Sample Solutions

CprE 281: Digital Logic
Midterm 1: Friday Sep 22, 2017

Student Name: _____ **Student ID Number:** _____

Lab Section:	**Mon 9-12(N)**	**Tue 11-2(U)**	**Wed 8-11(J)**	**Thur 11-2(Q)**	**Fri 11-2(G)**
(circle one)	**Mon 12-3(P)**	**Tue 2-5(M)**	**Wed 11-2(W)**	**Thur 11-2(V)**	
		Tue 2-5(Z)	**Wed 6-9(T)**	**Thur 2-5(L)**	
				Thur 5-8(K)	

1. True/False Questions (10 x 1p each = 10p)

(a) I forgot to write down my name, student ID, and lab section. TRUE / (FALSE)

(b) A 2-to-1 multiplexer can be built using only OR and NOT gates. (TRUE) / FALSE

(c) It is not possible to build a NOT gate with a 2-to-1 multiplexer. TRUE / (FALSE)

(d) The minimum expression for $f(x,y)=\sum m(0,1,3)+D(2)$ is 1. (TRUE) / FALSE

(e) $\overline{a \cdot c} + a \cdot c = 1$ (TRUE) / FALSE

(f) $\overline{\bar{x}\, y} = x\, \bar{y}$ TRUE / (FALSE)

(g) $m_0 + m_1 = \overline{M_0 \cdot M_1}$ (TRUE) / FALSE

(h) $\overline{x \cdot y} = \bar{x} + \bar{x}$ TRUE / (FALSE)

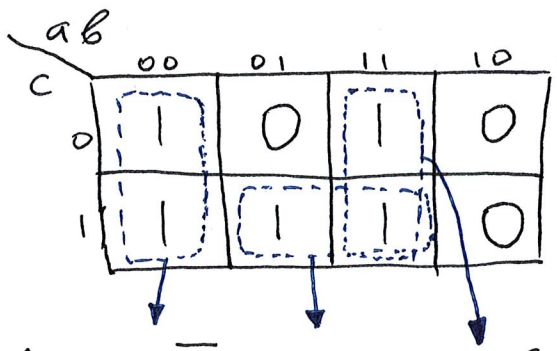
(i) The 'K' in K-map stands for 'Kilobyte.' TRUE / (FALSE)

(j) K-2SO is stronger than C-3PO. (TRUE) / FALSE

2. Three-Variable K-map (5p)

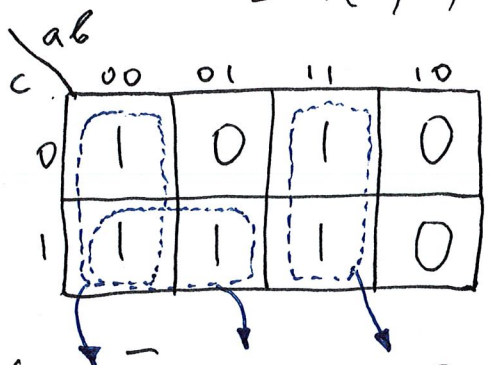
Draw the K-map and derive the minimum **SOP** expression for **f(a,b,c) = Π M(2, 4, 5)**.

This problem has two solutions. $= \Sigma m(0, 1, 3, 6, 7)$



$f = \bar{a}\bar{b} + bc + ab$

(solution #1)



$f = \bar{a}b + \bar{a}c + ab$

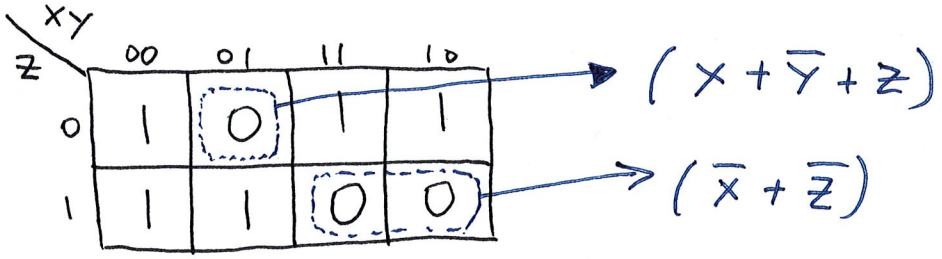
(solution #2)

3. Minimization (3 x 5p each = 15p)

(a) Draw the truth table for the Boolean function $f(x, y, z) = x\bar{z} + \bar{x}(z + \bar{y})$

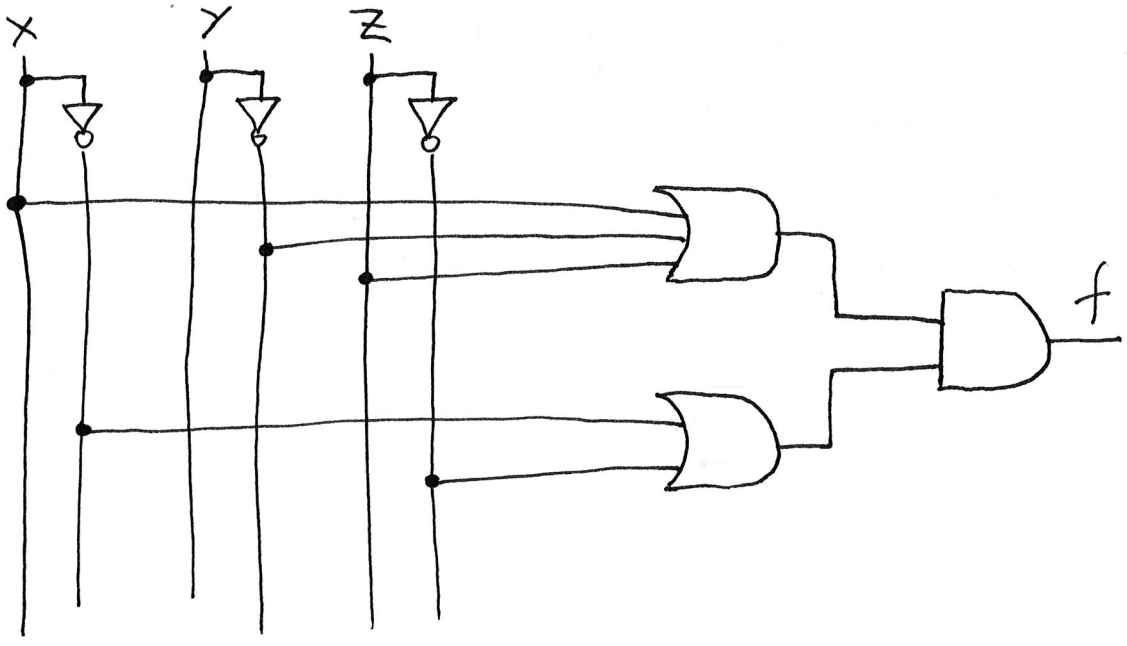
x	y	z	$x\bar{z}$	$\bar{x}$	$z + \bar{y}$	$\bar{x}(z + \bar{y})$	f
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	0	1	0	0	0
0	1	1	0	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	0	0	1	0	0
1	1	0	1	0	0	0	1
1	1	1	0	0	1	0	0

(b) Draw the K-map for this function and derive the minimum cost <u>POS</u> expression.



$$f = (x + \bar{y} + z) \cdot (\bar{x} + \bar{z})$$

(c) Draw the circuit diagram for the minimum cost <u>POS</u> expression.



4. Number Conversions (5 x 4p each = 20p)

(a) Convert 243_5 to decimal

$$2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0 = 2 \times 25 + 4 \times 5 + 3 \times 1$$
$$= 50 + 20 + 3$$
$$= 73_{10}$$

(b) Convert 193_{10} to binary

193 / 2 = 96 1
96 / 2 = 48 0
48 / 2 = 24 0
24 / 2 = 12 0
12 / 2 = 6 0
6 / 2 = 3 0
3 / 2 = 1 1
1 / 2 = 0 1

11000001_2

(c) Convert $DC5B_{16}$ to binary

$1101\ 1100\ 0101\ 1011_2$

D C 5 B

(d) Find the value of the base x in the following equation: $12_x = 1011_2$

$1011_2 = 11_{10}$

simplify ↓

$$1 \times X^1 + 2 \times X^0 = 11_{10}$$

$$X + 2 = 11 \implies X = 9$$

(e) Compute the following sum and write the answer in octal:

$1101001_2 + 81_{16} = ?_8$

padding: 001 101 001 + padding: 010 000 001 = $?_8$

convert to Binary ↓

1 5 1 + 2 0 1 = 352_8

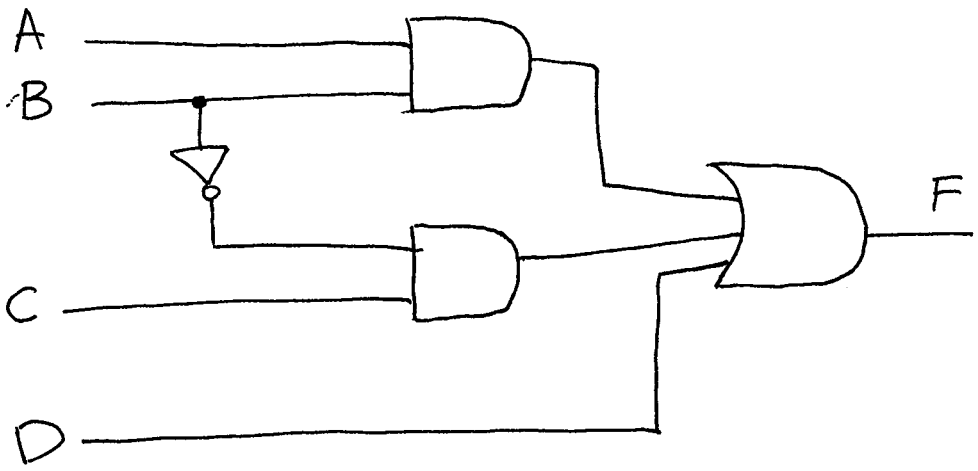
convert to octal ↓

5. Verilog to Circuit (2 x 5p each = 10p)

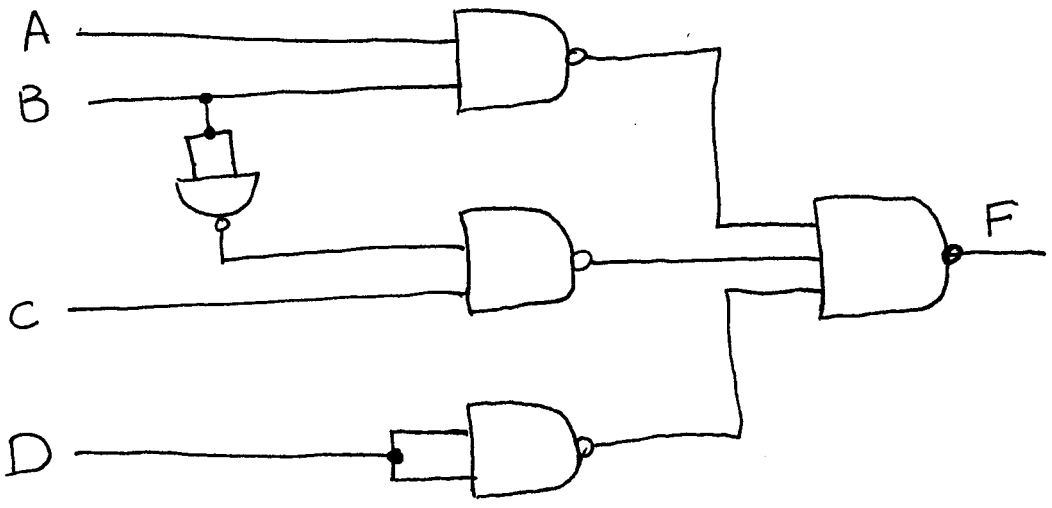
(a) Draw the circuit diagram that corresponds to the Verilog module shown below. Label all inputs and outputs.

```
module mystery (A, B, C, D, F);
    input A, B, C, D;
    output F;

    assign F = (A & B) | (~B & C) | D;
endmodule
```

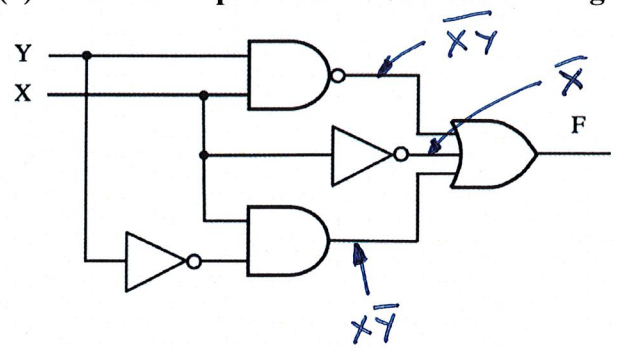


(b) Redraw the circuit from (a) using only NAND gates.



6. Circuit to Circuit Conversion (3 x 5p each = 15p)

(a) Write the expression for the function F given by this circuit (don't simplify it yet).



$$F = \overline{XY} + \overline{X} + X\overline{Y}$$

(b) Use the theorems of Boolean algebra to simplify the expression from part (a).

$$F = \overline{XY} + \overline{X} + X\overline{Y} \qquad (\text{De Morgan's})$$

$$= \overline{X} + \overline{Y} + \overline{X} + X\overline{Y} \qquad (\text{repeated } \overline{X})$$

$$= \overline{X} + \overline{Y} + X\overline{Y} \qquad (\text{factor } \overline{Y})$$

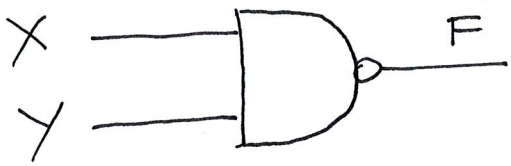
$$= \overline{X} + (\underbrace{1 + X}_{1})\overline{Y} \qquad (\text{Theorem 5e})$$

$$= \overline{X} + \overline{Y} \qquad (\text{De Morgan's})$$

$$= \overline{X \cdot Y}$$

(c) Draw the circuit for your expression from part (b). Label all inputs and outputs.

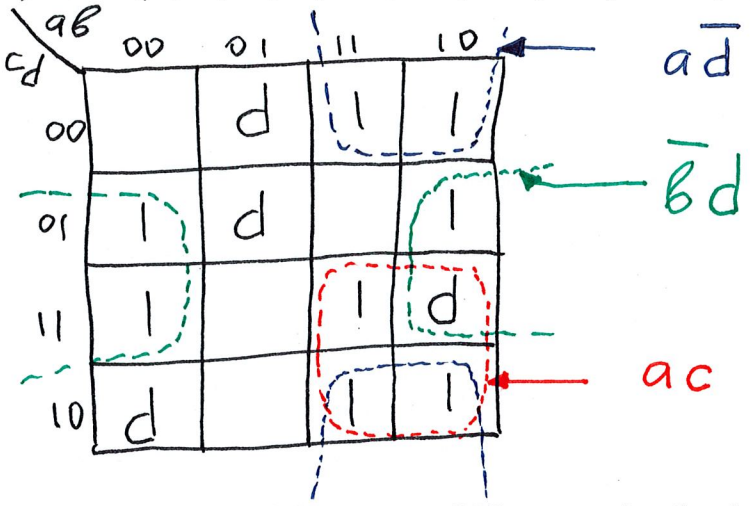
This simplifies to a NAND.



7. Derive the minimum SOP expression using a K-map (3 x 5p each = 15p)

(a) Draw the K-map for the following function

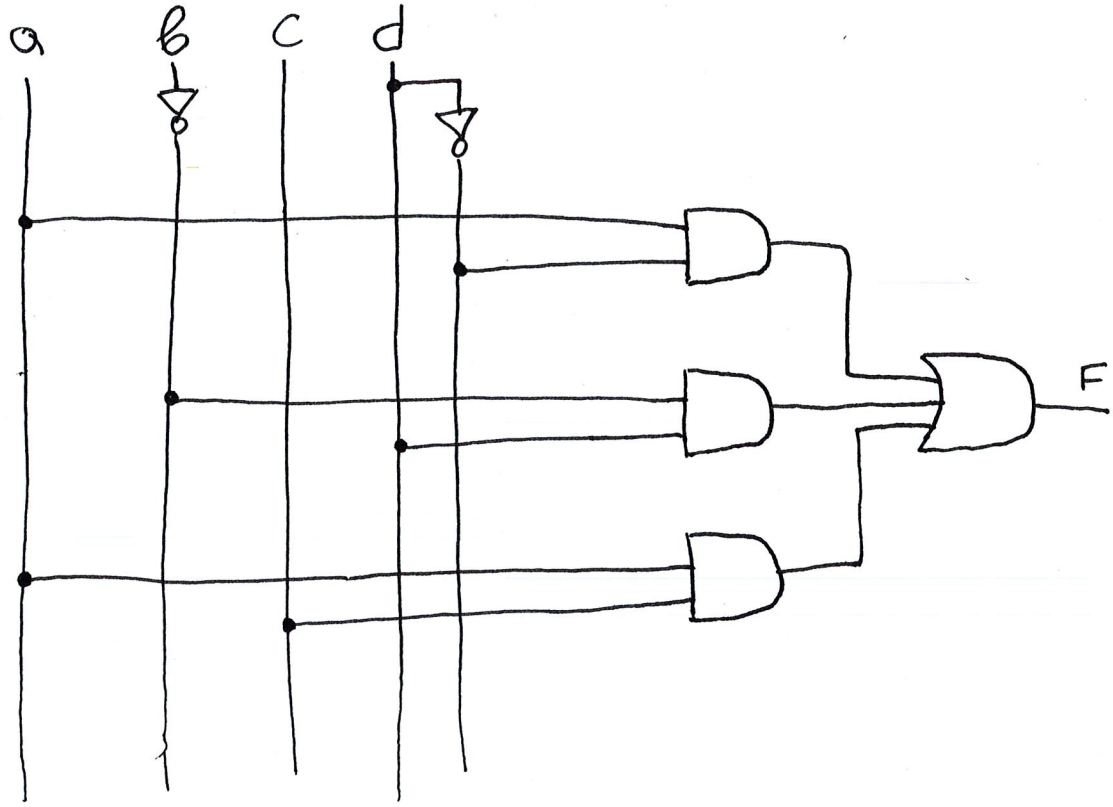
$$F(a,b,c,d)=\Sigma m(1, 3, 8, 9, 10, 12, 14, 15) + D(2, 4, 5, 11)$$



(b) Use the K-map to derive the minimum-cost SOP expression for the function F.

$$F = a\bar{d} + \bar{b}d + ac$$

(c) Draw the circuit diagram for the minimum expression from part (b).

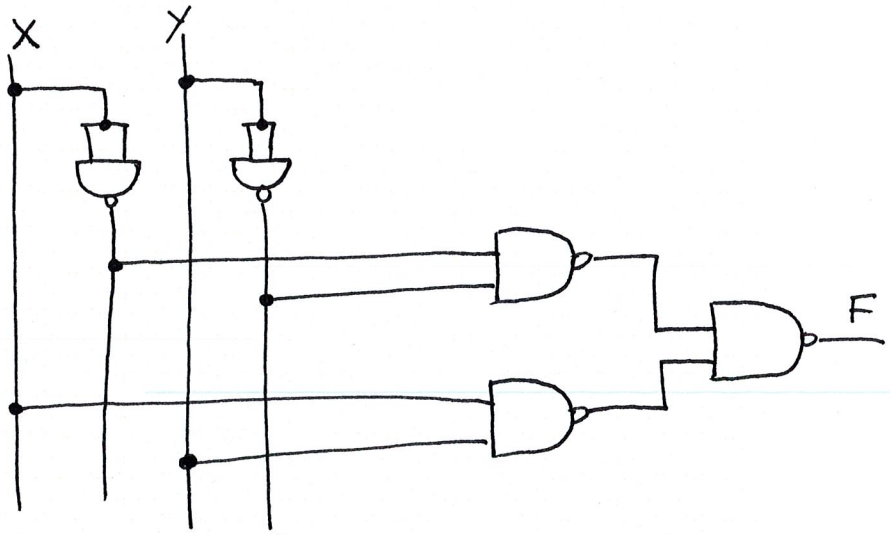


8. NAND/NOR Logic (2 x 5p each = 10p)

(a) Using only NAND gates, draw the logic circuit that corresponds to the truth table shown below. Hint: Start by writing and then modifying the expression for F.

X	Y	F	
0	0	1	← $\bar{X}\bar{Y}$
0	1	0	
1	0	0	
1	1	1	← XY

$$F = \bar{X}\bar{Y} + XY = \overline{\overline{\bar{X}\bar{Y}} + \overline{XY}} = \overline{\overline{\bar{X}\bar{Y}} \cdot \overline{XY}}$$



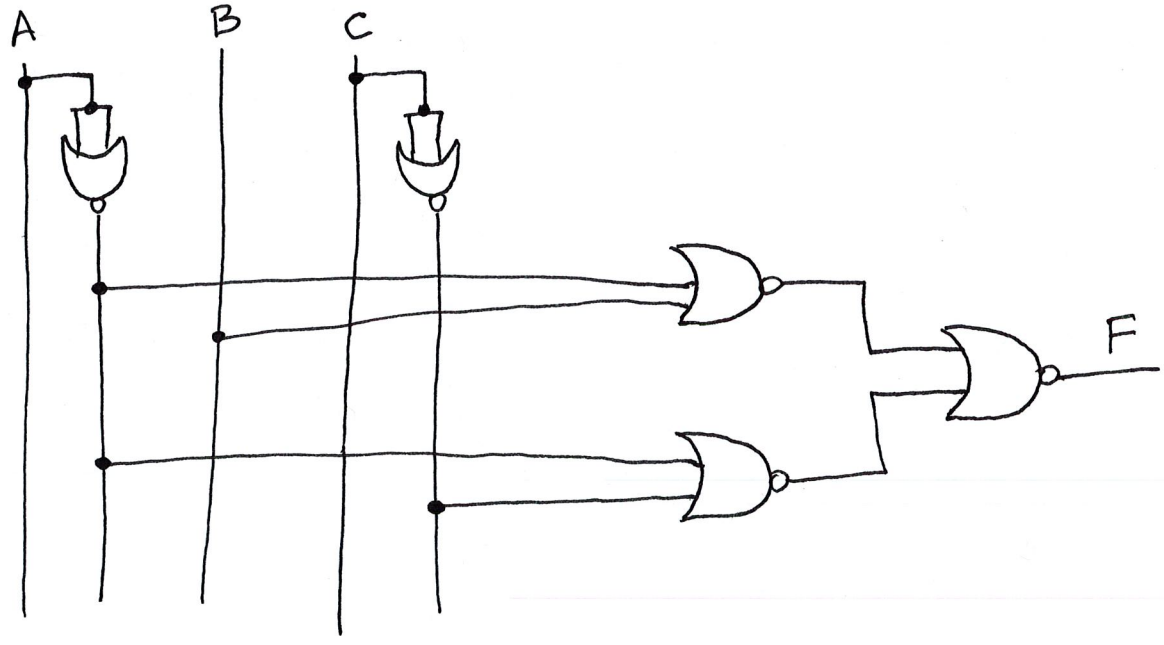
(b) Draw the circuit that corresponds to the following K-map using only NOR gates.

C \ AB	00	01	11	10
0	1	1	1	0
1	d	1	0	0

$(\bar{A} + B)$

$(\bar{A} + \bar{C})$

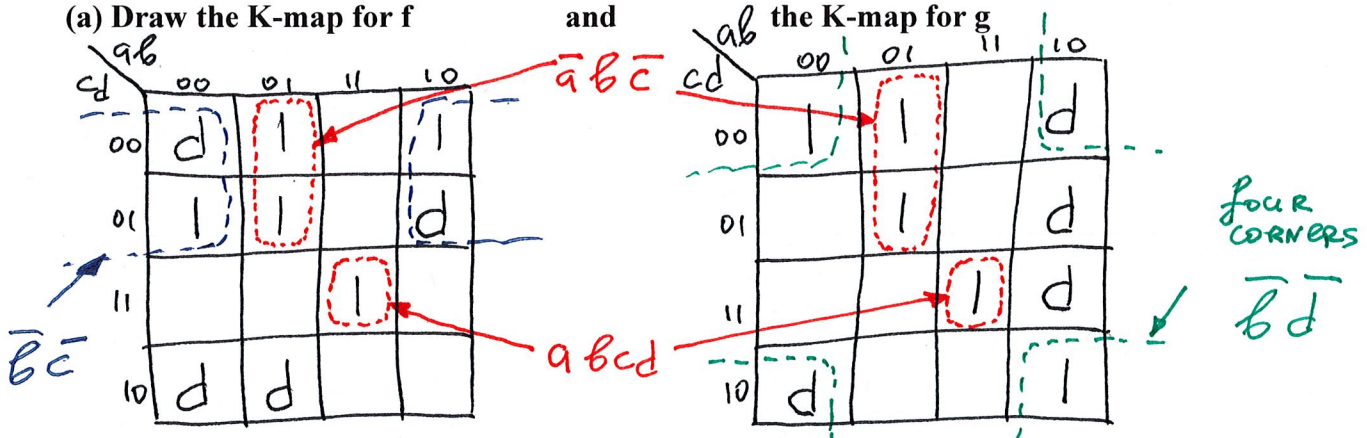
$$F = (\bar{A} + B) \cdot (\bar{A} + \bar{C}) = \overline{\overline{(\bar{A} + B) \cdot (\bar{A} + \bar{C})}} = \overline{\overline{\bar{A} + B} + \overline{\bar{A} + \bar{C}}}$$



9. Joint Optimization (3 x 5p each = 15p)
The outputs f and g of a two-output circuit are specified with the following expressions:

f (a, b, c, d) = Σm(1, 4, 5, 8, 15) + D(0, 2, 6, 9)
g (a, b, c, d) = Σm(0, 4, 5, 10, 15) + D(2, 8, 9, 11)

(a) Draw the K-map for f and the K-map for g

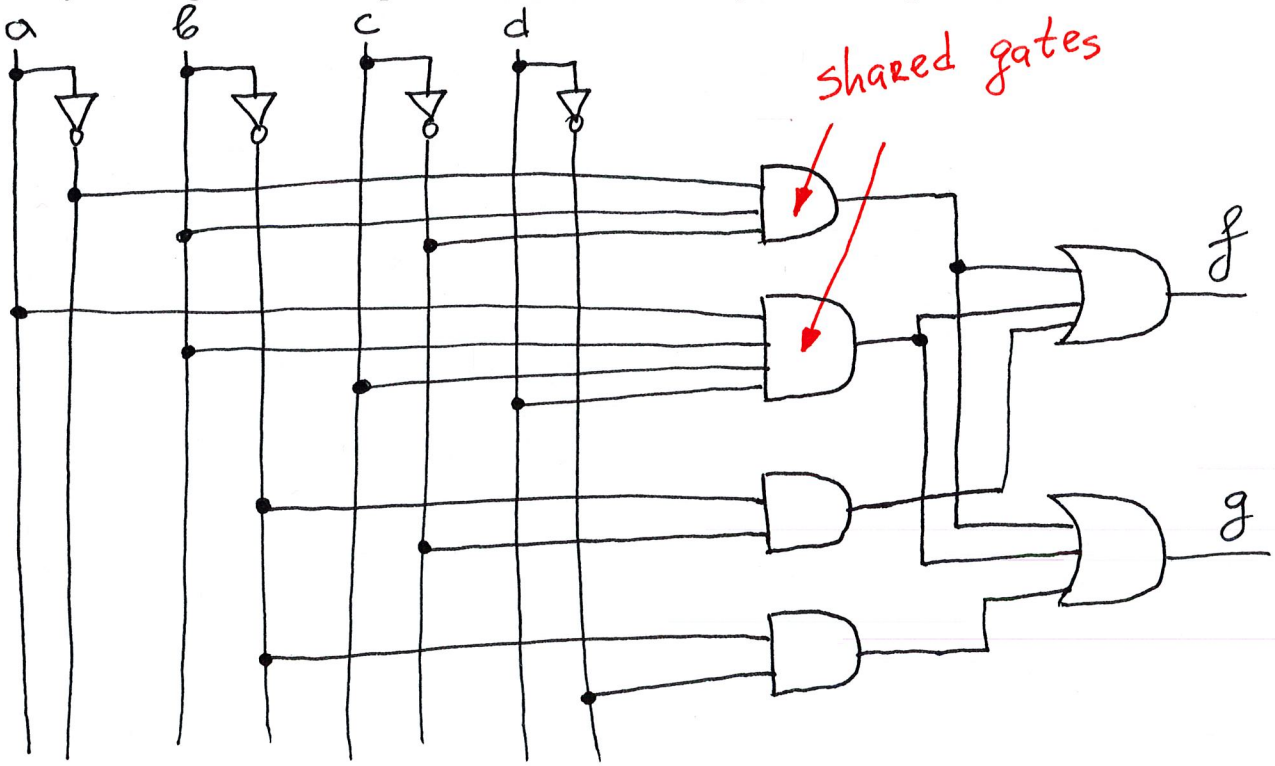


(b) Derive the jointly optimized SOP expressions for f and g such that the two expressions share two implicants. Note that these are not necessarily prime implicants.

$$f = \bar{a}b\bar{c} + abcd + \bar{b}\bar{c}$$

$$g = \bar{a}b\bar{c} + abcd + \bar{b}\bar{d}$$

(c) Draw the diagram for the jointly optimized circuit. Indicate which logic gates are shared by drawing arrows that point to them. Label all inputs and outputs.



10. Minimization with Theorems (15p)

Use the theorems of Boolean algebra to simplify the following Boolean function

$$f(w,x,y,z)=\underbrace{y(\bar{y}+z)}_{A}+\underbrace{(\bar{y}+w\bar{y}(xz+x\bar{z}))z}_{B}+\underbrace{\overline{(x\bar{y}+\overline{(x+y)}+\bar{z})}}_{C}+\underbrace{y(z+w\bar{x}(\bar{w}+z))}_{D}$$

Simplify A, B, C, and D separately and then combine the partial results.

$$A = y(\bar{y}+z) = \underbrace{y\bar{y}}_{0} + yz = yz$$

$$B = \left(\bar{y}+w\bar{y}(xz+x\bar{z})\right)z = \bar{y}z + w\bar{y}\left(x\underbrace{(z+\bar{z})}_{1}\right)z$$

$$= \bar{y}z + w\bar{y}xz = \bar{y}z\underbrace{(1+wx)}_{1} = \bar{y}z$$

$$C = \overline{x\bar{y} + \overline{(x+y)} + \bar{z}} = \overline{x\bar{y} + \bar{x}\bar{y} + \bar{z}} = \overline{\underbrace{(x+\bar{x})}_{1}\bar{y} + \bar{z}}$$

$$= \overline{\bar{y}+\bar{z}} = \bar{\bar{y}}\cdot\bar{\bar{z}} = yz$$

$$D = y(z + w\bar{x}(\bar{w}+z)) = yz + yw\bar{x}(\bar{w}+z)$$

$$= yz + \underbrace{yw\bar{x}\bar{w}}_{0} + ywx\bar{z} = yz(1+w\bar{x}) = yz$$

$$F = A + B + C + D$$

$$= yz + \bar{y}z + yz + yz \quad (yz \text{ repeated 3 times})$$

$$= yz + \bar{y}z$$

$$= \underbrace{(y+\bar{y})}_{1}z$$

$$= z$$

Question	Max	Score
1. True/False	10	
2. Three-variable K-map	5	
3. Minimization	15	
4. Number Conversions	20	
5. Verilog to Circuit	10	
6. Circuit to Circuit	15	
7. SOP with K-Map	15	
8. NAND/NOR Logic	10	
9. Joint Optimization	15	
10. Minimization	15	
TOTAL:	130	