## CprE 281: Digital Logic

Midterm 1: Friday Sep 21, 2018
Student Name: $\qquad$

## Student ID Number:

$\qquad$

| Lab Section: | Mon 12-3(P) | Tue 11-2(U) | Wed 8-11(J) | Thur 11-2(Q) | Fri 11-2(G) |
| :---: | :--- | :--- | :--- | :--- | :--- |
| (circle one) |  | Tue 2-5(M) | Wed 11-2(W) | Thur 11-2(V) |  |
|  |  | Tue 2-5(Z) | Wed 6-9(T) | Thur 2-5(L) |  |
|  |  |  |  | Thur 5-8(K) |  |

1. True/False Questions ( $10 \times 1 \mathrm{p}$ each $=10 \mathrm{p}$ )
(a) I forgot to write down my name, student ID, and lab section.
(b) A NOR gat can be built using one AND gate and two NOT gates.
(c) It is possible to build an AND gate with a 4-to-1 multiplexer.

TRUE / FALSE
(d) A two-input AND requires more transistors than a three-input OR.

TRUE / FALSE
(e) An XOR can be implemented with a 2-to-1 multiplexer and one NOT. TRUE / FALSE
(f) $\overline{x+y}+\bar{x} y+x \bar{y}+x y=1$.

TRUE / FALSE
(g) $\operatorname{XOR}(\mathrm{x}, 1)=\overline{\mathrm{x}}$

TRUE / FALSE
(h) $\operatorname{XOR}(\operatorname{XOR}(x, 0), 1)=x$

TRUE / FALSE
(i) A NAND can be implemented with fewer transistors than a NOR.

TRUE / FALSE
(j) Tatooine, Alderaan, and Jedha are all planets in the Star Wars universe. TRUE / FALSE
2. Three-Variable K-map (5p)

Draw the K-map and derive the minimum POS expression for $\mathbf{f}(\mathbf{a}, \mathbf{b}, \mathbf{c})=\sum \mathbf{m}(\mathbf{0}, \mathbf{4}, \mathbf{7})+\mathbf{D}(\mathbf{6})$.
3. Truth Tables ( $3 \times 5 p$ each $=15 p$ )
(a) Draw the truth table for the Boolean function $F$ that has the following K-Map:

(b) Prove that $(x+y) \bullet(x+\bar{y})=x$ using truth tables.
(c) Draw the truth table for the function $f(a, b, c)=\overline{\mathbf{a}} \mathbf{b}+\mathbf{a} \bar{c}+\overline{\mathbf{a}} \overline{\mathbf{b}} \mathbf{c}$.
4. Number Conversions ( $5 \times 4 p$ each $=20 p$ )
(a) Convert 21910 to binary
(b) Convert 11014 to decimal
(c) Find the values of the digits $x$ and $y$ in the equation: $X Y_{5}=1101_{2}$
(d) Convert 8513049 to ternary (base 3)
(e) Compute the following sums where all numbers are in base 5:
$\begin{array}{r}43 \\ +4 \\ \hline\end{array} \begin{array}{r}142 \\ \hline\end{array}$
5. Minimization ( $2 \times 5 p$ each $=10 p$ )

Consider the Boolean function $f(X, Y, Z)=((\overline{(\mathbf{Z}+\mathbf{Z})}+\mathbf{Y})+\mathbf{X})+\overline{(\mathbf{X}+\mathbf{Y})}$
(a) Draw the circuit diagram for this expression using only NOR gates.
(b) Use the theorems of Boolean algebra to simplify the expression from part (a).
6. Venn Diagrams ( $3 \times 5 p$ each $=15 p$ )
(a) Write the expression that is represented by each of the three Venn diagrams:

(A)

(B)

(C)

$$
A=
$$

$B=$
(b) Let $\mathbf{F}(\mathbf{X}, \mathbf{Y}, \mathbf{Z})=\mathbf{A}+\mathbf{B}+\mathbf{C}$. Use the expressions that you derived in part (a) to draw the K-map for the Boolean function $F$. Then use the K-map to derive the minimumcost SOP expression for $F$.
(c) Draw the circuit for your expression from part (b). Label all inputs and outputs.
7. Derive the minimum SOP expression using a K-map ( $3 \times 5 \mathrm{p}$ each $=15 \mathrm{p}$ )
(a) Draw the K-map for the following function

$$
\mathrm{F}(\mathrm{a}, \mathrm{~b}, \mathrm{c}, \mathrm{~d})=\operatorname{m}(0,1,7,10,11)+\mathrm{D}(2,4,6,8,9,14)
$$

(b) Use the K-map to derive the minimum-cost $\underline{\text { SOP }}$ expression for the function $F$.
(c) Draw the circuit diagram for the minimum expression from part (b).
8. NAND/NOR Logic ( $2 \times 5 p$ each $=10 p$ )
(a) Using only NOR gates, draw the logic circuit that corresponds to this K-map:

(b) Draw the circuit for $F(X, Y, Z)=\Pi M(1,2,4,6,7)$ using only NAND gates.
9. Seven-Segment Display ( $3 \times 5$ peach $=15 p$ ). The truth table for a Boolean function that convers its $\mathbf{4}$ binary inputs into a 7 -segment display code is given below.

|  | $x_{3}$ | $x_{2}$ | $x_{1}$ | $x_{0}$ | $a$ | $b$ | $c$ | d | $e$ | $f$ | $g$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| I | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 3 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 5 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 6 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 8 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 9 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

(a) Derive a minimum-cost SOP expression for the output $b$.
(b) Derive a minimum-cost POS expression for the output f.
(c) Derive a minimum-cost POS expression for the output g.

## 10. Minimization with Theorems (15p)

Use the theorems of Boolean algebra to simplify the following Boolean function
$f(x, y, z)=x y \bar{z}(x+y)+\bar{z}(\overline{x+\bar{y}})+x(\overline{\bar{x}+y z})+1(z+x \bar{y})+\overline{\bar{x}+z}$

| Question | Max | Score |
| :--- | ---: | ---: |
| 1. True/False | 10 |  |
| 2. Three-variable K-map | 5 |  |
| 3. Truth Tables | 15 |  |
| 4. Number Conversions | 20 |  |
| 5. Minimization | 10 |  |
| 6. Venn Diagrams | 15 |  |
| 7. SOP with K-Map | 15 |  |
| 8. NAND/NOR Logic | 10 |  |
| 9. Seven-Segment Display | 15 |  |
| 10. Minimization | 15 |  |
| TOTAL: | 130 |  |

