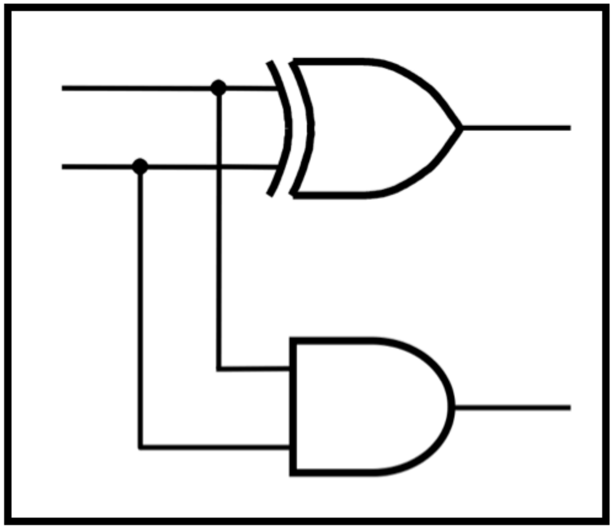


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Addition of Unsigned Numbers

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW5 is out

- It is due on Monday Oct 2 @ 4pm.

- Please write clearly on the first page (in BLOCK CAPITAL letters) the following three things:

    - Your First and Last Name
    - Your Student ID Number
    - Your Lab Section Letter

- Also, please
    - Staple your pages

Administrative Stuff

- Labs next week
- Mini-Project
- This is worth 3% of your grade (x2 labs)
- http://www.ece.iastate.edu/~alexs/classes/
  2017_Fall_281/labs/Project-Mini/

Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

Number Systems

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)

0-th digit
(least significant)

Number Systems

base

power

$$N = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_1 B^1 + d_0 B^0$$

n-th digit
(most significant)

0-th digit
(least significant)

The Decimal System

$$524_{10} = 5\times10^2+2\times10^1+4\times10^0$$

The Decimal System

$$524_{10} = 5\times10^2+2\times10^1+4\times10^0$$

$$= 5\times100+2\times10+4\times1$$

$$= 500+20+4$$

$$= 524_{10}$$

Another Way to Look at This

5	2	4

Another Way to Look at This

10^2	10^1	10^0
5	2	4

Another Way to Look at This

	10^2	10^1	10^0	labels
boxes	5	2	4	

Each box can contain only one digit and has only one label. From right to left, the labels are increasing powers of the base, starting from 0.

Base 7

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

Base 7

base

power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

Base 7

base power

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

most significant digit

least significant digit

Base 7

$$524_7 = 5 \times 7^2 + 2 \times 7^1 + 4 \times 7^0$$

$$= 5 \times 49 + 2 \times 7 + 4 \times 1$$

$$= 245 + 14 + 4$$

$$= 263_{10}$$

Another Way to Look at This

$$\begin{array}{ccc} 7^2 & 7^1 & 7^0 \\ \boxed{5} & \boxed{2} & \boxed{4} \end{array} \quad = \quad \begin{array}{ccc} 10^2 & 10^1 & 10^0 \\ \boxed{2} & \boxed{6} & \boxed{3} \end{array}$$

Binary Numbers (Base 2)

$$1001_2 = \quad 1 \times 2^3 \quad + \quad 0 \times 2^2 \quad + \quad 0 \times 2^1 \quad + \quad 1 \times 2^0$$

Binary Numbers (Base 2)

base power

$$1001_2 = \ 1 \times 2^3 \ + \ 0 \times 2^2 \ + \ 0 \times 2^1 \ + \ 1 \times 2^0$$

most significant bit least significant bit

Binary Numbers (Base 2)

$$\begin{aligned}1001_2 = \; & 1 \times 2^3 && + \; 0 \times 2^2 && + \; 0 \times 2^1 && + \; 1 \times 2^0 && = \\ = \; & 1 \times 8 && + \; 0 \times 4 && + \; 0 \times 2 && + \; 1 \times 1 && = \\ = \; & 8 && + \; 0 && + \; 0 && + \; 1 && = \\ = \; & 9_{10} \end{aligned}$$

Another Example

$$
\begin{aligned}
11101_2 &= 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = \\
&= 1 \times 16 + 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 = \\
&= 16 + 8 + 4 + 0 + 1 = 29_{10}
\end{aligned}
$$

Powers of 2

$$\begin{aligned}
2^{10} &= 1024 \\
2^{9} &= 512 \\
2^{8} &= 256 \\
2^{7} &= 128 \\
2^{6} &= 64 \\
2^{5} &= 32 \\
2^{4} &= 16 \\
2^{3} &= 8 \\
2^{2} &= 4 \\
2^{1} &= 2 \\
2^{0} &= 1
\end{aligned}$$

What is the value of this binary number?

- 0 0 1 0 1 1 0 0
- 0 0 1 0 1 1 0 0
- $0*2^7 + 0*2^6 + 1*2^5 + 0*2^4 + 1*2^3 + 1*2^2 + 0*2^1 + 0*2^0$
- 0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1
- 0*128 + 0*64 + 1*32 + 0*16 + 1*8 + 1*4 + 0*2 + 0*1
- 32+ 8 + 4 = 44 (in decimal)

Another Way to Look at This

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0	0	1	0	1	1	0	0

Binary numbers

Unsigned numbers

- **all bits represent the magnitude of a positive integer**

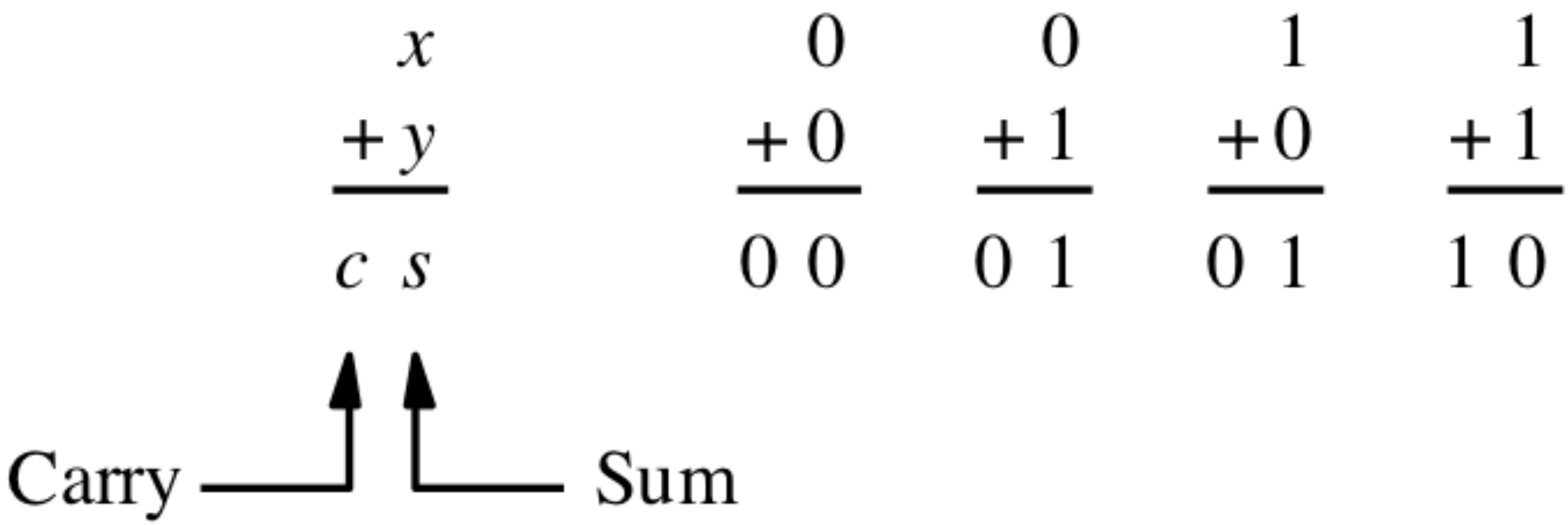
Signed numbers

- **left-most bit represents the sign of a number**

Decimal	Binary	Octal	Hexadecimal
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	01011	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10
17	10001	21	11
18	10010	22	12

Table 3.1. Numbers in different systems.

Adding two bits
(there are four possible cases)



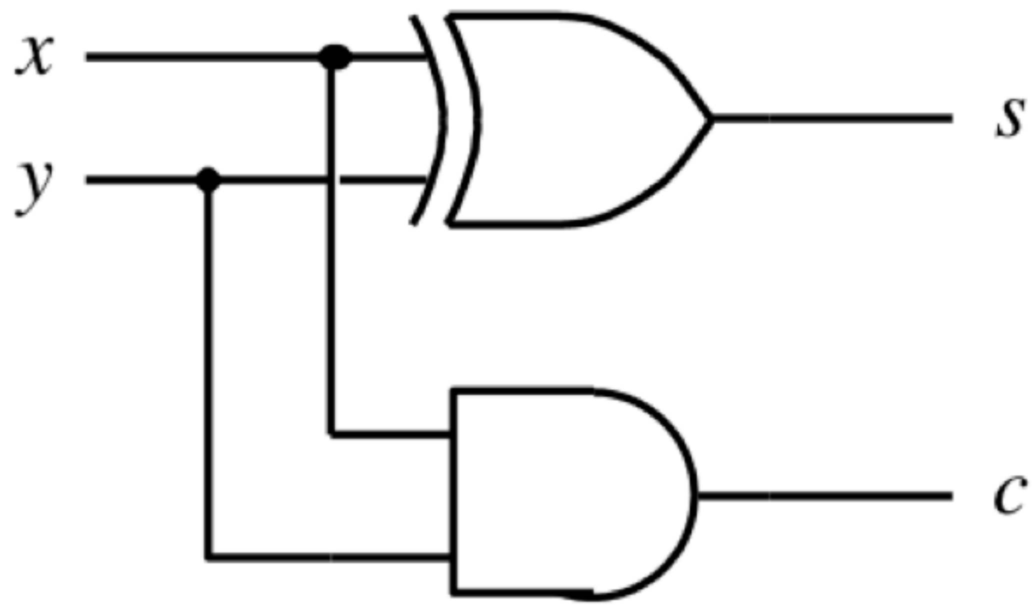
[Figure 3.1a from the textbook]

Adding two bits
(the truth table)

x	y	Carry c	Sum s
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

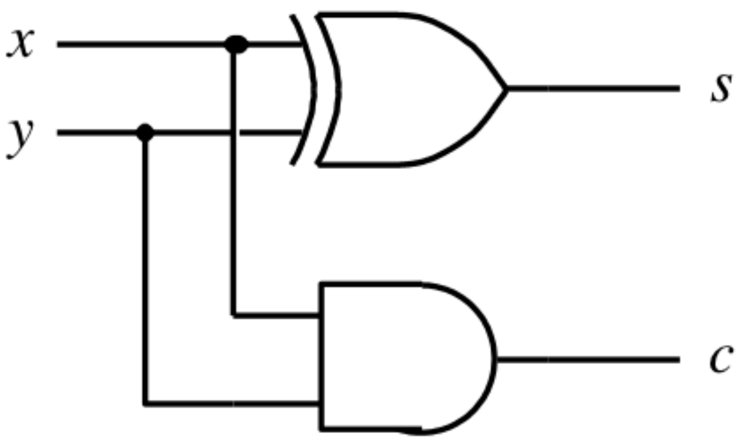
[Figure 3.1b from the textbook]

Adding two bits
(the logic circuit)



[Figure 3.1c from the textbook]

The Half-Adder



(c) Circuit

(d) Graphical symbol

[Figure 3.1c-d from the textbook]

Addition of multibit numbers

Generated carries $\longrightarrow$ 1 1 1 0

$$
\begin{array}{rcrr}
X = x_4x_3x_2x_1x_0 & & 0\,1\,1\,1\,1 & (15)_{10} \\
+\,Y = y_4y_3y_2y_1y_0 & & +\,0\,1\,0\,1\,0 & +\,(10)_{10} \\
\hline
S = s_4s_3s_2s_1s_0 & & 1\,1\,0\,0\,1 & (25)_{10}
\end{array}
$$

$$
\begin{array}{ccccc}
\cdots & c_{i+1} & c_i & \cdots \\
\cdots & \cdots & x_i & \cdots \\
\cdots & \cdots & y_i & \cdots \\
\hline
\cdots & \cdots & s_i & \cdots
\end{array}
$$

Bit position i

[Figure 3.2 from the textbook]

Analogy with addition in base 10

$$
\begin{array}{rccc}
 & x_2 & x_1 & x_0 \\
+ & y_2 & y_1 & y_0 \\
\hline
 & s_2 & s_1 & s_0
\end{array}
$$

Analogy with addition in base 10

$$\begin{array}{rccc} & 3 & 8 & 9 \\ + & 1 & 5 & 7 \\ \hline & 5 & 4 & 6 \end{array}$$

Analogy with addition in base 10

carry	0	1	1	0
		3	8	9
+		1	5	7
		5	4	6

Analogy with addition in base 10

$$
\begin{array}{rcccc}
 & c_3 & c_2 & c_1 & c_0 \\
 & & x_2 & x_1 & x_0 \\
+ & & y_2 & y_1 & y_0 \\
\hline
 & & s_2 & s_1 & s_0
\end{array}
$$

Problem Statement and Truth Table

…	c_{i+1}	c_i	…
…	…	x_i	…
…	…	y_i	…
…	…	s_i	…

[Figure 3.2b from the textbook]

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

[Figure 3.3a from the textbook]

Let's fill-in the two K-maps

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

c_i \ x_iy_i	00	01	11	10
0				
1				

$s_i =$

c_i \ x_iy_i	00	01	11	10
0				
1				

$c_{i+1} =$

[Figure 3.3a-b from the textbook]

Let’s fill-in the two K-maps

c_i	x_i	y_i	c_{i+1}	s_i
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

c_i \ $x_i y_i$	00	01	11	10
0		1		1
1	1		1	

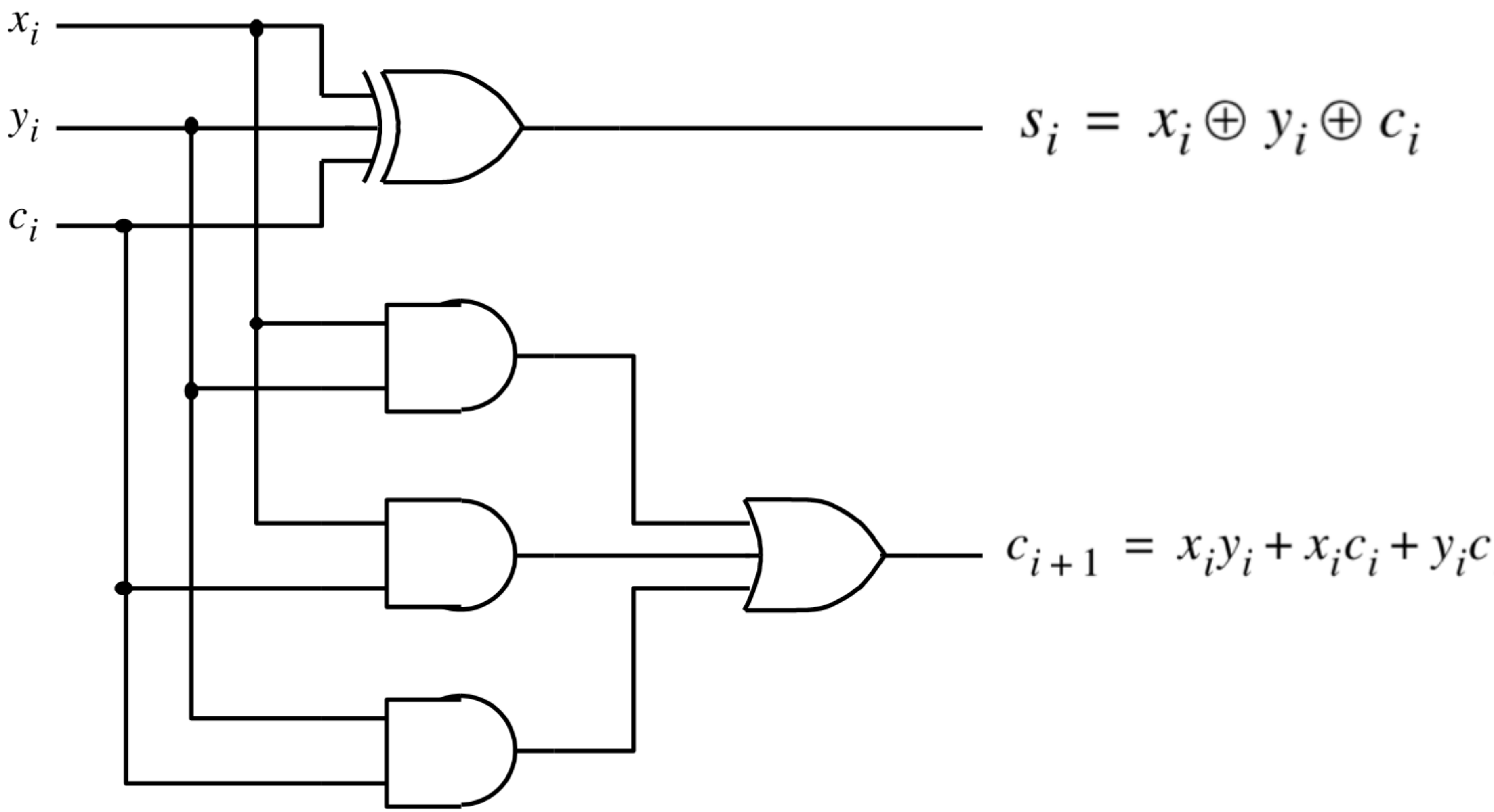
$$s_i = x_i \oplus y_i \oplus c_i$$

c_i \ $x_i y_i$	00	01	11	10
0			1	
1		1	1	1

$$c_{i+1} = x_i y_i + x_i c_i + y_i c_i$$

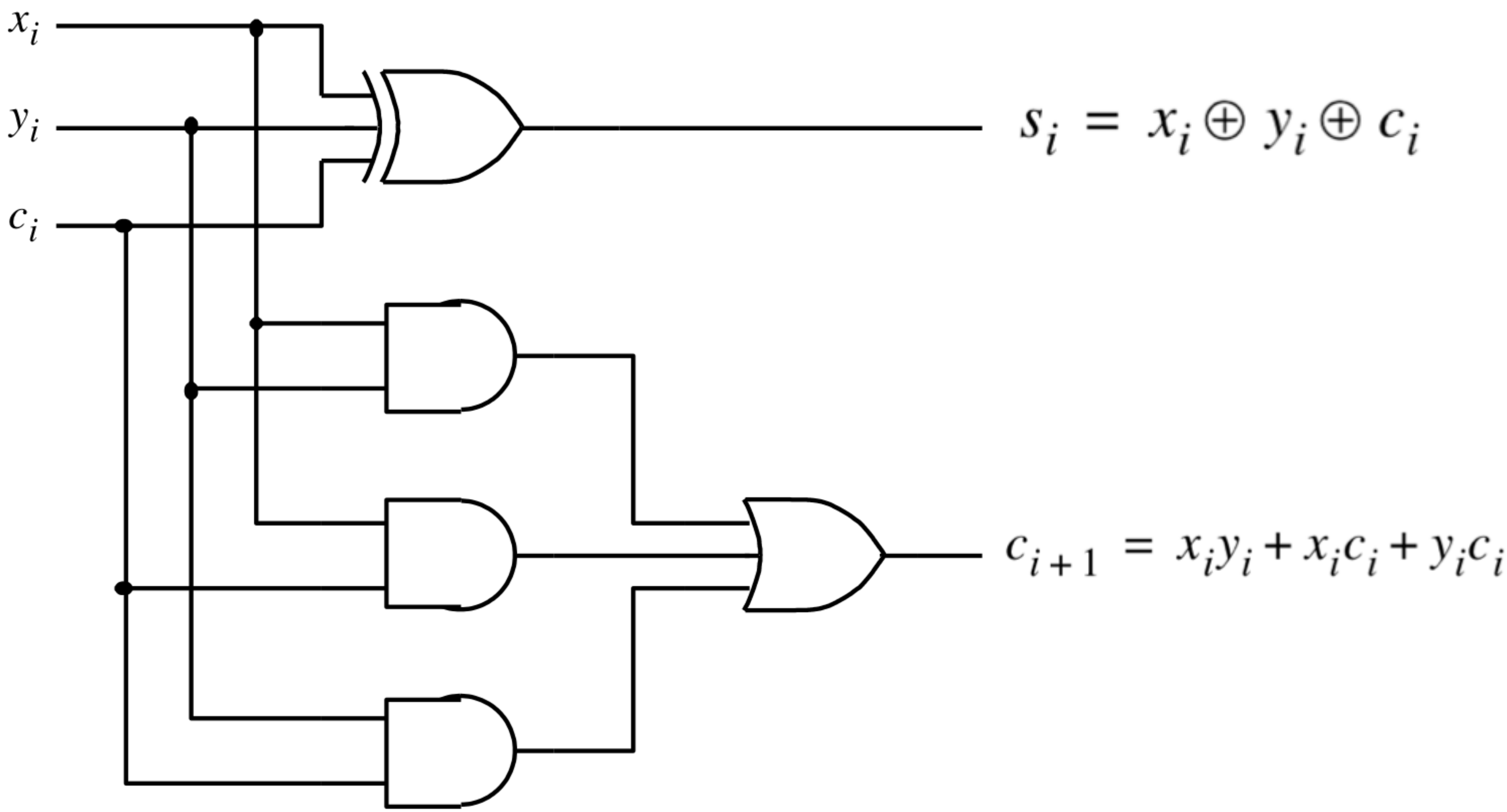
[Figure 3.3a-b from the textbook]

The circuit for the two expressions



[Figure 3.3c from the textbook]

This is called the Full-Adder



[Figure 3.3c from the textbook]

XOR Magic

$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$

XOR Magic

$$s_i = \overline{x}_i y_i \overline{c}_i + x_i \overline{y}_i \overline{c}_i + \overline{x}_i \overline{y}_i c_i + x_i y_i c_i$$

$$\begin{aligned} s_i &= (\overline{x}_i y_i + x_i \overline{y}_i)\overline{c}_i + (\overline{x}_i \overline{y}_i + x_i y_i) c_i \\ &= (x_i \oplus y_i)\overline{c}_i + \overline{(x_i \oplus y_i)} c_i \\ &= (x_i \oplus y_i) \oplus c_i \end{aligned}$$

XOR Magic

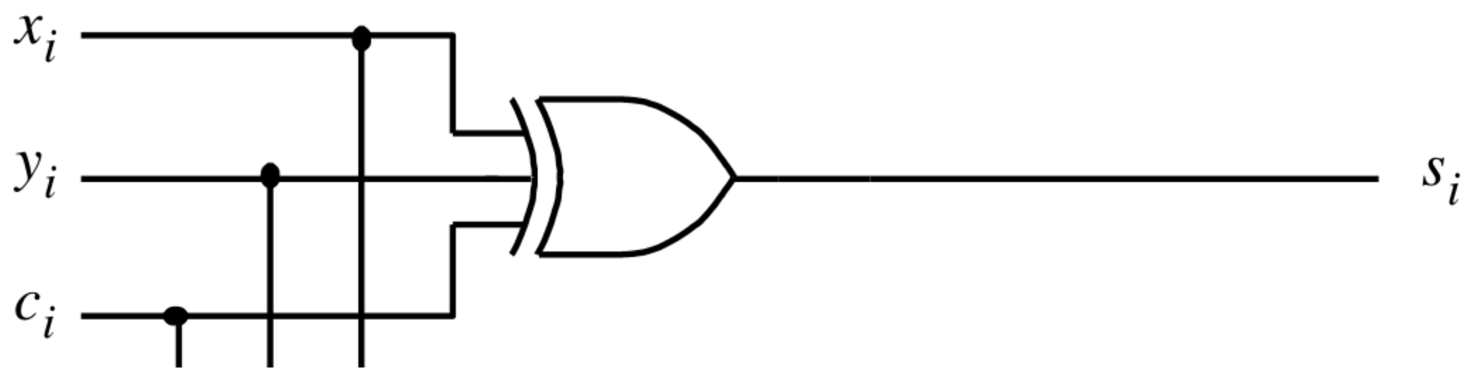
$$s_i = \bar{x}_i y_i \bar{c}_i + x_i \bar{y}_i \bar{c}_i + \bar{x}_i \bar{y}_i c_i + x_i y_i c_i$$

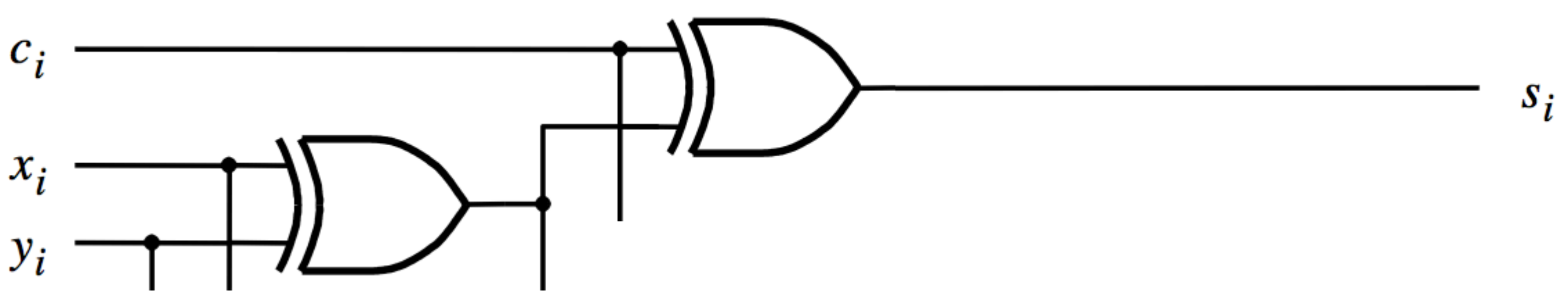
Can you prove this?

$$\begin{aligned} s_i &= (\bar{x}_i y_i + x_i \bar{y}_i)\bar{c}_i + (\bar{x}_i \bar{y}_i + x_i y_i) c_i \\ &= (x_i \oplus y_i)\bar{c}_i + \overline{(x_i \oplus y_i)} c_i \\ &= (x_i \oplus y_i) \oplus c_i \end{aligned}$$

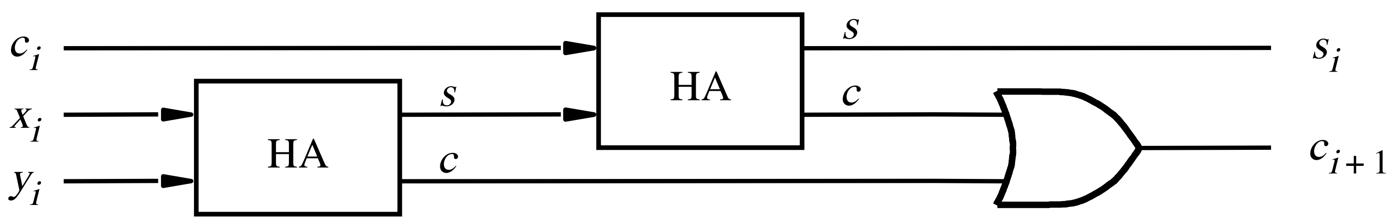
XOR Magic
(s_i can be implemented in two different ways)

$$s_i = x_i \oplus y_i \oplus c_i$$

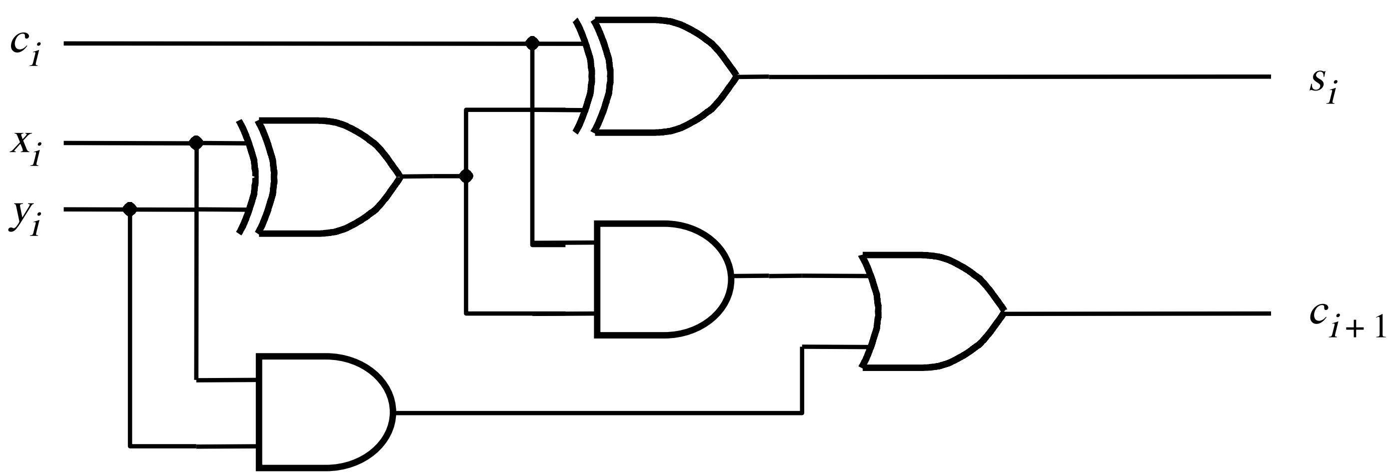




A decomposed implementation of the full-adder circuit



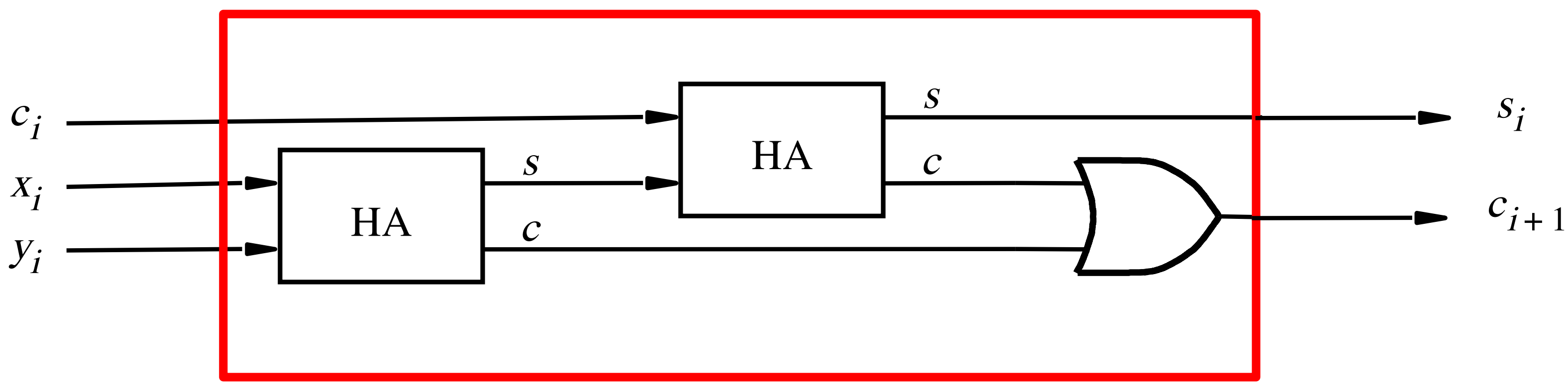
(a) Block diagram



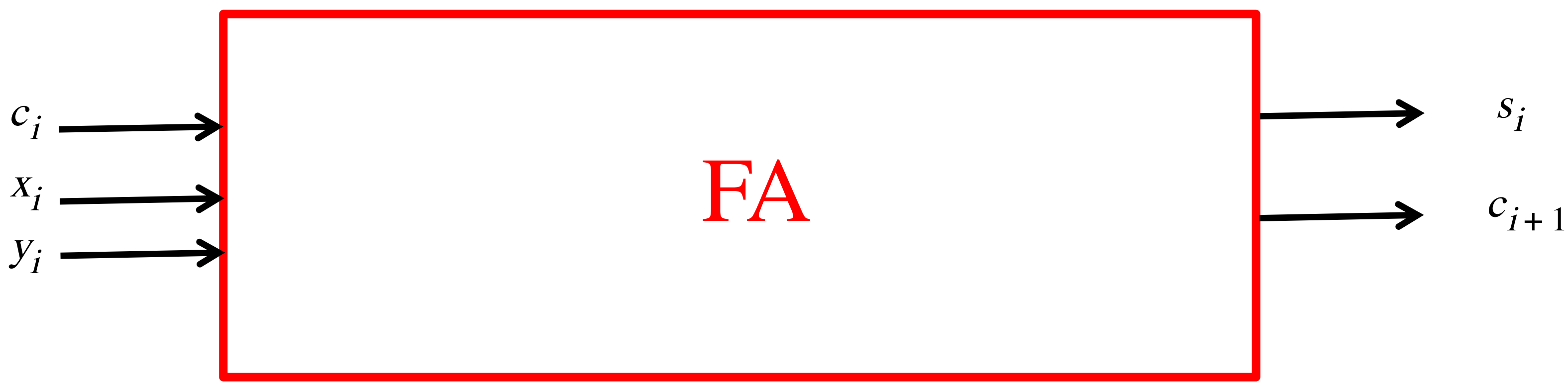
(b) Detailed diagram

[Figure 3.4 from the textbook]

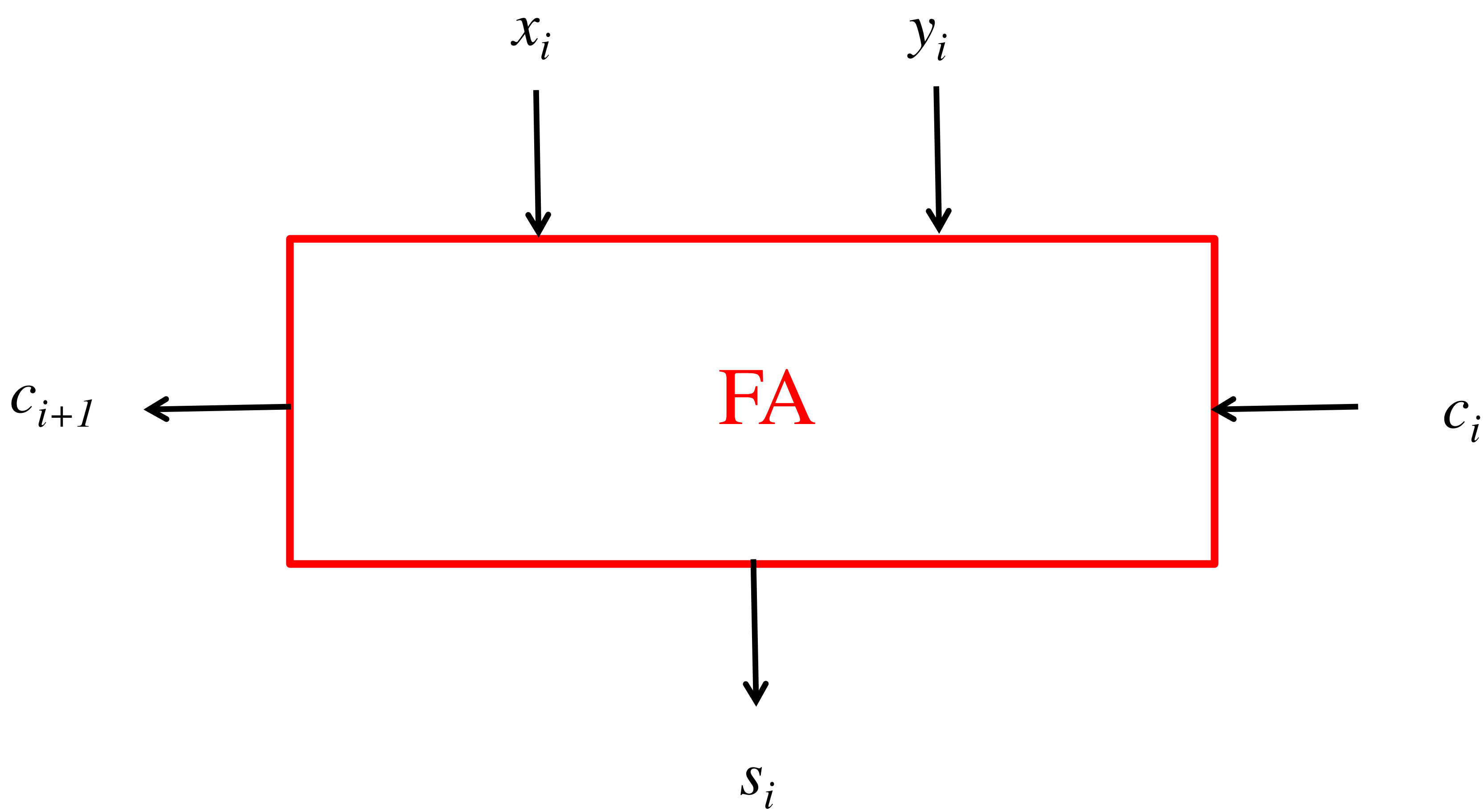
The Full-Adder Abstraction



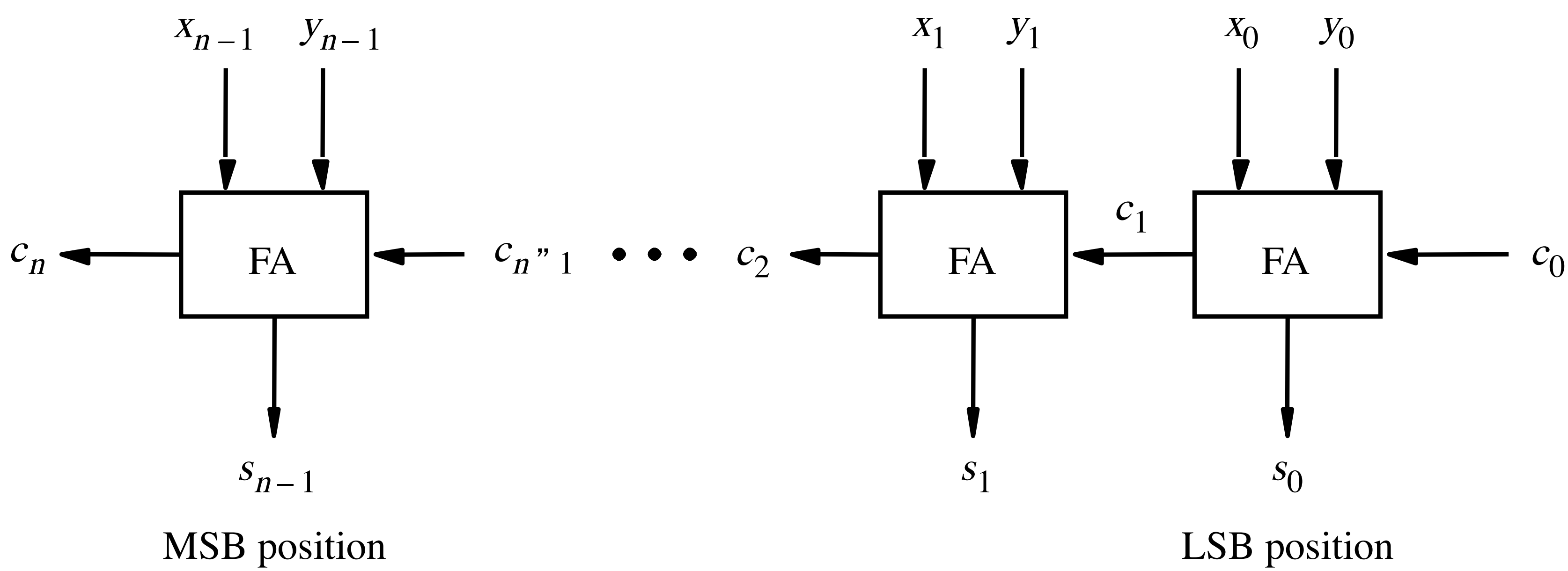
The Full-Adder Abstraction



We can place the arrows anywhere

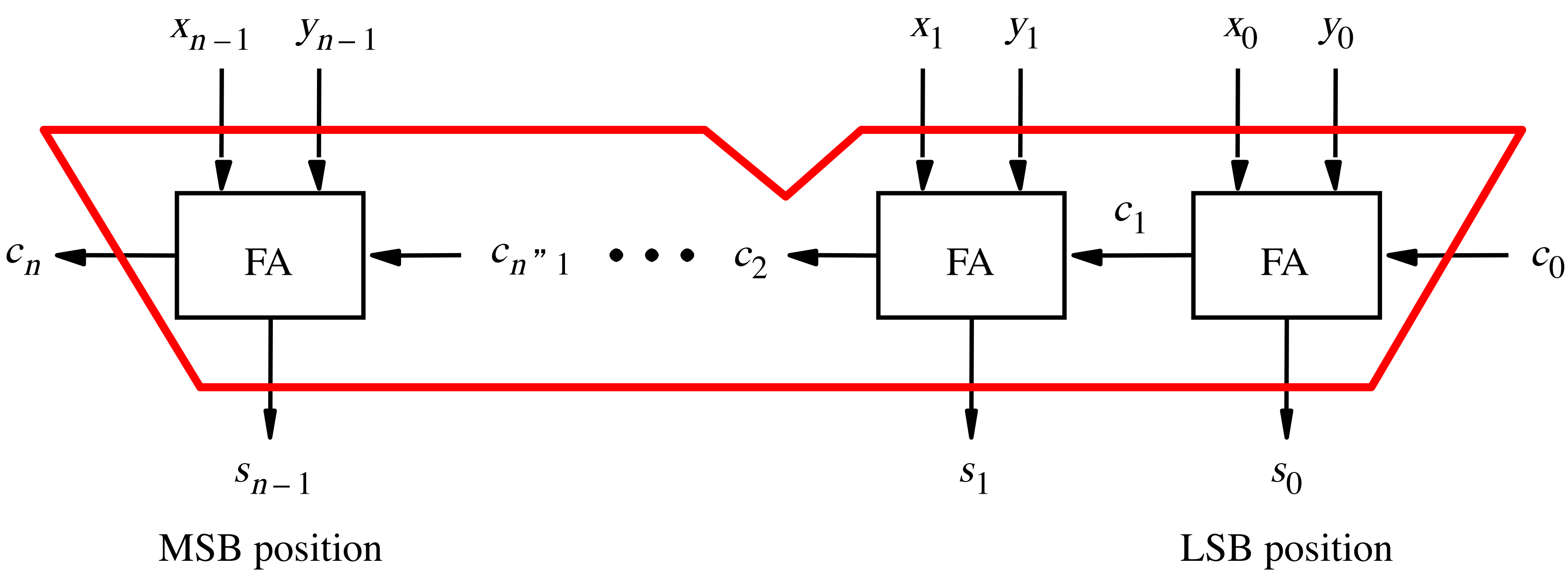


n-bit ripple-carry adder

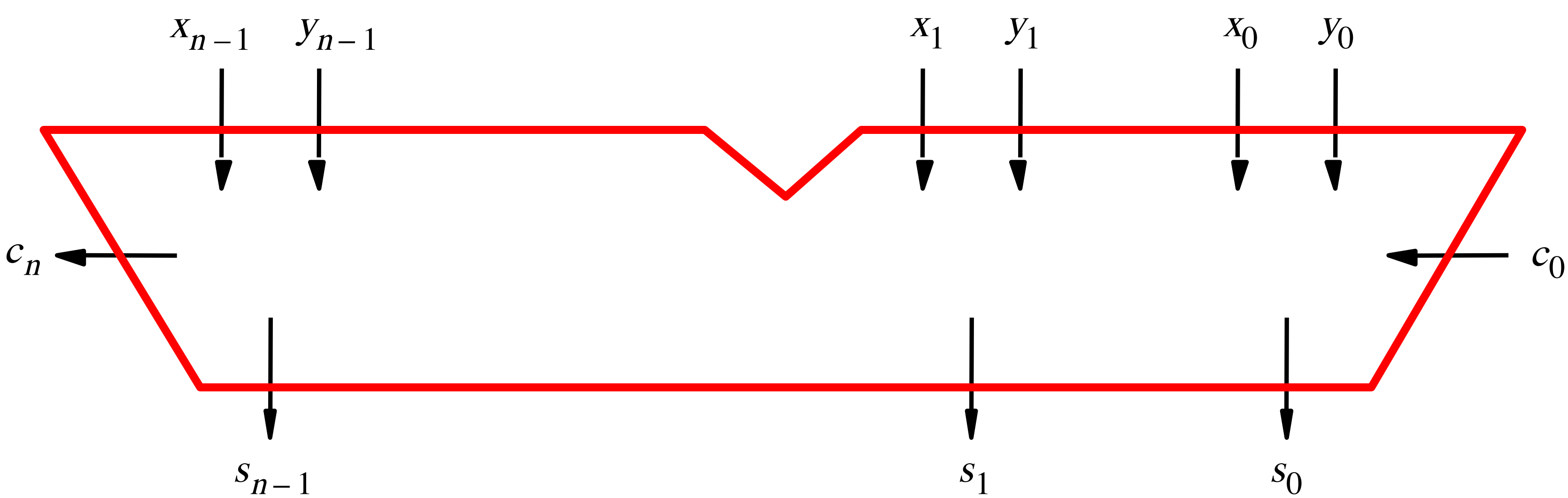


[Figure 3.5 from the textbook]

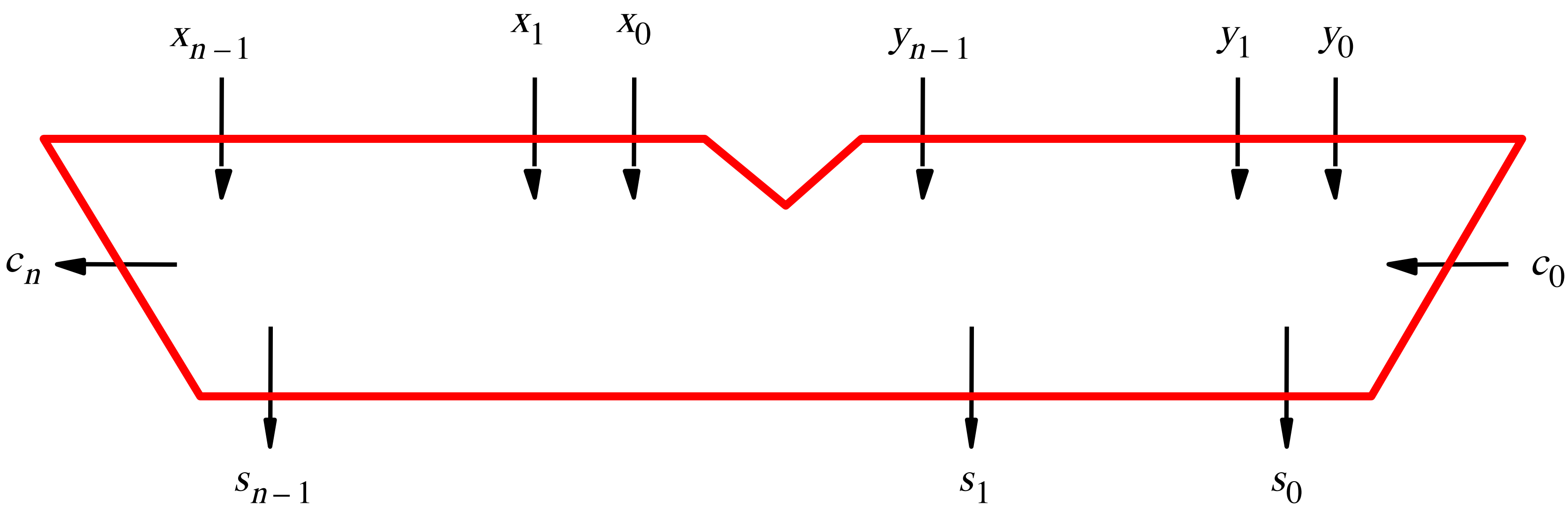
n-bit ripple-carry adder abstraction



n-bit ripple-carry adder abstraction



The x and y lines are typically grouped together for better visualization, but the underlying logic remains the same



Design Example:

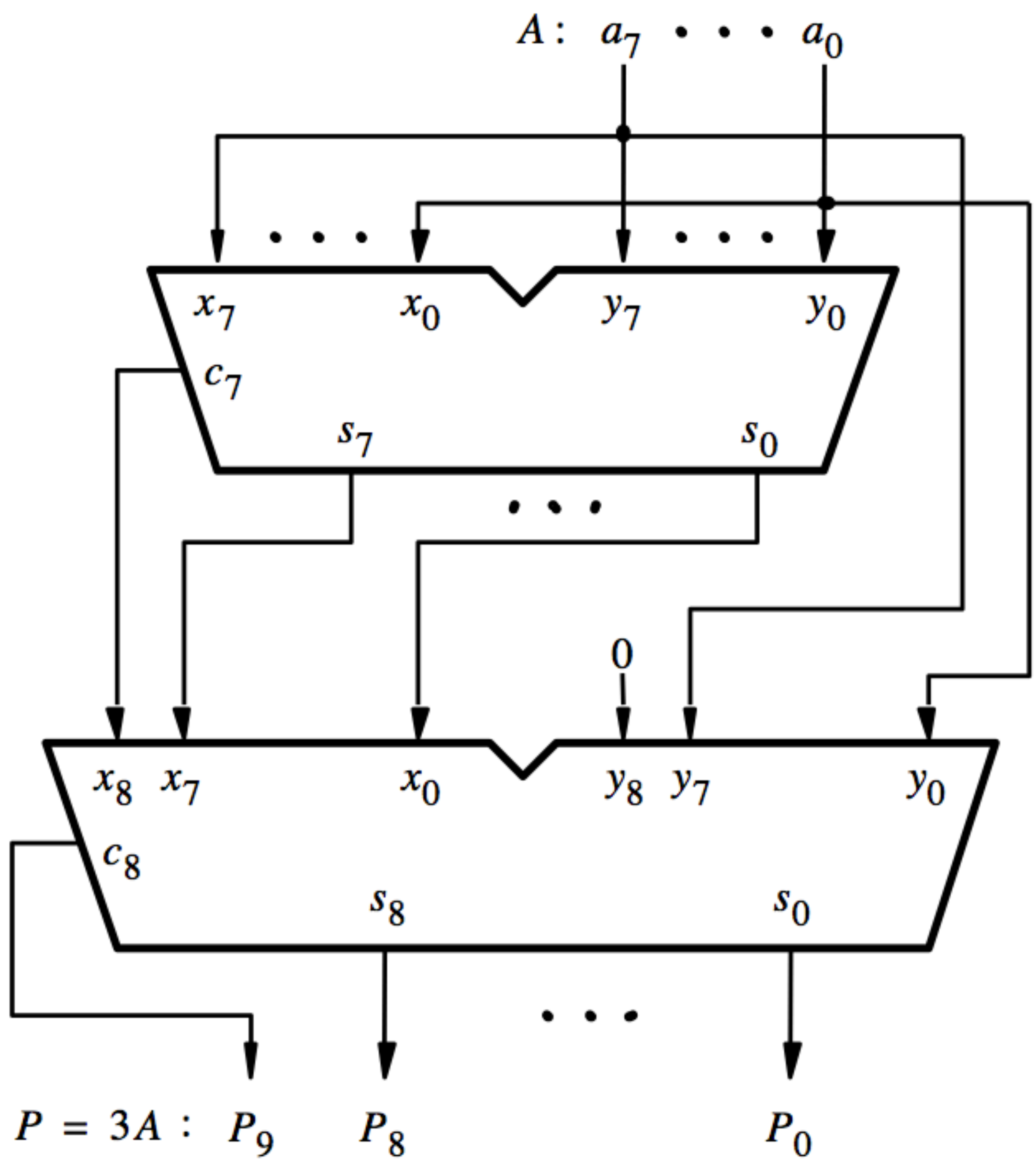
Create a circuit that multiplies a number by 3

How to Get 3A from A?

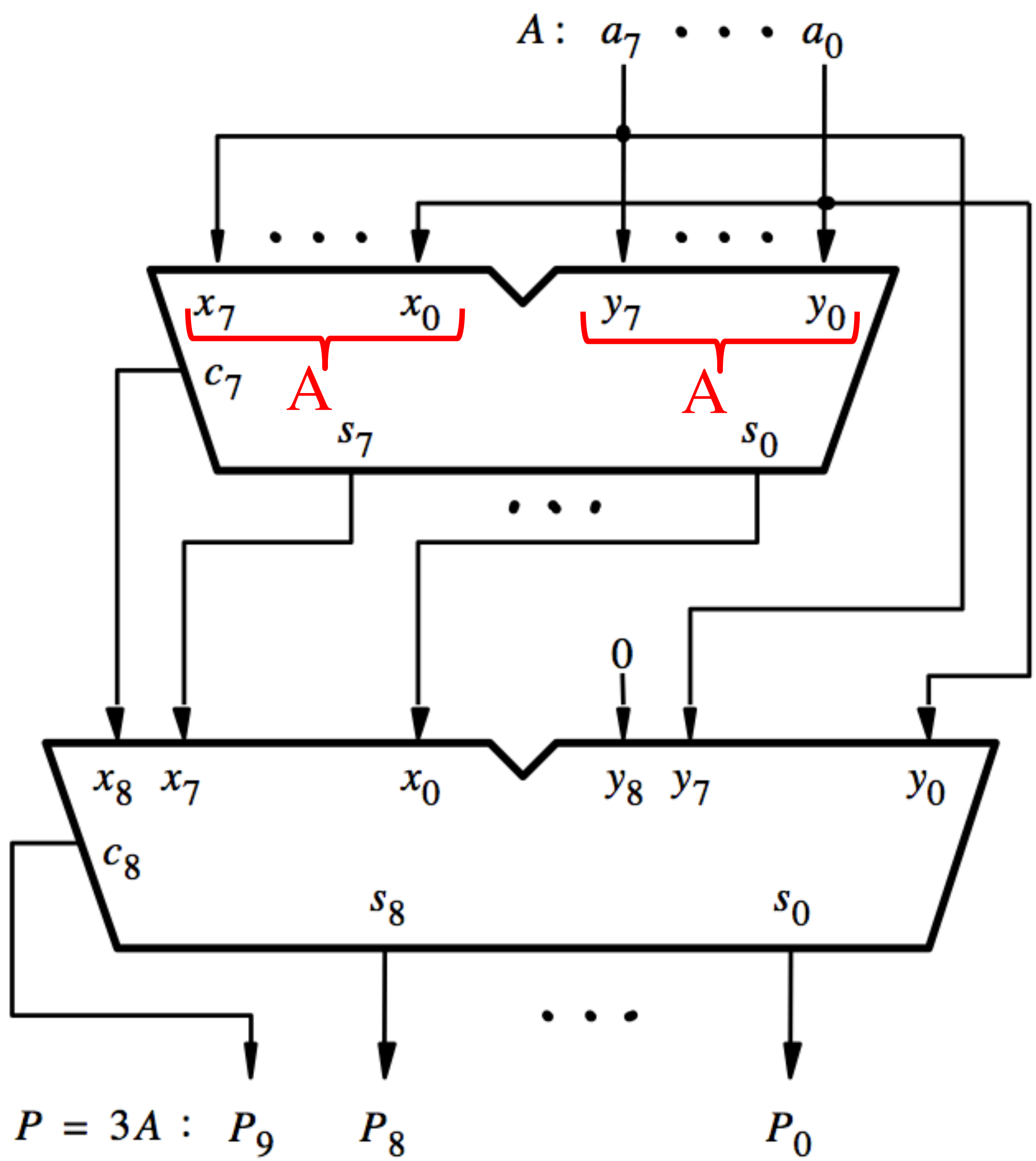
- $3A = A + A + A$

- $3A = (A+A) + A$

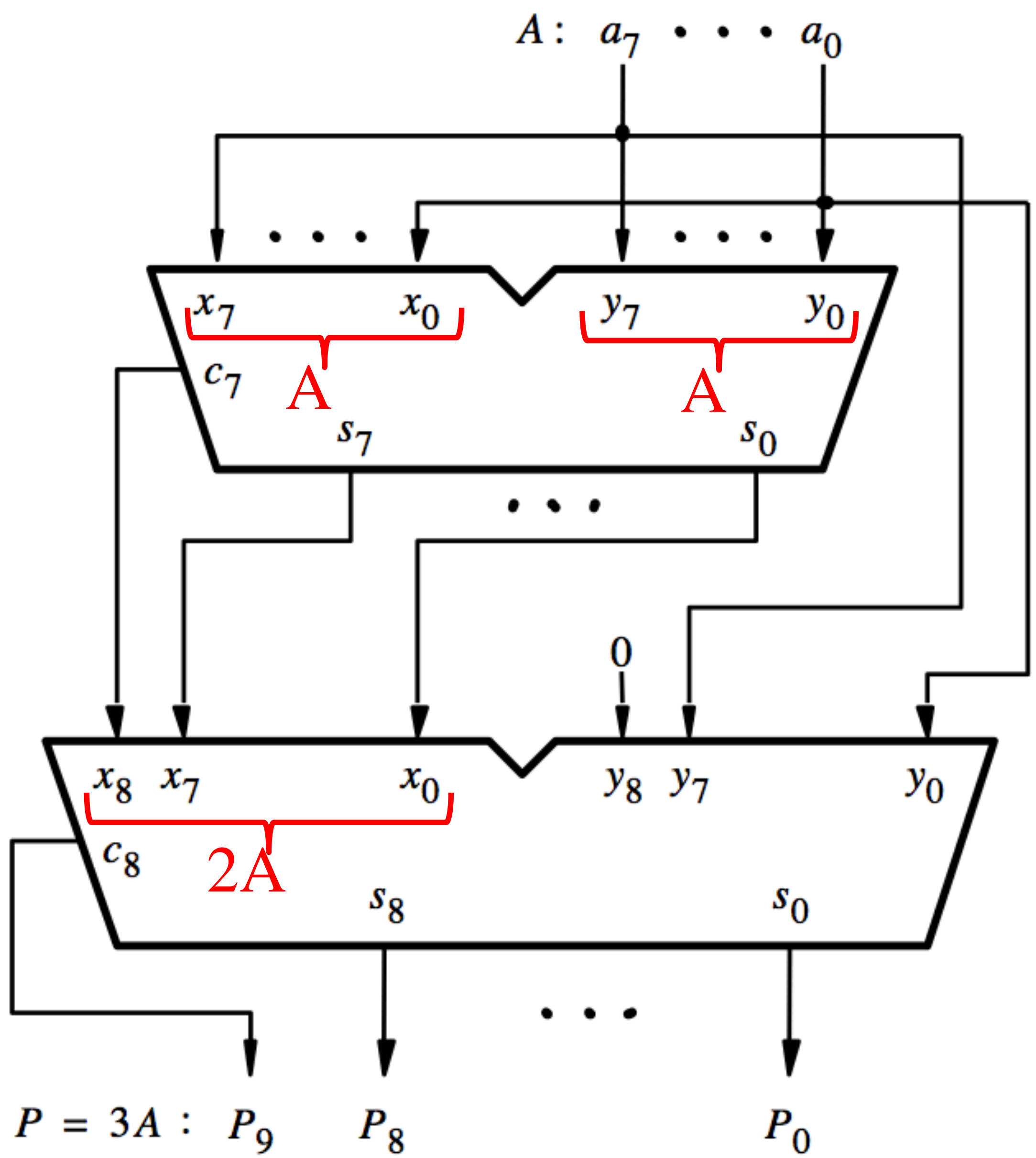
- $3A = 2A + A$



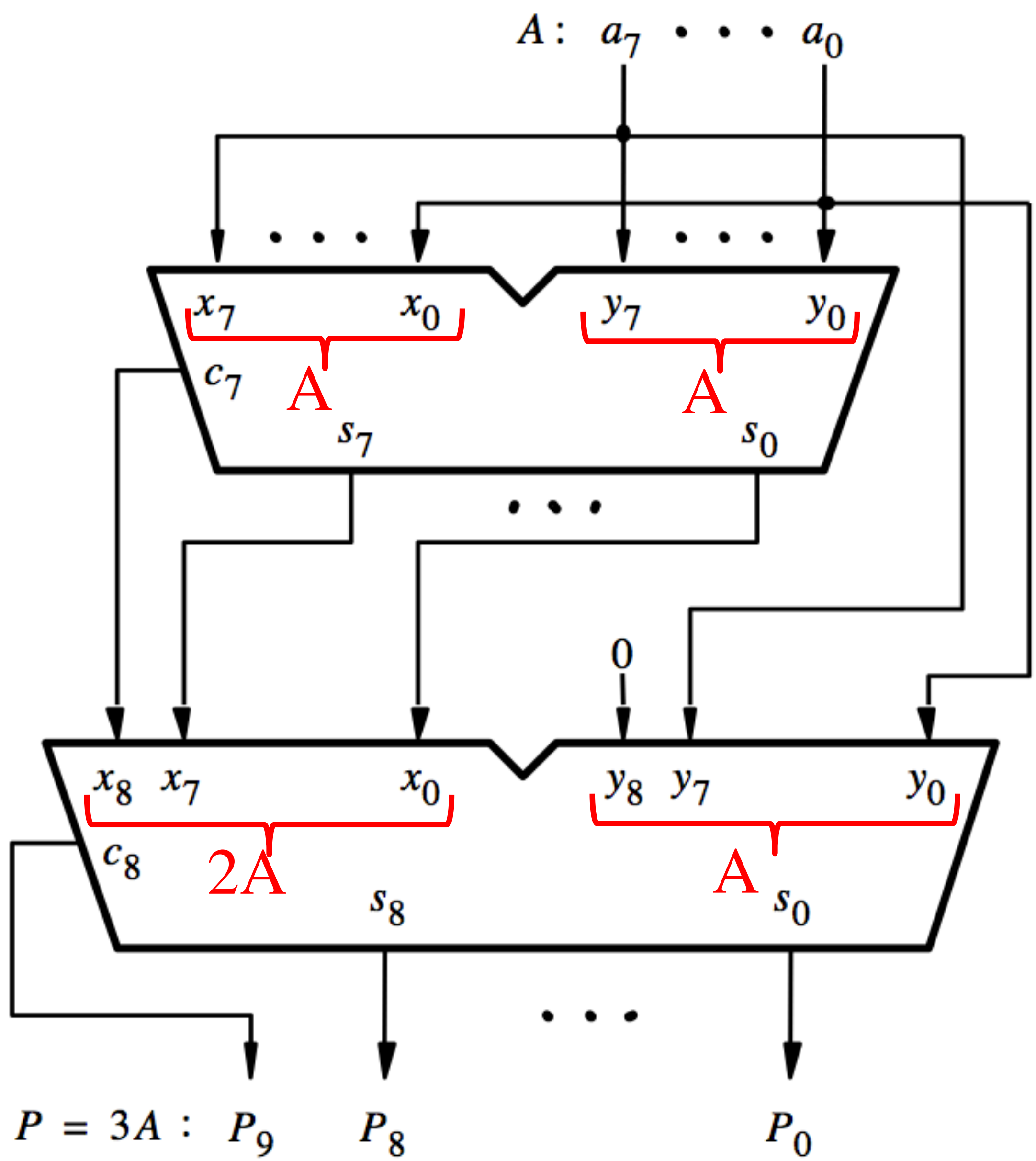
[Figure 3.6a from the textbook]



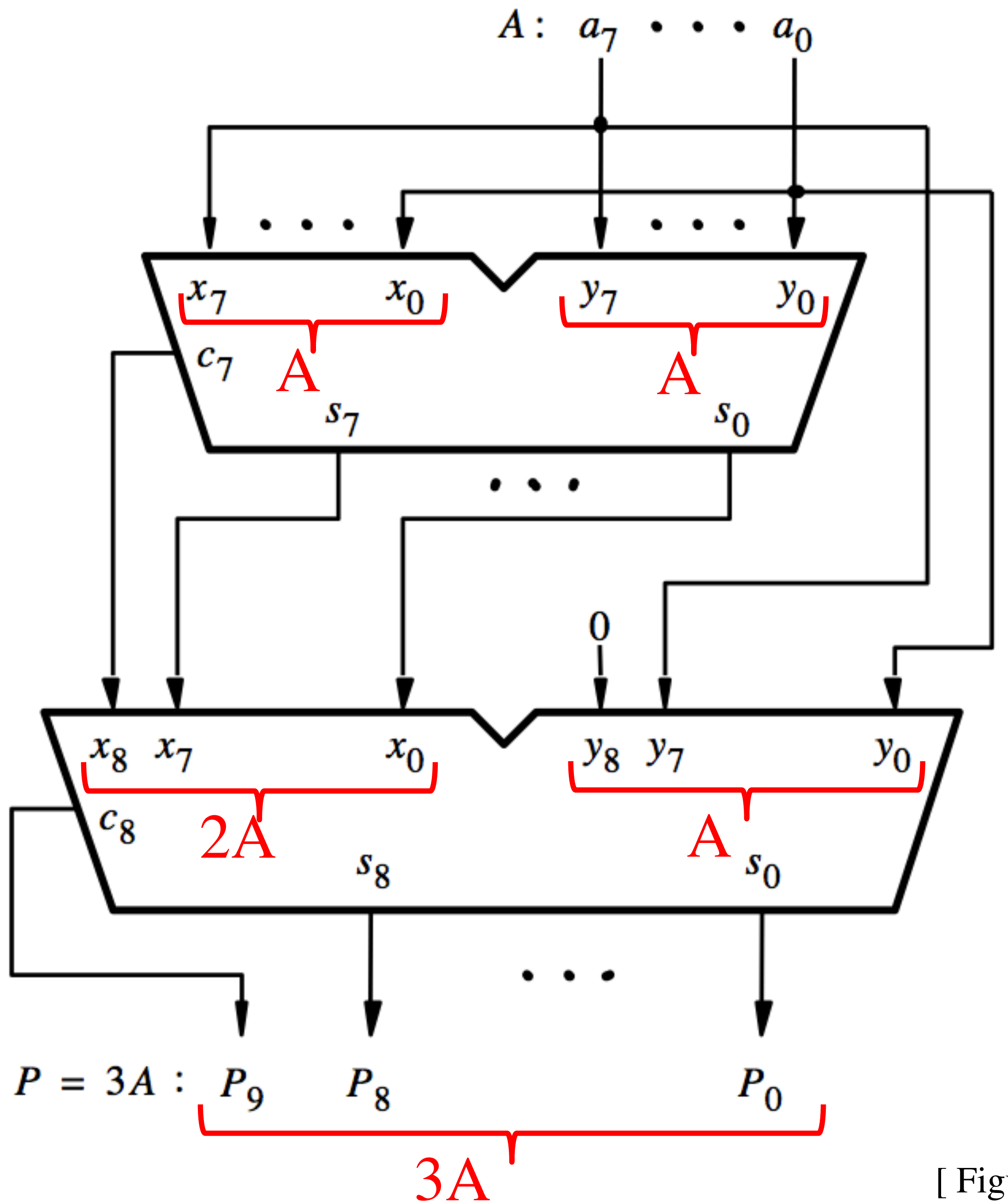
[Figure 3.6a from the textbook]



[Figure 3.6a from the textbook]



[Figure 3.6a from the textbook]



[Figure 3.6a from the textbook]

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = ?

542 x 10 = ?

1245 x 10 = ?

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = 40

542 x 10 = 5420

1245 x 10 = 12450

Decimal Multiplication by 10

What happens when we multiply a number by 10?

4 x 10 = 40

542 x 10 = 5420

1245 x 10 = 12450

You simply add a zero as the rightmost number

Binary Multiplication by 2

What happens when we multiply a number by 2?

011 times 2 = ?

101 times 2 = ?

110011 times 2 = ?

Binary Multiplication by 2

What happens when we multiply a number by 2?

011 times 2 = 0110

101 times 2 = 1010

110011 times 2 = 1100110

Binary Multiplication by 2

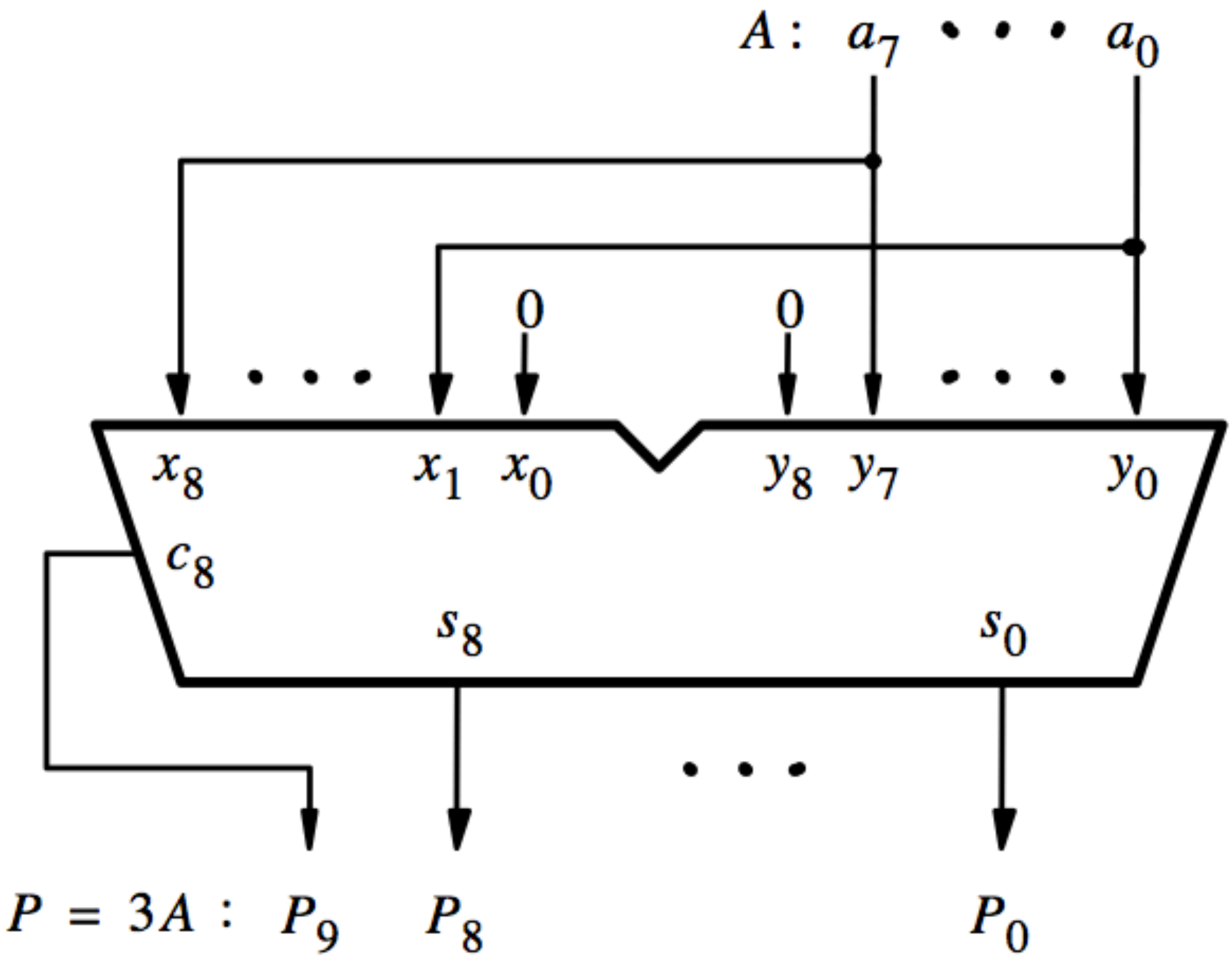
What happens when we multiply a number by 2?

011 times 2 = 0110

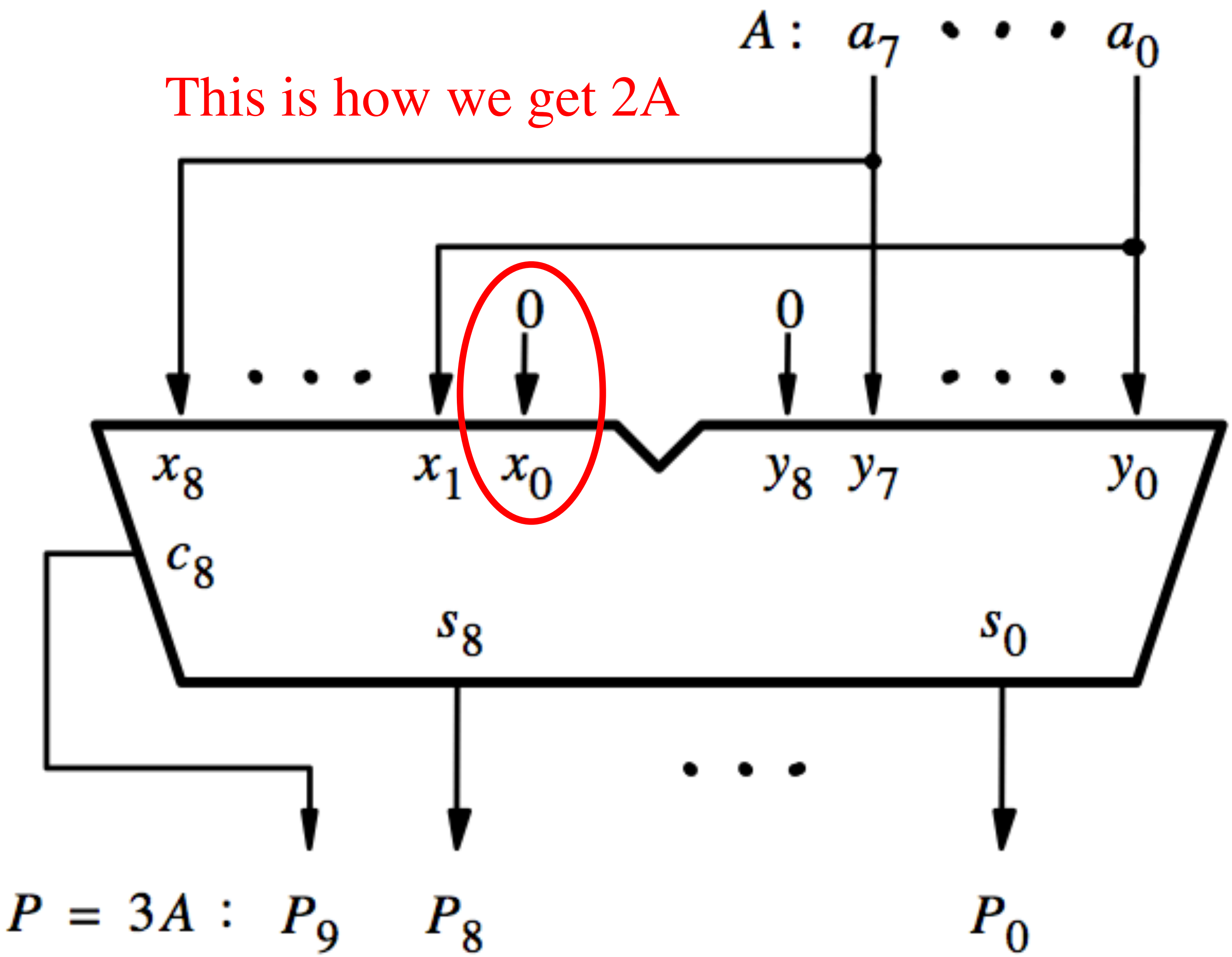
101 times 2 = 1010

110011 times 2 = 1100110

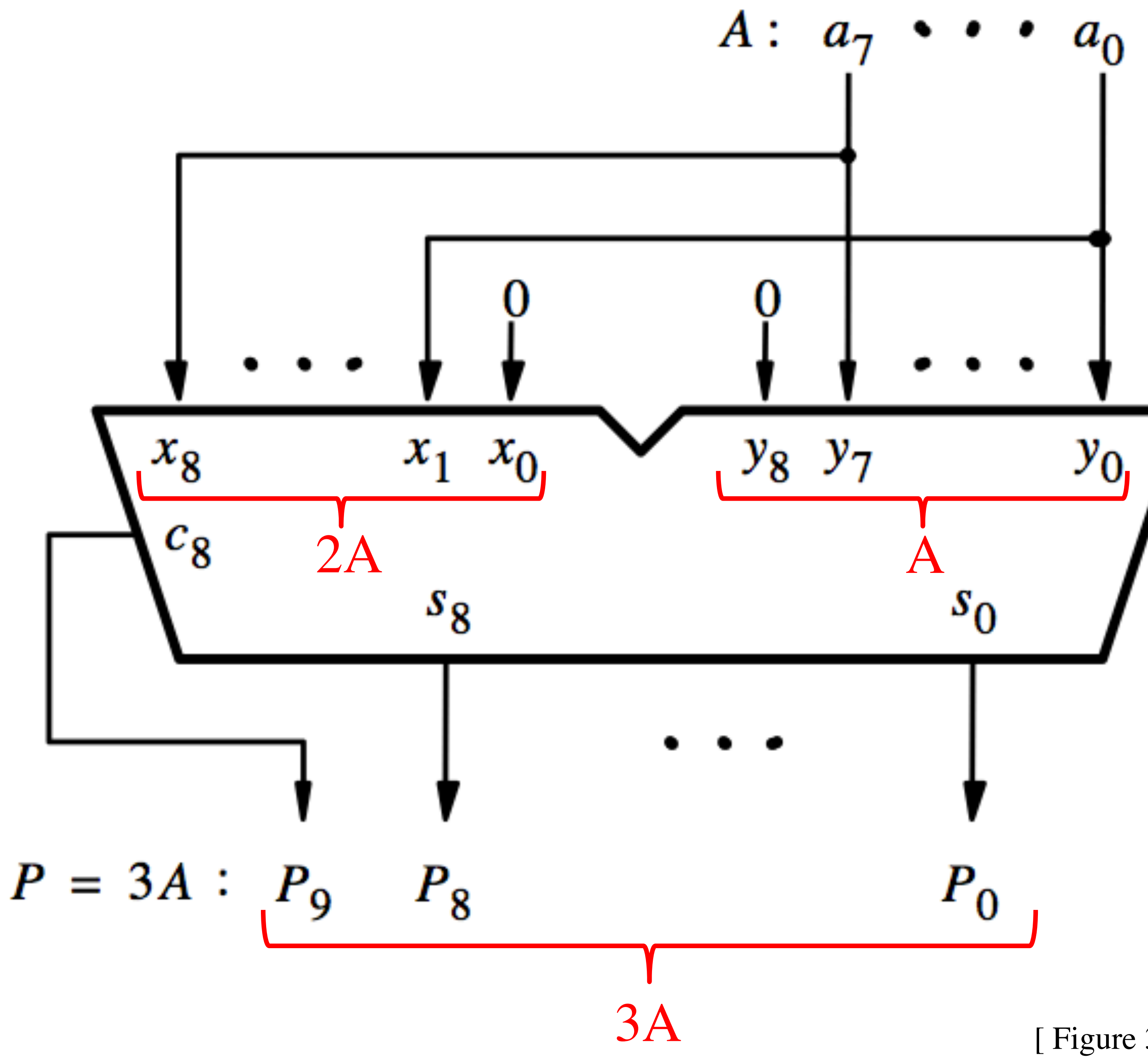
You simply add a zero as the rightmost number



[Figure 3.6b from the textbook]



[Figure 3.6b from the textbook]



[Figure 3.6b from the textbook]

Questions?

THE END