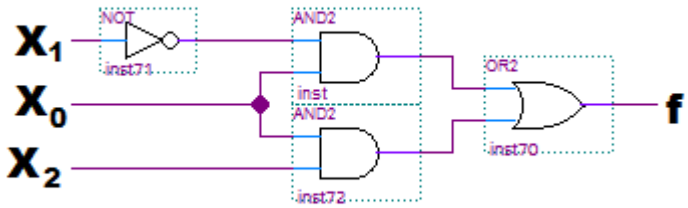
Cpr E 281 HW04

ELECTRICAL AND COMPUTER ENGINEERING
IOWA STATE UNIVERSITY

Minimization and Karnaugh Maps
Assigned: Week 4
Due Date: Sep. 17, 2018

P1 (10 points): Given the following circuit:



A: Write the SOP expression for the output f.
B: Show that the POS shorthand expression for f can be written as: $f(x_2, x_1, x_0) = \prod M(0,2,3,4,6)$
C: Assume that the inputs to the circuit (x_2, x_1, x_0) represent a three-bit integer X where (by convention) the subscript "0" (x_0) indicates the least significant bit (LSB). What is the value of f for the input X=6?
D: What decimal values of X will produce an output f=1?

P2 (9 points): Produce the simplified sum-of-products (SOP) expressions for the following K-maps:

A \ BC	00	01	11	10
0	0	1	1	0
1	0	1	1	0

WX \ YZ	00	01	11	10
00	0	1	0	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	1

WX \ YZ	00	01	11	10
00	1	0	0	0
01	1	0	0	0
11	1	0	0	0
10	0	0	0	1

P3 (12 points): Given the shorthand expression $F(a, b, c) = \prod M(0,1,2,5)$:
A: Use a K-Map to show that F may simplify to the product-of-sums (POS) expression $F = (a + c)(b + \bar{c})$.
B: A second expression is given as $F_2 = (a + b)(a + c)(b + \bar{c})$. Prove that these two expressions are equivalent; that is, prove that $F = F_2$.
C: What are the implications of implementing the circuit for F with the second expression $(a + b)(a + c)(b + \bar{c})$ instead of $(a + c)(b + \bar{c})$?

P4 (12 points): For each shorthand expression below, derive the simplest SOP expression:
A: $X_3(a, b, c, d) = \sum m(4,6,9,12,13,14)$
B: $X_2(a, b, c) = \sum m(0,1,5,7)$
C: $X_1(a, b, c, d) = \sum m(0,3,4,7,8,9,12,15)$

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P5 (12 points): For each expression below, derive the simplest POS expression:

A: $Y_3(P, Q, R) = \prod M(1,2,3,7)$

B: $Y_2(P, Q, R, S) = \prod M(0,2,5,7,8,10)$

C: $Y_1(P, Q, R, S) = \prod M(1,4,6,9,10)$

P6 (10 points): Given the shorthand expression $L(W, X, Y, Z) = \sum m(0,1,4,5,6,8,9,11,12,13,14,15)$:

A: Derive the simplified SOP expression for L, L_{SOP}.

B: Derive the simplified POS expression for L, L_{POS}.

C: Which expression will produce a circuit with a lower cost: L_{POS} or L_{SOP}? Why?

D: Is it valid to say that $L_{SOP} = L_{POS}$? Why?

P7 (15 points): Use Karnaugh Maps to convert the given expressions to <u>simplified SOP expressions</u>:

I: $Z_1(A, B, C, D) = A\bar{B}CD + \bar{A}BC\bar{D} + \bar{A}B\bar{C} + \bar{A}CD + ABD$

II: $Z_2(D, Q) = DQ + (D \oplus Q)$

III: $Z_3(a, b, c) = \prod M\,(2,3,4,6,7)$

P8 (20 points): The goal for this problem is to accept a four-bit integer X (x_3, x_2, x_1, x_0) and use the constitutent bits to derive two expressions: P_2=1 if X is a multiple of 2 (0, 2, 4…) and P_3=1 if X is a multiple of 3 (0, 3, 6…).

A: Show the K-map for P_2 and the K-map for P_3 given the four input bits for X (x_3, x_2, x_1, x_0)

B: Write a shorthand SOP expression for P_2 and show that the cost of the minimal SOP circuit for P_2 is 2.

C: Write a shorthand SOP expression for P_3 and show that the cost of the minimal SOP circuit for P_3 is 45 (when implemented separately from P_2).

D: Let P_6=1 if X is a multiple of 6. Show that its expression is $P_6 = \bar{x}_3\bar{x}_2\bar{x}_1\bar{x}_0 + \bar{x}_3x_2x_1\bar{x}_0 + x_3x_2\bar{x}_1\bar{x}_0$