

CprE 281: Digital Logic

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http://www.ece.iastate.edu/~alexs/classes/

Counters & Solved Problems

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Administrative Stuff

- Homework 9 is out
- It is due on Monday Nov 5

Counters

T Flip-Flop (circuit and graphical symbol)



The output of the T Flip-Flop divides the frequency of the clock by 2



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The first flip-flop changes on the positive edge of the clock



The first flip-flop changes on the positive edge of the clock The second flip-flop changes on the positive edge of Q_0



The first flip-flop changes on the positive edge of the clock The second flip-flop changes on the positive edge of Q_0

The third flip-flop changes on the positive edge of Q_1

















The first flip-flop changes on the positive edge of the clock



The first flip-flop changes on the positive edge of the clock The second flip-flop changes on the positive edge of \overline{Q}_0



The first flip-flop changes on the positive edge of the clock The second flip-flop changes on the positive edge of \overline{Q}_0 The third flip-flop changes on the positive edge of \overline{Q}_1





(b) Timing diagram















Synchronous Counters

A four-bit synchronous up-counter



A four-bit synchronous up-counter



The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

A four-bit synchronous up-counter



(a) Circuit



(b) Timing diagram

Derivation of the synchronous up-counter



[Table 5.1 from the textbook]

Derivation of the synchronous up-counter



$$T_0 = 1$$

 $T_1 = Q_0$
 $T_2 = Q_0 Q_1$
A four-bit synchronous up-counter



$$T_0 = 1$$

 $T_1 = Q_0$
 $T_2 = Q_0 Q_1$

In general we have

$$T_{0} = 1$$

$$T_{1} = Q_{0}$$

$$T_{2} = Q_{0} Q_{1}$$

$$T_{3} = Q_{0} Q_{1} Q_{2}$$

...

$$T_{n} = Q_{0} Q_{1} Q_{2} \dots Q_{n-1}$$

Synchronous v.s. Asynchronous Clear

2-Bit Synchronous Up-Counter (without clear capability)





2-Bit Synchronous Up-Counter (with asynchronous clear)



This is the same circuit but uses D Flip-Flops.

2-Bit Synchronous Up-Counter (with synchronous clear)



This counter can be cleared only on the positive clock edge.

Adding Enable Capability

A four-bit synchronous up-counter



[Figure 5.21 from the textbook]

Inclusion of Enable and Clear Capability



Inclusion of Enable and Clear Capability



Providing an enable input for a D flip-flop



(a) Using a multiplexer

 $\begin{array}{c|c} D & Q \\ \hline & D \\ \hline & Q \\ \hline & Clock \end{array} \xrightarrow{} \overline{Q} \end{array} \xrightarrow{} \overline{Q}$

(b) Clock gating

[Figure 5.56 from the textbook]

Synchronous Counter (with D Flip-Flops)

A 4-bit up-counter with D flip-flops



[[]Figure 5.23 from the textbook]

A 4-bit up-counter with D flip-flops



[[]Figure 5.23 from the textbook]

Equivalent to this circuit with T flip-flops



Equivalent to this circuit with T flip-flops



But has one extra output called Z, which can be used to connect two 4-bit counters to make an 8-bit counter.

When Z=1 the counter will go 0000 on the next clock edge, i.e., the outputs of all flip-flops are currently 1 (maximum count value).

Counters with Parallel Load

A counter with parallel-load capability



[Figure 5.24 from the textbook]

How to load the initial count value



Set the initial count on the parallel load lines (in this case 5).

How to zero a counter



How to zero a counter



Reset Synchronization

Motivation

- An n-bit counter counts from 0, 1, ..., 2ⁿ-1
- For example a 3-bit counter counts up as follow
 - 0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, ...

- What if we want it to count like this
 - **•** 0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0, 1, ...
- In other words, what is the cycle is not a power of 2?

What does this circuit do?



[Figure 5.25a from the textbook]

A modulo-6 counter with synchronous reset



(a) Circuit



(b) Timing diagram

[Figure 5.25 from the textbook]

A modulo-6 counter with asynchronous reset



[Figure 5.26 from the textbook]

A modulo-6 counter with asynchronous reset



Other Types of Counters (Section 5.11)

- 2: Parallel-load four-bit counter
 - Figure 5.24
- Each counts in binary
 - **0-9**
- Resets generated on 9
 - Reset by loading 0's
- Second digit enabled by a 9 on first counter





It is a counter with parallel-load capability



[Figure 5.24 from the textbook]

It is a counter with parallel-load capability






Zeroing the BCD counter



Zeroing the BCD counter



Setting "Clear" to 1, zeroes both counters.

How to zero a counter



Set all parallel load input lines to zero.

[Figure 5.24 from the textbook]

How to zero a counter



How to zero a counter



[Figure 5.24 from the textbook]

When the positive edge of the clock arrives, all outputs Loa are set to zero together.

When Clear = 0

























It is enabled only when the first counter is at 9.



At the same time the first counter is reset.



At the same time the first counter is reset.



























Ring Counter

How to build a 4-bit ring counter



To build a ring counter we start with a shift register.

How to build a 4-bit ring counter



Next, add a loop from the last flip-flop to the first...

How to build a 4-bit ring counter



... and remove the In input line.
How to build a 4-bit ring counter



Also, add a start input that goes inverted to preset_n of the first flip=flop and to clear_n of all remaining flip-flops.

How to build a 4-bit ring counter



Finally, extend the output lines that form the count number.

How to build a 4-bit ring counter



This is the final circuit diagram.

4-bit ring counter

- There is only one 1 on the outputs of the four flip-flops
- The counting sequence is: 1000, 0100, 0010, 0001, 1000, ...
- To reset the counter
 - set start to 1 for a short period of time
 - This sets the four outputs to 1000



To initialize the counter set Start to 1.



After the NOT gate, this 1 goes as 0 to preset_n of the first flip-flop and to clear_n of all remaining flip-flops.



This sets the output pattern to 1000, i.e., only the first bit is one and the rest are zeros.



Setting Start to 0 has no effect on the outputs, because both preset_n and clear_n are sensitive only to 0.



The initialization does not depend on the clock since both preset_n and clear_n bypass the gates of the latches in the flip-flops.



That last 0 loops back to the D input of the first flip-flop.



















It is back to the start of the counting sequence, which is: 1000, 0100, 0010, 0001.

n-bit ring counter



Ring Counter (alternative implementation)

Alternative version of a 4-bit ring counter

- This implementation uses 2-bit up-counter followed by a 2-to-4 decoder.
- The counter cycles through 00, 01, 10, 11, 00, ...
- Recall that the outputs of the decoder are one-hot encoded. Thus, there is only one 1 on its outputs.
- Because the output of the counter is the input to the decoder, the outputs of the decoder cycle through: 1000, 0100, 0010, 0001, 1000, ...
- This is the counting sequence for a ring counter.

Alternative version of a 4-bit ring counter



[Figure 5.28b from the textbook]

What are the components?



2-Bit Synchronous Up-Counter



2-Bit Synchronous Up-Counter



2-to-4 Decoder with Enable Input



[Figure 5.28b from the textbook]

2-to-4 Decoder with Enable Input



[Figure 4.14c from the textbook]

2-to-4 Decoder with Enable Input



(always enabled in this example)



[Figure 5.28b from the textbook]



To initialize this circuit set Start to 1, which sets the counter to 00.



This sets the outputs of the decoder to the start of the counting sequence.



When Start is equal to 0, clear_n has no effect.





The counter increments the count on the positive clock edge ...



... this updates the outputs of the decoder (which are one hot encoded).












It is back to the start of the counting sequence, which is: 1000, 0100, 0010, 0001.

Johnson Counter



To build a Johnson counter start with a shift register.



Next, add a loop from the \overline{Q} output of the last flip-flop to the first...



... and remove the In input line.



Also, add a Reset_n line that goes to clear_n of all flip-flops.



Finally, extend the output lines that form the count number.



This is the final circuit diagram.

4-Bit Johnson Counter

- Only 1-bit changes at a time
- Start with a reset of all flip-flops
- The counting sequence is: 0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001, 0000
- An n-bit Johnson counter has a counting sequence of length 2n





To initialize the Johnson counter set Reset_n to 0 ...



... this zeros the outputs of all flip-flops ...



... and also sets the Q output of the last flip-flop to 1.





To start counting, Reset_n needs to be set to 1.

























time -








Counting: How does it work?



It is back to the start of the counting sequence, which is: 0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001.

n-bit Johnson Counter



[Figure 5.29 from the textbook]

Timing Analysis of Flip-Flop Circuits (Section 5.15)

Timing Review

- t_{su}: setup time
- t_h: hold time
- t_{cQ} : propagation delay

Timing Example

- t_{su}: 0.6ns
- t_h: 0.4ns
- t_{cQ}: 0.8ns to 1.0ns
 - Which value to use?
- Logic gate delay: 1+0.1k
 - k is equal to the number of inputs
- $T_{min} = t_{su} + t_{cQ} + t_{not} = 0.6 + 1.0 + 1.1 = 2.7 ns$
- $F_{max} = 1/T_{min} = 370.37 \text{ MHz}$
- Check for hold violations
 - Fastest Q can change = $t_{cQ} + t_{not} = 0.8 + 1.1 = 1.9$ ns
 - 1.9ns > 0.4ns therefore no hold violations



Timing Example: 4-bit counter



Timing Example: 4-bit counter

- Look for longest path
 - Q0 to Q3
- Propagation delay of Q0
- 3 AND propagation delays
- 1 XOR propagation delay
- Setup delay for Q3
- $T_{min} = 1.0 + 3(1.2) + 1.2 + 0.6 = 6.4$ ns
- F_{max} = 1/6.4ns = 156.25MHz
- Check for hold violations
 - Fastest Q can change = $t_{cQ} + t_{XOR} = 0.8 + 1.2 = 2ns$
 - 2.0ns > 0.4ns therefore no hold violations

Timing Example: Clock Skew



Figure 5.68. A general example of clock skew.

Skew Timing Example: 4-bit counter

- Q3 now has a clock slew delay: 1.5ns
 - T = 1.0 + 3(1.2) + 1.2 + 0.6 1.5 = 4.9ns
- Now might not be the longest path
- Check Q0 to Q2
 - T = 1.0 + 2(1.2) + 1.2 + 0.6 = 5.2ns
- F_{max} = 1/5.2ns = 192.31 MHz

Example 5.22

Faster 4-bit Counter

- Want to increase the speed of the 4-bit counter
- Use a similar method as the one used in 4-bit adder
- Replace the series of 2-input AND gates with AND gates with more input lines.

A faster 4-bit counter



[Figure 5.75 from the textbook]

Faster 4-bit Counter

• Longest path: Q0 to Q3

•
$$T_{min} = t_{cQ0} + t_{AND} + t_{XOR} + t_{su}$$

= 1.0 + 1.4 + 1.2 + 0.6 = 4.2ns

Reaction Timer Circuit (Section 5.14)

Problem Statement

- Want to design a reaction timer
- The circuit turns on a light (LED)
- A person then presses a switch
- Measure the elapsed time from when the LED is turned on until the switch is pressed

Clock Divider

- Input: 102.4kHz
- Output: 100Hz
- 10-bit Counter to divide
- Output Frequency = 102.4k / 2¹⁰ = 100Hz

A reaction-timer circuit



[Figure 5.61 from the textbook]

Functionality of circuit

- Push switch
 - Nominally 1
- DFF to keep track of the state
- Two-digit BCD counter
 - Output goes to converters to a 7-segment display
- Start-up
 - Assert the Reset signal
 - Clears counter
 - Clears flip-flop
 - Assert w=1 for one cycle
 - Once switch is hit
 - Clears flip-flop
 - Stops counting

Push-button switch, LED, and 7-segment displays



[Figure 5.61c from the textbook]

Examples of Solved Problems (Section 5.17)

Example 5.18

Circuit for Example 5.18



[Figure 5.70 from the textbook]

Example 5.19

Circuit for Example 5.19



	Q(t+1)	К	J
No Change	Q(t)	0	0
Reset	0	1 0	0
Set	1		1
Complemen	Q'(t)	1	1

Summary of the behavior of the circuit

Time interval		FF0			FF1	-
	J_0	K_0	Q_0	J_1	K_1	Q_1
Clear	1	1	0	0	1	0
t_1	1	1	1	1	1	0
t_2	0	1	0	0	1	1
t_3	1	1	0	0	1	0
t_4	1	1	1	1	1	0

Example 5.20

Vending Machine Example

- Inputs N, D, Q, Coin, Resetn
 - N, D, Q: nickel, dime, quarter
 - Coin: pulsed when a coin is entered
 - Used to store values into register
 - Resetn: resets the register value to zero
- Add up new coin with old value
 - Store new sum into old value register
- See if total is above thirty cents
 - If so output Z goes high

Circuit for Example 5.20



Questions?

THE END