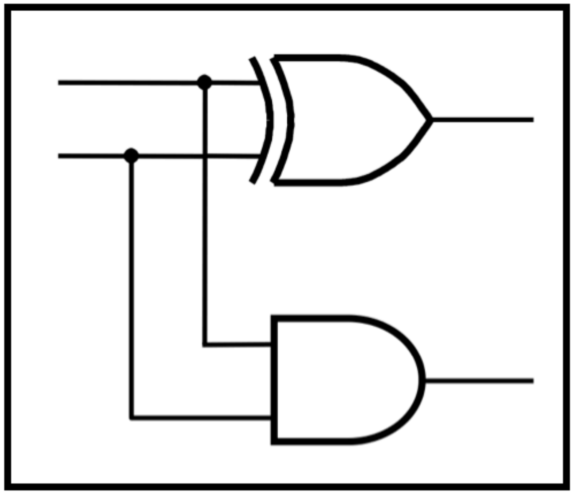


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Signed Numbers

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW5 is out

- It is due on Monday Oct 3 @ 4pm.

- Please write clearly on the first page (in block capital letters) the following three things:

  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter

- Also, please staple all of your pages together.

Administrative Stuff

- Labs Next Week

- Mini-Project

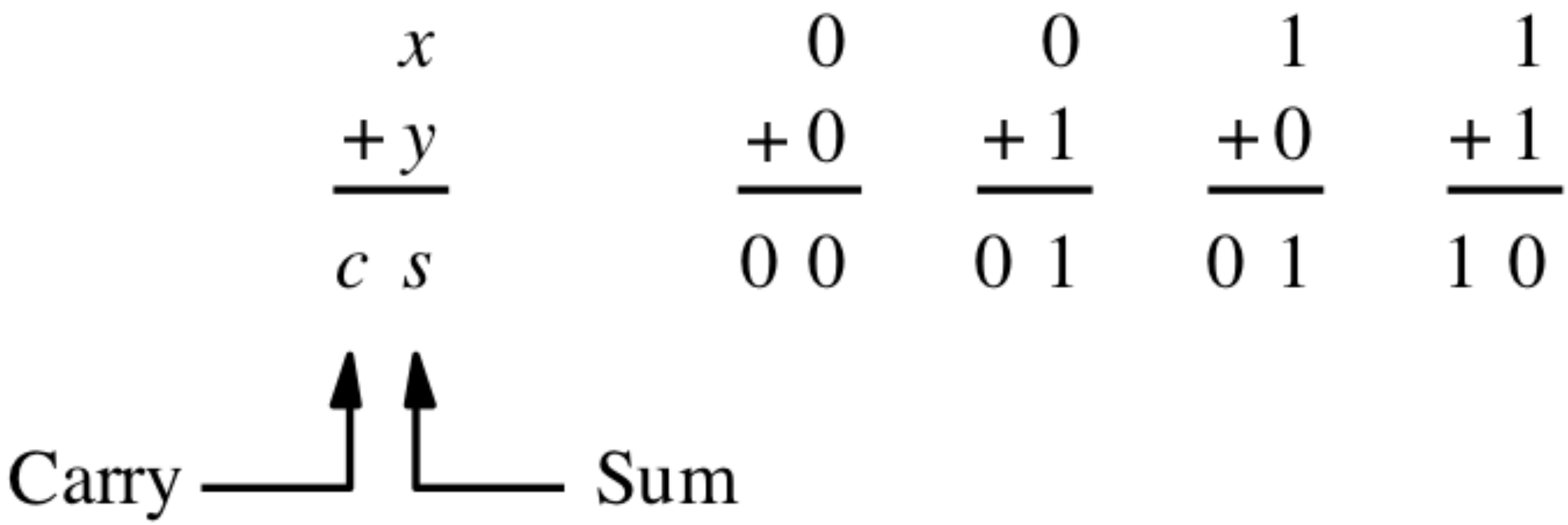
- This one is worth 3% of your grade.

- Make sure to get all the points.

- http://www.ece.iastate.edu/~alexs/classes/2016_Fall_281/labs/Project-Mini/

Quick Review

Adding two bits
(there are four possible cases)



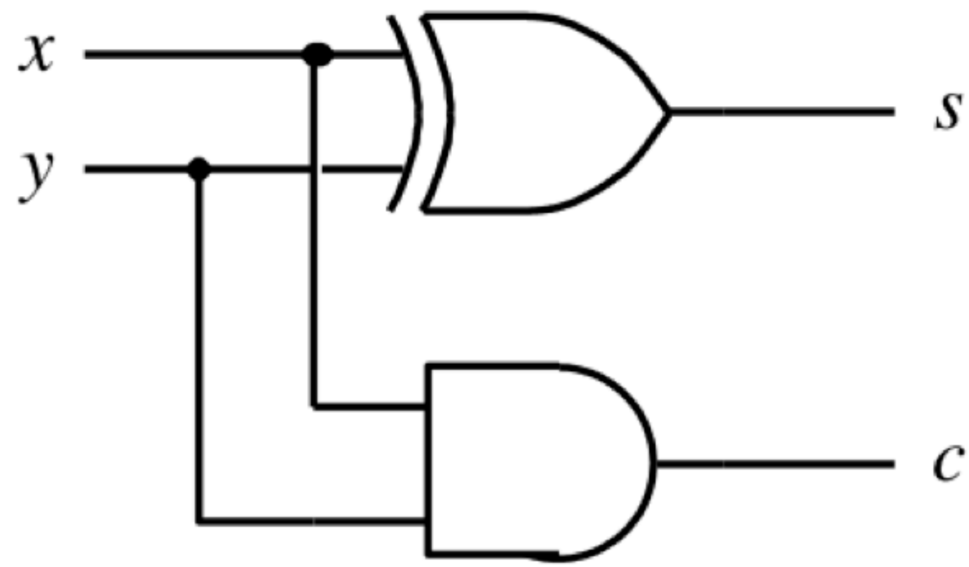
[Figure 3.1a from the textbook]

Adding two bits
(the truth table)

		Carry	Sum
x	*y*	*c*	.
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

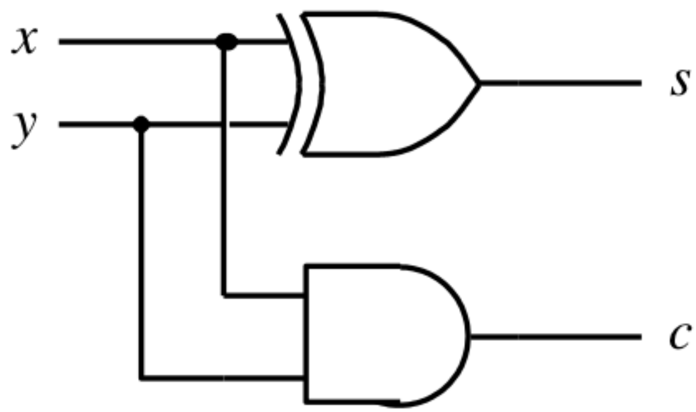
[Figure 3.1b from the textbook]

Adding two bits
(the logic circuit)

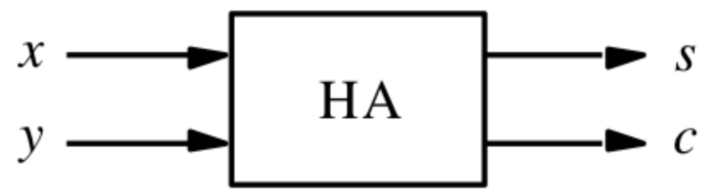


[Figure 3.1c from the textbook]

The Half-Adder



(c) Circuit

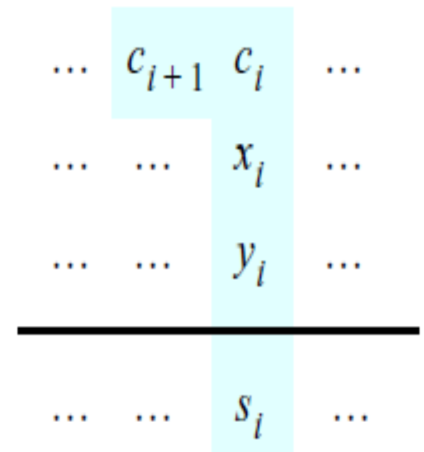


(d) Graphical symbol

[Figure 3.1c-d from the textbook]

Addition of multibit numbers

Generated carries →	1 1 1 0	
$X = x_4x_3x_2x_1x_0$	0 1 1 1 1	$(15)_{10}$
$+\,Y = y_4y_3y_2y_1y_0$	+ 0 1 0 1 0	+ $(10)_{10}$
$S = s_4s_3s_2s_1s_0$	1 1 0 0 1	$(25)_{10}$



Bit position *i*

[Figure 3.2 from the textbook]

Analogy with addition in base 10

$$\begin{array}{r} x_2\ x_1\ x_0 \\ +\quad y_2\ y_1\ y_0 \\ \hline s_2\ s_1\ s_0 \end{array}$$

Analogy with addition in base 10

$$\begin{array}{r} 3\ \ 8\ \ 9 \\ +\ \ 1\ \ 5\ \ 7 \\ \hline 5\ \ 4\ \ 6 \end{array}$$

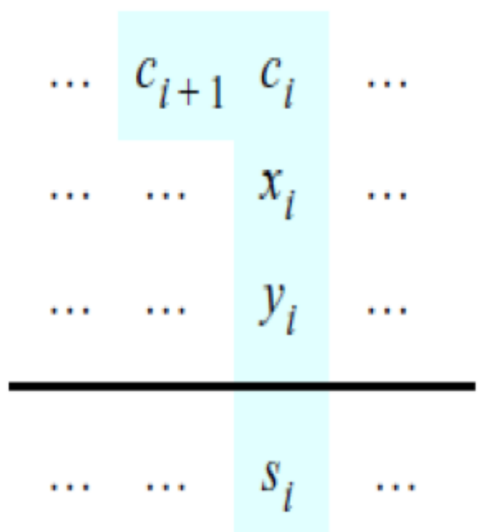
Analogy with addition in base 10

carry	0	1	1	0
		3	8	9
+		1	5	7
		5	4	6

Analogy with addition in base 10

$$
\begin{array}{rcccc}
 & c_3 & c_2 & c_1 & c_0 \\
 & & x_2 & x_1 & x_0 \\
+ & & y_2 & y_1 & y_0 \\
\hline
 & & s_2 & s_1 & s_0
\end{array}
$$

Problem Statement and Truth Table



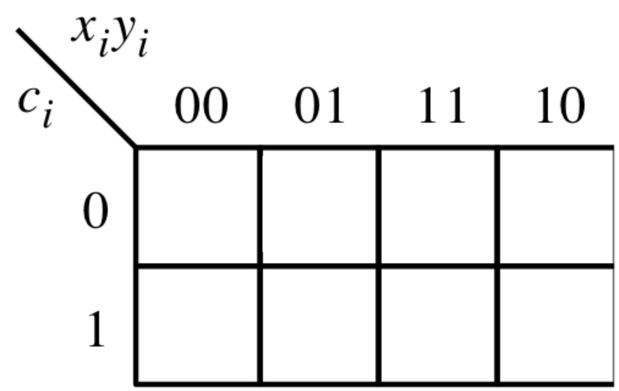
c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

[Figure 3.2b from the textbook]

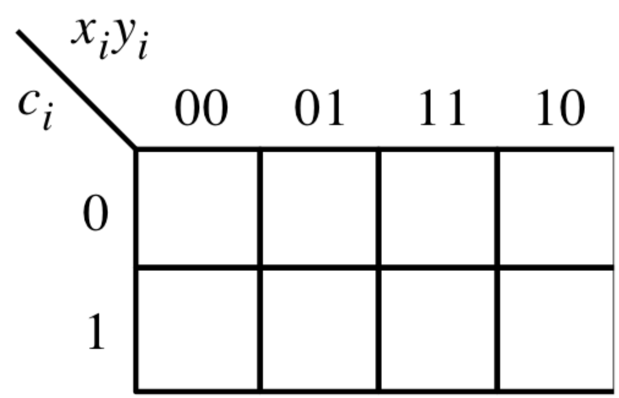
[Figure 3.3a from the textbook]

Let’s fill-in the two K-maps

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



$s_i =$



$c_{i+1} =$

[Figure 3.3a-b from the textbook]

Let’s fill-in the two K-maps

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

c_i \ x_iy_i	00	01	11	10
0		1		1
1	1		1	

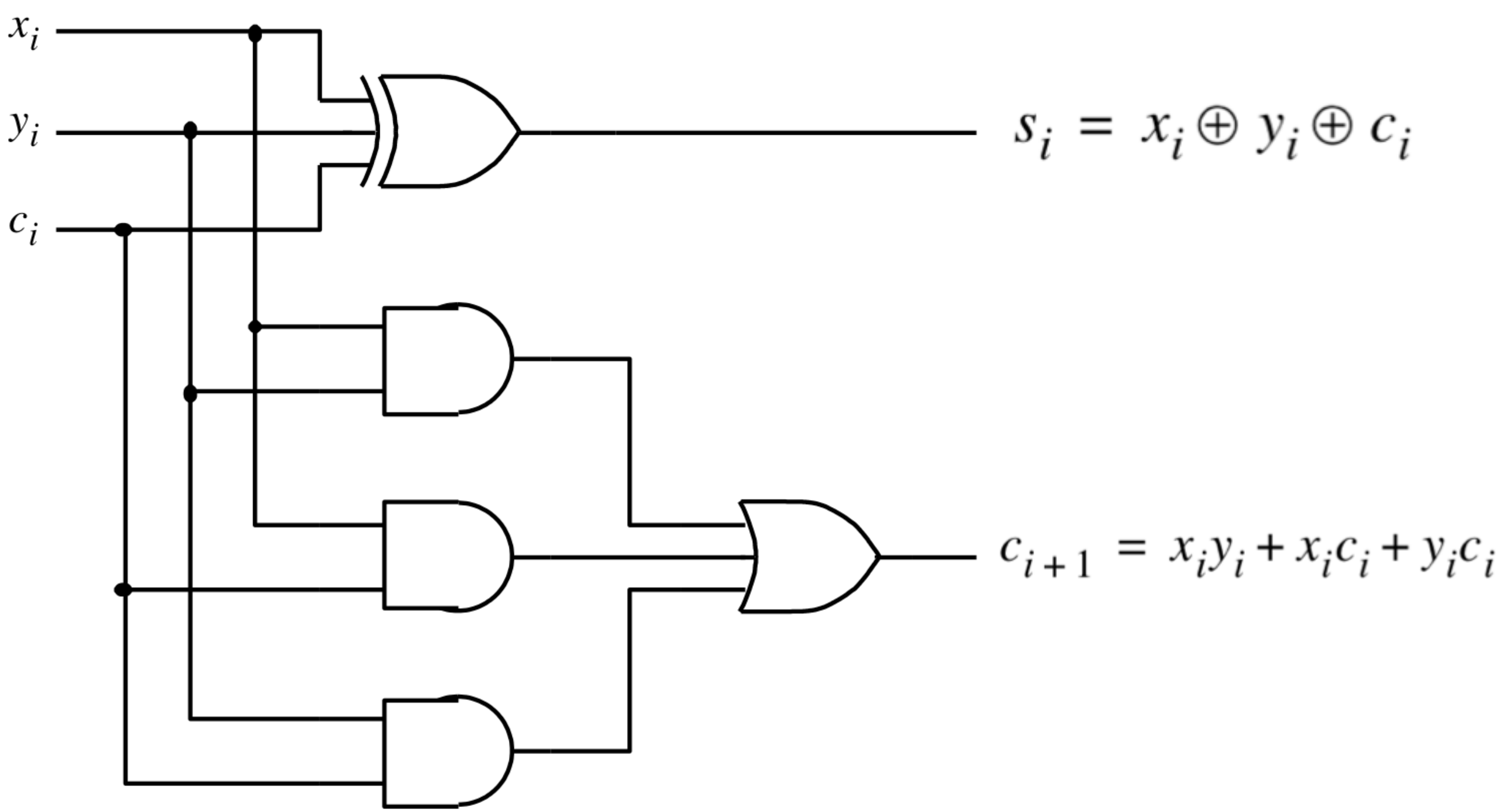
$$s_i = x_i \oplus y_i \oplus c_i$$

c_i \ x_iy_i	00	01	11	10
0			1	
1		1	1	1

$$c_{i+1} = x_iy_i + x_ic_i + y_ic_i$$

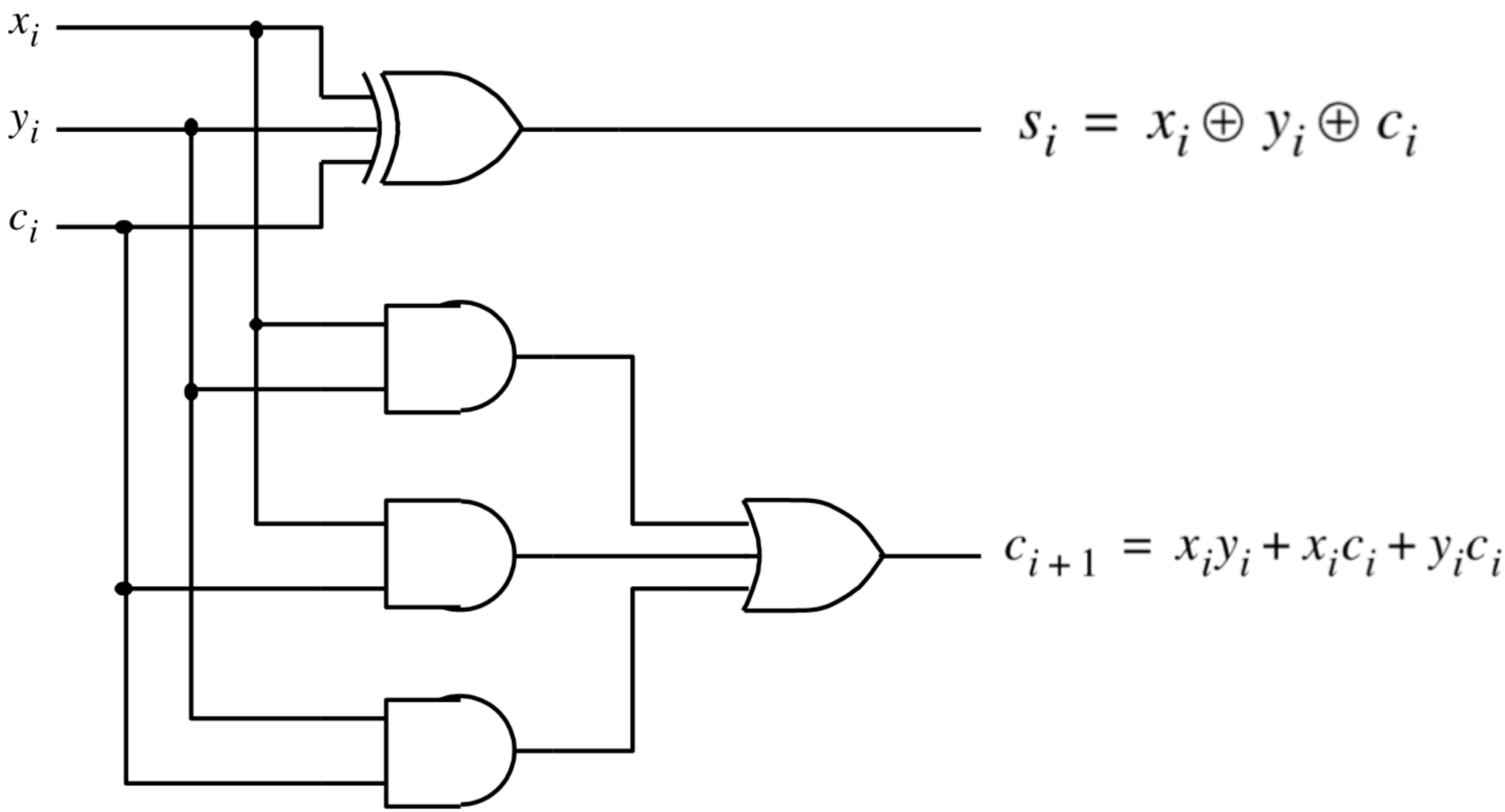
[Figure 3.3a-b from the textbook]

The circuit for the two expressions



[Figure 3.3c from the textbook]

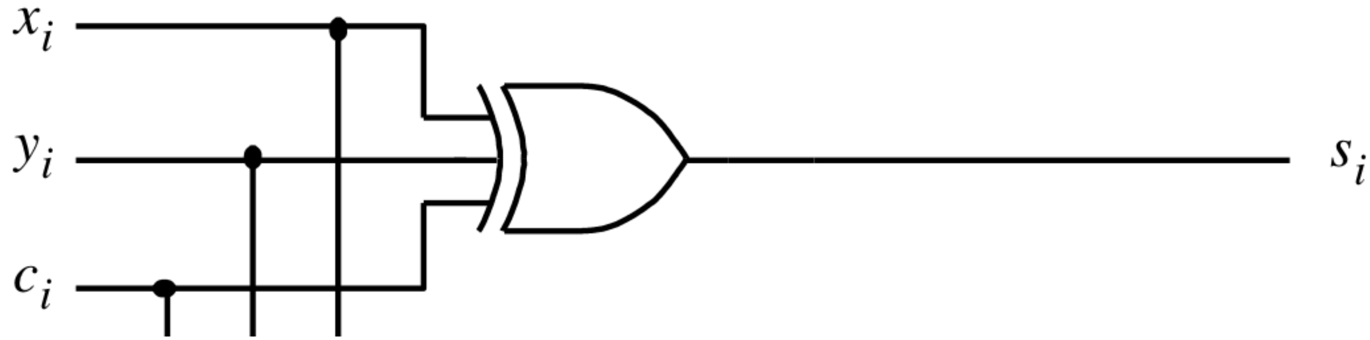
This is called the Full-Adder

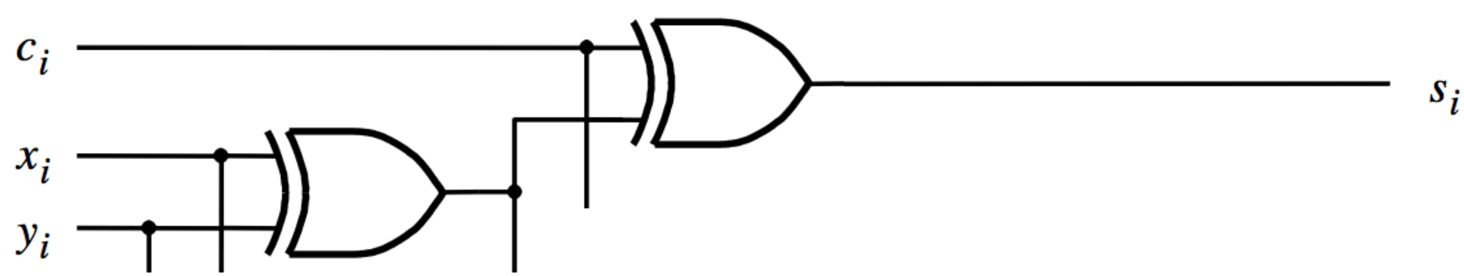


[Figure 3.3c from the textbook]

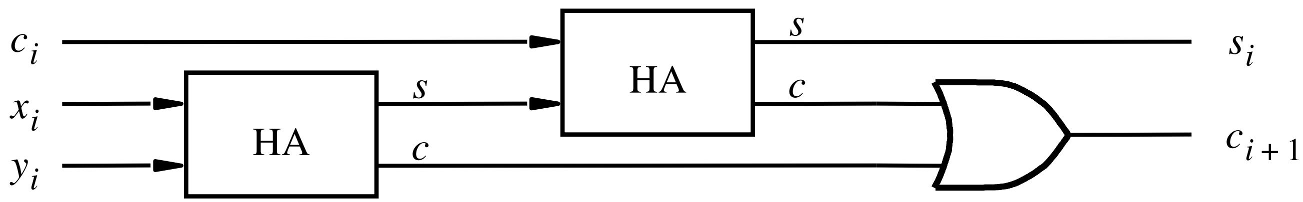
XOR Magic
(s_i can be implemented in two different ways)

$$s_i = x_i \oplus y_i \oplus c_i$$

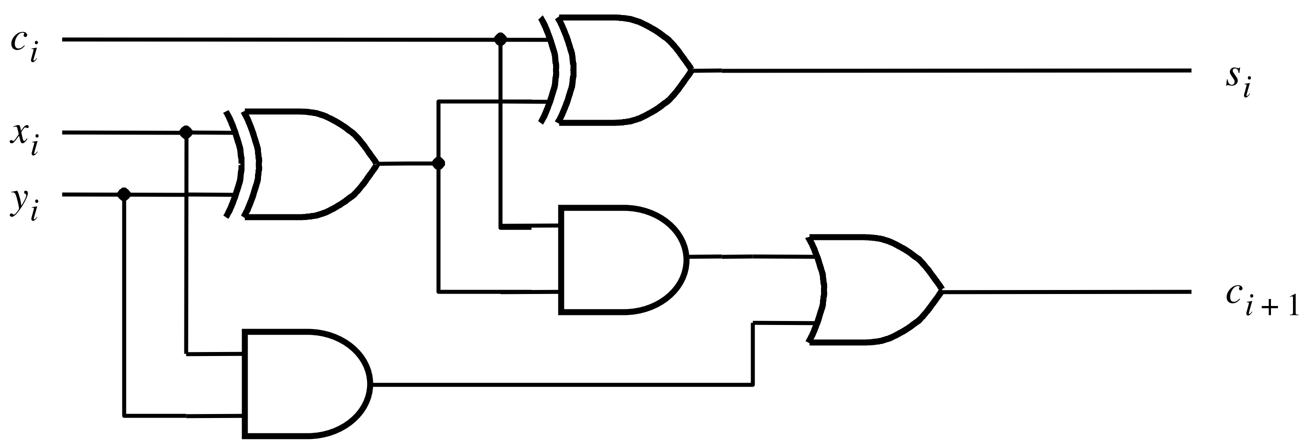




A decomposed implementation of the full-adder circuit



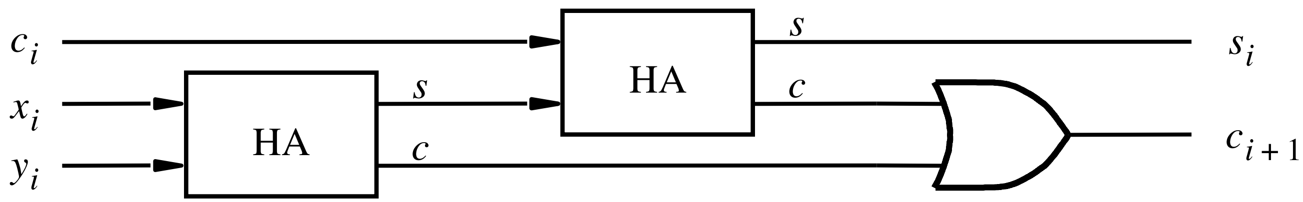
(a) Block diagram



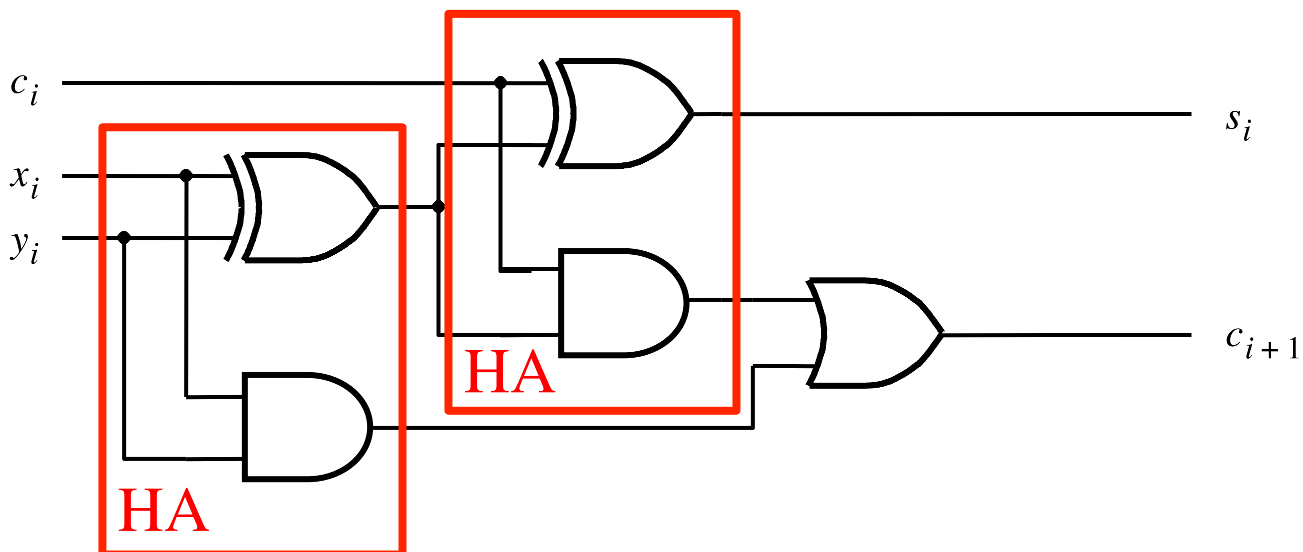
(b) Detailed diagram

[Figure 3.4 from the textbook]

A decomposed implementation of the full-adder circuit



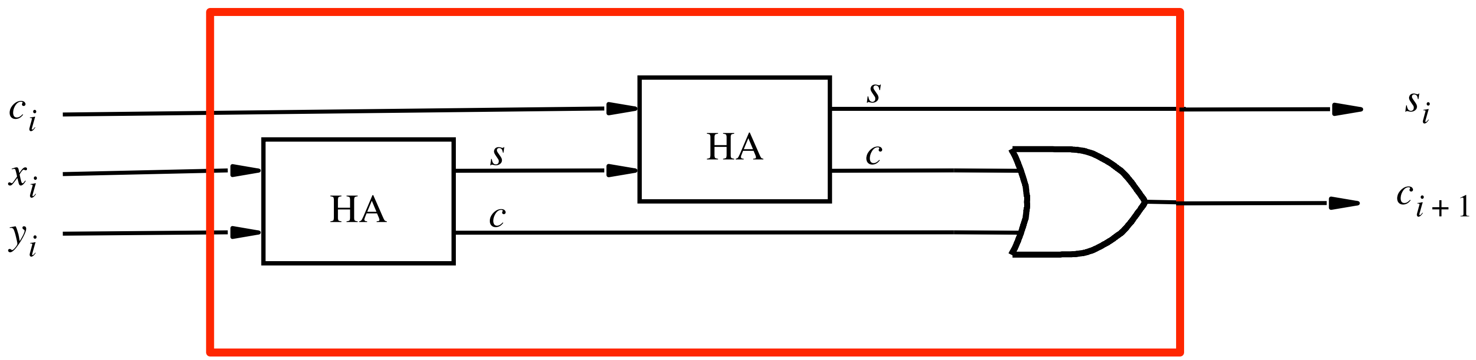
(a) Block diagram



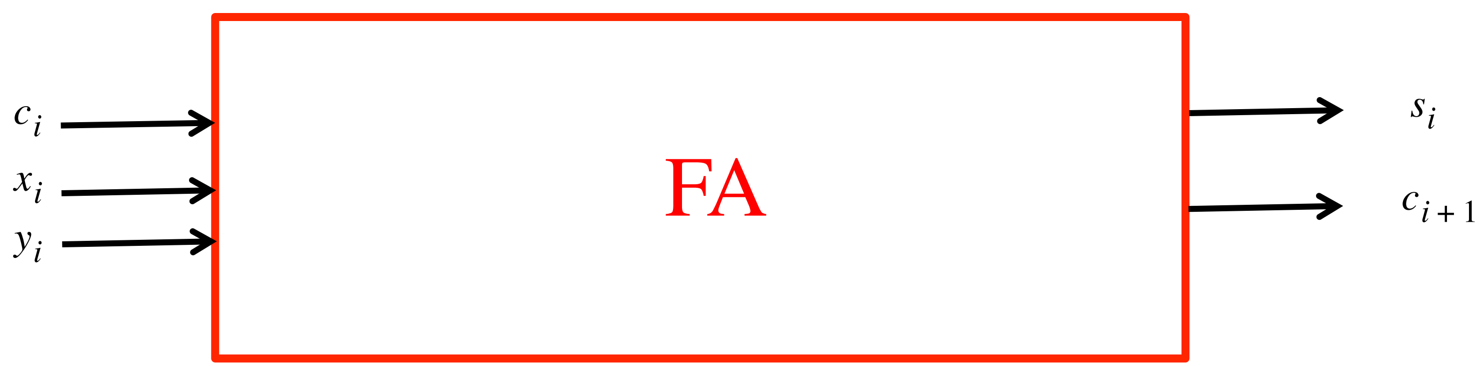
(b) Detailed diagram

[Figure 3.4 from the textbook]

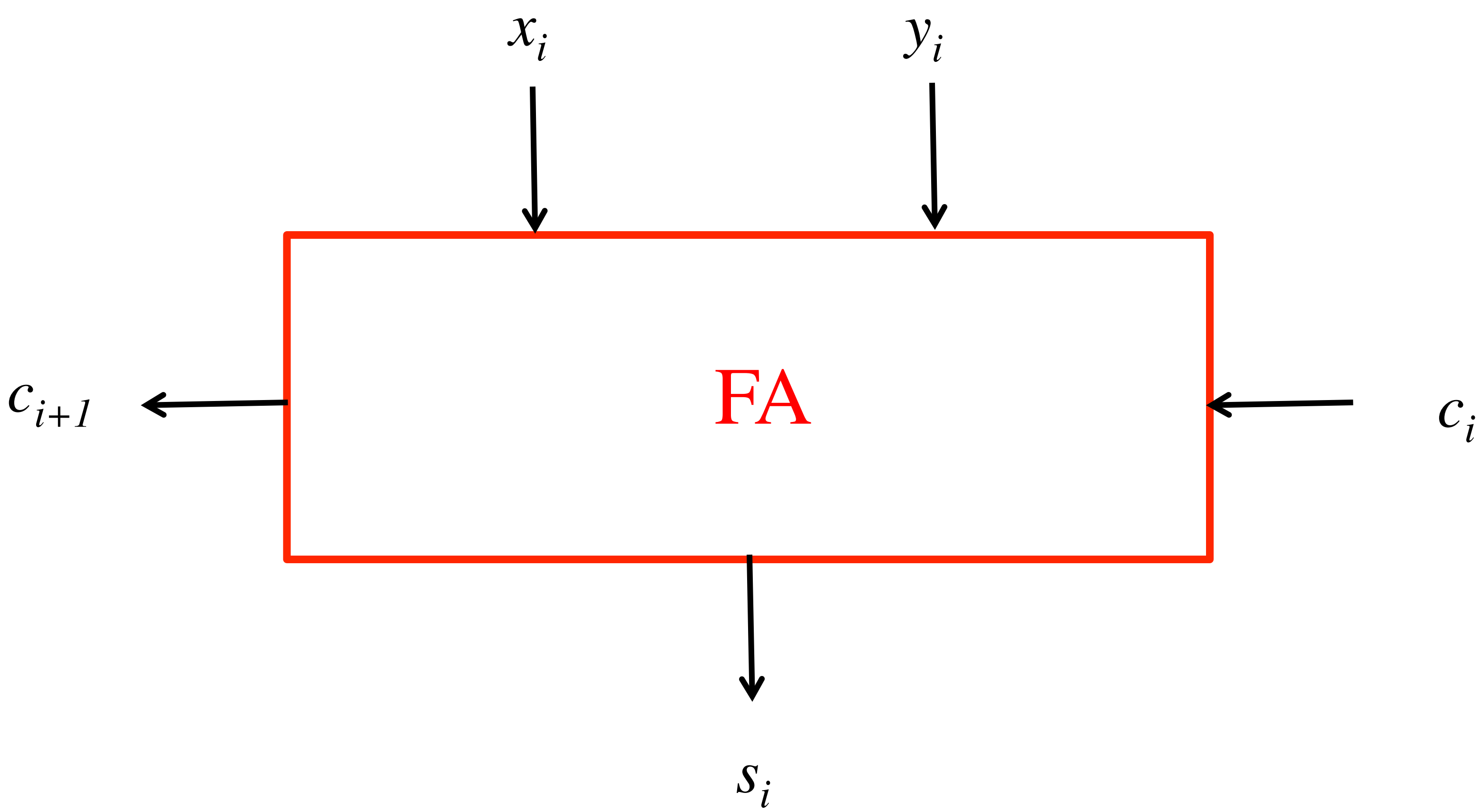
The Full-Adder Abstraction



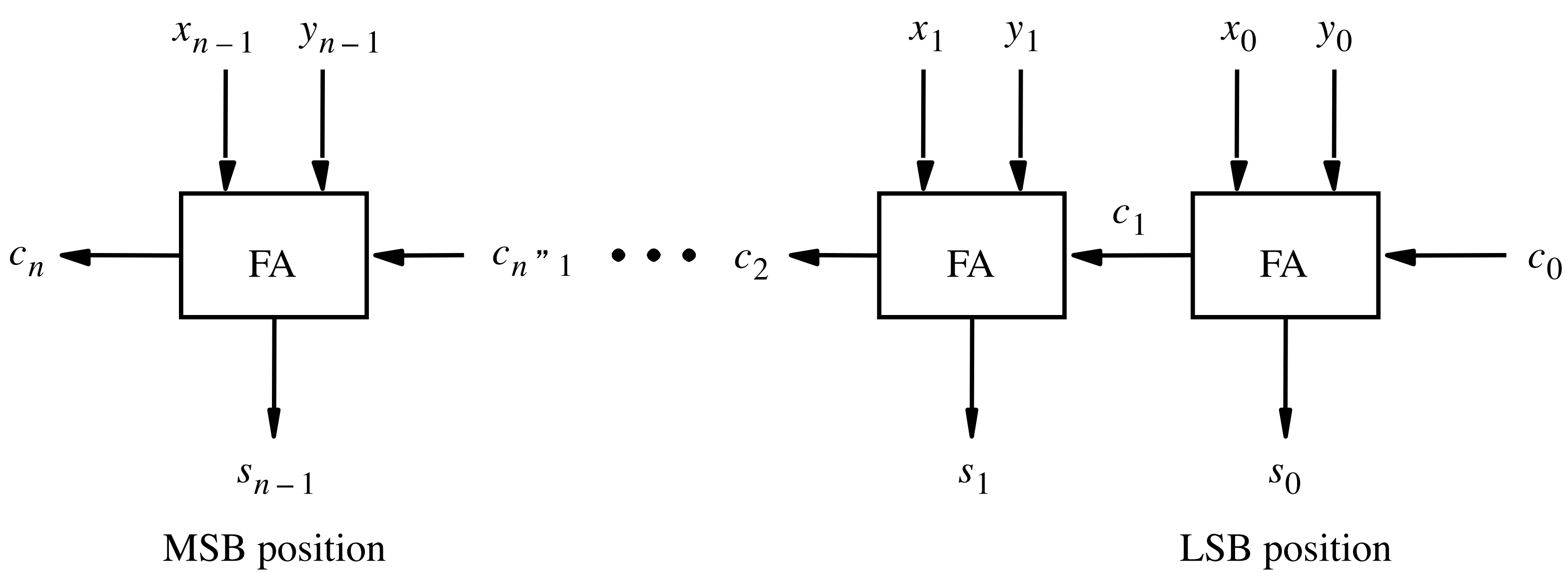
The Full-Adder Abstraction



We can place the arrows anywhere

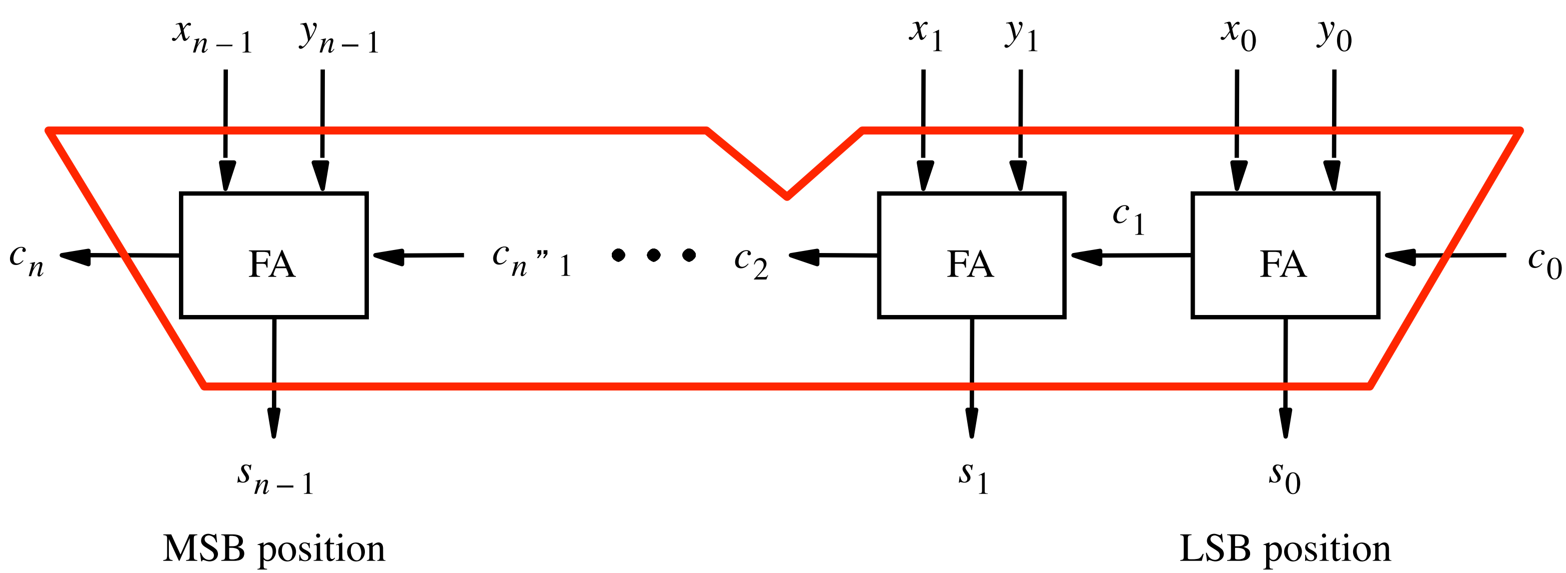


n-bit ripple-carry adder

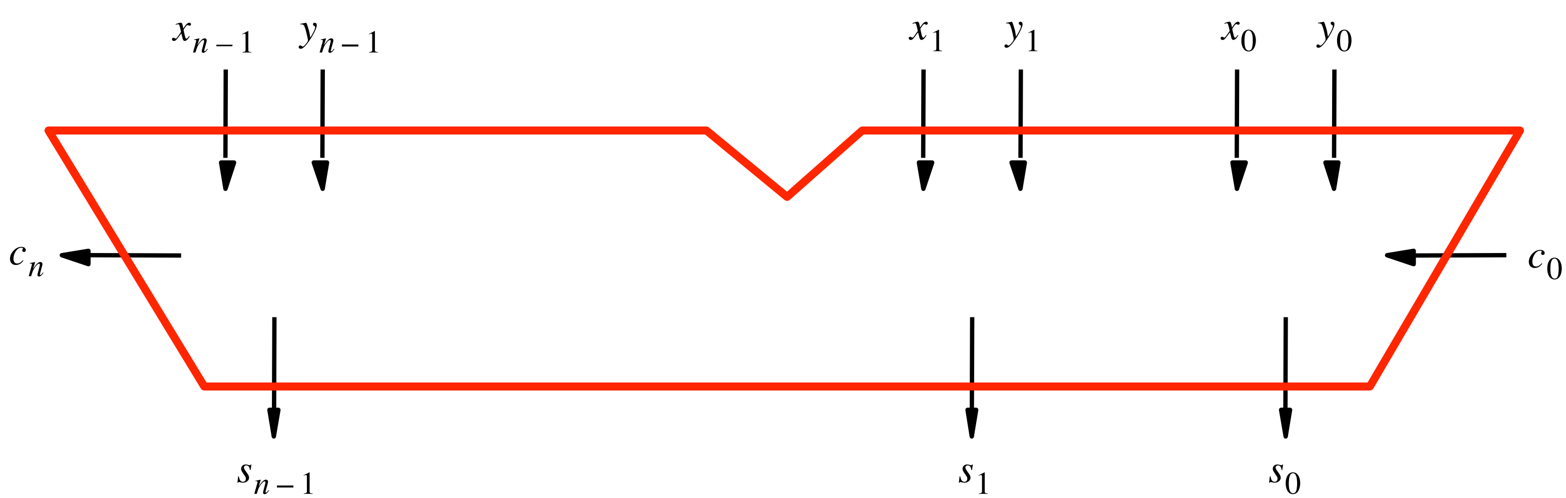


[Figure 3.5 from the textbook]

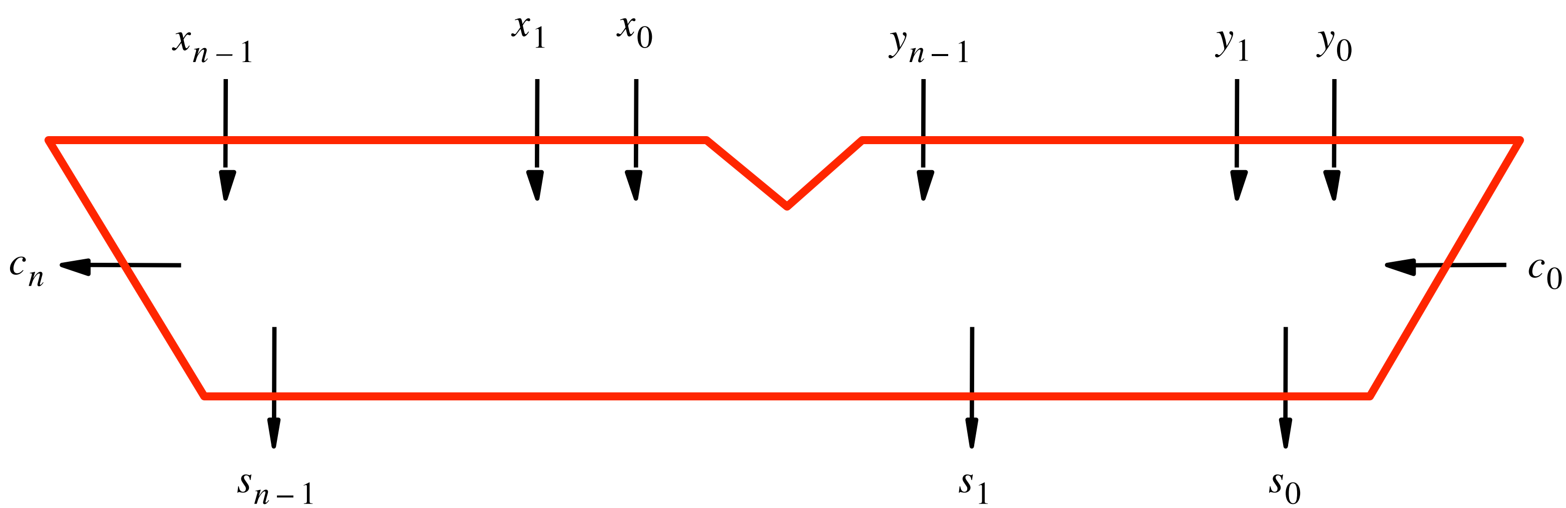
n-bit ripple-carry adder abstraction



n-bit ripple-carry adder abstraction



The x and y lines are typically grouped together for better visualization, but the underlying logic remains the same



Math Review: Subtraction

$$\begin{array}{r} 39 \\ -\ 15 \\ \hline ?? \end{array}$$

Math Review: Subtraction

$$\begin{array}{r} 39 \\ -\ 15 \\ \hline 24 \end{array}$$

Math Review: Subtraction

$$\begin{array}{r} 82 \\ -\ 61 \\ \hline ?? \end{array} \qquad \begin{array}{r} 48 \\ -\ 26 \\ \hline ?? \end{array} \qquad \begin{array}{r} 32 \\ -\ 11 \\ \hline ?? \end{array}$$

Math Review: Subtraction

$$\begin{array}{r} 82 \\ -\ 61 \\ \hline 21 \end{array} \qquad \begin{array}{r} 48 \\ -\ 26 \\ \hline 22 \end{array} \qquad \begin{array}{r} 32 \\ -\ 11 \\ \hline 21 \end{array}$$

Math Review: Subtraction

$$\begin{array}{r} 82 \\ -\ 64 \\ \hline ?? \end{array} \qquad \begin{array}{r} 48 \\ -\ 29 \\ \hline ?? \end{array} \qquad \begin{array}{r} 32 \\ -\ 13 \\ \hline ?? \end{array}$$

Math Review: Subtraction

$$\begin{array}{r} 82 \\ -\ 64 \\ \hline 18 \end{array} \qquad \begin{array}{r} 48 \\ -\ 29 \\ \hline 19 \end{array} \qquad \begin{array}{r} 32 \\ -\ 13 \\ \hline 19 \end{array}$$

The problems in which row are easier to calculate?

$$\begin{array}{r} 82 \\ -\ 61 \\ \hline ?? \end{array} \qquad \begin{array}{r} 48 \\ -\ 26 \\ \hline ?? \end{array} \qquad \begin{array}{r} 32 \\ -\ 11 \\ \hline ?? \end{array}$$

$$\begin{array}{r} 82 \\ -\ 64 \\ \hline ?? \end{array} \qquad \begin{array}{r} 48 \\ -\ 29 \\ \hline ?? \end{array} \qquad \begin{array}{r} 32 \\ -\ 13 \\ \hline ?? \end{array}$$

The problems in which row are easier to calculate?

$$\begin{array}{r} 82 \\ -\ 61 \\ \hline 21 \end{array} \qquad \begin{array}{r} 48 \\ -\ 26 \\ \hline 22 \end{array} \qquad \begin{array}{r} 32 \\ -\ 11 \\ \hline 21 \end{array}$$

Why?

$$\begin{array}{r} 82 \\ -\ 64 \\ \hline 18 \end{array} \qquad \begin{array}{r} 48 \\ -\ 29 \\ \hline 19 \end{array} \qquad \begin{array}{r} 32 \\ -\ 13 \\ \hline 19 \end{array}$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

Another Way to Do Subtraction

$$82 – 64 = 82 + 100 – 100 - 64$$

$$= 82 + (100 – 64) - 100$$

$$= 82 + (99 + 1 – 64) - 100$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

$$= 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

$$82 - 64 = 82 + 100 - 100 - 64$$

$$= 82 + (100 - 64) - 100$$

$$= 82 + (99 + 1 - 64) - 100$$

Does not require borrows

$$= 82 + (99 - 64) + 1 - 100$$

9's Complement
(subtract each digit from 9)

$$\begin{array}{r} 99 \\ -\ 64 \\ \hline 35 \end{array}$$

10’s Complement
(subtract each digit from 9 and add 1 to the result)

$$
\begin{array}{r}
99 \\
-\ 64 \\
\hline
35 + 1 = 36
\end{array}
$$

Another Way to Do Subtraction

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

$$= 82 + 35 + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

$$= 82 + 36 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

$$= 82 + 35 + 1 - 100$$

$$= 82 + 36 - 100$$ // Add the first two.

$$= 118 - 100$$

Another Way to Do Subtraction

9's complement

$$82 - 64 = 82 + (99 - 64) + 1 - 100$$

10's complement

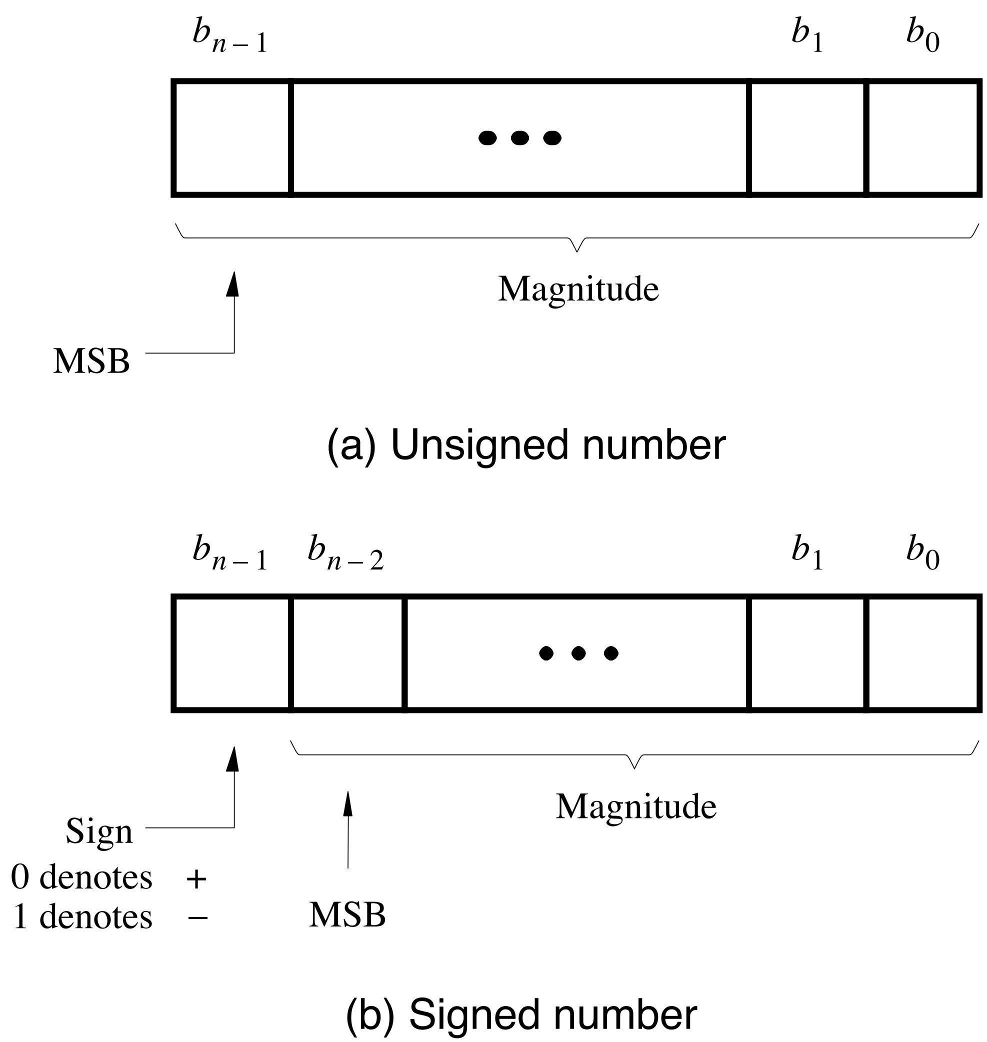
$$= 82 + 35 + 1 - 100$$

$$= 82 + 36 - 100$$ // Add the first two.

$$= 118 - 100$$ // Just delete the leading 1.
// No need to subtract 100.

$$= 18$$

Formats for representation of integers



[Figure 3.7 from the textbook]

Negative numbers can be represented in following ways

- Sign and magnitude
- 1's complement
- 2's complement

1's complement

Let K be the negative equivalent of an n-bit positive number P.

Then, in 1's complement representation K is obtained by subtracting P from $2^n - 1$, namely

$$K = (2^n - 1) - P$$

This means that K can be obtained by inverting all bits of P.

Find the 1's complement of …

0 1 0 1

0 0 1 0

0 0 1 1

0 1 1 1

Find the 1's complement of …

0 1 0 1

1 0 1 0

0 0 1 0

1 1 0 1

0 0 1 1

1 1 0 0

0 1 1 1

1 0 0 0

Just flip 1's to 0's and vice versa.

A) Example of 1's complement addition

(+5)	0 1 0 1
+ (+2)	+ 0 0 1 0
(+7)	0 1 1 1

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

[Figure 3.8 from the textbook]

A) Example of 1's complement addition

$$\begin{array}{r} (+5) \\ +\,(+2) \\ \hline (+7) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,0\,0\,1\,0 \\ \hline 0\,1\,1\,1 \end{array}$$

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

B) Example of 1's complement addition

```
   (- 5)       1 0 1 0
 + (+2)      + 0 0 1 0
 -------     ---------
   (- 3)       1 1 0 0
```

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

[Figure 3.8 from the textbook]

B) Example of 1's complement addition

(- 5)		1 0 1 0	
+ (+2)	+	0 0 1 0	
(- 3)		1 1 0 0	

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

C) Example of 1's complement addition

$$\begin{array}{r} (+5) \\ +(-2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,0\,1\,0 \end{array}$$

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

[Figure 3.8 from the textbook]

C) Example of 1's complement addition

$$\begin{array}{r} (+5) \\ +(-2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,0\,1\,0 \end{array}$$

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

C) Example of 1's complement addition

$$\begin{array}{r} (+5) \\ +(-2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,0\,1\,0 \end{array}$$

But this is 2!

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

C) Example of 1's complement addition

$$
\begin{array}{rr}
(+5) & 0\,1\,0\,1 \\
+(-2) & +\,1\,1\,0\,1 \\
\hline
(+3) & 1\,0\,0\,1\,0 \\
 & 1 \\
\hline
 & 0\,0\,1\,1
\end{array}
$$

We need to perform one more addition to get the result.

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

C) Example of 1's complement addition

$$\begin{array}{r} (+5) \\ +(-2) \\ \hline (+3) \end{array}$$

$$\begin{array}{r} 0\;1\;0\;1 \\ +\;1\;1\;0\;1 \\ \hline 1\;0\;0\;1\;0 \\ 1 \\ \hline 0\;0\;1\;1 \end{array}$$

We need to perform one more addition to get the result.

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

D) Example of 1’s complement addition

$$\begin{array}{rr} & (-5) \\ + & (-2) \\ \hline & (-7) \end{array} \qquad \begin{array}{rr} & 1010 \\ + & 1101 \\ \hline & 10111 \end{array}$$

$b_3b_2b_1b_0$	1’s complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

[Figure 3.8 from the textbook]

D) Example of 1's complement addition

$$
\begin{array}{rr}
 & (-5) \\
+ & (-2) \\
\hline
 & (-7)
\end{array}
\qquad
\begin{array}{rr}
 & 1\,0\,1\,0 \\
+ & 1\,1\,0\,1 \\
\hline
1 & 0\,1\,1\,1
\end{array}
$$

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

D) Example of 1's complement addition

$$\begin{array}{r} (-5) \\ +\,(-2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\;1\,1\,0\,1 \\ \hline 1\;\;0\,1\,1\,1 \end{array}$$

But this is +7!

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

D) Example of 1's complement addition

$$\begin{array}{r} (-5) \\ +\,(-2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\,1\,1\,0\,1 \\ \hline 1\,0\,1\,1\,1 \\ 1 \\ \hline 1\,0\,0\,0 \end{array}$$

We need to perform one more addition to get the result.

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

D) Example of 1's complement addition

$$\begin{array}{r} (-5) \\ +\,(-2) \\ \hline (-7) \end{array} \qquad \begin{array}{r} 1\,0\,1\,0 \\ +\,1\,1\,0\,1 \\ \hline 1\;0\,1\,1\,1 \\ 1 \\ \hline 1\,0\,0\,0 \end{array}$$

We need to perform one more addition to get the result.

$b_3b_2b_1b_0$	1's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−7
1001	−6
1010	−5
1011	−4
1100	−3
1101	−2
1110	−1
1111	−0

2' s complement

Let K be the negative equivalent of an n-bit positive number P.

Then, in 2' s complement representation K is obtained by subtracting P from 2^n , namely

$$K = 2^n - P$$

Deriving 2’s complement

For a positive n-bit number P, let K_1 and K_2 denote its 1’s and 2’s complements, respectively.

$$K_1 = (2^n - 1) - P$$

$$K_2 = 2^n - P$$

Since $K_2 = K_1 + 1$, it is evident that in a logic circuit the 2’s complement can computed by inverting all bits of P and then adding 1 to the resulting 1’s-complement number.

Find the 2’s complement of …

0 1 0 1

0 0 1 0

0 1 0 0

0 1 1 1

Find the 2’s complement of …

0 1 0 1	0 0 1 0
1 0 1 0	1 1 0 1
0 1 0 0	0 1 1 1
1 0 1 1	1 0 0 0

Invert all bits.

Find the 2's complement of …

$$\begin{array}{rcccc} & 0 & 1 & 0 & 1 \\ & 1 & 0 & 1 & 0 \\ + & & & & 1 \\ \hline & 1 & 0 & 1 & 1 \end{array}$$

$$\begin{array}{rcccc} & 0 & 0 & 1 & 0 \\ & 1 & 1 & 0 & 1 \\ + & & & & 1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{rcccc} & 0 & 1 & 0 & 0 \\ & 1 & 0 & 1 & 1 \\ + & & & & 1 \\ \hline & 1 & 1 & 0 & 0 \end{array}$$

$$\begin{array}{rcccc} & 0 & 1 & 1 & 1 \\ & 1 & 0 & 0 & 0 \\ + & & & & 1 \\ \hline & 1 & 0 & 0 & 1 \end{array}$$

Then add 1.

Quick Way to find 2's complement

- Scan the binary number from right to left
- Copy all bits that are 0 from right to left
- Stop at the first 1
- Copy that 1 as well
- Invert all remaining bits

Find the 2's complement of …

0 1 0 1

0 0 1 0

0 1 0 0

0 1 1 1

Find the 2's complement of …

0 1 0 1

. . . .

0 0 1 0

. . . 0

0 1 0 0

. . 0 0

0 1 1 1

. . . .

Copy all bits that are 0 from right to left.

Find the 2's complement of …

0 1 0 1
. . . 1

0 0 1 0
. . 1 0

0 1 0 0
. 1 0 0

0 1 1 1
. . . 1

Stop at the first 1. Copy that 1 as well.

Find the 2's complement of …

0 1 0 1
1 0 1 1

0 0 1 0
1 1 1 0

0 1 0 0
1 1 0 0

0 1 1 1
1 0 0 1

Invert all remaining bits.

Interpretation of four-bit signed integers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	−0	−7	−8
1001	−1	−6	−7
1010	−2	−5	−6
1011	−3	−4	−5
1100	−4	−3	−4
1101	−5	−2	−3
1110	−6	−1	−2
1111	−7	−0	−1

[Table 3.2 from the textbook]

Interpretation of four-bit signed integers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	−0	−7	−8
1001	−1	−6	−7
1010	−2	−5	−6
1011	−3	−4	−5
1100	−4	−3	−4
1101	−5	−2	−3
1110	−6	−1	−2
1111	−7	−0	−1

Notice that in this representation there are two zeros!

Interpretation of four-bit signed integers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	−0	−7	−8
1001	−1	−6	−7
1010	−2	−5	−6
1011	−3	−4	−5
1100	−4	−3	−4
1101	−5	−2	−3
1110	−6	−1	−2
1111	−7	−0	−1

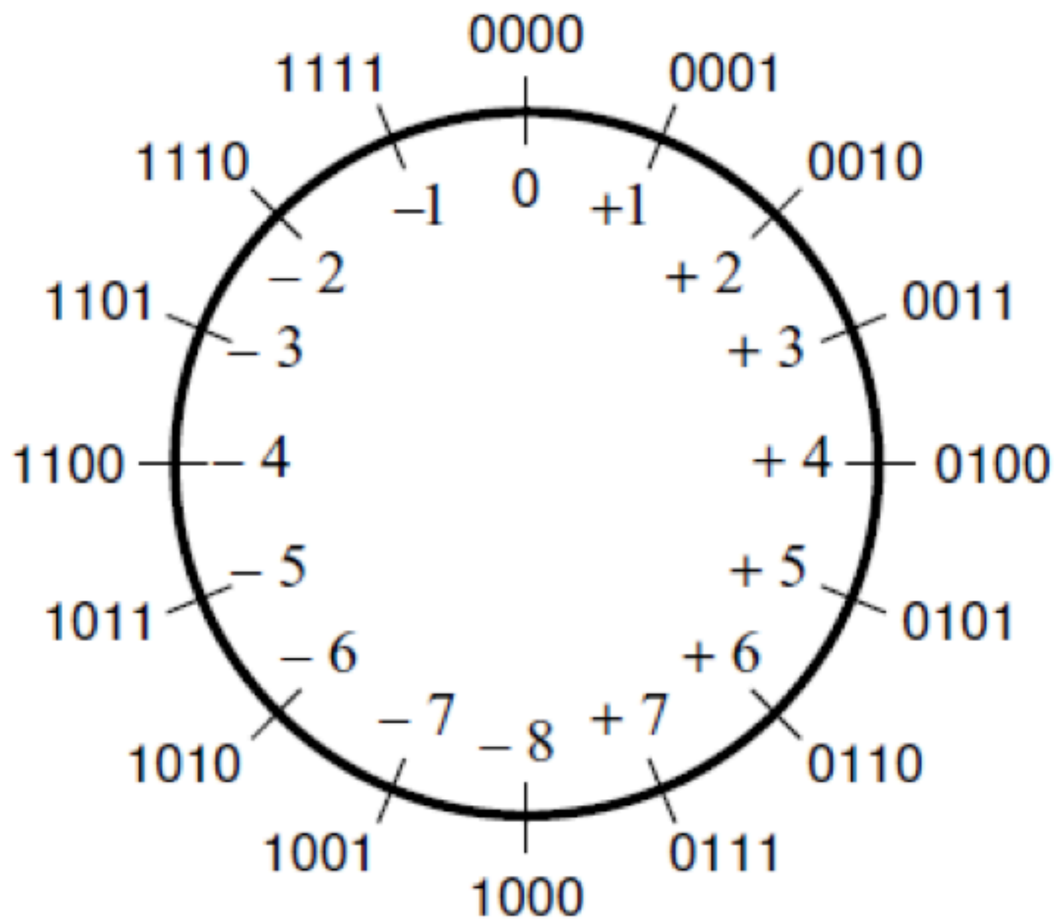
There are two zeros in this representation as well!

Interpretation of four-bit signed integers

$b_3b_2b_1b_0$	Sign and magnitude	1's complement	2's complement
0111	+7	+7	+7
0110	+6	+6	+6
0101	+5	+5	+5
0100	+4	+4	+4
0011	+3	+3	+3
0010	+2	+2	+2
0001	+1	+1	+1
0000	+0	+0	+0
1000	−0	−7	−8
1001	−1	−6	−7
1010	−2	−5	−6
1011	−3	−4	−5
1100	−4	−3	−4
1101	−5	−2	−3
1110	−6	−1	−2
1111	−7	−0	−1

In this representation there is one more negative number.

The number circle for 2's complement



[Figure 3.11a from the textbook]

A) Example of 2's complement addition

$(+5)$	0 1 0 1
$+ (+2)$	+ 0 0 1 0
$(+7)$	0 1 1 1

$b_3b_2b_1b_0$	2's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−8
1001	−7
1010	−6
1011	−5
1100	−4
1101	−3
1110	−2
1111	−1

[Figure 3.9 from the textbook]

B) Example of 2' s complement addition

(–5)	1 0 1 1
+ (+2)	+ 0 0 1 0
(–3)	1 1 0 1

$b_3b_2b_1b_0$	2's complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−8
1001	−7
1010	−6
1011	−5
1100	−4
1101	−3
1110	−2
1111	−1

[Figure 3.9 from the textbook]

C) Example of 2’s complement addition

(+ 5)	0 1 0 1
+ (–2)	+ 1 1 1 0
(+ 3)	1 0 0 1 1

↑ ignore (the leftmost 1 of 1 0 0 1 1)

$b_3b_2b_1b_0$	2’s complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−8
1001	−7
1010	−6
1011	−5
1100	−4
1101	−3
1110	−2
1111	−1

[Figure 3.9 from the textbook]

D) Example of 2’s complement addition

```
    (–5)        1 0 1 1
 +  (–2)     +  1 1 1 0
 -------     ----------
    (–7)      1 1 0 0 1
              ↑
            ignore
```

$b_3b_2b_1b_0$	2’s complement
0111	+7
0110	+6
0101	+5
0100	+4
0011	+3
0010	+2
0001	+1
0000	+0
1000	−8
1001	−7
1010	−6
1011	−5
1100	−4
1101	−3
1110	−2
1111	−1

[Figure 3.9 from the textbook]

Example of 2's complement subtraction

$$\begin{array}{r} (+5) \\ -\ (+2) \\ \hline (+3) \end{array} \qquad \begin{array}{r} 0\,1\,0\,1 \\ -\ 0\,0\,1\,0 \\ \hline \end{array} \qquad \Longrightarrow \qquad \begin{array}{r} 0\,1\,0\,1 \\ +\ 1\,1\,1\,0 \\ \hline 1\,0\,0\,1\,1 \end{array}$$

ignore

$\Longrightarrow$ means take the 2's complement

[Figure 3.10 from the textbook]

Example of 2's complement subtraction

(-5)	1011	$\Rightarrow$	1011
$-\,(+2)$	$-\;0010$		$+\;1110$
(-7)			11001

ignore (leftmost carry bit)

[Figure 3.10 from the textbook]

Example of 2’s complement subtraction

(+5)	0 1 0 1		0 1 0 1
− (−2)	− 1 1 1 0	⇒	+ 0 0 1 0
(+7)			0 1 1 1

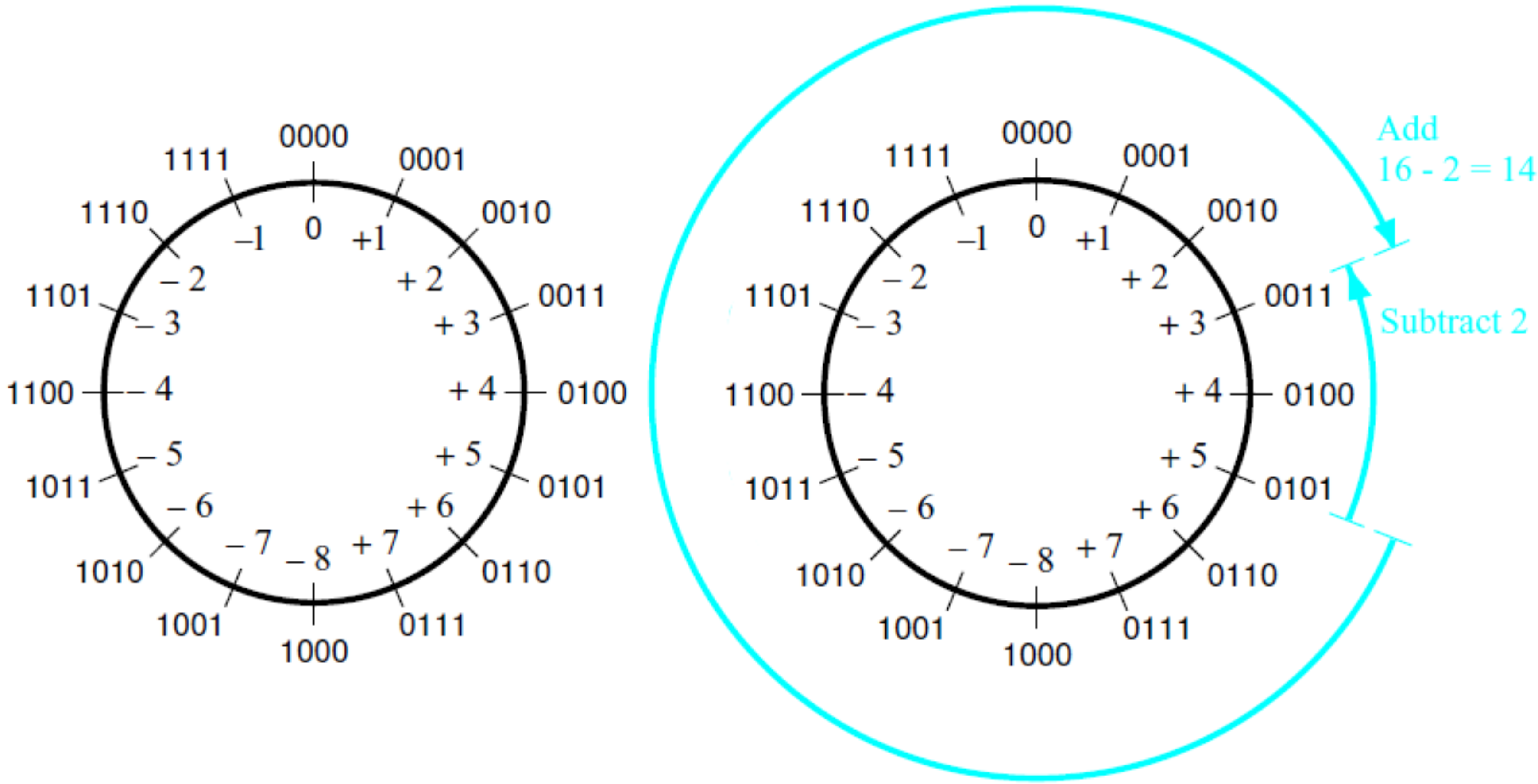
[Figure 3.10 from the textbook]

Example of 2's complement subtraction

$$
\begin{array}{rr}
 & (-5) \\
- & (-2) \\
\hline
 & (-3)
\end{array}
\qquad
\begin{array}{rr}
 & 1\,0\,1\,1 \\
- & 1\,1\,1\,0 \\
\hline
\end{array}
\qquad \Longrightarrow \qquad
\begin{array}{rr}
 & 1\,0\,1\,1 \\
+ & 0\,0\,1\,0 \\
\hline
 & 1\,1\,0\,1
\end{array}
$$

[Figure 3.10 from the textbook]

Graphical interpretation of four-bit 2's complement numbers



(a) The number circle (b) Subtracting 2 by adding its 2's complement

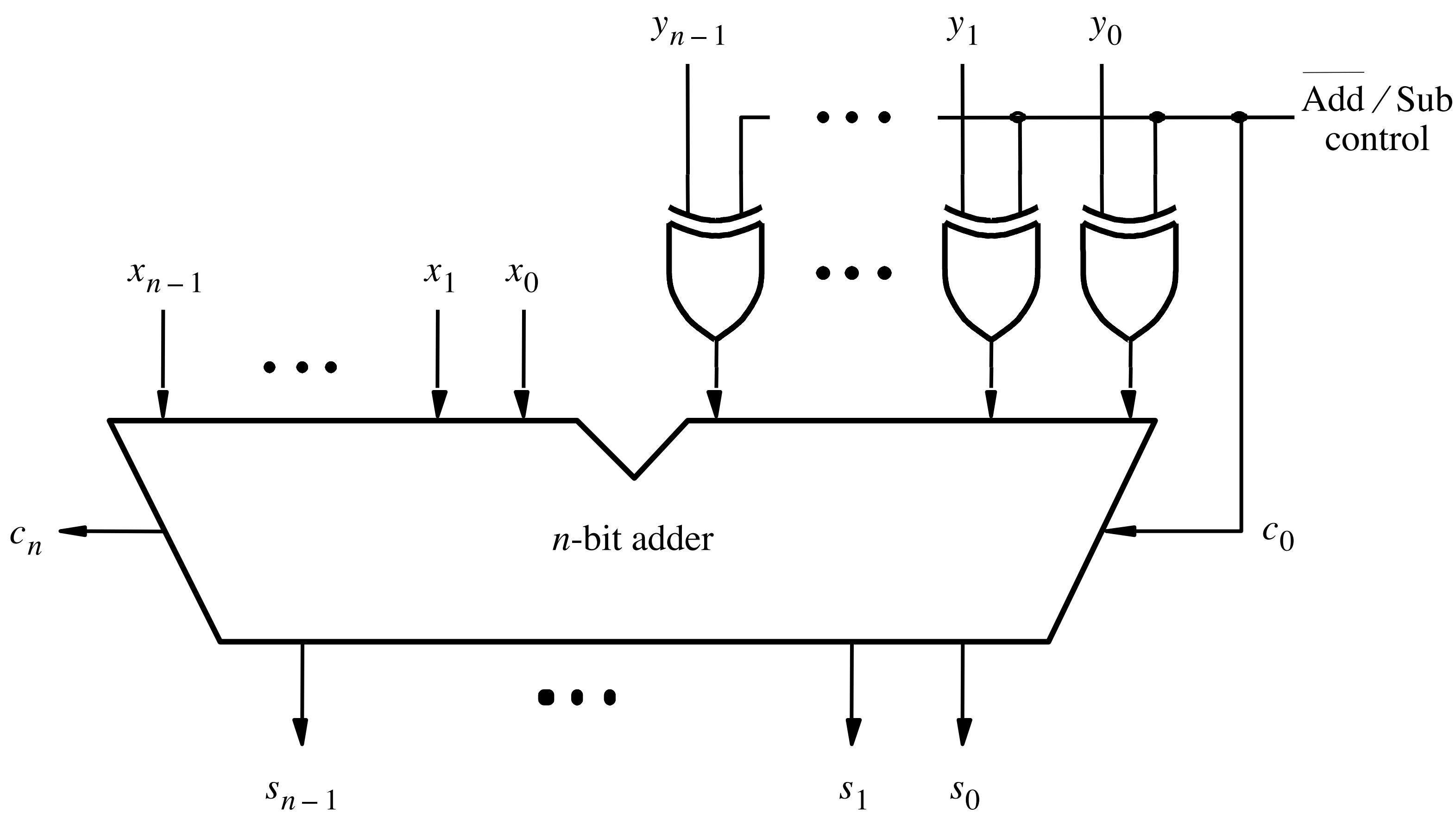
[Figure 3.11 from the textbook]

Take-Home Message

- **Subtraction can be performed by simply adding the 2's complement of the second number, regardless of the signs of the two numbers.**

- **Thus, the same adder circuit can be used to perform both addition and subtraction !!!**

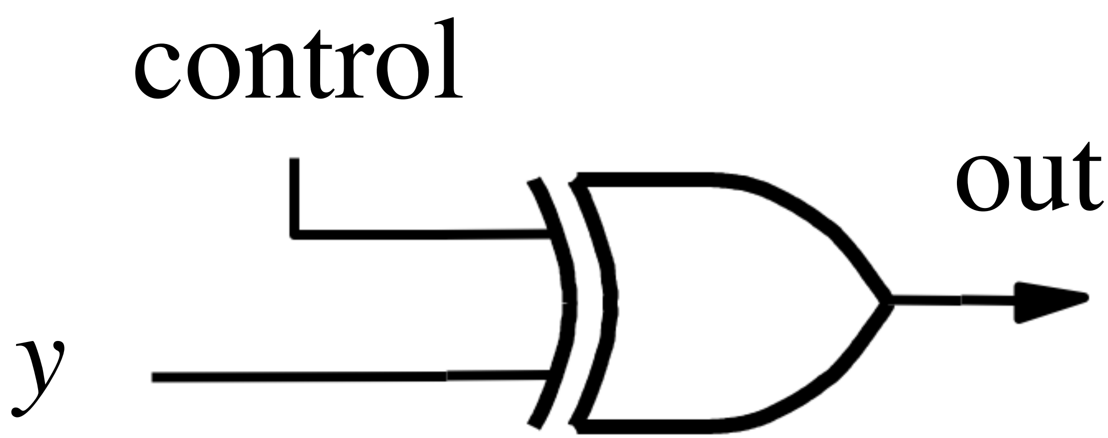
Adder/subtractor unit



[Figure 3.12 from the textbook]

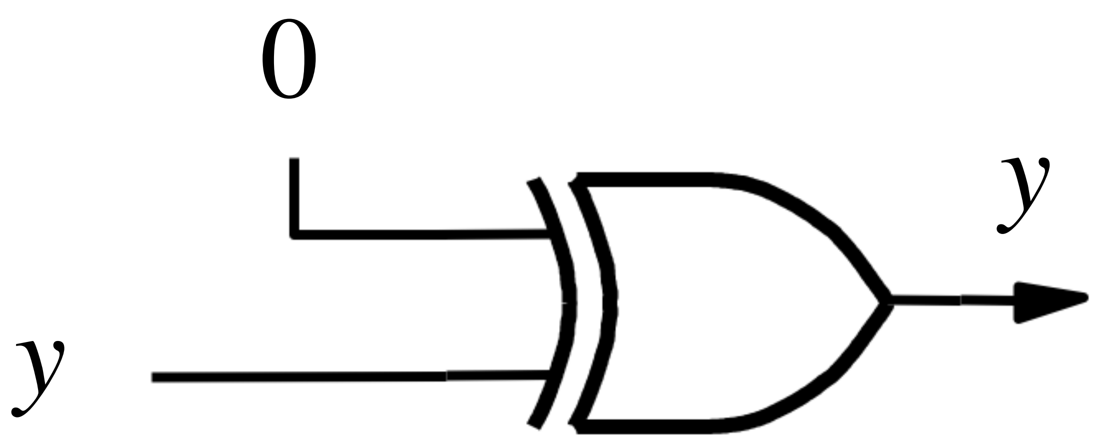
XOR Tricks

control	y	out
0	0	0
0	1	1
1	0	1
1	1	0



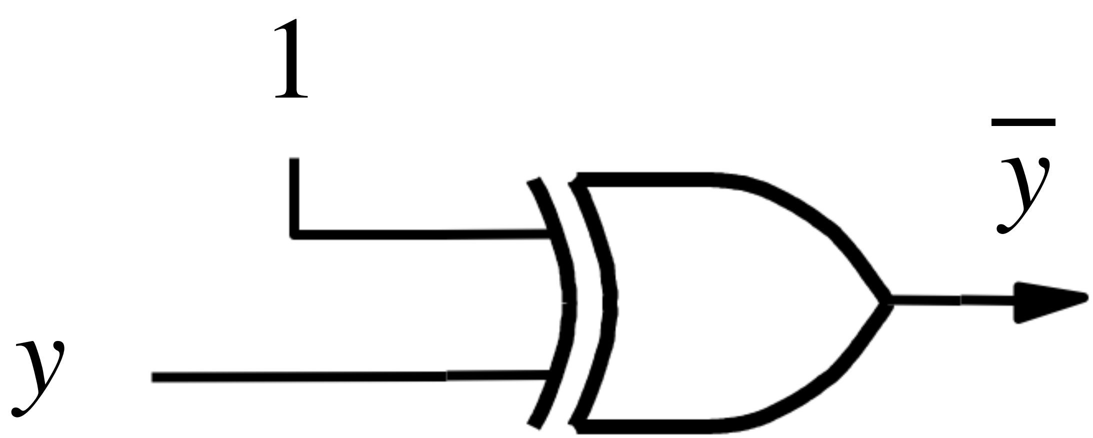
XOR as a repeater

control	y	out
0	0	0
0	1	1

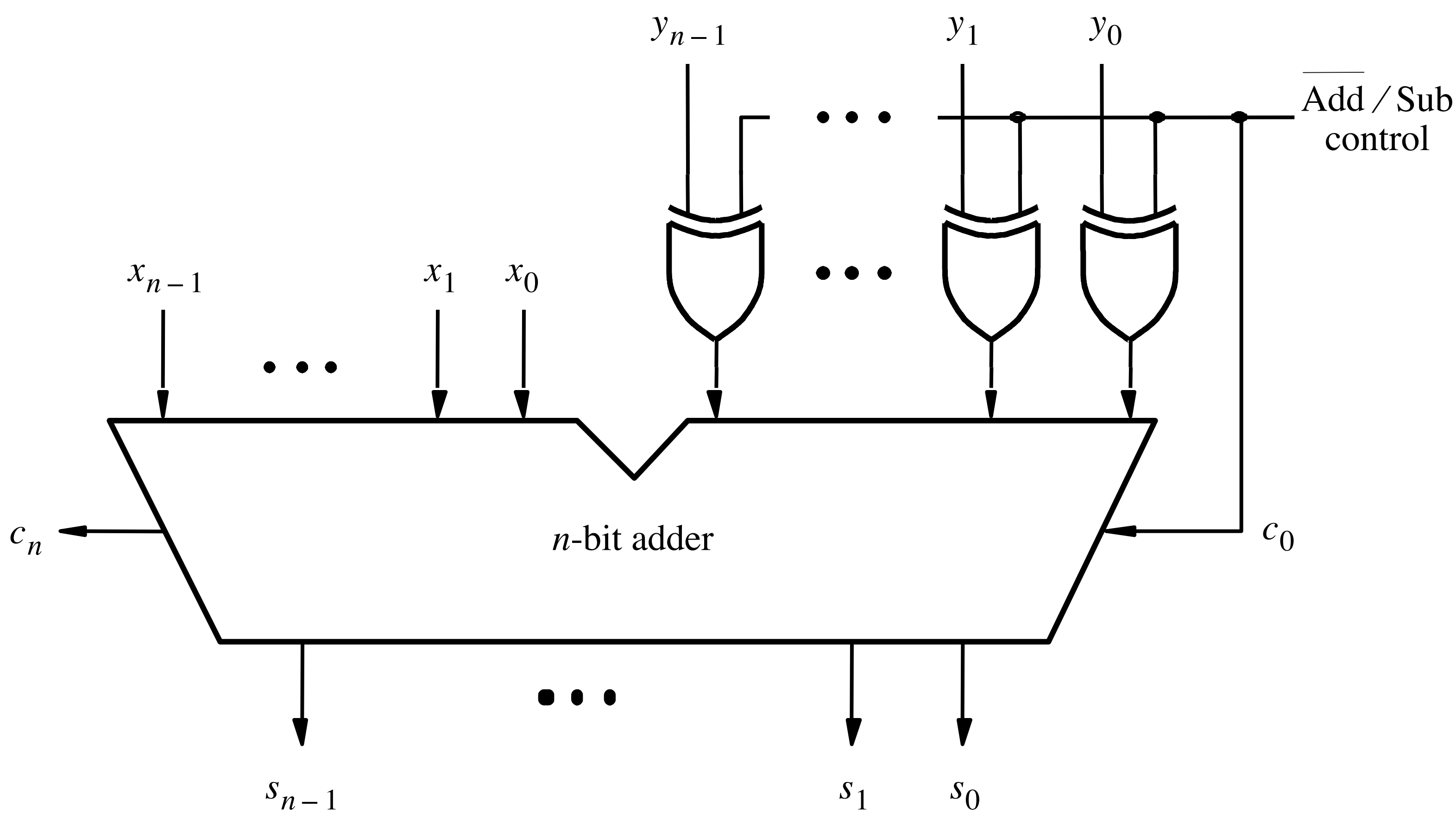


XOR as an inverter

control	y	out
1	0	1
1	1	0

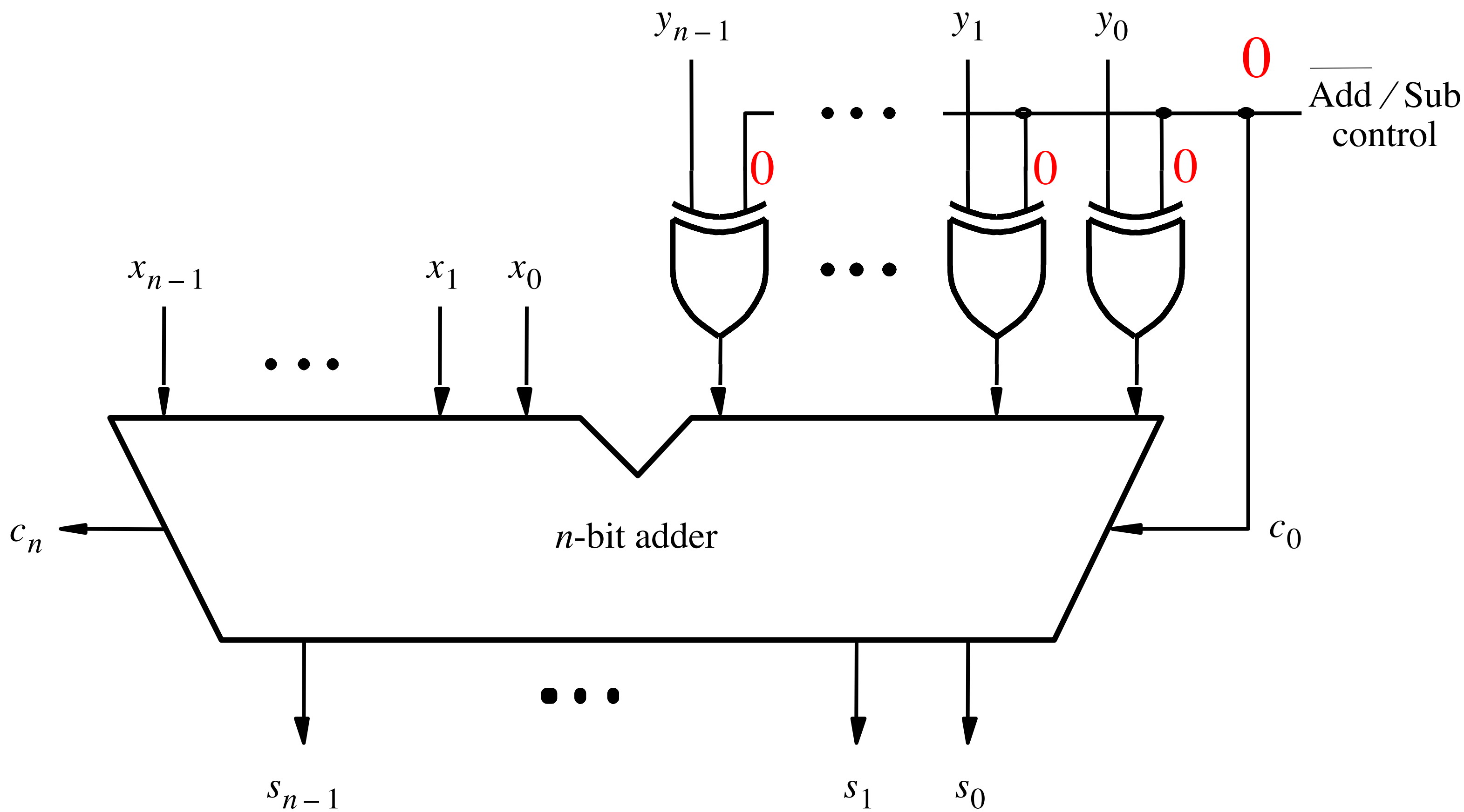


Addition: when control = 0



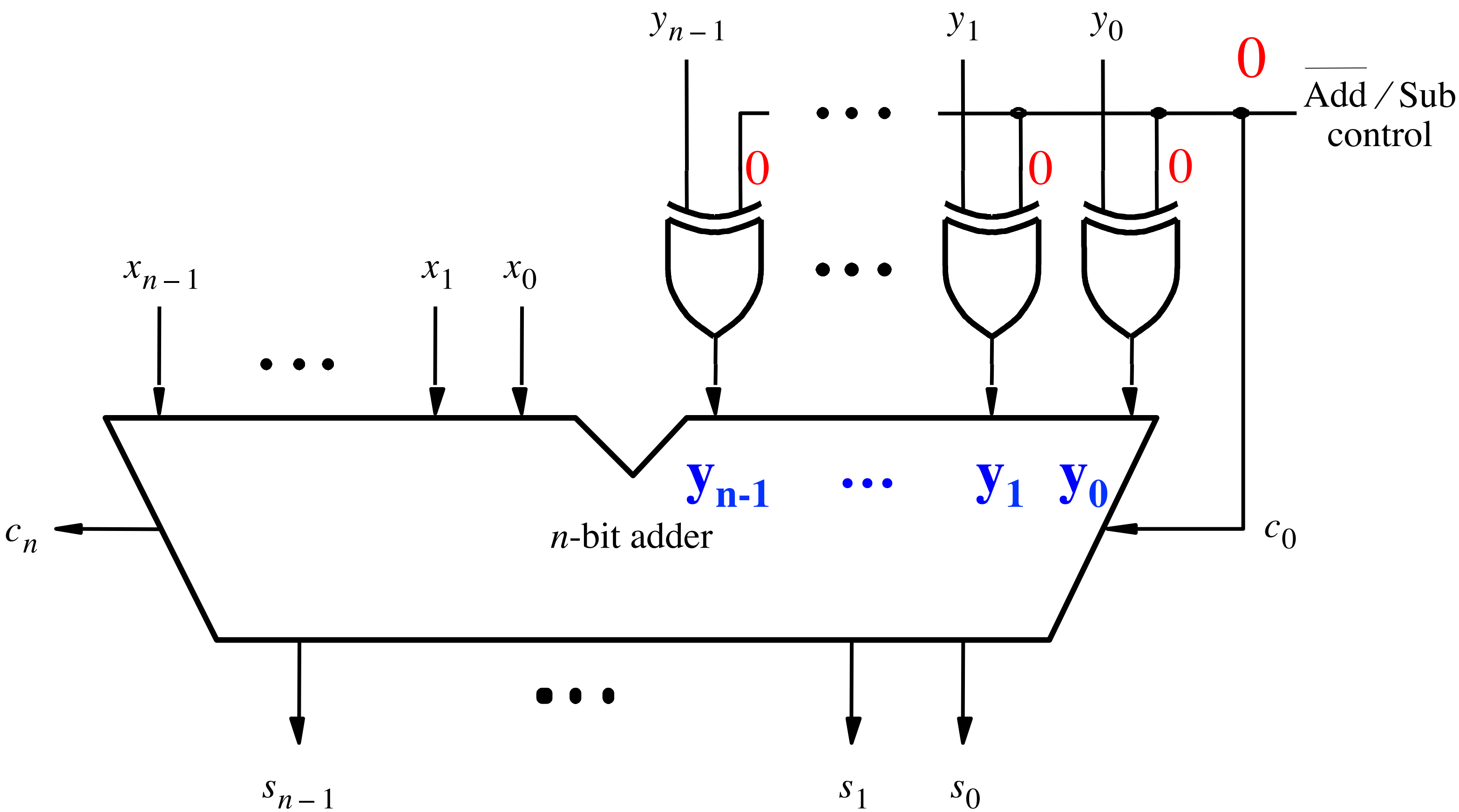
[Figure 3.12 from the textbook]

Addition: when control = 0



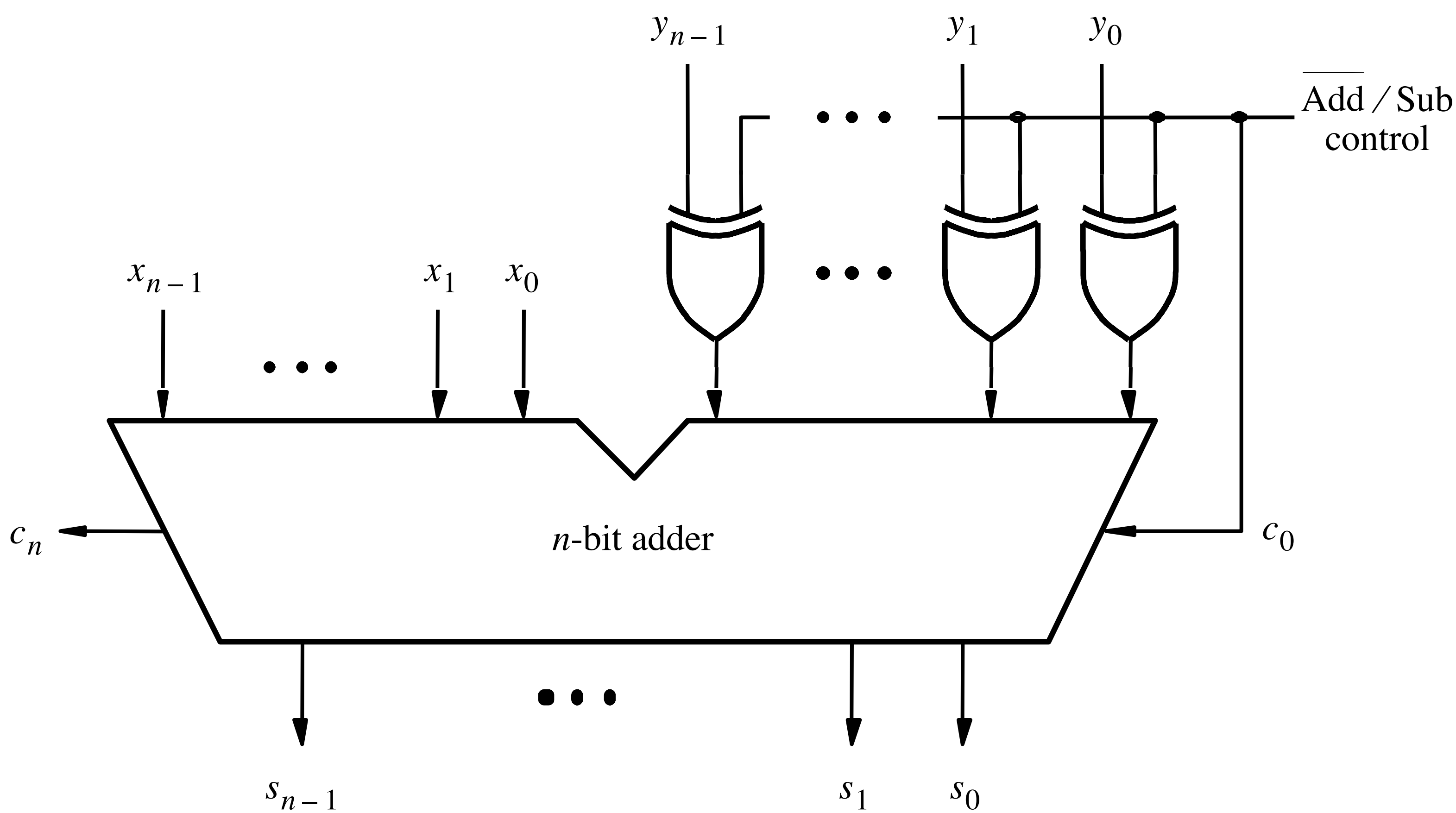
[Figure 3.12 from the textbook]

Addition: when control = 0



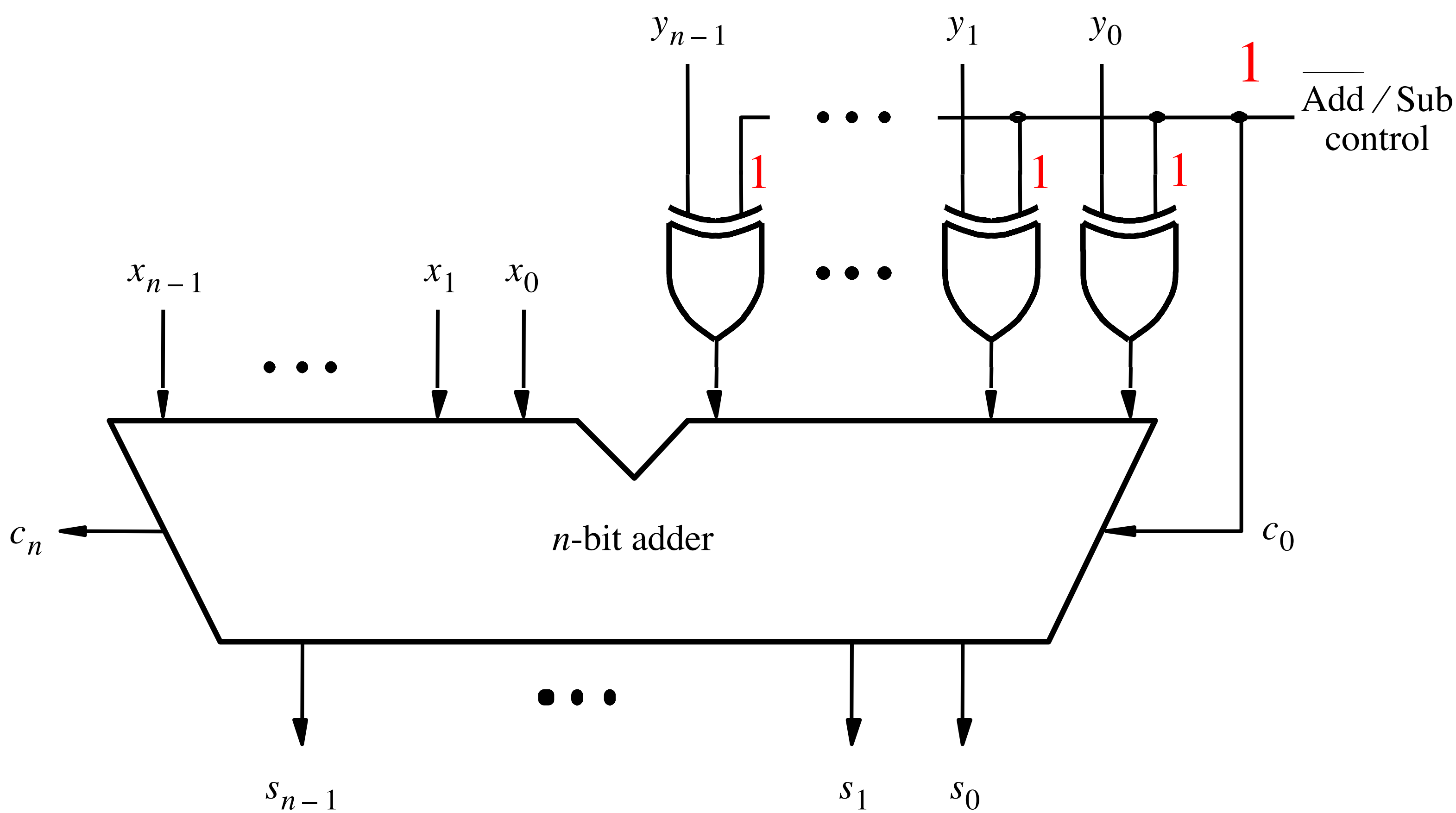
[Figure 3.12 from the textbook]

Subtraction: when control = 1



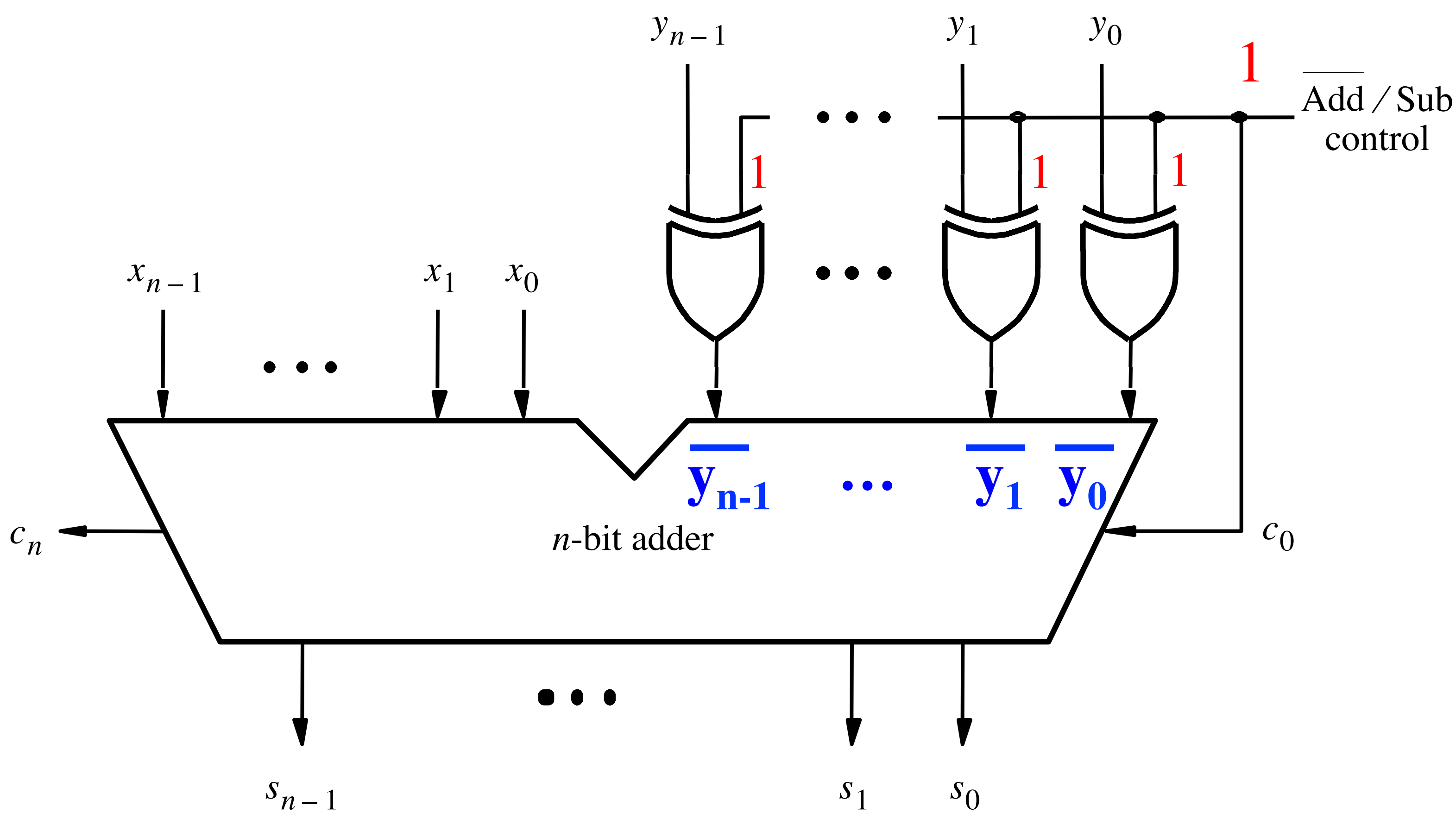
[Figure 3.12 from the textbook]

Subtraction: when control = 1



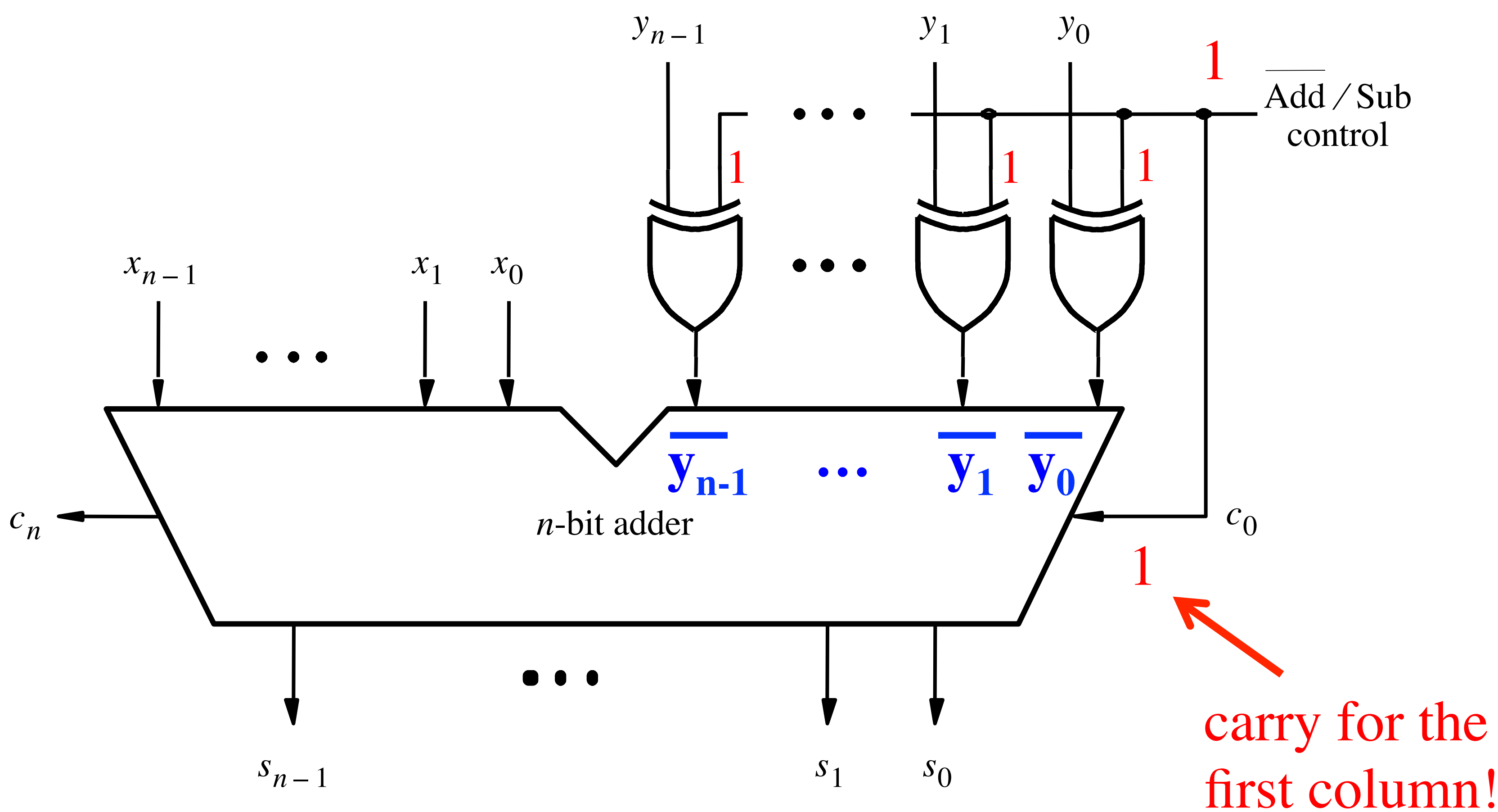
[Figure 3.12 from the textbook]

Subtraction: when control = 1



[Figure 3.12 from the textbook]

Subtraction: when control = 1



[Figure 3.12 from the textbook]

Examples of determination of overflow

$$
\begin{array}{r}
(+7) \\
+\ (+2) \\
\hline
(+9)
\end{array}
\qquad
\begin{array}{rr}
 & 0\,1\,1\,1 \\
+ & 0\,0\,1\,0 \\
\hline
 & 1\,0\,0\,1
\end{array}
\qquad\qquad
\begin{array}{r}
(-7) \\
+\ (+2) \\
\hline
(-5)
\end{array}
\qquad
\begin{array}{rr}
 & 1\,0\,0\,1 \\
+ & 0\,0\,1\,0 \\
\hline
 & 1\,0\,1\,1
\end{array}
$$

$$
\begin{array}{r}
(+7) \\
+\ (-2) \\
\hline
(+5)
\end{array}
\qquad
\begin{array}{rr}
 & 0\,1\,1\,1 \\
+ & 1\,1\,1\,0 \\
\hline
 & 1\,0\,1\,0\,1
\end{array}
\qquad\qquad
\begin{array}{r}
(-7) \\
+\ (-2) \\
\hline
(-9)
\end{array}
\qquad
\begin{array}{rr}
 & 1\,0\,0\,1 \\
+ & 1\,1\,1\,0 \\
\hline
 & 1\,0\,1\,1\,1
\end{array}
$$

[Figure 3.13 from the textbook]

Examples of determination of overflow

	0 1 1 0 0		0 0 0 0 0
(+ 7)	0 1 1 1	(–7)	1 0 0 1
+ (+ 2)	+ 0 0 1 0	+ (+ 2)	+ 0 0 1 0
(+ 9)	1 0 0 1	(–5)	1 0 1 1

	1 1 1 0 0		1 0 0 0 0
(+ 7)	0 1 1 1	(–7)	1 0 0 1
+ (– 2)	+ 1 1 1 0	+ (–2)	+ 1 1 1 0
(+ 5)	1 0 1 0 1	(–9)	1 0 1 1 1

Include the carry bits: $c_4\ c_3\ c_2\ c_1\ c_0$

Examples of determination of overflow

Decimal		Binary
	Carries	[0 1] 1 0 0
(+7)		0 1 1 1
+ (+2)	+	0 0 1 0
(+9)		1 0 0 1

Decimal		Binary
	Carries	[0 0] 0 0 0
(–7)		1 0 0 1
+ (+2)	+	0 0 1 0
(–5)		1 0 1 1

Decimal		Binary
	Carries	[1 1] 1 0 0
(+7)		0 1 1 1
+ (–2)	+	1 1 1 0
(+5)		1 0 1 0 1

Decimal		Binary
	Carries	[1 0] 0 0 0
(–7)		1 0 0 1
+ (–2)	+	1 1 1 0
(–9)		1 0 1 1 1

Include the carry bits: [c_4 c_3] c_2 c_1 c_0

Examples of determination of overflow

$c_4 = 0$
$c_3 = 1$

$$\begin{array}{r} (+7) \\ +\ (+2) \\ \hline (+9) \end{array} \qquad \begin{array}{rr} & \boxed{0\ 1}\,1\ 0\ 0 \\ & 0\ 1\ 1\ 1 \\ + & 0\ 0\ 1\ 0 \\ \hline & 1\ 0\ 0\ 1 \end{array}$$

$$\begin{array}{r} (-7) \\ +\ (+2) \\ \hline (-5) \end{array} \qquad \begin{array}{rr} & \boxed{0\ 0}\,0\ 0\ 0 \\ & 1\ 0\ 0\ 1 \\ + & 0\ 0\ 1\ 0 \\ \hline & 1\ 0\ 1\ 1 \end{array}$$

$c_4 = 0$
$c_3 = 0$

$c_4 = 1$
$c_3 = 1$

$$\begin{array}{r} (+7) \\ +\ (-2) \\ \hline (+5) \end{array} \qquad \begin{array}{rr} & \boxed{1\ 1}\,1\ 0\ 0 \\ & 0\ 1\ 1\ 1 \\ + & 1\ 1\ 1\ 0 \\ \hline & 1\ 0\ 1\ 0\ 1 \end{array}$$

$$\begin{array}{r} (-7) \\ +\ (-2) \\ \hline (-9) \end{array} \qquad \begin{array}{rr} & \boxed{1\ 0}\,0\ 0\ 0 \\ & 1\ 0\ 0\ 1 \\ + & 1\ 1\ 1\ 0 \\ \hline & 1\ 0\ 1\ 1\ 1 \end{array}$$

$c_4 = 1$
$c_3 = 0$

Include the carry bits: $\boxed{c_4\ c_3}\,c_2\ c_1\ c_0$

Examples of determination of overflow

$c_4 = 0$
$c_3 = 1$

$$\begin{array}{rr} & (+7) \\ + & (+2) \\ \hline & (+9) \end{array} \qquad \begin{array}{rr} & \boxed{0\,1}\,1\,0\,0 \\ & 0\,1\,1\,1 \\ + & 0\,0\,1\,0 \\ \hline & 1\,0\,0\,1 \end{array}$$

$c_4 = 0$
$c_3 = 0$

$$\begin{array}{rr} & (-7) \\ + & (+2) \\ \hline & (-5) \end{array} \qquad \begin{array}{rr} & \boxed{0\,0}\,0\,0\,0 \\ & 1\,0\,0\,1 \\ + & 0\,0\,1\,0 \\ \hline & 1\,0\,1\,1 \end{array}$$

$c_4 = 1$
$c_3 = 1$

$$\begin{array}{rr} & (+7) \\ + & (-2) \\ \hline & (+5) \end{array} \qquad \begin{array}{rr} & \boxed{1\,1}\,1\,0\,0 \\ & 0\,1\,1\,1 \\ + & 1\,1\,1\,0 \\ \hline & 1\,0\,1\,0\,1 \end{array}$$

$c_4 = 1$
$c_3 = 0$

$$\begin{array}{rr} & (-7) \\ + & (-2) \\ \hline & (-9) \end{array} \qquad \begin{array}{rr} & \boxed{1\,0}\,0\,0\,0 \\ & 1\,0\,0\,1 \\ + & 1\,1\,1\,0 \\ \hline & 1\,0\,1\,1\,1 \end{array}$$

Overflow occurs only in these two cases.

Examples of determination of overflow

$c_4 = 0$
$c_3 = 1$

		0 1 1 0 0
(+7)		0 1 1 1
+ (+2)	+	0 0 1 0
(+9)		1 0 0 1

		0 0 0 0 0
(–7)		1 0 0 1
+ (+2)	+	0 0 1 0
(–5)		1 0 1 1

$c_4 = 0$
$c_3 = 0$

$c_4 = 1$
$c_3 = 1$

		1 1 1 0 0
(+7)		0 1 1 1
+ (–2)	+	1 1 1 0
(+5)		1 0 1 0 1

		1 0 0 0 0
(–7)		1 0 0 1
+ (–2)	+	1 1 1 0
(–9)		1 0 1 1 1

$c_4 = 1$
$c_3 = 0$

$$\text{Overflow} = c_3\overline{c_4} + \overline{c_3}c_4$$

Examples of determination of overflow

$c_4 = 0$
$c_3 = 1$

$$\begin{array}{r} (+7) \\ +\ (+2) \\ \hline (+9) \end{array} \qquad \begin{array}{rr} & \boxed{0\ 1}\,1\ 0\ 0 \\ & 0\ 1\ 1\ 1 \\ + & 0\ 0\ 1\ 0 \\ \hline & 1\ 0\ 0\ 1 \end{array}$$

$$\begin{array}{r} (-7) \\ +\ (+2) \\ \hline (-5) \end{array} \qquad \begin{array}{rr} & \boxed{0\ 0}\,0\ 0\ 0 \\ & 1\ 0\ 0\ 1 \\ + & 0\ 0\ 1\ 0 \\ \hline & 1\ 0\ 1\ 1 \end{array}$$

$c_4 = 0$
$c_3 = 0$

$c_4 = 1$
$c_3 = 1$

$$\begin{array}{r} (+7) \\ +\ (-2) \\ \hline (+5) \end{array} \qquad \begin{array}{rr} & \boxed{1\ 1}\,1\ 0\ 0 \\ & 0\ 1\ 1\ 1 \\ + & 1\ 1\ 1\ 0 \\ \hline & 1\ 0\ 1\ 0\ 1 \end{array}$$

$$\begin{array}{r} (-7) \\ +\ (-2) \\ \hline (-9) \end{array} \qquad \begin{array}{rr} & \boxed{1\ 0}\,0\ 0\ 0 \\ & 1\ 0\ 0\ 1 \\ + & 1\ 1\ 1\ 0 \\ \hline & 1\ 0\ 1\ 1\ 1 \end{array}$$

$c_4 = 1$
$c_3 = 0$

$$\text{Overflow} = \underbrace{c_3\bar{c}_4 + \bar{c}_3 c_4}_{\text{XOR}}$$

Calculating overflow for 4-bit numbers with only three significant bits

$$\begin{aligned} \text{Overflow} &= c_3\overline{c}_4 + \overline{c}_3 c_4 \\ &= c_3 \oplus c_4 \end{aligned}$$

Calculating overflow for n-bit numbers with only n-1 significant bits

$$\text{Overflow} = c_{n-1} \oplus c_n$$

Another way to look at the overflow issue

$$X = x_3 x_2 x_1 x_0$$

$$Y = y_3 y_2 y_1 y_0$$

$$S = s_3 s_2 s_1 s_0$$

Another way to look at the overflow issue

$$\begin{array}{r} X = x_3x_2x_1x_0 \\ Y = y_3y_2y_1y_0 \\ \hline S = s_3s_2s_1s_0 \end{array}$$

If both numbers that we are adding have the same sign but the sum does not, then we have an overflow.

$$\text{Overflow} = x_3y_3\overline{s}_3 + \overline{x}_3\overline{y}_3s_3$$

Questions?

THE END