

## CprE 281: Digital Logic

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http://www.ece.iastate.edu/~alexs/classes/

# Counters \& Solved Problems 

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## Administrative Stuff

- Homework 9 is out
- It is due on Monday Nov 5


## Counters

## T Flip-Flop (circuit and graphical symbol)


[ Figure 5.15a,c from the textbook ]

## The output of the T Flip-Flop divides the frequency of the clock by 2



## The output of the T Flip-Flop divides the frequency of the clock by 2



## A three-bit down-counter


[ Figure 5.20 from the textbook]

## A three-bit down-counter



The first flip-flop changes
on the positive edge of the clock

## A three-bit down-counter



The first flip-flop changes on the positive edge of the clock

The second flip-flop changes on the positive edge of $\mathrm{Q}_{0}$

## A three-bit down-counter



The first flip-flop changes on the positive edge of the clock

The second flip-flop changes on the positive edge of $\mathrm{Q}_{0}$

The third flip-flop changes on the positive edge of $\mathrm{Q}_{1}$

## A three-bit down-counter


(a) Circuit

(b) Timing diagram
[ Figure 5.20 from the textbook]

## A three-bit down-counter


(a) Circuit

(b) Timing diagram

## A three-bit down-counter


(a) Circuit

(b) Timing diagram

## A three-bit down-counter



## A three-bit down-counter


(a) Circuit

(b) Timing diagram

## A three-bit down-counter



## A three-bit up-counter


[ Figure 5.19 from the textbook]

## A three-bit up-counter



The first flip-flop changes
on the positive edge of the clock

## A three-bit up-counter



The first flip-flop changes on the positive edge of the clock

The second flip-flop changes
on the positive edge of $\overline{\mathrm{Q}}_{0}$

## A three-bit up-counter



The first flip-flop changes on the positive edge of the clock

The second flip-flop changes The third flip-flop changes on the positive edge of $\overline{\mathrm{Q}}_{0}$ on the positive edge of $\overline{\mathrm{Q}}_{1}$

## A three-bit up-counter


(a) Circuit

[ Figure 5.19 from the textbook ]

## A three-bit up-counter


[ Figure 5.19 from the textbook ]

## A three-bit up-counter


(a) Circuit

(b) Timing diagram
[ Figure 5.19 from the textbook ]

## A three-bit up-counter


(a) Circuit

(b) Timing diagram

## A three-bit up-counter


(a) Circuit

(b) Timing diagram

## A three-bit up-counter


(a) Circuit

(b) Timing diagram

## A three-bit up-counter


(a) Circuit

(b) Timing diagram

## A three-bit up-counter


(b) Timing diagram

## A three-bit up-counter


[ Figure 5.19 from the textbook ]

## Synchronous Counters

## A four-bit synchronous up-counter



## A four-bit synchronous up-counter



The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

## A four-bit synchronous up-counter


(a) Circuit

(b) Timing diagram
[ Figure 5.21 from the textbook ]

## Derivation of the synchronous up-counter

| Clock cycle | $\mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}$ |  |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 |  |  |  |
| 4 | 0 | 1 | 1 |
| 5 | 1 | 0 | 0 |
| 6 | 1 | 0 | 1 |
| 7 | 1 | 1 | 0 |
| 8 | 1 | 1 | 1 |

## Derivation of the synchronous up-counter

| Clock cycle | $\mathrm{Q}_{2} \mathrm{Q}_{1} \mathrm{Q}_{0}$ |  |  |
| :---: | :--- | :--- | :--- |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |
| 8 | 0 | 0 | 0 |$] \quad$| $\mathrm{Q}_{1}$ changes |
| :--- |

$$
\begin{aligned}
& \mathrm{T}_{0}=1 \\
& \mathrm{~T}_{1}=\mathrm{Q}_{0} \\
& \mathrm{~T}_{2}=\mathrm{Q}_{0} \mathrm{Q}_{1}
\end{aligned}
$$

## A four-bit synchronous up-counter



$$
\begin{aligned}
& \mathrm{T}_{0}=1 \\
& \mathrm{~T}_{1}=\mathrm{Q}_{0} \\
& \mathrm{~T}_{2}=\mathrm{Q}_{0} \mathrm{Q}_{1}
\end{aligned}
$$

[ Figure 5.21 from the textbook]

## In general we have

$$
\begin{aligned}
& \mathrm{T}_{0}=1 \\
& \mathrm{~T}_{1}=\mathrm{Q}_{0} \\
& \mathrm{~T}_{2}=\mathrm{Q}_{0} \mathrm{Q}_{1} \\
& \mathrm{~T}_{3}=\mathrm{Q}_{0} \mathrm{Q}_{1} \mathrm{Q}_{2} \\
& \ldots \\
& \mathrm{~T}_{\mathrm{n}}=\mathrm{Q}_{0} \mathrm{Q}_{1} \mathrm{Q}_{2} \ldots \mathrm{Q}_{\mathrm{n}-1}
\end{aligned}
$$

## Synchronous v.s. Asynchronous Clear

## 2-Bit Synchronous Up-Counter (without clear capability)



## 2-Bit Synchronous Up-Counter (with asynchronous clear)



## 2-Bit Synchronous Up-Counter (with asynchronous clear)



This is the same circuit but uses D Flip-Flops.

## 2-Bit Synchronous Up-Counter (with synchronous clear)



This counter can be cleared only on the positive clock edge.

## Adding Enable Capability

## A four-bit synchronous up-counter


[ Figure 5.21 from the textbook]

## Inclusion of Enable and Clear Capability


[ Figure 5.22 from the textbook]

## Inclusion of Enable and Clear Capability


[ Figure 5.22 from the textbook]

## Providing an enable input for a D flip-flop


(a) Using a multiplexer

(b) Clock gating

## Synchronous Counter (with D Flip-Flops)

## A 4-bit up-counter with D flip-flops


[ Figure 5.23 from the textbook]

## A 4-bit up-counter with D flip-flops


[ Figure 5.23 from the textbook]

## Equivalent to this circuit with T flip-flops



## Equivalent to this circuit with T flip-flops



But has one extra output called Z , which can be used to connect two 4 -bit counters to make an 8 -bit counter.

When $\mathrm{Z}=1$ the counter will go 0000 on the next clock edge, i.e., the outputs of all flip-flops are currently 1 (maximum count value).

## Counters with Parallel Load

## A counter with parallel-load capability


[ Figure 5.24 from the textbook ]

## How to load the initial count value

Set the initial count on the parallel load lines
(in this case 5).


## How to zero a counter

Set "Load" to 1, to open the


## How to zero a counter



## Reset Synchronization

## Motivation

- An n-bit counter counts from $0,1, \ldots, \mathbf{2}^{\mathrm{n}}-\mathbf{1}$
- For example a 3-bit counter counts up as follow
- $0,1,2,3,4,5,6,7,0,1,2, \ldots$
- What if we want it to count like this
- 0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0, 1, ...
- In other words, what is the cycle is not a power of 2?


## What does this circuit do?


[ Figure 5.25a from the textbook ]

## A modulo-6 counter with synchronous reset


(a) Circuit

(b) Timing diagram

## A modulo-6 counter with asynchronous reset


(a) Circuit

(b) Timing diagram

## A modulo-6 counter with asynchronous reset


(b) Timing diagram
[ Figure 5.26 from the textbook ]

## Other Types of Counters (Section 5.11)

## A two-digit BCD counter

- 2: Parallel-load four-bit counter
- Figure 5.24
- Each counts in binary
- 0-9
- Resets generated on 9
- Reset by loading 0’s
- Second digit enabled by a 9 on first counter


## A two-digit BCD counter


[ Figure 5.27 from the textbook ]

## A two-digit BCD counter


[ Figure 5.27 from the textbook ]

## It is a counter with parallel-load capability


[ Figure 5.24 from the textbook ]

## It is a counter with parallel-load capability



## A two-digit BCD counter



## A two-digit BCD counter



## Zeroing the BCD counter


[ Figure 5.27 from the textbook ]

## Zeroing the BCD counter


[ Figure 5.27 from the textbook ]

## How to zero a counter

Set all parallel load input lines to zero.

[ Figure 5.24 from the textbook ]

## How to zero a counter

Set "Load" to 1, to open the "1" line of the multiplexers. Load

## How to zero a counter

When the positive edge of the clock arrives, all outputs are set to zero together.

[ Figure 5.24 from the textbook ]

## When Clear = 0

 on the feedback connections.

## Enabling the second counter


[ Figure 5.27 from the textbook ]

## Enabling the second counter



0

## Enabling the second counter

 most of the time.

## Enabling the second counter

 most of the time.

## Enabling the second counter



2

## Enabling the second counter

 most of the time.

## Enabling the second counter

 most of the time.

## Enabling the second counter



6

## Enabling the second counter

 most of the time.

## Enabling the second counter



0

## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



-     -         - 


## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



-     -         - 


## Enabling the second counter



## Enabling the second counter



9

## Enabling the second counter



## Enabling the second counter



## Enabling the second counter



## Ring Counter

## How to build a 4-bit ring counter



To build a ring counter we start with a shift register.

## How to build a 4-bit ring counter



Next, add a loop from the last flip-flop to the first...

## How to build a 4-bit ring counter


... and remove the In input line.

## How to build a 4-bit ring counter



Also, add a start input that goes inverted to preset_n of the first flip=flop and to clear_n of all remaining flip-flops.

## How to build a 4-bit ring counter



Finally, extend the output lines that form the count number.

## How to build a 4-bit ring counter



This is the final circuit diagram.

## 4-bit ring counter

- There is only one 1 on the outputs of the four flip-flops
- The counting sequence is: $\mathbf{1 0 0 0}, \mathbf{0 1 0 0}, \mathbf{0 0 1 0}, \mathbf{0 0 0 1}, \mathbf{1 0 0 0}, \ldots$
- To reset the counter
- set start to 1 for a short period of time
- This sets the four outputs to 1000


## 4-bit ring counter: How does it work



To initialize the counter set Start to 1 .

## 4-bit ring counter: How does it work



After the NOT gate, this 1 goes as 0 to preset_n of the first flip-flop and to clear_n of all remaining flip-flops.

## 4-bit ring counter: How does it work



This sets the output pattern to 1000 , i.e., only the first bit is one and the rest are zeros.

## 4-bit ring counter: How does it work



Setting Start to 0 has no effect on the outputs, because both preset_n and clear_n are sensitive only to 0 .

## 4-bit ring counter: How does it work



The initialization does not depend on the clock since both preset_n and clear_n bypass the gates of the latches in the flip-flops.

## 4-bit ring counter: How does it work



That last 0 loops back to the D input of the first flip-flop.

## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



## 4-bit ring counter: How does it work



It is back to the start of the counting sequence, which is: $1000,0100,0010,0001$.

## n-bit ring counter


[ Figure 5.28a from the textbook]

## Ring Counter (alternative implementation)

## Alternative version of a 4-bit ring counter

- This implementation uses 2-bit up-counter followed by a 2-to-4 decoder.
- The counter cycles through 00, 01, 10, 11, 00, ...
- Recall that the outputs of the decoder are one-hot encoded. Thus, there is only one 1 on its outputs.
- Because the output of the counter is the input to the decoder, the outputs of the decoder cycle through: 1000, 0100, 0010, 0001, 1000, ...
- This is the counting sequence for a ring counter.


## Alternative version of a 4-bit ring counter


[ Figure 5.28b from the textbook ]

## What are the components?



## 2-Bit Synchronous Up-Counter



## 2-Bit Synchronous Up-Counter



## 2-to-4 Decoder with Enable Input


[ Figure 5.28b from the textbook ]

## 2-to-4 Decoder with Enable Input


[ Figure 4.14c from the textbook]

## 2-to-4 Decoder with Enable Input


(always enabled in this example)
[ Figure 4.14c from the textbook ]

## How Does It Work?


[ Figure 5.28b from the textbook ]

## How Does It Work?



To initialize this circuit set Start to 1 , which sets the counter to 00 .

## How Does It Work?



This sets the outputs of the decoder to the start of the counting sequence.

## How Does It Work?



When Start is equal to 0, clear_n has no effect.

## How Does It Work?



## How Does It Work?



The counter increments the count on the positive clock edge ...

## How Does It Work?


... this updates the outputs of the decoder (which are one hot encoded).

## How Does It Work?



## How Does It Work?



## How Does It Work?



## How Does It Work?



## How Does It Work?



## How Does It Work?



It is back to the start of the counting sequence, which is: $1000,0100,0010,0001$.

## Johnson Counter

## How to build a 4-bit Johnson counter



To build a Johnson counter start with a shift register.

## How to build a 4-bit Johnson counter



Next, add a loop from the $\overline{\mathrm{Q}}$ output of the last flip-flop to the first...

## How to build a 4-bit Johnson counter


... and remove the In input line.

## How to build a 4-bit Johnson counter



Also, add a Reset_n line that goes to clear_n of all flip-flops.

## How to build a 4-bit Johnson counter



Finally, extend the output lines that form the count number.

## How to build a 4-bit Johnson counter



This is the final circuit diagram.

## 4-Bit Johnson Counter

- Only 1-bit changes at a time
- Start with a reset of all flip-flops
- The counting sequence is:

$$
0000,1000,1100,1110,1111,0111,0011,0001,0000
$$

- An n-bit Johnson counter has a counting sequence of length $2 n$


## Initialization: How does it work?



## Initialization: How does it work?



To initialize the Johnson counter set Reset_n to 0 ...

## Initialization: How does it work?


... this zeros the outputs of all flip-flops ...

## Initialization: How does it work?


... and also sets the $\overline{\mathrm{Q}}$ output of the last flip-flop to 1 .

## Counting: How does it work?



## Counting: How does it work?



To start counting, Reset_n needs to be set to 1 .

## Counting: How does it work?



## Counting: How does it work?



Clock


## Counting: How does it work?



## Counting: How does it work?



## Counting: How does it work?



Clock


## Counting: How does it work?



Clock


## Counting: How does it work?



Clock


## Counting: How does it work?



Clock


## Counting: How does it work?



Clock


Reset_n $\square$

## Counting: How does it work?



Clock

time $\longrightarrow$

## Counting: How does it work?



Clock


Reset_n $\square$

## Counting: How does it work?



Clock

time $\longrightarrow$

## Counting: How does it work?



Clock


## Counting: How does it work?



Clock


## Counting: How does it work?



Clock $\square$


## Counting: How does it work?



Clock $\square$


## Counting: How does it work?



It is back to the start of the counting sequence, which is: $0000,1000,1100,1110,1111,0111,0011,0001$.

## n-bit Johnson Counter


[ Figure 5.29 from the textbook]

# Timing Analysis of Flip-Flop Circuits (Section 5.15) 

## Timing Review

- $\mathrm{t}_{\text {su }}$ : setup time
- $t_{h}$ : hold time
- $\mathbf{t}_{\mathrm{cQ}}$ : propagation delay


## Timing Example

- $t_{\text {su }}: 0.6 \mathrm{~ns}$
- $t_{h}: 0.4 n s$
- $t_{c Q}$ : 0.8ns to 1.0 ns
- Which value to use?

- Logic gate delay: $1+0.1 \mathrm{k}$
- $k$ is equal to the number of inputs
- $\mathrm{T}_{\text {min }}=\mathrm{t}_{\mathrm{su}}+\mathrm{t}_{\mathrm{cQ}}+\mathrm{t}_{\mathrm{not}}=0.6+1.0+1.1=2.7 \mathrm{~ns}$
- $F_{\text {max }}=1 / \mathrm{T}_{\text {min }}=370.37 \mathrm{MHz}$
- Check for hold violations
- Fastest $Q$ can change $=t_{c Q}+t_{\text {not }}=0.8+1.1=1.9 \mathrm{~ns}$
- $1.9 \mathrm{~ns}>0.4 \mathrm{~ns}$ therefore no hold violations


## Timing Example: 4-bit counter


[ Figure 5.67 from the textbook ]

## Timing Example: 4-bit counter

- Look for longest path
- Q0 to Q3
- Propagation delay of Q0
- 3 AND propagation delays
- 1 XOR propagation delay
- Setup delay for Q3
- $\mathrm{T}_{\text {min }}=1.0+3(1.2)+1.2+0.6=6.4 \mathrm{~ns}$
- $F_{\text {max }}=1 / 6.4 \mathrm{~ns}=156.25 \mathrm{MHz}$
- Check for hold violations
- Fastest $Q$ can change $=t_{c Q}+t_{\text {XOR }}=0.8+1.2=2 n s$
- 2.0ns $\boldsymbol{>} \mathbf{0 . 4 n s}$ therefore no hold violations


## Timing Example: Clock Skew



Figure 5.68. A general example of clock skew.

## Skew Timing Example: 4-bit counter

- Q3 now has a clock slew delay: 1.5 ns
- $\mathrm{T}=1.0$ + 3(1.2) +1.2 + 0.6 - $1.5=4.9 \mathrm{~ns}$
- Now might not be the longest path
- Check Q0 to Q2
- $\mathrm{T}=1.0+2(1.2)+1.2+0.6=5.2 \mathrm{~ns}$
- $F_{\text {max }}=1 / 5.2 \mathrm{~ns}=192.31 \mathrm{MHz}$


## Example 5.22

## Faster 4-bit Counter

- Want to increase the speed of the 4-bit counter
- Use a similar method as the one used in 4-bit adder
- Replace the series of 2-input AND gates with AND gates with more input lines.


## A faster 4-bit counter


[ Figure 5.75 from the textbook ]

## Faster 4-bit Counter

- Longest path: Q0 to Q3
- $\mathrm{T}_{\text {min }}=\mathrm{t}_{\mathrm{cQO}}+\mathrm{t}_{\mathrm{AND}}+\mathrm{t}_{\mathrm{XOR}}+\mathrm{t}_{\mathrm{su}}$

$$
=1.0+1.4+1.2+0.6=4.2 \mathrm{~ns}
$$

- $F_{\text {max }}=1 / 4.2 \mathrm{~ns}=238.1 \mathrm{MHz} \boldsymbol{>}$ 156.25 MHz


# Reaction Timer Circuit (Section 5.14) 

## Problem Statement

- Want to design a reaction timer
- The circuit turns on a light (LED)
- A person then presses a switch
- Measure the elapsed time from when the LED is turned on until the switch is pressed


## Clock Divider

- Input: 102.4kHz
- Output: 100Hz
- 10-bit Counter to divide
- Output Frequency $=102.4 \mathrm{k} / \mathbf{2}^{10}=100 \mathrm{~Hz}$


## A reaction-timer circuit


(a) Clock divider
(b) LED circuit

## Functionality of circuit

- Push switch
- Nominally 1
- DFF to keep track of the state
- Two-digit BCD counter
- Output goes to converters to a 7-segment display
- Start-up
- Assert the Reset signal
- Clears counter
- Clears flip-flop
- Assert w=1 for one cycle
- Once switch is hit
- Clears flip-flop
- Stops counting


## Push-button switch, LED, and 7-segment displays


[ Figure 5.61c from the textbook ]

## Examples of Solved Problems (Section 5.17)

## Example 5.18

## Circuit for Example 5.18


(a) Circuit

(b) Timing diagram
[ Figure 5.70 from the textbook]

## Example 5.19

## Circuit for Example 5.19



| $\mathbf{J}$ | $\mathbf{K}$ | $\mathbf{Q}(\mathbf{t}+\mathbf{1})$ |  |
| :---: | :---: | :---: | :---: |
| 0 | 0 | $\mathbf{Q}(\mathbf{t})$ | No Change |
| 0 | 1 | 0 | Reset |
| 1 | 0 | 1 | Set |
| 1 | 1 | $\mathbf{Q}^{\prime}(\mathbf{t})$ | Complement |

[ Figure 5.71 from the textbook]

## Summary of the behavior of the circuit

| Time | FF 0 |  |  | FF 1 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $J_{0}$ | $K_{0}$ | $\mathrm{Q}_{0}$ | $J_{1}$ | $K_{1}$ | $\mathrm{Q}_{1}$ |
| Clear | 1 | 1 | 0 | 0 | 1 | 0 |
| $t_{1}$ | 1 | 1 | 1 | 1 | 1 | 0 |
| $t_{2}$ | 0 | 1 | 0 | 0 | 1 | 1 |
| $t_{3}$ | 1 | 1 | 0 | 0 | 1 | 0 |
| $t_{4}$ | 1 | 1 | 1 | 1 | 1 | 0 |

[ Figure 5.72 from the textbook ]

## Example 5.20

## Vending Machine Example

- Inputs N, D, Q, Coin, Resetn
- N, D, Q: nickel, dime, quarter
- Coin: pulsed when a coin is entered
- Used to store values into register
- Resetn: resets the register value to zero
- Add up new coin with old value
- Store new sum into old value register
- See if total is above thirty cents
- If so output Z goes high


## Circuit for Example 5.20


[ Figure 5.73 from the textbook]

## Questions?

## THE END

