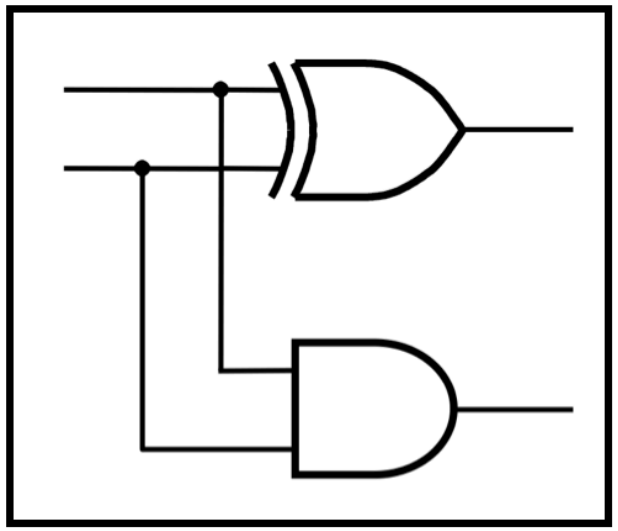


# CprE 281: Digital Logic

**Instructor: Alexander Stoytchev**

**http://www.ece.iastate.edu/~alexs/classes/**

# Designing a Counter
# (Using the Sequential Circuit Approach)

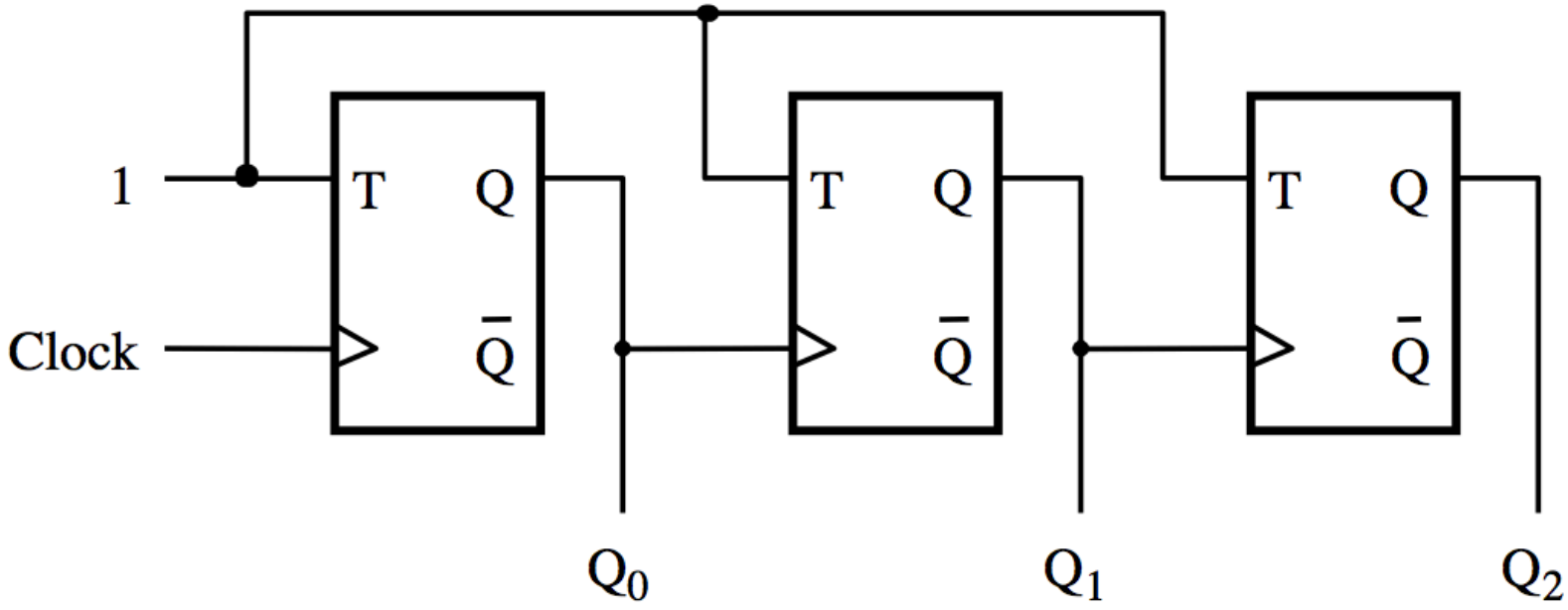
*CprE 281: Digital Logic*
*Iowa State University, Ames, IA*

# Example:
# Implement a modulo-8 counter

# Mini Review

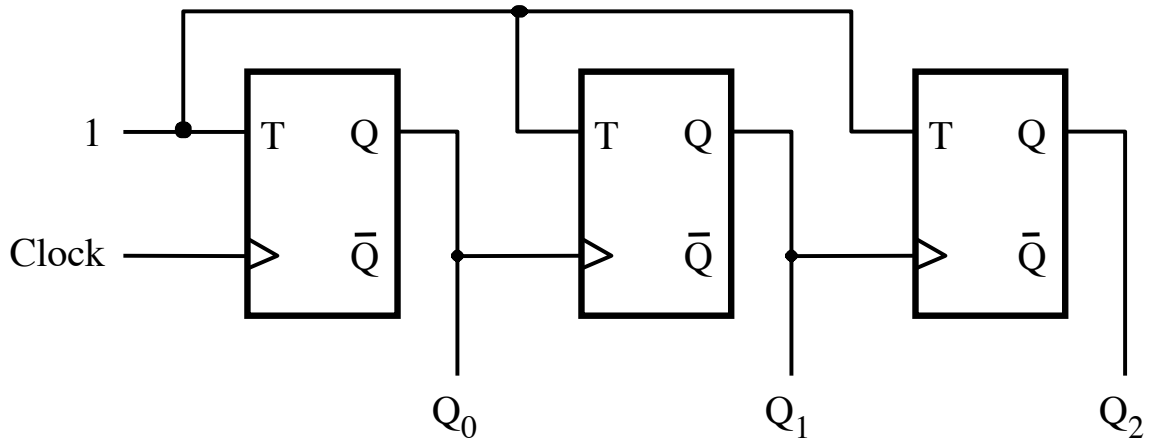
# Asynchronous Counters

# A three-bit down-counter

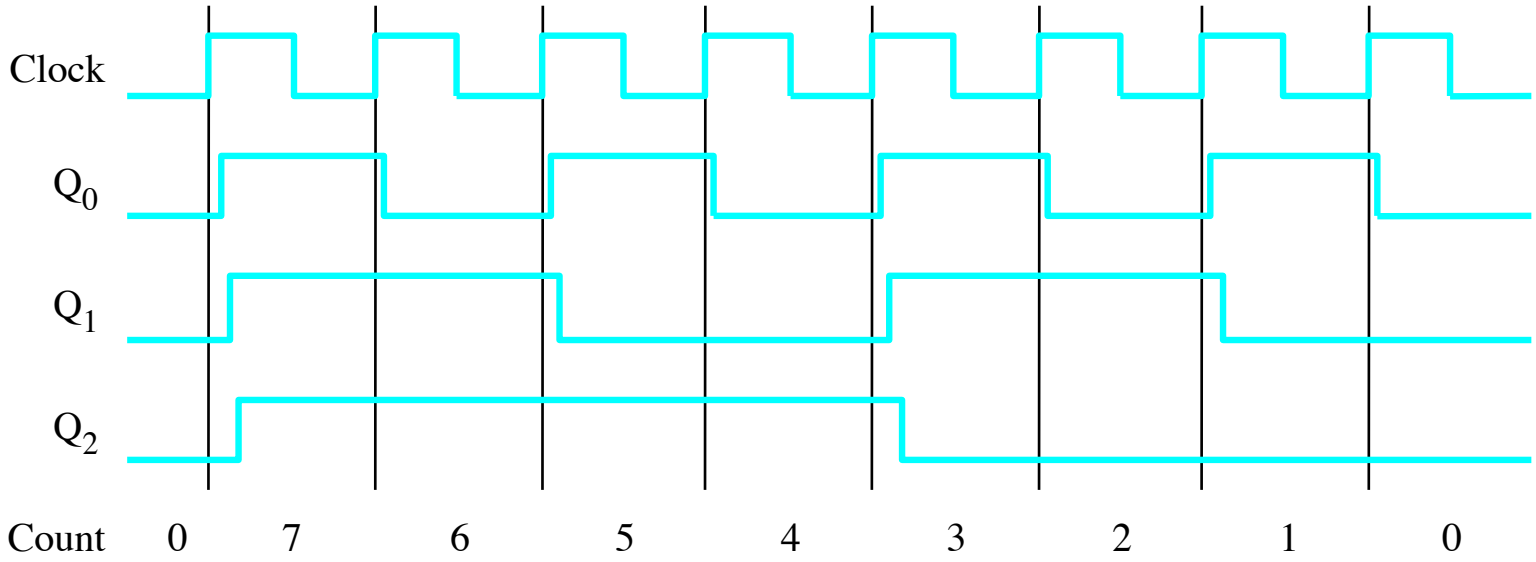



[ Figure 5.20 from the textbook ]

# A three-bit down-counter



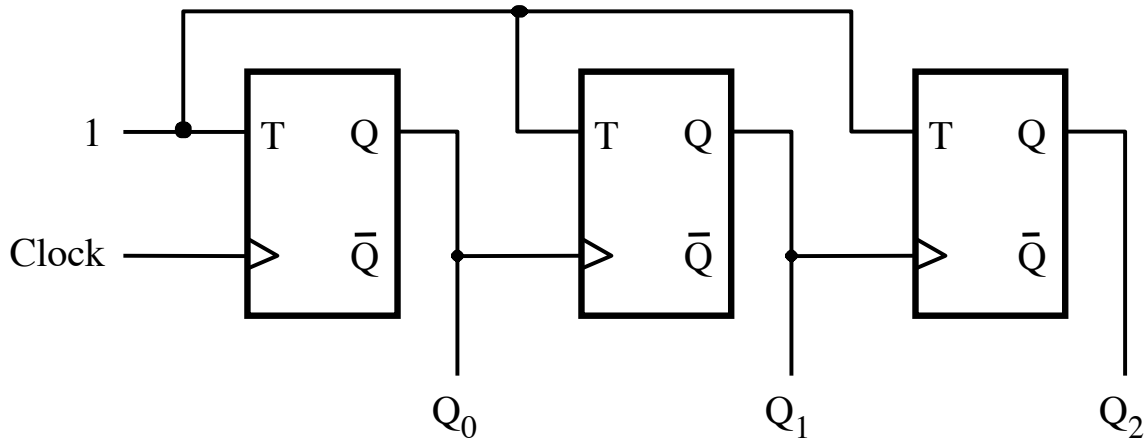

(a) Circuit




(b) Timing diagram

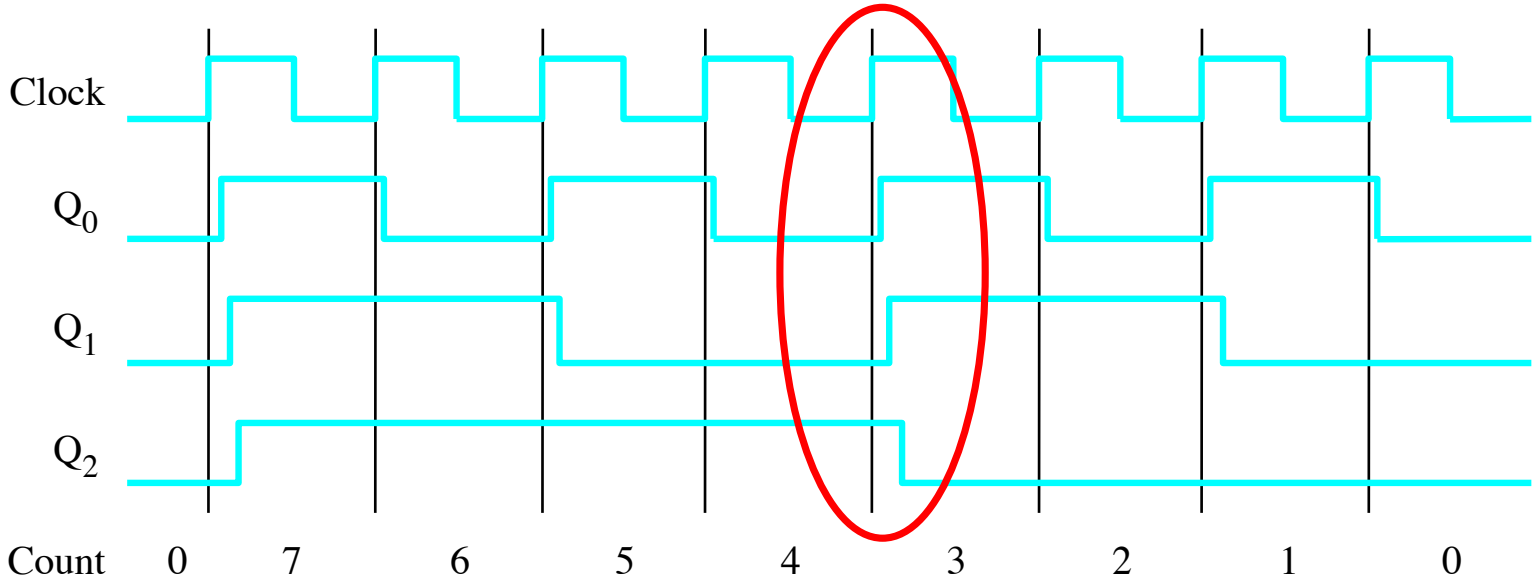
[ Figure 5.20 from the textbook ]

# A three-bit down-counter




The propagation delays get longer

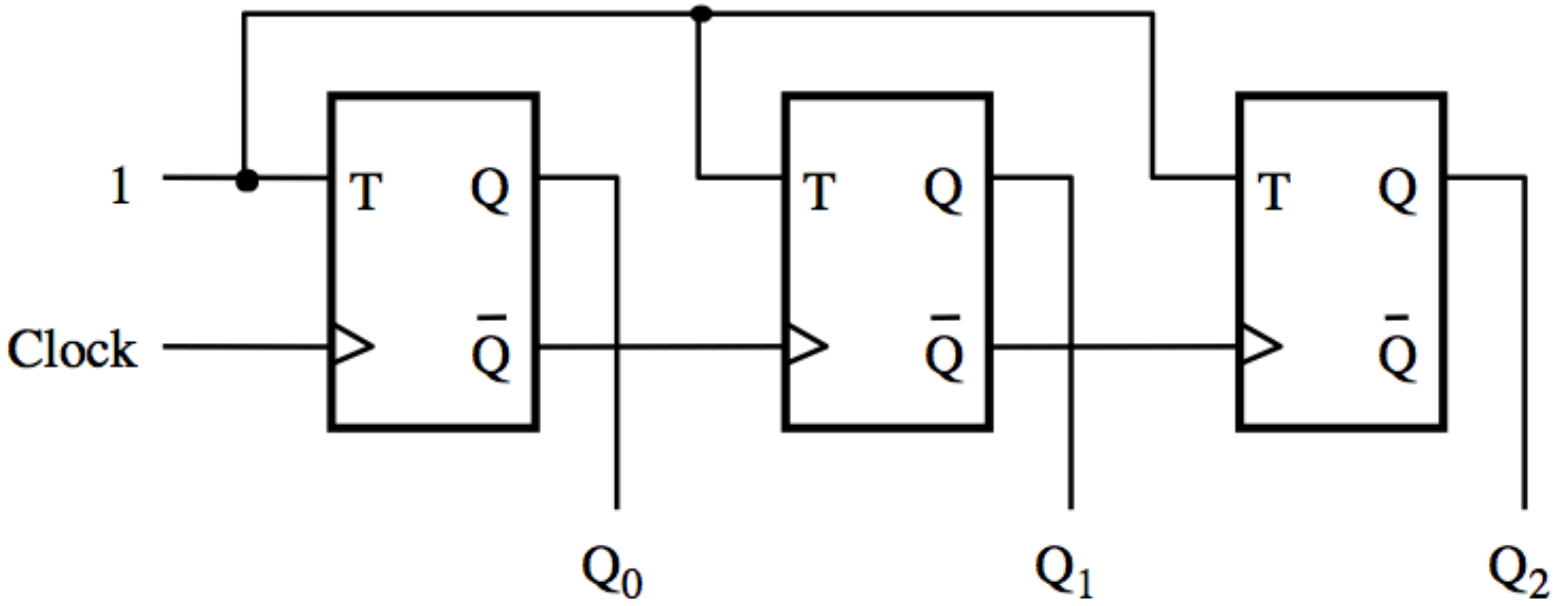
(a) Circuit




(b) Timing diagram

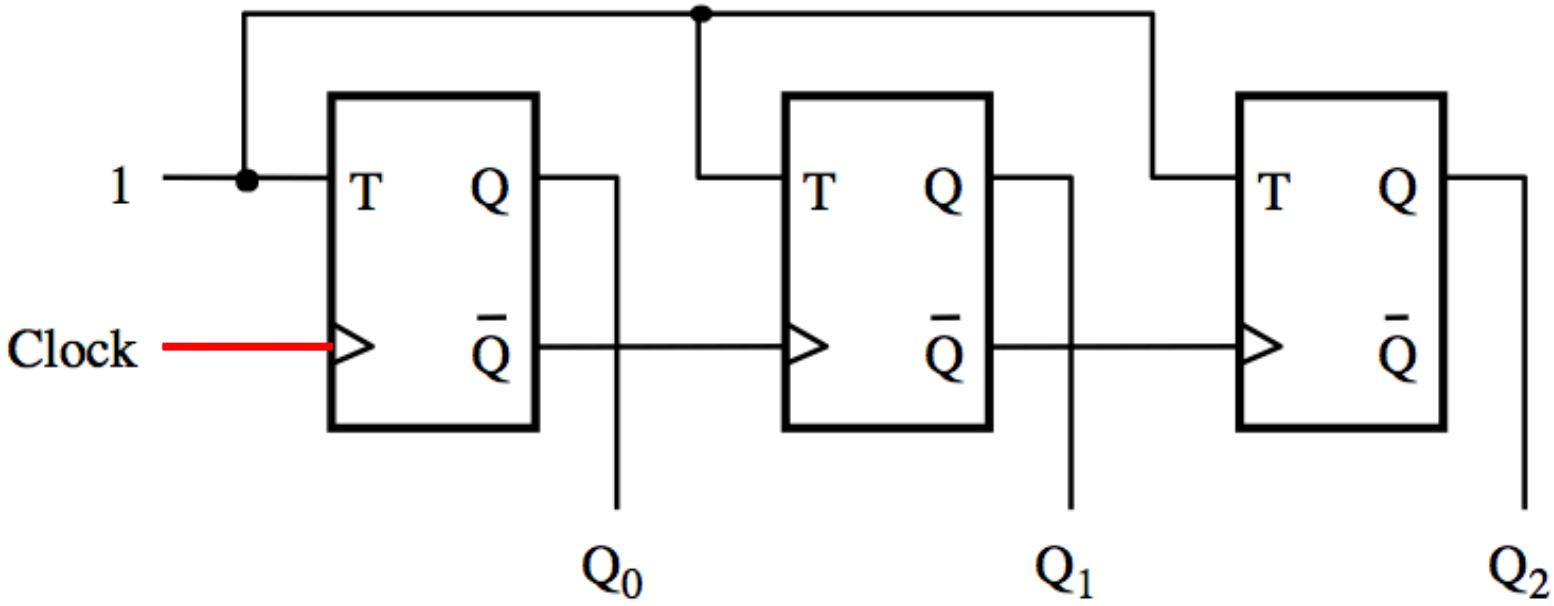
[ Figure 5.20 from the textbook ]

# A three-bit up-counter




[ Figure 5.19 from the textbook ]

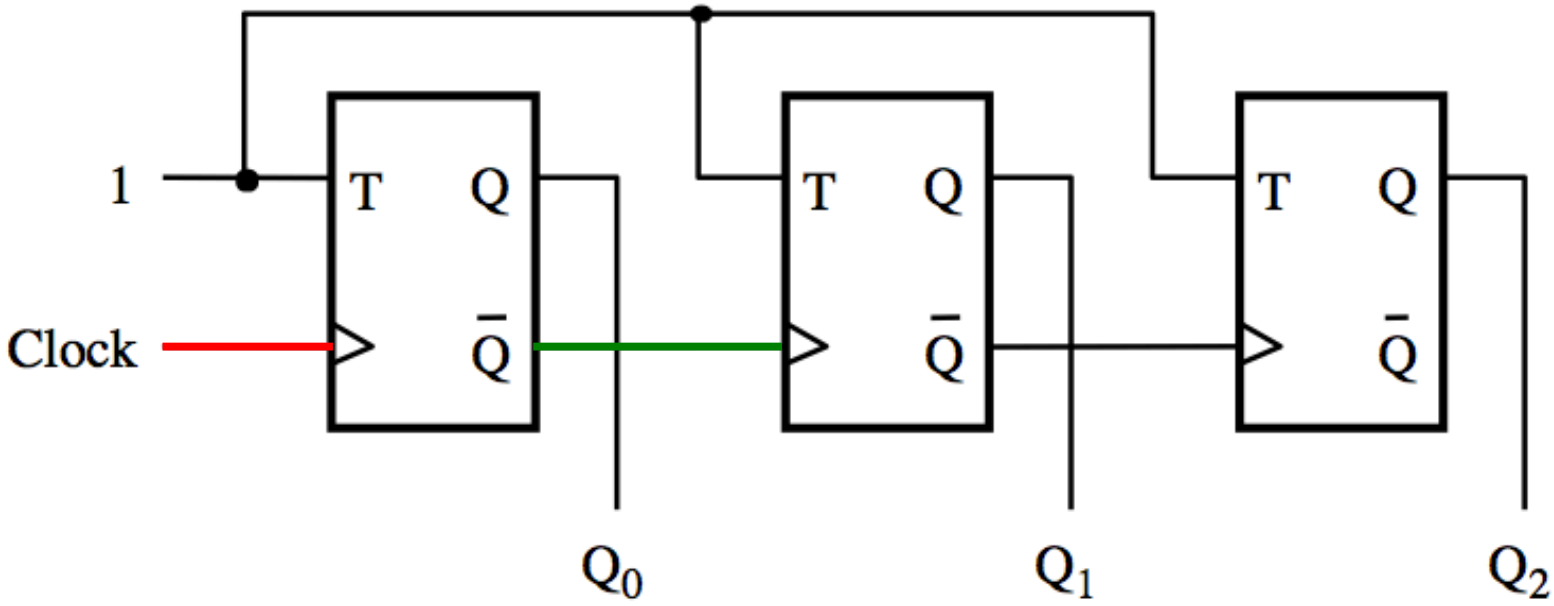
# A three-bit up-counter




The first flip-flop changes
on the positive edge of the clock

[ Figure 5.19 from the textbook ]

# A three-bit up-counter

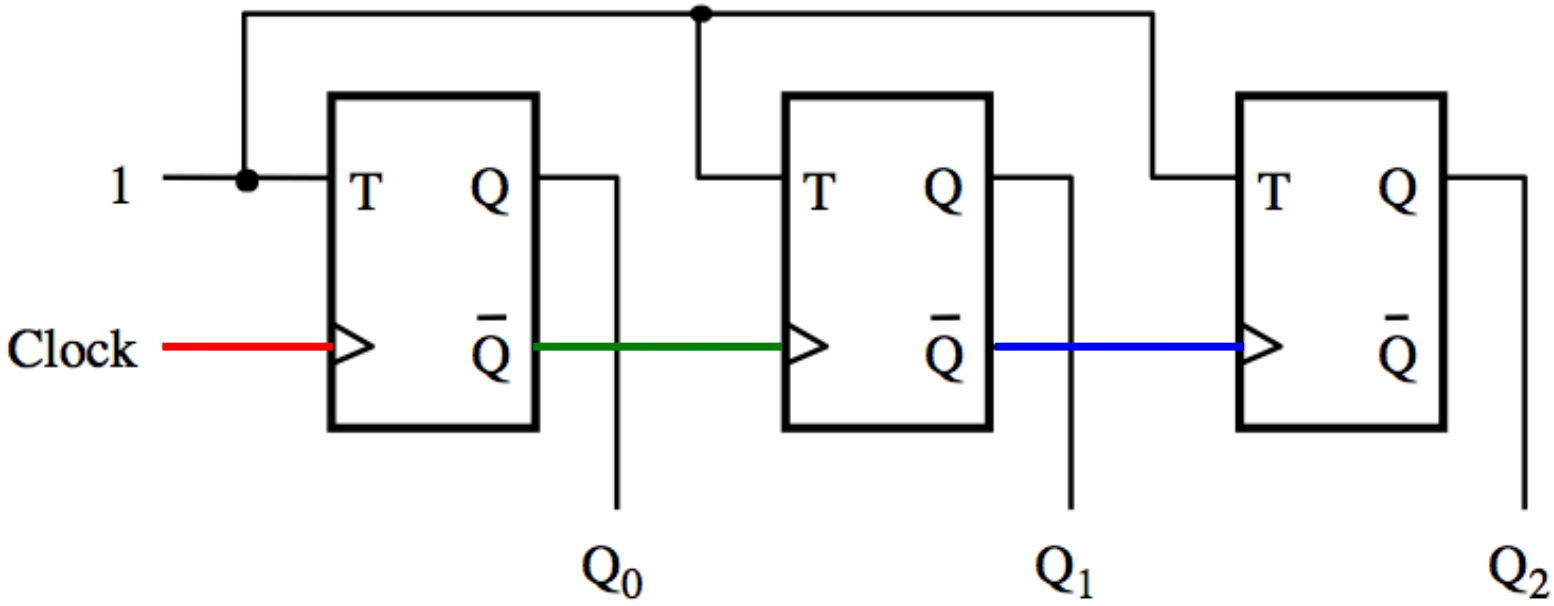



The first flip-flop changes
on the positive edge of the clock

The second flip-flop changes
on the positive edge of $\overline{Q}_0$

[ Figure 5.19 from the textbook ]

# A three-bit up-counter



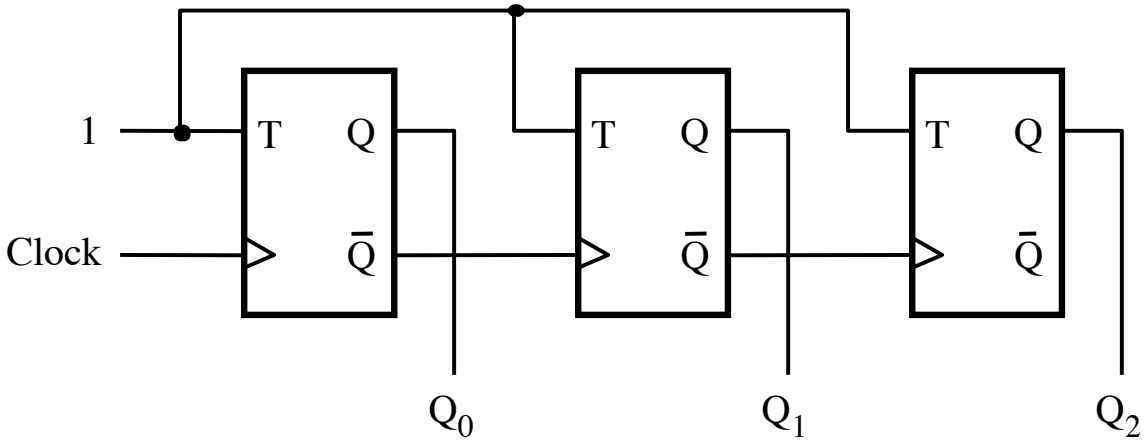

The first flip-flop changes on the positive edge of the clock

The second flip-flop changes on the positive edge of $\overline{Q}_0$

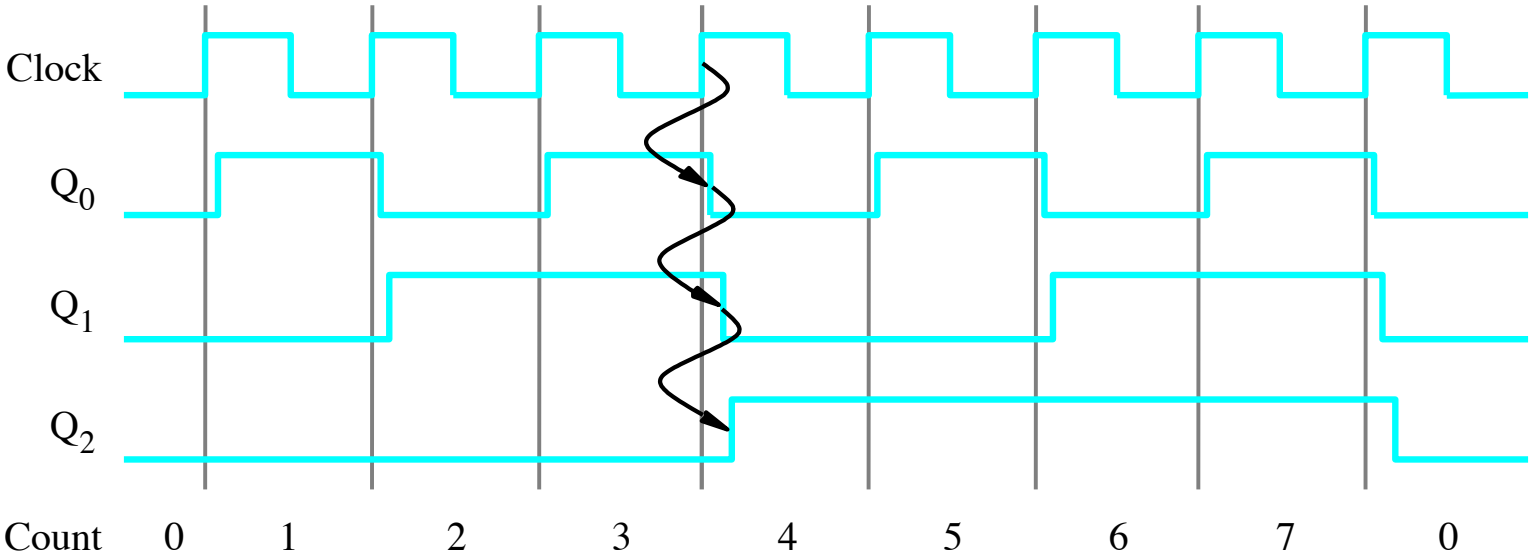
The third flip-flop changes on the positive edge of $\overline{Q}_1$

[ Figure 5.19 from the textbook ]

# A three-bit up-counter



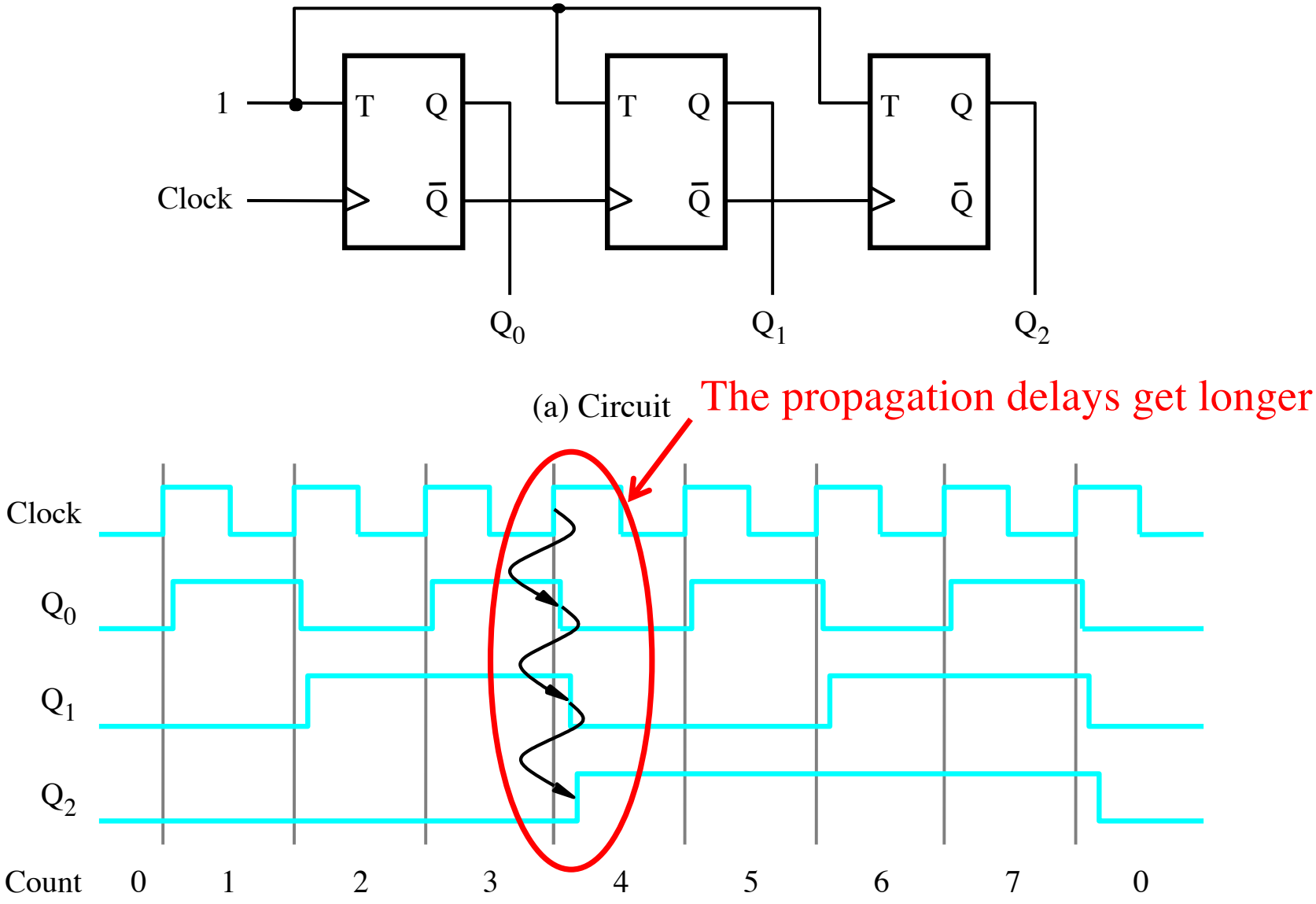

(a) Circuit




(b) Timing diagram

[ Figure 5.19 from the textbook ]

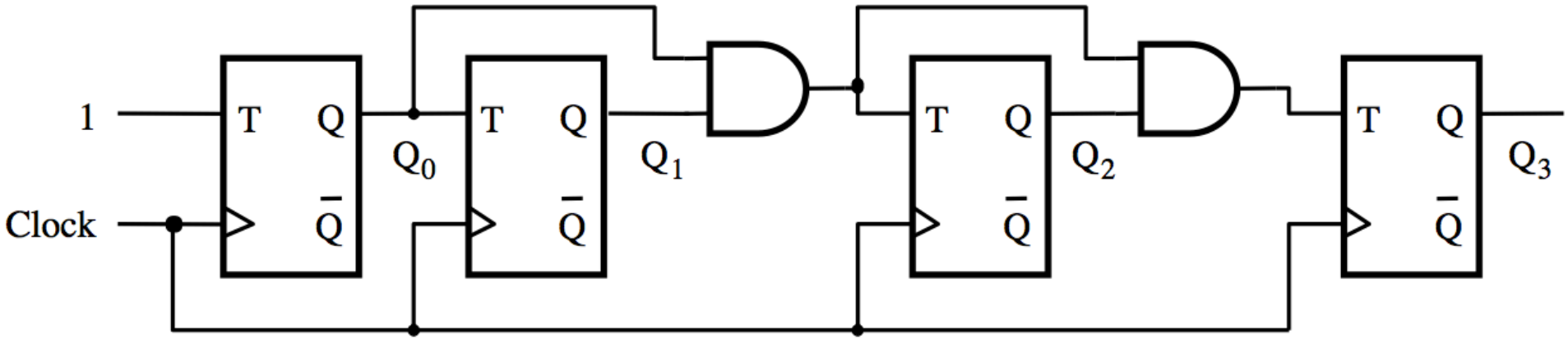
# A three-bit up-counter




[ Figure 5.19 from the textbook ]

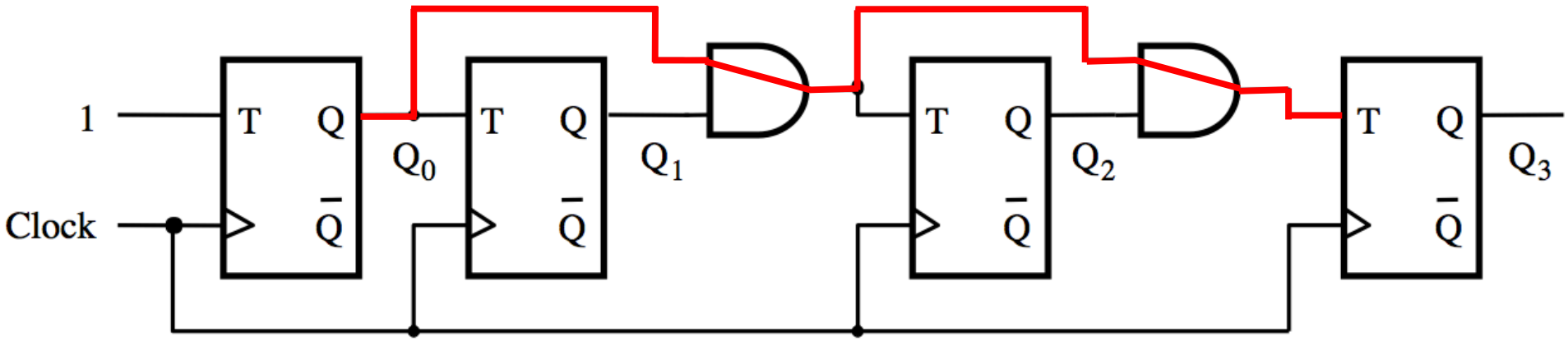
# Synchronous Counters

# A four-bit synchronous up-counter




[ Figure 5.21 from the textbook ]

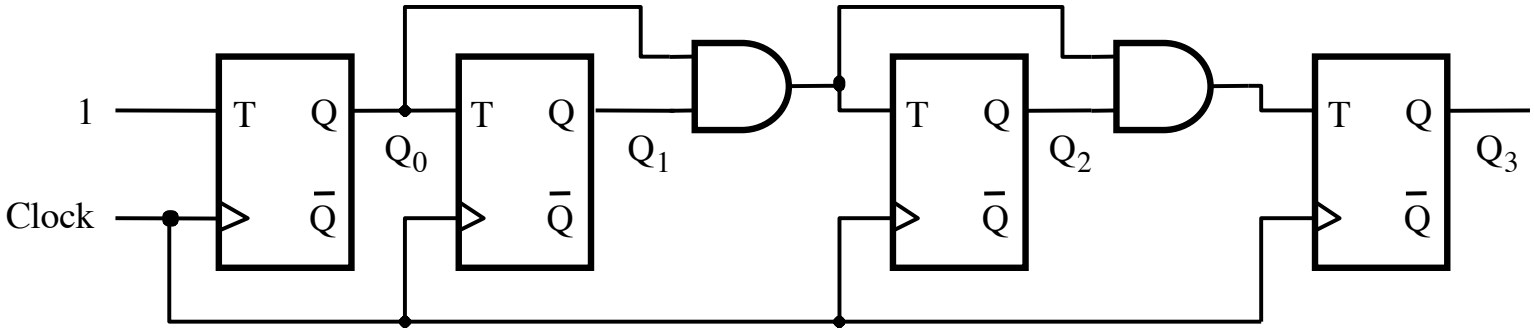
# A four-bit synchronous up-counter



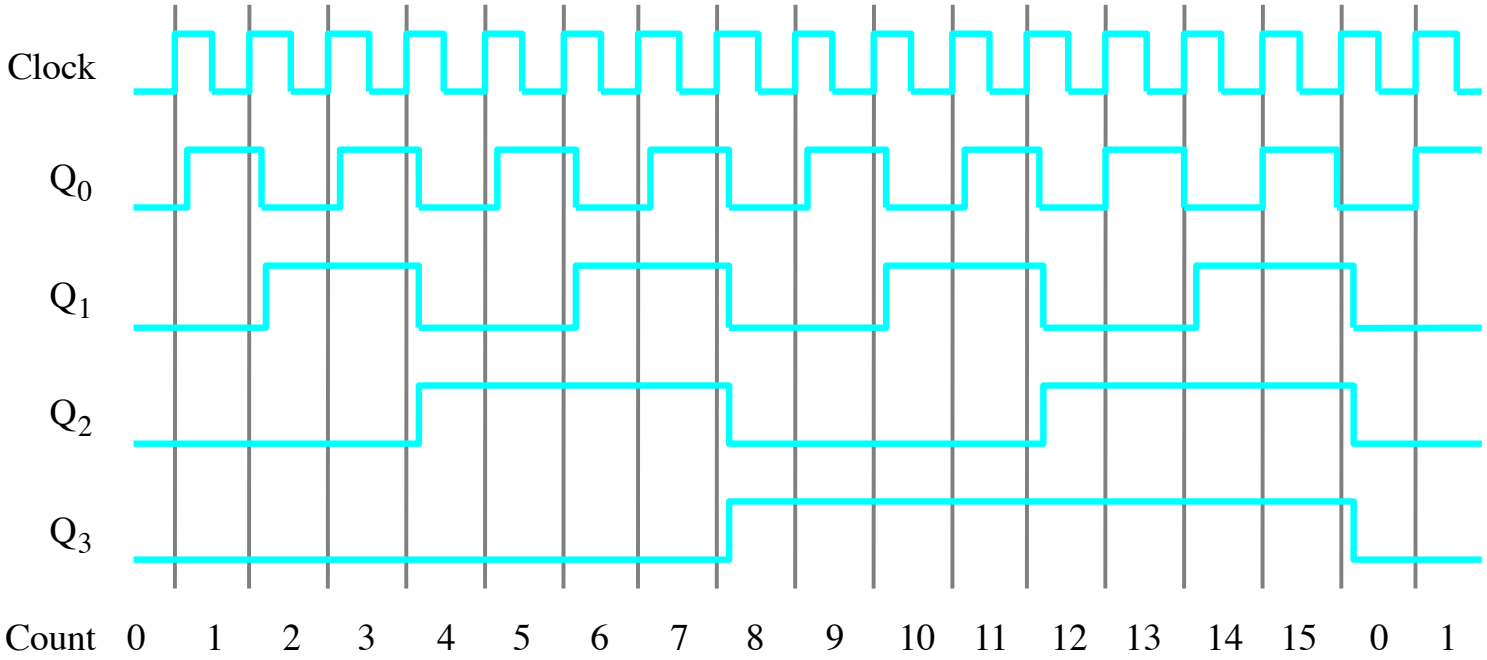

The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

[ Figure 5.21 from the textbook ]

# A four-bit synchronous up-counter




(a) Circuit

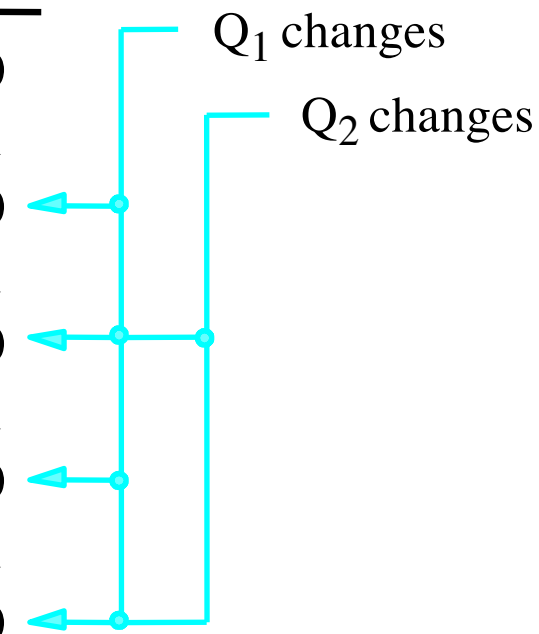



(b) Timing diagram

[ Figure 5.21 from the textbook ]

# Derivation of the synchronous up-counter

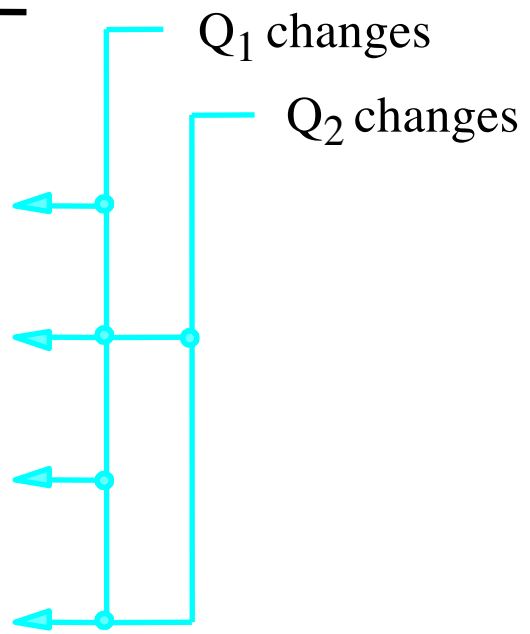
| Clock cycle | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |
| 8 | 0 | 0 | 0 |




[ Table 5.1 from the textbook ]

# Derivation of the synchronous up-counter

| Clock cycle | $Q_2$ | $Q_1$ | $Q_0$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 |
| 2 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 |
| 4 | 1 | 0 | 0 |
| 5 | 1 | 0 | 1 |
| 6 | 1 | 1 | 0 |
| 7 | 1 | 1 | 1 |
| 8 | 0 | 0 | 0 |



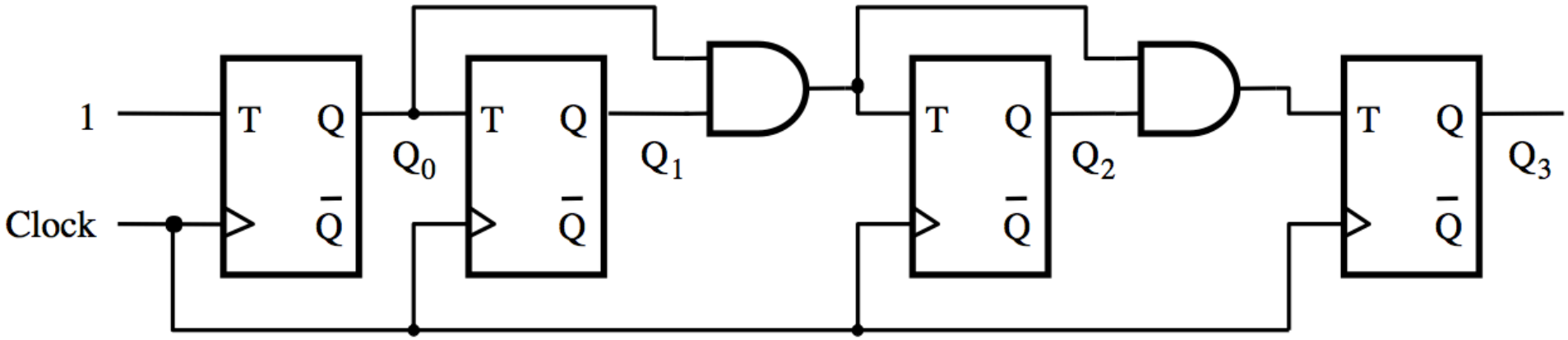

$T_0 = 1$

$T_1 = Q_0$

$T_2 = Q_0 Q_1$

[ Table 5.1 from the textbook ]

# A four-bit synchronous up-counter




$T_0 = 1$

$T_1 = Q_0$

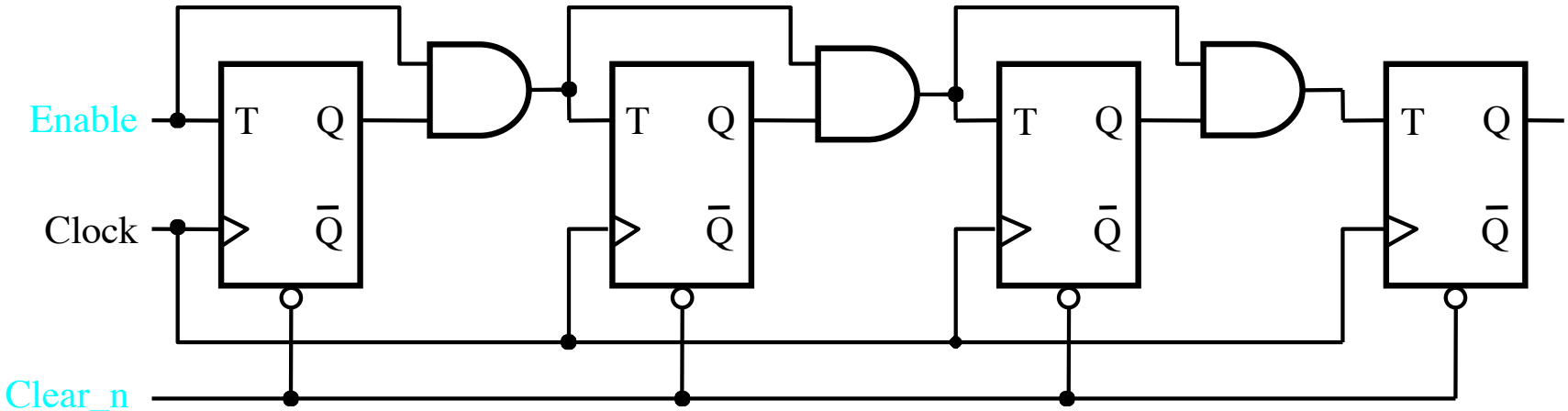
$T_2 = Q_0 Q_1$

[ Figure 5.21 from the textbook ]

# In general we have

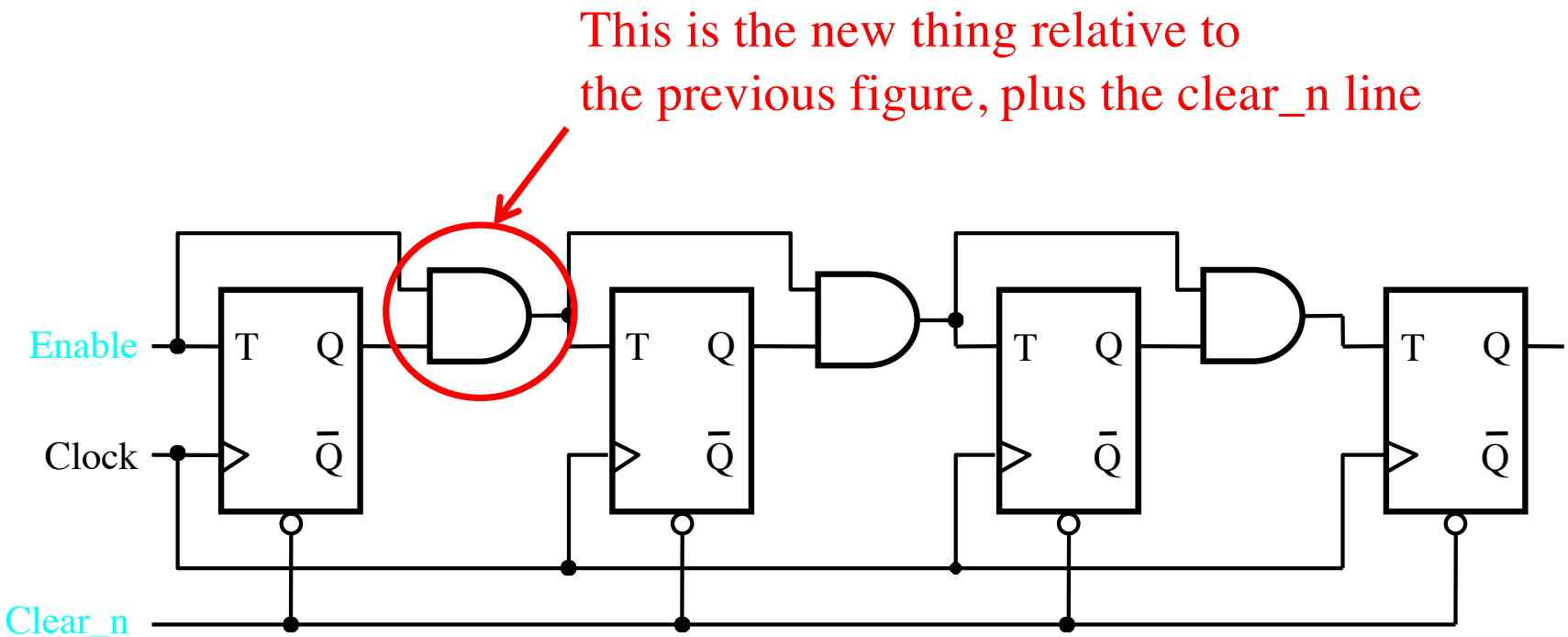
$$T_0 = 1$$
$$T_1 = Q_0$$
$$T_2 = Q_0\, Q_1$$
$$T_3 = Q_0\, Q_1\, Q_2$$
$$\cdots$$
$$T_n = Q_0\, Q_1\, Q_2 \cdots Q_{n-1}$$

# Inclusion of Enable and Clear capability



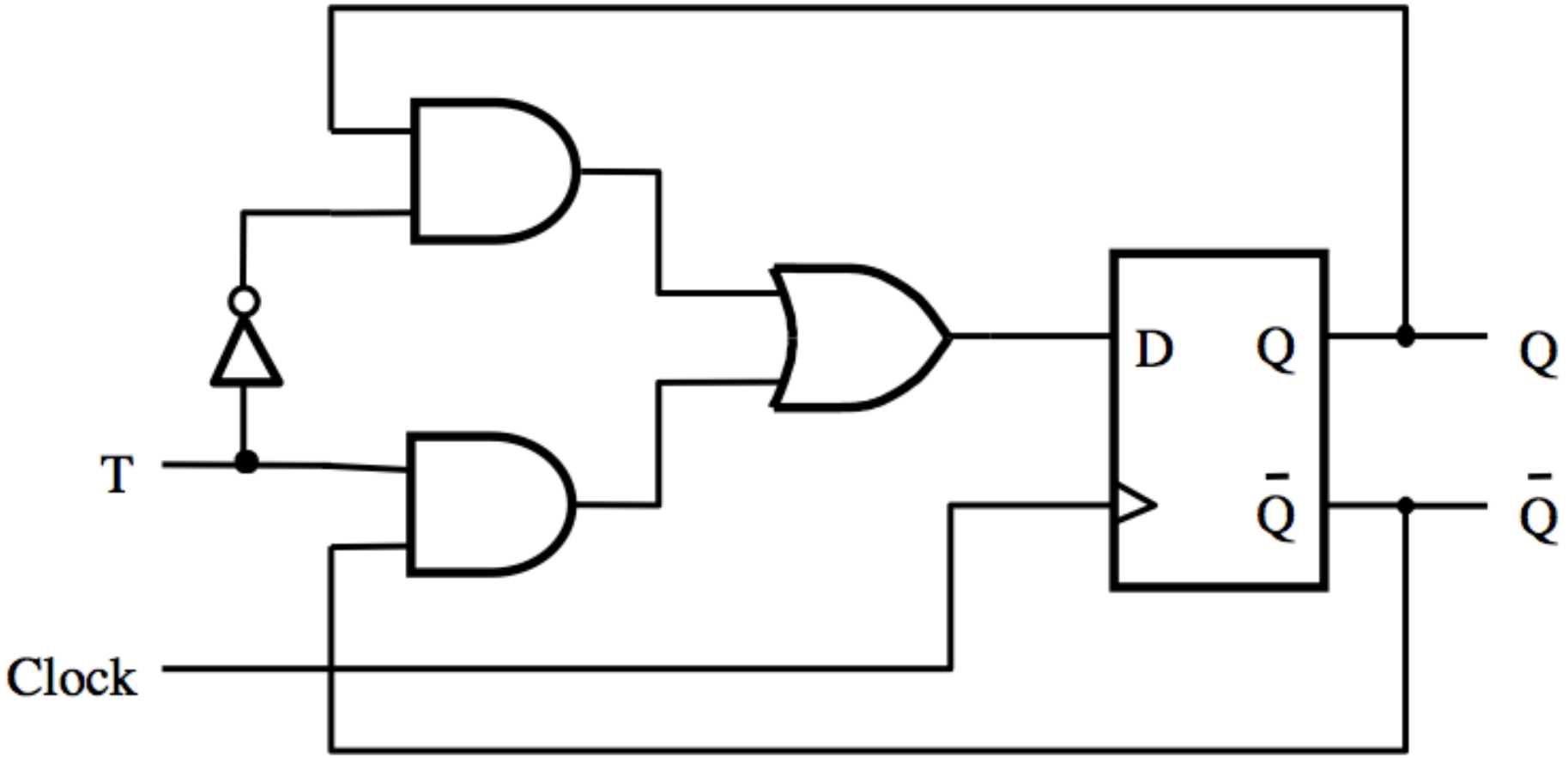

[ Figure 5.22 from the textbook ]

# Inclusion of Enable and Clear capability



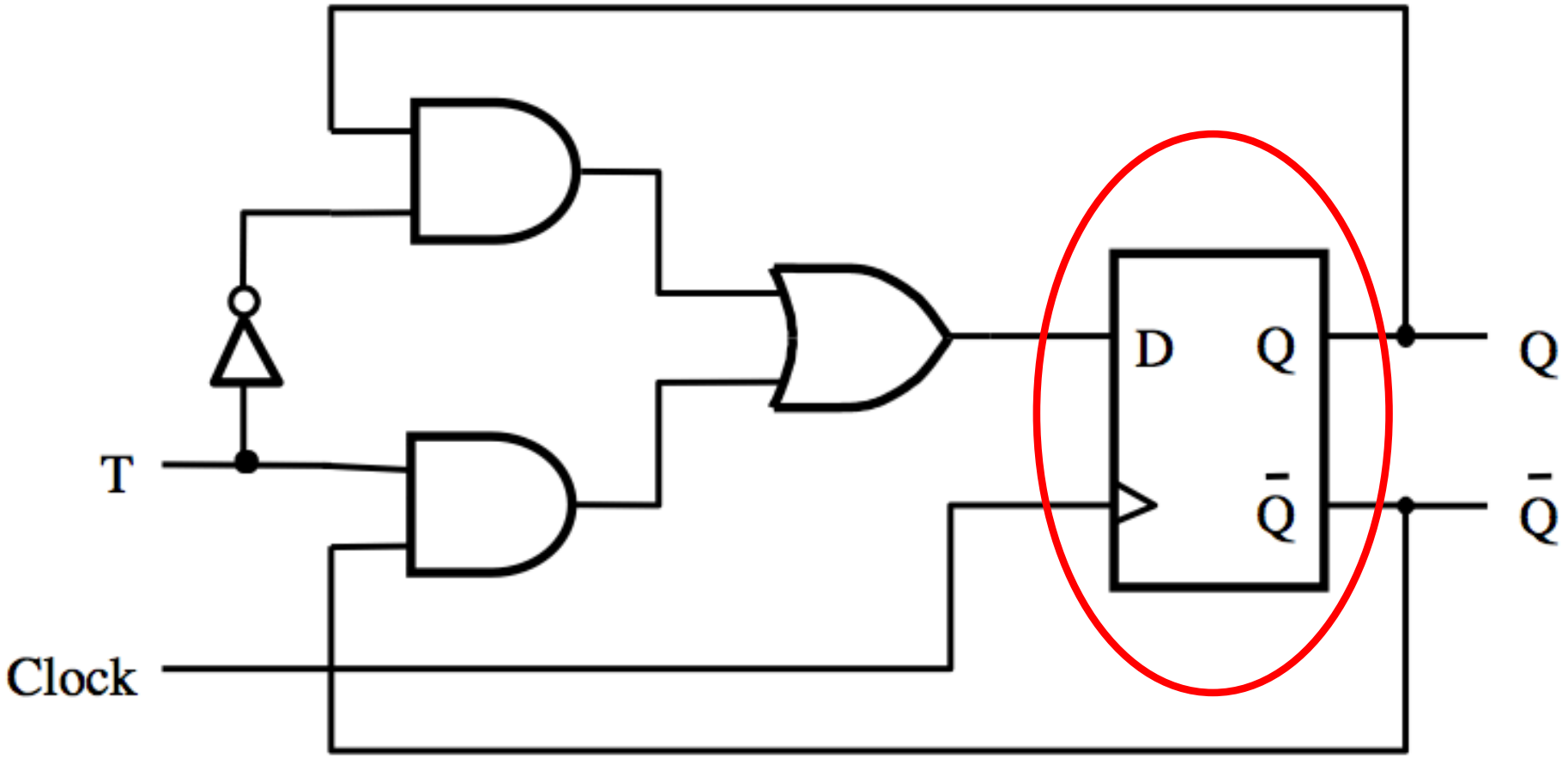

[ Figure 5.22 from the textbook ]

# T Flip-Flop




[ Figure 5.15a from the textbook ]

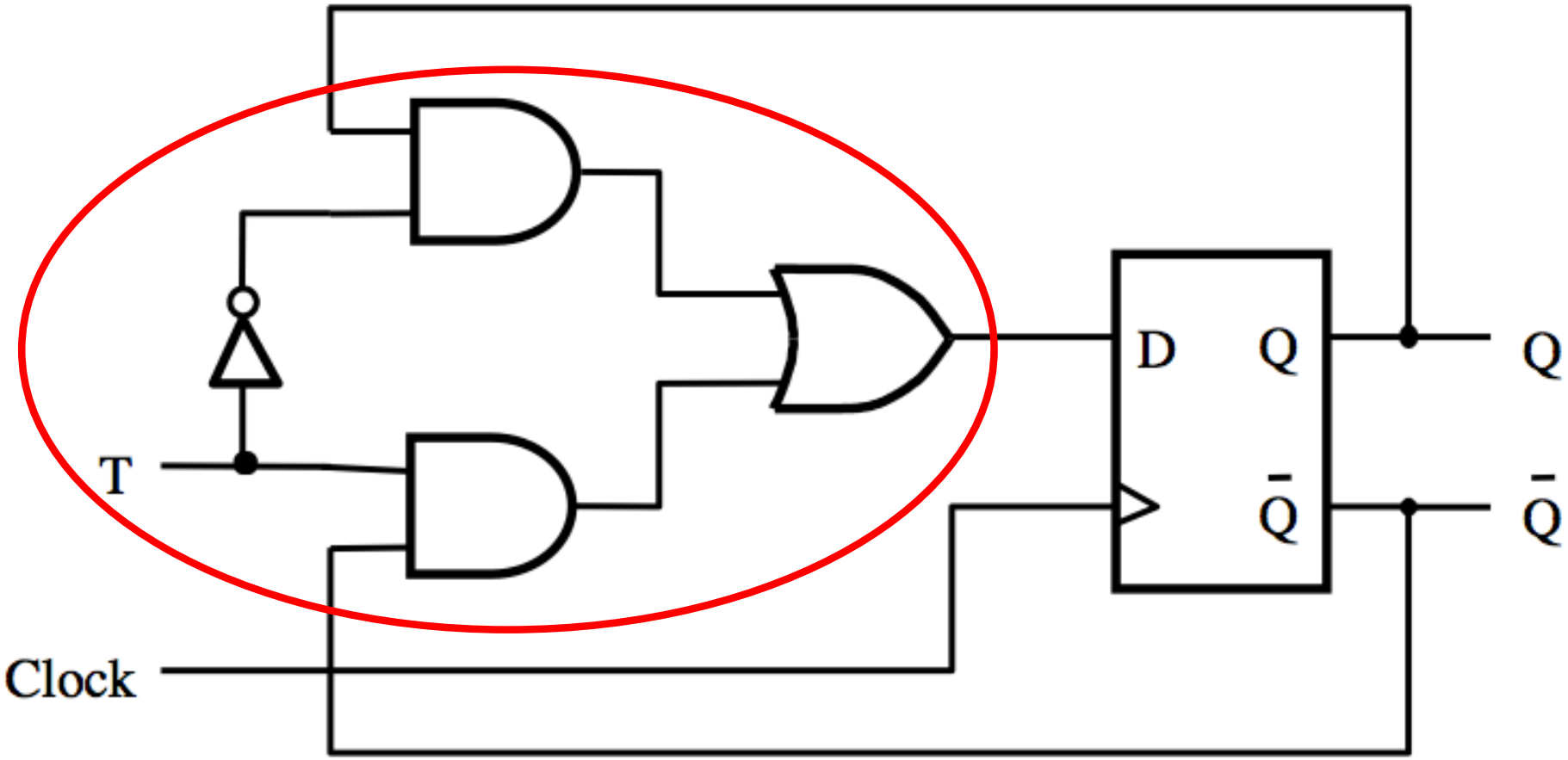
# T Flip-Flop




Positive-edge-triggered
D Flip-Flop

[ Figure 5.15a from the textbook ]

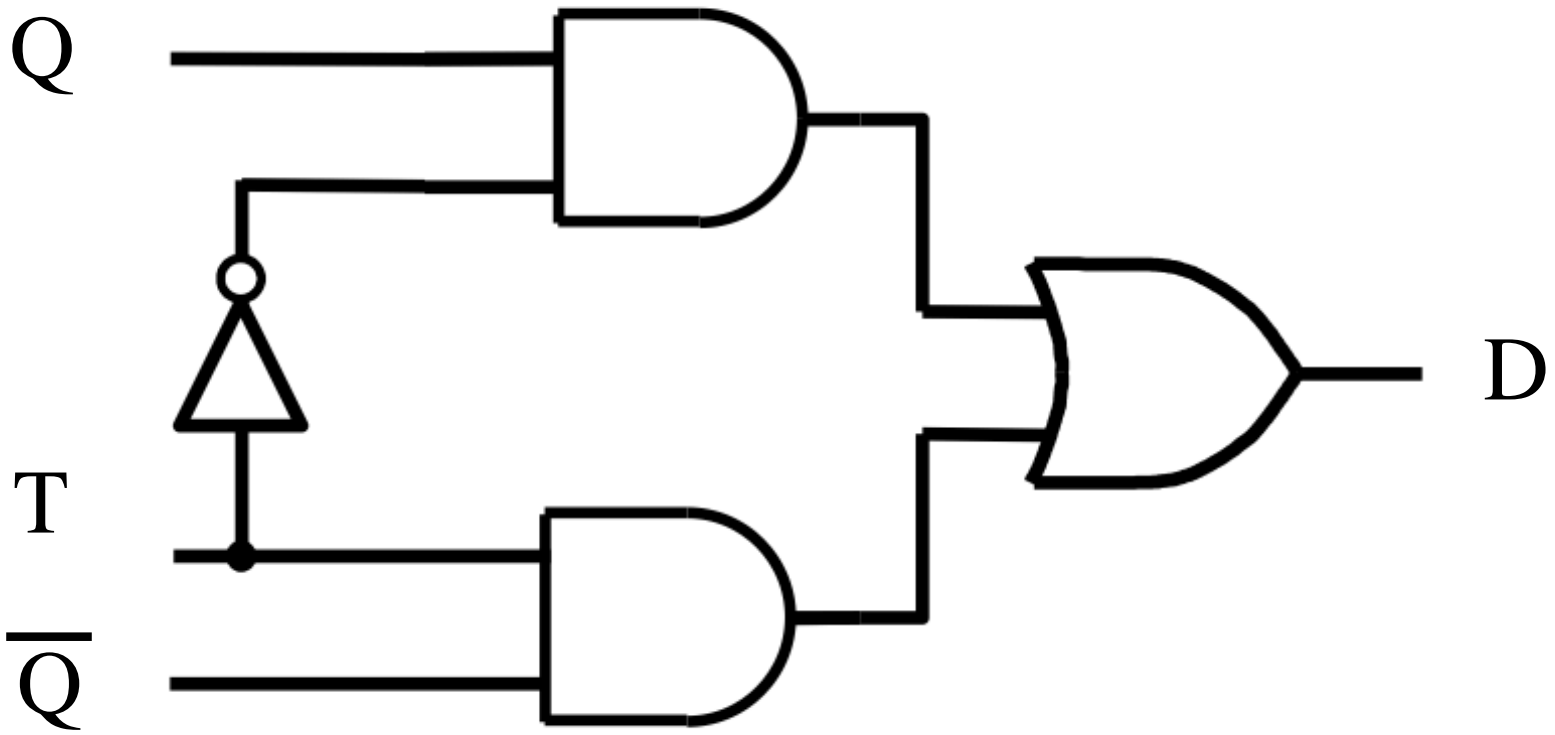
# T Flip-Flop



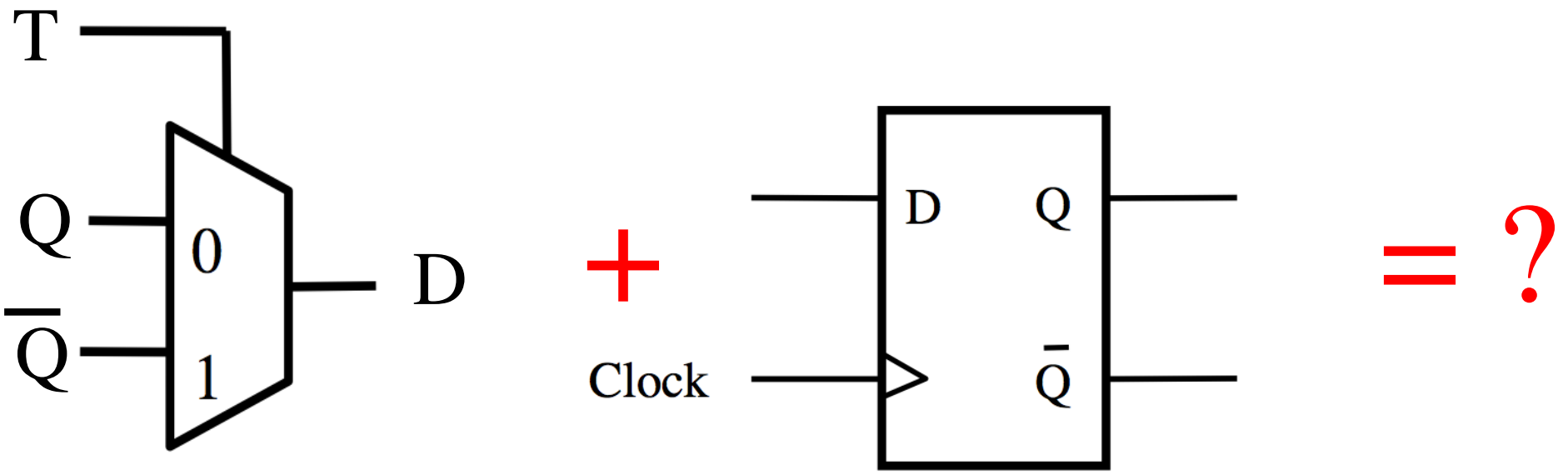

2-to-1 multiplexer

[ Figure 5.15a from the textbook ]

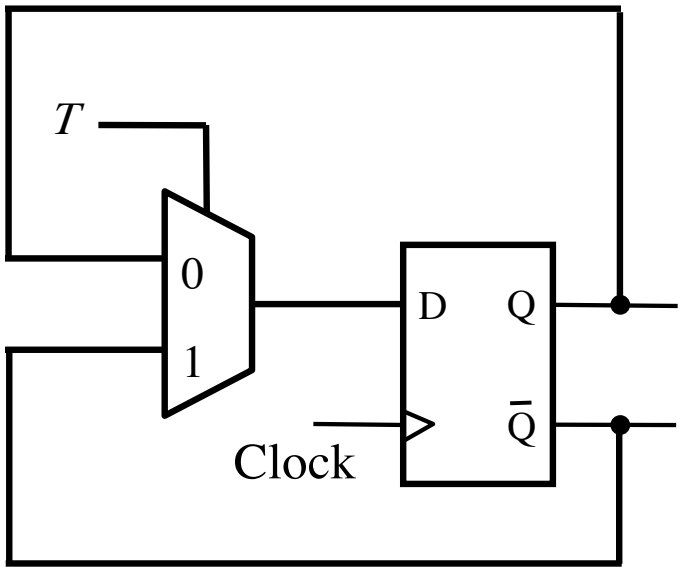
# 2-to-1 Multiplexer



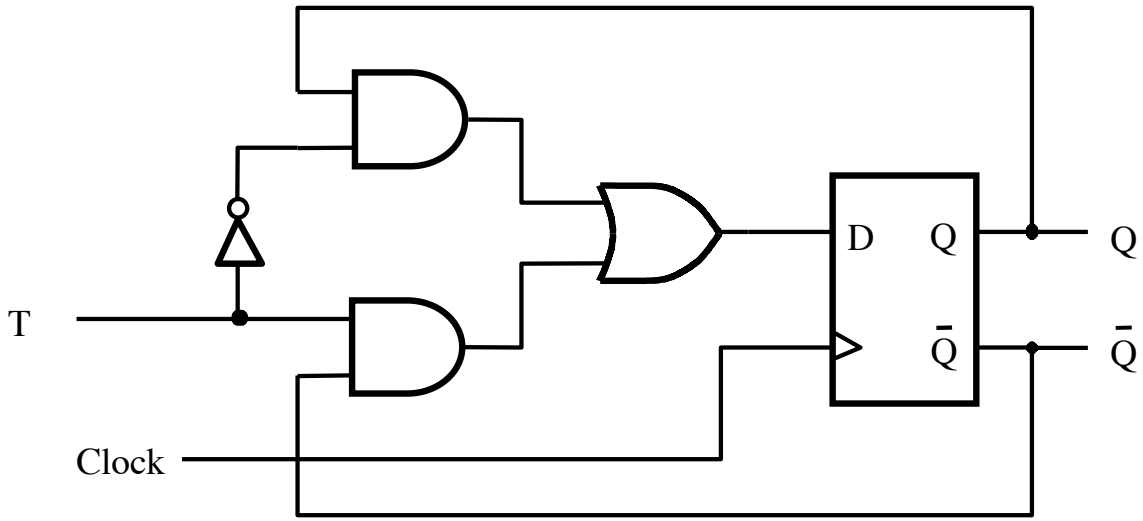
# What is this?



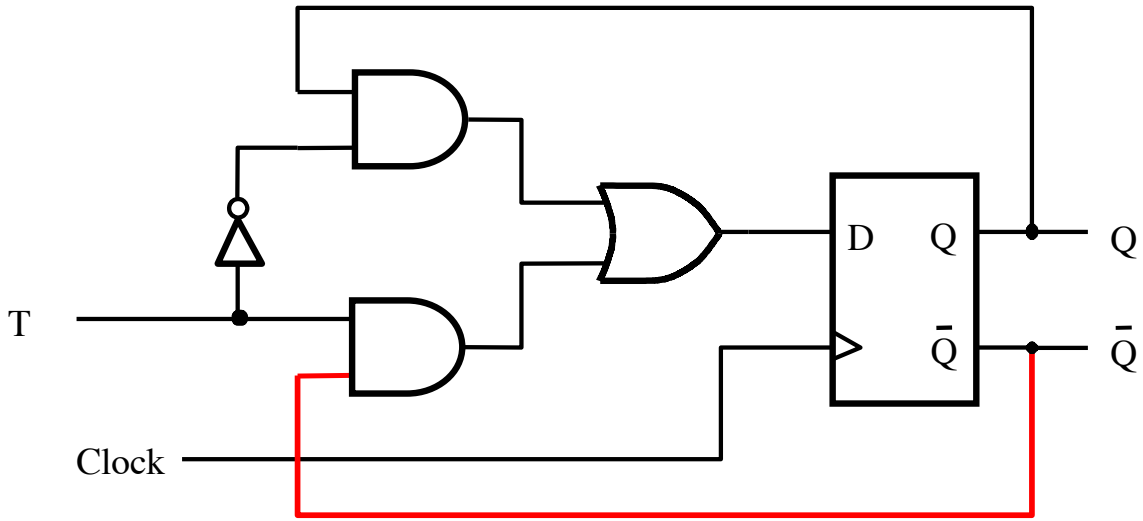
# T Flip-Flop



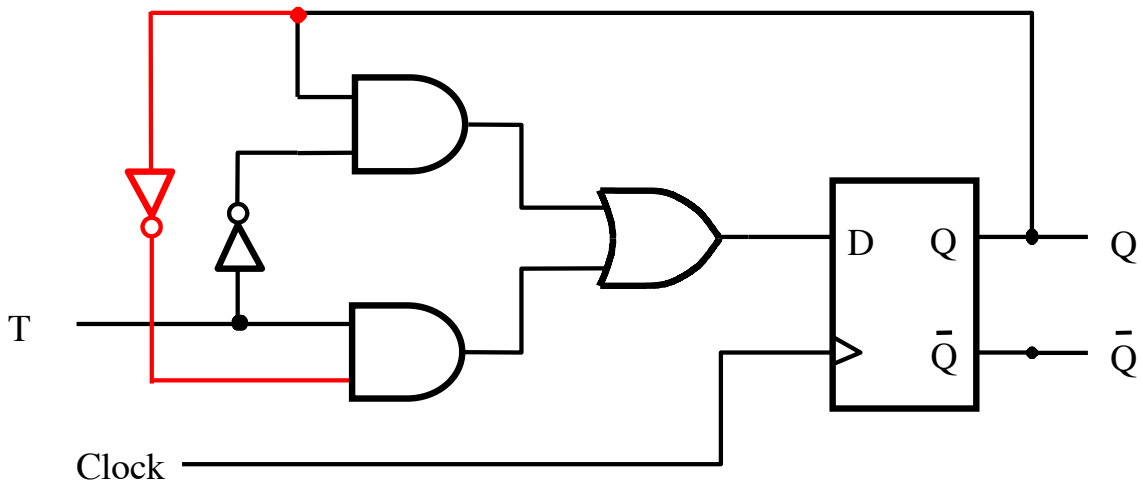
# T Flip-Flop



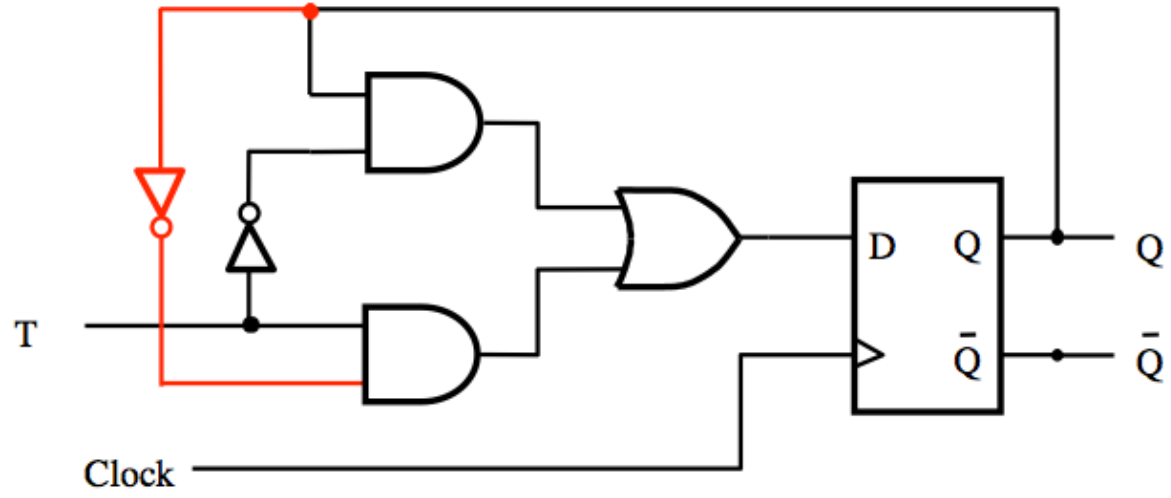
# T Flip-Flop

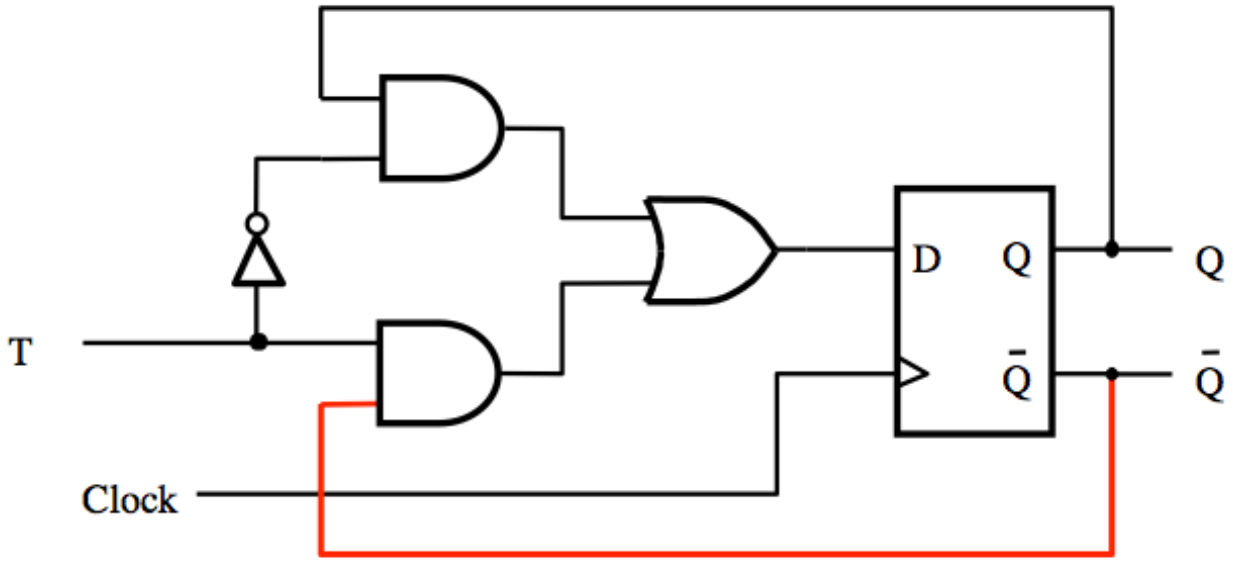


# T Flip-Flop

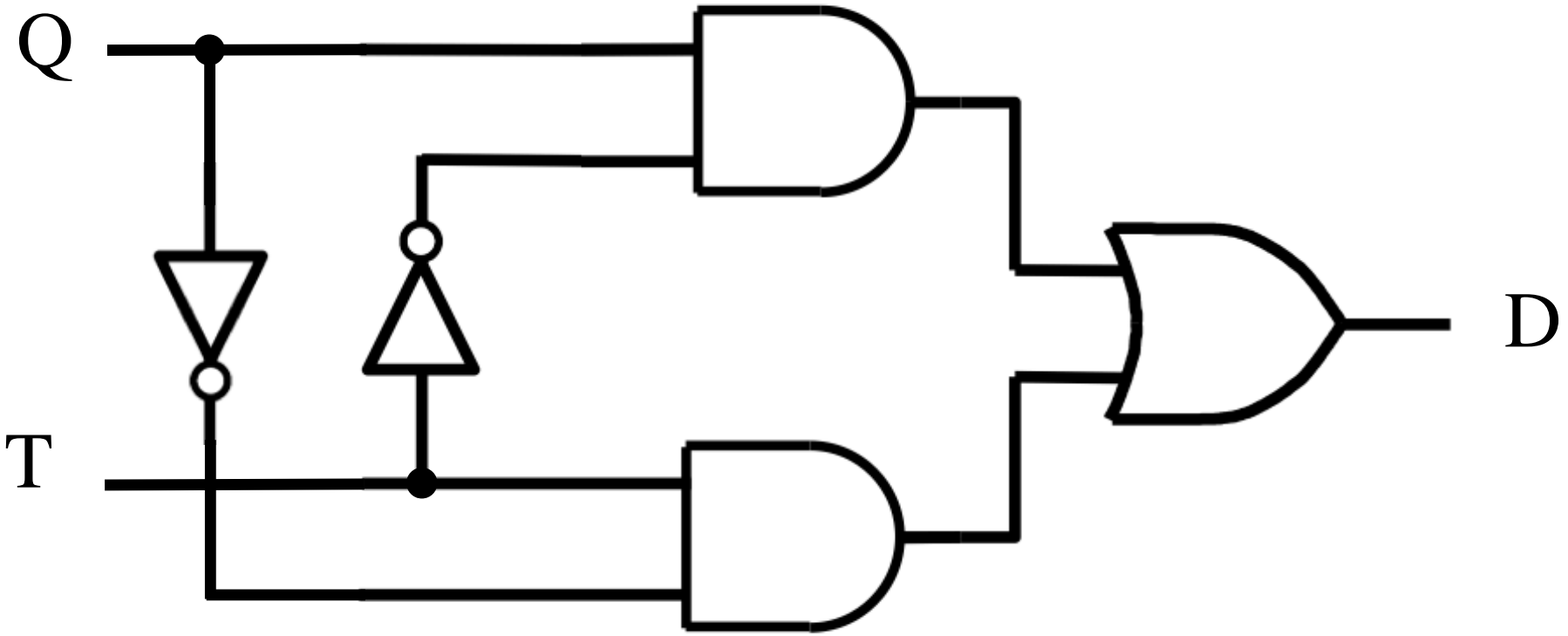


# These two circuits are equivalent

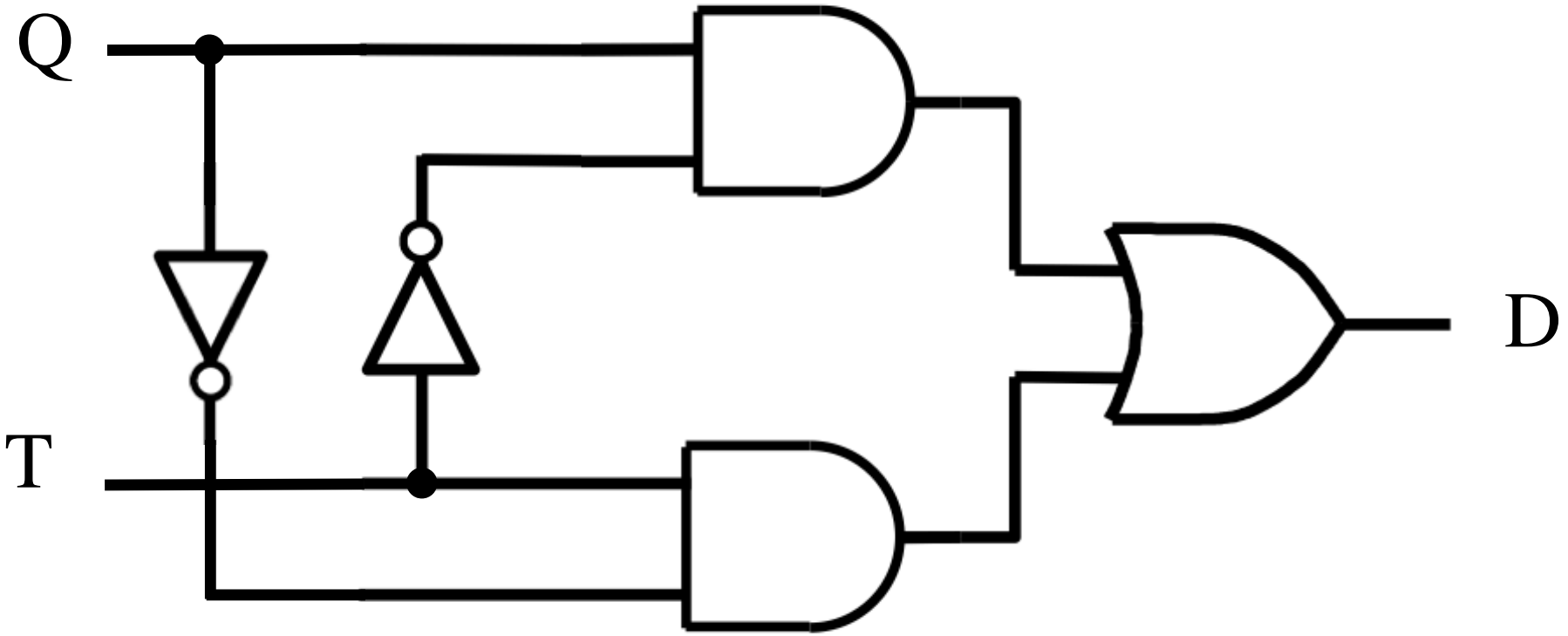


# What is this?

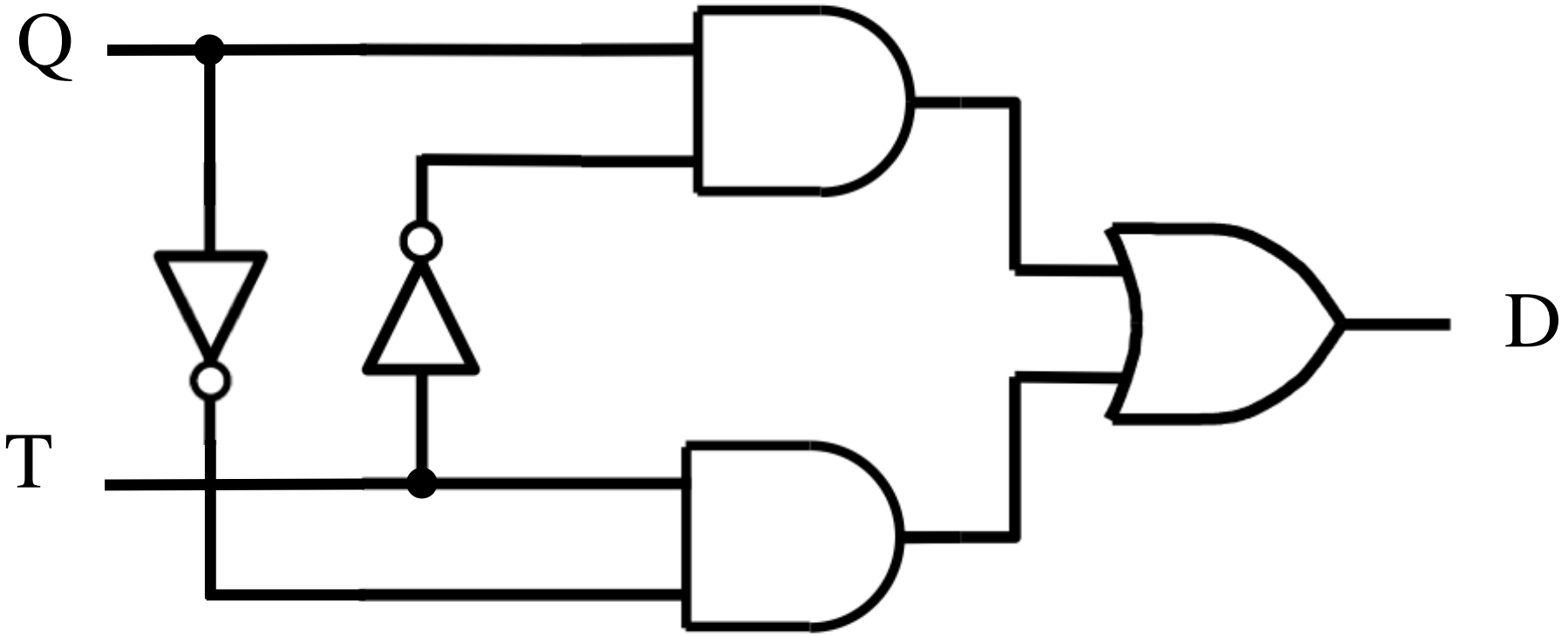


# What is this?



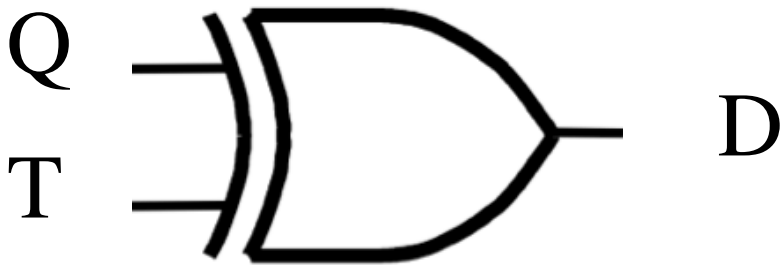

$$D = Q\overline{T} + \overline{Q}T$$

# What is this?



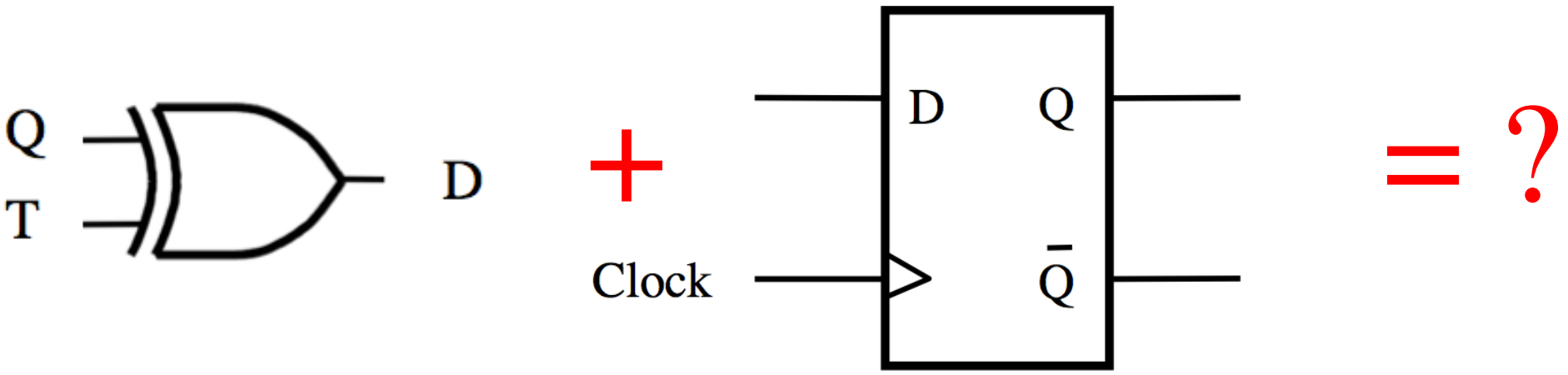

$$D = Q \oplus T$$

# What is this?

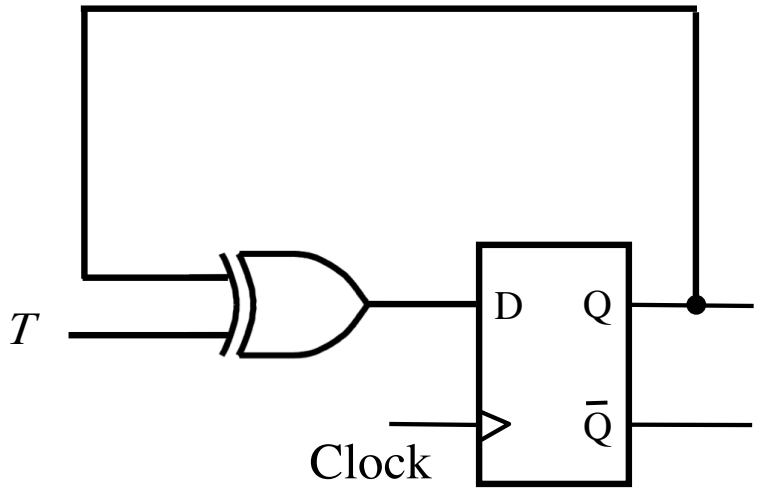



$$D = Q \oplus T$$

# What is this?

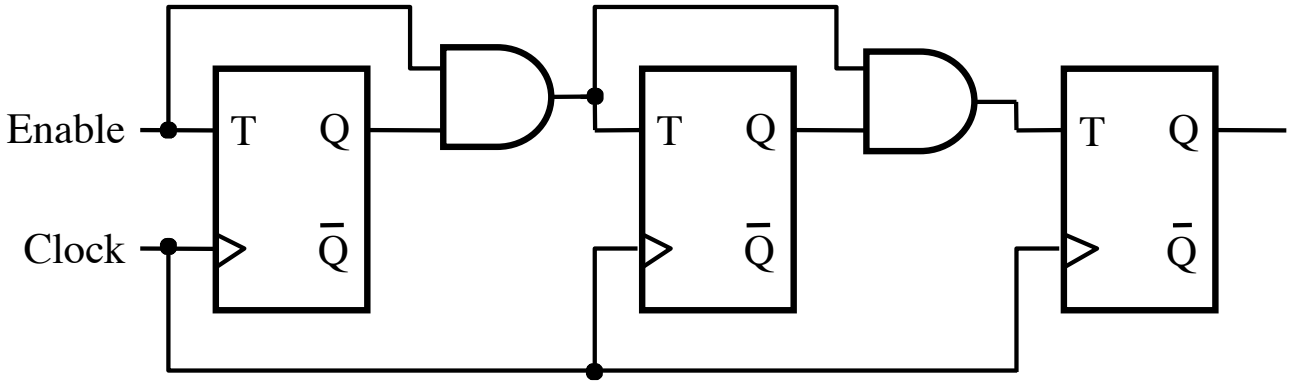


# T Flip-Flop

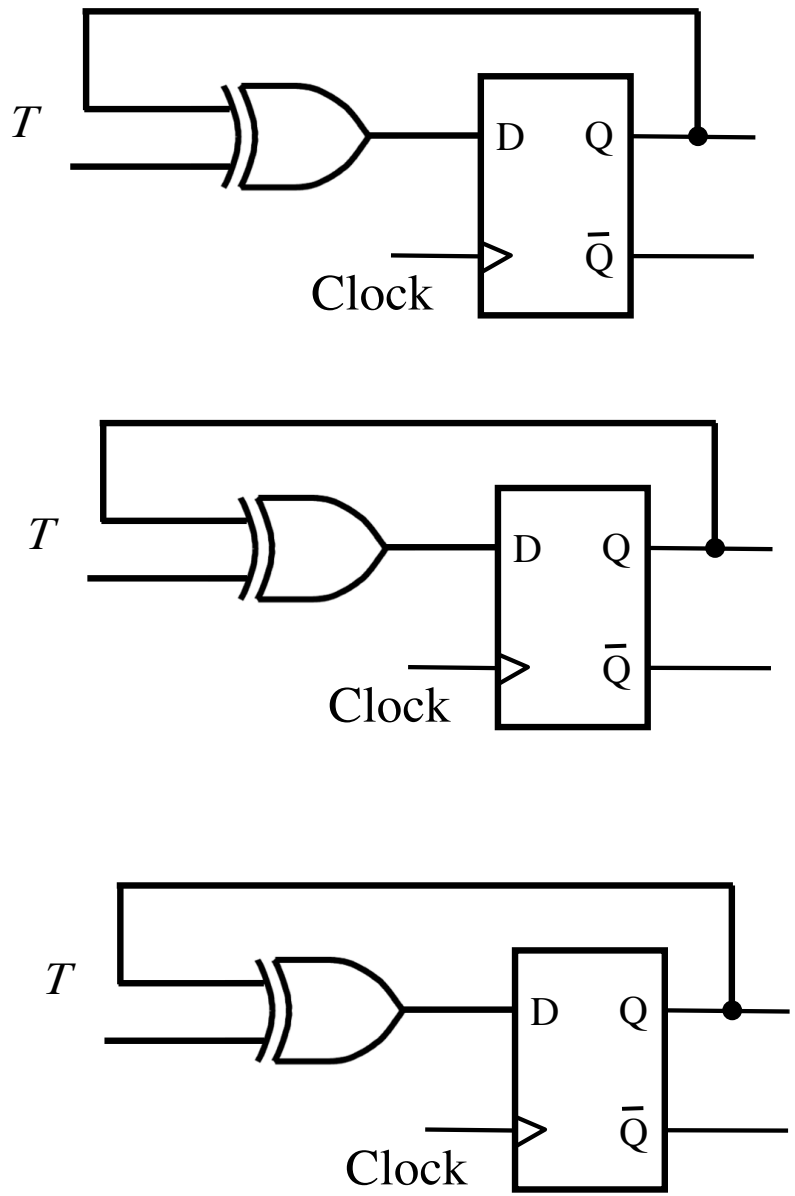


# Synchronous Counter with D Flip-Flops

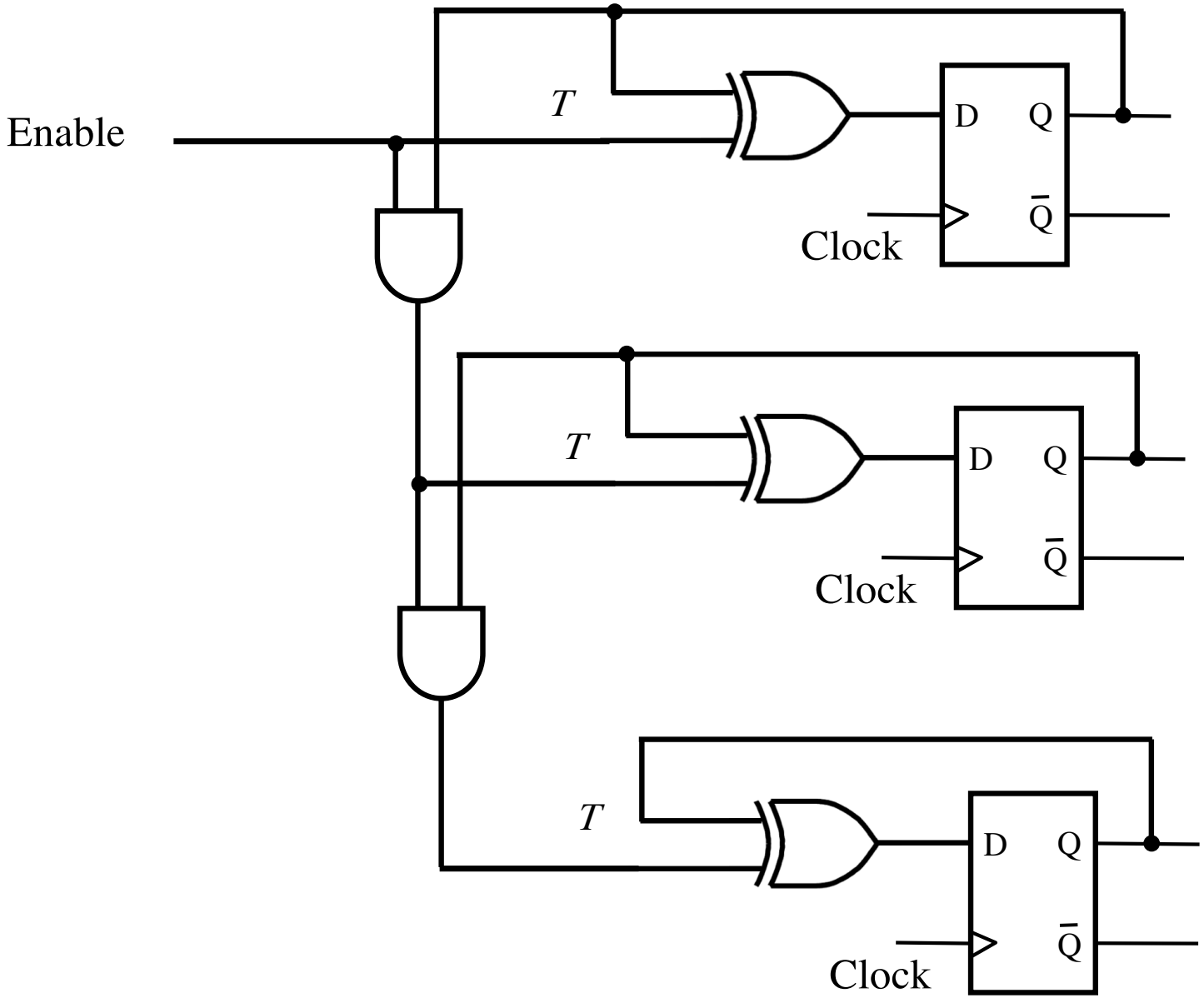
# A three-bit up-counter with T flip-flops



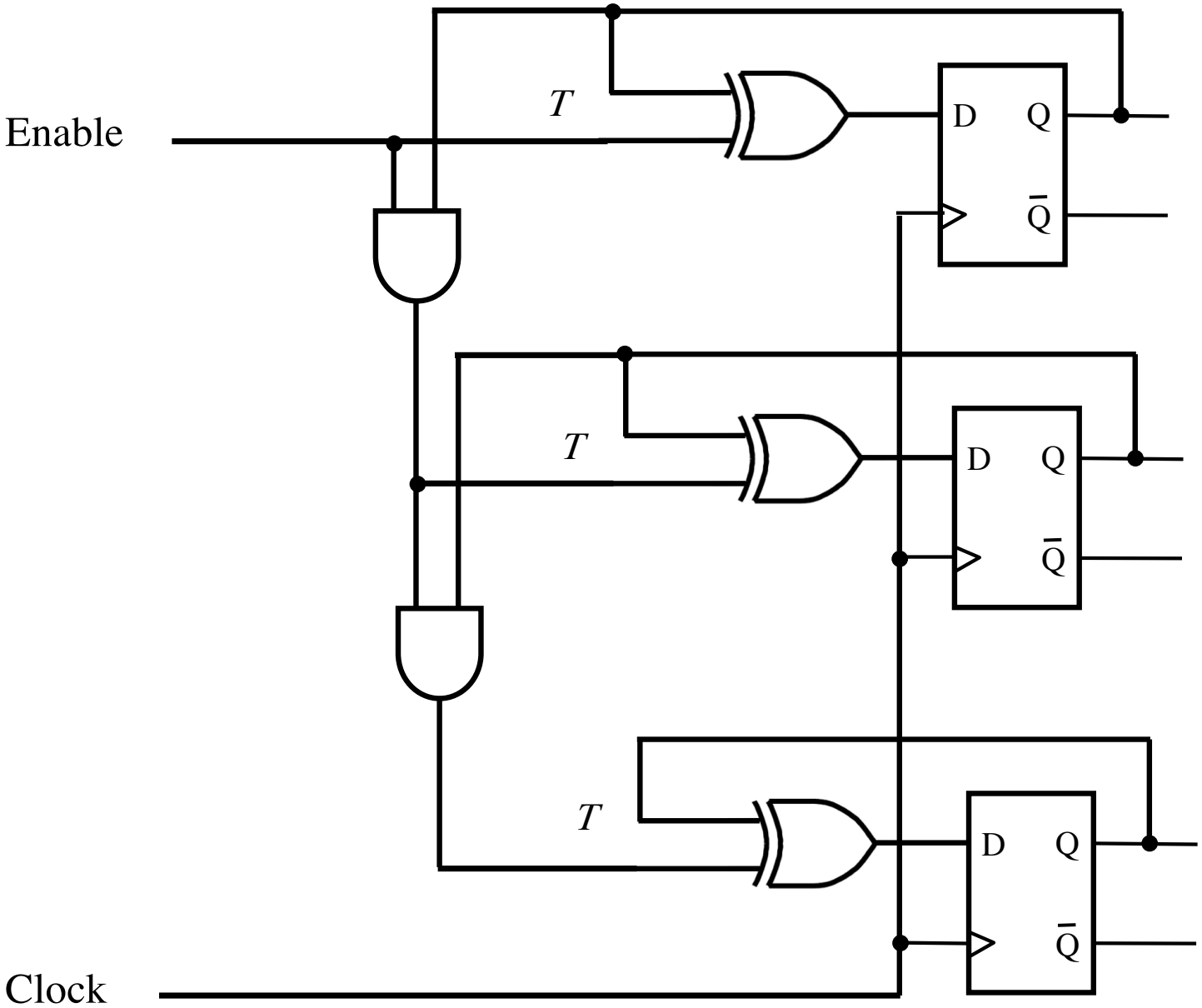
# A three-bit up-counter with D flip-flops



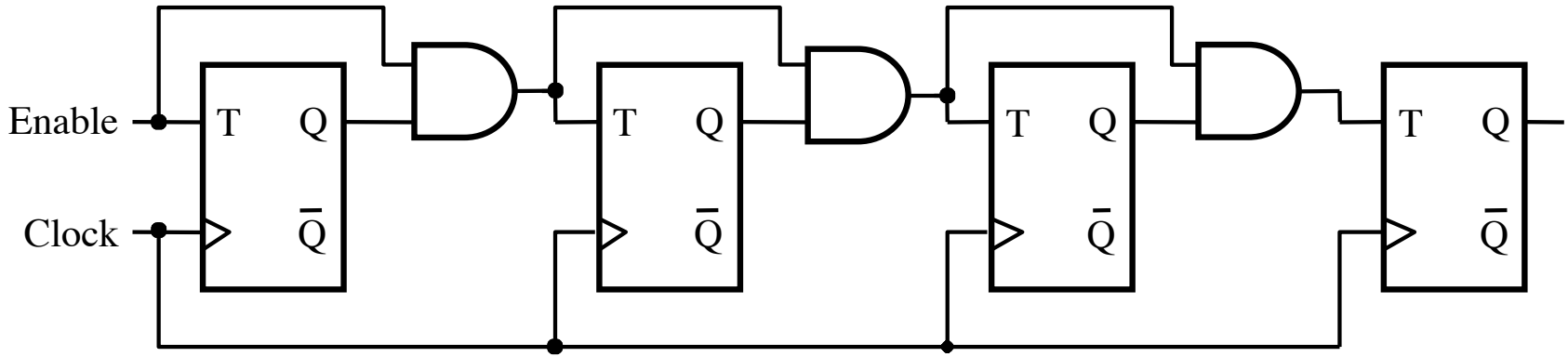
# A three-bit up-counter with D flip-flops



# A three-bit up-counter with D flip-flops

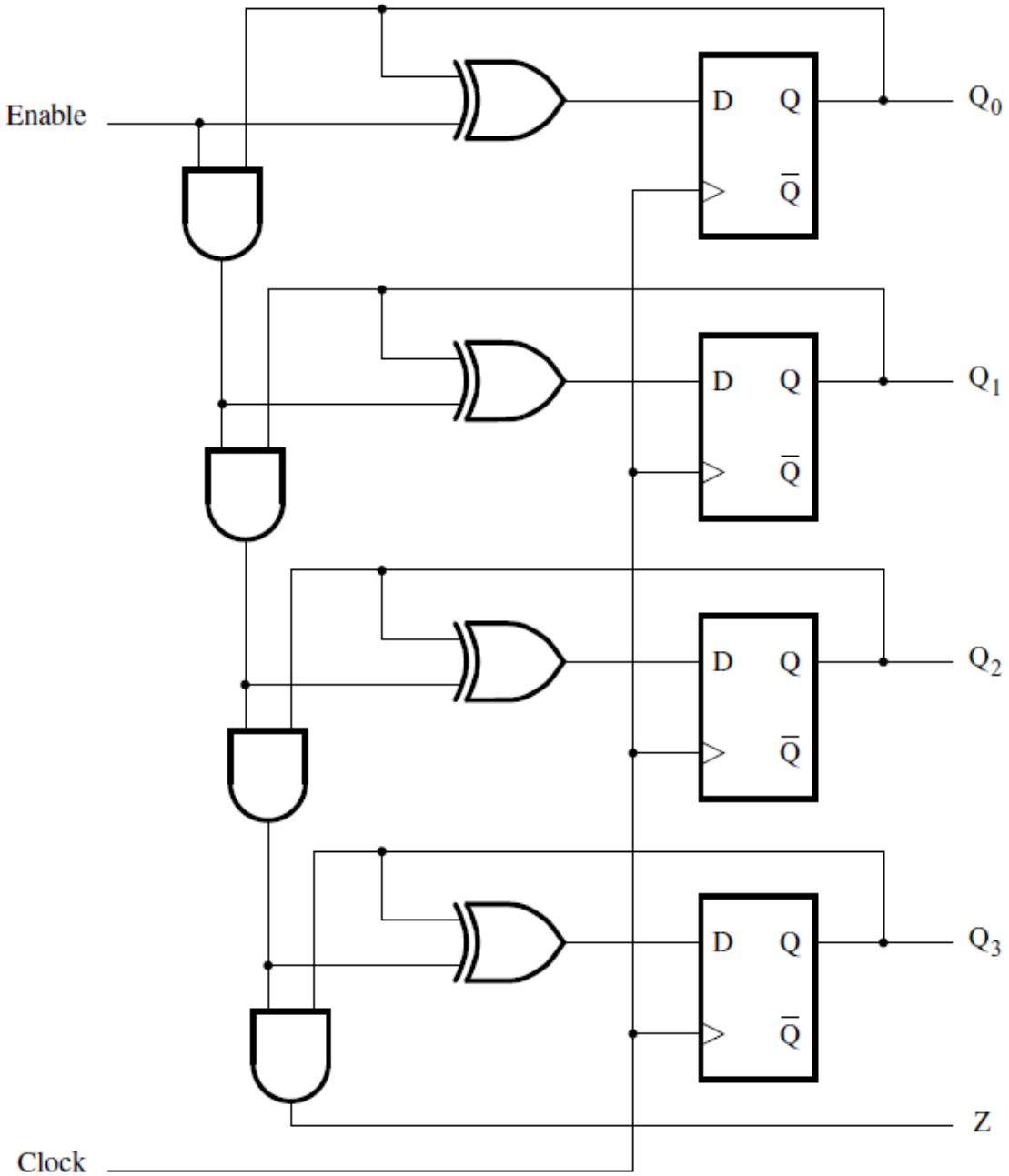


# A four-bit up-counter with T flip-flops




[ Figure 5.22 from the textbook (Modified) ]

# A four-bit up-counter with D flip-flops




[ Figure 5.23 from the textbook ]

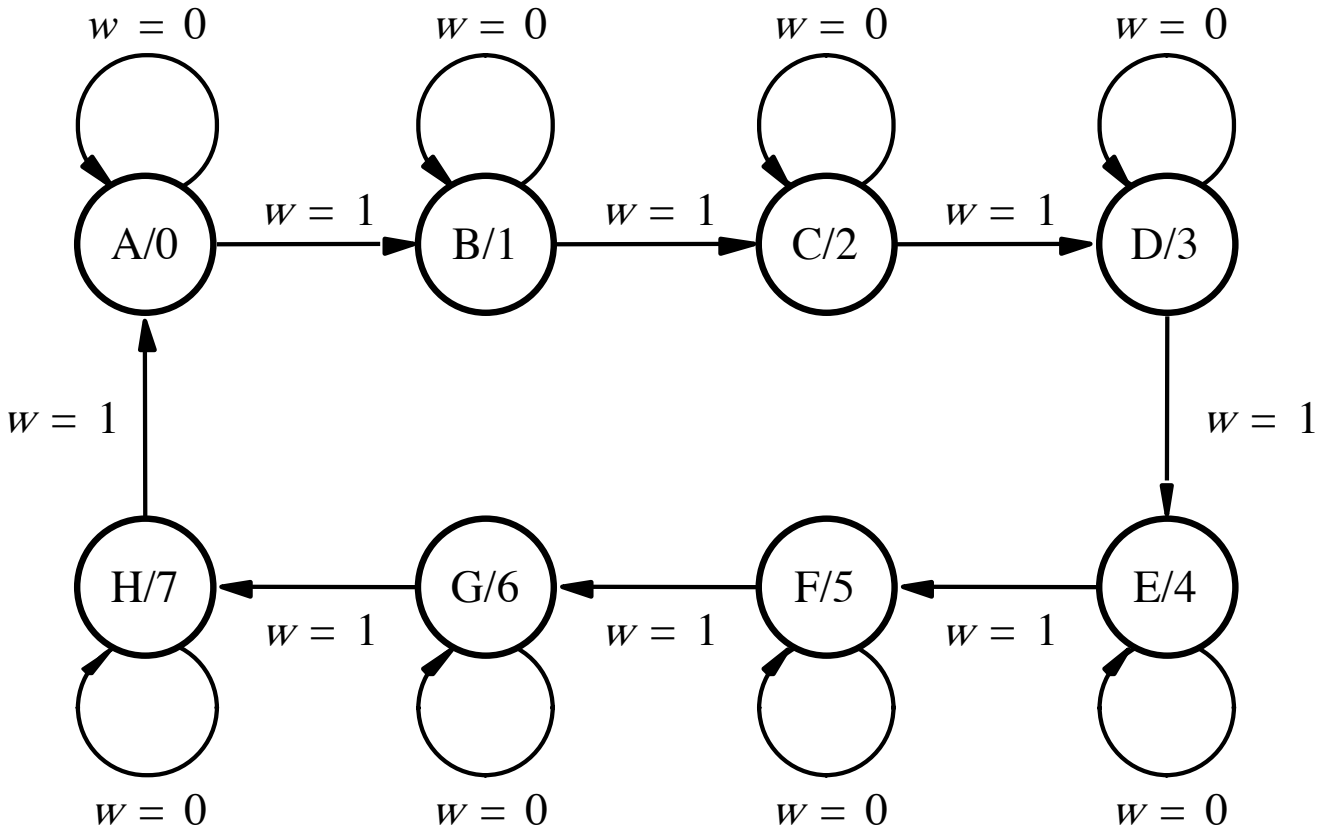
# End of Mini Review

# Goal

- **Implement a modulo-8 counter using the sequential circuit approach**
- **In other words, the counting sequence must be**

  **0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, …**
- **The count changes based on the input signal w:**
  - **If w=0, then the count remains the same**
  - **If w=1, then the count is advanced by one**

# State diagram for the counter




[ Figure 6.60 from the textbook ]

# State table for the counter

| Present state | Next state | | Output |
|---|---|---|---|
| | $w = 0$ | $w = 1$ | |
| A | A | B | 0 |
| B | B | C | 1 |
| C | C | D | 2 |
| D | D | E | 3 |
| E | E | F | 4 |
| F | F | G | 5 |
| G | G | H | 6 |
| H | H | A | 7 |

[ Figure 6.61 from the textbook ]

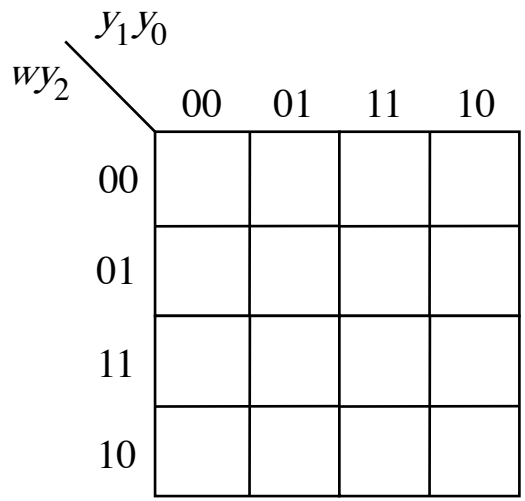
# State-assigned table for the counter

| | Present state $y_2y_1y_0$ | Next state | | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| | | $w = 0$ | $w = 1$ | |
| | | $Y_2Y_1Y_0$ | $Y_2Y_1Y_0$ | |
| A | 000 | 000 | 001 | 000 |
| B | 001 | 001 | 010 | 001 |
| C | 010 | 010 | 011 | 010 |
| D | 011 | 011 | 100 | 011 |
| E | 100 | 100 | 101 | 100 |
| F | 101 | 101 | 110 | 101 |
| G | 110 | 110 | 111 | 110 |
| H | 111 | 111 | 000 | 111 |

[ Figure 6.62 from the textbook ]

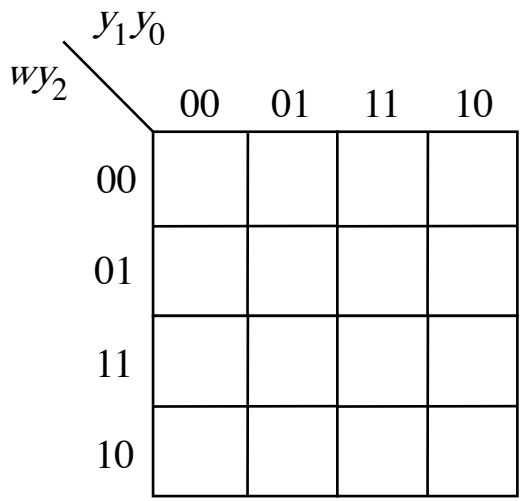
# K-map for $Y_0$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 000 | 001 | 000 |
| B | 001 | 001 | 010 | 001 |
| C | 010 | 010 | 011 | 010 |
| D | 011 | 011 | 100 | 011 |
| E | 100 | 100 | 101 | 100 |
| F | 101 | 101 | 110 | 101 |
| G | 110 | 110 | 111 | 110 |
| H | 111 | 111 | 000 | 111 |



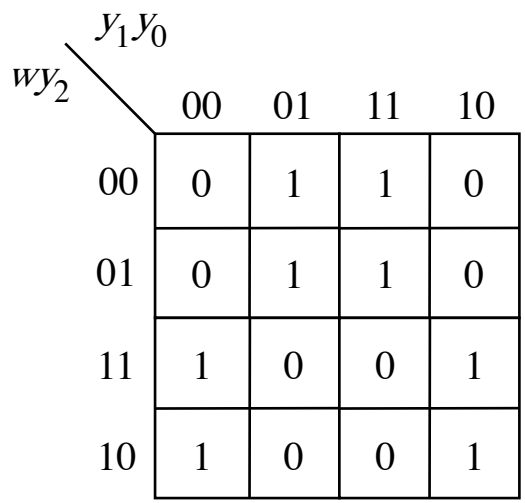
# K-map for $Y_0$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 1 | 000 |
| B | 001 | 1 | 0 | 001 |
| C | 010 | 0 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 1 | 100 |
| F | 101 | 1 | 0 | 101 |
| G | 110 | 0 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |



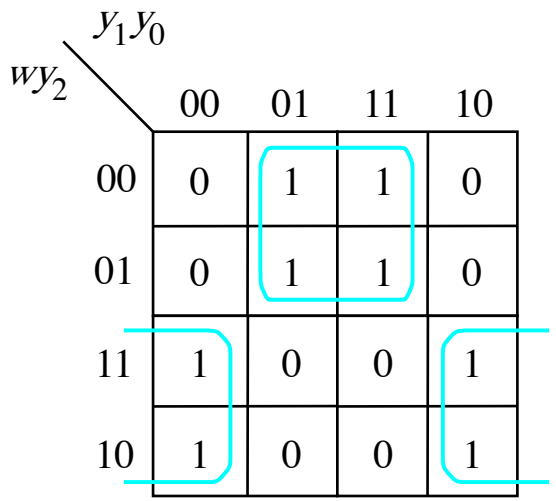
# K-map for $Y_0$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 1 | 000 |
| B | 001 | 1 | 0 | 001 |
| C | 010 | 0 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 1 | 100 |
| F | 101 | 1 | 0 | 101 |
| G | 110 | 0 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |



# K-map for $Y_0$

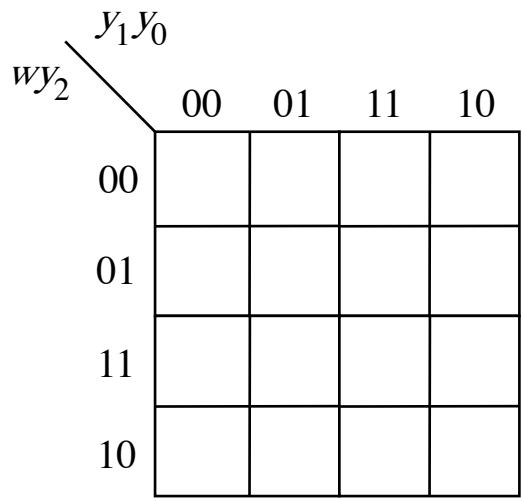
| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 1 | 000 |
| B | 001 | 1 | 0 | 001 |
| C | 010 | 0 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 1 | 100 |
| F | 101 | 1 | 0 | 101 |
| G | 110 | 0 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |




$$Y_0 = \bar{w}y_0 + w\bar{y}_0$$

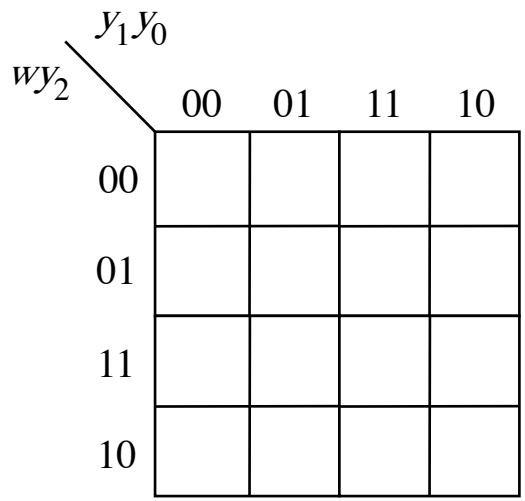
# K-map for $Y_1$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 000 | 001 | 000 |
| B | 001 | 001 | 010 | 001 |
| C | 010 | 010 | 011 | 010 |
| D | 011 | 011 | 100 | 011 |
| E | 100 | 100 | 101 | 100 |
| F | 101 | 101 | 110 | 101 |
| G | 110 | 110 | 111 | 110 |
| H | 111 | 111 | 000 | 111 |



# K-map for $Y_1$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 1 | 001 |
| C | 010 | 1 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 0 | 100 |
| F | 101 | 0 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |



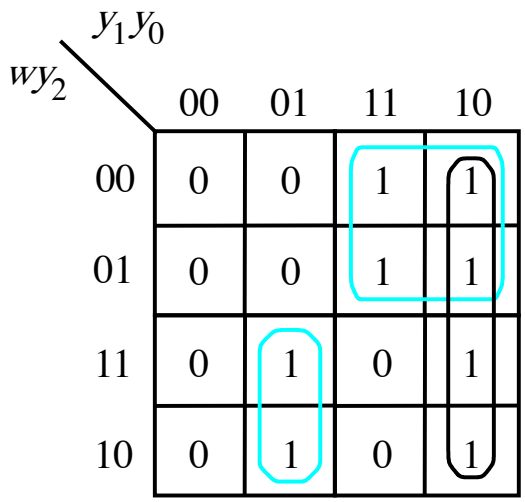
# K-map for $Y_1$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 1 | 001 |
| C | 010 | 1 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 0 | 100 |
| F | 101 | 0 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 0 | 1 | 0 | 1 |
| 10 | 0 | 1 | 0 | 1 |

# K-map for $Y_1$

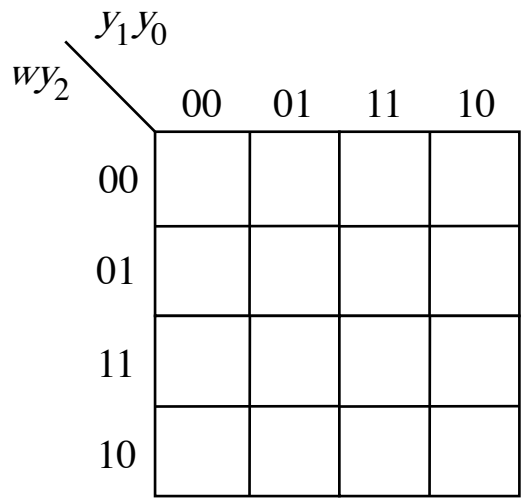
| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 1 | 001 |
| C | 010 | 1 | 1 | 010 |
| D | 011 | 1 | 0 | 011 |
| E | 100 | 0 | 0 | 100 |
| F | 101 | 0 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |




$$Y_1 = \bar{w}y_1 + y_1\bar{y}_0 + wy_0\bar{y}_1$$

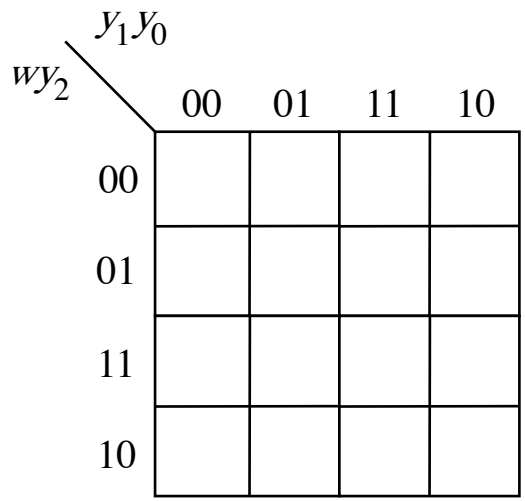
# K-map for $Y_2$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 000 | 001 | 000 |
| B | 001 | 001 | 010 | 001 |
| C | 010 | 010 | 011 | 010 |
| D | 011 | 011 | 100 | 011 |
| E | 100 | 100 | 101 | 100 |
| F | 101 | 101 | 110 | 101 |
| G | 110 | 110 | 111 | 110 |
| H | 111 | 111 | 000 | 111 |



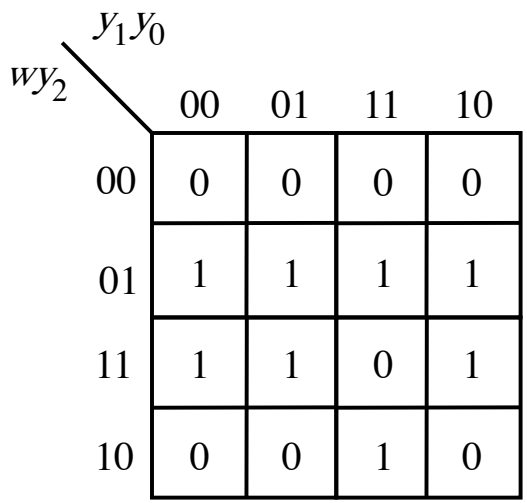
# K-map for $Y_2$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 0 | 001 |
| C | 010 | 0 | 0 | 010 |
| D | 011 | 0 | 1 | 011 |
| E | 100 | 1 | 1 | 100 |
| F | 101 | 1 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |



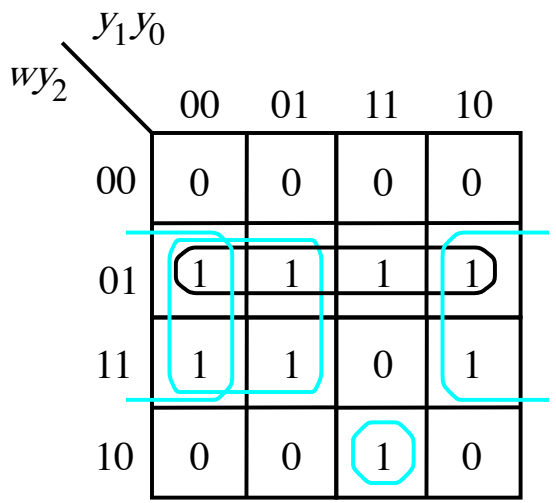
# K-map for $Y_2$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 0 | 001 |
| C | 010 | 0 | 0 | 010 |
| D | 011 | 0 | 1 | 011 |
| E | 100 | 1 | 1 | 100 |
| F | 101 | 1 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |



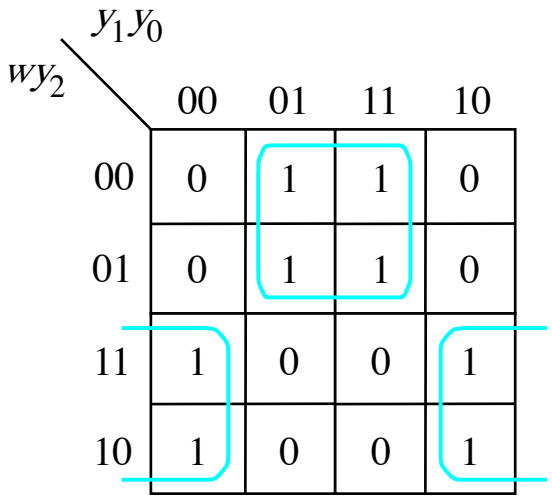
# K-map for $Y_2$

| | Present state $y_2y_1y_0$ | Next state $w = 0$ $Y_2Y_1Y_0$ | Next state $w = 1$ $Y_2Y_1Y_0$ | Count $z_2z_1z_0$ |
|---|---|---|---|---|
| A | 000 | 0 | 0 | 000 |
| B | 001 | 0 | 0 | 001 |
| C | 010 | 0 | 0 | 010 |
| D | 011 | 0 | 1 | 011 |
| E | 100 | 1 | 1 | 100 |
| F | 101 | 1 | 1 | 101 |
| G | 110 | 1 | 1 | 110 |
| H | 111 | 1 | 0 | 111 |

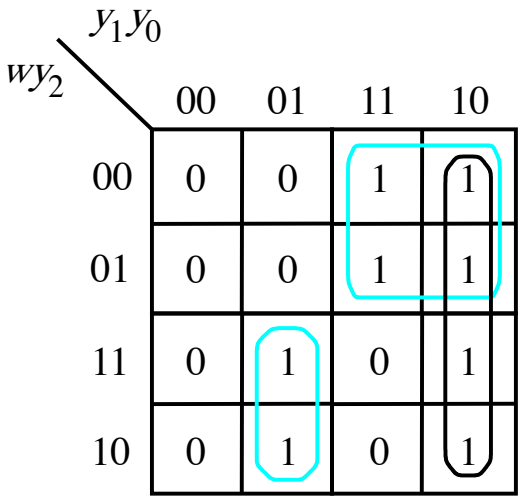



$$Y_2 = \bar{w}y_2 + \bar{y}_0y_2 + \bar{y}_1y_2 + wy_0y_1\bar{y}_2$$

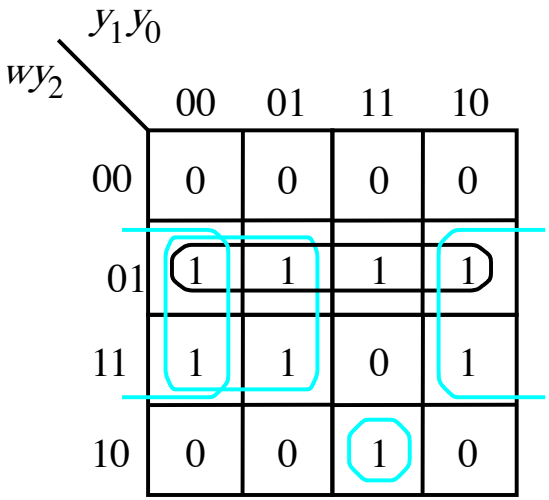
# Karnaugh maps for D flip-flops for the counter




$$Y_0 = \bar{w}y_0 + w\bar{y}_0$$



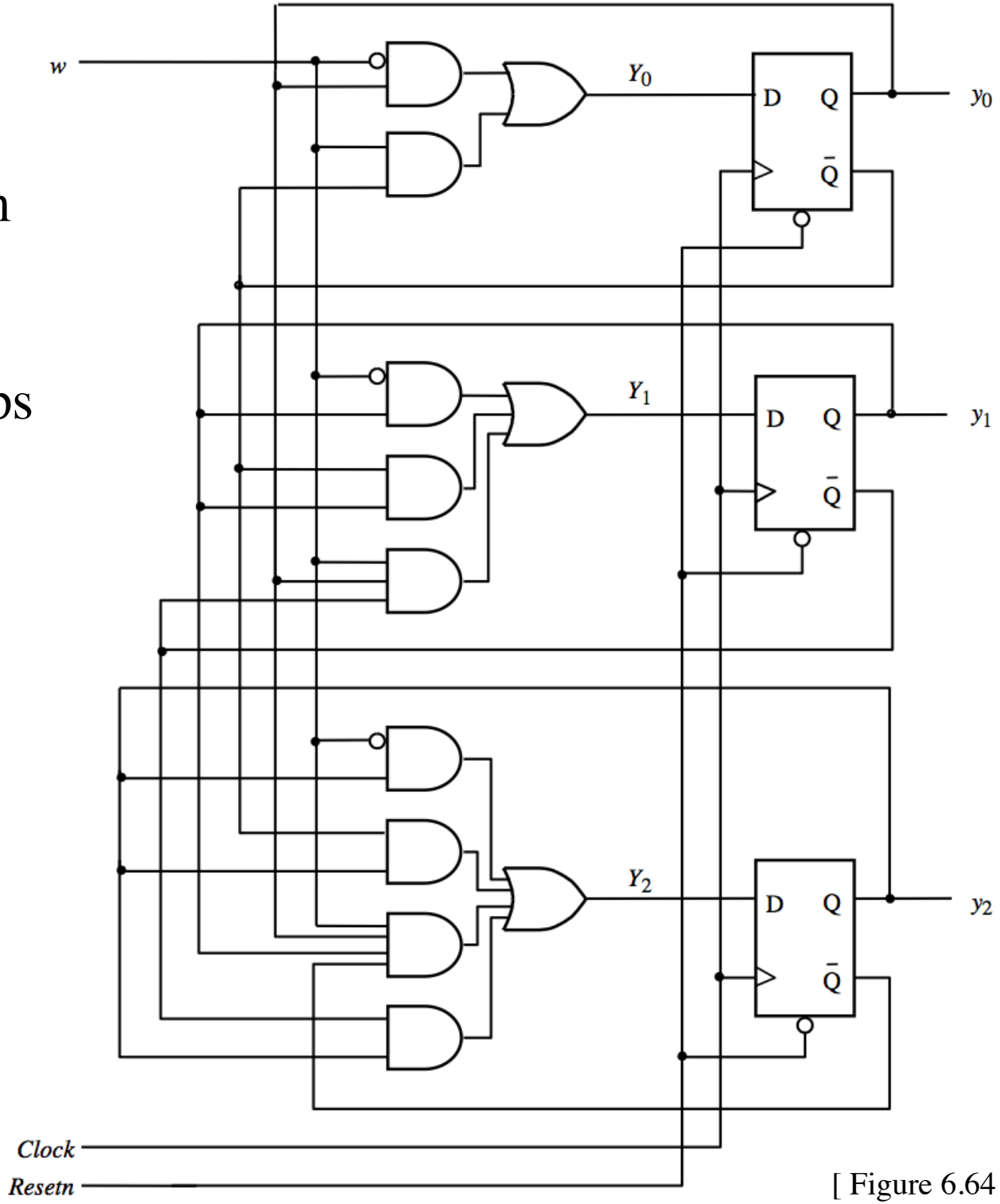

$$Y_1 = \bar{w}y_1 + y_1\bar{y}_0 + wy_0\bar{y}_1$$




$$Y_2 = \bar{w}y_2 + y_0y_2 + \bar{y}_1y_2 + wy_0\bar{y}_1\bar{y}_2$$

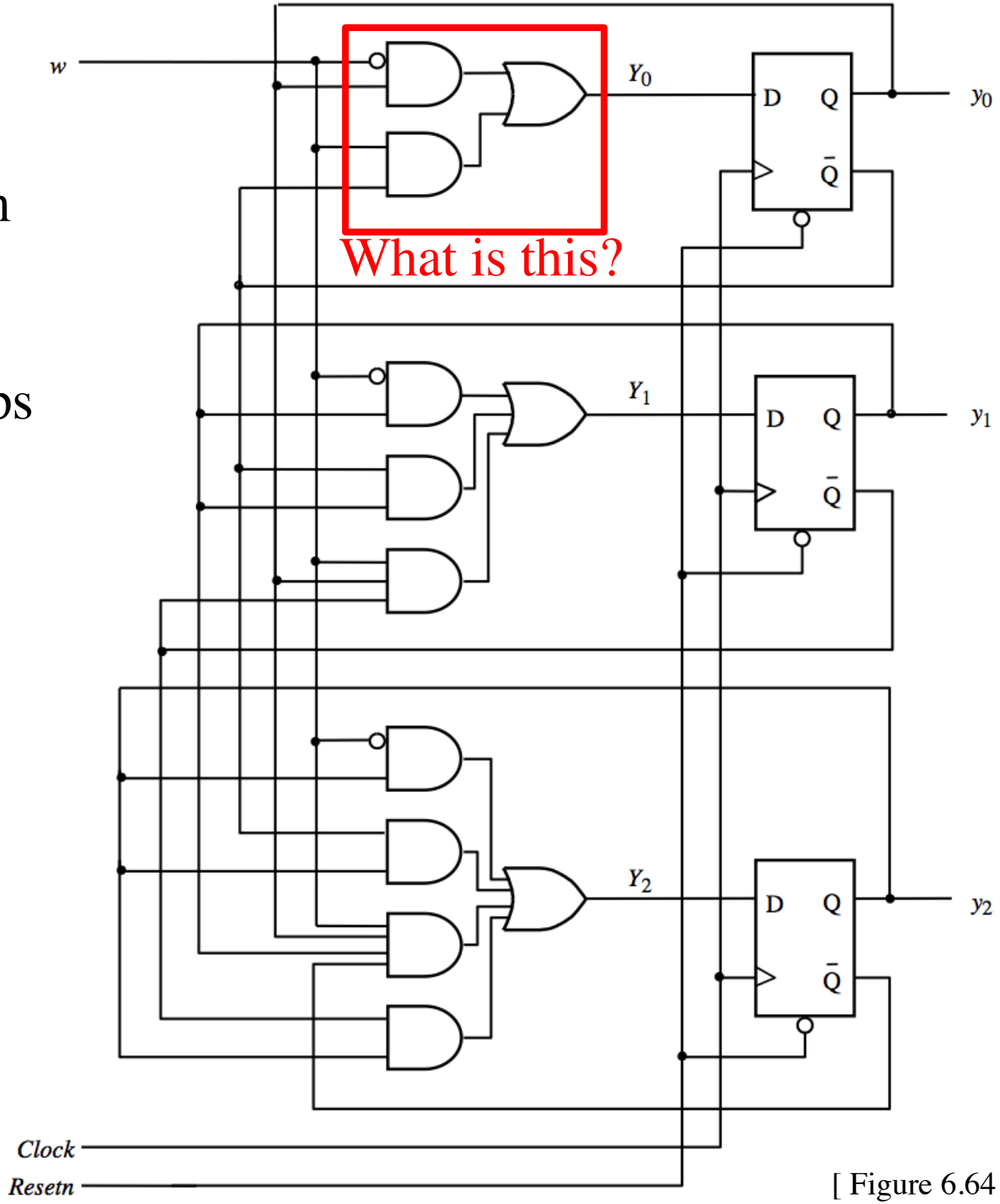
[ Figure 6.63 from the textbook ]

Circuit diagram for the counter implemented with D flip-flops



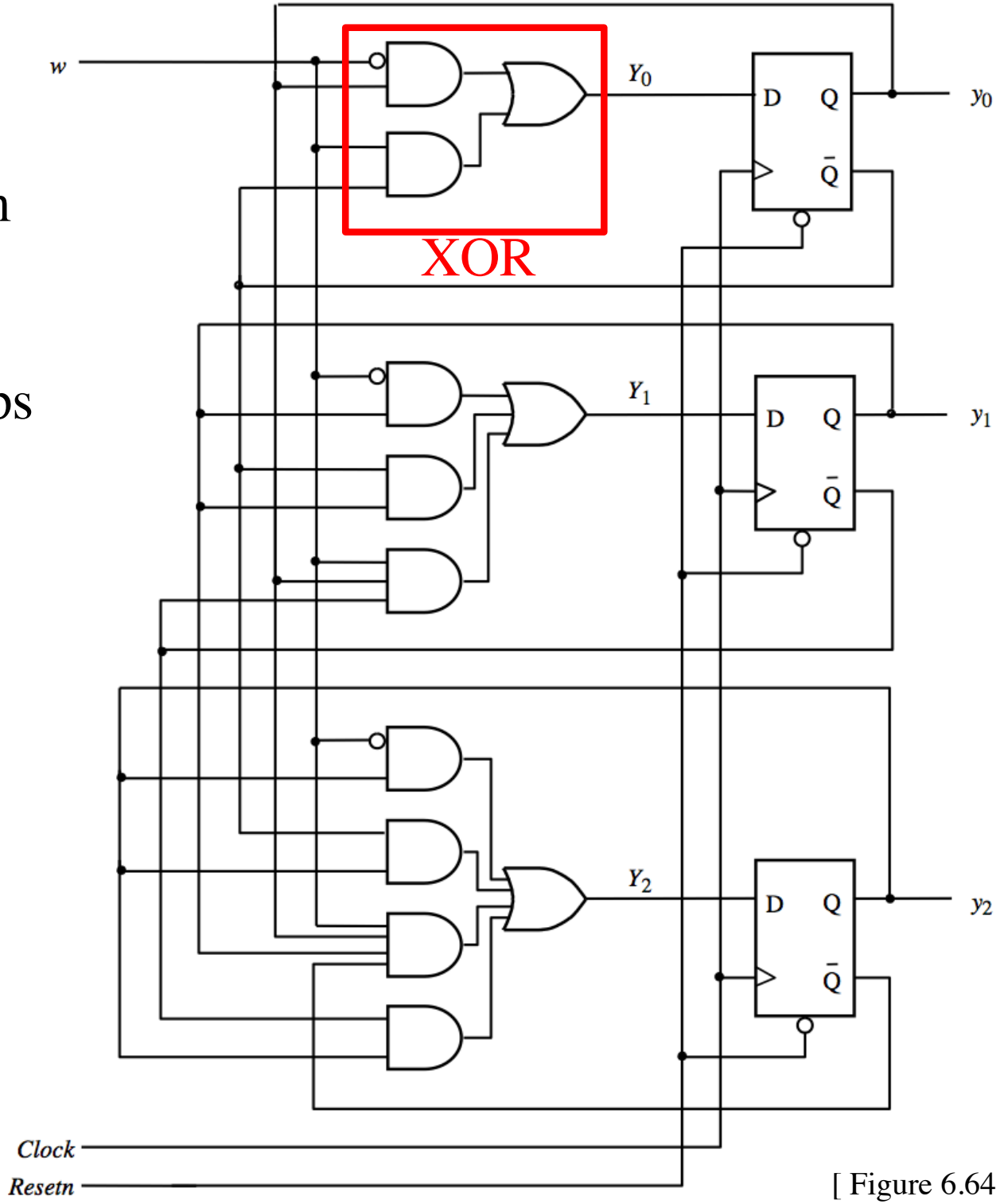

[ Figure 6.64 from the textbook ]

Circuit diagram for the counter implemented with D flip-flops




[ Figure 6.64 from the textbook ]

Circuit diagram for the counter implemented with D flip-flops




[ Figure 6.64 from the textbook ]

# We can simplify all three expressions

$$Y_0 = \bar{w} y_0 + w \bar{y}_0$$

$$Y_1 = \bar{w} y_1 + y_1 \bar{y}_0 + w y_0 \bar{y}_1$$

$$Y_2 = \bar{w} y_2 + \bar{y}_0 y_2 + \bar{y}_1 y_2 + w y_0 y_1 \bar{y}_2$$

# We can simplify all three expressions

$$Y_0 = \bar{w}y_0 + w\bar{y}_0$$

$$\begin{aligned} D_0 &= \overline{w}y_0 + w\overline{y}_0 \\ &= w \oplus y_0 \end{aligned}$$

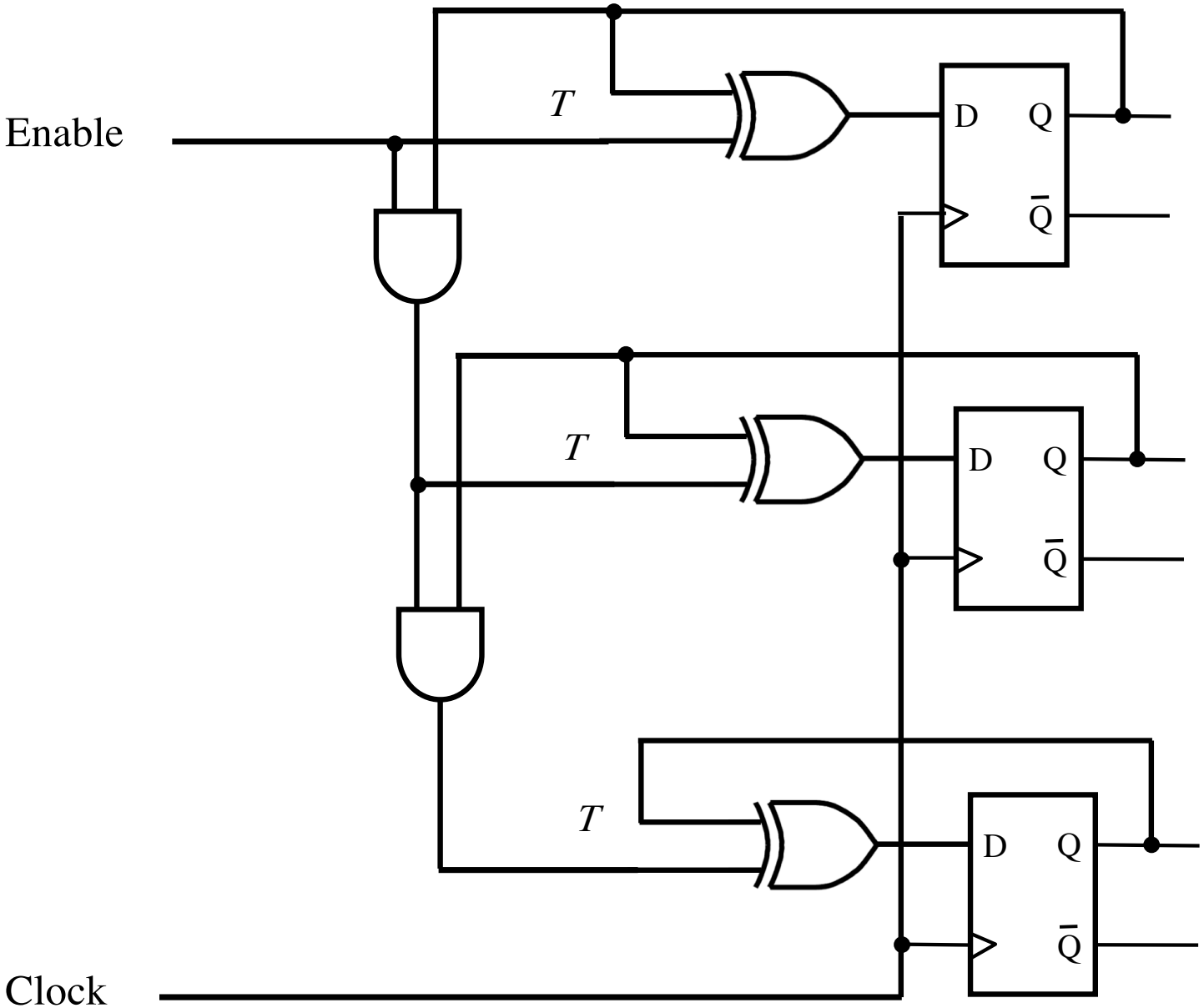
$$Y_1 = \bar{w}y_1 + y_1\bar{y}_0 + wy_0\bar{y}_1$$

$$\begin{aligned} D_1 &= \overline{w}y_1 + y_1\overline{y}_0 + wy_0\overline{y}_1 \\ &= (\overline{w} + \overline{y}_0)y_1 + wy_0\overline{y}_1 \\ &= \overline{wy}_0y_1 + wy_0\overline{y}_1 \\ &= wy_0 \oplus y_1 \end{aligned}$$

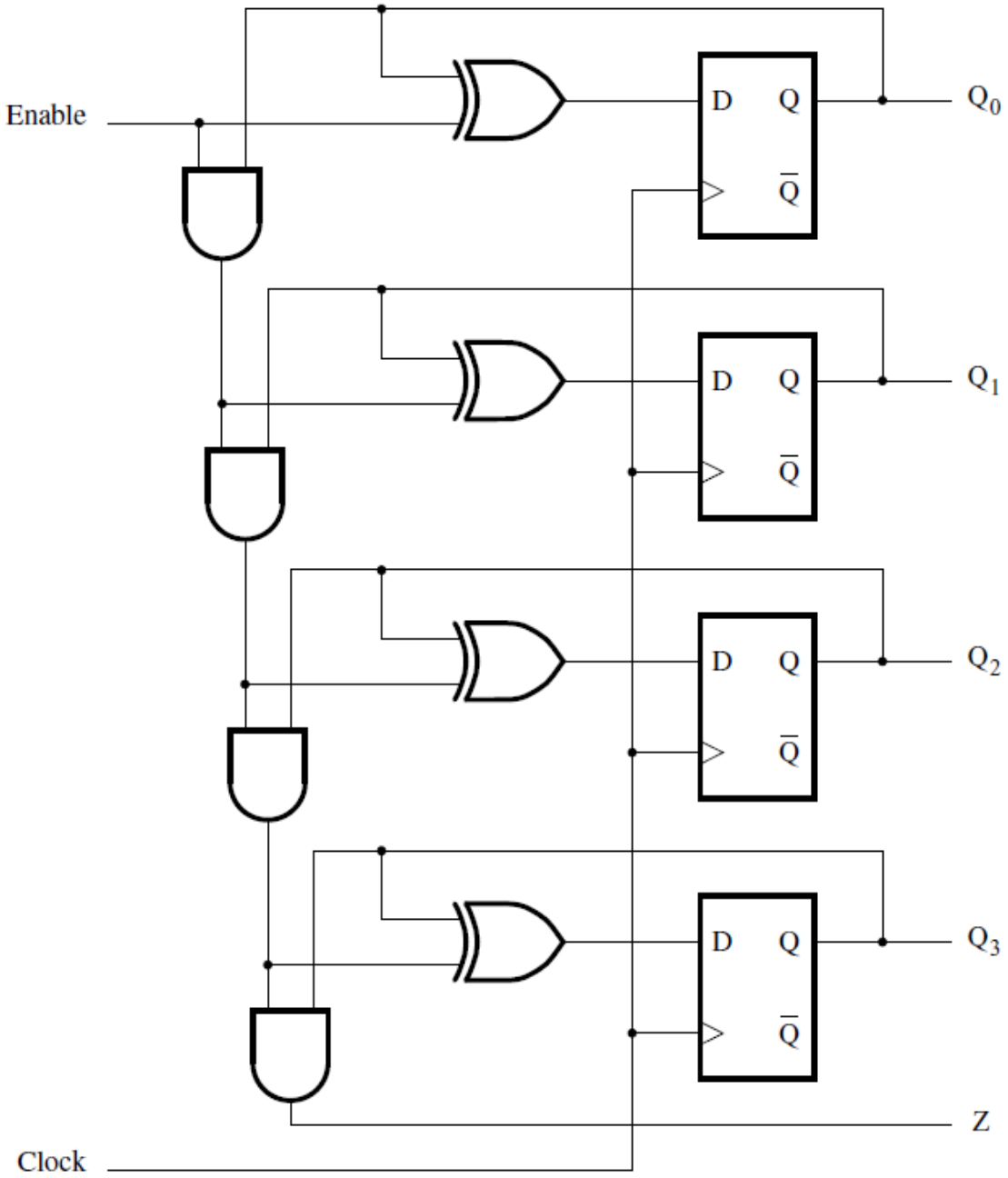
$$Y_2 = \bar{w}y_2 + \bar{y}_0y_2 + \bar{y}_1y_2 + wy_0y_1\bar{y}_2$$

$$\begin{aligned} D_2 &= \overline{w}y_2 + \overline{y}_0y_2 + \overline{y}_1y_2 + wy_0y_1\overline{y}_2 \\ &= (\overline{w} + \overline{y}_0 + \overline{y}_1)y_2 + wy_0y_1\overline{y}_2 \\ &= \overline{wy_0y}_1y_2 + wy_0y_1\overline{y}_2 \\ &= wy_0y_1 \oplus y_2 \end{aligned}$$

# A three-bit counter with D flip-flops



# A four-bit counter with D flip-flops




[ Figure 5.23 from the textbook ]

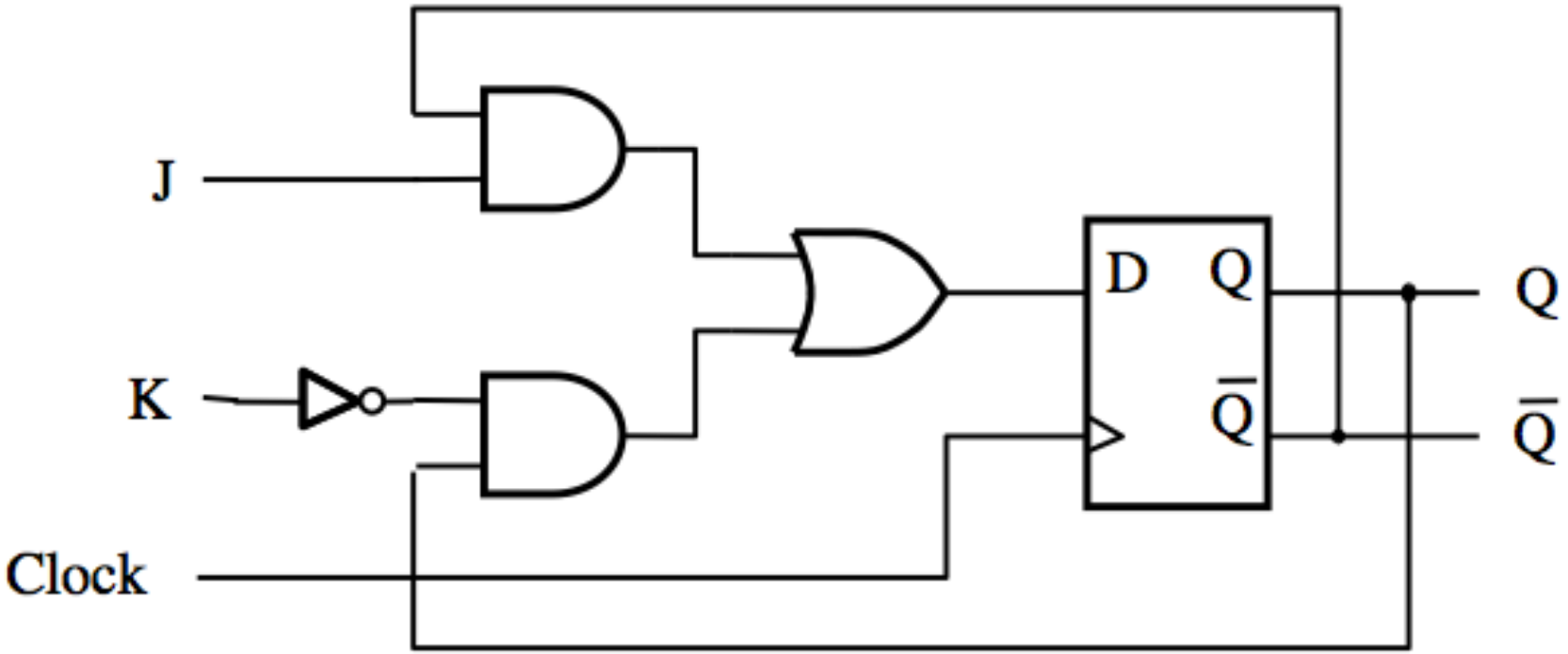
# Summary

- **The up-counters that we studied in Chapter 5 can now be derived using the sequential circuit approach**

- **We get the same circuit diagrams as before**

# Example 2:
# Implement a modulo-8 counter using JK Flip-Flops

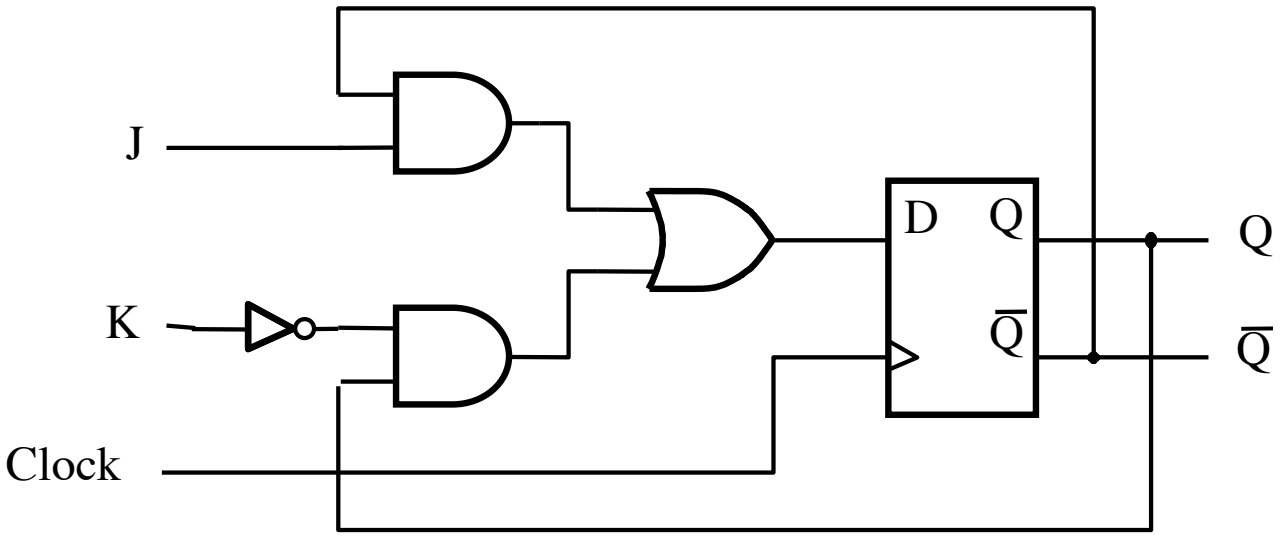
# JK Flip-Flop




$$D = J\overline{Q} + \overline{K}Q$$

[ Figure 5.16a from the textbook ]

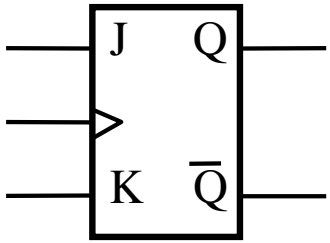
# JK Flip-Flop




(a) Circuit

| J | K | Q(t+1) |
|---|---|---|
| 0 | 0 | Q(t) |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | $\overline{Q}(t)$ |

(b) Truth table




(c) Graphical symbol

[ Figure 5.16 from the textbook ]

# JK Flip-Flop
# (How it Works)

**A versatile circuit that can be used both as a SR flip-flop and as a T flip flop**

**If J=0 and K =0 it stays in the same state**

**Just like SR It can be set and reset**

**J=S and K=R**

**If J=K then it behaves as a T flip-flop**

# Transition Rules in terms of J and K

Current State of the Flip-flop: Q(t)

Next State of the Flip-flop: Q(t+1)

| J | K | Q(t+1) |
|---|---|---|
| 0 | 0 | Q(t) |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | $\overline{Q}(t)$ |

- **From 0 to 0** **J=0 and K= d**
- **From 0 to 1** **J=1 and K= d**
- **From 1 to 0** **J=d and K= 1**
- **From 1 to 1** **J=d and K= 0**

# Transition Rules in terms of J and K

Current State of the Flip-flop: Q(t)

Next State of the Flip-flop: Q(t+1)

| J | K | Q(t+1) |
|---|---|---|
| 0 | 0 | Q(t) |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | $\overline{Q}(t)$ |

- **From 0 to 0** **J=0 and K= d**
- **From 0 to 1** **J=1 and K= d**
- **From 1 to 0** **J=d and K= 1**
- **From 1 to 1** **J=d and K= 0**

| Q(t) → Q(t+1) | J K |
|---|---|
| 0 → 0 | 0 d |
| 0 → 1 | 1 d |
| 1 → 0 | d 1 |
| 1 → 1 | d 0 |

# Excitation table for the counter with JK flip-flops

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

[ Figure 6.65 from the textbook ]

# Excitation table for the counter with JK flip-flops

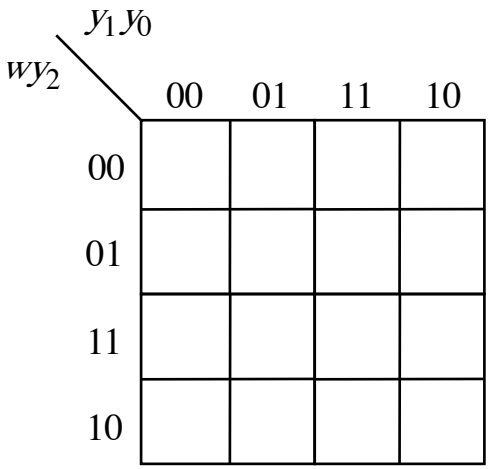
| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

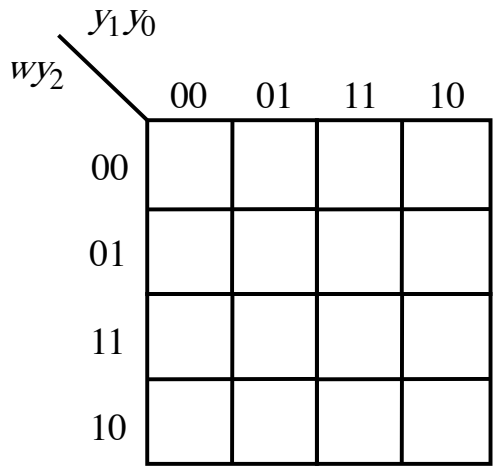
| Q(t) → Q(t+1) | J K |
|---|---|
| 0 → 0 | 0 d |
| 0 → 1 | 1 d |
| 1 → 0 | d 1 |
| 1 → 1 | d 0 |

[ Figure 6.65 from the textbook ]

# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |



# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

$J_0$ →

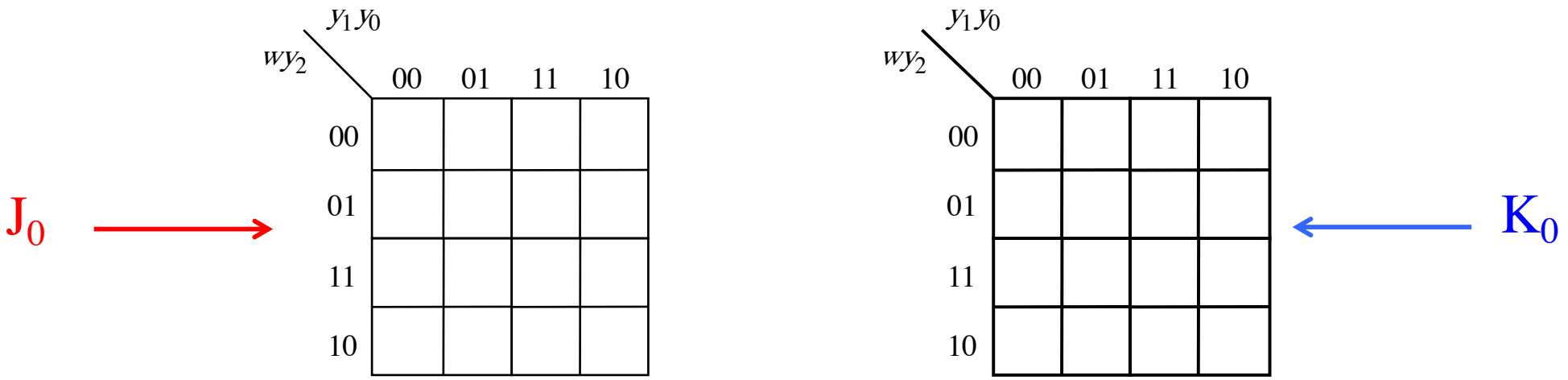
| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

← $K_0$

# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |



# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

$J_0$ →

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | d | d | 0 |
| 01 | 0 | d | d | 0 |
| 11 | 1 | d | d | 1 |
| 10 | 1 | d | d | 1 |

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

← $K_0$

# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

$J_0$ →

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | d | d | 0 |
| 01 | 0 | d | d | 0 |
| 11 | 1 | d | d | 1 |
| 10 | 1 | d | d | 1 |

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | d | 0 | 0 | d |
| 01 | d | 0 | 0 | d |
| 11 | d | 1 | 1 | d |
| 10 | d | 1 | 1 | d |

← $K_0$

# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

$J_0$ →

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | d | d | 0 |
| 01 | 0 | d | d | 0 |
| 11 | 1 | d | d | 1 |
| 10 | 1 | d | d | 1 |

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | d | 0 | 0 | d |
| 01 | d | 0 | 0 | d |
| 11 | d | 1 | 1 | d |
| 10 | d | 1 | 1 | d |

← $K_0$

# Karnaugh maps for the first JK flip-flop

| | Present state $y_2y_1y_0$ | Flip-flop inputs | | | | | | | Count $z_2z_1z_0$ |
|---|---|---|---|---|---|---|---|---|---|
| | | $w = 0$ | | | | $w = 1$ | | | |
| | | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | $Y_2Y_1Y_0$ | $J_2K_2$ | $J_1K_1$ | $J_0K_0$ | |
| A | 000 | 000 | 0d | 0d | 0d | 001 | 0d | 0d | 1d | 000 |
| B | 001 | 001 | 0d | 0d | d0 | 010 | 0d | 1d | d1 | 001 |
| C | 010 | 010 | 0d | d0 | 0d | 011 | 0d | d0 | 1d | 010 |
| D | 011 | 011 | 0d | d0 | d0 | 100 | 1d | d1 | d1 | 011 |
| E | 100 | 100 | d0 | 0d | 0d | 101 | d0 | 0d | 1d | 100 |
| F | 101 | 101 | d0 | 0d | d0 | 110 | d0 | 1d | d1 | 101 |
| G | 110 | 110 | d0 | d0 | 0d | 111 | d0 | d0 | 1d | 110 |
| H | 111 | 111 | d0 | d0 | d0 | 000 | d1 | d1 | d1 | 111 |

$J_0$

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | d | d | 0 |
| 01 | 0 | d | d | 0 |
| 11 | 1 | d | d | 1 |
| 10 | 1 | d | d | 1 |

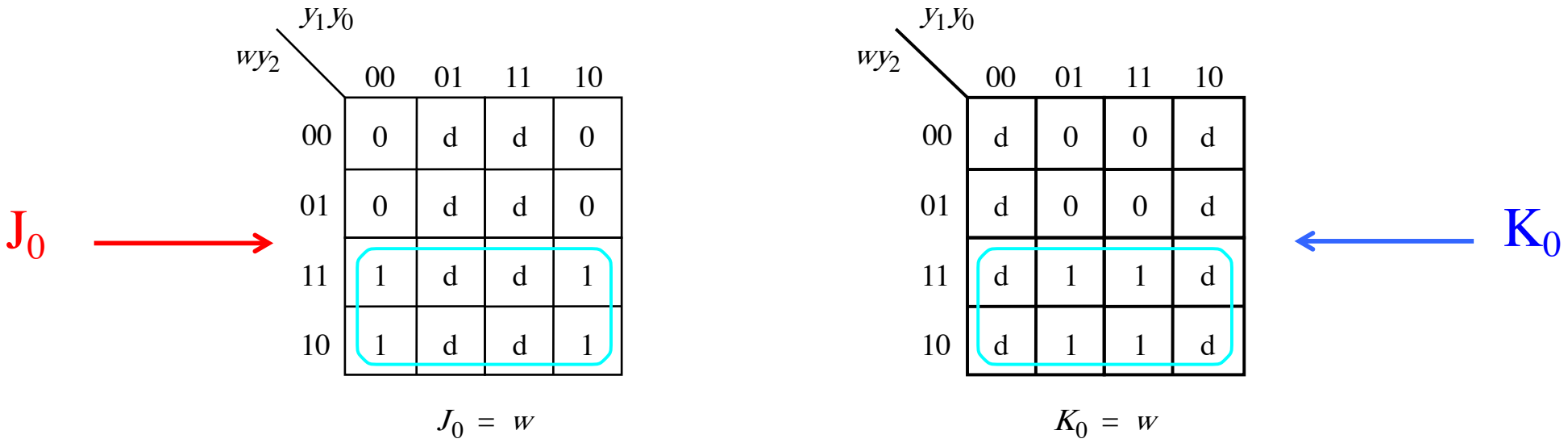
$J_0 = w$

| $wy_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | d | 0 | 0 | d |
| 01 | d | 0 | 0 | d |
| 11 | d | 1 | 1 | d |
| 10 | d | 1 | 1 | d |

$K_0$

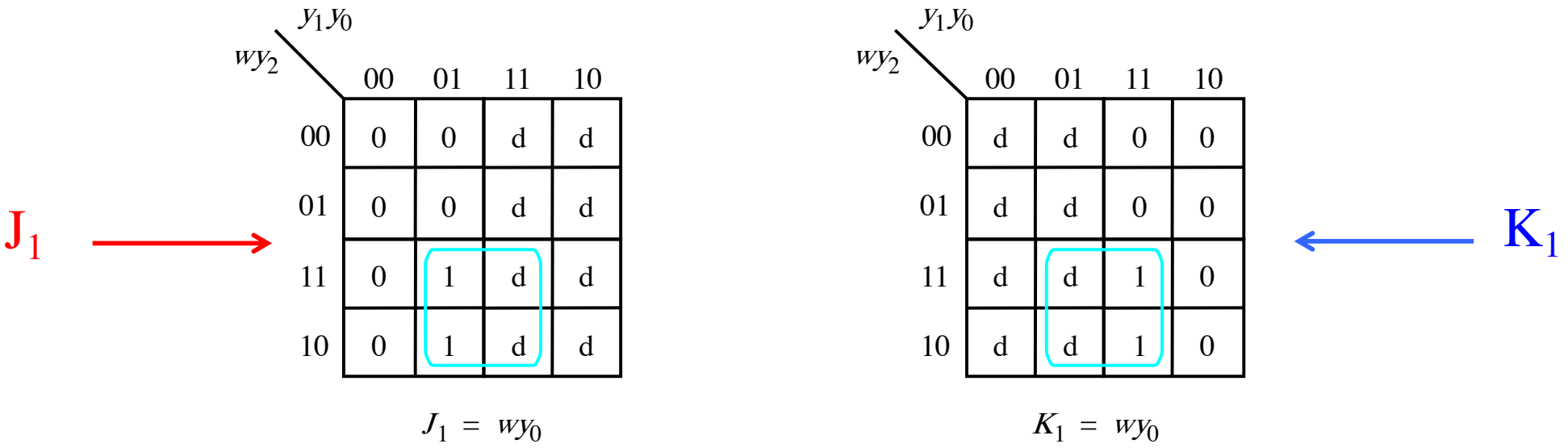
$K_0 = w$

# Karnaugh maps for the first JK flip-flop



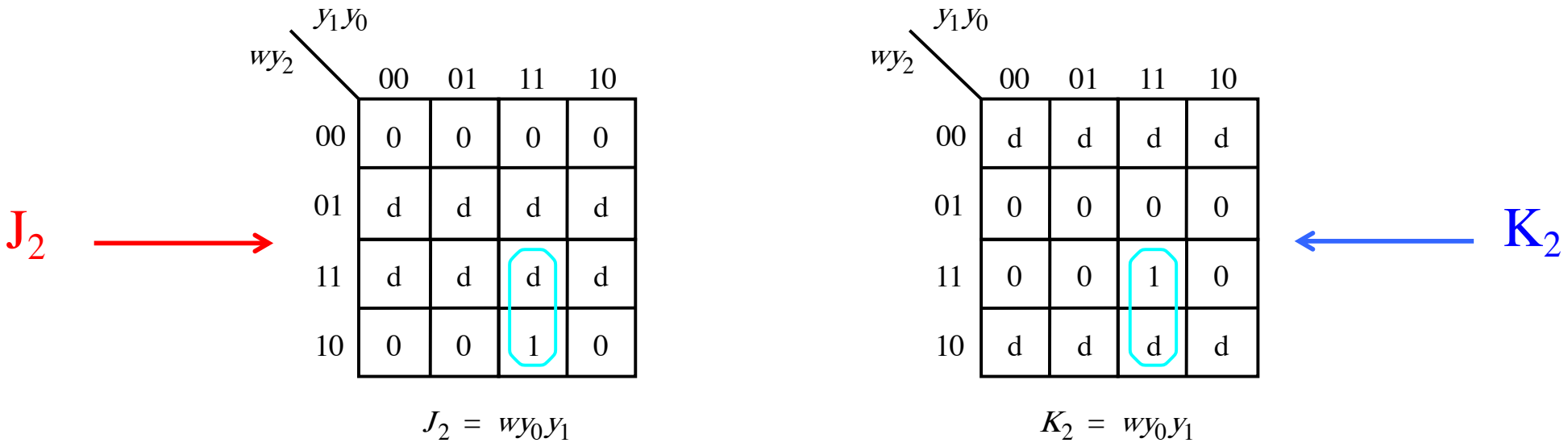

[ Figure 6.66 from the textbook ]

# Karnaugh maps for the second JK flip-flop




[ Figure 6.66 from the textbook ]

# Karnaugh maps for the third JK flip-flop



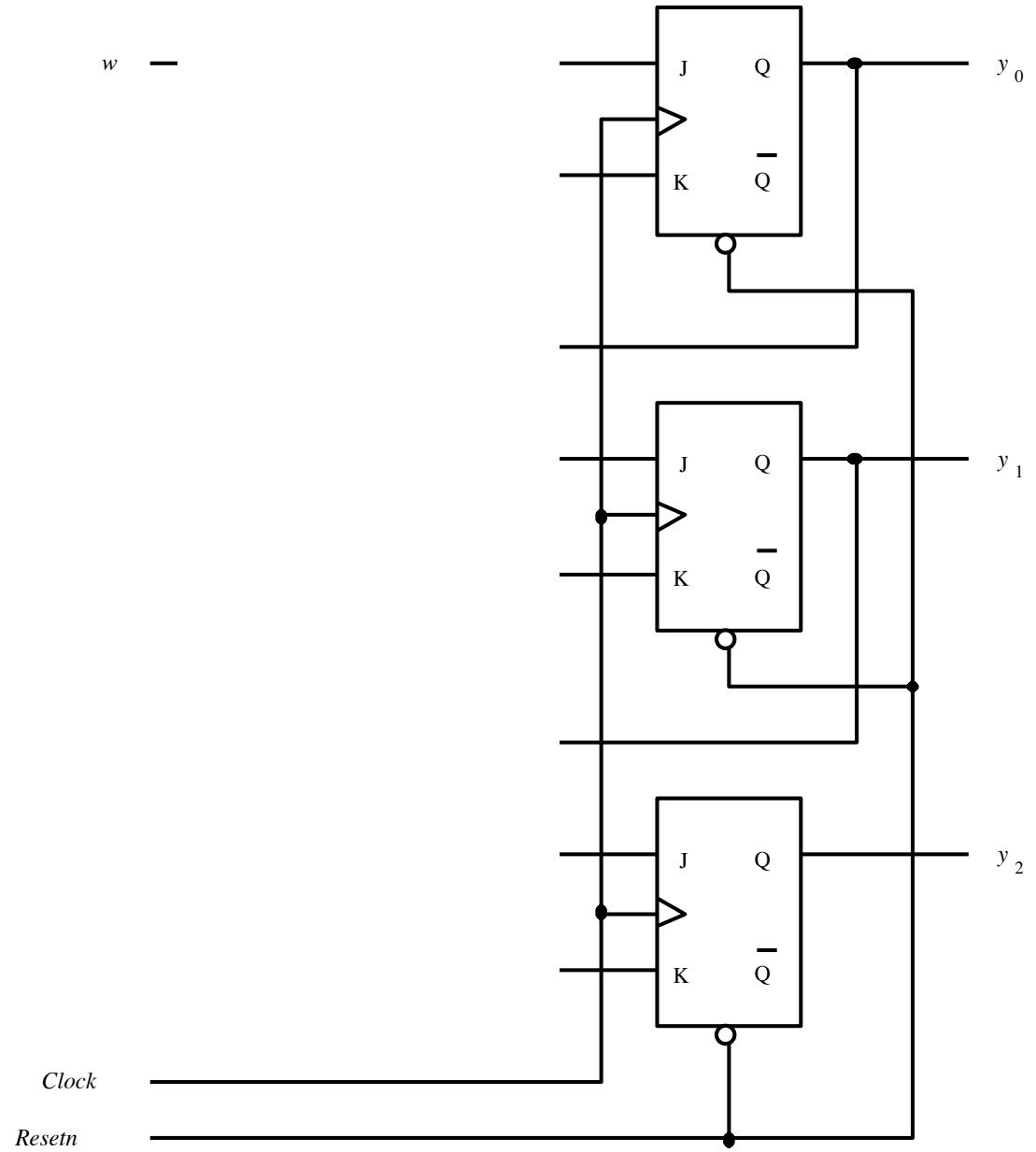

[ Figure 6.66 from the textbook ]

# Circuit diagram using JK flip-flops

$$J_0 = w$$
$$K_0 = w$$

$$J_1 = wy_0$$
$$K_1 = wy_0$$

$$J_2 = wy_0y_1$$
$$K_2 = wy_0y_1$$

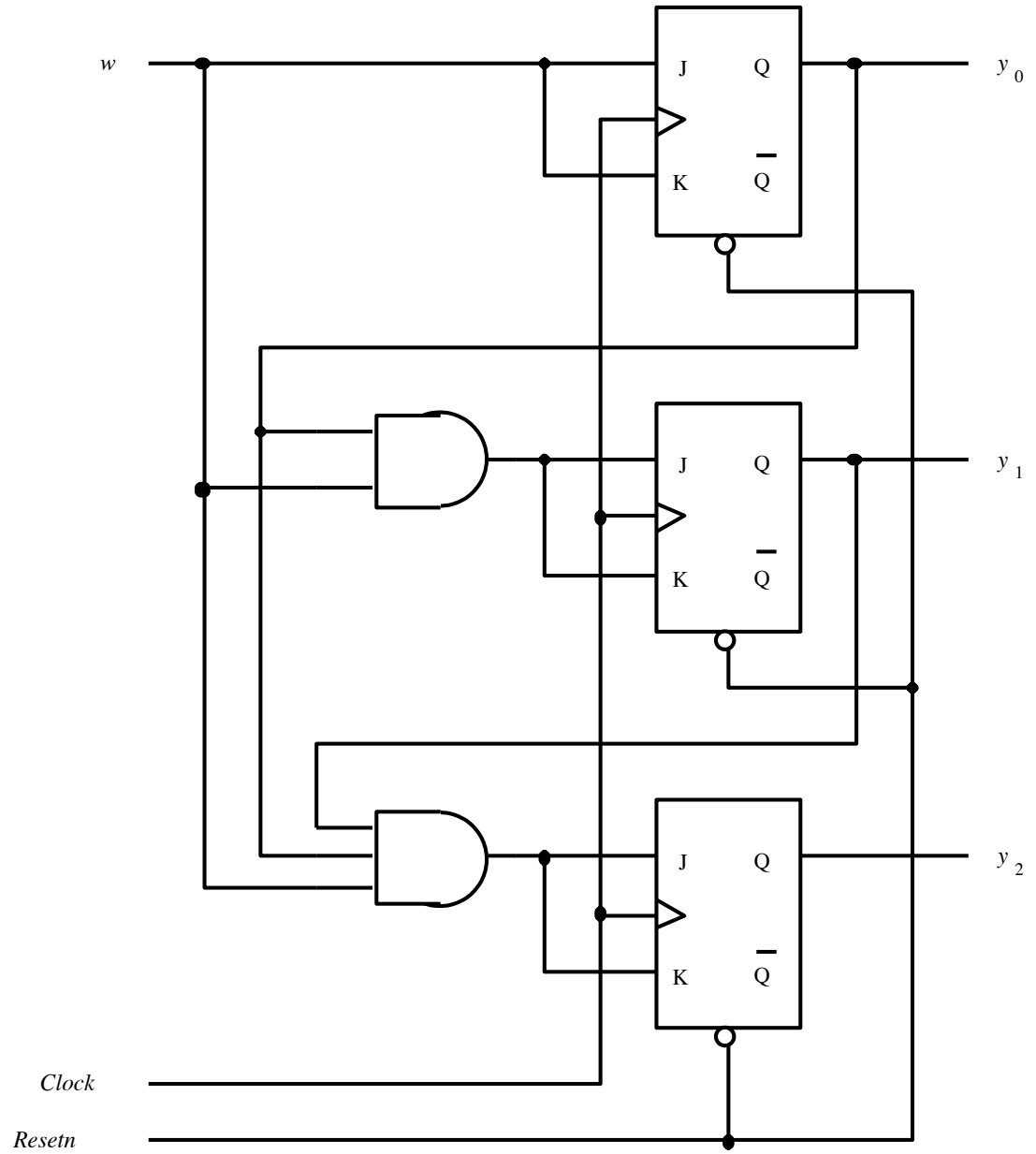


# Circuit diagram using JK flip-flops

$$J_0 = w$$
$$K_0 = w$$

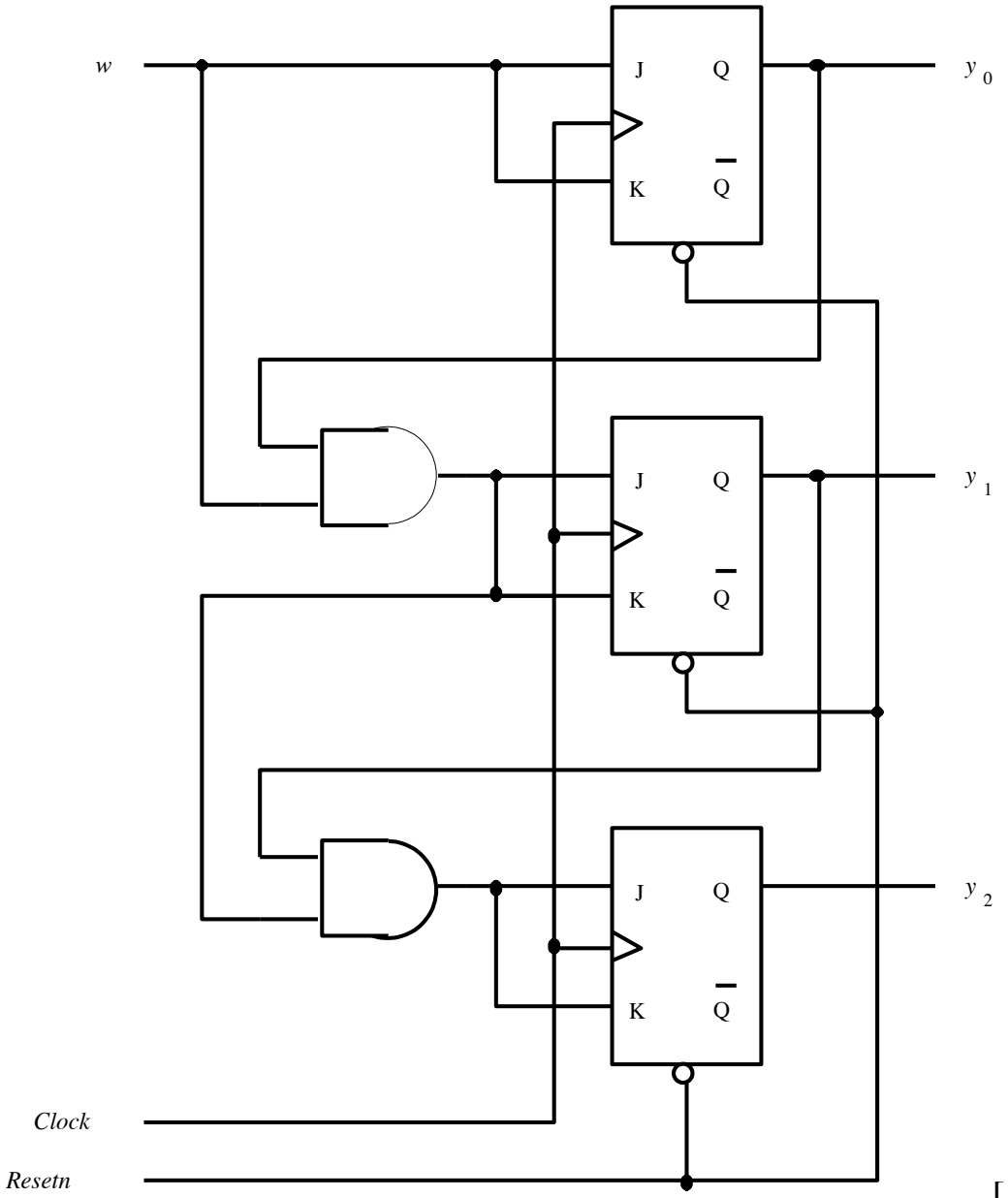
$$J_1 = wy_0$$
$$K_1 = wy_0$$

$$J_2 = wy_0y_1$$
$$K_2 = wy_0y_1$$




[ Figure 6.67 from the textbook ]

# Factored-form implementation of the counter




[ Figure 6.68 from the textbook ]

# Another Example
# (A Different “Counter”)

# Goal

- **Implement a 3-bit counter using the sequential circuit approach that counts the pulses on the input line w.**
- **The counter must count in the following sequence:**
  **0, 4, 2, 6, 1, 5, 3, 7, 0, 4, 2, …**
- **The count must be represented directly by the flip-flop values. No extra gates are allowed.**
- **In other words, count = $Q_2$ $Q_1$ $Q_0$**
- **The count changes based on the input signal w:**
  - **If w=0, then the count remains the same**
  - **If w=1, then the count is advanced by one**

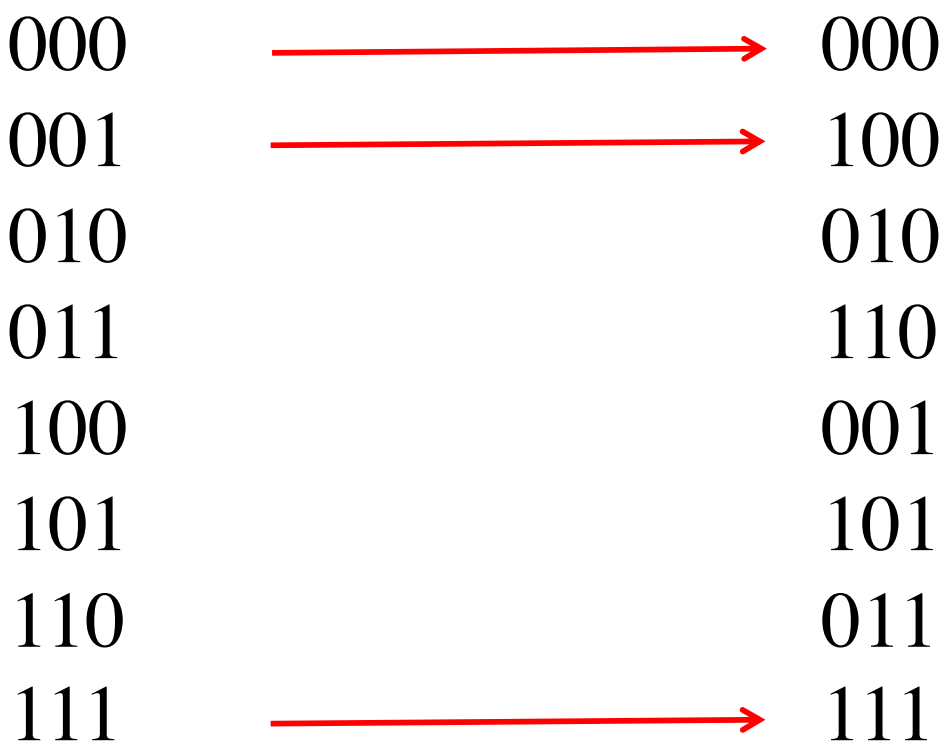
# Goal

- **Implement a 3-bit counter using the sequential circuit approach that counts the pulses on the input line w.**
- **The counter must count in the following sequence:**

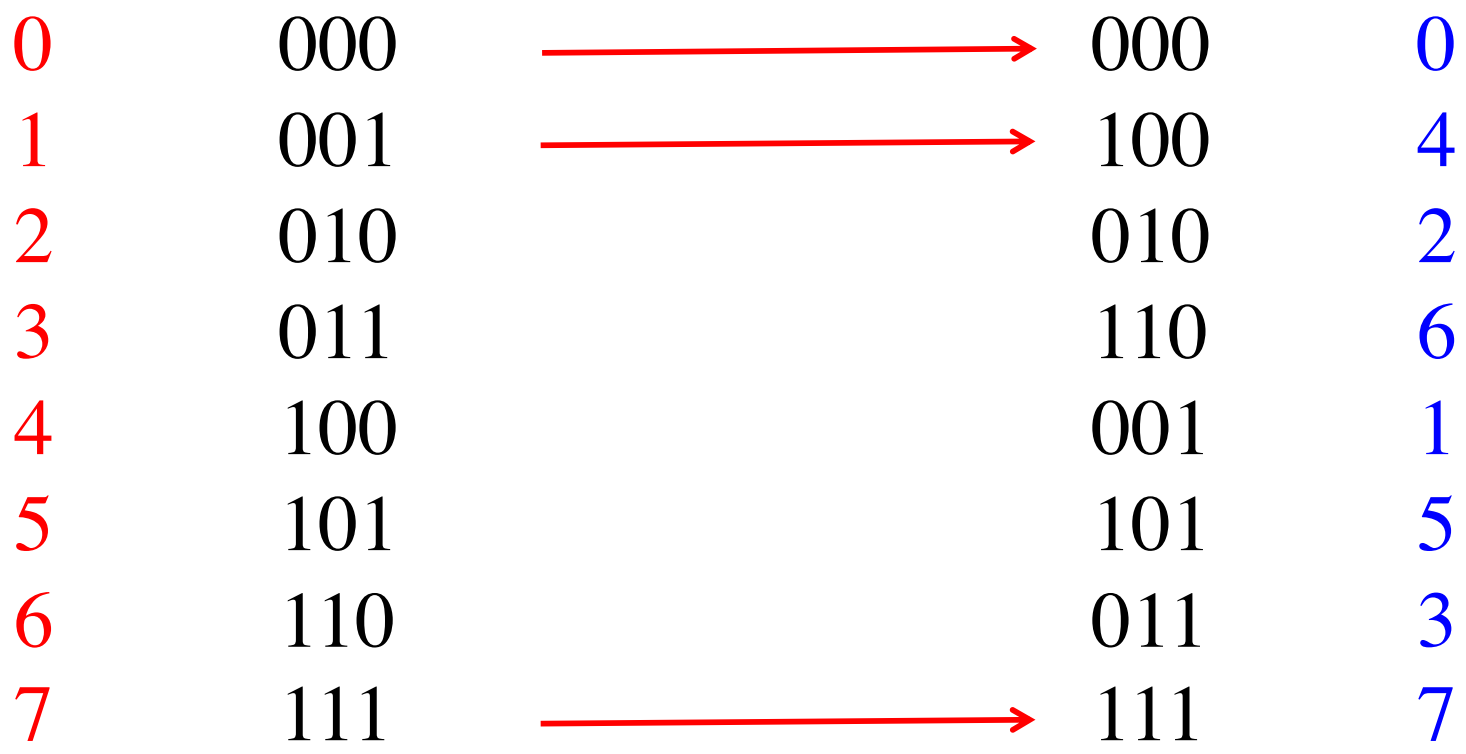
  **0, 4, 2, 6, 1, 5, 3, 7, 0, 4, 2, …**
- **The count must be represented directly by the flip-flop values. No extra gates are allowed.**
- **In other words, count = $Q_2$ $Q_1$ $Q_0$**
- **The count changes based on the input signal w:**
  - **If w=0, then the count remains the same**
  - **If w=1, then the count is advanced by one**

Clock = w

# By flipping the order of the bits we get



# By flipping the order of the bits we get



# State table for the counterlike example

| Present state | Next state | Output $z_2z_1z_0$ |
|---|---|---|
| A | B | 000 |
| B | C | 100 |
| C | D | 010 |
| D | E | 110 |
| E | F | 001 |
| F | G | 101 |
| G | H | 011 |
| H | A | 111 |

[ Figure 6.69 from the textbook ]

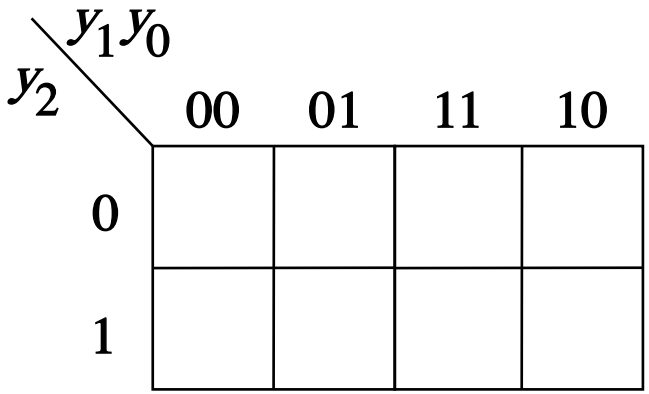
# State-assigned table for this example

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |

[ Figure 6.70 from the textbook ]

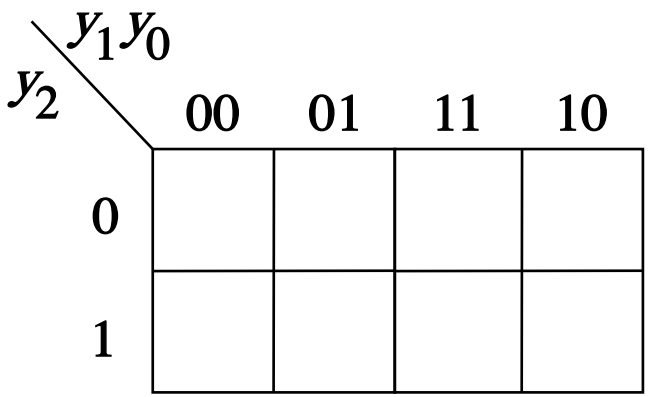
# K-maps for $Y_2$, $Y_1$, and $Y_0$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



# K-maps for $Y_2$, $Y_1$, and $Y_0$

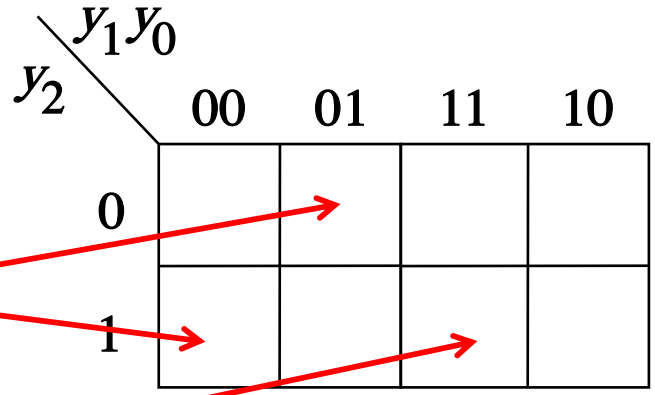
| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |




Notice that these are scrambled

# K-maps for $Y_2$, $Y_1$, and $Y_0$

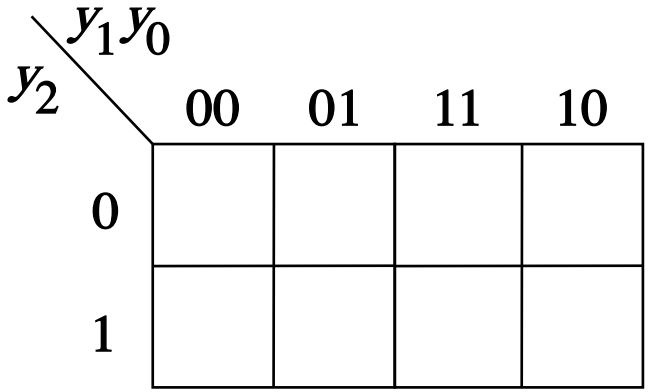
| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |




Notice that these are scrambled

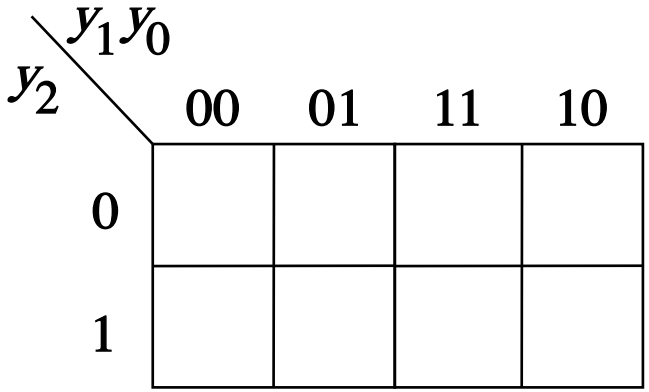
# K-map for $Y_2$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



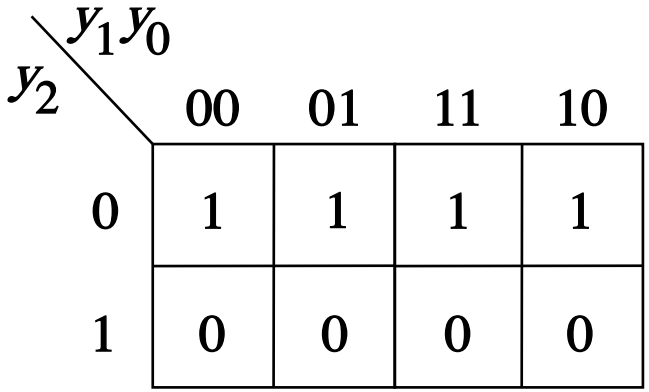
# K-map for $Y_2$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



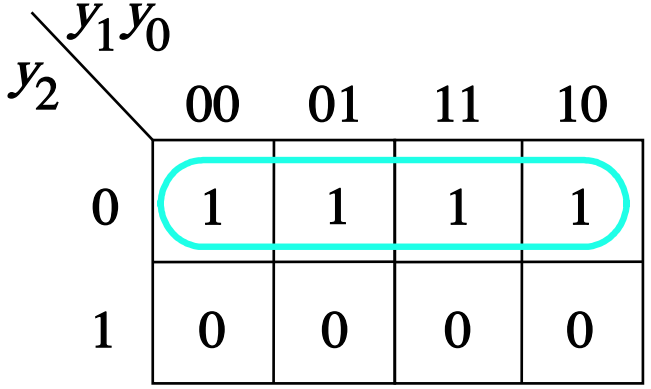
# K-map for $Y_2$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



# K-map for $Y_2$

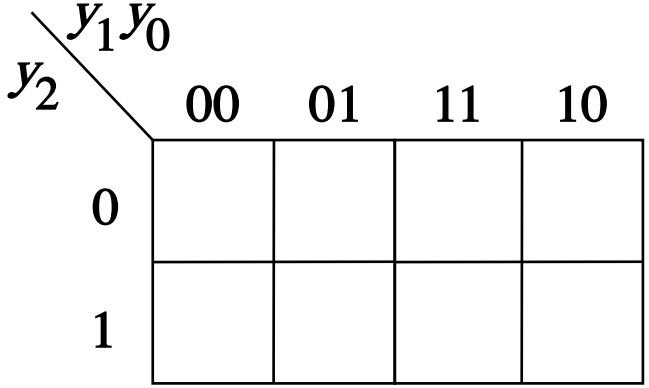
| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |




$$Y_2 = \overline{y_2}$$

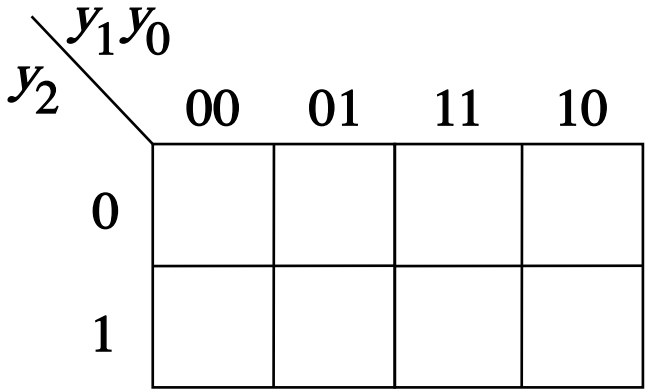
# K-map for $Y_1$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



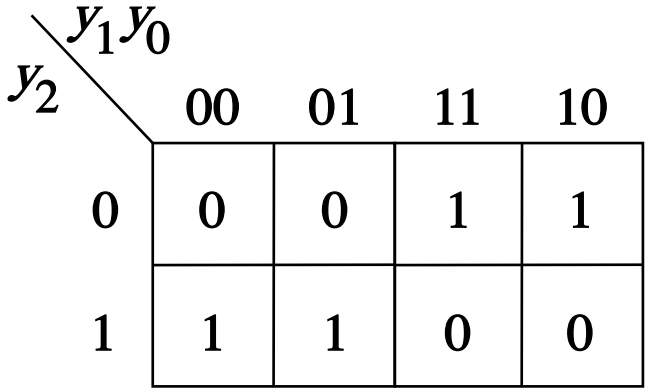
# K-map for $Y_1$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



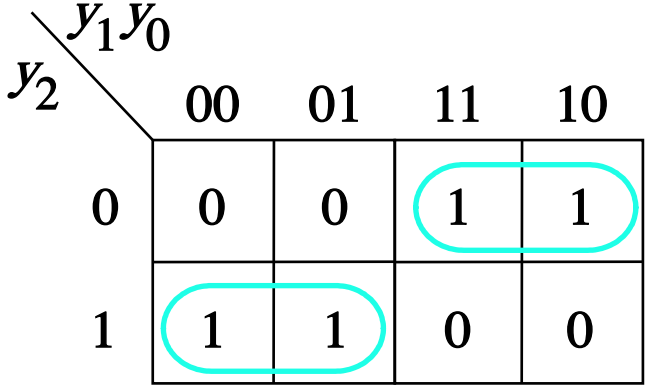
# K-map for $Y_1$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



# K-map for $Y_1$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |

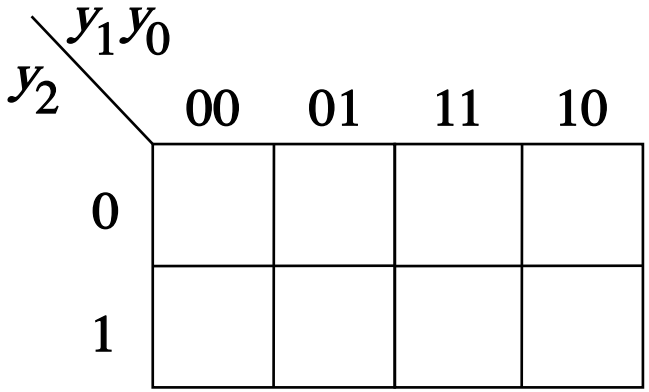



$$Y_1 = y_2\overline{y_1} + \overline{y_2}\, y_1$$

XOR

# K-map for $Y_0$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



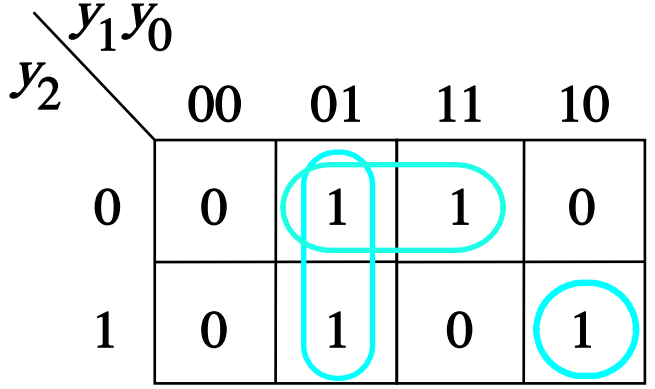
# K-map for $Y_0$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |

| $y_2$ \ $y_1y_0$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |

# K-map for $Y_0$

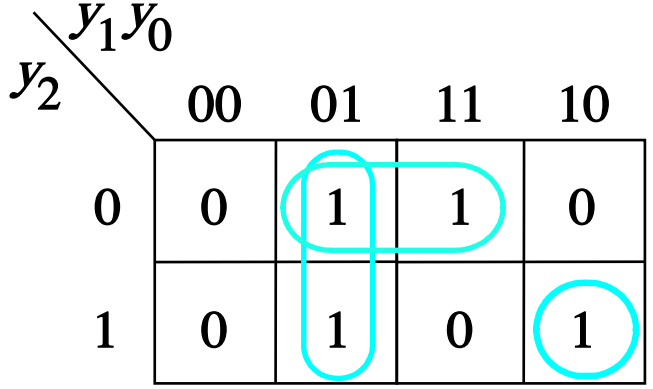
| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |




$$Y_0 = \overline{y}_1 y_0 + \overline{y}_2 y_0 + y_2 y_1 \overline{y}_0$$

# K-map for $Y_0$

| Present state $y_2y_1y_0$ | Next state $Y_2Y_1Y_0$ | Output $z_2z_1z_0$ |
|---|---|---|
| 000 | 100 | 000 |
| 100 | 010 | 100 |
| 010 | 110 | 010 |
| 110 | 001 | 110 |
| 001 | 101 | 001 |
| 101 | 011 | 101 |
| 011 | 111 | 011 |
| 111 | 000 | 111 |



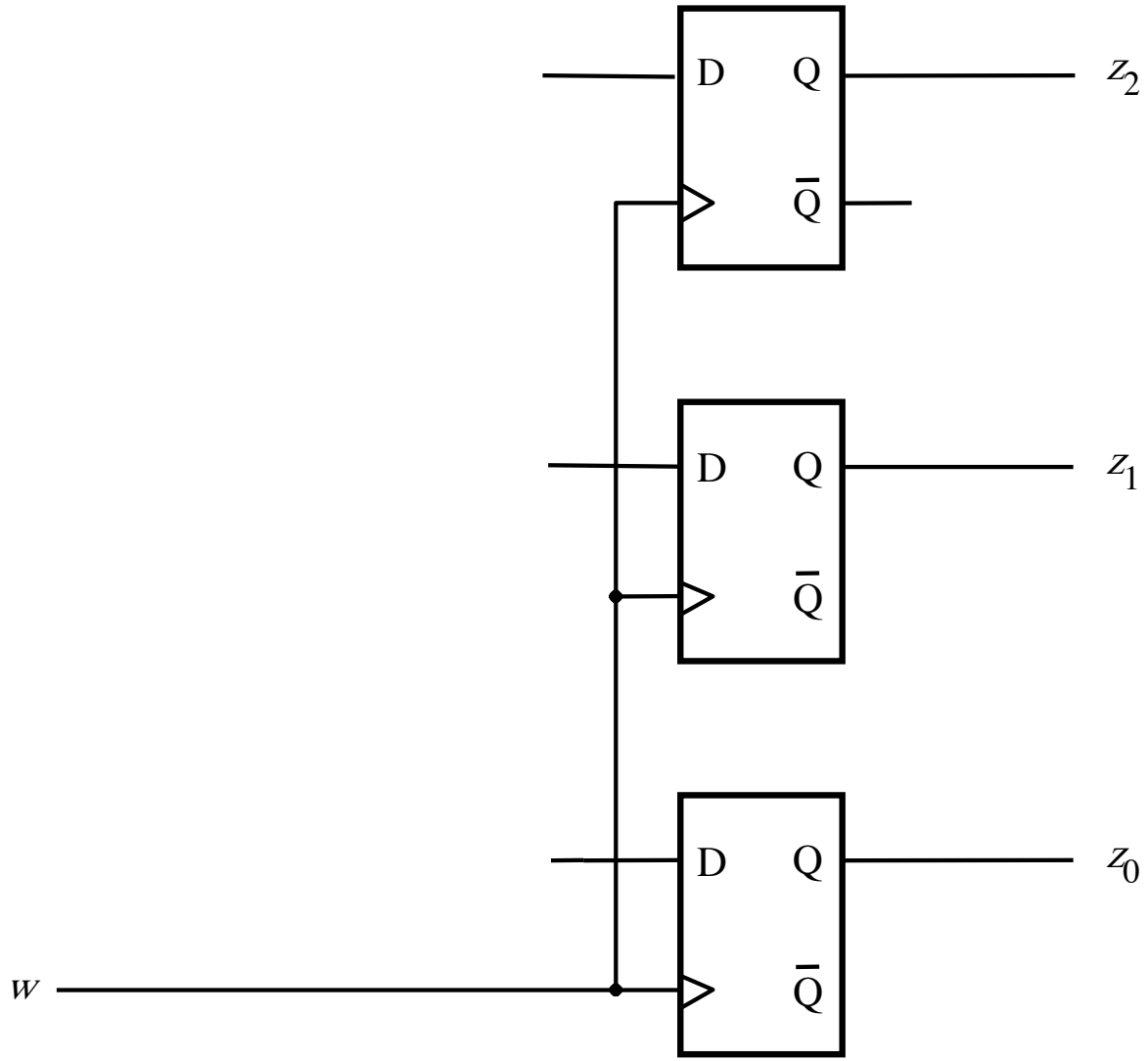

$$Y_0 = \overline{y}_1 y_0 + \overline{y}_2 y_0 + y_2 y_1 \overline{y}_0$$

$$= (\overline{y}_1 + \overline{y}_2)\, y_0 + y_2 y_1 \overline{y}_0$$

$$= (\overline{y_1 y_2})\, y_0 + (y_2 y_1)\overline{y}_0$$

$$= (y_1 y_2) \oplus y_0$$

# Let's Draw the Circuit for this example

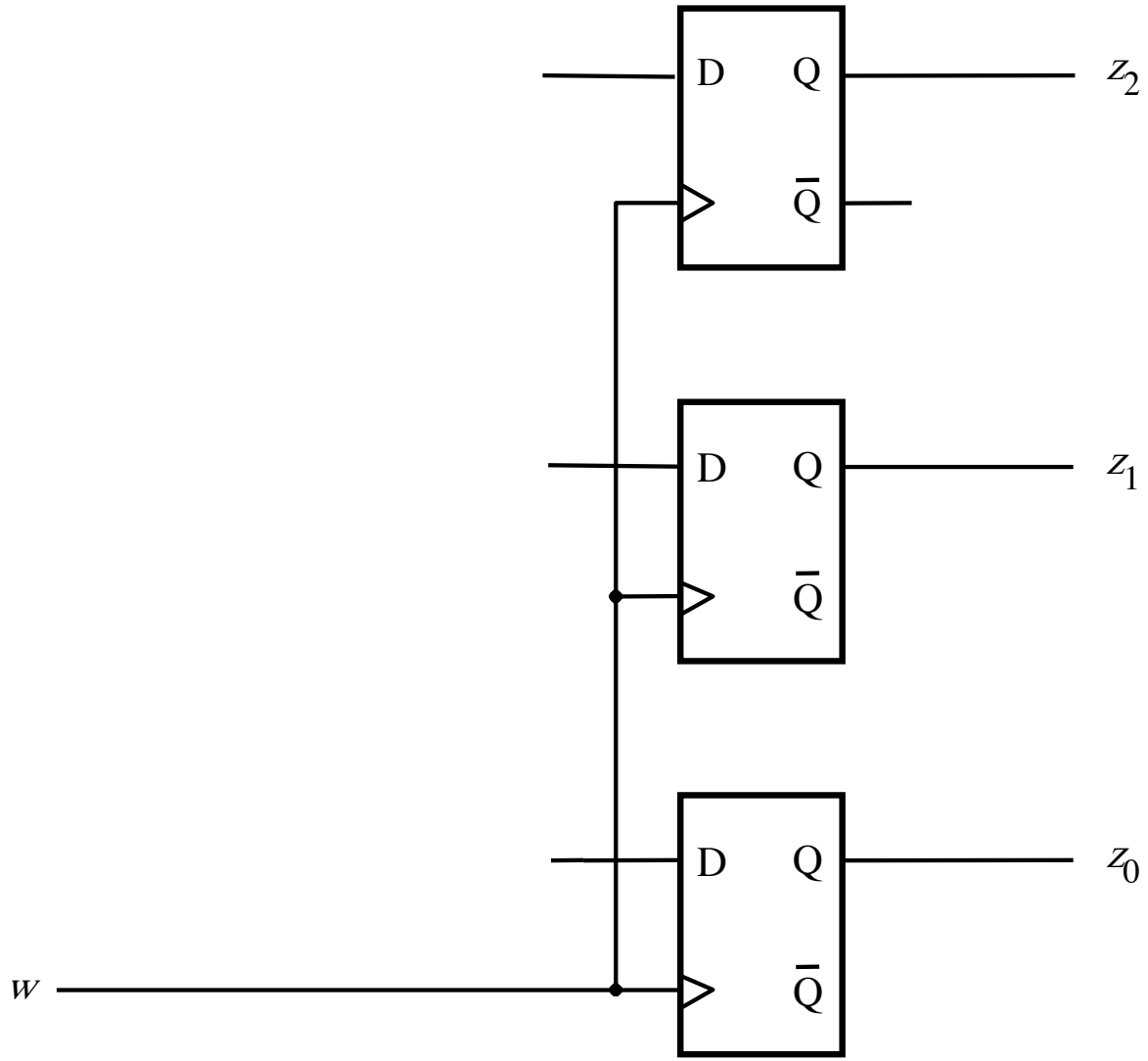


# Let's Draw the Circuit for this example

$Y_2 = \overline{y_2}$

$Y_1 = y_1 \oplus y_2$

$Y_0 = ( y_1 y_2 ) \oplus y_0$

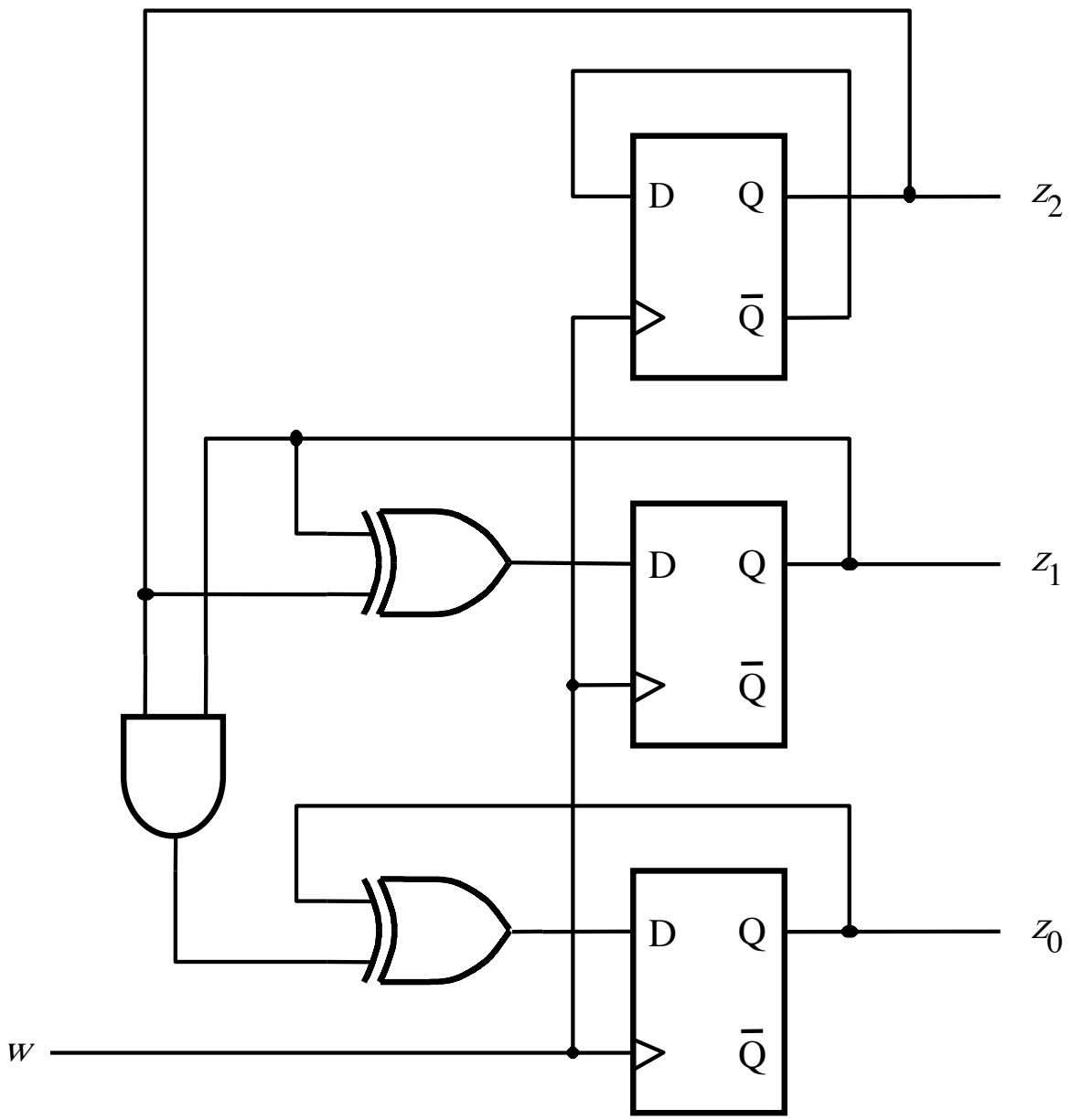


# The Circuit for this example

$Y_2 = \overline{y_2}$

$Y_1 = y_1 \oplus y_2$

$Y_0 = ( y_1 y_2 ) \oplus y_0$




[ Figure 6.71 from the textbook ]

# Questions?

# THE END