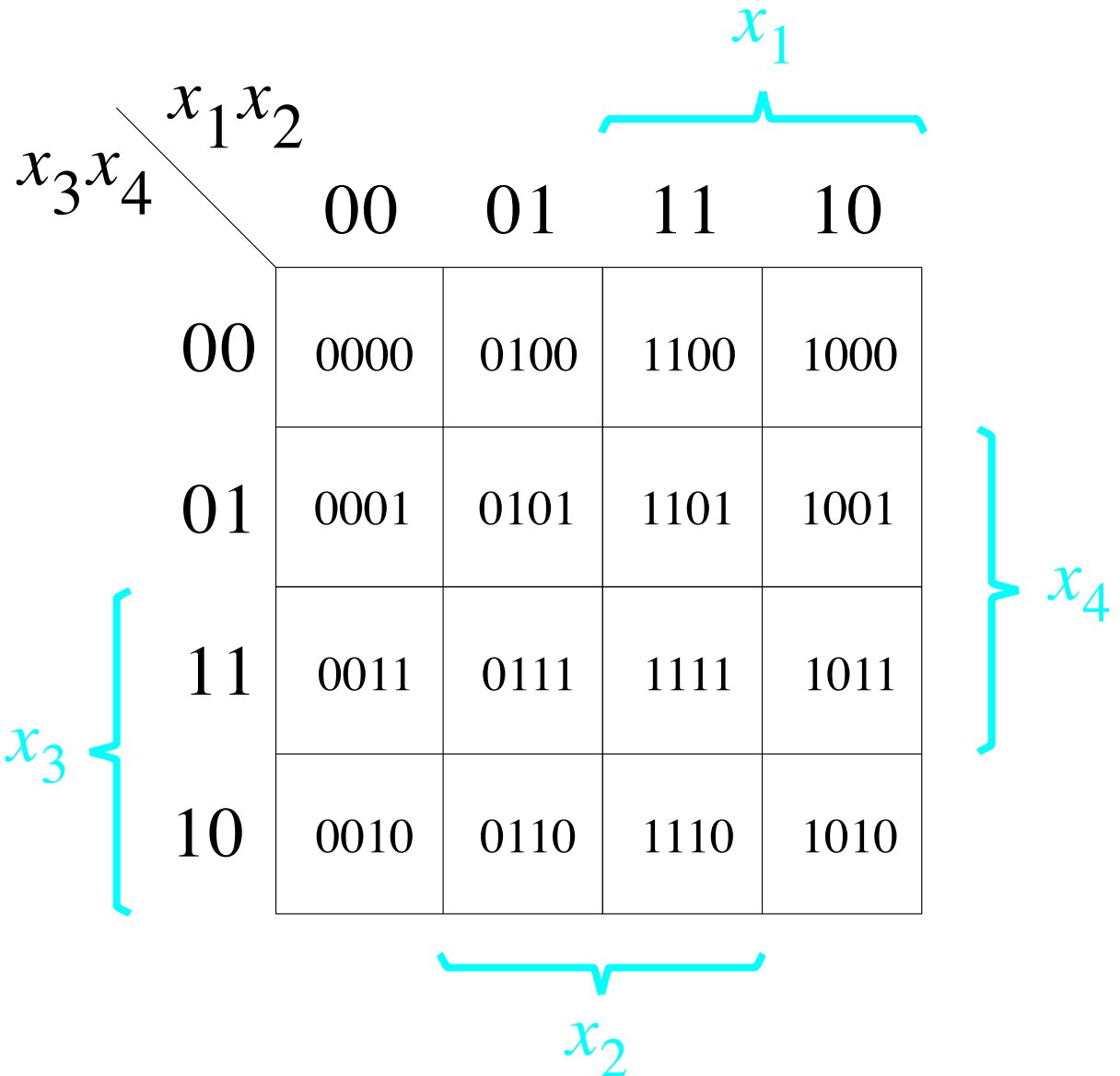
Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

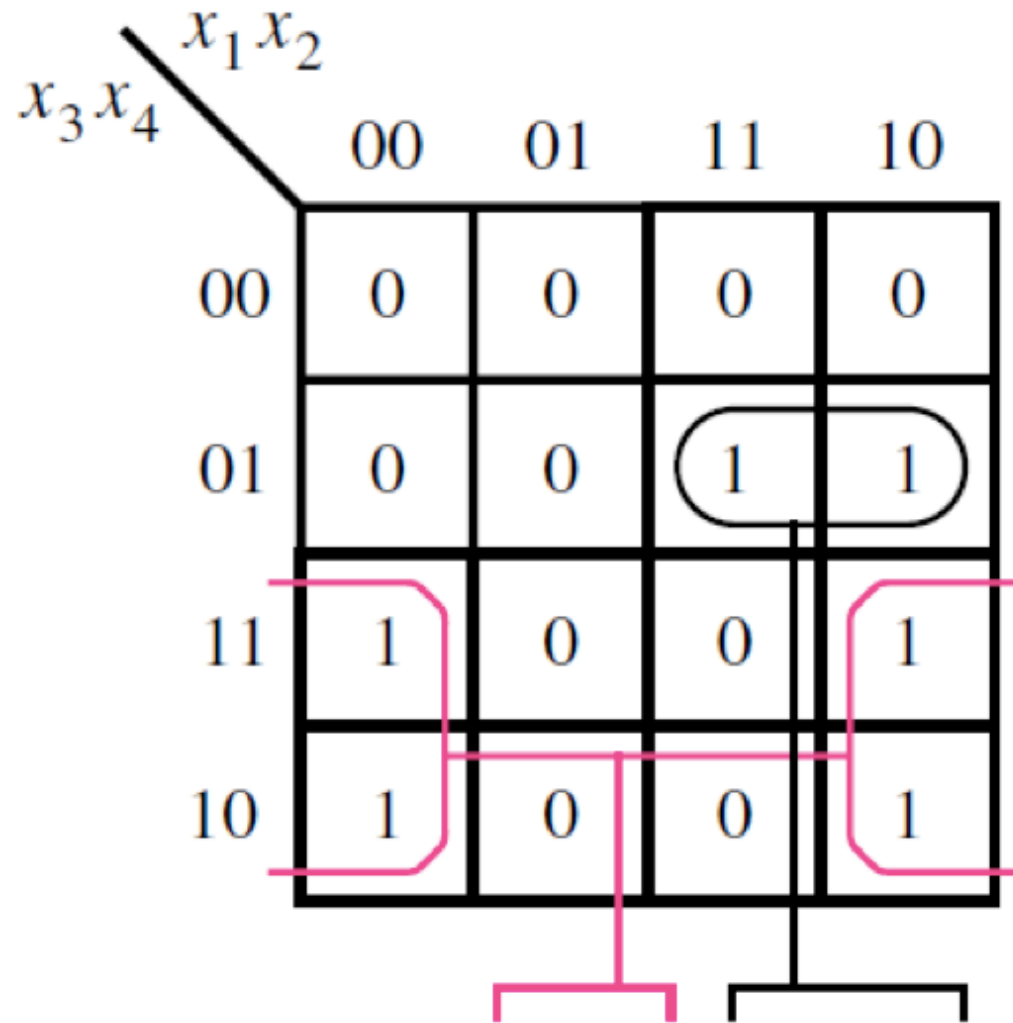
adjacent
columns

		x_1x_2			
		00	01	11	10
x_3x_4	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

adjacent
columns

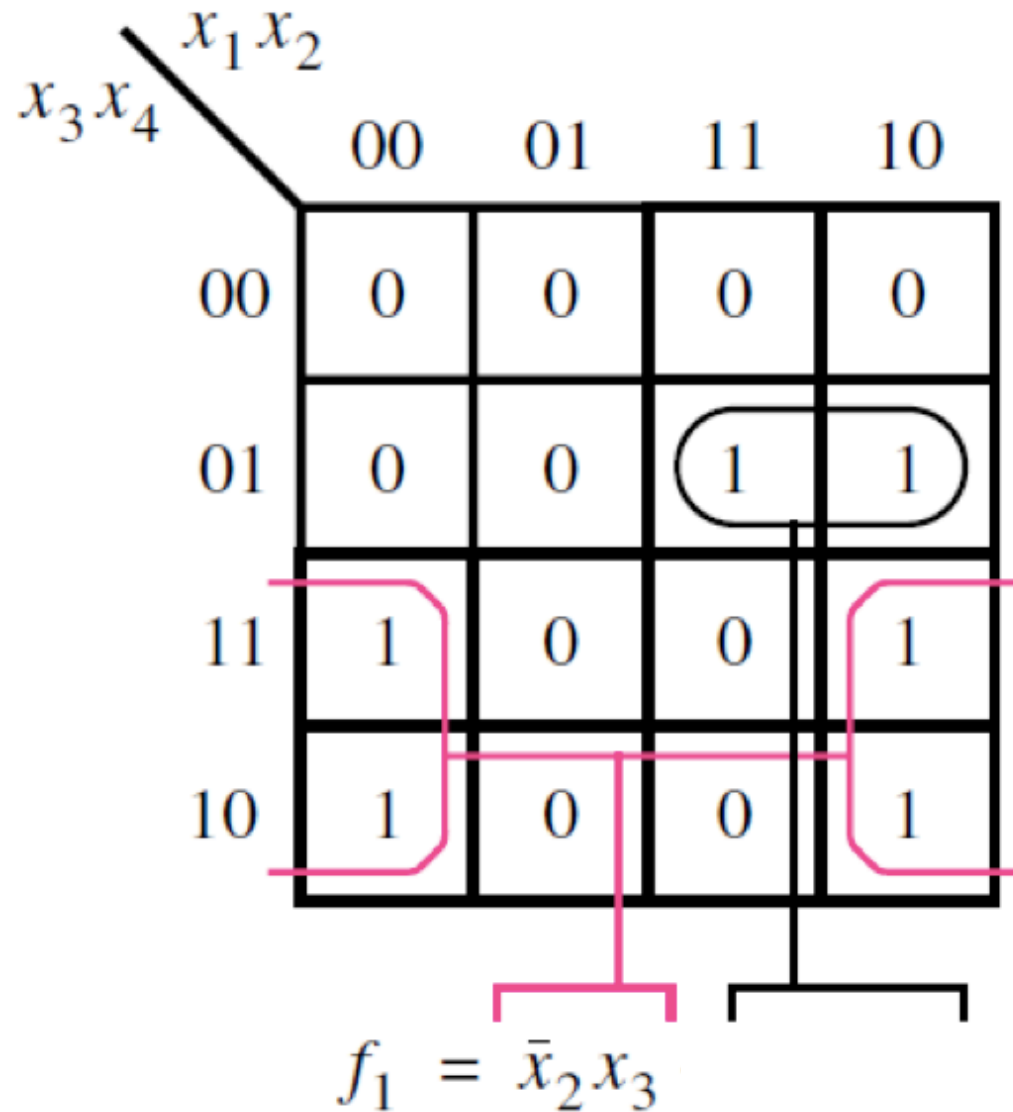
adjacent
rows

Example of a four-variable Karnaugh map



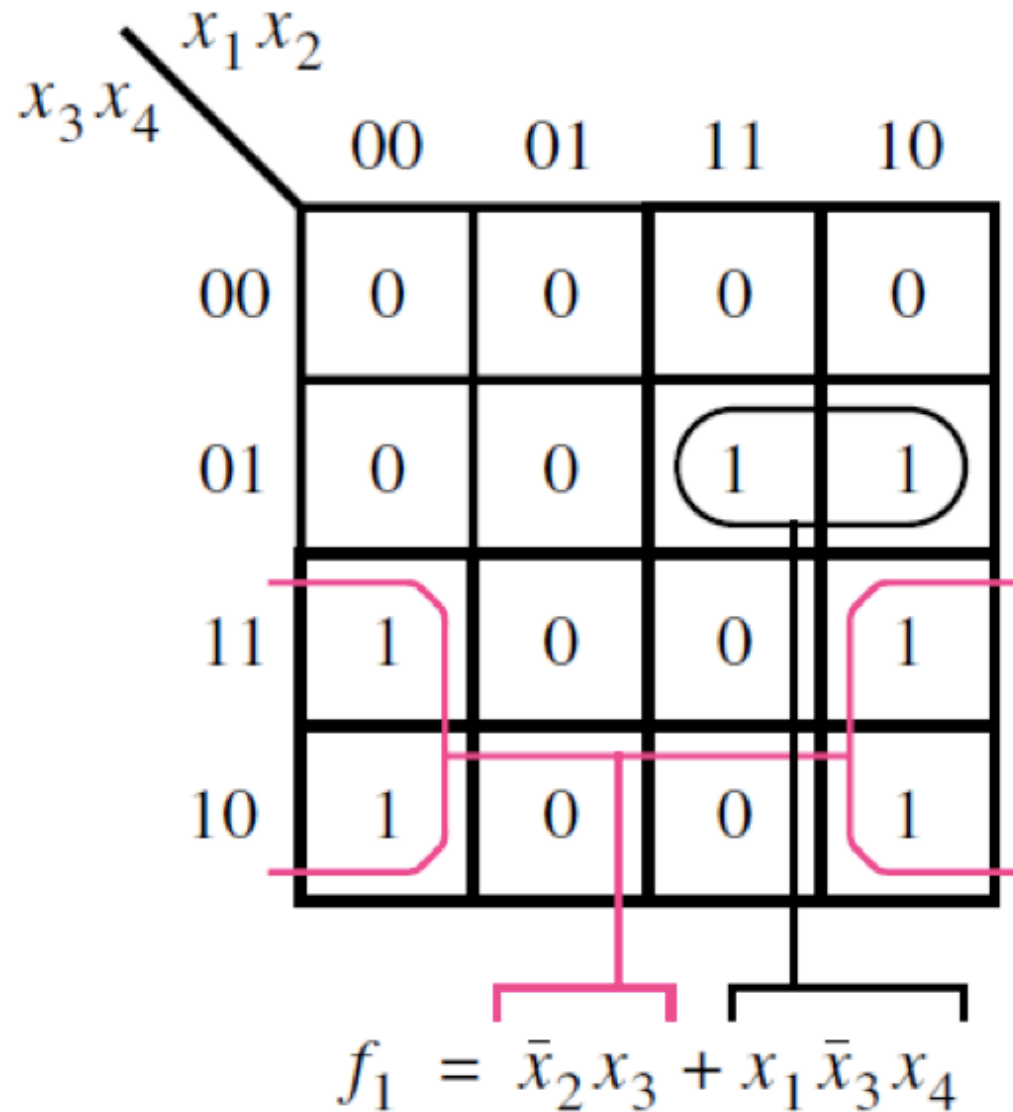
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map

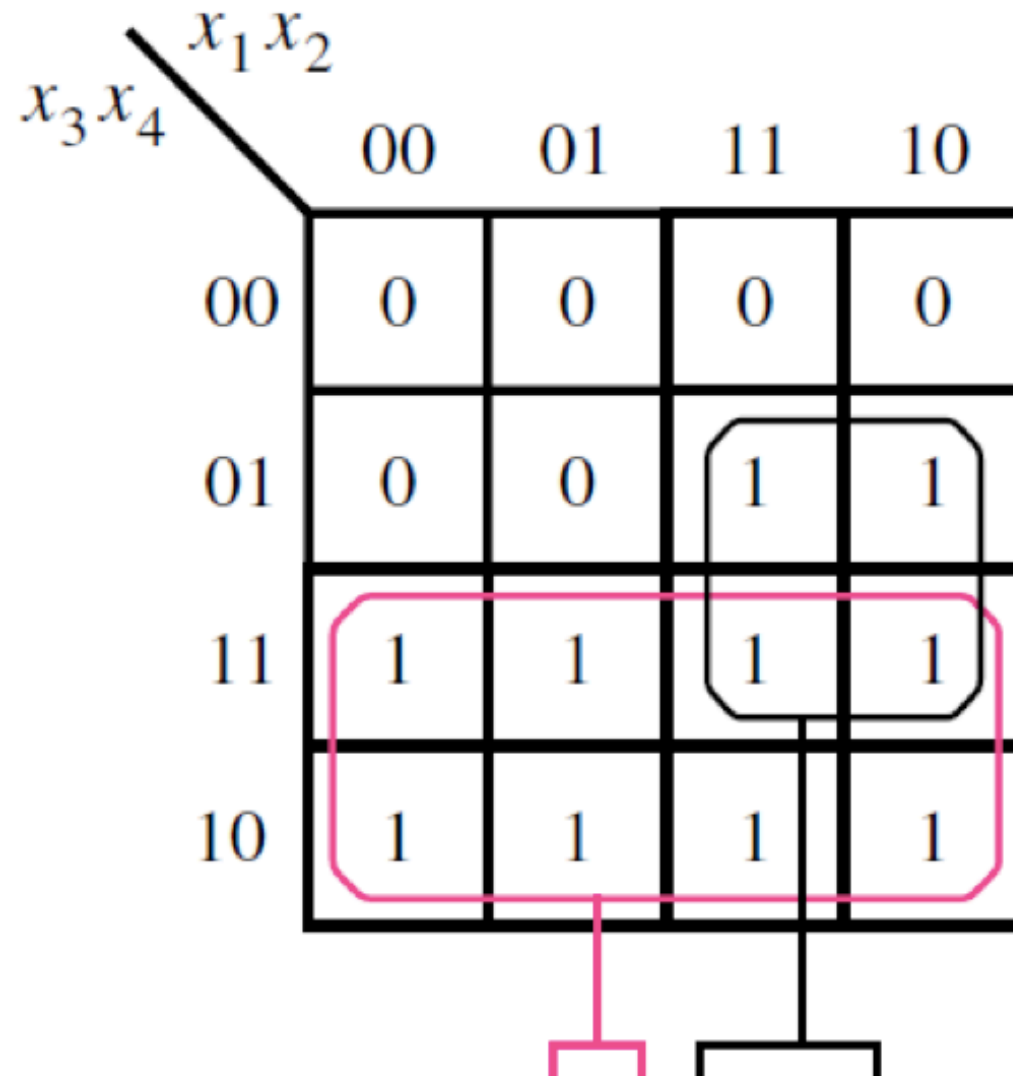


[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map

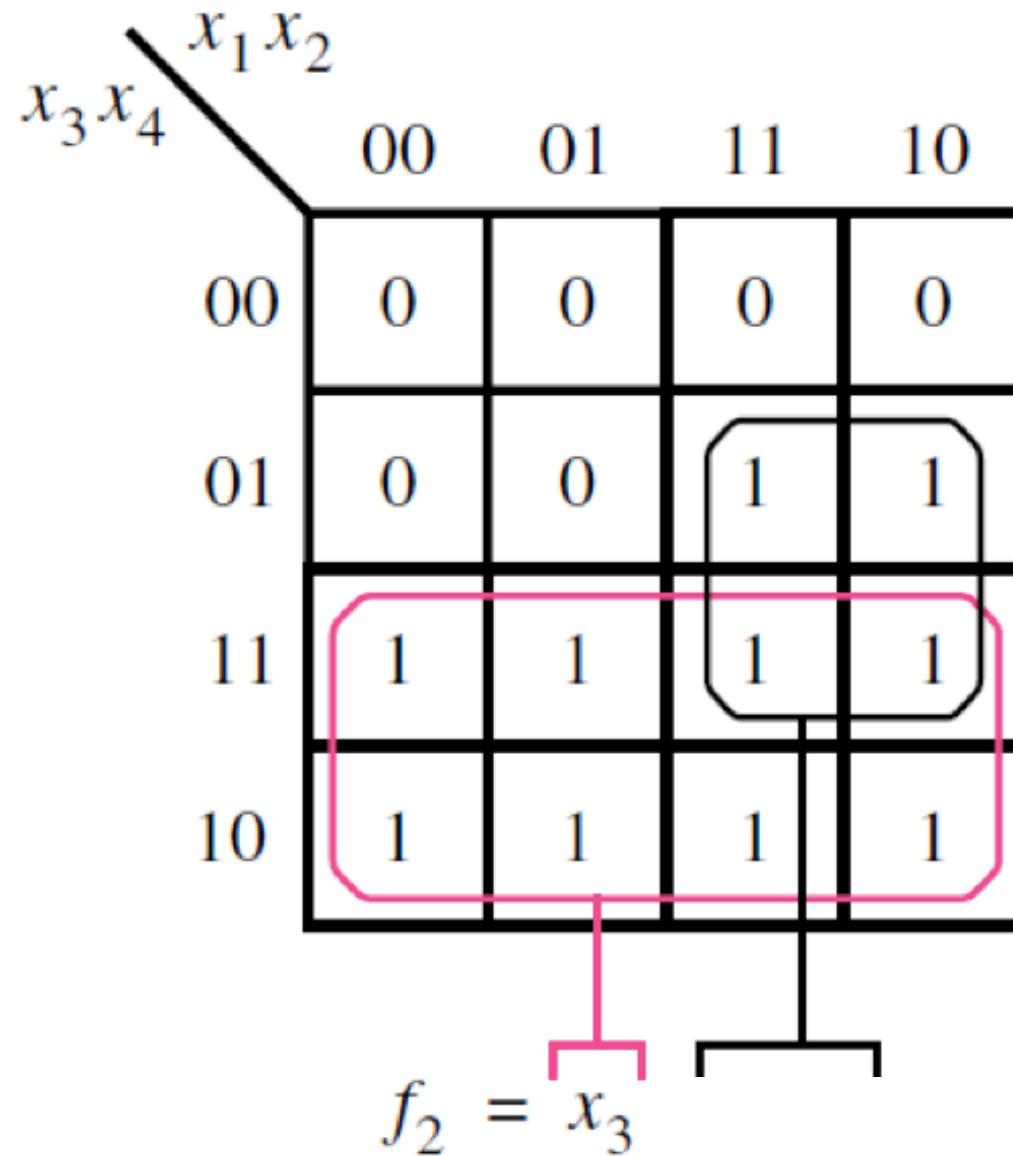


Example of a four-variable Karnaugh map



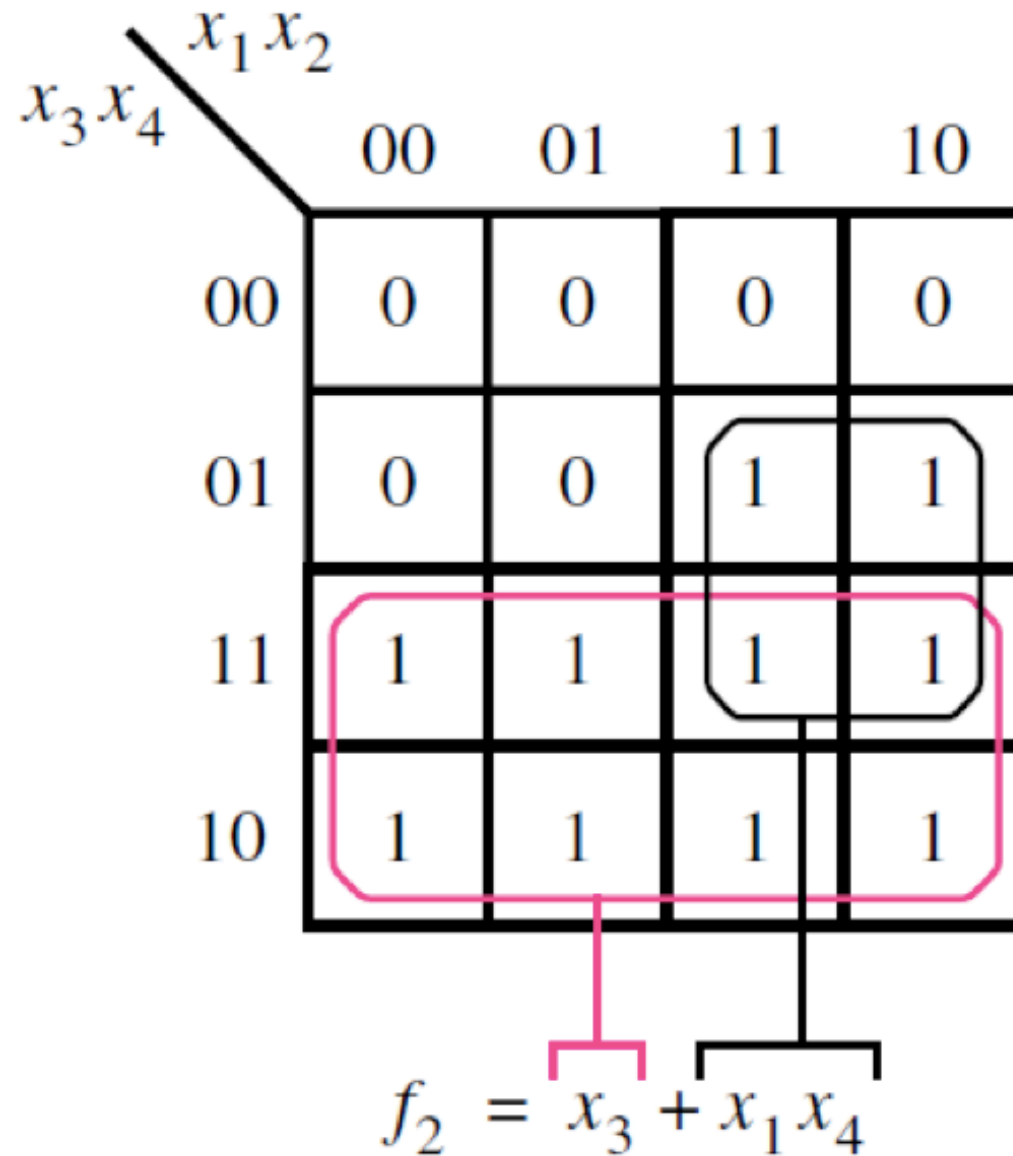
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



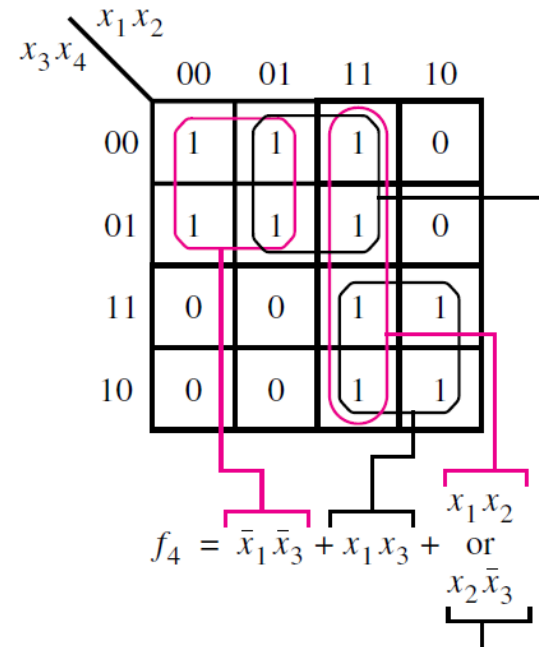
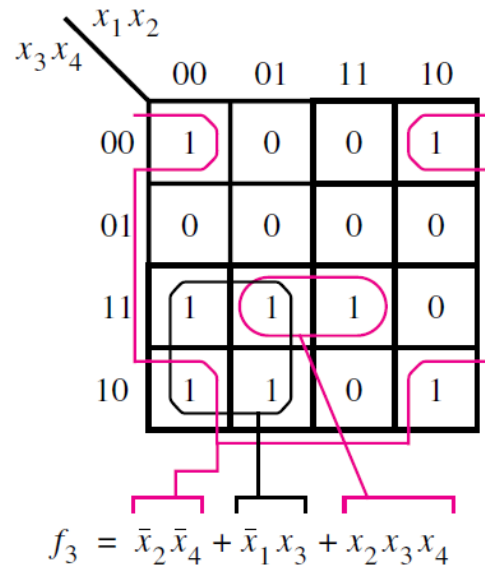
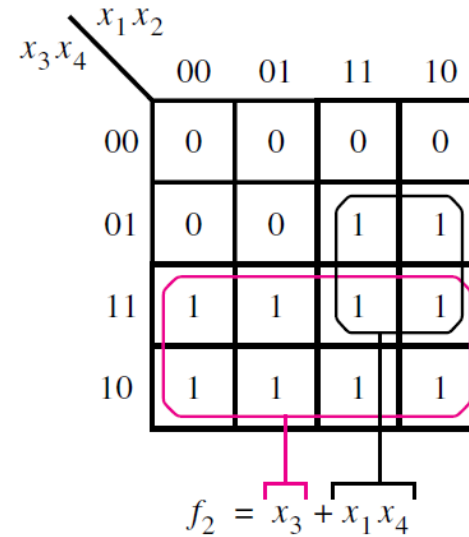
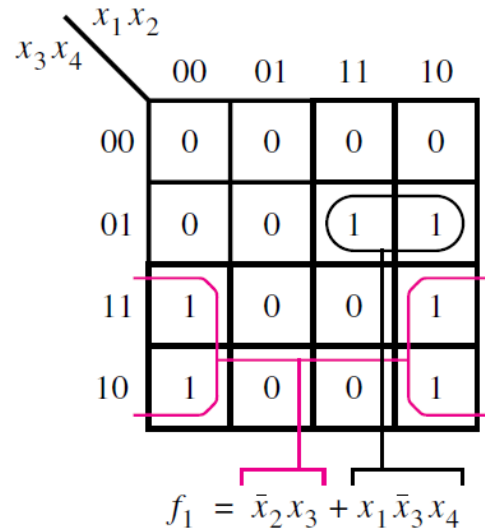
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Other Four-Variable K-map Examples



[Figure 2.54 from the textbook]

Strategy For Minimization

Grouping Rules

- **Group “1”s with rectangles**
- **Both sides a power of 2:**
 - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
 - **Try to use as few groups as possible to cover all “1”s.**
 - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don't use a 2x1 even if that is enough).**

Terminology

Literal: a variable, complemented or uncomplemented

Some Examples:

- \bar{X}_1
- X_2

Terminology

- **Implicant: product term that indicates the input combinations for which the function output is 1**

- **Example**

- \bar{x}_1 - indicates that $\bar{x}_1\bar{x}_2$ and \bar{x}_1x_2 yield output of 1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	0

Terminology

- **Prime Implicant**
 - Implicant that cannot be combined into another implicant with fewer literals
 - Some Examples

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	1	0

Not prime

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	1	0

Prime

Terminology

- **Essential Prime Implicant**
 - Prime implicant that includes a minterm not covered by any other prime implicant
 - Some Examples

		x_1x_2			
		00	01	11	10
x_3	0	0	1	1	1
	1	1	1	0	0

The Karnaugh map shows a 2x4 grid of cells. The columns are labeled 00, 01, 11, and 10 for x_1x_2 . The rows are labeled 0 and 1 for x_3 . The cells contain the following values: (0,00)=0, (0,01)=1, (0,11)=1, (0,10)=1, (1,00)=1, (1,01)=1, (1,11)=0, (1,10)=0. A blue circle highlights the 1s in the 01 column (minterms 010 and 101). A red circle highlights the 1s in the 11 column (minterms 011 and 101). Another red circle highlights the 1s in the 10 column (minterms 010 and 100). A third red circle highlights the 1s in the 00 column (minterms 100 and 110).

Terminology

- **Cover**

- **Collection of implicants that account for all possible input valuations where output is 1**

- **Ex. $x_1' x_2 x_3 + x_1 x_2 x_3' + x_1 x_2' x_3'$**

- **Ex. $x_1' x_2 x_3 + x_1 x_3'$**

		$x_1 x_2$			
		00	01	11	10
x_3	0	0	0	1	1
	1	0	1	0	0

Example

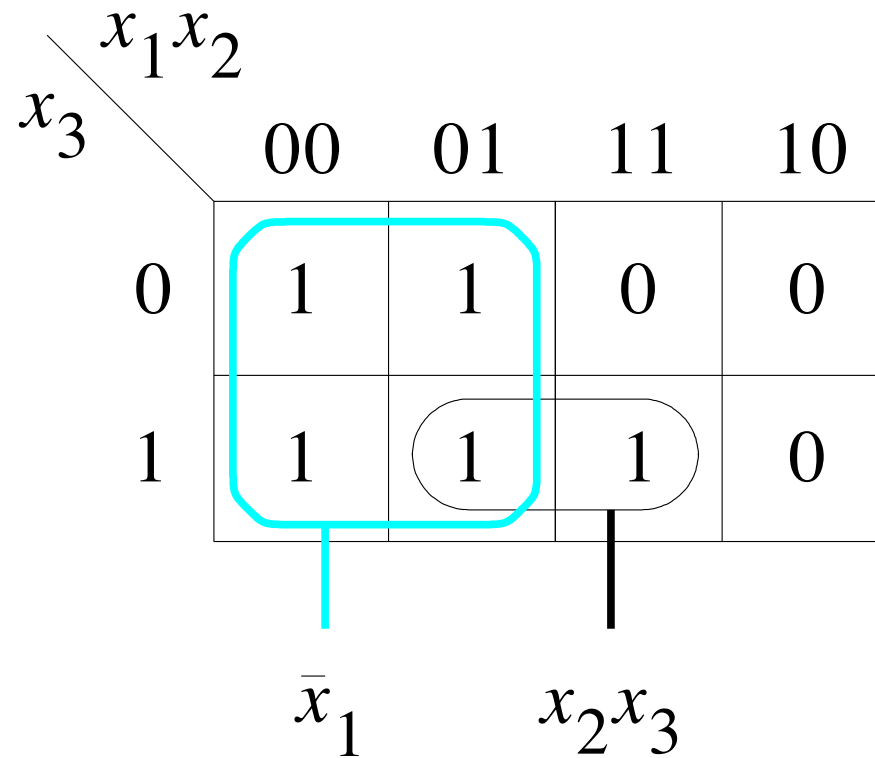
- Give the Number of
 - Implicants?
 - Prime Implicants?
 - Essential Prime Implicants?

$x_3 \backslash x_1 x_2$	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Why concerned with minimization?

- **Simplified function**
- **Reduce the cost of the circuit**
 - **Cost: Gates + Inputs**
 - **Transistors**

Three-variable function $f(x_1, x_2, x_3) = \Sigma m(0, 1, 2, 3, 7)$

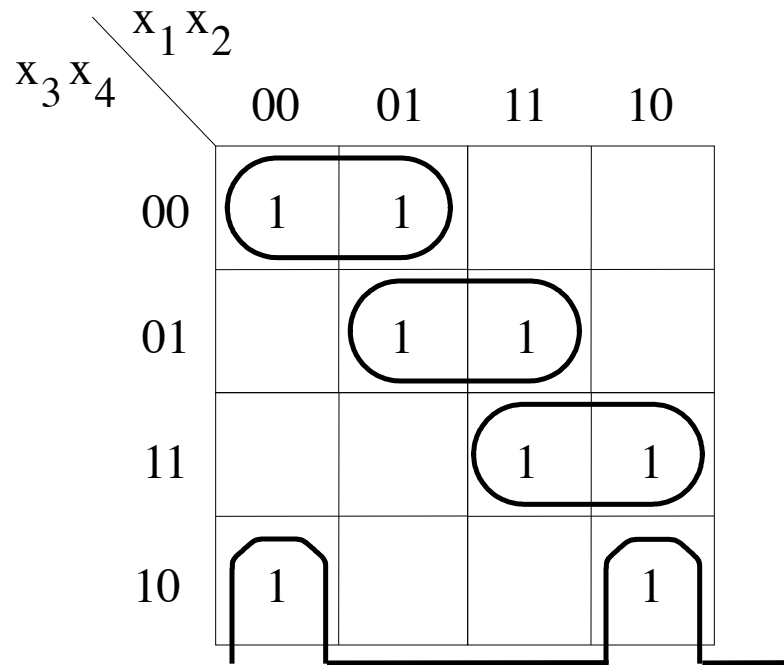


[Figure 2.56 from the textbook]

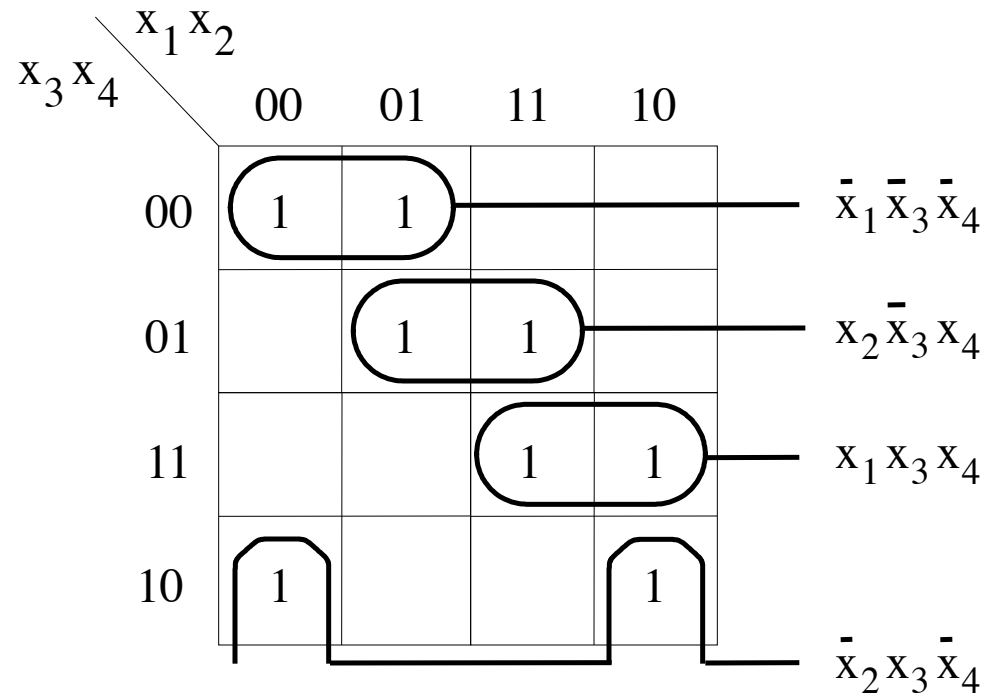
Example

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	1	1		
	01		1	1	
	11			1	1
	10	1			1

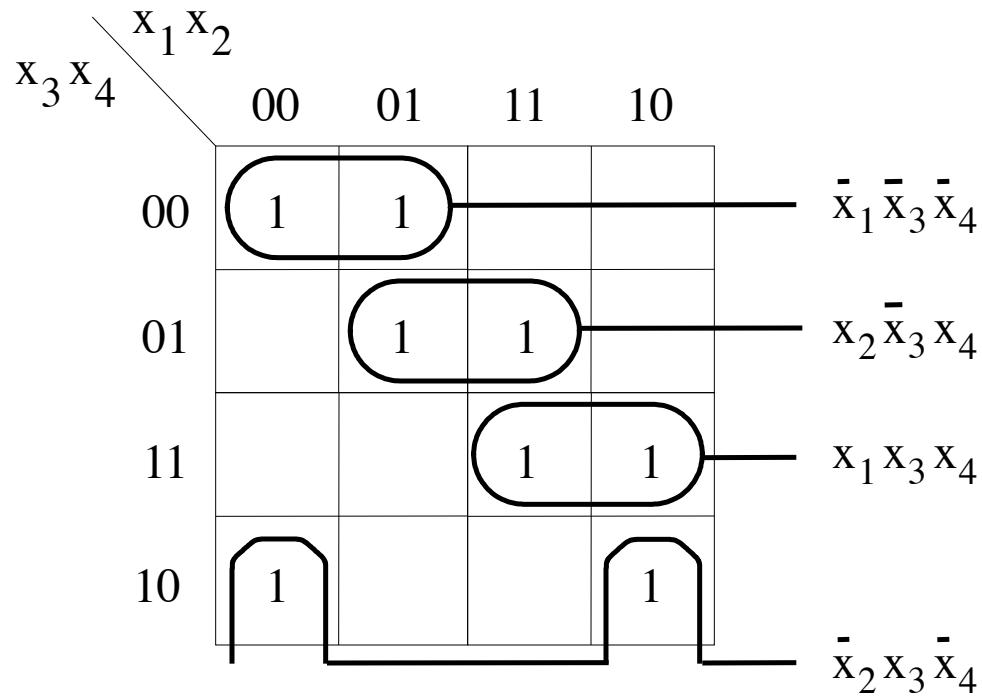
Example



Example



Example

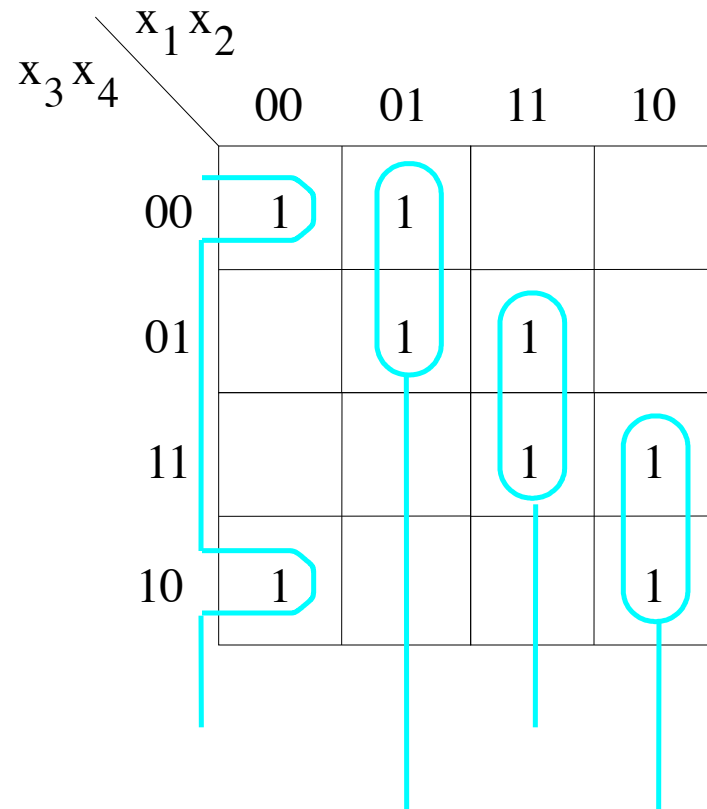


$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

Example: Another Solution

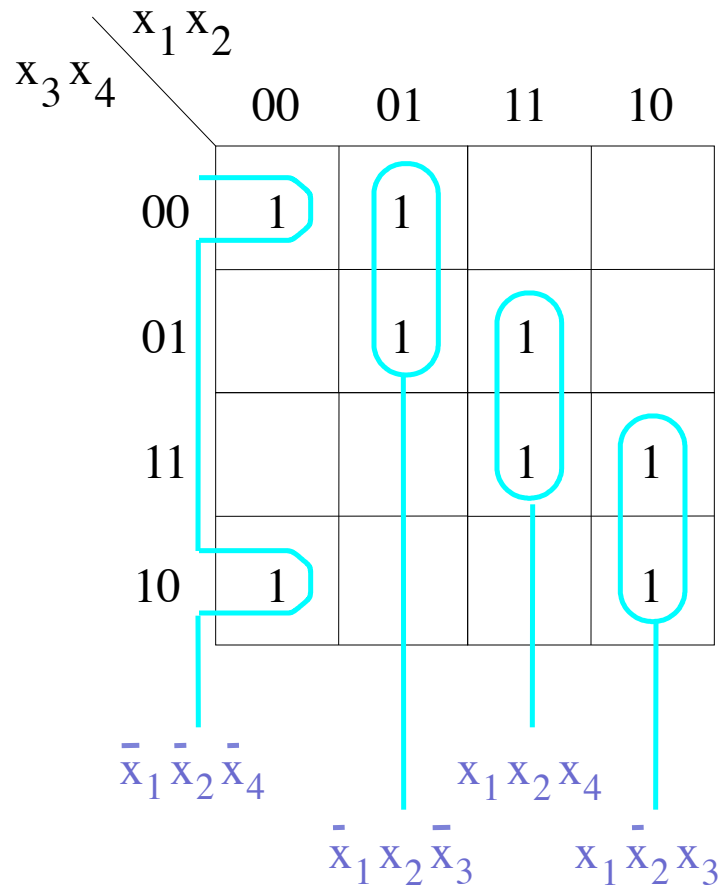
$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	1	1		
01		1	1	
11			1	1
10	1			1

Example: Another Solution

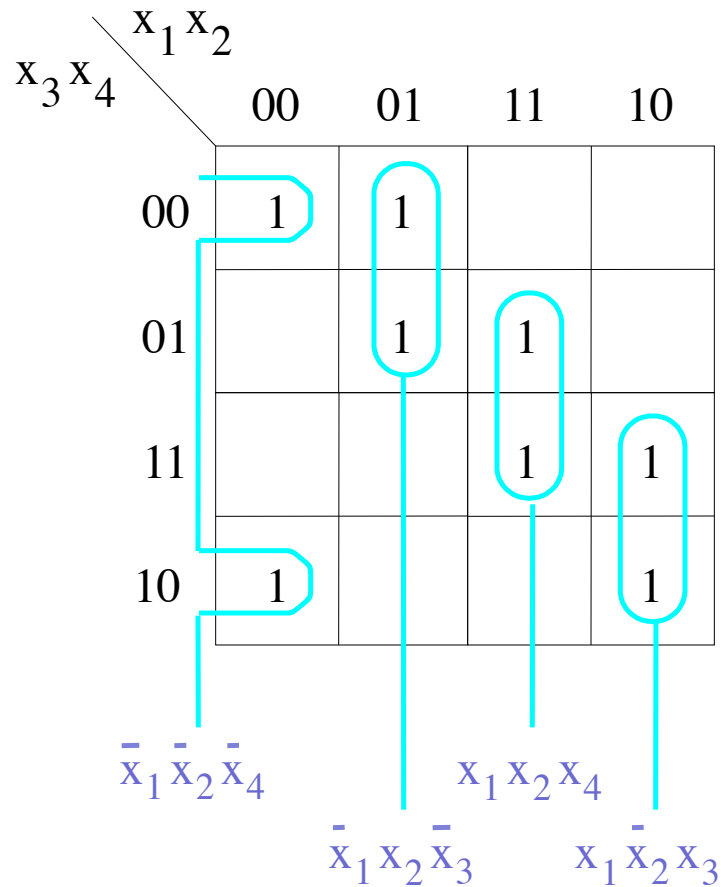


[Figure 2.59 from the textbook]

Example: Another Solution

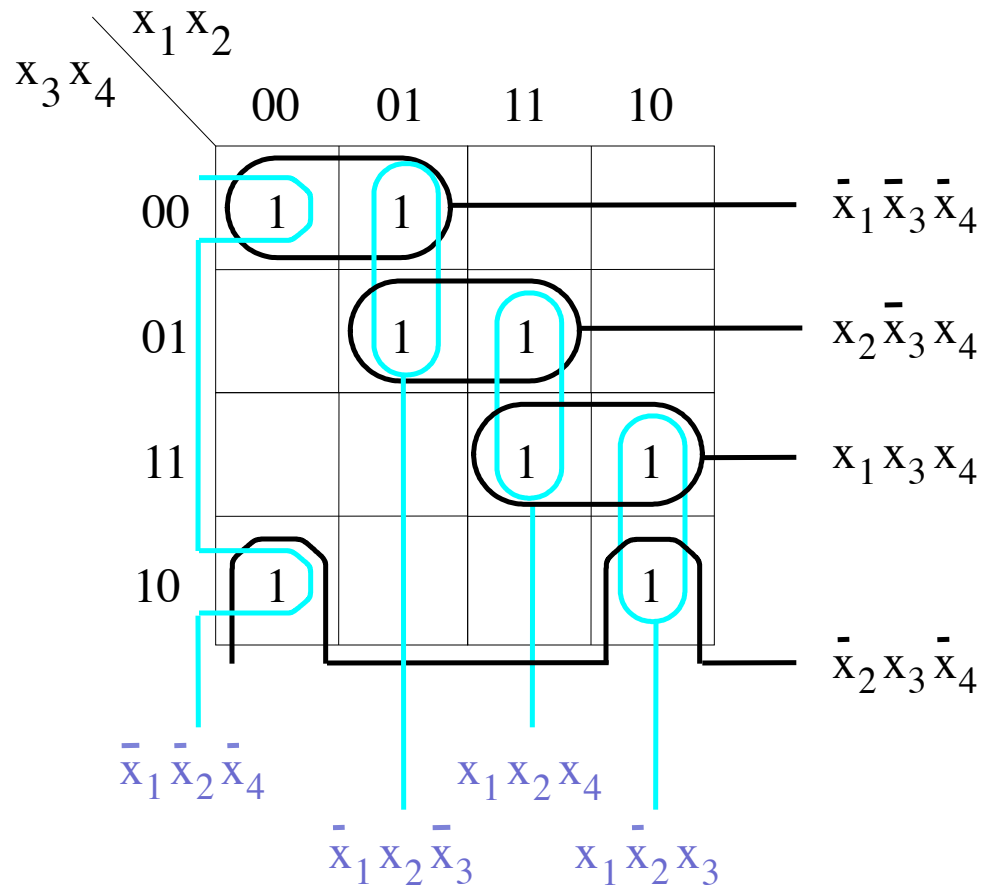


Example: Another Solution



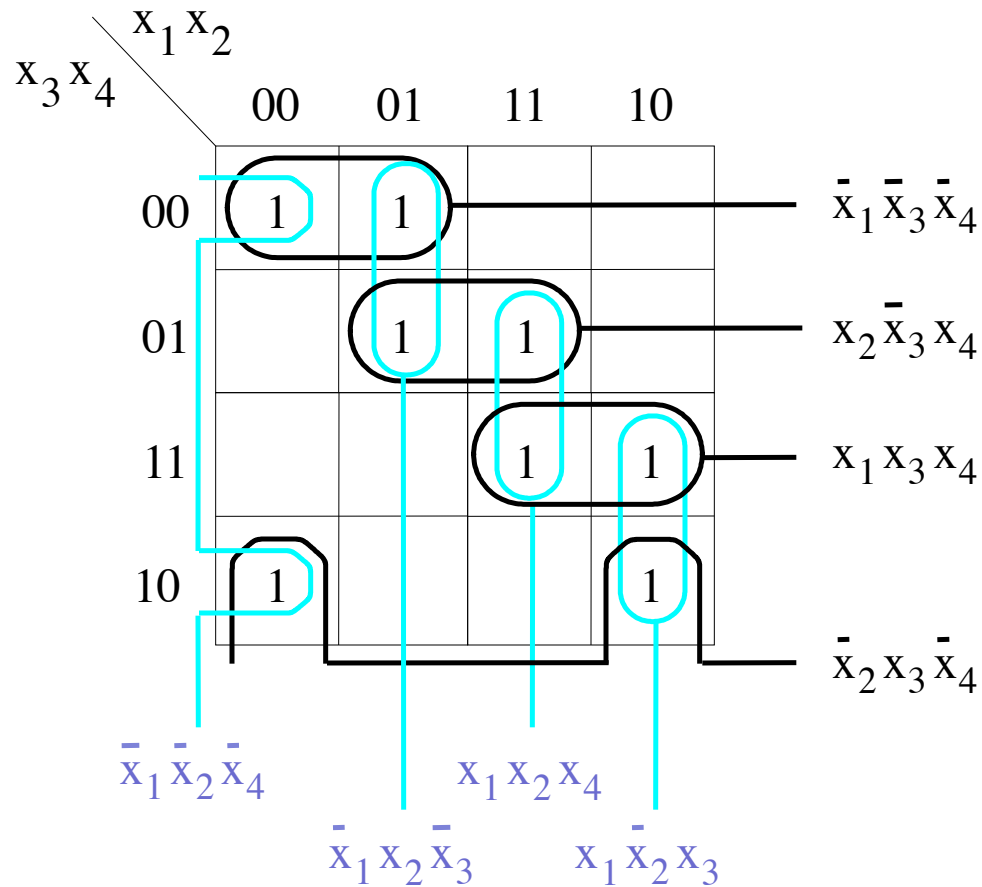
$$f = \bar{x}_1 \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

Example: Another Solution



[Figure 2.59 from the textbook]

Example: Another Solution



$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

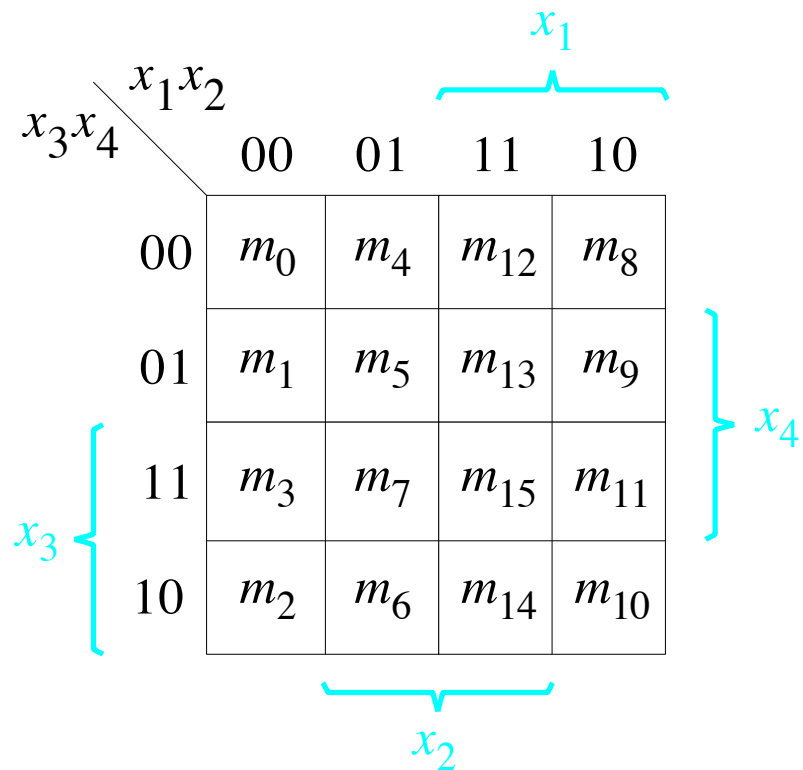
$$f = \bar{x}_1 \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

Example:
Incompletely Specified Function

Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1	x_2	x_3	x_4	f
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}



Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1	x_2	x_3	x_4	f
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}

		x_1x_2			
		00	01	11	10
x_3x_4	00	0	1	d	0
	01	0	1	d	0
	11	0	0	d	0
	10	1	1	d	1

SOP implementation

A Karnaugh map for a 4-variable function with variables x_1, x_2, x_3, x_4 . The horizontal axis represents x_1x_2 with values 00, 01, 11, 10. The vertical axis represents x_3x_4 with values 00, 01, 11, 10. The map contains the following values:

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

Two prime implicants are circled in cyan:

- A vertical rectangle covering the cells (01, 00), (01, 01), (01, 11), and (01, 10), labeled $x_2\bar{x}_3$.
- A horizontal rectangle covering the cells (10, 00), (10, 01), (10, 11), and (10, 10), labeled $x_3\bar{x}_4$.

(a) SOP implementation

POS implementation

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

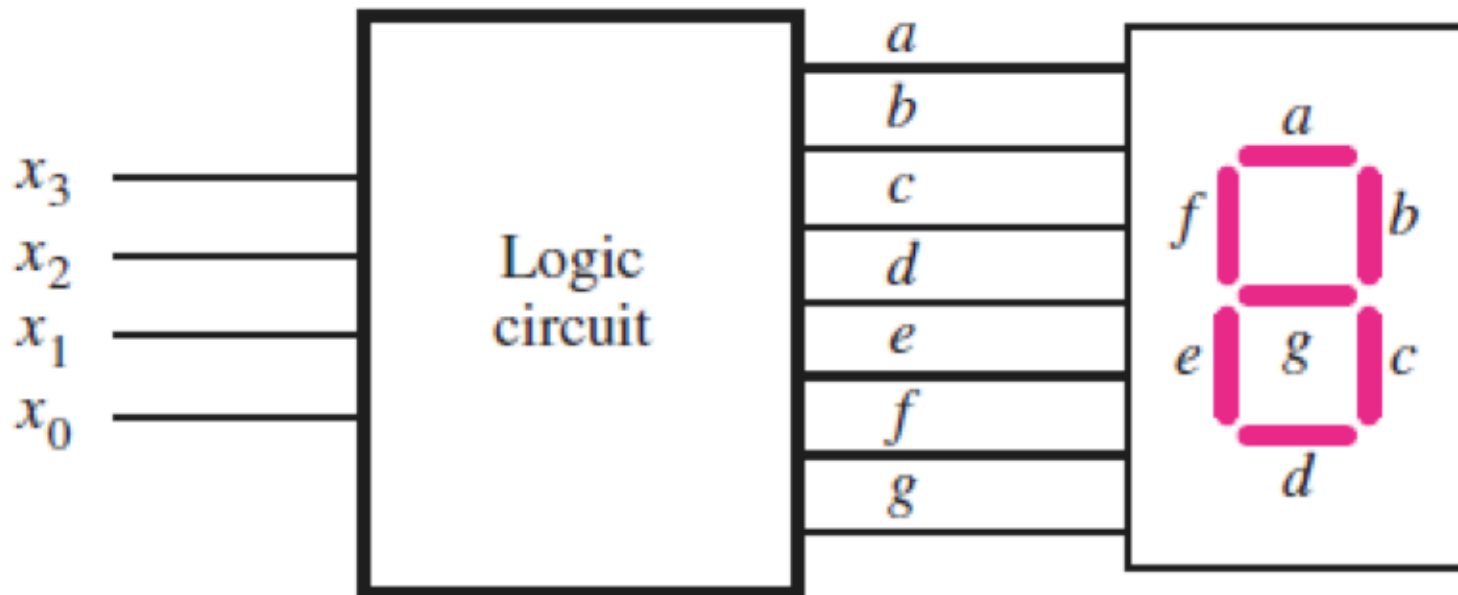
$(x_2 + x_3)$

$(\bar{x}_3 + \bar{x}_4)$

(b) POS implementation

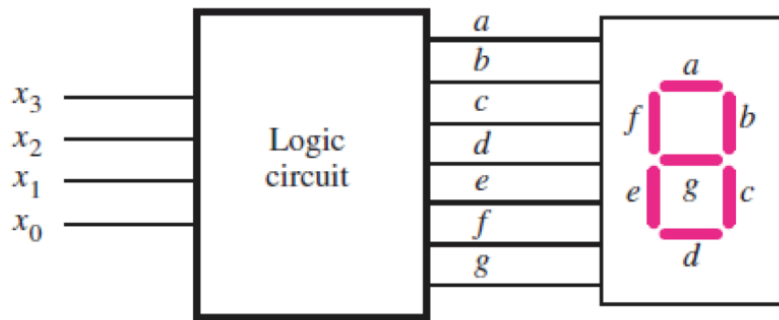
Example:
A circuit with multiple outputs

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0							
1011	1	0	1	1							
1100	1	1	0	0							
1101	1	1	0	1							
1110	1	1	1	0							
1111	1	1	1	1							

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000-1001	0	0	0	0	1	1	1	1	1	1	0
	0	0	0	1	0	1	1	0	0	0	0
	0	0	1	0	1	1	0	1	1	0	1
	0	0	1	1	1	1	1	1	0	0	1
	0	1	0	0	0	1	1	0	0	1	1
	0	1	0	1	1	0	1	1	0	1	1
	0	1	1	0	1	0	1	1	1	1	1
	0	1	1	1	1	1	1	0	0	0	0
	1	0	0	0	1	1	1	1	1	1	1
	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000-1001	0	0	0	0	1	1	1	1	1	1	0
	0	0	0	1	0	1	1	0	0	0	0
	0	0	1	0	1	1	0	1	1	0	1
	0	0	1	1	1	1	1	1	0	0	1
	0	1	0	0	0	1	1	0	0	1	1
	0	1	0	1	1	0	1	1	0	1	1
	0	1	1	0	1	0	1	1	1	1	1
	0	1	1	1	1	1	1	1	0	0	0
	1	0	0	0	1	1	1	1	1	1	1
	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
	1	0	1	1	d	d	d	d	d	d	d
	1	1	0	0	d	d	d	d	d	d	d
	1	1	0	1	d	d	d	d	d	d	d
	1	1	1	0	d	d	d	d	d	d	d
	1	1	1	1	d	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00				
01				
11				
10				

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d

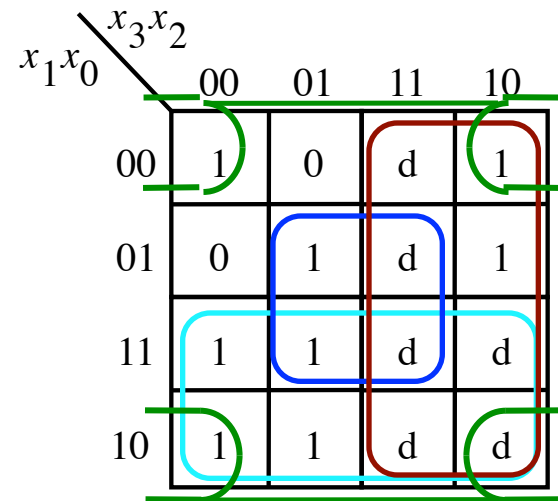
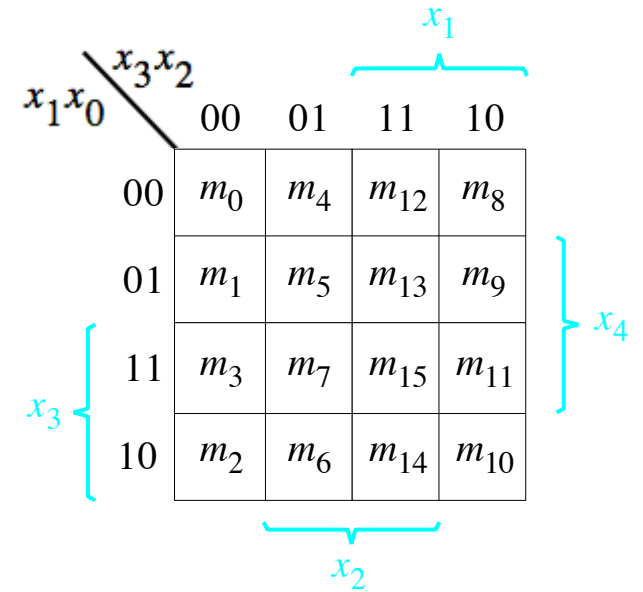
$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	1	0	d	1
01	0	1	d	1
11	1	1	d	d
10	1	1	d	d

Seven-Segment Indicator

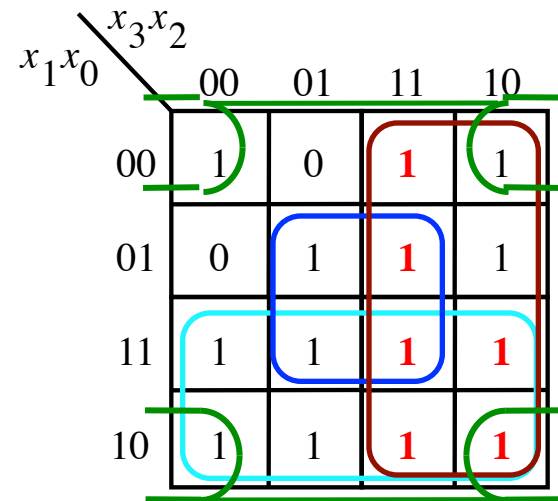
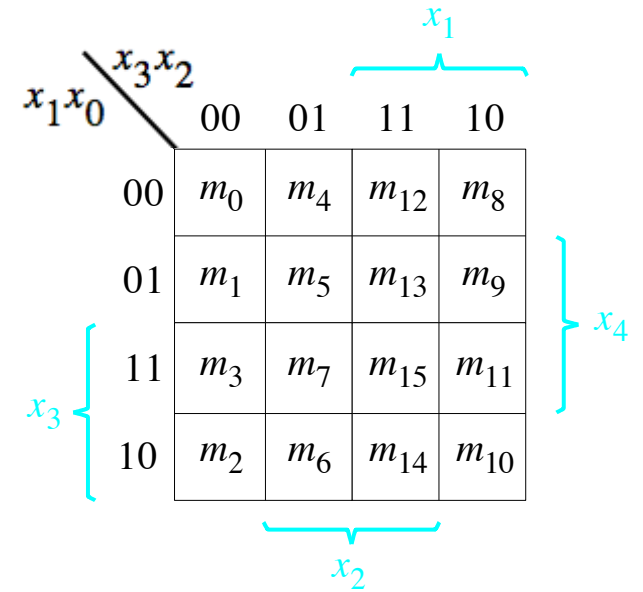
C
O
M
B
I
N
E
D
C
O
D
E

x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d



Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d



In this case all d's were treated as 1's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000-1001	0	0	0	0	1	1	1	1	1	1	0
	0	0	0	1	0	1	1	0	0	0	0
	0	0	1	0	1	1	0	1	1	0	1
	0	0	1	1	1	1	1	1	0	0	1
	0	1	0	0	0	1	1	0	0	1	1
	0	1	0	1	1	0	1	1	0	1	1
	0	1	1	0	1	0	1	1	1	1	1
	0	1	1	1	1	1	1	0	0	0	0
	1	0	0	0	1	1	1	1	1	1	1
	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	1	d	d	d	d	d	d
	1	0	1	1	1	d	d	d	d	d	d
	1	1	0	0	1	d	d	d	d	d	d
	1	1	0	1	1	d	d	d	d	d	d
	1	1	1	0	1	d	d	d	d	d	d
	1	1	1	1	1	d	d	d	d	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d

		x_1			
		00	01	11	10
x_3	x_3x_2	00	01	11	10
	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}	

		x_1			
		00	01	11	10
x_3	x_3x_2	00	01	11	10
	00				
	01				
	11				
	10				

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d

		x_3x_2			
		00	01	11	10
x_1x_0	00	m_0	m_4	m_{12}	m_8
	01	m_1	m_5	m_{13}	m_9
	11	m_3	m_7	m_{15}	m_{11}
	10	m_2	m_6	m_{14}	m_{10}

		x_3x_2			
		00	01	11	10
x_1x_0	00	1	0	d	1
	01	0	0	d	0
	11	0	0	d	d
	10	1	1	d	d

Seven-Segment Indicator

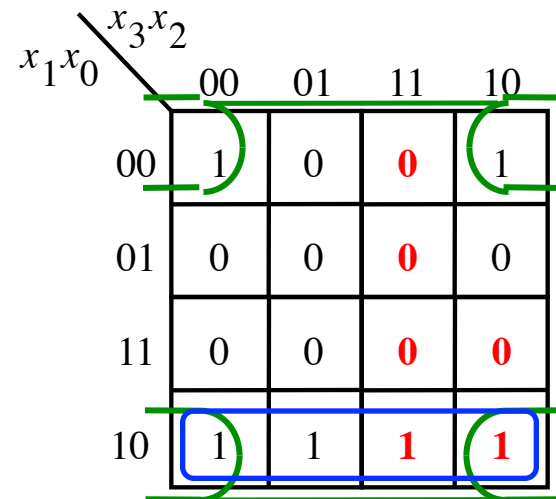
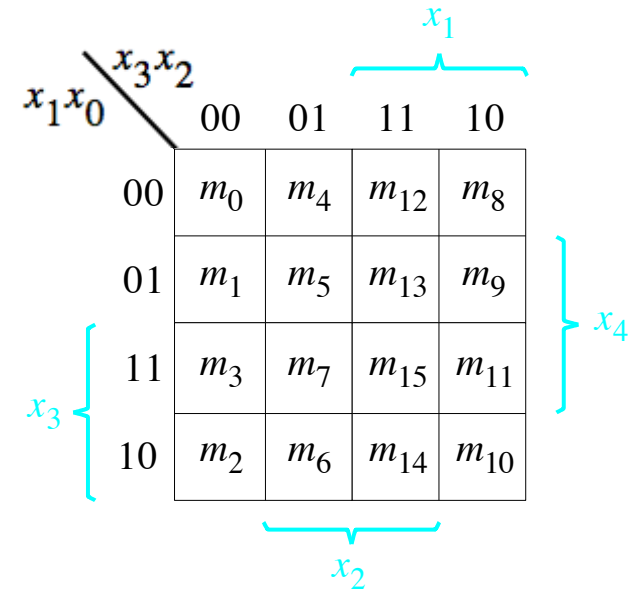
	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

$x_1x_0 \backslash x_3x_2$	00	01	11	10
00	1	0	d	1
01	0	0	d	0
11	0	0	d	d
10	1	1	d	d

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	1	d	d
1011	1	0	1	1	1	d	d	d	0	d	d
1100	1	1	0	0	1	d	d	d	0	d	d
1101	1	1	0	1	1	d	d	d	0	d	d
1110	1	1	1	0	1	d	d	d	1	d	d
1111	1	1	1	1	1	d	d	d	0	d	d

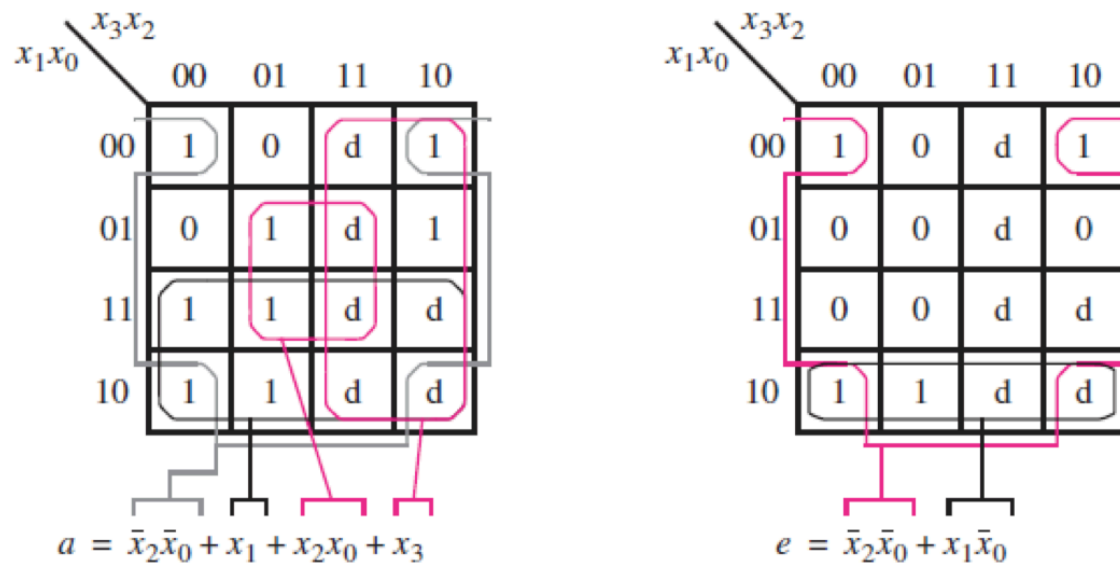


In this case some d's were treated as 1's, others as 0's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table



(c) The Karnaugh maps for outputs a and e .

Another Example

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

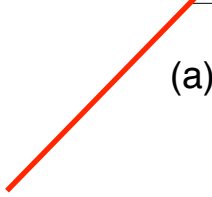
	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$\bar{x}_1 x_3$



	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$\bar{x}_1 x_3$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\overline{x_1} x_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$$\overline{x_1} x_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\overline{x_1} x_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$$\overline{x_1} x_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\bar{X}_1 \bar{X}_3$$

$$\bar{X}_1 X_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$$\bar{X}_1 X_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\bar{X}_1 X_3$$

$$X_1 \bar{X}_3$$

$$X_2 \bar{X}_3 X_4$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$$X_2 X_3 X_4$$

$$\bar{X}_1 X_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\bar{X}_1 \bar{X}_3$$

$$\bar{X}_2 \bar{X}_3 X_4$$

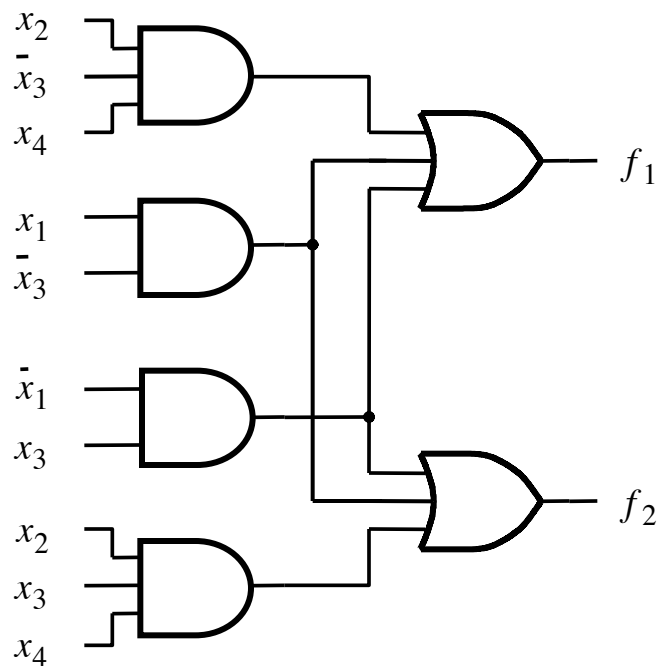
$$\bar{X}_1 X_3$$

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$$X_2 X_3 X_4$$

$$\bar{X}_1 X_3$$



(c) Combined circuit for f_1 and f_2

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\bar{X}_1 \bar{X}_3$$

$$\bar{X}_2 \bar{X}_3 X_4$$

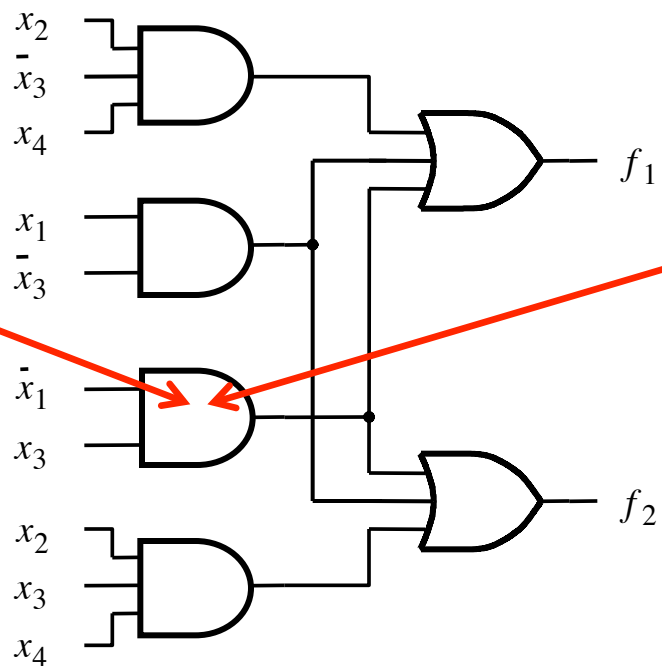
	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$$X_2 X_3 X_4$$

$$\bar{X}_1 X_3$$

$$\bar{X}_1 X_3$$



(c) Combined circuit for f_1 and f_2

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

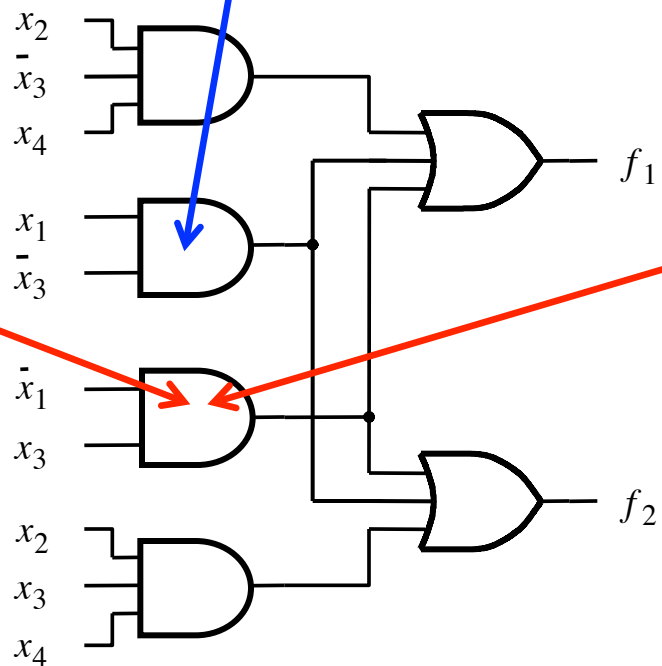
$\bar{X}_1 \bar{X}_3$

$\bar{X}_2 \bar{X}_3 X_4$

$X_2 X_3 X_4$

$\bar{X}_1 \bar{X}_3$

$\bar{X}_1 \bar{X}_3$



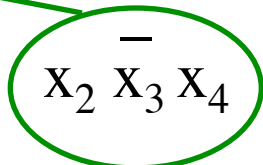
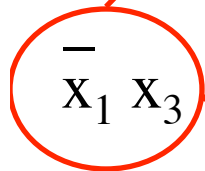
(c) Combined circuit for f_1 and f_2

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

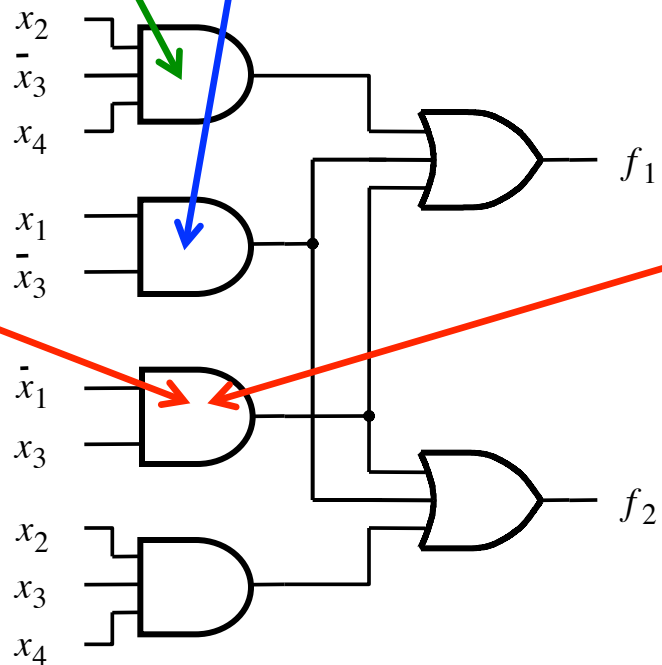
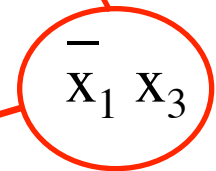
(a) Function f_1

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2



$X_2 X_3 X_4$



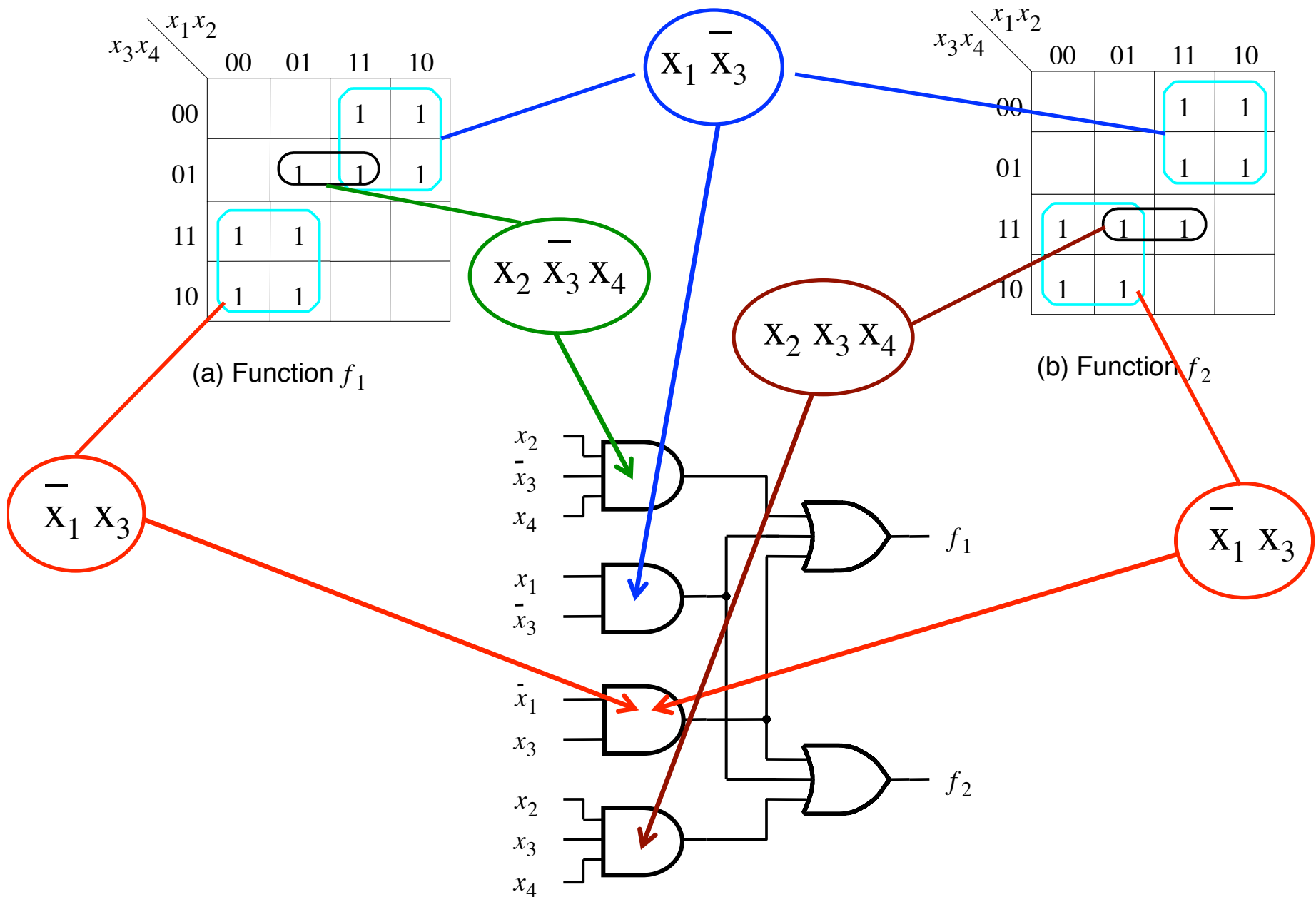
(c) Combined circuit for f_1 and f_2

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2



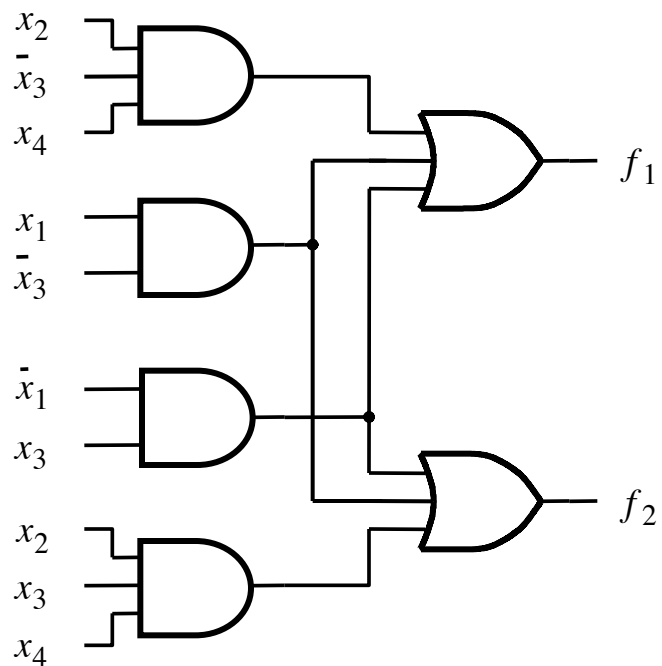
(c) Combined circuit for f_1 and f_2

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

	x_1x_2			
x_3x_4	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

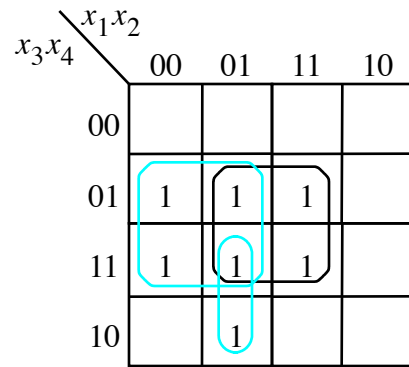
(b) Function f_2



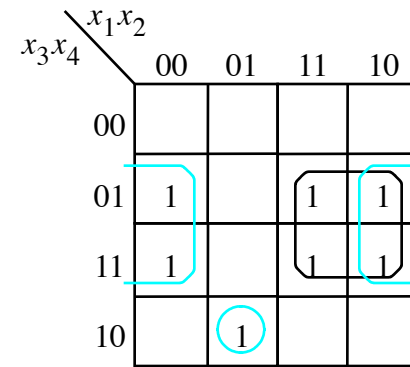
(c) Combined circuit for f_1 and f_2

Yet Another Example

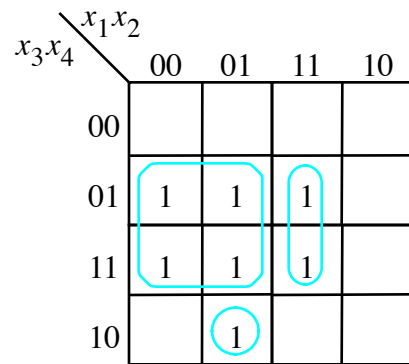
Individual vs Joint Optimization



(a) Optimal realization of f_3

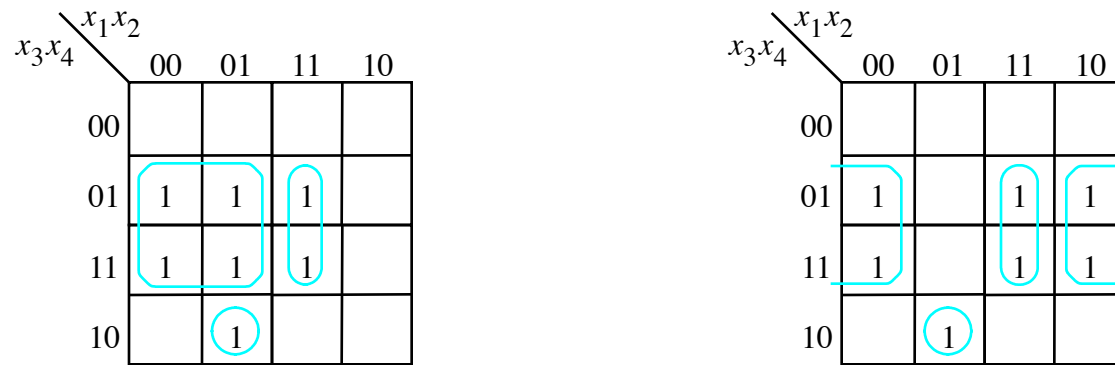


(b) Optimal realization of f_4

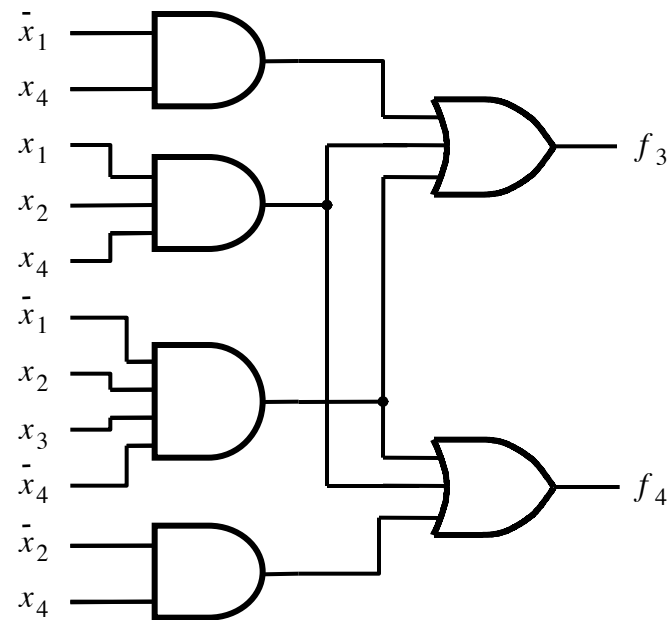


(c) Optimal realization of f_3 and f_4 together

Individual vs Joint Optimization

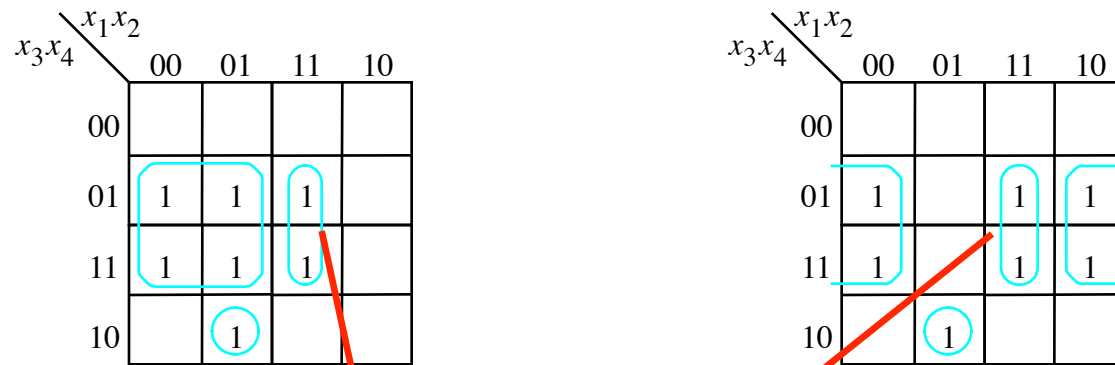


(c) Optimal realization of f_3 and f_4 together

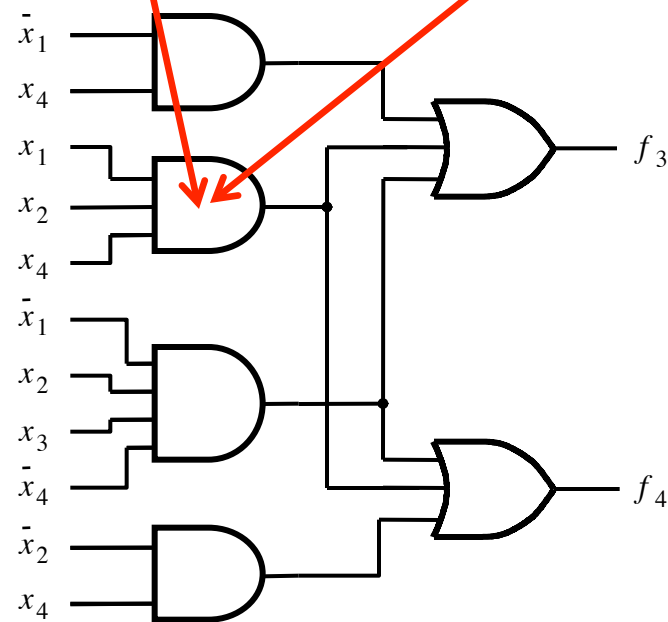


(d) Combined circuit for f_3 and f_4

Individual vs Joint Optimization

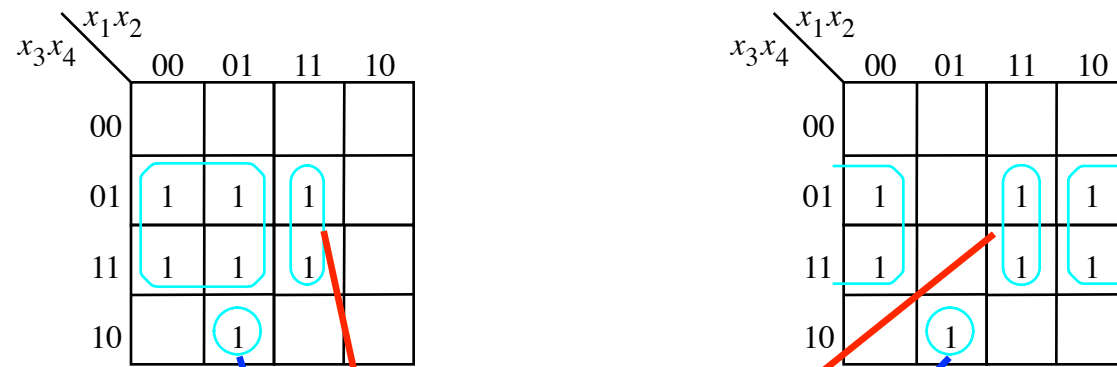


(c) Optimal realization of f_3 and f_4 together

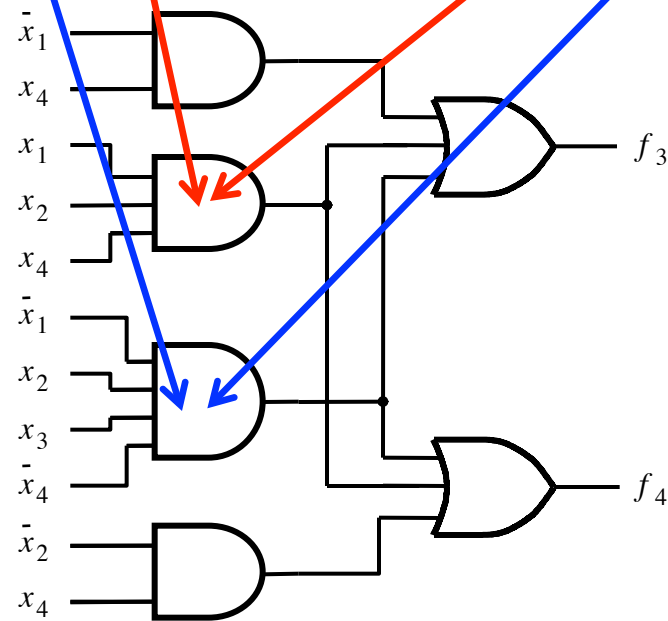


(d) Combined circuit for f_3 and f_4

Individual vs Joint Optimization



(c) Optimal realization of f_3 and f_4 together



(d) Combined circuit for f_3 and f_4

Questions?

THE END