



# **CprE 281: Digital Logic**

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**<http://www.ece.iastate.edu/~alexs/classes/>**

# Karnaugh Maps

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# **Administrative Stuff**

- **HW4 is out**
- **It is due on Monday Sep 17 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

# **Administrative Stuff**

- **Sample homework solutions are posted on Canvas**
- **Look under 'Files'**

# **Quick Review**

# Do You Still Remember This Boolean Algebra Theorem?

14a.  $x \cdot y + x \cdot \bar{y} = x$

Combining

14b.  $(x + y) \cdot (x + \bar{y}) = x$

# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	
0	1	
1	0	
1	1	

# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	0
1	1	1



# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0 0 0
0	1	0 0 0
1	0	0 1 1
1	1	1 1 0

# Let's prove 14.a

$x$	$y$	$\mathbf{x \cdot y + x \cdot \bar{y} = x}$
0	0	0
0	1	0
1	0	1
1	1	1

# Let's prove 14.a

$x$	$y$	$\mathbf{x} \cdot \mathbf{y} + \mathbf{x} \cdot \bar{\mathbf{y}} = \mathbf{x}$
0	0	0
0	1	0
1	0	1
1	1	1

They are equal.



# Motivation

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

**An approach for simplifying logic expressions.**

**How do we guarantee we have reached the minimum SOP/POS representation?**

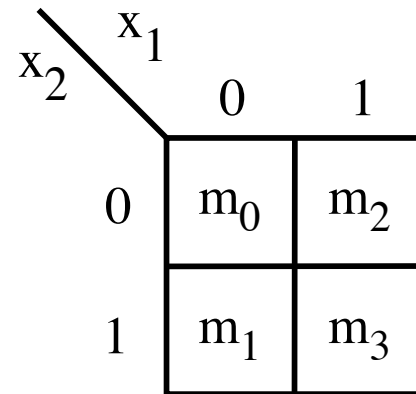
# Two-Variable K-Map

# Karnaugh Map (K-map)

- View the function in a visual form
- Same information as truth table
- Easier to group minterms

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



(b) Karnaugh map



# Minterms

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	1

# Minterm Example

$x_1$	$x_2$	
0	0	0
0	1	1
1	0	0
1	1	1

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

# Minterm Example

$x_1$	$x_2$	
0	0	0
0	1	1
1	0	0
1	1	1

$x_1$	$x_2$	$m_0$	$m_1$	$m_2$	$m_3$	$m_1 + m_3$
0	0	1	0	0	0	0
0	1	0	1	0	0	1
1	0	0	0	1	0	0
1	1	0	0	0	1	1

$$\bar{x}_1x_2 + x_1x_2 = x_2$$

# Grouping Example

		$x_1$	
	$x_2$		
		0	1
0		1	0
1		0	0

$m_0$

		$x_1$	
	$x_2$		
		0	1
0		0	0
1		1	0

$m_1$

# Grouping Example

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$m_0$

+

	$x_1$	0	1
$x_2$	0	0	0
	1	1	0

$m_1$

=

	$x_1$	0	1
$x_2$	0	1	0
	1	1	0

$m_0 + m_1$

# Grouping Example

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$m_0$

+

	$x_1$	0	1
$x_2$	0	0	0
	1	1	0

$m_1$

=

	$x_1$	0	1
$x_2$	0	1	0
	1	1	0

$m_0 + m_1$

# Grouping Example

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$m_0$

$\bar{x}_1\bar{x}_2$

+

	$x_1$	0	1
$x_2$	0	0	0
	1	1	0

$m_1$

$\bar{x}_1x_2$

=

	$x_1$	0	1
$x_2$	0	1	0
	1	1	0

$m_0 + m_1$

$\bar{x}_1$

Property 14a (Combining)

# Grouping Rules

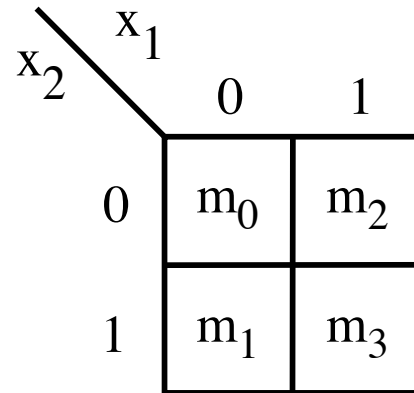
- **Group “1”s with rectangles**
- **Both sides a power of 2:**
  - **1x1, 1x2, 2x1, 2x2, 1x4, 4x1, 2x4, 4x2, 4x4**
- **Can use the same minterm more than once**
- **Can wrap around the edges of the map**
- **Some rules in selecting groups:**
  - **Try to use as few groups as possible to cover all “1”s.**
  - **For each group, try to make it as large as you can (i.e., if you can use a 2x2, don't use a 2x1 even if that is enough).**



# Two-Variable K-map

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



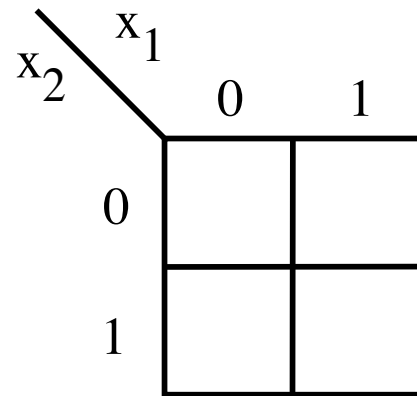
(b) Karnaugh map

# Step-By-Step Example

$x_1$	$x_2$	
0	0	1
0	1	1
1	0	0
1	1	1

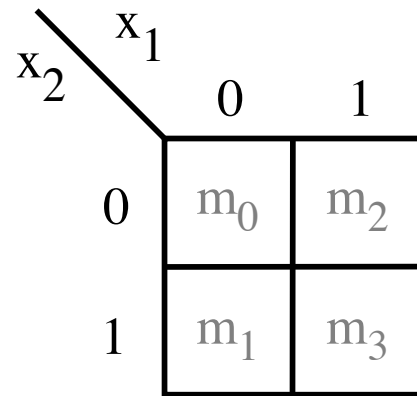
# 1. Draw The Map

$x_1$	$x_2$	
0	0	1
0	1	1
1	0	0
1	1	1



# 2. Fill The Map

	$x_1$	$x_2$	
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1



## 2. Fill The Map

	$x_1$	$x_2$	
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2$	$x_1$	0	1
0		1	0
1		1	1

# 3. Group

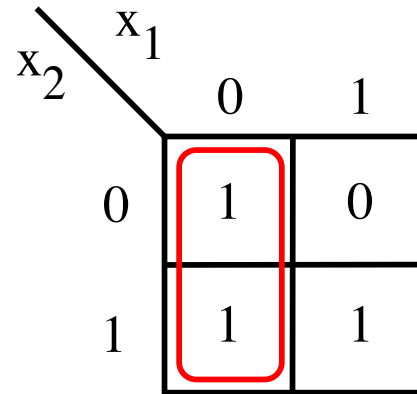
	$x_1$	$x_2$	
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2$	$x_1$	0	1
0		1	0
1		1	1

# 3. Group

	$x_1$	$x_2$	
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

$x_2$	$x_1$	0	1
0		1	0
1		1	1



# 3. Group

	$x_1$	$x_2$	
$m_0$	0	0	1
$m_1$	0	1	1
$m_2$	1	0	0
$m_3$	1	1	1

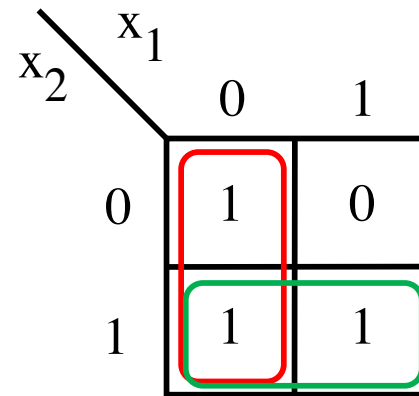
	$x_1$	0	1
0		1	0
1		1	1



# 4. Write The Expression

$x_1$	$x_2$	
0	0	1
0	1	1
1	0	0
1	1	1

$x_2$	$x_1$	0	1
0		1	0
1		1	1



# 4. Write The Expression

$x_1$	$x_2$	
0	0	1
0	1	1
1	0	0
1	1	1

$x_2 \backslash x_1$	0	1
0	1	0
1	1	1

$$\bar{x}_1 + x_2$$

# Writing The Expression

- Find which variable is constant

		$x_1$	
		0	1
$x_2$	0	1	0
	1	1	0

$\bar{x}_1$  is constant

# Writing The Expression

- Find which variable is constant

		$x_1$	
		0	1
$x_2$	0	0	1
	1	0	1

$x_1$  is constant



# These are all valid groupings

	$x_1$	
$x_2$		
	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$

	$x_1$	
$x_2$		
	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$

	$x_1$	
$x_2$		
	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$

	$x_1$	
$x_2$		
	0	1
0	$m_0$	$m_2$
1	$m_1$	$m_3$

# These are also valid

	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

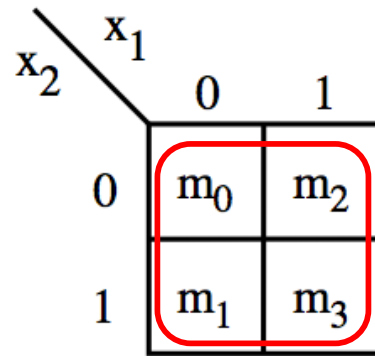
	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

But try to use larger rectangles if possible.

# This one is valid too



In this case the result is the constant function 1.



# Why are these two not valid?

	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

	$x_1$		
$x_2$		0	1
0		$m_0$	$m_2$
1		$m_1$	$m_3$

# Let's Find Out

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$m_0$

	$x_1$	0	1
$x_2$	0	0	0
	1	0	1

$m_3$

# Let's Find Out

	$x_1$	0	1
$x_2$			
0		1	0
1		0	0

$m_0$

+

	$x_1$	0	1
$x_2$			
0		0	0
1		0	1

$m_3$

=

	$x_1$	0	1
$x_2$			
0		1	0
1		0	1

$m_0 + m_3$

# Let's Find Out

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$m_0$

+

	$x_1$	0	1
$x_2$	0	0	0
	1	0	1

$m_3$

=

	$x_1$	0	1
$x_2$	0	1	0
	1	0	1

$m_0 + m_3$

# Let's Find Out

The image shows three 2x2 Karnaugh maps for variables  $x_1$  and  $x_2$ . The first map, labeled  $m_0$ , has a 1 in the top-left cell (0,0) and 0s elsewhere. The second map, labeled  $m_3$ , has 0s in the top row and a 1 in the bottom-right cell (1,1). The third map, labeled  $m_0 + m_3$ , has 1s in the top-left and bottom-right cells. The 1s in the first two maps are highlighted with red boxes.

$$\begin{array}{c} \begin{array}{cc} x_1 & \\ x_2 \diagdown & \begin{array}{cc} 0 & 1 \\ \hline 0 & 1 \\ \hline 1 & 0 \end{array} \end{array} \\ m_0 \end{array} + \begin{array}{c} \begin{array}{cc} x_1 & \\ x_2 \diagdown & \begin{array}{cc} 0 & 1 \\ \hline 0 & 0 \\ \hline 1 & 1 \end{array} \end{array} \\ m_3 \end{array} = \begin{array}{c} \begin{array}{cc} x_1 & \\ x_2 \diagdown & \begin{array}{cc} 0 & 1 \\ \hline 0 & 1 \\ \hline 1 & 1 \end{array} \end{array} \\ m_0 + m_3 \end{array}$$
$$\overline{x_1}\overline{x_2} + x_1x_2 = \overline{x_1}\overline{x_2} + x_1x_2$$

We can't use Property 14a here. This can't be simplified.

# Three-Variable K-Map

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

$x_3$	$x_1 x_2$	00	01	11	10
0		$m_0$	$m_2$	$m_6$	$m_4$
1		$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
<hr/>			
0	1	0	$m_2$
0	1	1	$m_3$
<hr/>			
1	0	0	$m_4$
1	0	1	$m_5$
<hr/>			
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

	$x_1 x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$	
1	$m_1$	$m_3$	$m_7$	$m_5$	

(b) Karnaugh map

**Notice the placement of**

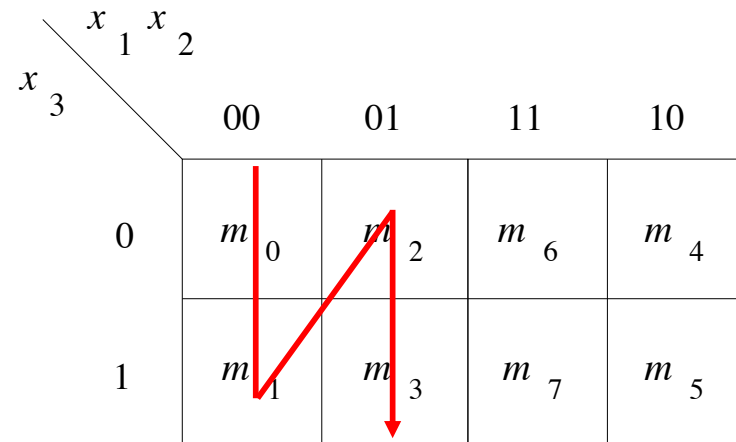
- **Variables**
- **Binary pair values**
- **Minterms**



# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
<hr/>			
0	1	0	$m_2$
0	1	1	$m_3$
<hr/>			
1	0	0	$m_4$
1	0	1	$m_5$
<hr/>			
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table



(b) Karnaugh map

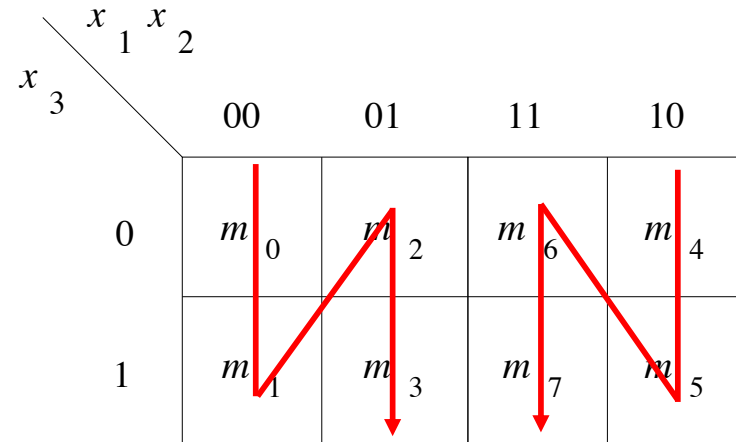
**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
<hr/>			
0	1	0	$m_2$
0	1	1	$m_3$
<hr/>			
1	0	0	$m_4$
1	0	1	$m_5$
<hr/>			
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table



(b) Karnaugh map

**Notice the placement of**

- **Variables**
- **Binary pair values**
- **Minterms**

# Gray Code

- **Sequence of binary codes**
- **Two neighboring lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	00	010	110	100
	1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$	$s$	$x_1$				
			00	01	11	10
0	000	010	110	100		
1	001	011	111	101		

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$	$s x_1$			
	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These two neighbors  
differ only in the FIRST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These four neighbors  
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$	$s$	$x_1$				
			00	01	11	10
0			000	010	110	100
1			001	011	111	101

These two neighbors  
differ only in the FIRST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$	$s x_1$			
	00	01	11	10
0	000	010	110	100
1	001	011	111	101

These four neighbors  
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

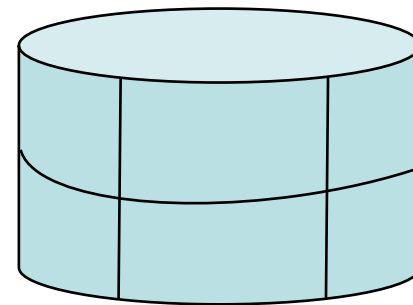


# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$



adjacent  
columns



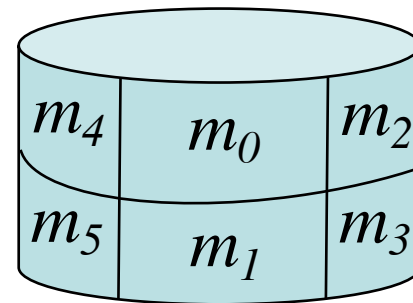
As if the K-map were  
drawn on a cylinder

# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$



adjacent  
columns



As if the K-map were  
drawn on a cylinder

# These are valid groupings

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

# These are valid groupings

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

# These are valid groupings

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

# These are valid groupings

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

$x_3 \backslash x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$
1	$m_1$	$m_3$	$m_7$	$m_5$

# These are valid groupings

	$x_1x_2$	00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

	$x_1x_2$	00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

# This is a valid grouping

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

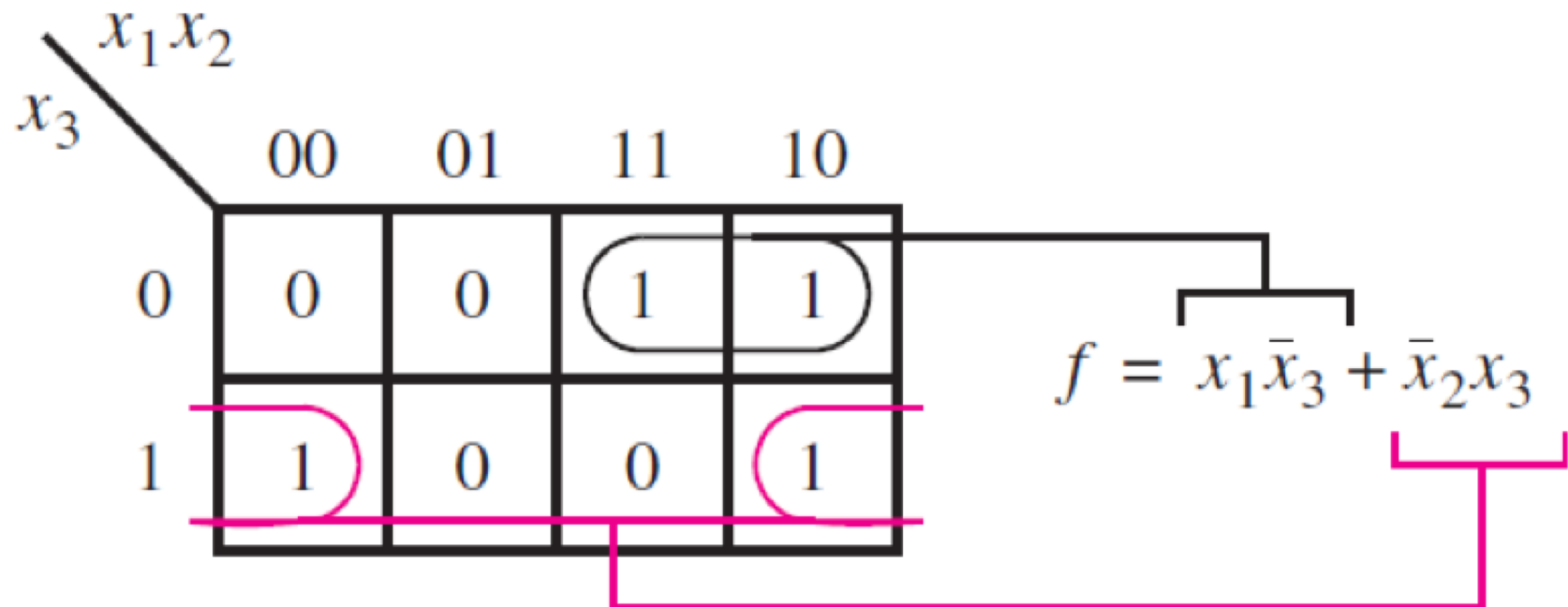


# Some invalid groupings

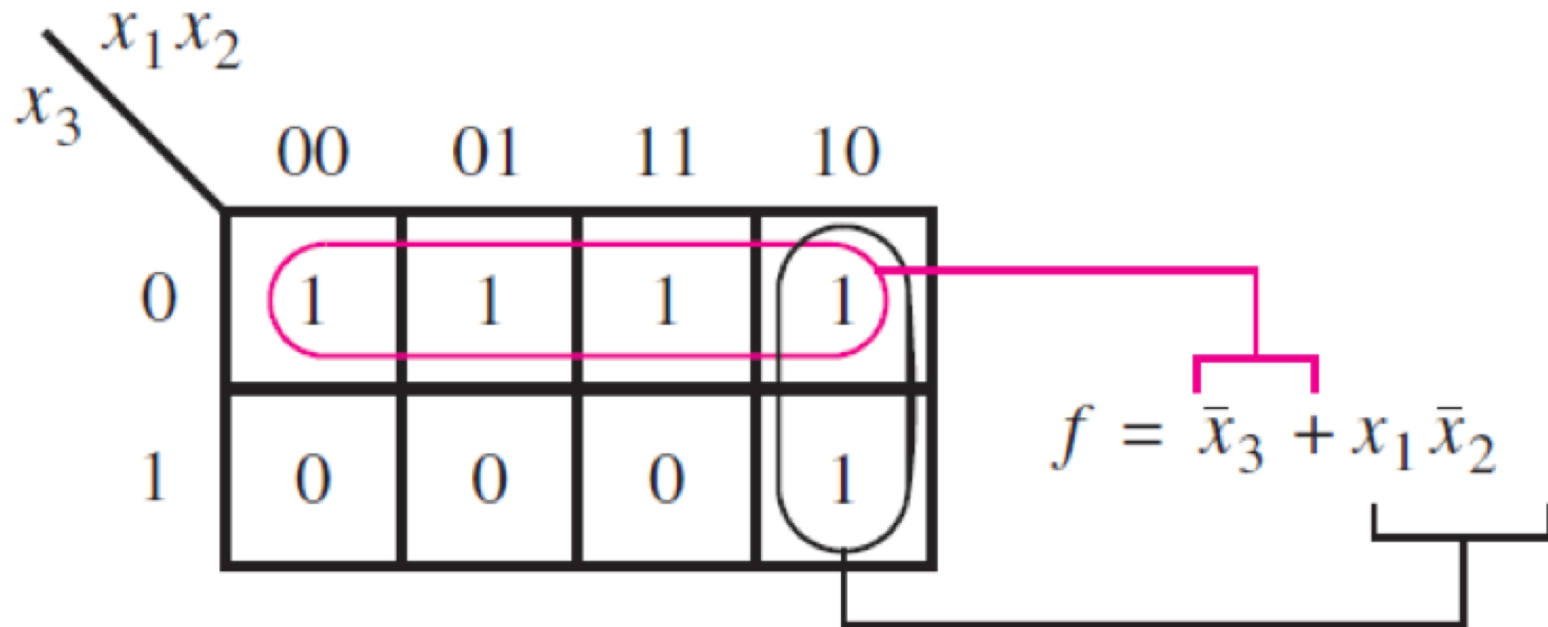
		$x_1x_2$			
$x_3$		00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$	
1	$m_1$	$m_3$	$m_7$	$m_5$	

		$x_1x_2$			
$x_3$		00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$	
1	$m_1$	$m_3$	$m_7$	$m_5$	

# Examples of three-variable Karnaugh maps

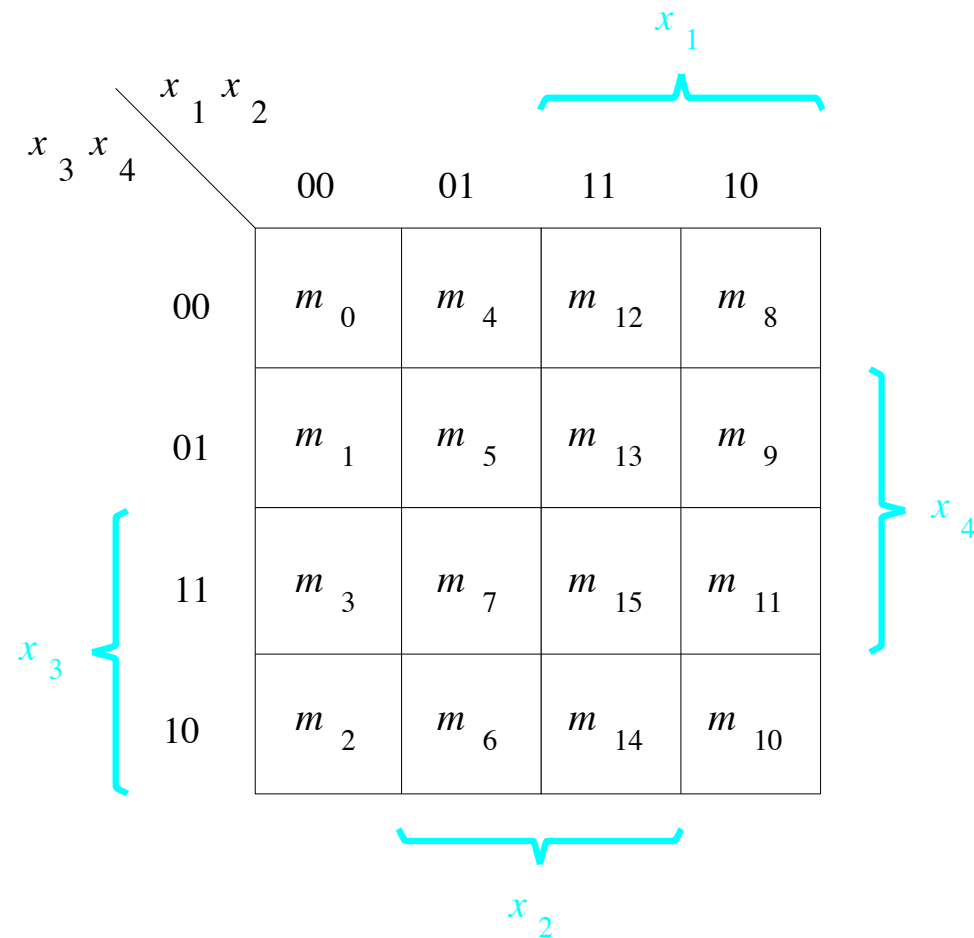


# Examples of three-variable Karnaugh maps



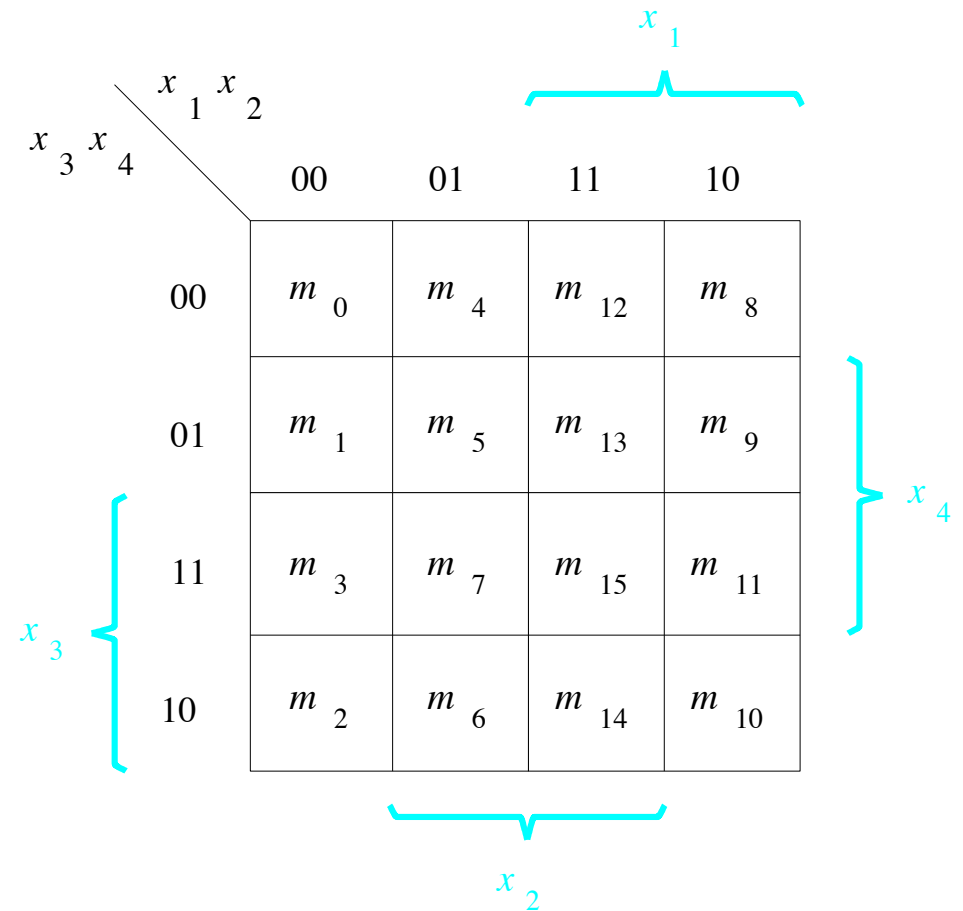
# Four-Variable K-Map

# A four-variable Karnaugh map



# A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



# Adjacency Rules

$x_3$	$x_1x_2$	00	01	11	10
0	$m_0$	$m_2$	$m_6$	$m_4$	
1	$m_1$	$m_3$	$m_7$	$m_5$	

adjacent  
columns

$x_3x_4$	$x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$	
01	$m_1$	$m_5$	$m_{13}$	$m_9$	
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$	
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$	

adjacent  
columns

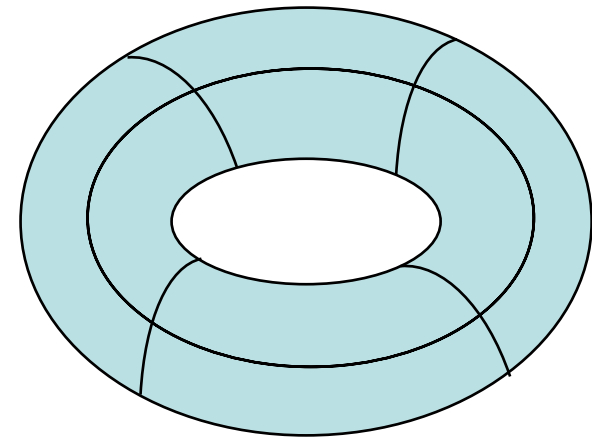
adjacent  
rows

# Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

adjacent  
rows

adjacent  
columns



As if the K-map were  
drawn on a torus

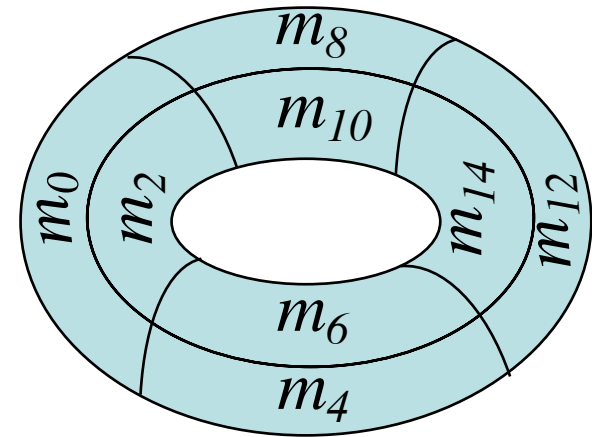


# Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

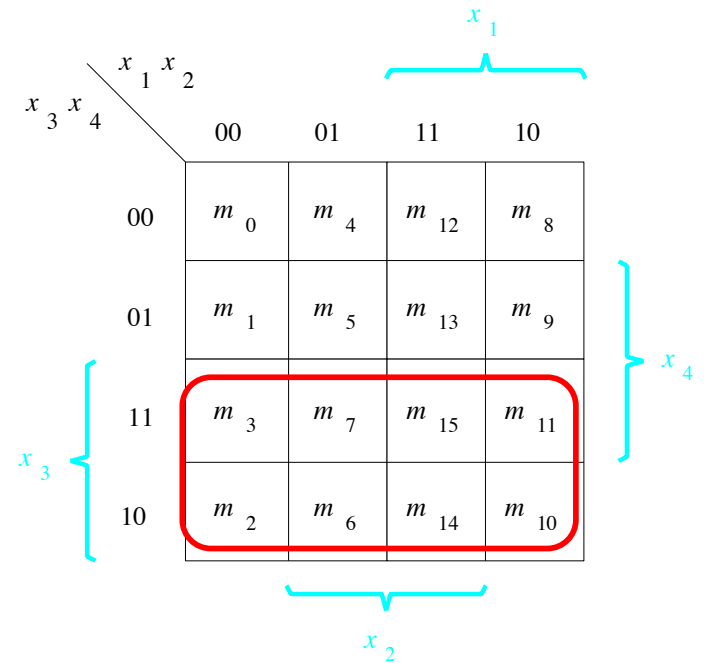
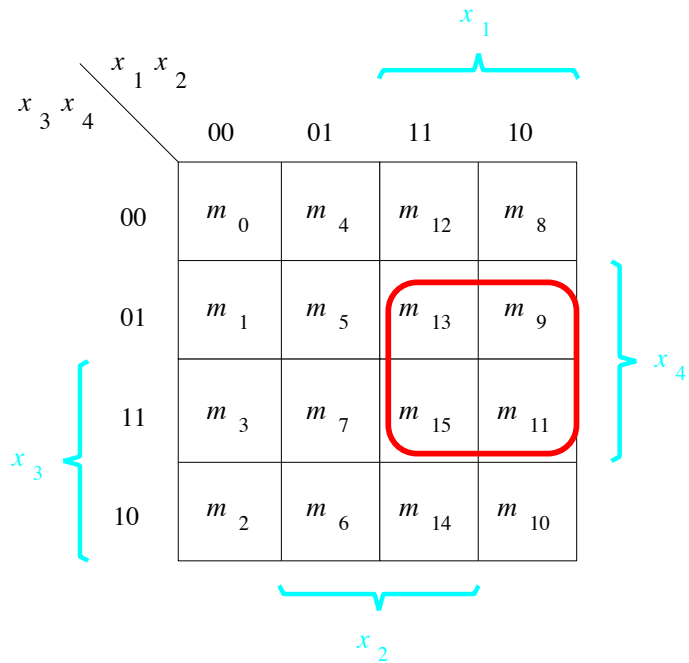
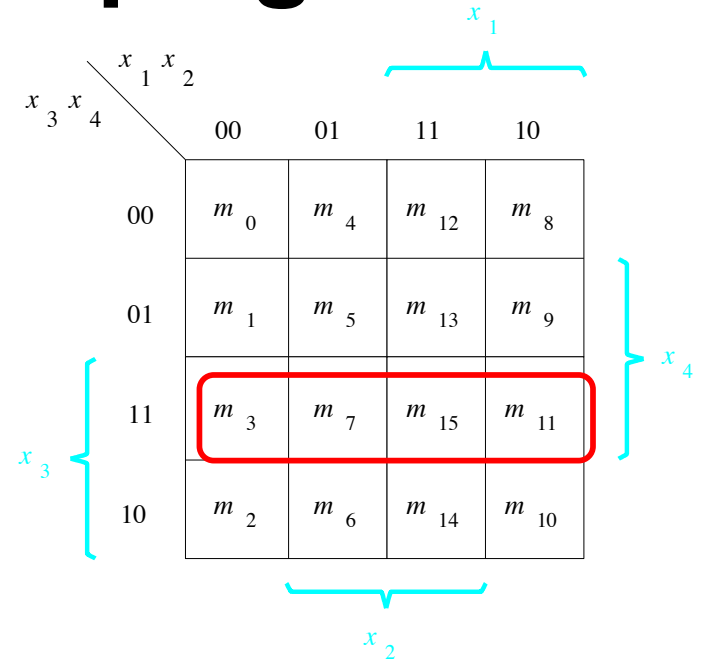
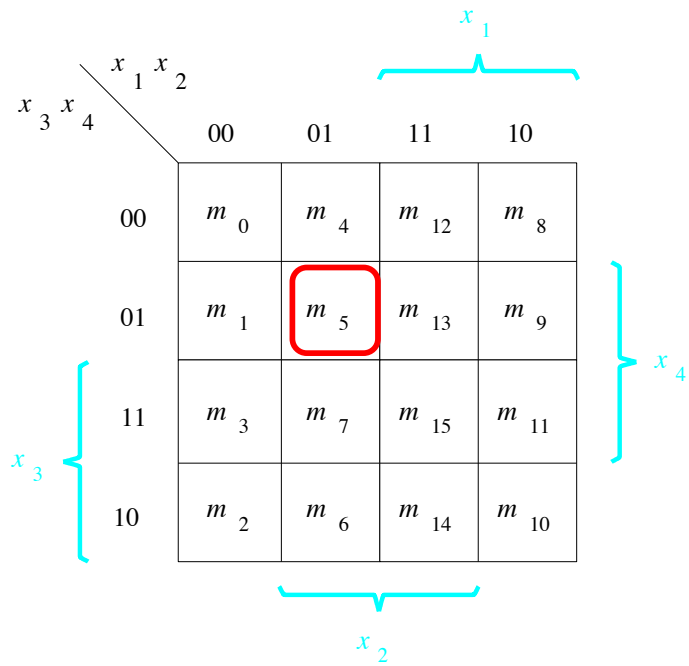
adjacent  
rows

adjacent  
columns

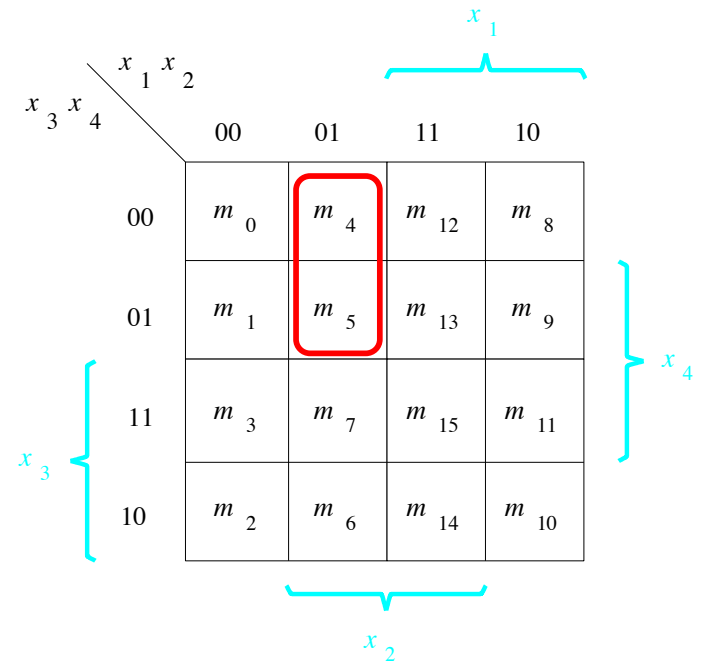
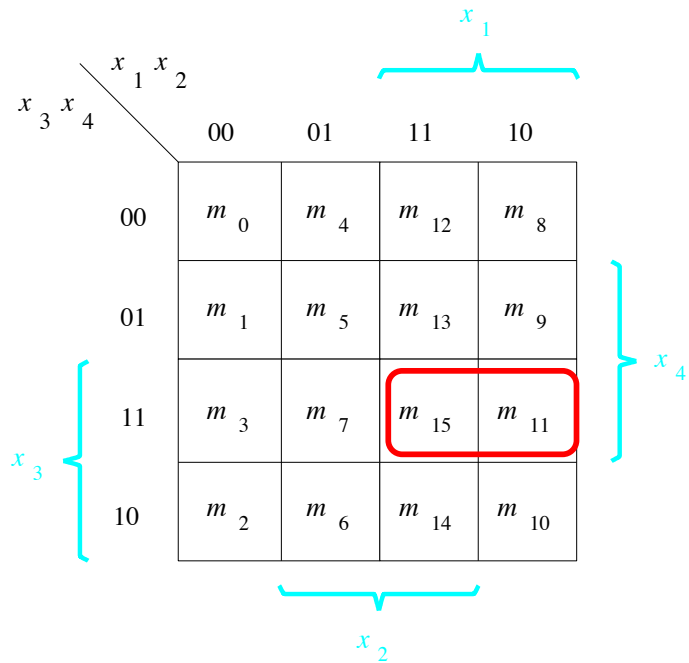
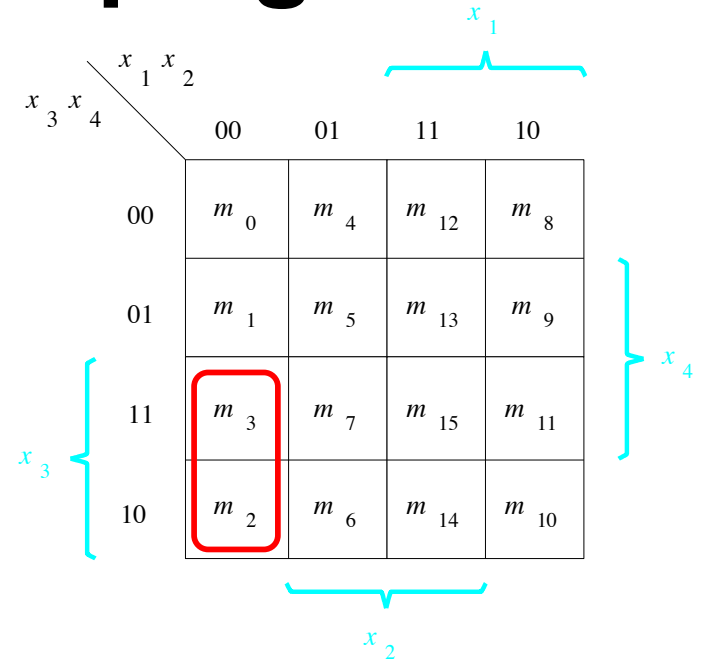
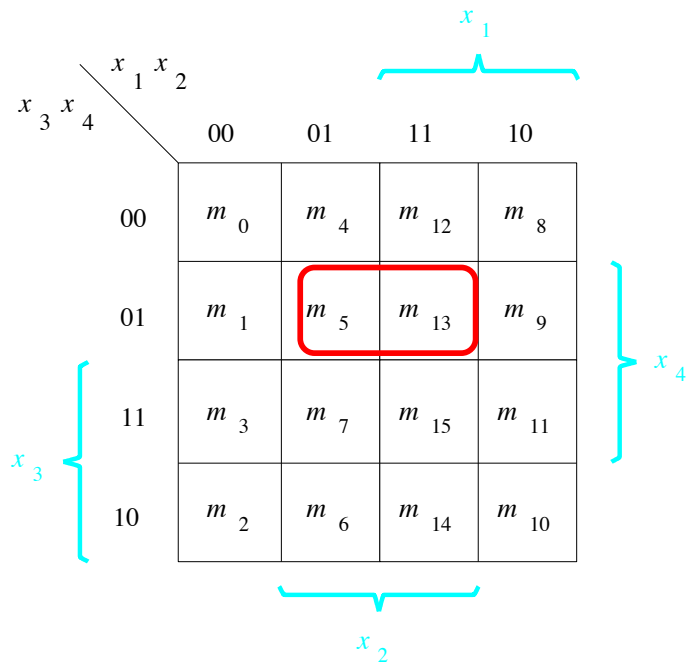


As if the K-map were  
drawn on a torus

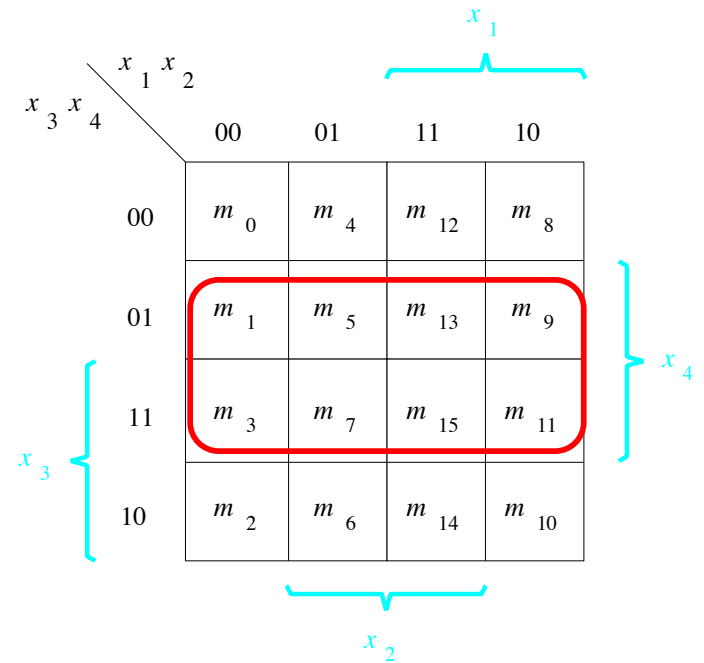
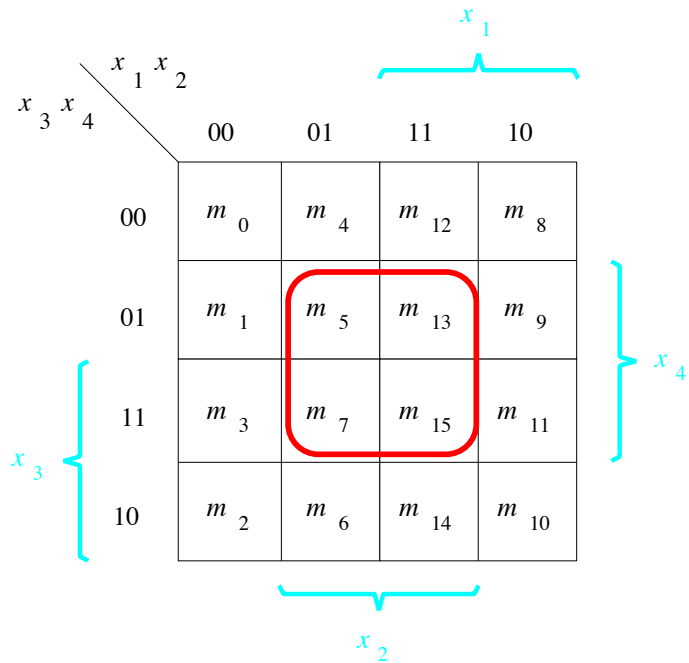
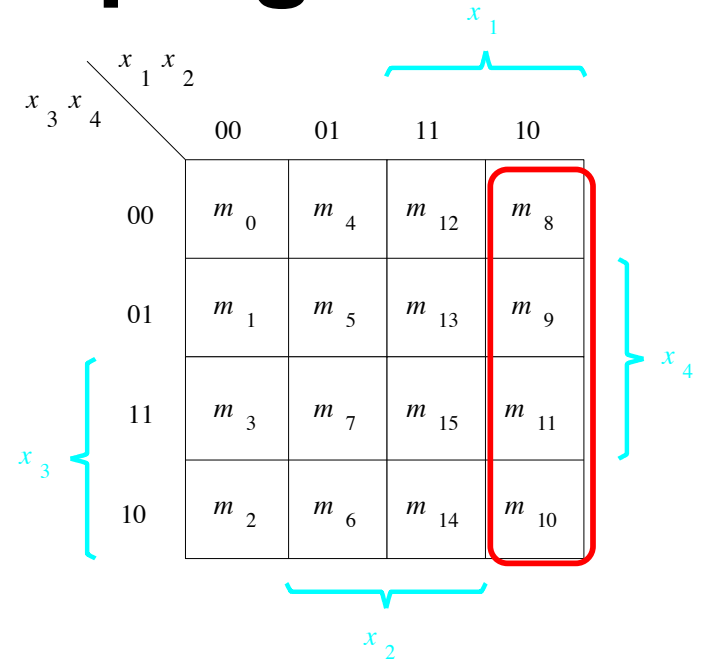
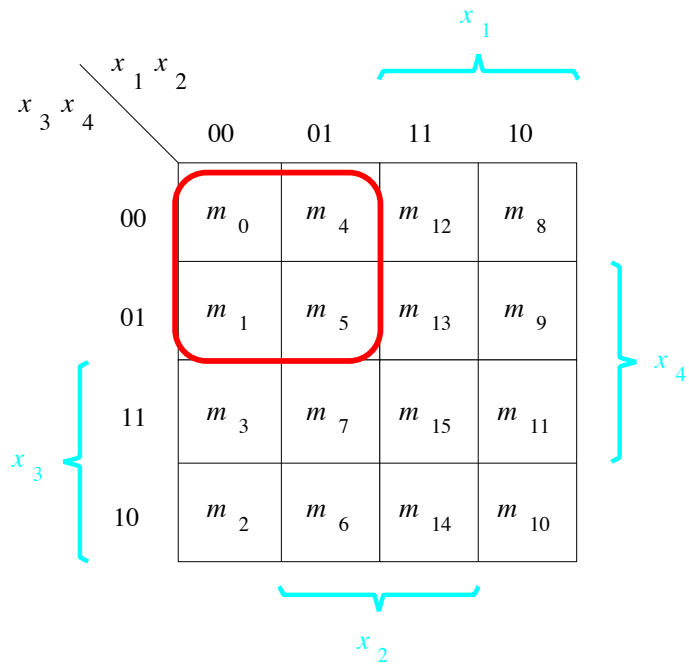
# Some Valid Groupings



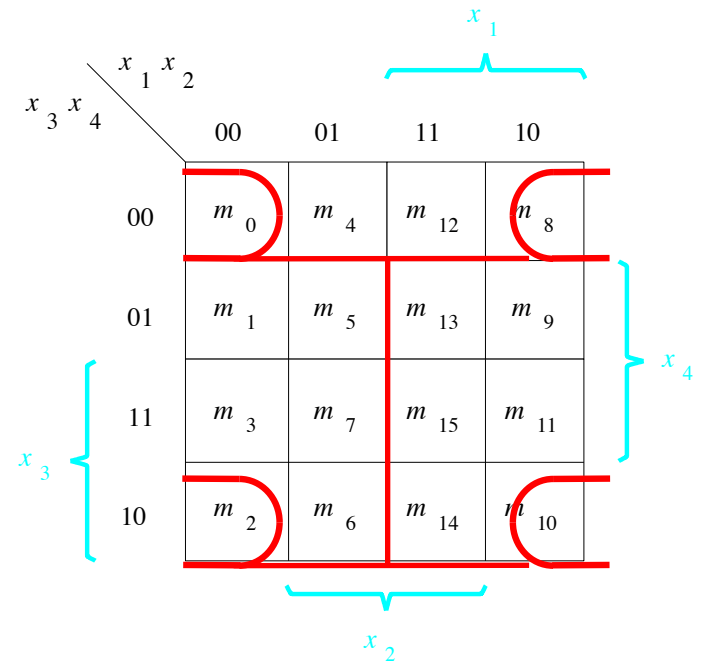
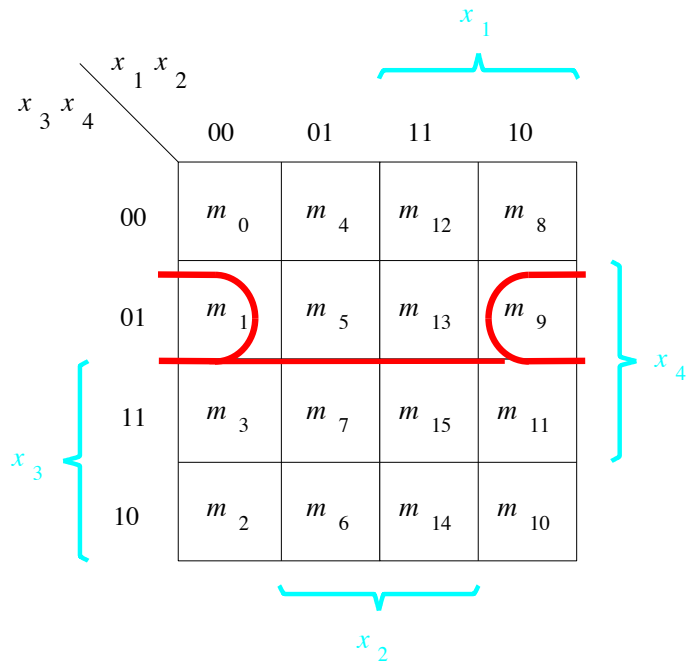
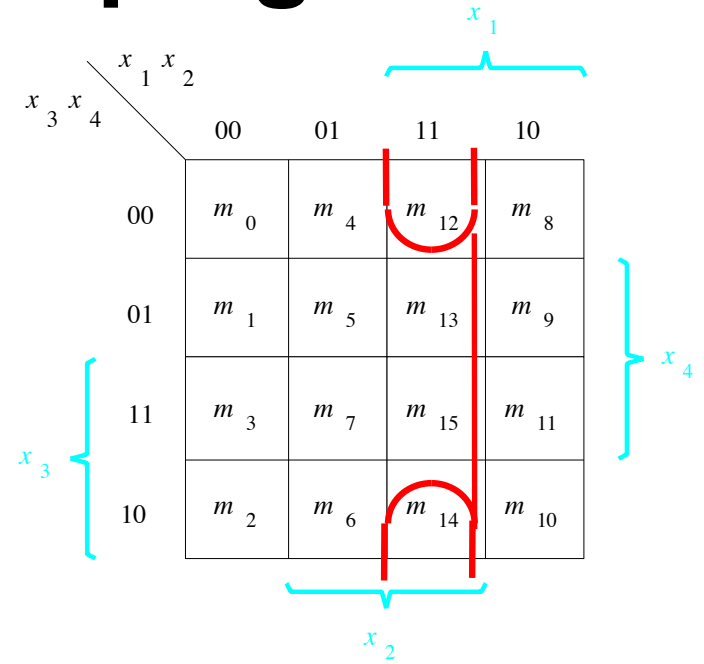
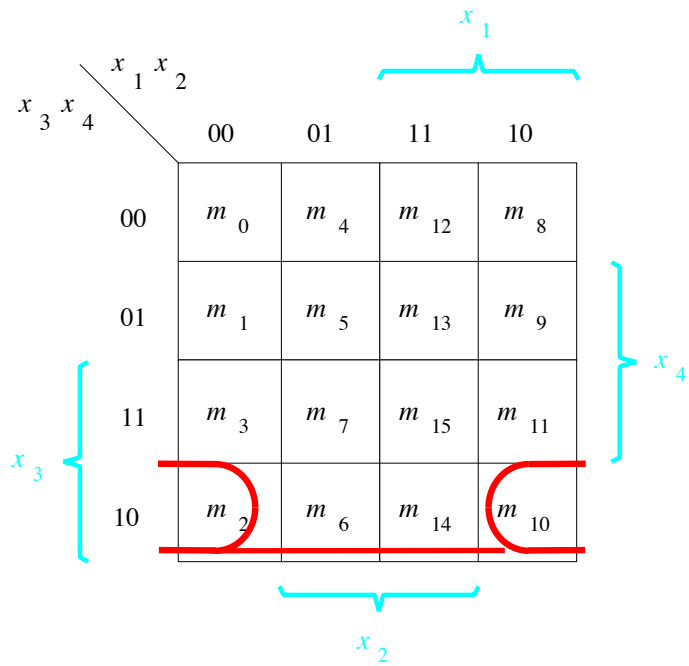
# Some Valid Groupings



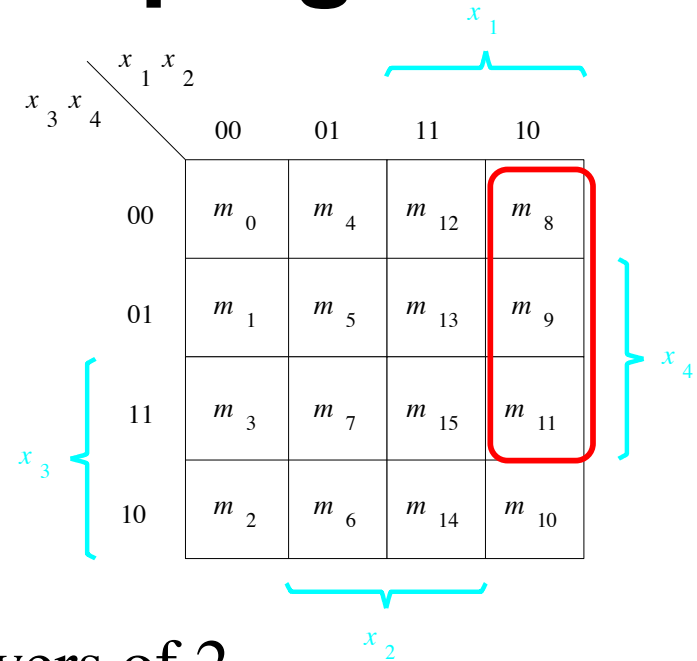
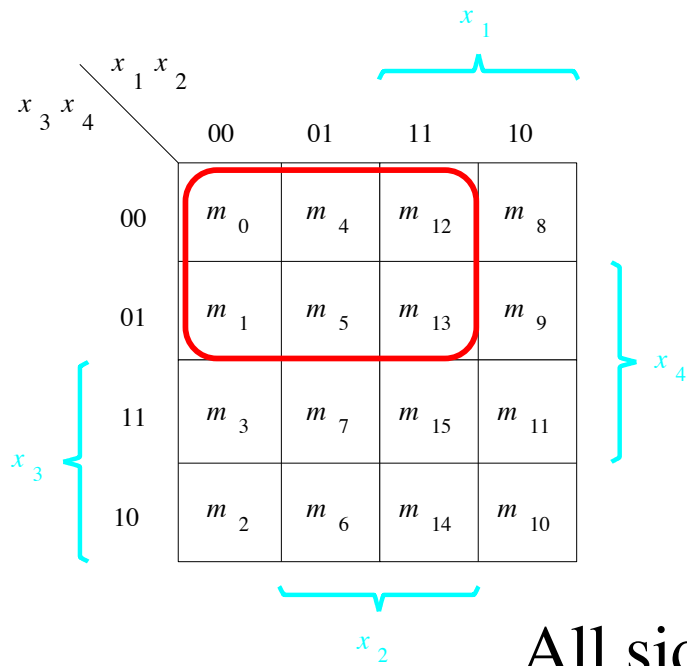
# Some Valid Groupings



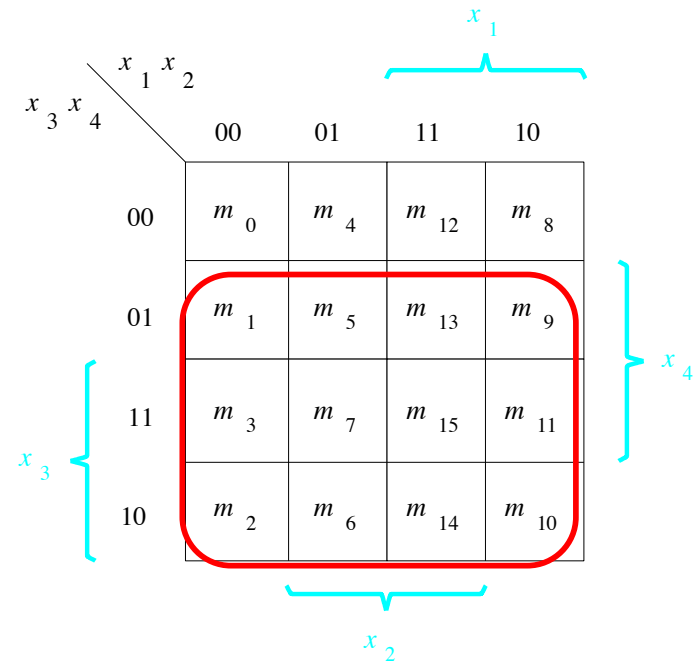
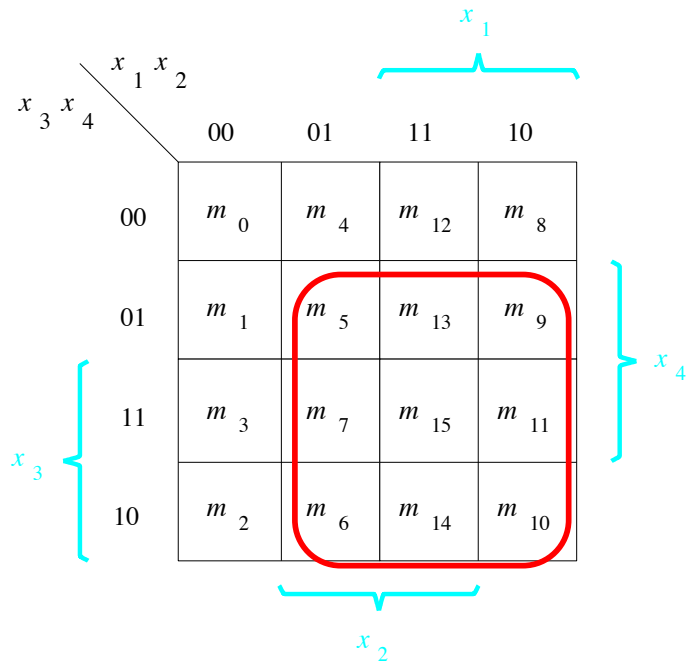
# Some Valid Groupings



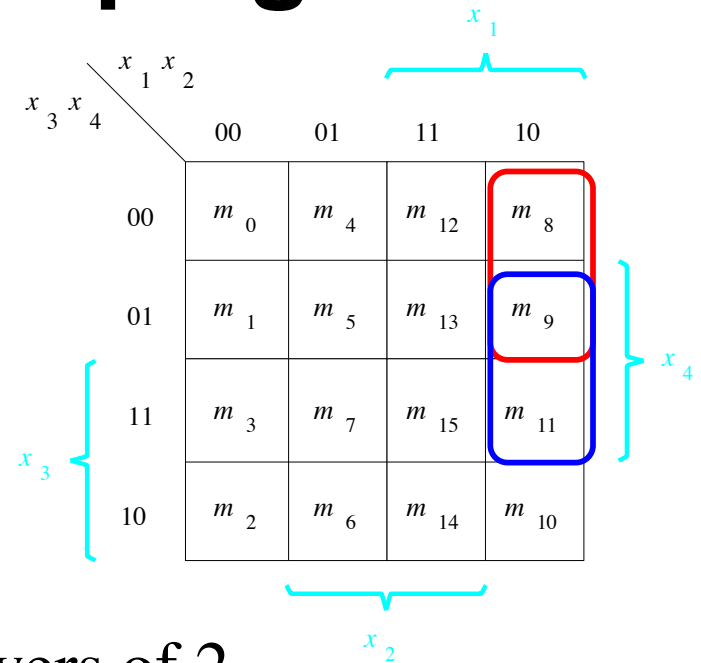
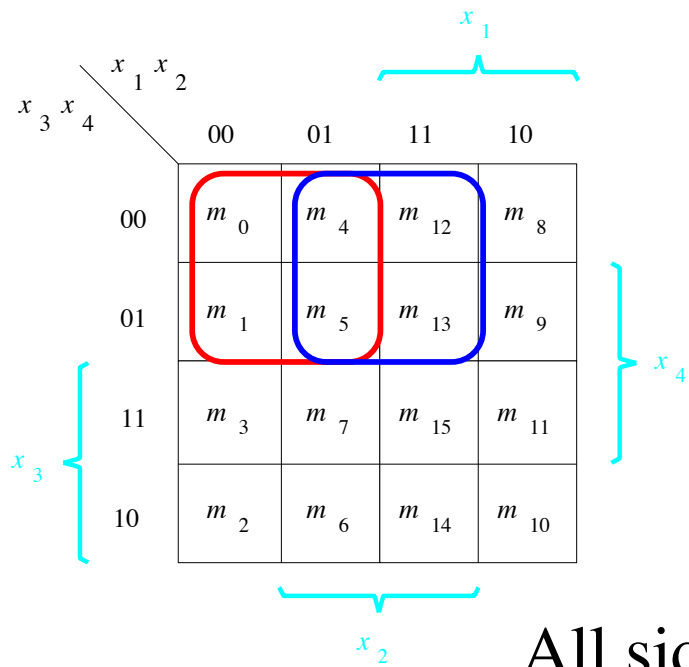
# Some Invalid Groupings



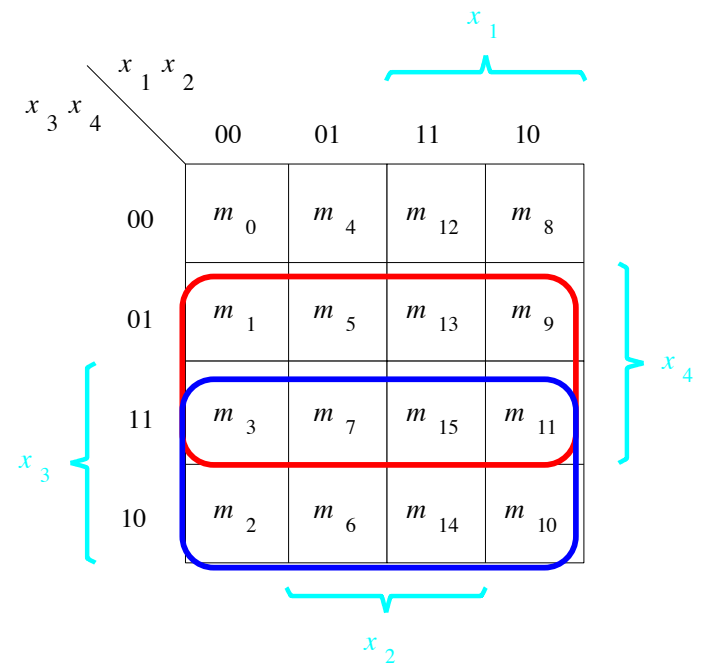
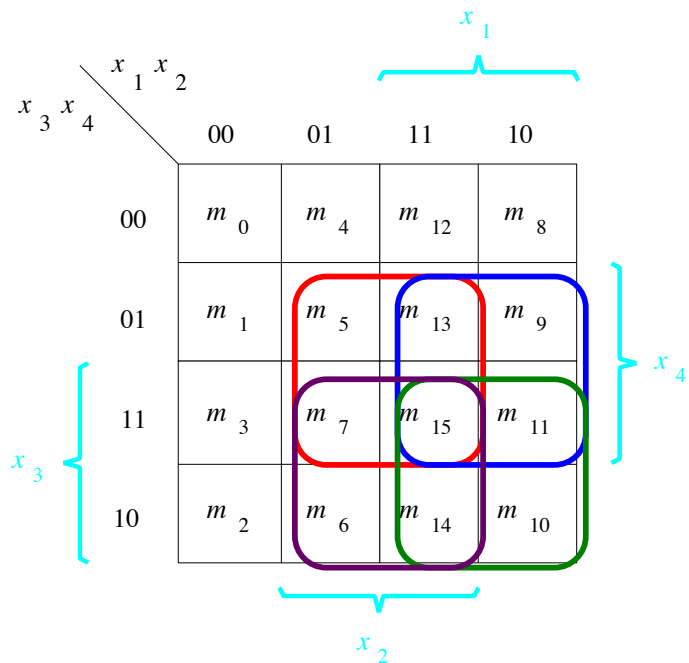
All sides must be powers of 2.



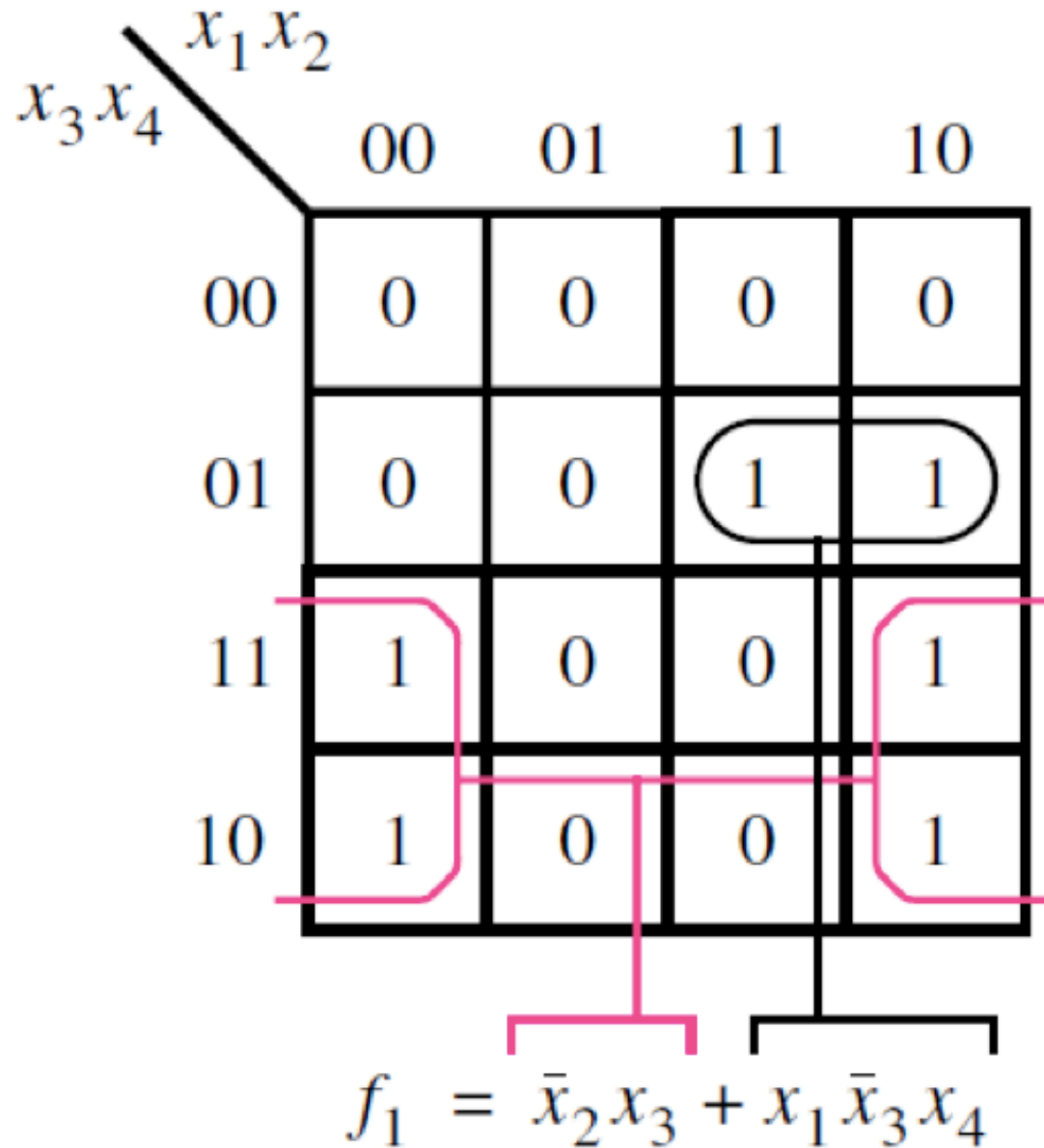
# Some **valid** Groupings



All sides must be powers of 2.

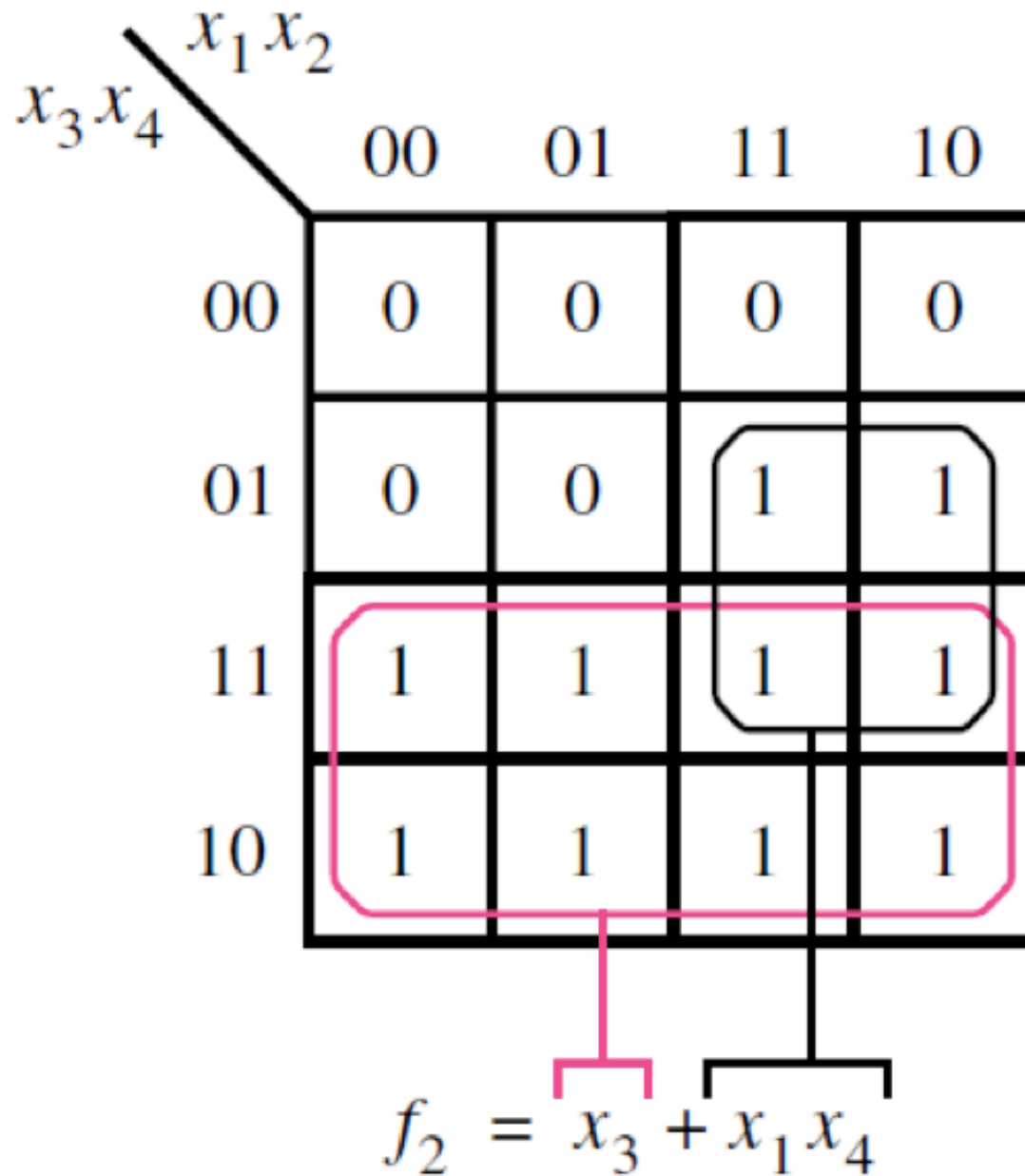


# Example of a four-variable Karnaugh map

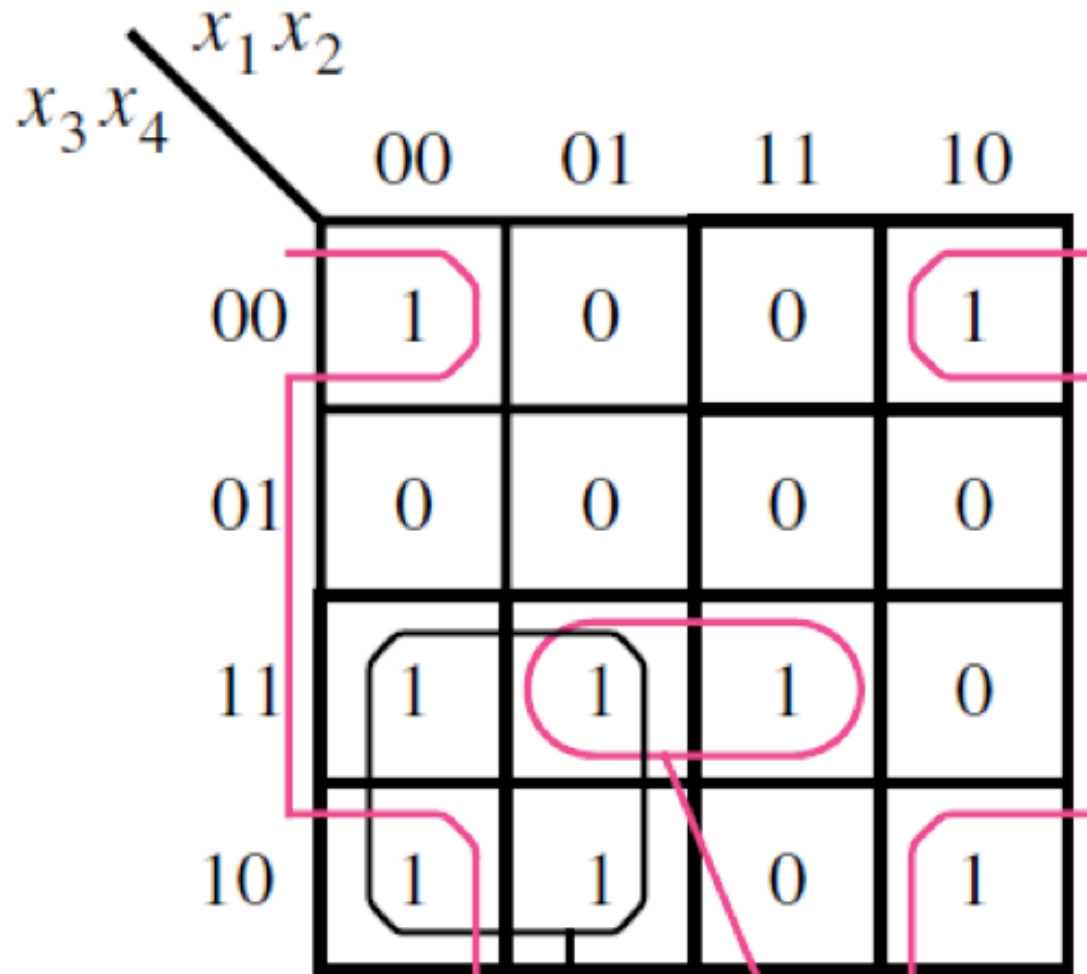




# Example of a four-variable Karnaugh map

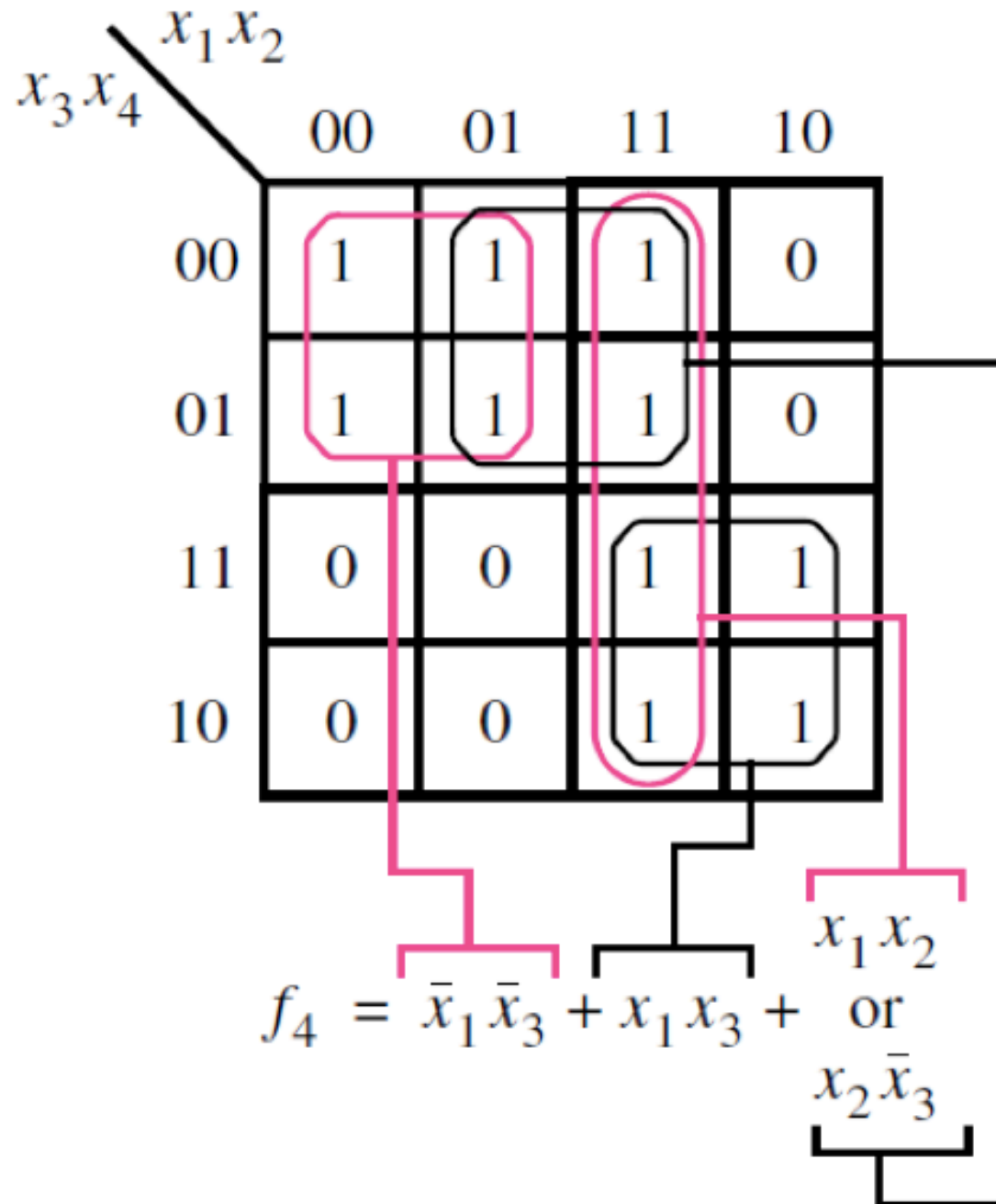


# Example of a four-variable Karnaugh map



$$f_3 = \bar{x}_2\bar{x}_4 + \bar{x}_1x_3 + x_2x_3x_4$$

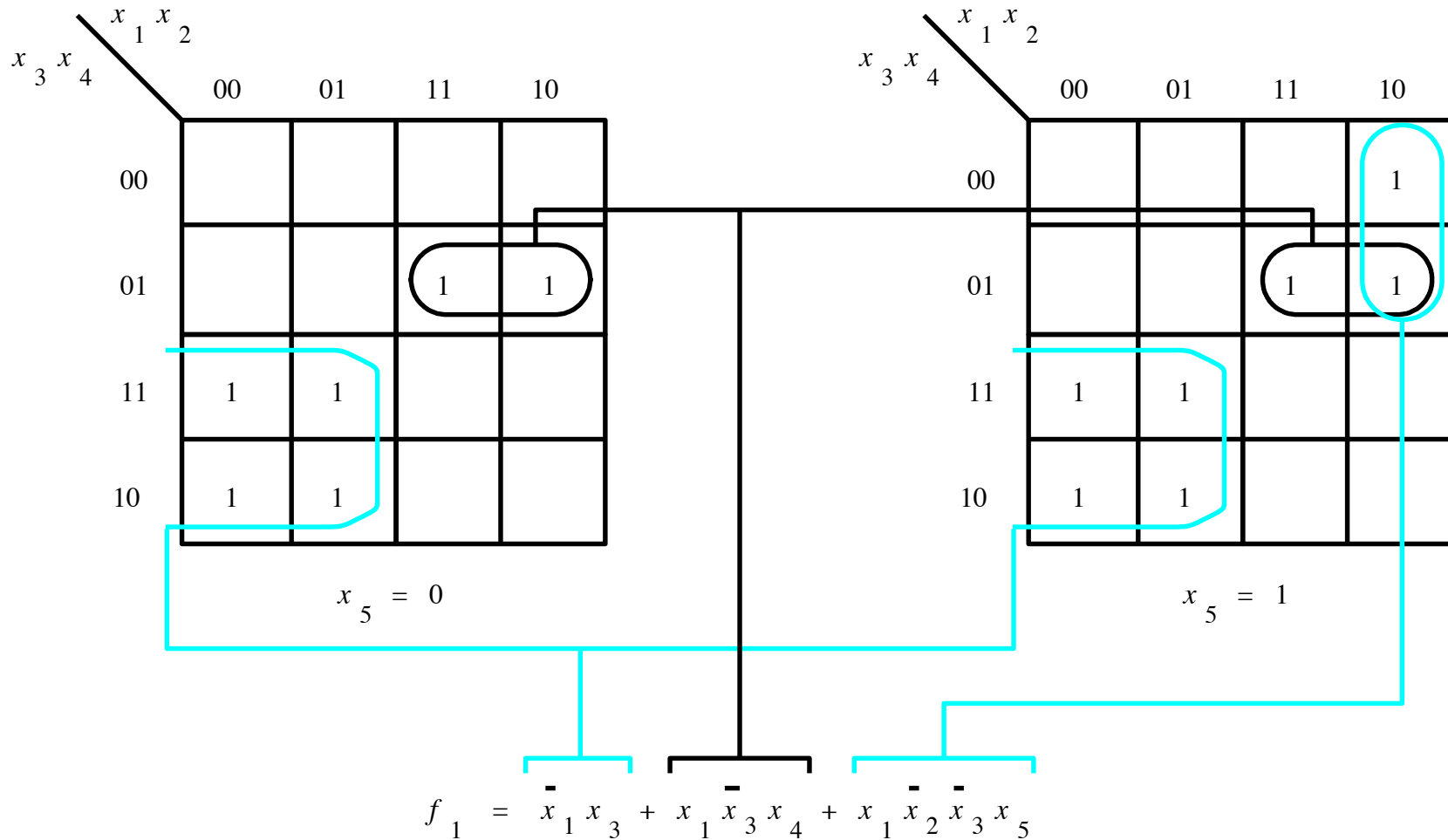
# Example of a four-variable Karnaugh map



[ Figure 2.54 from the textbook ]

# Five-Variable K-Map

# A five-variable Karnaugh map



[ Figure 2.55 from the textbook ]

**Questions?**

**THE END**