Sample Solutions

CprE 281: Digital Logic
Midterm 1: Friday Sep 21, 2018

Student Name: ______________________ **Student ID Number:** ____________

Lab Section:	**Mon 12-3(P)**	**Tue 11-2(U)**	**Wed 8-11(J)**	**Thur 11-2(Q)**	**Fri 11-2(G)**
(circle one)		**Tue 2-5(M)**	**Wed 11-2(W)**	**Thur 11-2(V)**	
		Tue 2-5(Z)	**Wed 6-9(T)**	**Thur 2-5(L)**	
				Thur 5-8(K)	

1. **True/False Questions (10 x 1p each = 10p)**

(a) I forgot to write down my name, student ID, and lab section. TRUE / FALSE

(b) A NOR gat can be built using one AND gate and two NOT gates. TRUE / FALSE

(c) It is possible to build an AND gate with a 4-to-1 multiplexer. TRUE / FALSE

(d) A two-input AND requires more transistors than a three-input OR. TRUE / FALSE

(e) An XOR can be implemented with a 2-to-1 multiplexer and one NOT. TRUE / FALSE

(f) $\overline{x+y} + \overline{x}y + x\overline{y} + xy = 1.$ TRUE / FALSE

(g) XOR (x, 1) = $\overline{x}$ TRUE / FALSE

(h) XOR (XOR (x, 0), 1) = x TRUE / FALSE

(i) A NAND can be implemented with fewer transistors than a NOR. TRUE / FALSE

(j) $\underbrace{\text{Tatooine}}_{\text{planet}}$, $\underbrace{\text{Alderaan}}_{\text{planet}}$, and $\underbrace{\text{Jedha}}_{\text{moon}}$ are all planets in the Star Wars universe. TRUE / FALSE

2. **Three-Variable K-map (5p)**

Draw the K-map and derive the minimum **POS** expression for **f(a,b,c)=Σm(0,4,7)+D(6).**

c \ ab	00	01	11	10
0	1	0	d	1
1	0	0	1	0

$$f = (b+\bar{c})\cdot(a+\bar{b})$$

3. Truth Tables (3 x 5p each = 15p)

(a) Draw the truth table for the Boolean function F that has the following K-Map:

X \ YZ	00	01	11	10
0	0	1	0	0
1	0	1	1	1

X Y Z	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	0
1 0 0	0
1 0 1	1
1 1 0	1
1 1 1	1

(b) Prove that $(x + y) \bullet (x + \overline{y}) = x$ using truth tables.

x y	$x + y$	$x + \overline{y}$	$(x+y) \cdot (x+\overline{y})$	x
0 0	0	1	0	0
0 1	1	0	0	0
1 0	1	1	1	1
1 1	1	1	1	1

equal

(c) Draw the truth table for the function $f(a,b,c) = \overline{a}\,b + a\,\overline{c} + \overline{a}\,\overline{b}\,c$.

a b c	$\overline{a}b$	$+ a\overline{c}$	$+ \overline{a}\overline{b}c$	f
0 0 0	0	0	0	0
0 0 1	0	0	1	1
0 1 0	1	0	0	1
0 1 1	1	0	0	1
1 0 0	0	1	0	1
1 0 1	0	0	0	0
1 1 0	0	1	0	1
1 1 1	0	0	0	0

4. Number Conversions (5 x 4p each = 20p)

(a) Convert 219_{10} to binary

$$
\begin{array}{rcrc}
219/2 & = & 109 & 1 \\
109/2 & = & 54 & 1 \\
54/2 & = & 27 & 0 \\
27/2 & = & 13 & 1 \\
13/2 & = & 6 & 1 \\
6/2 & = & 3 & 0 \\
3/2 & = & 1 & 1 \\
1/2 & = & 0 & 1
\end{array}
\quad \uparrow
$$

$$11011011_2 = 219_{10}$$

(b) Convert 1101_4 to decimal

$$1 \times 4^3 + 1 \times 4^2 + 0 \times 4^1 + 1 \times 4^0 = 64 + 16 + 0 + 1 = 81_{10}$$

(c) Find the values of the digits x and y in the equation: $XY_5 = \underbrace{1101_2}_{13_{10}}$

$$X \times 5^1 + Y \times 5^0 = 13$$

$$\Downarrow$$

$$5X + Y = 13 \qquad \text{(given that } X, Y \in \{0, 1, 2, 3, 4\}\text{)}$$

$$\Downarrow$$

$$X = 2 \quad \text{and} \quad Y = 3$$

(d) Convert 851304_9 to ternary (base 3)

$$\overbrace{22}\ \overbrace{12}\ \overbrace{01}\ \overbrace{10}\ \overbrace{00}\ \overbrace{11}{}_3$$

base 3	base 9
00	0
01	1
02	2
10	3
11	4
12	5
20	6
21	7
22	8

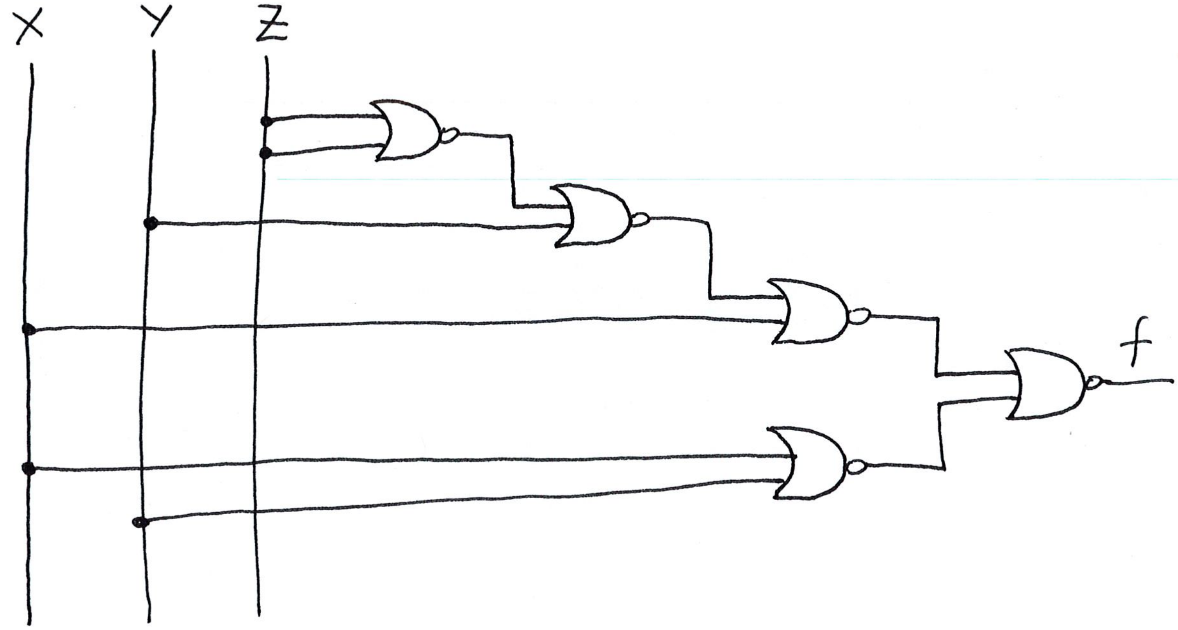
(e) Compute the following sums where all numbers are in base 5:

$$
\begin{array}{r}
\overset{\bullet}{4}3 \\
+\quad 4 \\
\hline
102
\end{array}
\qquad\qquad
\begin{array}{r}
\overset{\bullet}{1}\overset{\bullet}{4}2 \\
+\quad 13 \\
\hline
210
\end{array}
$$

5. Minimization (2 x 5p each = 10p)

Consider the Boolean function $f(X, Y, Z) = \overline{\overline{(\overline{(\overline{(Z + Z)} + Y)} + X)} + \overline{(X + Y)}}$

(a) Draw the circuit diagram for this expression using <u>only NOR</u> gates.



(b) Use the theorems of Boolean algebra to simplify the expression from part (a).

$$f = \overline{\overline{(\overline{(\underbrace{\overline{(Z + Z)}}_{\text{repeated}} + Y)} + X)} + \overline{(X+Y)}} \qquad \text{(De Morgan's)}$$

$$= \overline{\overline{(\overline{(\overline{Z} + Y)} + X)}} \bullet \overline{\overline{(X+Y)}} \qquad \text{(remove double negation)}$$

$$= ((\overline{\overline{Z} + Y}) + X) \bullet (X + Y) \qquad \text{(De Morgan's)}$$

$$= (\overline{\overline{Z}} \bullet \overline{Y} + X) \bullet (X + Y) \qquad \text{(remove double negation and expand)}$$

$$= Z\overline{Y}X + \underbrace{Z\overline{Y}Y}_{0} + \underbrace{XX}_{X} + XY$$

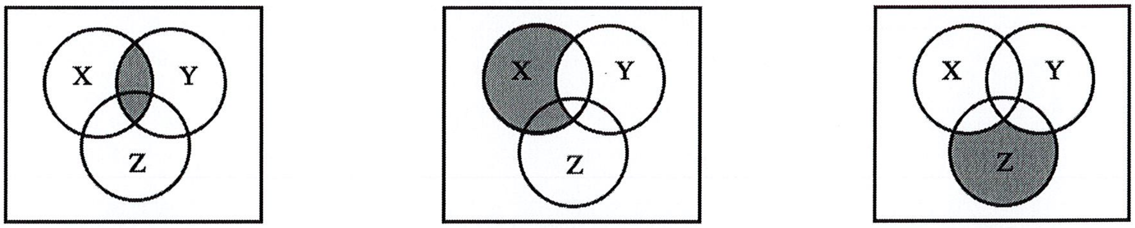
$$= X\overline{Y}Z + X + XY$$

$$= X(\underbrace{\overline{Y}Z + 1 + Y}_{1})$$

$$= X$$

6. Venn Diagrams (3 x 5p each = 15p)

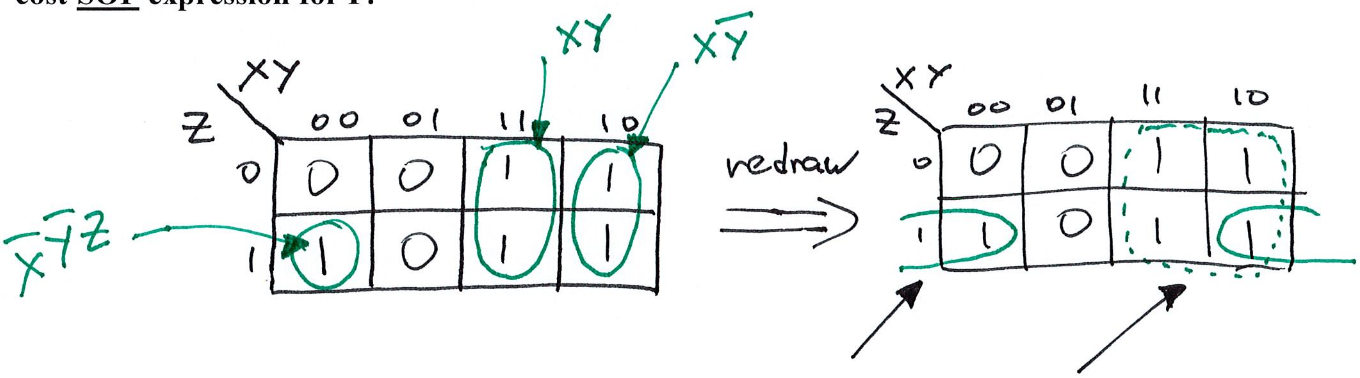
(a) Write the expression that is represented by each of the three Venn diagrams:



(A) **(B)** **(C)**

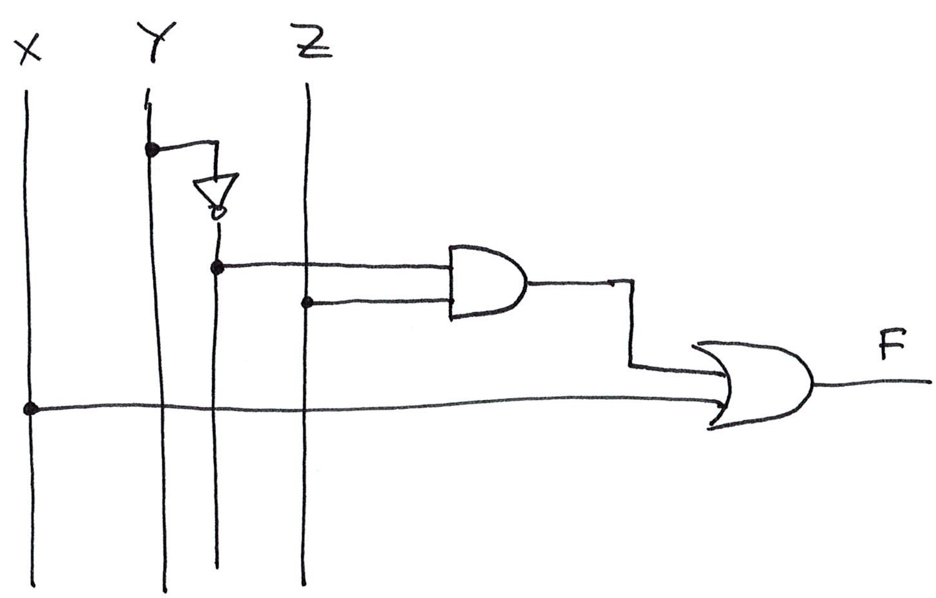
A = $X \cdot Y$ **B =** $X \cdot \overline{Y}$ **C =** $\overline{X}\,\overline{Y}\,Z$

(b) Let F(X, Y, Z) = A + B + C. Use the expressions that you derived in part (a) to draw the K-map for the Boolean function F. Then use the K-map to derive the minimum-cost SOP expression for F.



$F = \overline{Y}Z + X$

(c) Draw the circuit for your expression from part (b). Label all inputs and outputs.



7. Derive the minimum SOP expression using a K-map (3 x 5p each = 15p)

(a) Draw the K-map for the following function

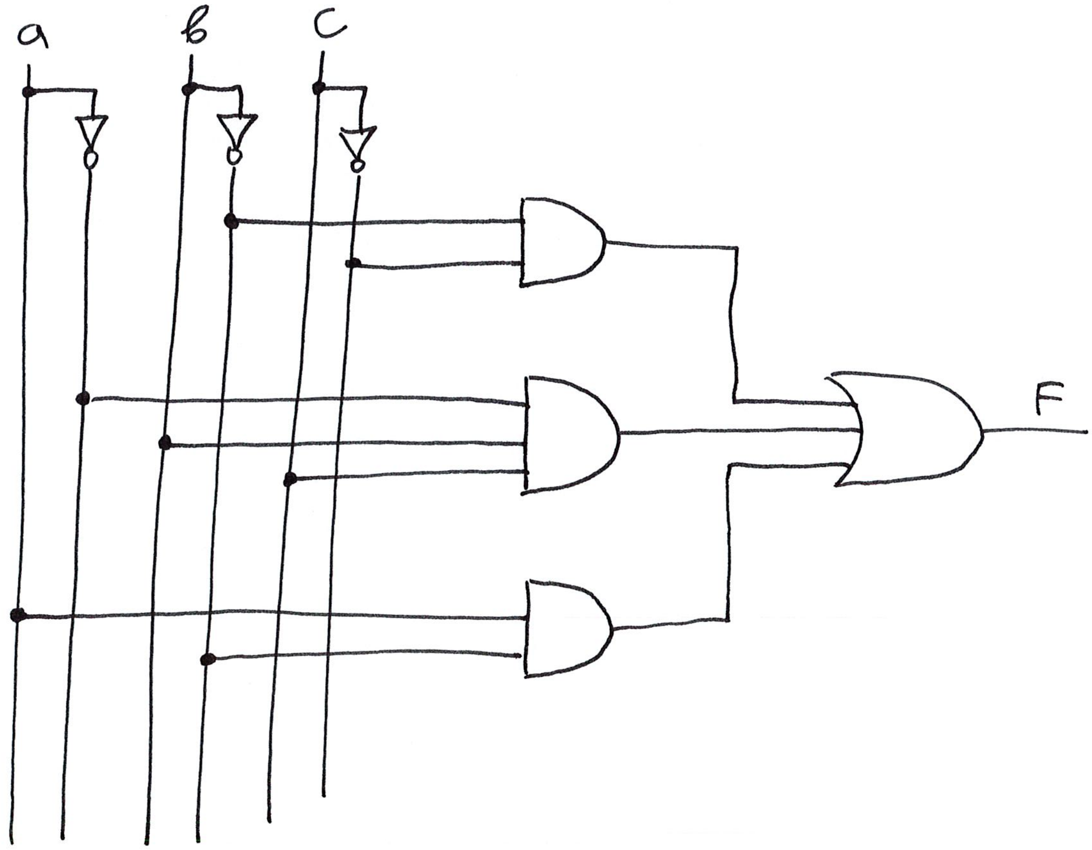
$$F(a,b,c,d)=\Sigma m(0, 1, 7, 10, 11) + D(2, 4, 6, 8, 9, 14)$$

cd \ ab	00	01	11	10
00	1	d	0	d
01	1	0	0	d
11	0	1	0	1
10	d	d	d	1

(b) Use the K-map to derive the minimum-cost SOP expression for the function F.

$$F = \bar{b}\bar{c} + \bar{a}bc + a\bar{b}$$

(c) Draw the circuit diagram for the minimum expression from part (b).



8. NAND/NOR Logic (2 x 5p each = 10p)

(a) Using only NOR gates, draw the logic circuit that corresponds to this K-map:

B \ A	0	1
0	0	1
1	1	0

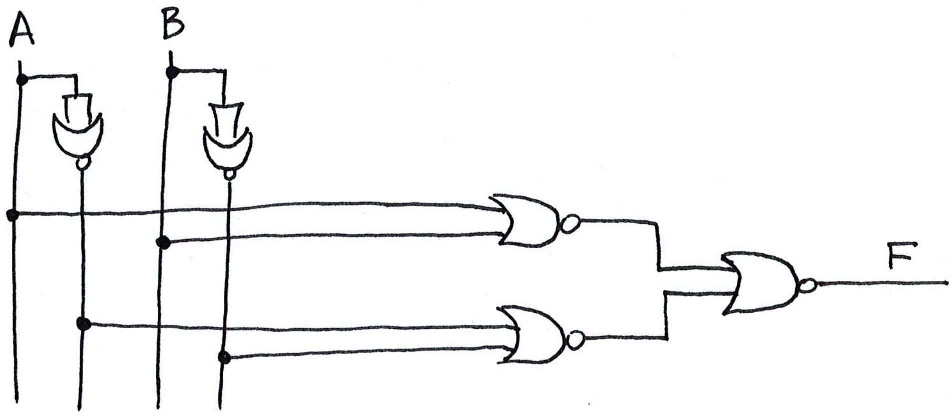
$(A+B)$

$(\bar{A}+\bar{B})$

$$F = (A+B)\cdot(\bar{A}+\bar{B})$$

$$= \overline{\overline{(A+B)\cdot(\bar{A}+\bar{B})}}$$

$$= \overline{\overline{(A+B)} + \overline{(\bar{A}+\bar{B})}}$$



(b) Draw the circuit for F(X, Y, Z) = Π M (1, 2, 4, 6, 7) using only NAND gates.

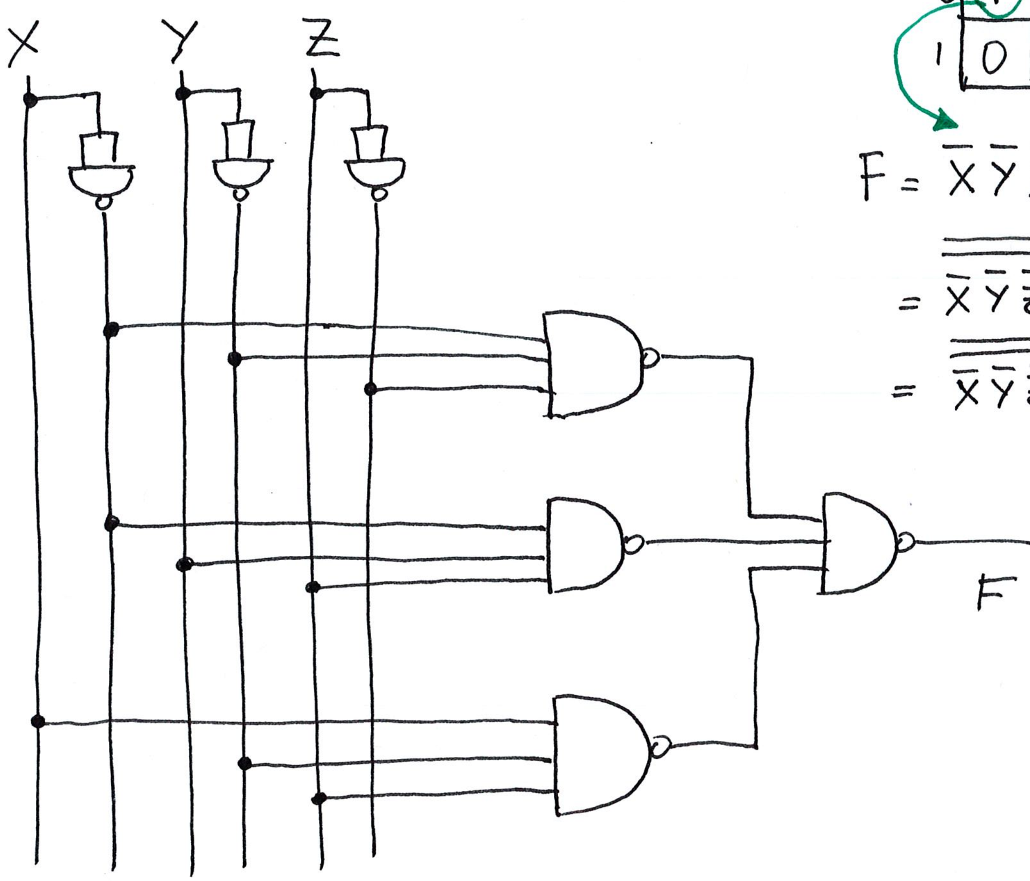
$$= \Sigma m(0,3,5)$$

Z \ XY	00	01	11	10
0	1	0	0	0
1	0	1	0	1

$$F = \bar{X}\bar{Y}\bar{Z} + \bar{X}YZ + X\bar{Y}Z$$

$$= \overline{\overline{\bar{X}\bar{Y}\bar{Z} + \bar{X}YZ + X\bar{Y}Z}}$$

$$= \overline{\overline{\bar{X}\bar{Y}\bar{Z}}\cdot\overline{\bar{X}YZ}\cdot\overline{X\bar{Y}Z}}$$



9. Seven-Segment Display (3 x 5p each = 15p). The truth table for a Boolean function that convers its 4 binary inputs into a 7-segment display code is given below.

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

six rows of d's

(a) Derive a minimum-cost SOP expression for the output b.

x_1x_0 \ x_3x_2	00	01	11	10
00	1	1	d	1
01	1	0	d	1
11	1	1	d	d
10	1	0	d	d

$\overline{x_2}$

$\overline{x_1}\,\overline{x_0}$

$x_1 x_0$

$$b = \overline{x_2} + \overline{x_1}\,\overline{x_0} + x_1 x_0$$

(b) Derive a minimum-cost POS expression for the output f.

x_1x_0 \ x_3x_2	00	01	11	10
00	1	1	d	1
01	0	1	d	1
11	0	0	d	d
10	0	1	d	d

$(\overline{x_0} + x_2 + x_3)$

$(\overline{x_0} + \overline{x_1})$

$(\overline{x_1} + x_2)$

$$f = (\overline{x_0} + x_2 + x_3)\cdot(\overline{x_0} + \overline{x_1})\cdot(\overline{x_1} + x_2)$$

(c) Derive a minimum-cost POS expression for the output g.

x_1x_0 \ x_3x_2	00	01	11	10
00	0	1	d	1
01	0	1	d	1
11	1	0	d	d
10	1	1	d	d

$(x_1 + x_2 + x_3)$

$(\overline{x_0} + \overline{x_1} + \overline{x_2})$

$$g = (x_1 + x_2 + x_3)\cdot(\overline{x_0} + \overline{x_1} + \overline{x_2})$$

10. Minimization with Theorems (15p)

Use the theorems of Boolean algebra to simplify the following Boolean function

$$f(x, y, z) = \underbrace{x\,y\,\bar{z}\,(x+y)}_{A} + \underbrace{\bar{z}\,(\overline{x+\bar{y}})}_{B} + \underbrace{x\,(\overline{\bar{x}+yz})}_{C} + \underbrace{1\,(z+x\bar{y})}_{D} + \underbrace{\overline{\bar{x}+z}}_{E}$$

$$A = xy\bar{z}\,(x+y) = xy\bar{z}x + xy\bar{z}y = xy\bar{z} + xy\bar{z} = xy\bar{z}$$

repeated repeated repeated

$$B = \bar{z}\,(\overline{x+\bar{y}}) = \bar{z}\,(\bar{x}\cdot\bar{\bar{y}}) = \bar{z}\,(\bar{x}\cdot y) = \bar{x}y\bar{z}$$

$$C = x\,(\overline{\bar{x}+yz}) = x\,(\bar{\bar{x}}\cdot\overline{yz}) = x\,(x\cdot(\bar{y}+\bar{z})) = x\bar{y} + x\bar{z}$$

$$D = 1\,(z+x\bar{y}) = z + x\bar{y}$$

$$E = \overline{\bar{x}+z} = \bar{\bar{x}}\cdot\bar{z} = x\bar{z}$$

$$f = A + B + C + D + E$$

repeated term

$$= xy\bar{z} + \bar{x}y\bar{z} + x\bar{y} + x\bar{z} + z + x\bar{y} + x\bar{z}$$

repeated term

$$= y\bar{z}\,(\underbrace{x+\bar{x}}_{1}) + x\bar{y} + x\bar{z} + z$$

$$= y\bar{z} + x\bar{y} + \underbrace{x\bar{z} + z}_{x+z} \qquad \text{// Theorem 16.a}$$

$$= y\bar{z} + x\bar{y} + x + z \qquad \text{// Theorem 16.a}$$

$y+z$

$$= \underbrace{y + x\bar{y}}_{y+x} + x + z \qquad \text{// Theorem 16.a}$$

$$= y + x + x + z$$

repeated

$$= x + y + z$$

Question	Max	Score
1. True/False	10	
2. Three-variable K-map	5	
3. Truth Tables	15	
4. Number Conversions	20	
5. Minimization	10	
6. Venn Diagrams	15	
7. SOP with K-Map	15	
8. NAND/NOR Logic	10	
9. Seven-Segment Display	15	
10. Minimization	15	
TOTAL:	130	