CprE 281: Digital Logic
Midterm 2: Friday Oct 27, 2017
Student Name: $\qquad$ Student ID Number: $\qquad$
Lab Section: Mon 9-12(N) Tue 11-2(U) Wed 8-11(J) Thur 11-2(Q) Fri 11-2(G)
(circle one) Mon 12-3(P) Tue 2-5(M) Wed 11-2(W) Thur 11-2(V)
Tue 2-5(Z) Wed 6-9(T) Thur 2-5(L)
Thur 5-8(K)

1. $\quad$ True/False Questions ( $10 \times 1 \mathrm{p}$ each $=10 \mathrm{p}$ )
(a) I forgot to write down my name and student ID number and lab section. TRUE / FALSE
(b) When $\mathrm{K}=\overline{\mathrm{J}}$ a JK flip-flop behaves just like a D flip-flop.

TRUE / FALSE
(c) Any Boolean function can be implemented using only 1-to-4 demultiplexers.
(d) The outputs of a demultiplexer are one-hot encoded.

TRUE / FALSE
(e) A D-Latch changes its value only when the clock is 1 .

TRUE / FALSE
(f) $\operatorname{XOR}(\operatorname{XOR}(\mathrm{A}, \mathrm{B}), \mathrm{B})=\mathrm{A}$.

TRUE / FALSE
(g) In 4-bit 2's complement: $1010+0111$ results in an overflow.

TRUE / FALSE
(h) In 16-bit 2's complement: FACE $_{16}<$ CAFE $_{16}$

TRUE / FALSE
(i) A D-flip flop and a NOT gate can be used to implement a T-flip flop.

TRUE / FALSE
(j) According to George Lucas, Princes Leia got her Ph.D. at age 19.

TRUE / FALSE
2. Prime Numbers (5p)

Let $f(a, b, c, d)$ be a Boolean function that is equal to 1 if and only if the 4-bit number abcd is prime. Use a K-map to derive the minimum SOP expression for the function $f$.

## 3. Binary Addition and Subtraction ( $4 \times 3 p$ each $=12 p$ )

Convert the following integers into binary numbers and perform the addition or subtraction using 2's complement if necessary. Write your answers and all intermediary steps to the right of each problem. Use 5-bit numbers for all problems and indicate if any bits need to be ignored.

```
    (+2)
+
    (+6)
```

------
(+7)
(+4)

## 4. JK Flip-Flop (8pt)

Complete the wiring diagram below to implement a negative-edge-triggered JK flip-flop. Clearly label all inputs and outputs.

5. Code Converter ( $5 \mathrm{p}+5 \mathrm{p}=10 \mathrm{p}$ ).

Your task is to design a code converter with two inputs ( $\mathrm{X}_{1}$ and $\mathrm{X}_{0}$ ) and four outputs $\left(Y_{3}, Y_{2}, Y_{1}\right.$, and $\left.Y_{0}\right)$ that has the following truth table:

| $\mathbf{X}_{1}$ | $\mathbf{X}_{0}$ | $\mathbf{Y}_{3}$ | $\mathbf{Y}_{2}$ | $\mathbf{Y}_{1}$ | $\mathbf{Y}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

a) Derive the correct Boolean expressions for the four outputs.

$$
Y_{3}=
$$

$$
\mathbf{Y}_{2}=
$$

$$
\mathbf{Y}_{1}=
$$

$$
\mathbf{Y}_{0}=
$$

b) Draw the full wiring diagram of this code converter using basic logic gates. Label all inputs and outputs.

## 6. Computations with Adders ( $5 \times 3 p$ each $=15 p$ )

Let $A=A_{1} A_{0}$ be a 2-bit unsigned binary number. Let $B=B_{5} B_{4} B_{3} B_{2} B_{1} B_{0}$ be the result of the operation, interpreted as a number in 2's complement representation. For each of the following, write a formula for the value of $B$. The problem in a) is already solved.
a)


$$
B=A+1
$$

c)


$$
B=
$$

e)

$$
B=
$$


b)

$B=$
f)


$$
B=
$$


$B=$

## 7. Flip-Flops and Timing Diagrams ( $3 \times 5 \mathrm{p}=15 \mathrm{p}$ )

Complete the timing diagram for the specified flip-flop such that the output $Q$ will be as indicated. Assume that the input signal can change only on the vertical lines. Also, assume that the setup time $t_{s u}$ and the hold time $t_{h}$ are each equal to the width of one square.
a) Complete the timing diagram for the $D$ input to a negative-edge triggered $D$ flip-flop.

D

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Clock



## Q


b) Complete the timing diagram for the $\mathbf{T}$ input to a negative-edge triggered $\mathbf{T}$ flip-flop.

## T

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Clock

c) Complete the timing diagram for the $\mathbf{J}$ input to a negative-edge triggered JK flip-flop.

J


K


Clock


Q

8. Multiplexers ( $3 \times 5 \mathrm{p}$ each $=15 p$ )
a) Draw the truth table for the function $f(x, y, z)=\overline{\mathbf{x}} \overline{\mathbf{y}} \mathbf{z}+\overline{\mathbf{x}} \mathbf{y} \overline{\mathbf{z}}+\mathbf{x} \overline{\mathbf{y}} \overline{\mathbf{z}}$.
b) Implement this function using only 2-to-1 multiplexers and no other logic gates. Assume that the signals $x, y$, and $z$ are available only in their non-inverted form as well as the constants 0 and 1. Clearly label all inputs, outputs, and pins.
c) Implement this function using a 3-to-8 decoder (with enable) and one OR gate. Clearly label all inputs, outputs, and pins of the decoder.

## 9. Alternative Implementation ( 10 p )

Implement the logic gates OR, XOR, and NAND in three different ways: 1) using a 2-to-1 multiplexer; 2) using AND-OR logic; and 3) using OR-AND logic. In this problem, you are not allowed to use any other logic gates. Assume that both $x$ and $y$ are available in their inverted and non-inverted form, along with the constants 0 and 1 . If some implementation is not possible, then indicate that with words. Label all inputs and outputs.
a) Implement in three different ways: $f=\operatorname{OR}(x, y)$.

b) Implement in three different ways: $\mathbf{f}=\operatorname{XOR}(x, y)$.

c) Implement in three different ways: $f=\operatorname{NAND}(x, y)$.

10. Mystery Circuit $(5 p+10 p=15 p)$
(a) The truth table and the graphical symbol for a mystery circuit are shown below. Implement this circuit using a 4-to-1 multiplexer and any number of basic logic gates (AND, OR, NOT). Assume that the input variables are available only in their regular (i.e., non-inverted form) as well as the constants 0 and 1 . Label all inputs, outputs, and pins.

Truth table

| $\mathbf{S}_{\mathbf{1}}$ | $\mathbf{S}_{\mathbf{0}}$ | $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{y}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{0}$ | $\mathbf{1}$ |
| $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ | $\mathbf{1}$ |

(b) Implement a 2-to-1 multiplexer using only copies of the mystery circuit and no other logic gates. Please use the graphical symbol of the mystery circuit to denote each copy. You can assume that the input variables are available only in their regular (i.e., non-inverted form) as well as the constants 0 and 1 . Label all inputs, outputs, and pins of your circuit.

## 11. Parallel Load and Shift Left/Right Register (15p)

Complete the following circuit diagram to implement a 4-bit register that has both parallel load and shift left/right functionality. The register has two control inputs ( $\mathrm{C}_{1}$ and $\mathrm{C}_{0}$ ), four parallel input lines ( $\mathbf{I}_{3}, I_{2}, I_{1}$, and $\left.I_{0}\right)$, and four output lines $\left(Q_{3}, Q_{2}, Q_{1}\right.$, and $\left.Q_{0}\right)$. Depending on the values of $\mathrm{C}_{1}$ and $\mathrm{C}_{0}$, the register performs one of the following four operations:

| $\mathrm{C}_{1}$ | $\mathrm{C}_{0}$ | Operation |
| :---: | :---: | :---: |
| 0 | 0 | Hold the current value (i.e., $\mathrm{Q}_{3} \mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}$ are not changed) |
| 0 | 1 | Shift left (i.e., new $Q_{3}=Q_{2}$, new $Q_{2}=Q_{1}$, new $Q_{1}=Q_{0}$, new $Q_{0}=I_{0}$ ) |
| 1 | 0 | Shift right (i.e., new $Q_{3}=I_{3}$, new $Q_{2}=Q_{3}$, new $Q_{1}=Q_{2}$, new $Q_{0}=Q_{1}$ ) |
| 1 | 1 | Load new data (i.e., new $Q_{3}=I_{3}$, new $Q_{2}=I_{2}$, new $Q_{1}=I_{1}$, new $Q_{0}=I_{0}$ ) |

Clearly label all inputs, outputs, and pins.


| Question | Max | Score |
| :--- | ---: | :--- |
| 1. True/False | 10 |  |
| 2. Prime Numbers | 5 |  |
| 3. Addition/Subtraction | 12 |  |
| 4. JK Flip-Flop | 8 |  |
| 5. Code Converter | 10 |  |
| 6. Computations with Adders | 15 |  |
| 7. Flip-Flops | 15 |  |
| 8. Multiplexers | 15 |  |
| 9. Alternative Implementation | 10 |  |
| 10. Mystery Circuit | 15 |  |
| 11. Register | 15 |  |
| TOTAL: | 130 |  |

