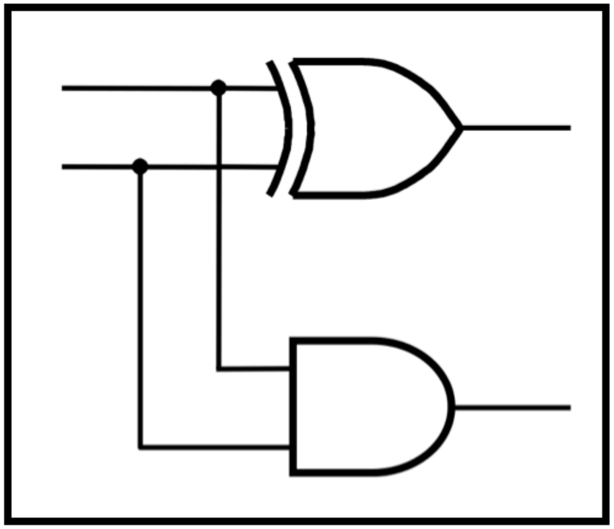


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Algorithmic State Machine (ASM) Charts

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **Homework 12 is out**

- **It is due on Monday Dec 3 @ 4pm**

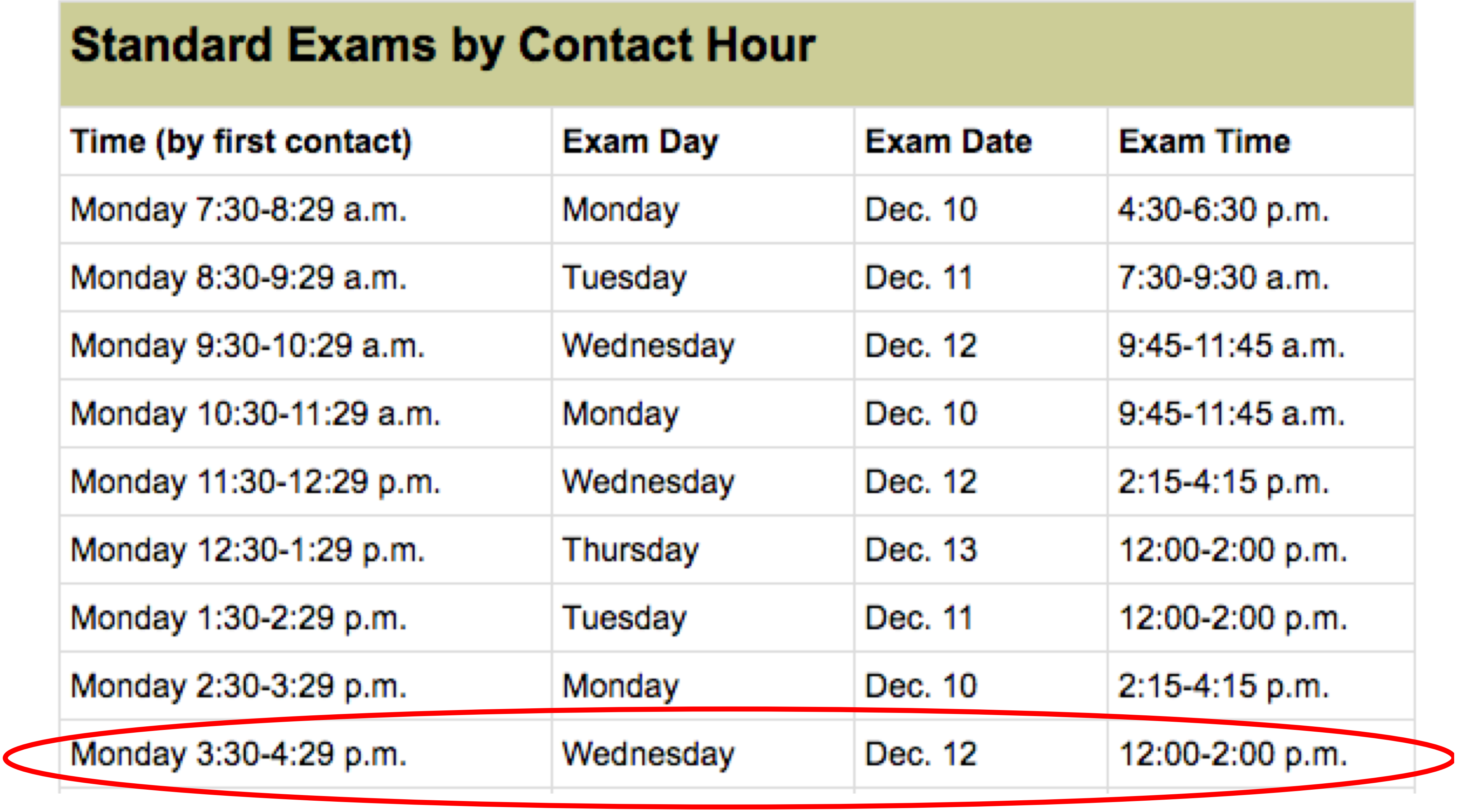
Administrative Stuff

- The FINAL exam is scheduled for

- Wednesday Dec 12 @ 12:00 – 2:00 PM

- It will be in this room.

http://www.registrar.iastate.edu/students/exams/fallexams



Standard Exams by Contact Hour

Time (by first contact)	Exam Day	Exam Date	Exam Time
Monday 7:30-8:29 a.m.	Monday	Dec. 10	4:30-6:30 p.m.
Monday 8:30-9:29 a.m.	Tuesday	Dec. 11	7:30-9:30 a.m.
Monday 9:30-10:29 a.m.	Wednesday	Dec. 12	9:45-11:45 a.m.
Monday 10:30-11:29 a.m.	Monday	Dec. 10	9:45-11:45 a.m.
Monday 11:30-12:29 p.m.	Wednesday	Dec. 12	2:15-4:15 p.m.
Monday 12:30-1:29 p.m.	Thursday	Dec. 13	12:00-2:00 p.m.
Monday 1:30-2:29 p.m.	Tuesday	Dec. 11	12:00-2:00 p.m.
Monday 2:30-3:29 p.m.	Monday	Dec. 10	2:15-4:15 p.m.
Monday 3:30-4:29 p.m.	Wednesday	Dec. 12	12:00-2:00 p.m.

Final Exam Format

- The exam will cover: Chapter 1 to Chapter 6, and Sections 7.1-7.2
- Emphasis will be on Chapter 5, 6, and 7
- The exam will be open book and open notes.
- You can bring up to 5 pages of handwritten or typed notes plus your textbook.

Final Exam Format

- The exam will be out of 130 points
- You need 95 points to get an A on this exam
- It will be great if you can score more than 100 points.
  - but you can’t roll over your extra points ☹

Topics for the Final Exam

- K-maps for 2, 3, and 4 variables
- Multiplexers (circuits and function)
- Synthesis of logic functions using multiplexers
- Shannon’s Expansion Theorem
- 1’s complement and 2’s complement representation
- Addition and subtraction of binary numbers
- Circuits for adding and subtracting
- Serial adder
- Latches (circuits, behavior, timing diagrams)
- Flip-Flops (circuits, behavior, timing diagrams)
- Counters (up, down, synchronous, asynchronous)
- Registers and Register Files

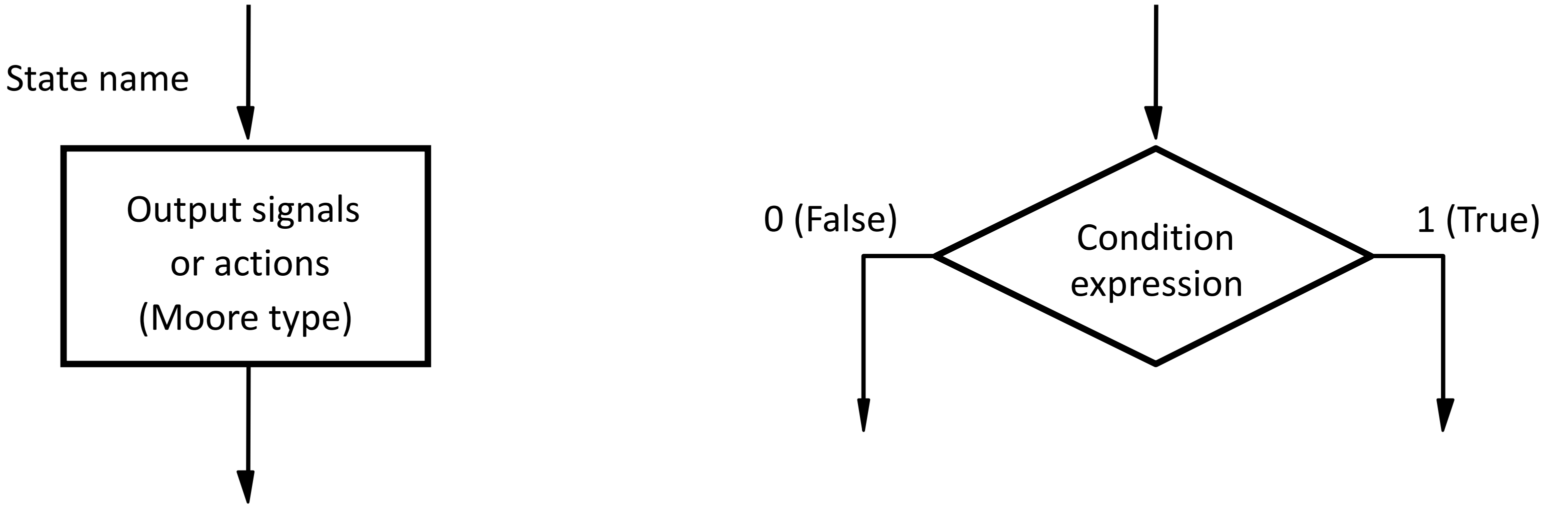
Topics for the Final Exam

- Synchronous Sequential Circuits
- FSMs
- Moore Machines
- Mealy Machines
- State diagrams, state tables, state-assigned tables
- State minimization
- Designing a counter
- Arbiter Circuits
- Reverse engineering a circuit
- ASM Charts
- Register Machines
- Bus structure and Simple Processors
- Something from Star Wars

Reading Material for Next Lecture

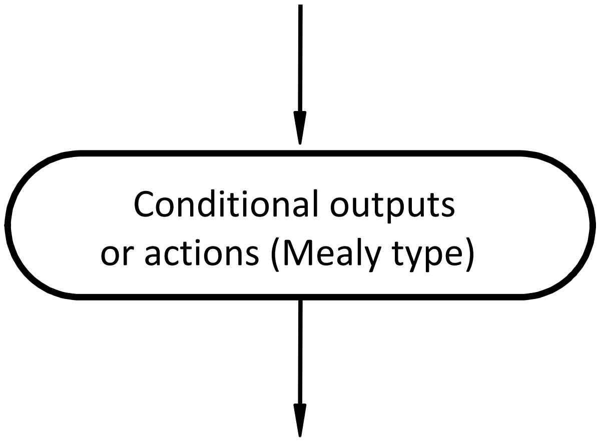
- **“The Seven Secrets of Computer Power Revealed” by Daniel Dennett.**
- **This is Chapter 24 in his book
“Intuition Pumps and Other Tools for Thinking”, 2013**

Elements used in ASM charts



(a) State box

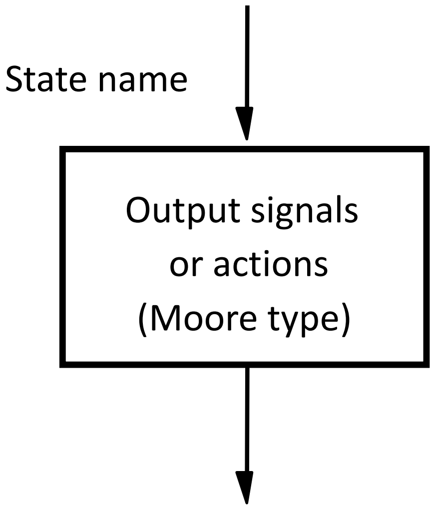
(b) Decision box



(c) Conditional output box

[Figure 6.81 from the textbook]

State Box

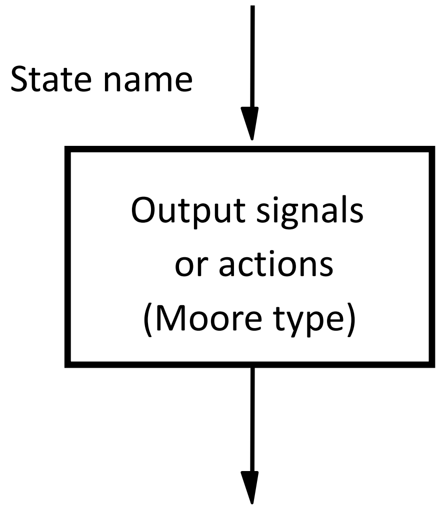


State name

Output signals
or actions
(Moore type)

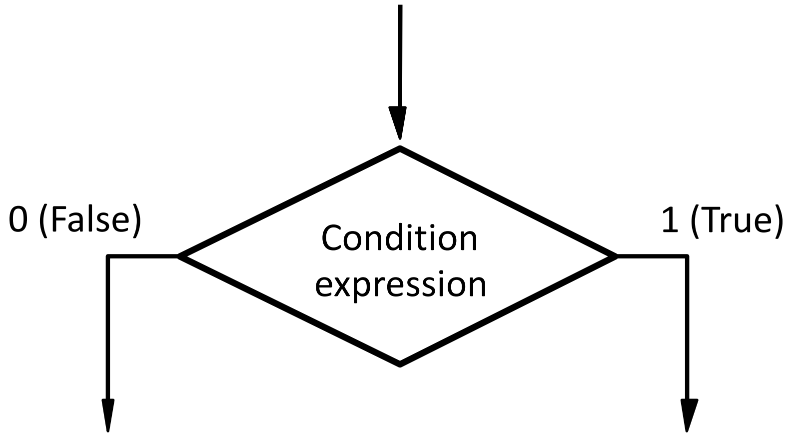
[Figure 6.81a from the textbook]

State Box



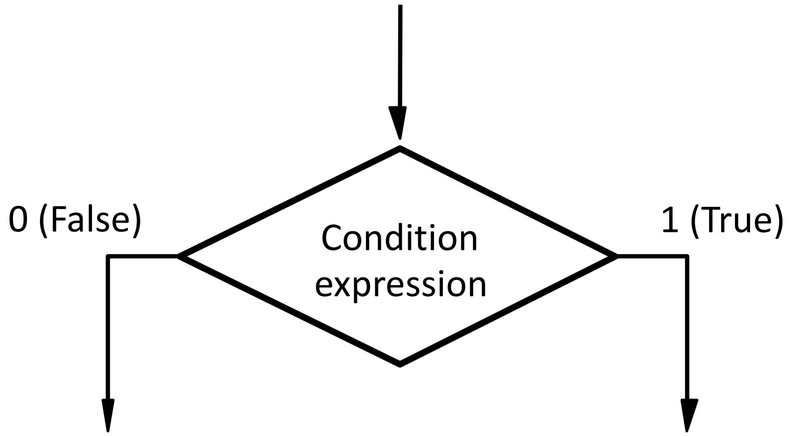
- Indicated with a rectangle
- Equivalent to a node in the State diagram
- The name of the state is written outside the box
- Moore-type outputs are written inside the box
- Only the output that must be set to 1 is written (by default, if an output is not listed it is set to 0)

Decision Box



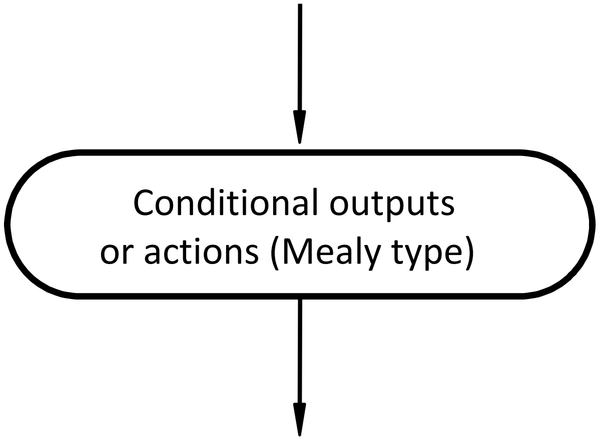
[Figure 6.81b from the textbook]

Decision Box



- Indicated with a diamond shape
- Used for a condition expression that must be tested
- The exit path is chosen based on the outcome of the test
- The condition is on one or more inputs to the FSM
- Shortcut notation: w means “is w equal to 1?”

Conditional Output Box

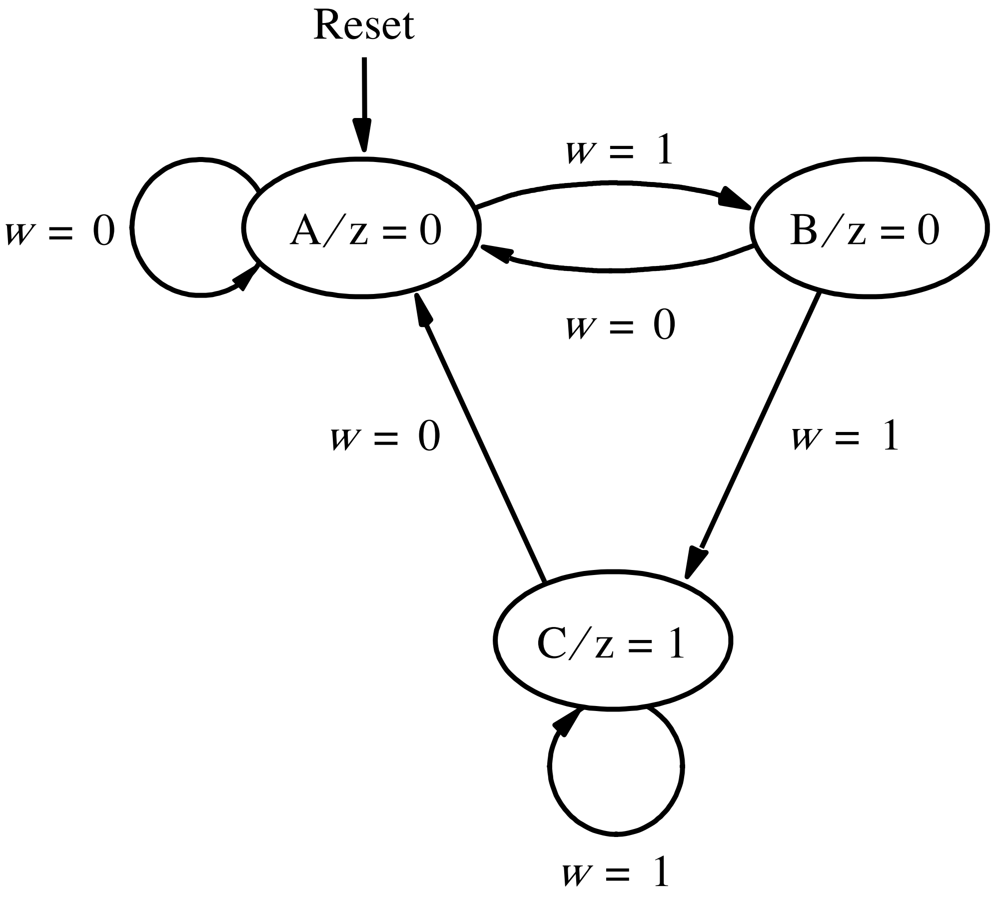


- **Indicated with an oval shape**
- **Used for a Mealy-type output signals**
- **The outputs depend on the state variables and inputs**
- **The condition that determines when such outputs are generated is placed in a separate decision box**

[Figure 6.81c from the textbook]

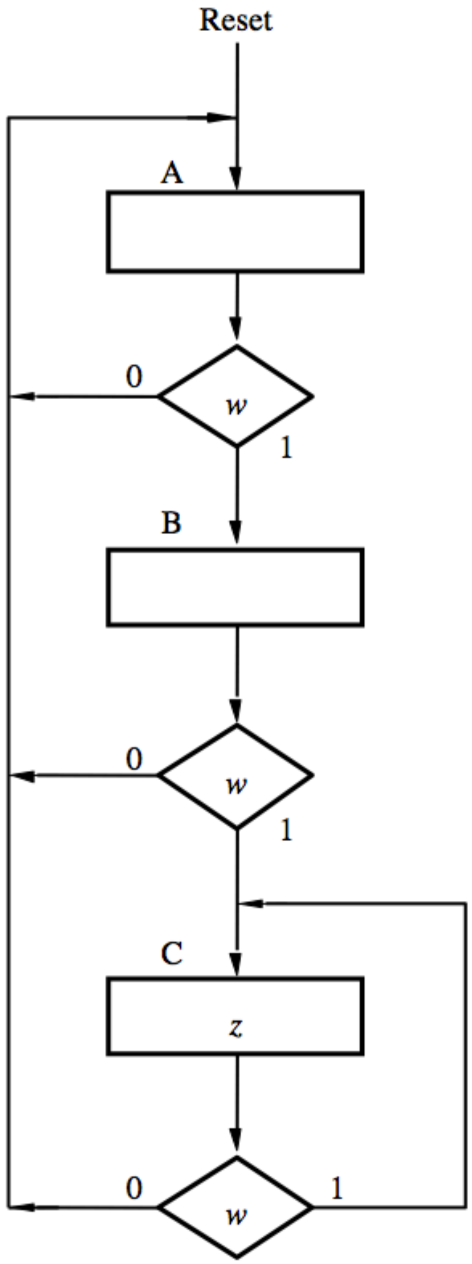
Some Examples

FSM



[Figure 6.3 from the textbook]

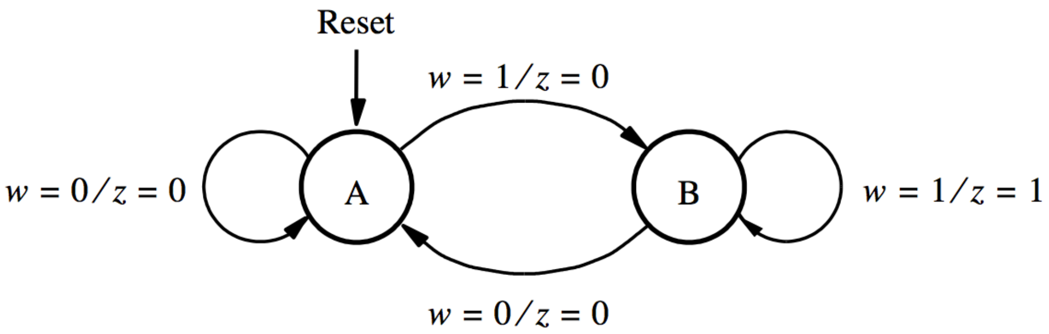
ASM chart



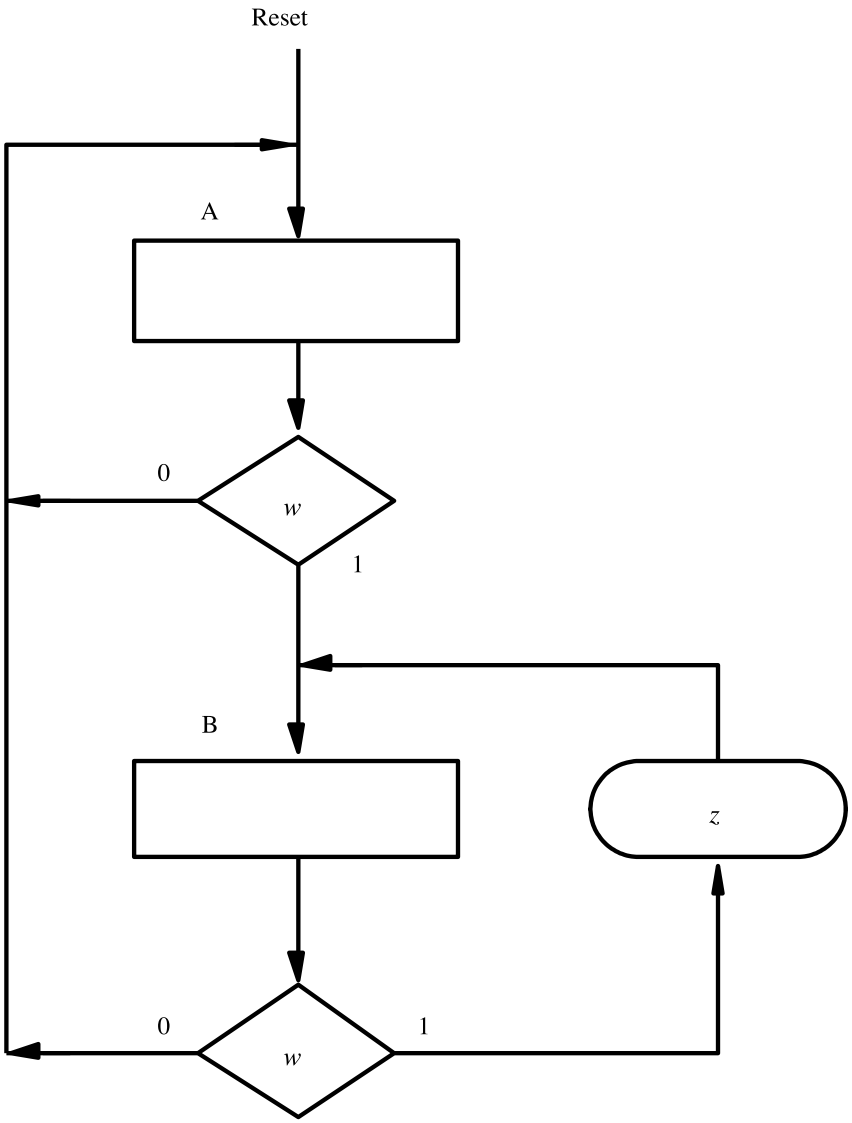
[Figure 6.82 from the textbook]

FSM

ASM chart

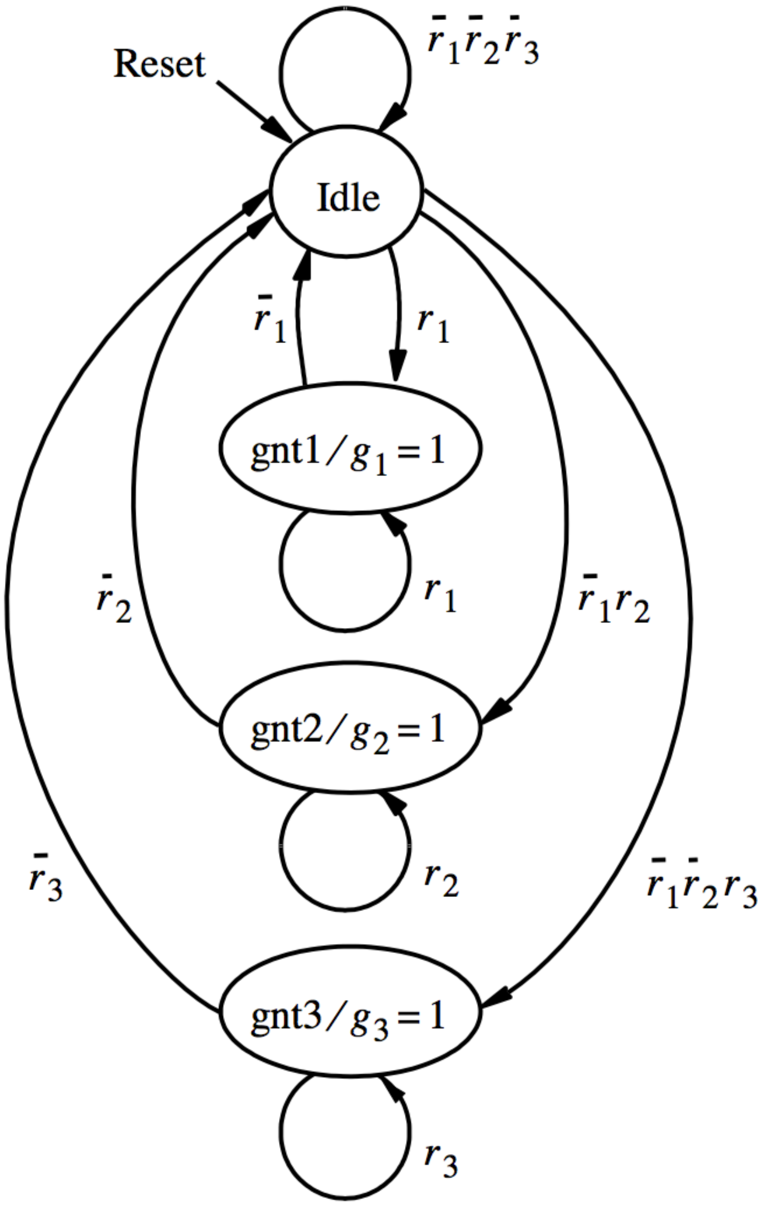


[Figure 6.23 from the textbook]



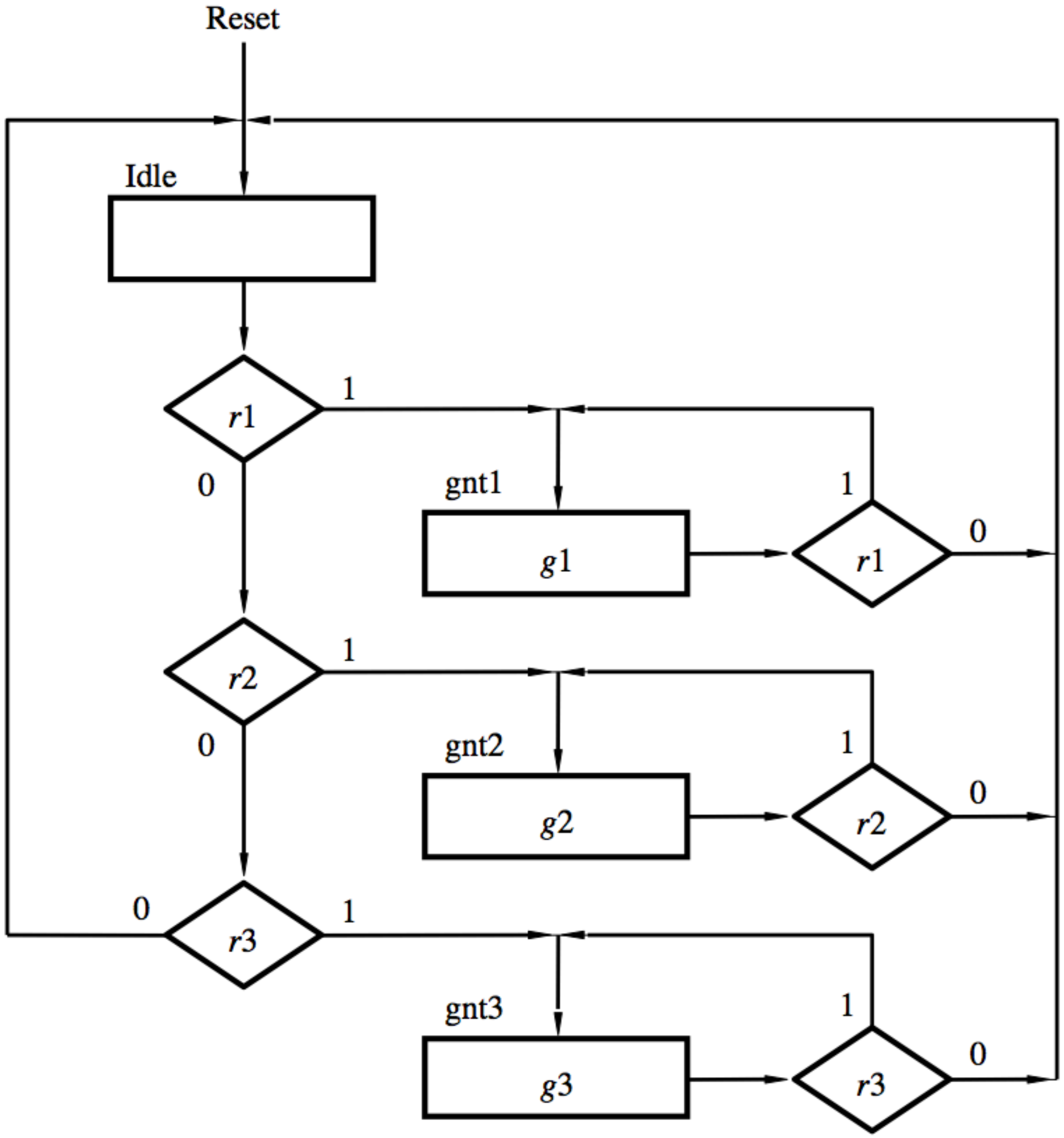
[Figure 6.83 from the textbook]

FSM



[Figure 6.73 from the textbook]

ASM chart

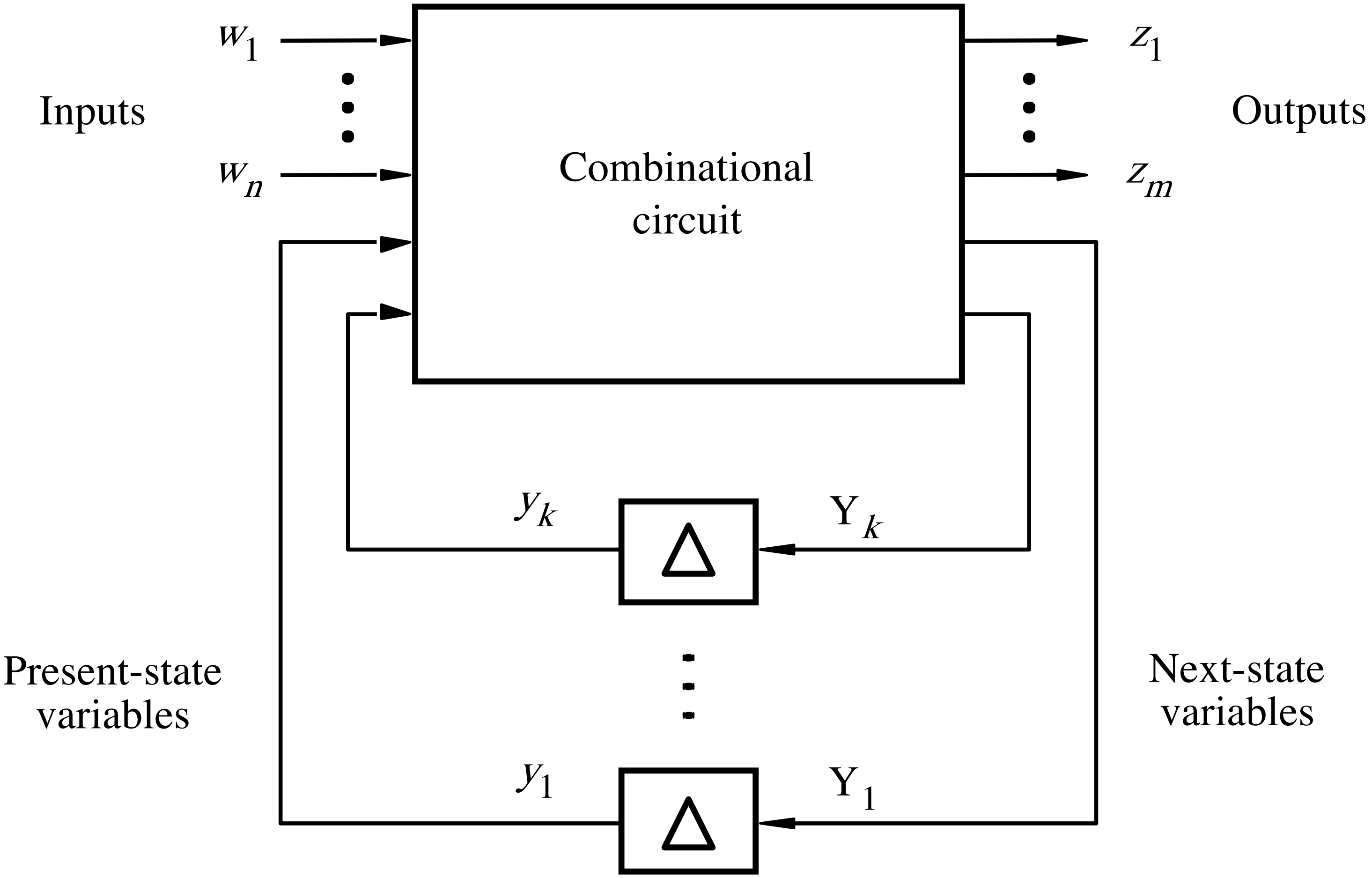


[Figure 6.84 from the textbook]

ASM Chart is different from a Flow Chart

- The ASM chart implicitly includes timing info
- It is assumed that the underlying FSM changes from one state to another on every active clock edge
- Flow charts don’t make that assumption.

The general model for a sequential circuit



[Figure 6.85 from the textbook]

The general model for a sequential circuit

$$M = (W, Z, S, \varphi, \lambda)$$

- W, Z, and S are finite, nonempty sets of inputs, outputs, and states, respectively.
- φ is the state transition function, such that $S(t+1) = \varphi[W(t), S(t)]$.
- λ is the output function, such that $\lambda(t) = \lambda[S(t)]$ for the Moore model and $\lambda(t) = \lambda[W(t), S(t)]$ for the Mealy model.

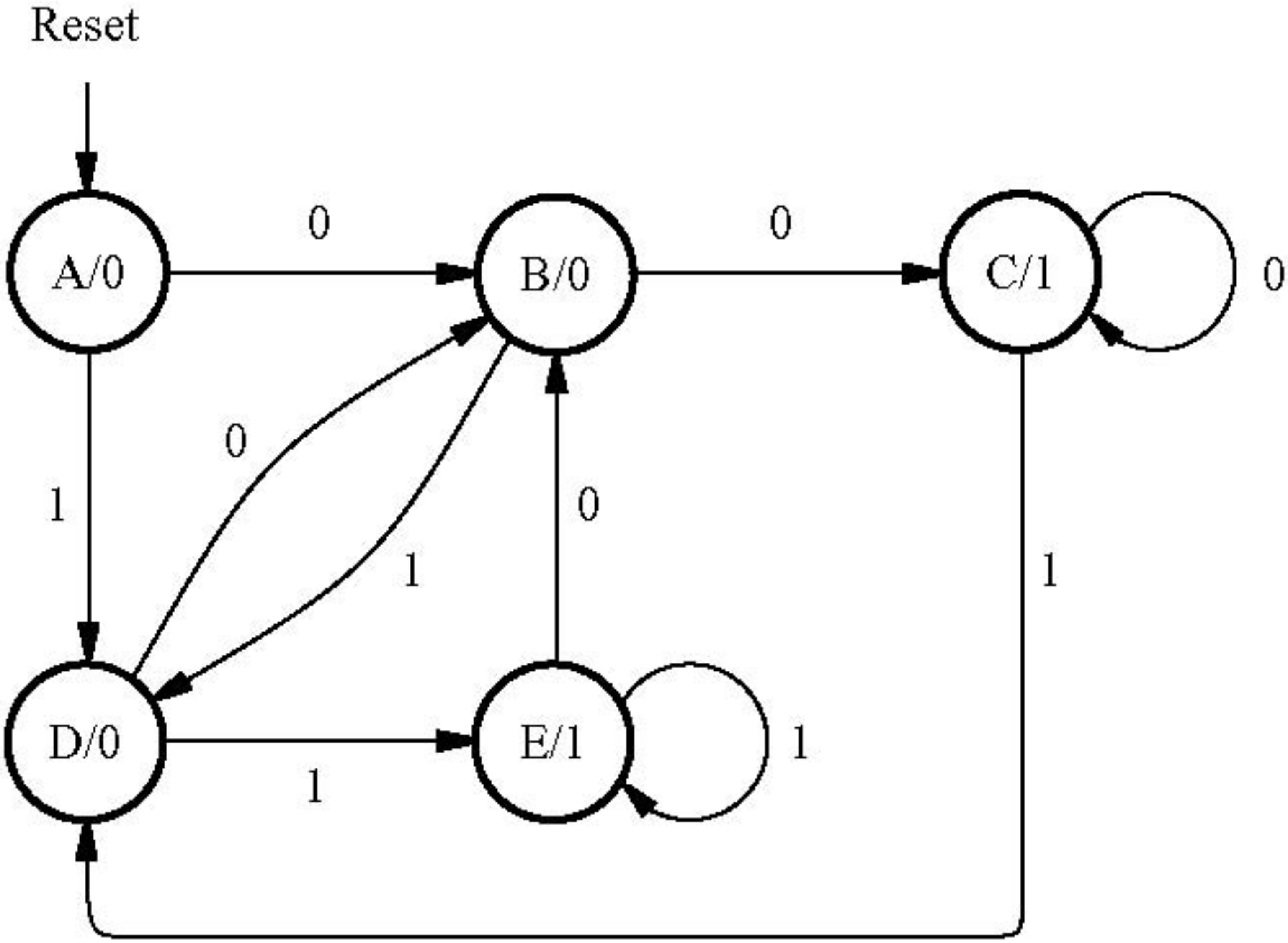
Examples of Solved Problems

Example 6.12

Goal

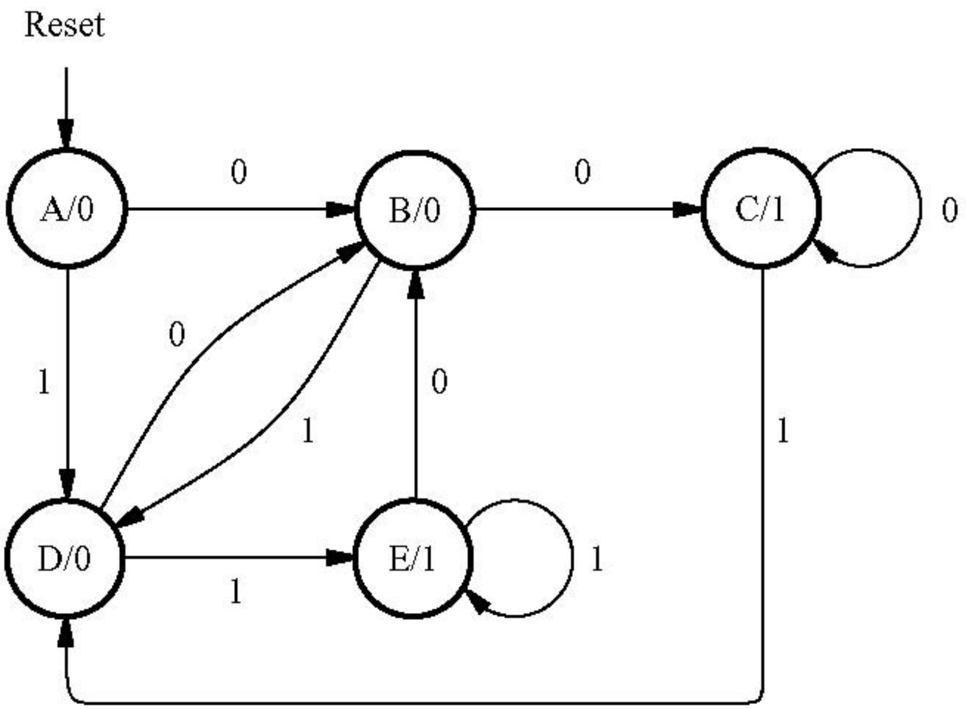
- Design an FSM that detects if the previous two values of the input w were equal to 00 or 11.
- If either condition is true then the output z should be set to 1; otherwise to 0.

State Diagram



[Figure 6.86 from the textbook]

State Table for the FSM

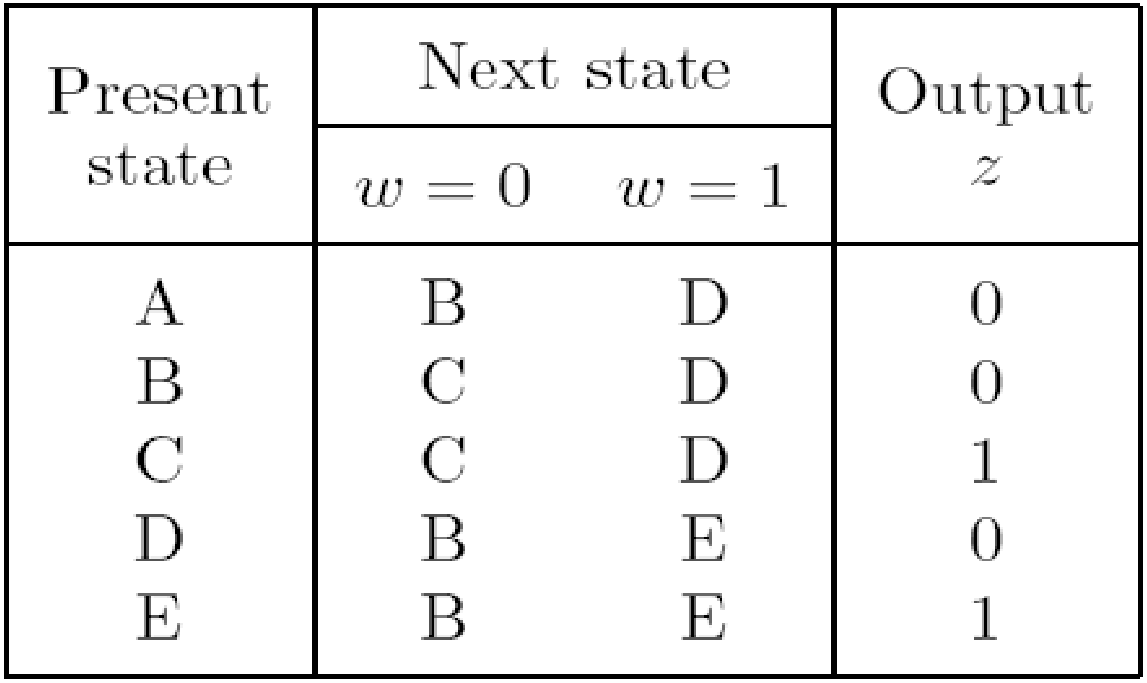


[Figure 6.86 from the textbook]

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	D	0
B	C	D	0
C	C	D	1
D	B	E	0
E	B	E	1

[Figure 6.87 from the textbook]

State Table for the FSM



Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	D	0
B	C	D	0
C	C	D	1
D	B	E	0
E	B	E	1

[Figure 6.87 from the textbook]

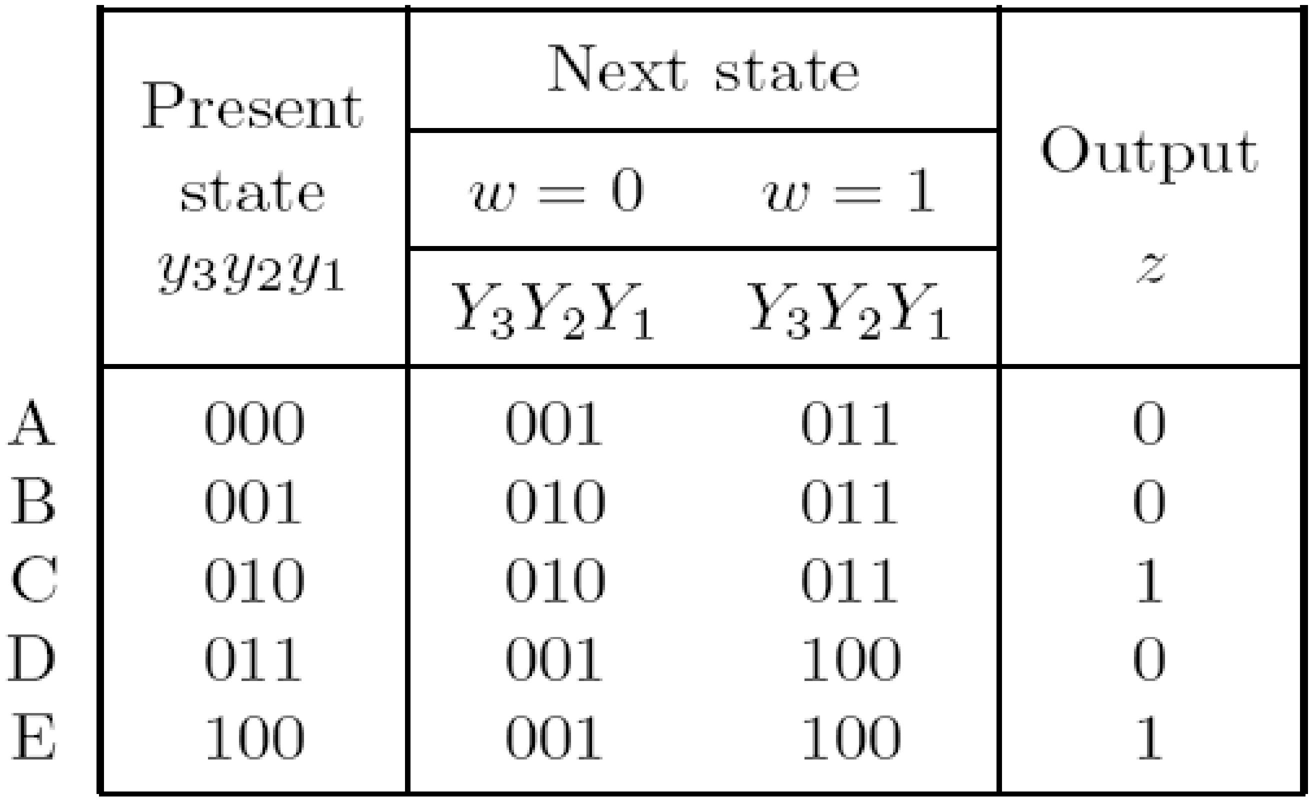
State-Assigned Table for the FSM

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	D	0
B	C	D	0
C	C	D	1
D	B	E	0
E	B	E	1

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

[Figure 6.88 from the textbook]

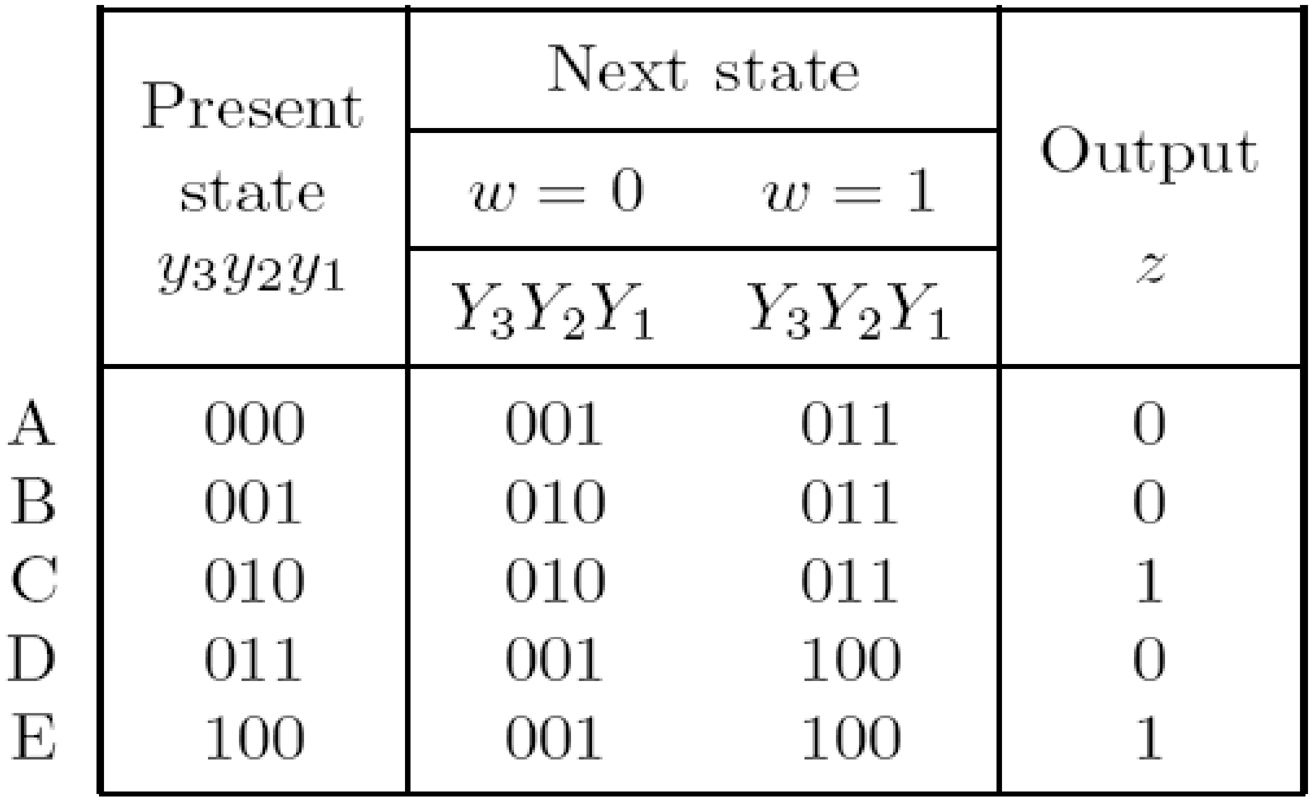
State-Assigned Table for the FSM



	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

[Figure 6.88 from the textbook]

State-Assigned Table for the FSM

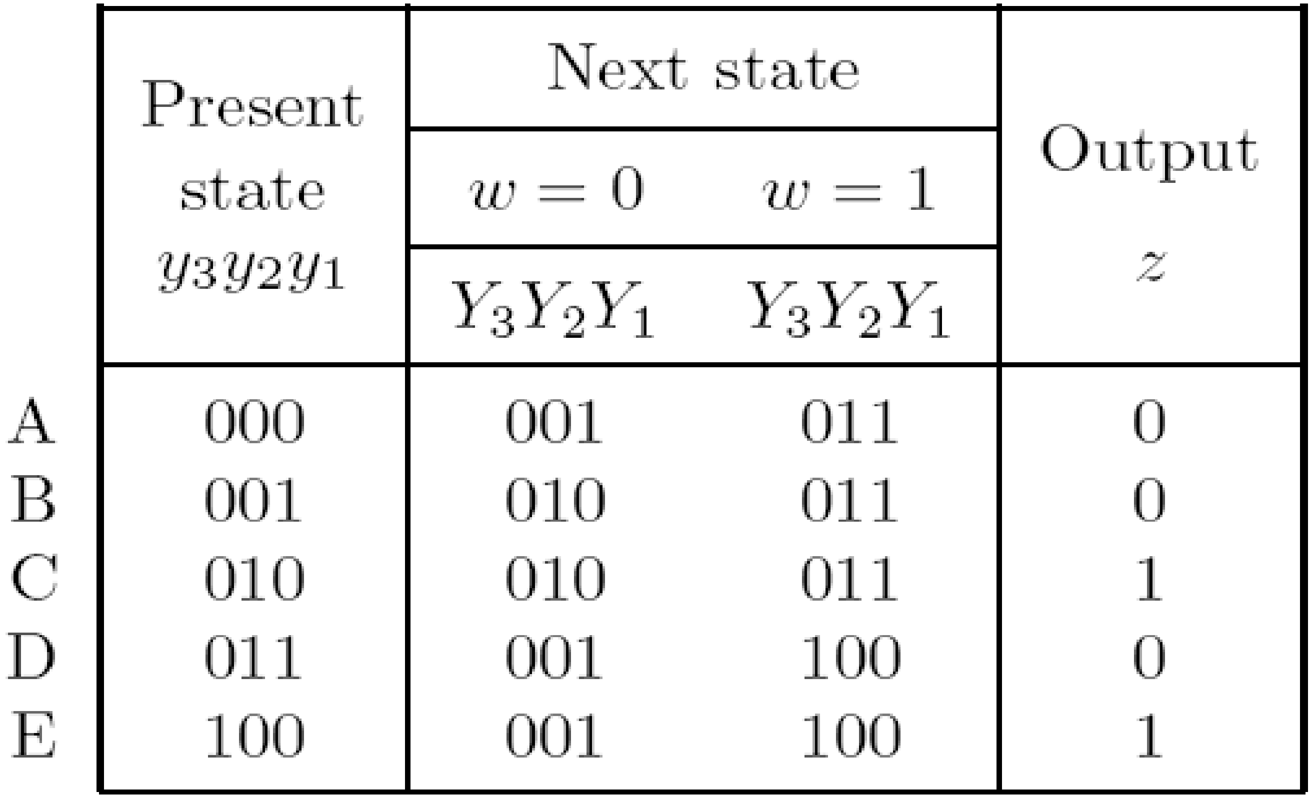


	Present state $y_3y_2y_1$	Next state		Output z
		$w=0$	$w=1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

$$z = y_3 + \overline{y}_1 y_2$$

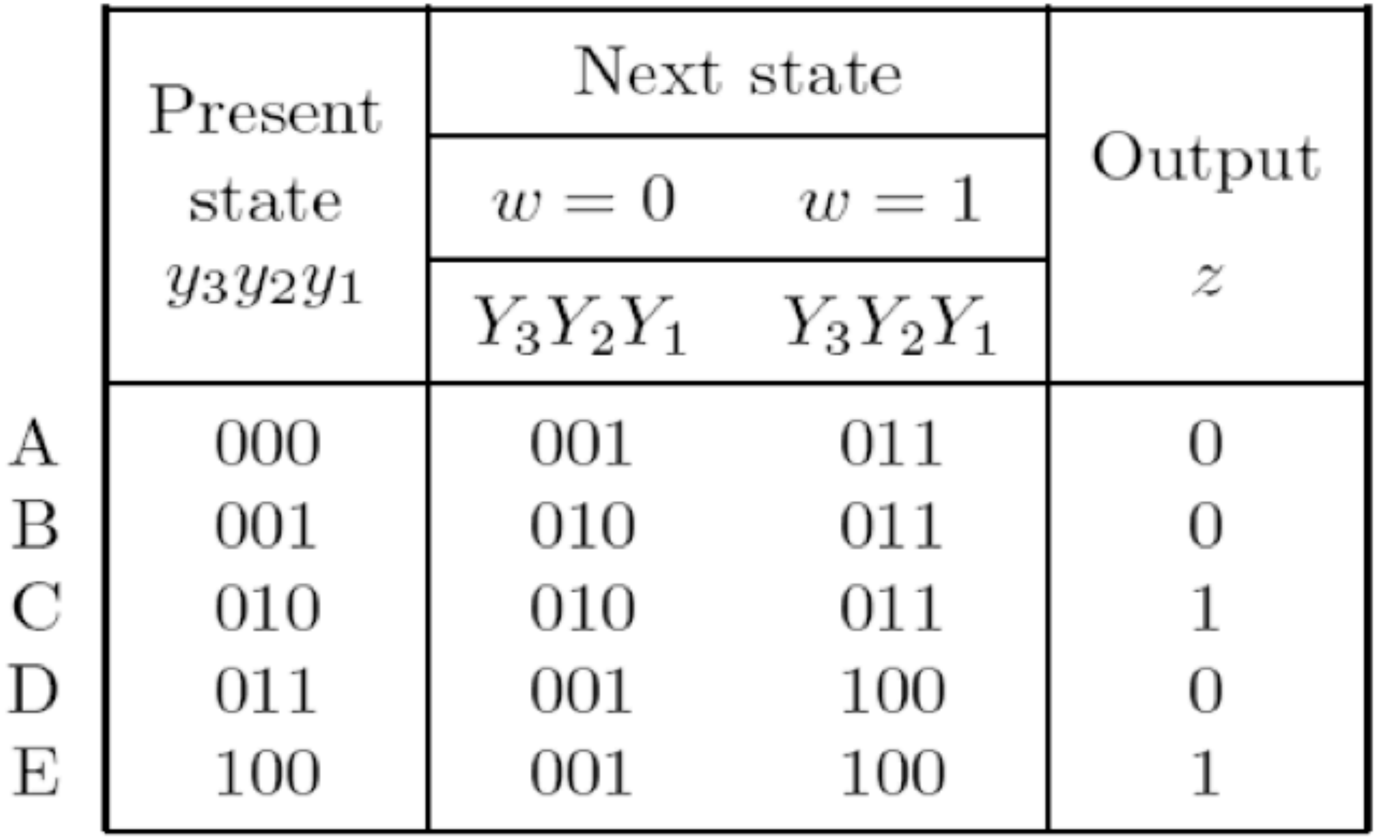
How can we derive this expression?

State-Assigned Table for the FSM



	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

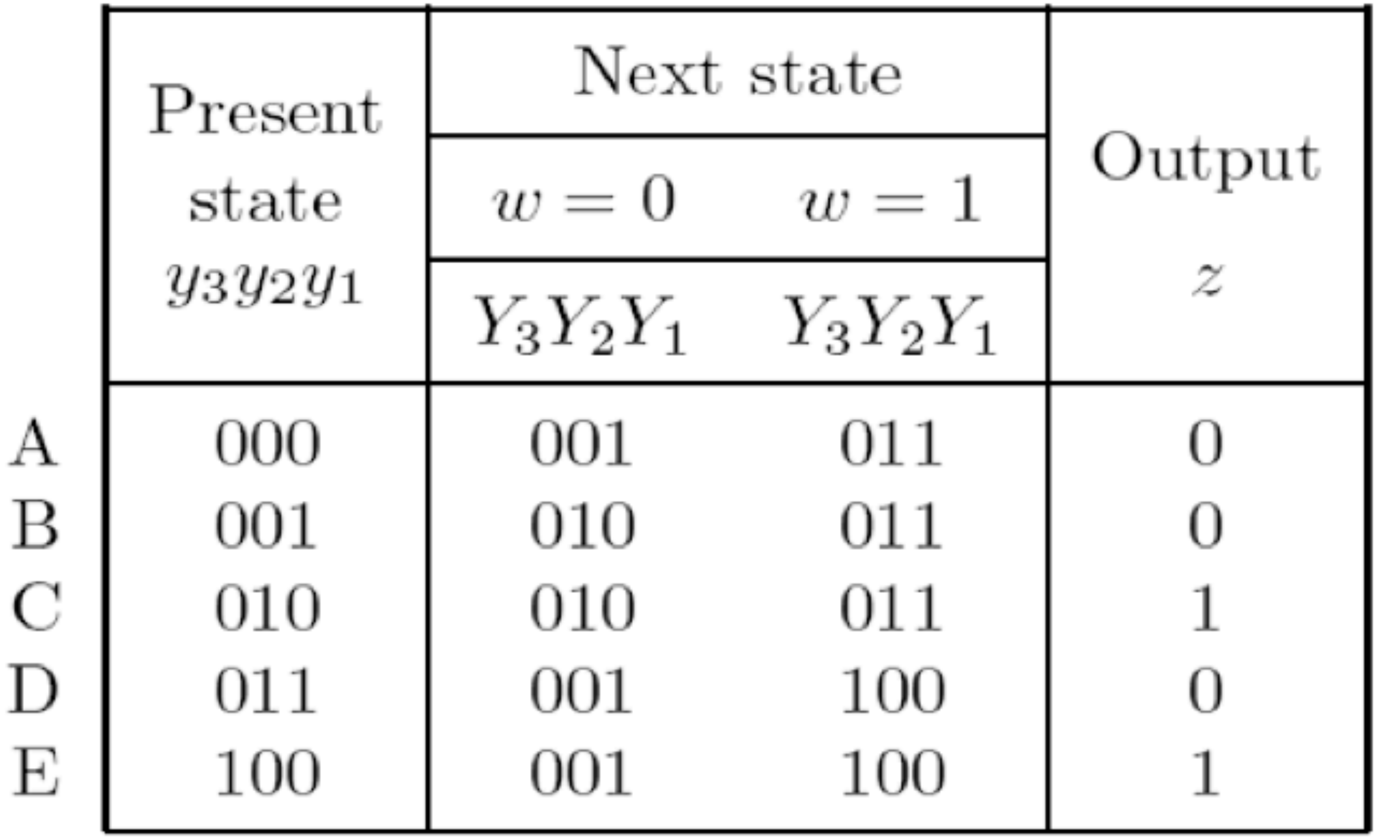
Truth Table for the Output z



	Present state $y_3y_2y_1$	Next state $w = 0$ $Y_3Y_2Y_1$	Next state $w = 1$ $Y_3Y_2Y_1$	Output z
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

y_3	y_2	y_1	z
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

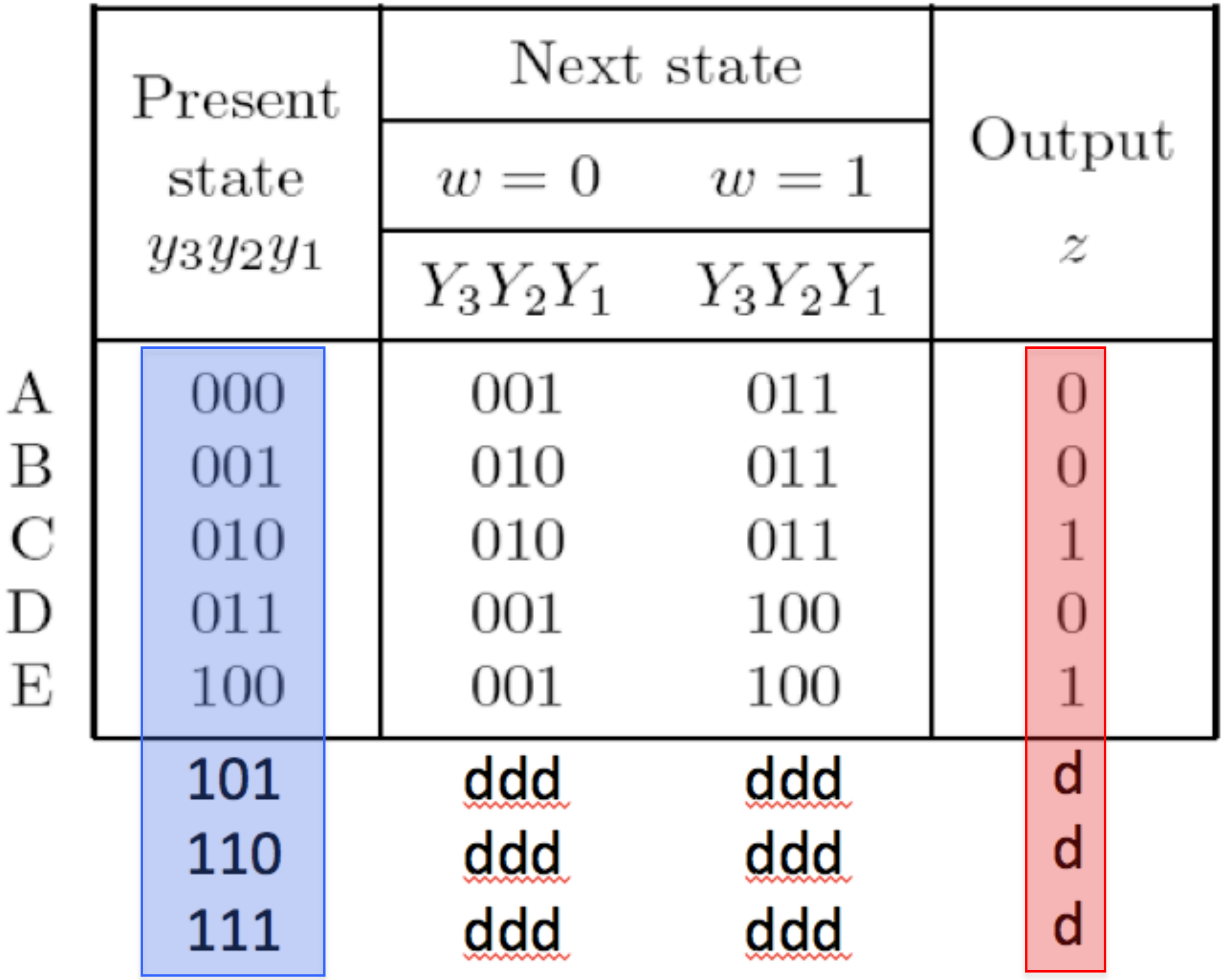
Truth Table for the Output z



	Present state $y_3y_2y_1$	Next state		Output z
		$w=0$	$w=1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

y_3	y_2	y_1	z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	d
1	1	0	d
1	1	1	d

Truth Table for the Output z



	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_3y_2y_1$	$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	z
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

y_3	y_2	y_1	z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	d
1	1	0	d
1	1	1	d

K-Map for the Output z

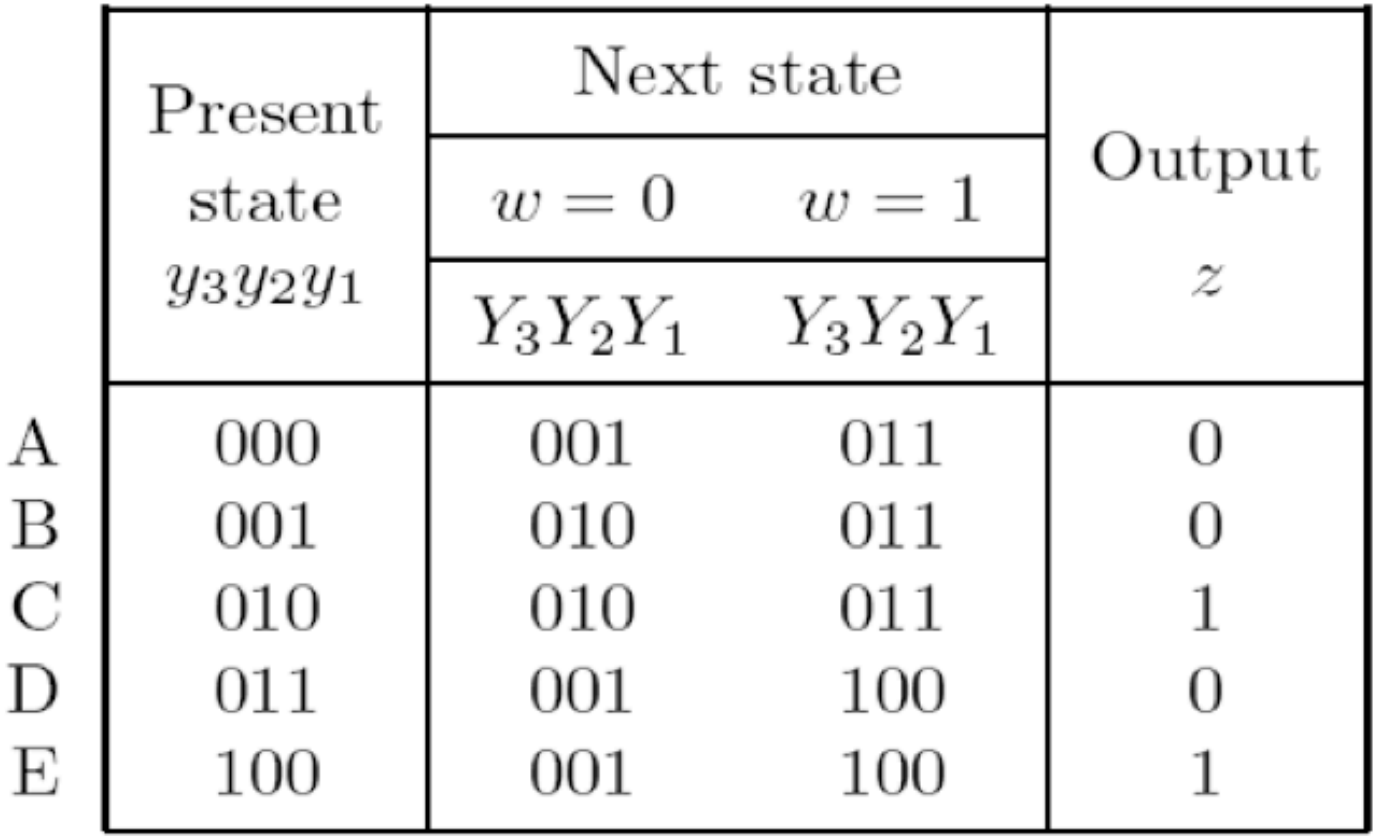
	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

z

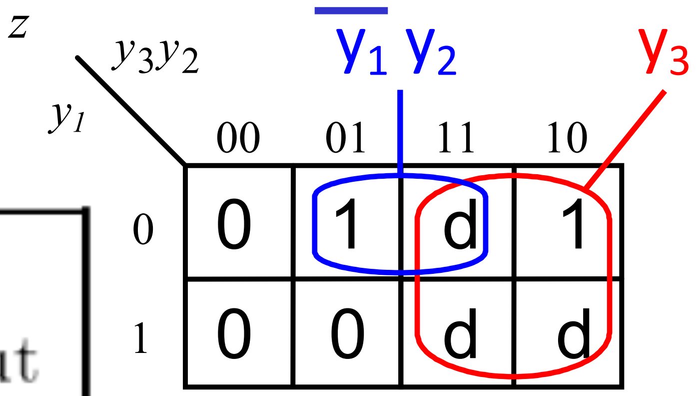
y_1 \ y_3y_2	00	01	11	10
0	0	1	d	1
1	0	0	d	d

y_3	y_2	y_1	z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	d
1	1	0	d
1	1	1	d

The Expression for the Output z



	Present state $y_3y_2y_1$	Next state $w = 0$ $Y_3Y_2Y_1$	Next state $w = 1$ $Y_3Y_2Y_1$	Output z
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d



z y_1 \ y_3y_2	00	01	11	10
0	0	1	d	1
1	0	0	d	d

y_3	y_2	y_1	z
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	d
1	1	0	d
1	1	1	d

State-Assigned Table for the FSM

	Present state $y_3y_2y_1$	Next state		Output z
		$w=0$	$w=1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

$$Y_1 = w\overline{y}_1\overline{y}_3 + w\overline{y}_2\overline{y}_3 + \overline{w}y_1y_2 + \overline{w}\overline{y}_1\overline{y}_2$$

$$Y_2 = y_1\overline{y}_2 + \overline{y}_1y_2 + w\overline{y}_2\overline{y}_3$$

$$Y_3 = wy_3 + wy_1y_2$$

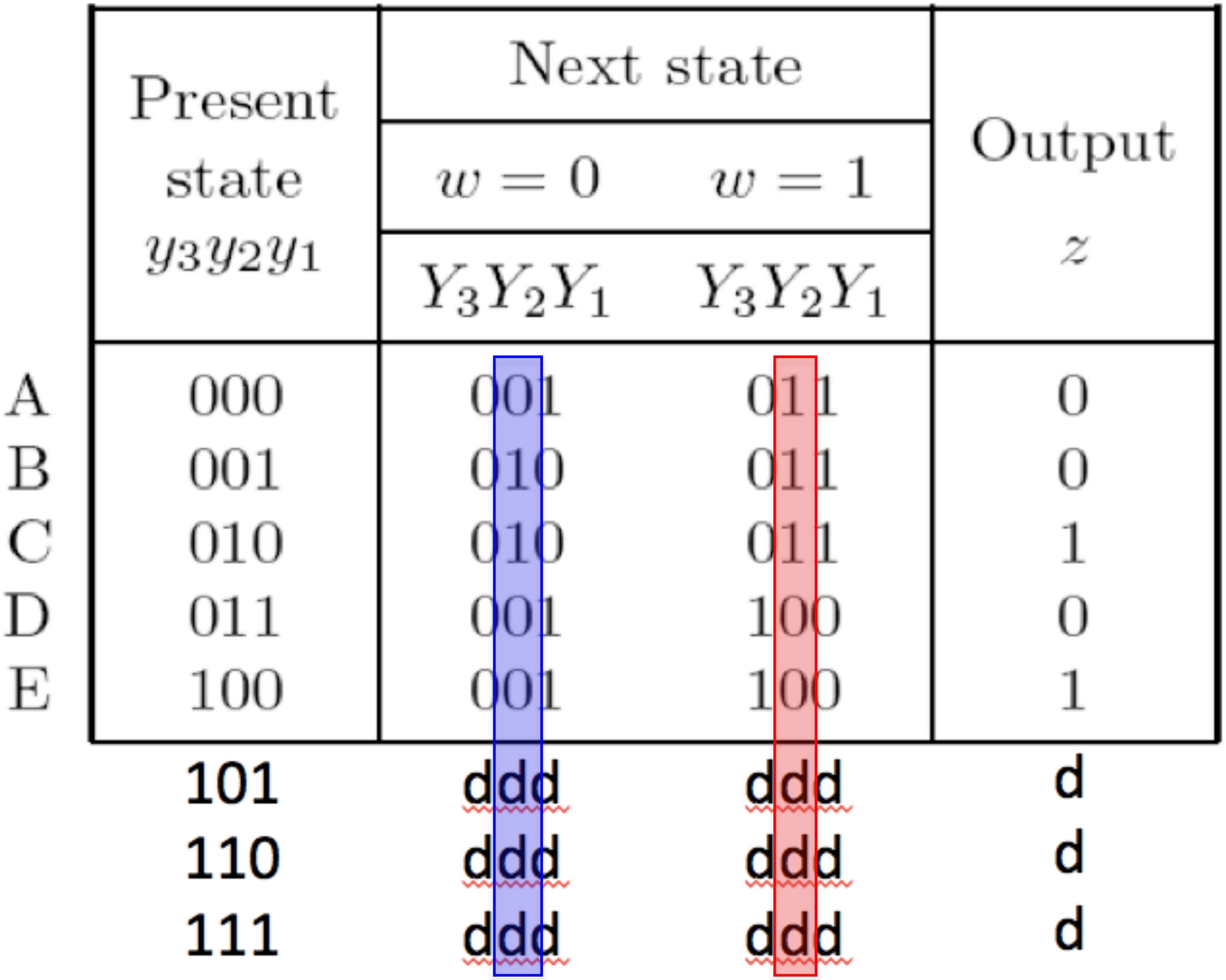
How can we derive these expressions?

Truth Table for Y_3

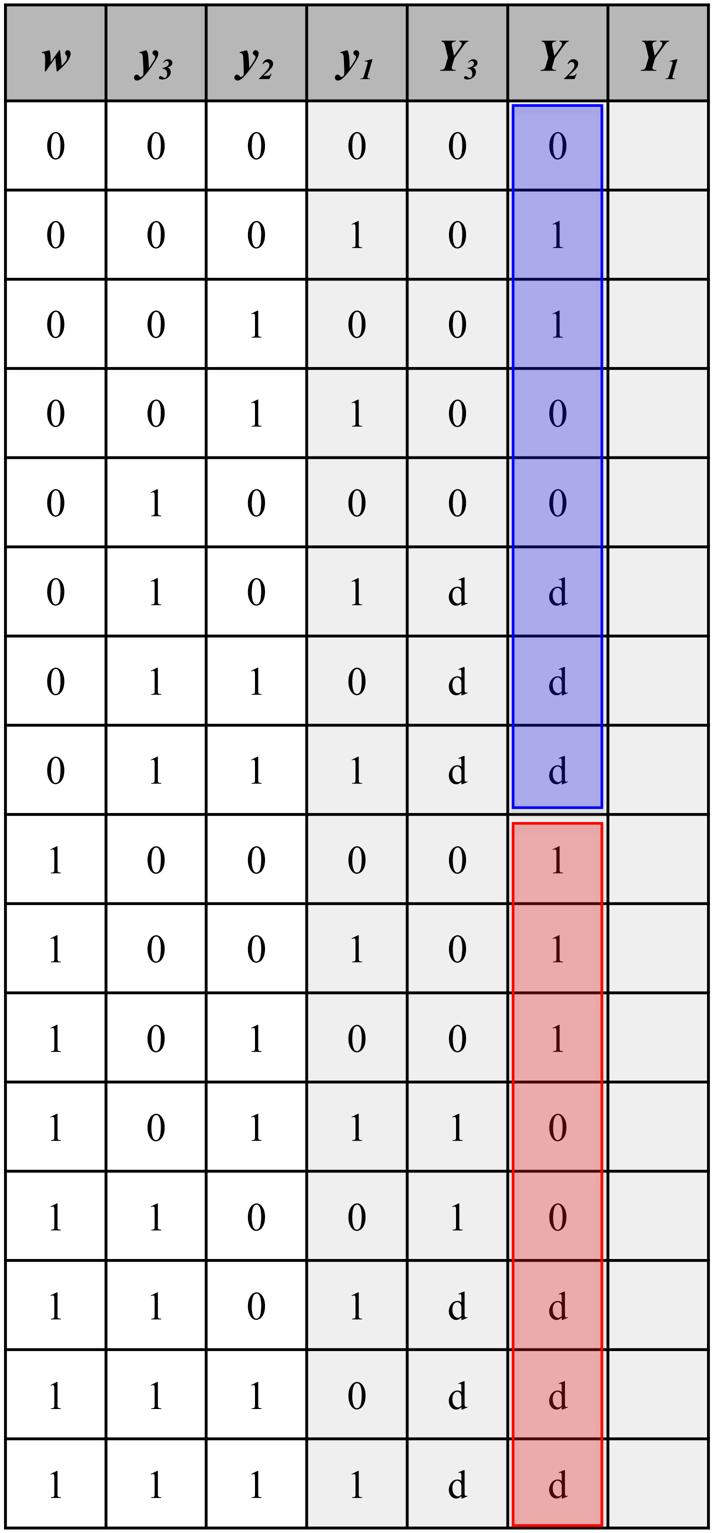
	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_3y_2y_1$	$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	z
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0		
0	0	0	1	0		
0	0	1	0	0		
0	0	1	1	0		
0	1	0	0	0		
0	1	0	1	d		
0	1	1	0	d		
0	1	1	1	d		
1	0	0	0	0		
1	0	0	1	0		
1	0	1	0	0		
1	0	1	1	1		
1	1	0	0	1		
1	1	0	1	d		
1	1	1	0	d		
1	1	1	1	d		

Truth Table for Y_2

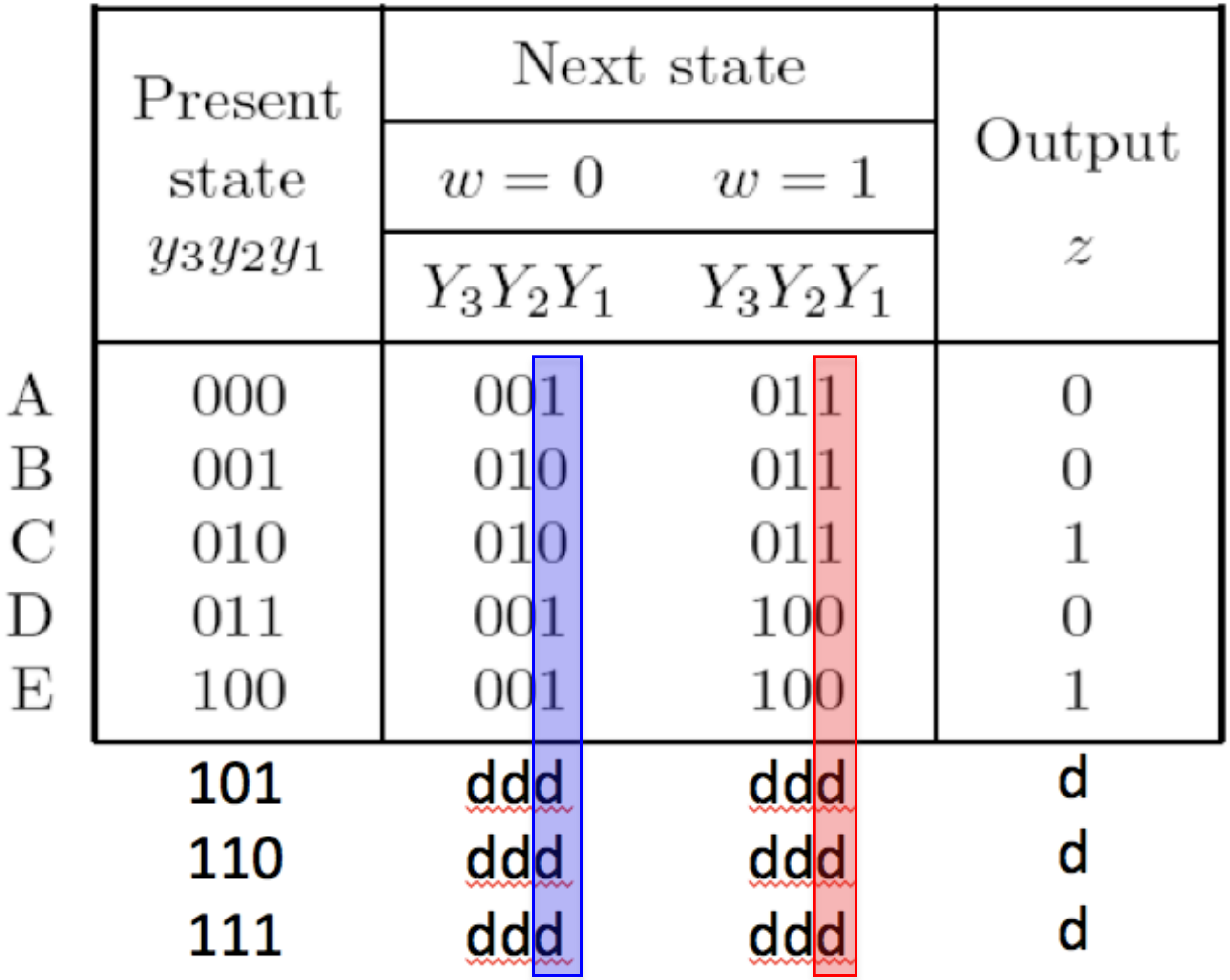


	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d



w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0	0	
0	0	0	1	0	1	
0	0	1	0	0	1	
0	0	1	1	0	0	
0	1	0	0	0	0	
0	1	0	1	d	d	
0	1	1	0	d	d	
0	1	1	1	d	d	
1	0	0	0	0	1	
1	0	0	1	0	1	
1	0	1	0	0	1	
1	0	1	1	1	0	
1	1	0	0	1	0	
1	1	0	1	d	d	
1	1	1	0	d	d	
1	1	1	1	d	d	

Truth Table for Y_1



	Present state $y_3y_2y_1$	Next state $w = 0$ $Y_3Y_2Y_1$	Next state $w = 1$ $Y_3Y_2Y_1$	Output z
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1
	101	ddd	ddd	d
	110	ddd	ddd	d
	111	ddd	ddd	d

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	d	d	d
0	1	1	0	d	d	d
0	1	1	1	d	d	d
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	1	0	0
1	1	0	0	1	0	0
1	1	0	1	d	d	d
1	1	1	0	d	d	d
1	1	1	1	d	d	d

K-Maps for Y_3, Y_2, Y_1

Y_3

$w\,y_3$

$y_2\,y_1$

	00	01	11	10
00	**0**	**0**	**1**	**0**
01	**0**	**d**	**d**	**0**
11	**0**	**d**	**d**	**1**
10	**0**	**d**	**d**	**0**

Y_2

$w\,y_3$

$y_2\,y_1$

	00	01	11	10
00	**0**	**0**	**0**	**1**
01	**1**	**d**	**d**	**1**
11	**0**	**d**	**d**	**0**
10	**1**	**d**	**d**	**1**

Y_1

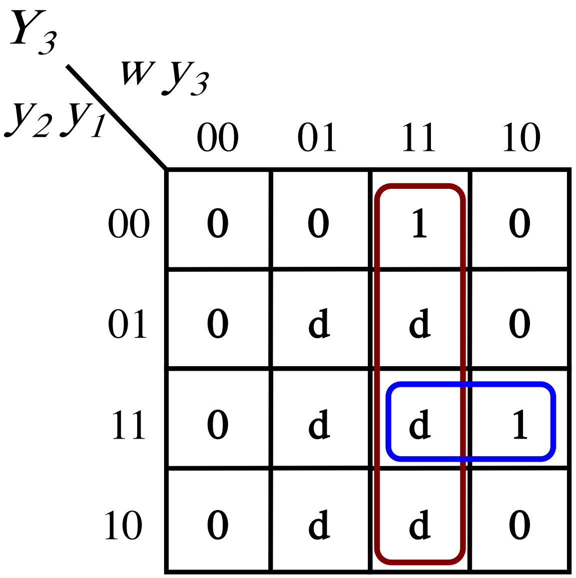
$w\,y_3$

$y_2\,y_1$

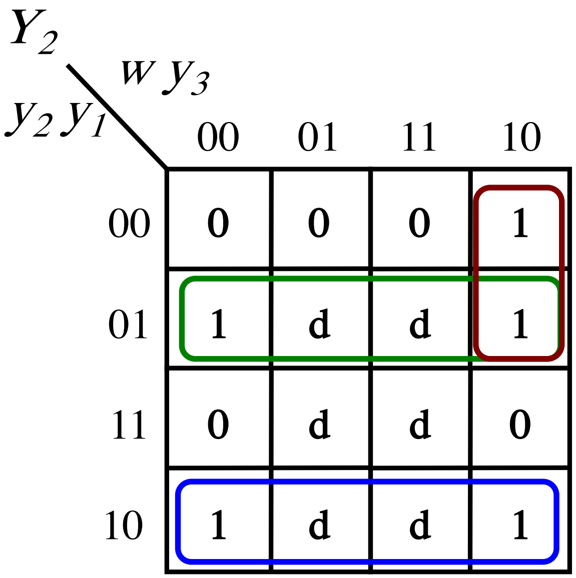
	00	01	11	10
00	**1**	**1**	**0**	**1**
01	**0**	**d**	**d**	**1**
11	**1**	**d**	**d**	**0**
10	**0**	**d**	**d**	**1**

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	d	d	d
0	1	1	0	d	d	d
0	1	1	1	d	d	d
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	1	0	0
1	1	0	0	1	0	0
1	1	0	1	d	d	d
1	1	1	0	d	d	d
1	1	1	1	d	d	d

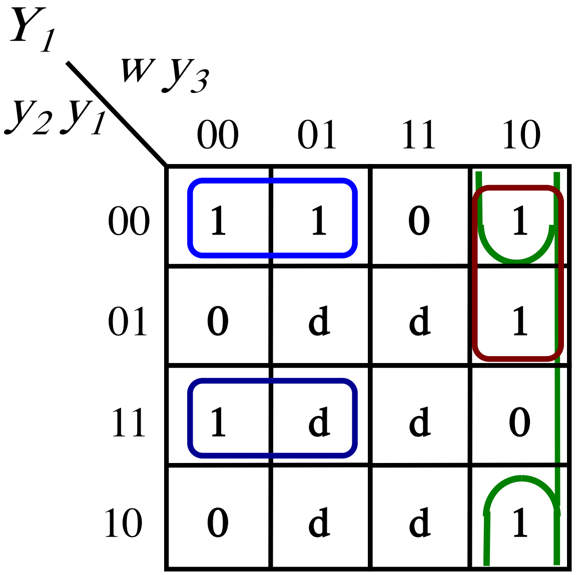
K-Maps for Y_3, Y_2, Y_1



	00	01	11	10
00	0	0	1	0
01	0	d	d	0
11	0	d	d	1
10	0	d	d	0



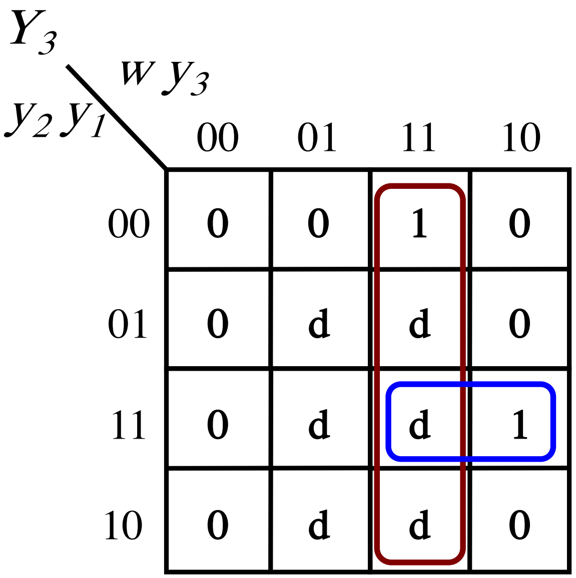
	00	01	11	10
00	0	0	0	1
01	1	d	d	1
11	0	d	d	0
10	1	d	d	1



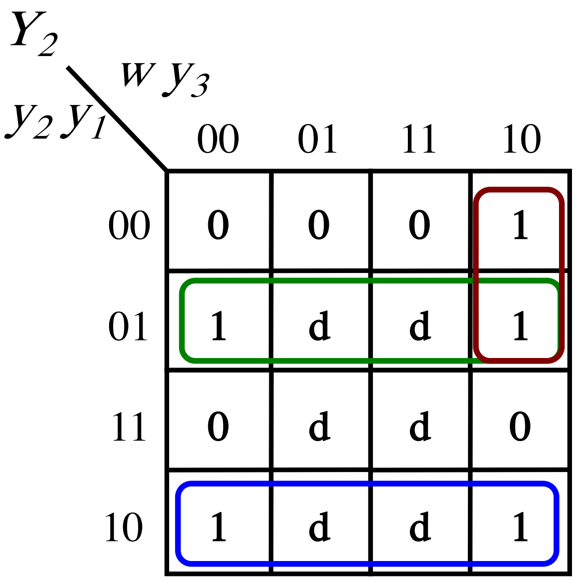
	00	01	11	10
00	1	1	0	1
01	0	d	d	1
11	1	d	d	0
10	0	d	d	1

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	d	d	d
0	1	1	0	d	d	d
0	1	1	1	d	d	d
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	1	0	0
1	1	0	0	1	0	0
1	1	0	1	d	d	d
1	1	1	0	d	d	d
1	1	1	1	d	d	d

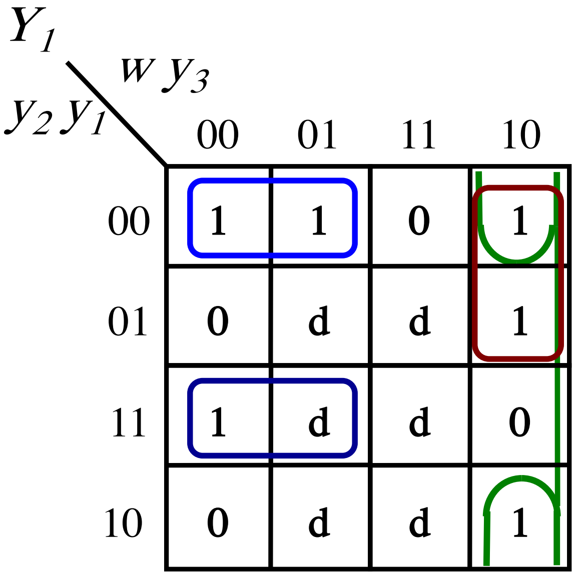
Expressions for Y_3, Y_2, Y_1



	00	01	11	10
00	0	0	1	0
01	0	d	d	0
11	0	d	d	1
10	0	d	d	0



	00	01	11	10
00	0	0	0	1
01	1	d	d	1
11	0	d	d	0
10	1	d	d	1



	00	01	11	10
00	1	1	0	1
01	0	d	d	1
11	1	d	d	0
10	0	d	d	1

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	0	0	1
0	0	0	1	0	1	0
0	0	1	0	0	1	0
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	d	d	d
0	1	1	0	d	d	d
0	1	1	1	d	d	d
1	0	0	0	0	1	1
1	0	0	1	0	1	1
1	0	1	0	0	1	1

$$Y_1 = w\bar{y}_1\bar{y}_3 + w\bar{y}_2\bar{y}_3 + \bar{w}y_1y_2 + \bar{w}\bar{y}_1\bar{y}_2$$

$$Y_2 = y_1\bar{y}_2 + \bar{y}_1y_2 + w\bar{y}_2\bar{y}_3$$

$$Y_3 = wy_3 + wy_1y_2$$

Next State and Output Expressions

$$Y_1 = w\overline{y}_1\overline{y}_3 + w\overline{y}_2\overline{y}_3 + \overline{w}y_1y_2 + \overline{w}\overline{y}_1\overline{y}_2$$

$$Y_2 = y_1\overline{y}_2 + \overline{y}_1y_2 + w\overline{y}_2\overline{y}_3$$

$$Y_3 = wy_3 + wy_1y_2$$

$$z = y_3 + \overline{y}_1y_2$$

An Improved State-Assigned Table

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

[Figure 6.87 from the textbook]

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

↑

B,C, D, E – when y_3=1

[Figure 6.89 from the textbook]

An Improved State-Assigned Table

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	D	0
B	C	D	0
C	C	D	1
D	B	E	0
E	B	E	1

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	B	D	0
B	C	D	0
C	C	D	1
D	B	E	0
E	B	E	1

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	001	011	0
B	001	010	011	0
C	010	010	011	1
D	011	001	100	0
E	100	001	100	1

[Figure 6.87 from the textbook]

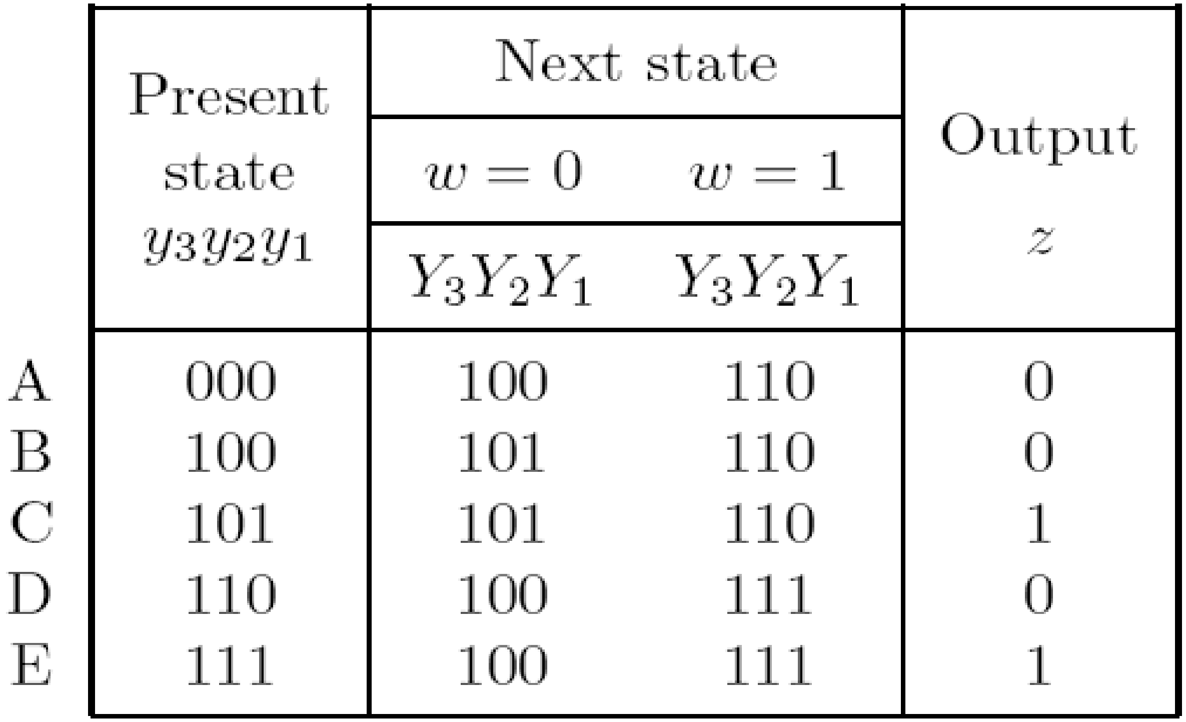
	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

↑

B,C, D, E – when y_3=1

[Figure 6.89 from the textbook]

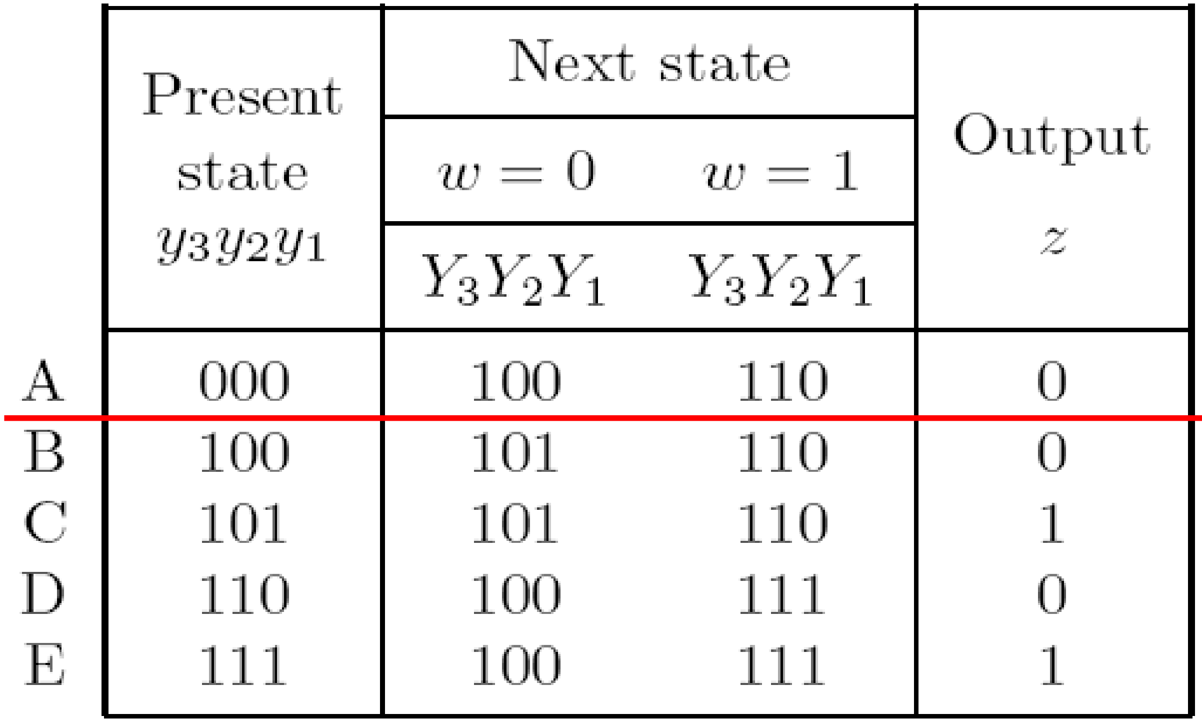
An Improved State-Assigned Table



	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

[Figure 6.89 from the textbook]

An Improved State-Assigned Table



	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_3y_2y_1$	$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	z
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

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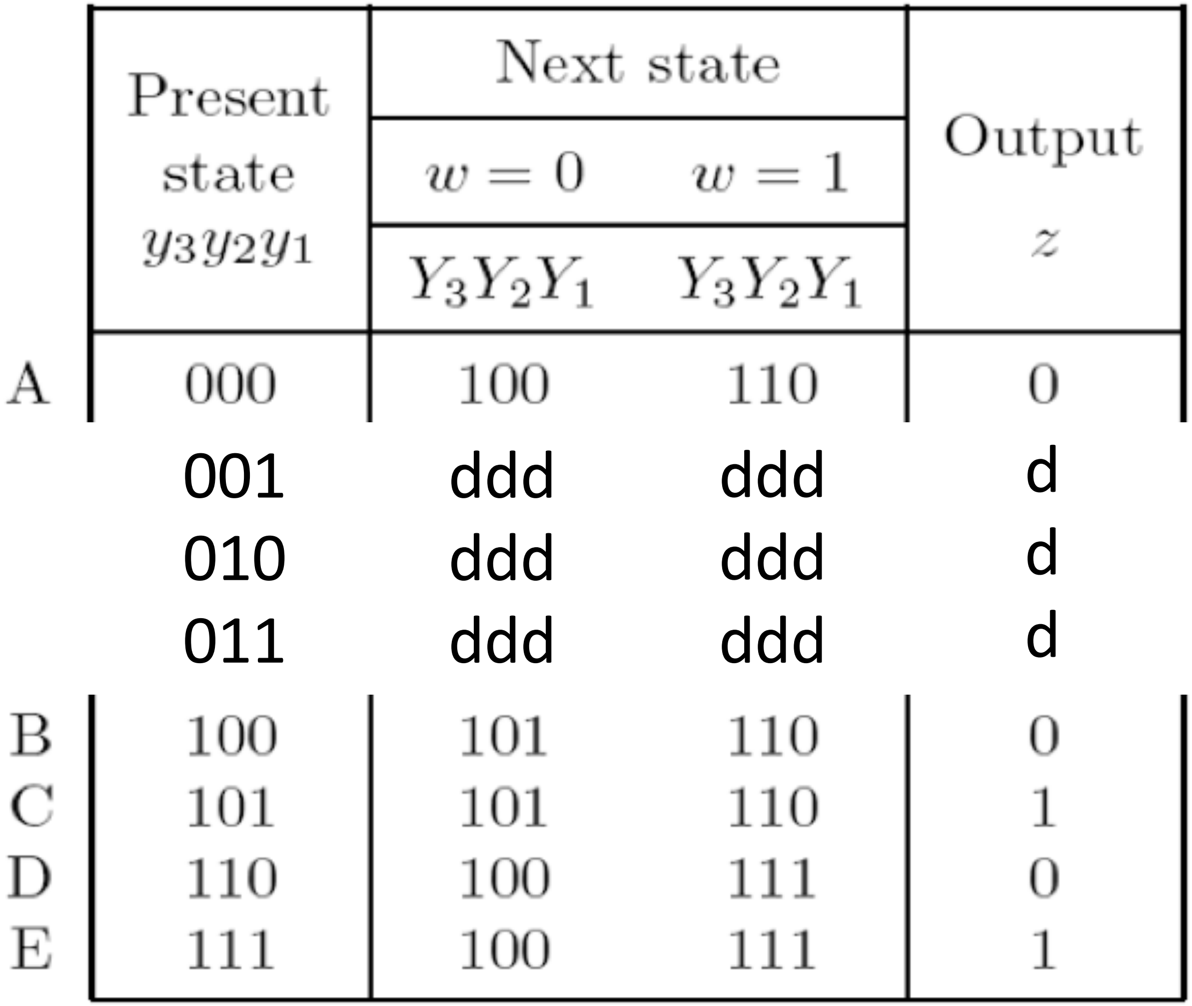
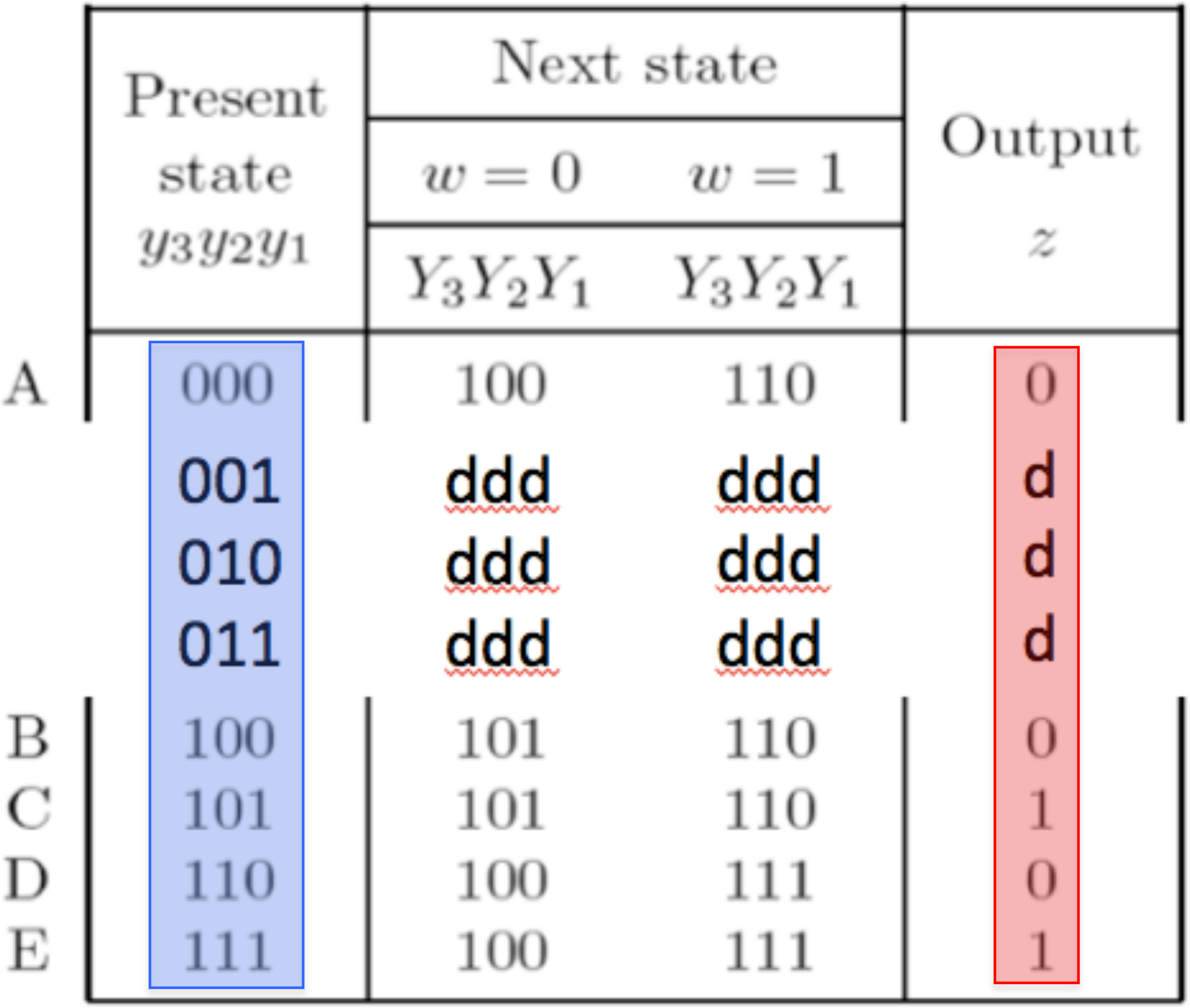
An Improved State-Assigned Table

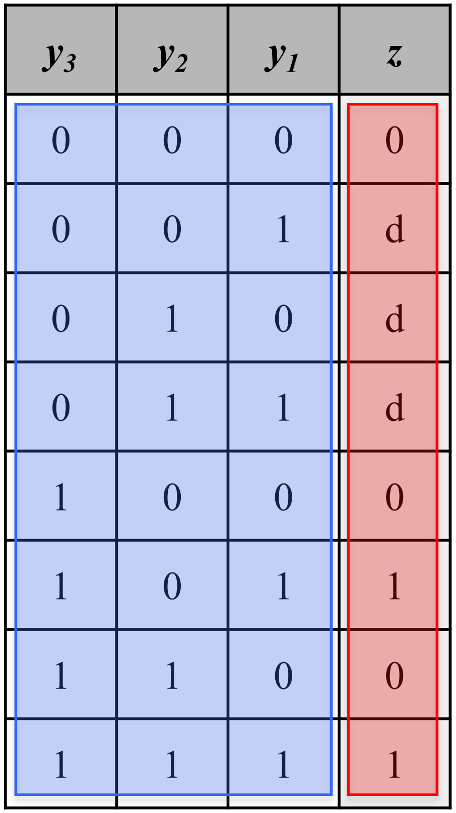


	Present state	Next state		Output
		$w=0$	$w=1$	
	$y_3y_2y_1$	$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	z
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

Truth Table for the Output z



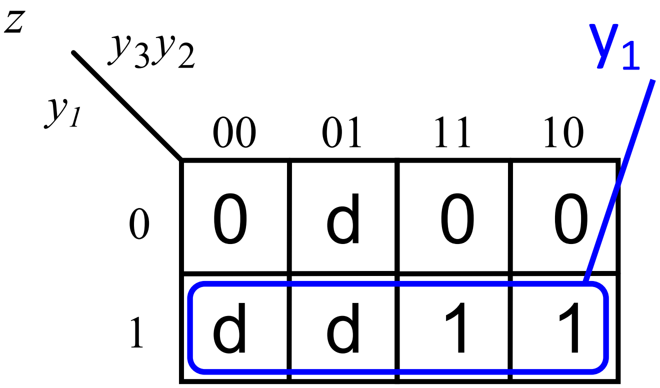
	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_3y_2y_1$	$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	z
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1



y_3	y_2	y_1	z
0	0	0	0
0	0	1	d
0	1	0	d
0	1	1	d
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Expression for the Output z

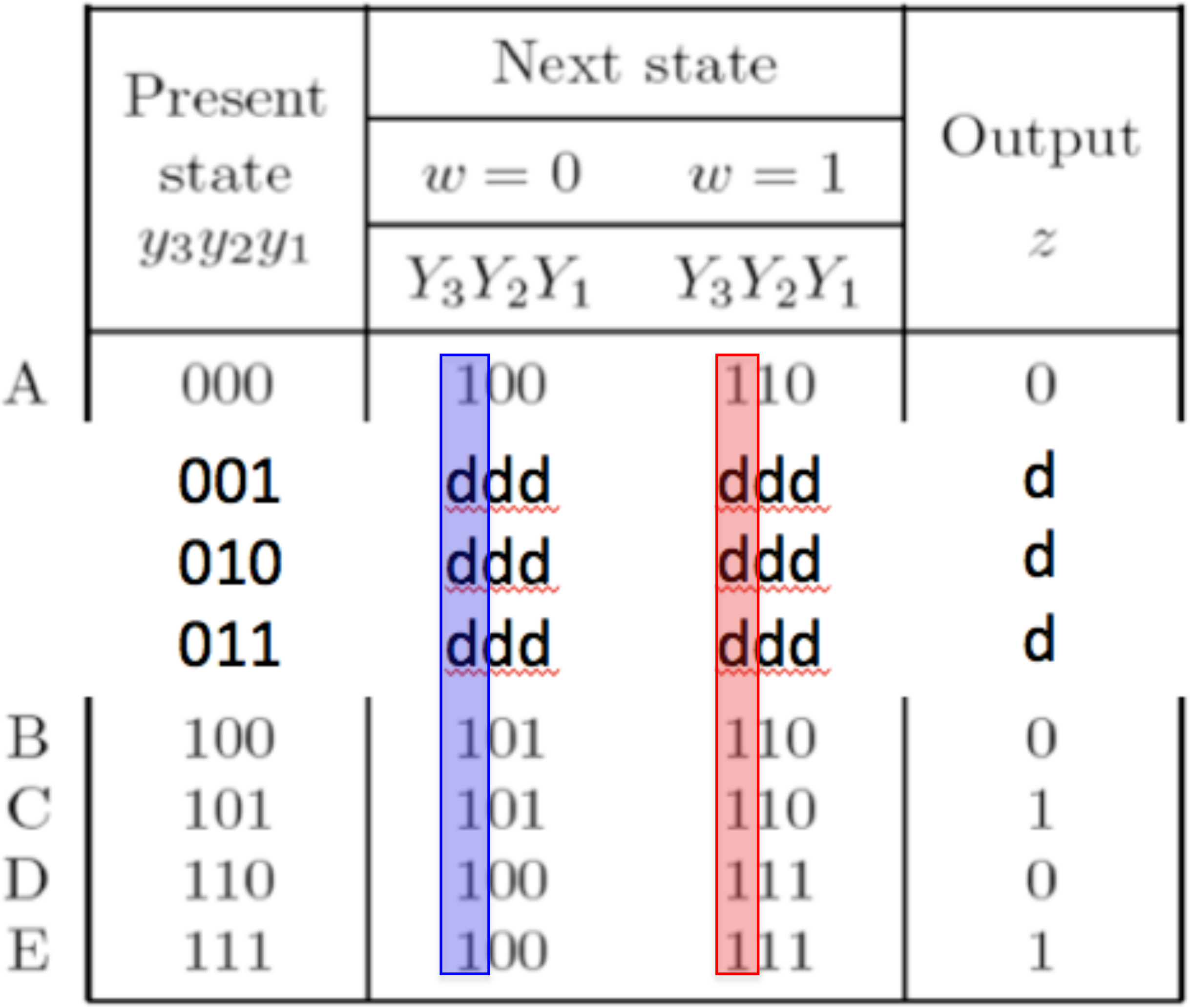
	Present state $y_3y_2y_1$	Next state $w=0$ $Y_3Y_2Y_1$	Next state $w=1$ $Y_3Y_2Y_1$	Output z
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1



y_1 \ y_3y_2	00	01	11	10
0	0	d	0	0
1	d	d	1	1

y_3	y_2	y_1	z
0	0	0	0
0	0	1	d
0	1	0	d
0	1	1	d
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Truth Table for Y_3



	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1		
0	0	0	1	d		
0	0	1	0	d		
0	0	1	1	d		
0	1	0	0	1		
0	1	0	1	1		
0	1	1	0	1		
0	1	1	1	1		
1	0	0	0	1		
1	0	0	1	d		
1	0	1	0	d		
1	0	1	1	d		
1	1	0	0	1		
1	1	0	1	1		
1	1	1	0	1		
1	1	1	1	1		

Truth Table for Y_2

	Present state $y_3y_2y_1$	Next state $w=0$ $Y_3Y_2Y_1$	Next state $w=1$ $Y_3Y_2Y_1$	Output z
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1	0	
0	0	0	1	d	d	
0	0	1	0	d	d	
0	0	1	1	d	d	
0	1	0	0	1	0	
0	1	0	1	1	0	
0	1	1	0	1	0	
0	1	1	1	1	0	
1	0	0	0	1	1	
1	0	0	1	d	d	
1	0	1	0	d	d	
1	0	1	1	d	d	
1	1	0	0	1	1	
1	1	0	1	1	1	
1	1	1	0	1	1	
1	1	1	1	1	1	

Truth Table for Y_1

	Present state $y_3y_2y_1$	Next state $w = 0$ $Y_3Y_2Y_1$	Next state $w = 1$ $Y_3Y_2Y_1$	Output z
A	000	100	110	0
	001	ddd	ddd	d
	010	ddd	ddd	d
	011	ddd	ddd	d
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1	0	0
0	0	0	1	d	d	d
0	0	1	0	d	d	d
0	0	1	1	d	d	d
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	1	0
1	0	0	1	d	d	d
1	0	1	0	d	d	d
1	0	1	1	d	d	d
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	1	1

K-Maps for Y_3, Y_2, Y_1

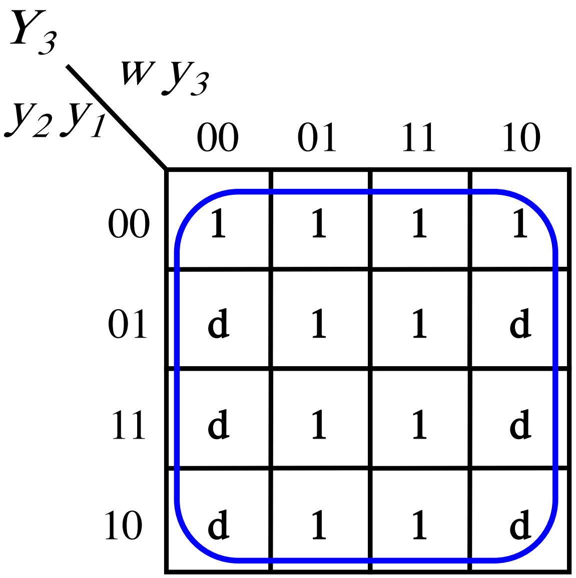
Y_3 $w y_3$ $y_2 y_1$	00	01	11	10
00	1	1	1	1
01	d	1	1	d
11	d	1	1	d
10	d	1	1	d

Y_2 $w y_3$ $y_2 y_1$	00	01	11	10
00	0	0	1	1
01	d	0	1	d
11	d	0	1	d
10	d	0	1	d

Y_1 $w y_3$ $y_2 y_1$	00	01	11	10
00	0	1	0	0
01	d	1	0	d
11	d	0	1	d
10	d	0	1	d

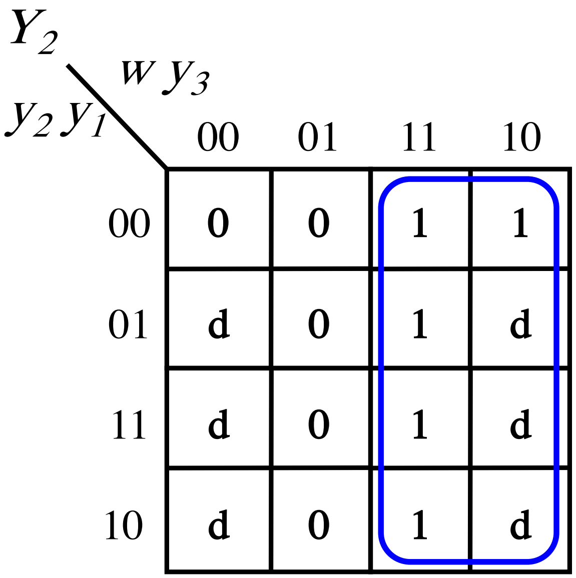
w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1	0	0
0	0	0	1	d	d	d
0	0	1	0	d	d	d
0	0	1	1	d	d	d
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	1	0
1	0	0	1	d	d	d
1	0	1	0	d	d	d
1	0	1	1	d	d	d
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	1	1

K-Maps for Y_3, Y_2, Y_1



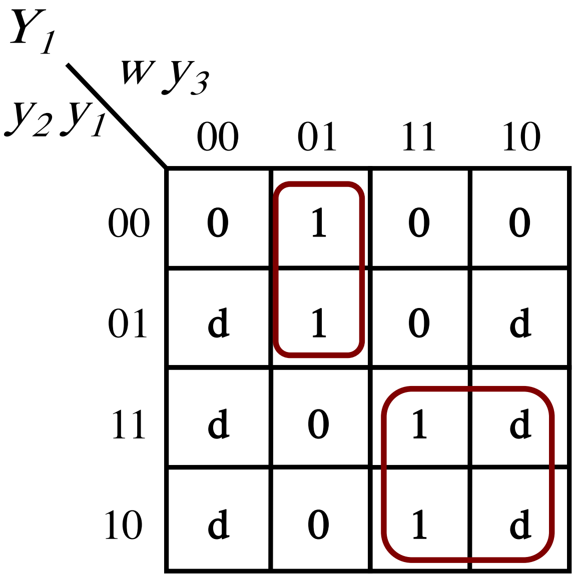
Y_3

$y_2 y_1$ \ $w y_3$	00	01	11	10
00	1	1	1	1
01	d	1	1	d
11	d	1	1	d
10	d	1	1	d



Y_2

$y_2 y_1$ \ $w y_3$	00	01	11	10
00	0	0	1	1
01	d	0	1	d
11	d	0	1	d
10	d	0	1	d

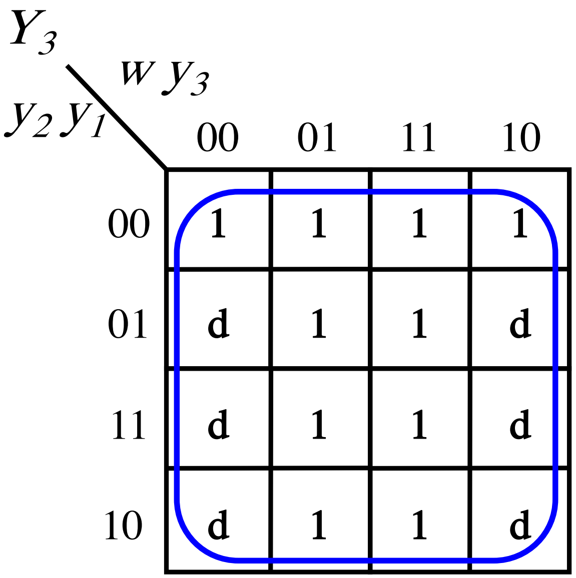


Y_1

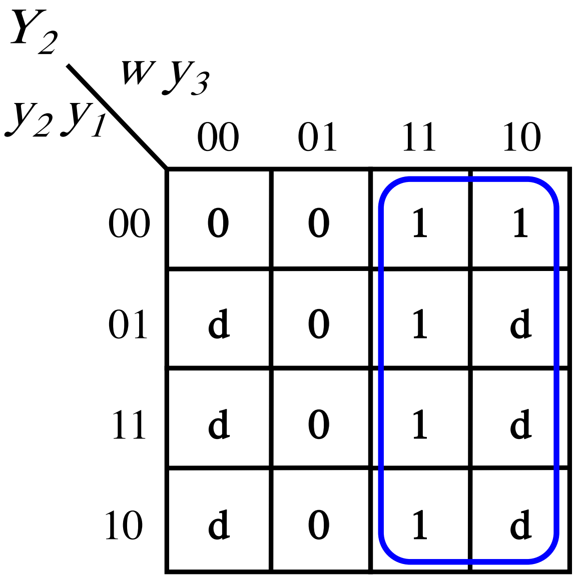
$y_2 y_1$ \ $w y_3$	00	01	11	10
00	0	1	0	0
01	d	1	0	d
11	d	0	1	d
10	d	0	1	d

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1	0	0
0	0	0	1	d	d	d
0	0	1	0	d	d	d
0	0	1	1	d	d	d
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	1	0
1	0	0	1	d	d	d
1	0	1	0	d	d	d
1	0	1	1	d	d	d
1	1	0	0	1	1	0
1	1	0	1	1	1	0
1	1	1	0	1	1	1
1	1	1	1	1	1	1

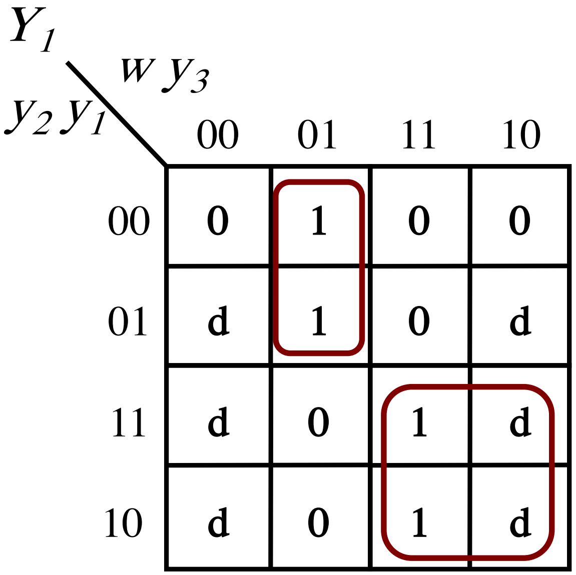
K-Maps for Y_3, Y_2, Y_1



	00	01	11	10
00	1	1	1	1
01	d	1	1	d
11	d	1	1	d
10	d	1	1	d



	00	01	11	10
00	0	0	1	1
01	d	0	1	d
11	d	0	1	d
10	d	0	1	d



	00	01	11	10
00	0	1	0	0
01	d	1	0	d
11	d	0	1	d
10	d	0	1	d

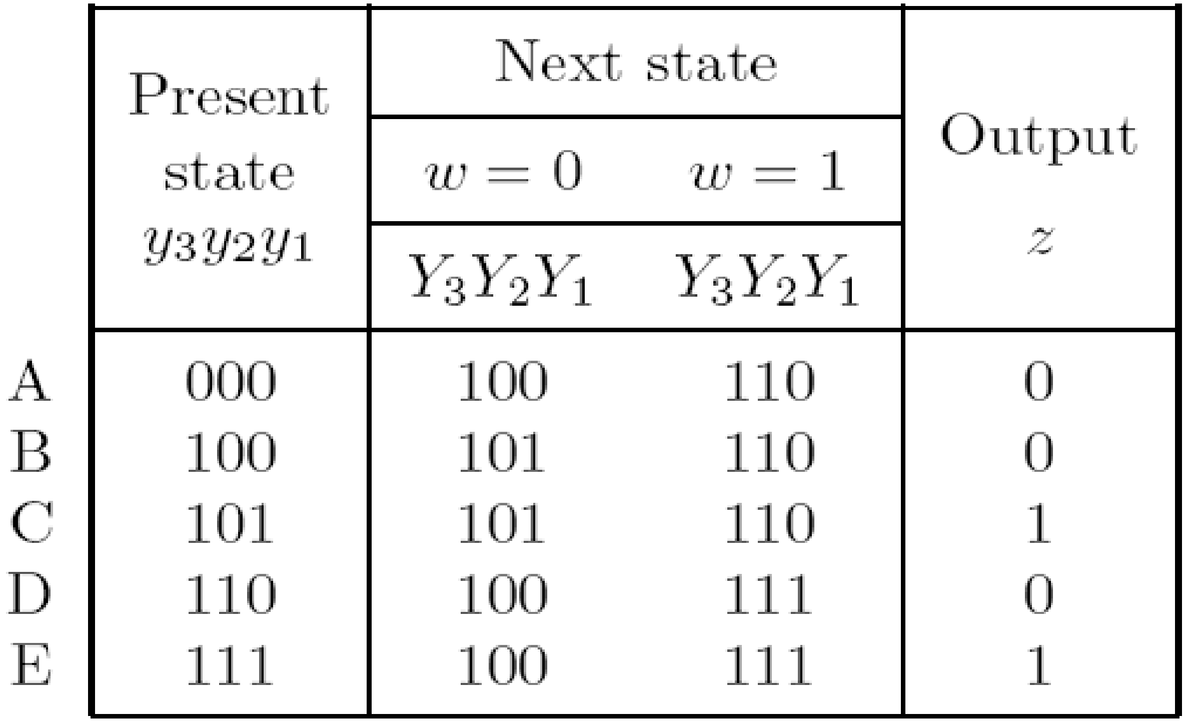
$$Y_1 = wy_2 + \overline{w}y_3\overline{y}_2$$

$$Y_2 = w$$

$$Y_3 = 1$$

w	y_3	y_2	y_1	Y_3	Y_2	Y_1
0	0	0	0	1	0	0
0	0	0	1	d	d	d
0	0	1	0	d	d	d
0	0	1	1	d	d	d
0	1	0	0	1	0	1
0	1	0	1	1	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	1	1	0
1	0	0	1	d	d	d
1	0	1	0	d	d	d
1	0	1	1	d	d	d
		0	0	1	1	0
		0	1	1	1	0
		1	0	1	1	1
		1	1	1	1	1

An Improved State-Assigned Table



	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

$$Y_1 = wy_2 + \overline{w}y_3\overline{y}_2$$

$$Y_2 = w$$

$$Y_3 = 1$$

$$z = y_1$$

An Improved State-Assigned Table

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

$$Y_1 = wy_2 + \overline{w}\cancel{y_3}\overline{y}_2$$

$$Y_2 = w$$

$$\cancel{Y_3 = 1}$$

$$z = y_1$$

Example 6.13

Goal

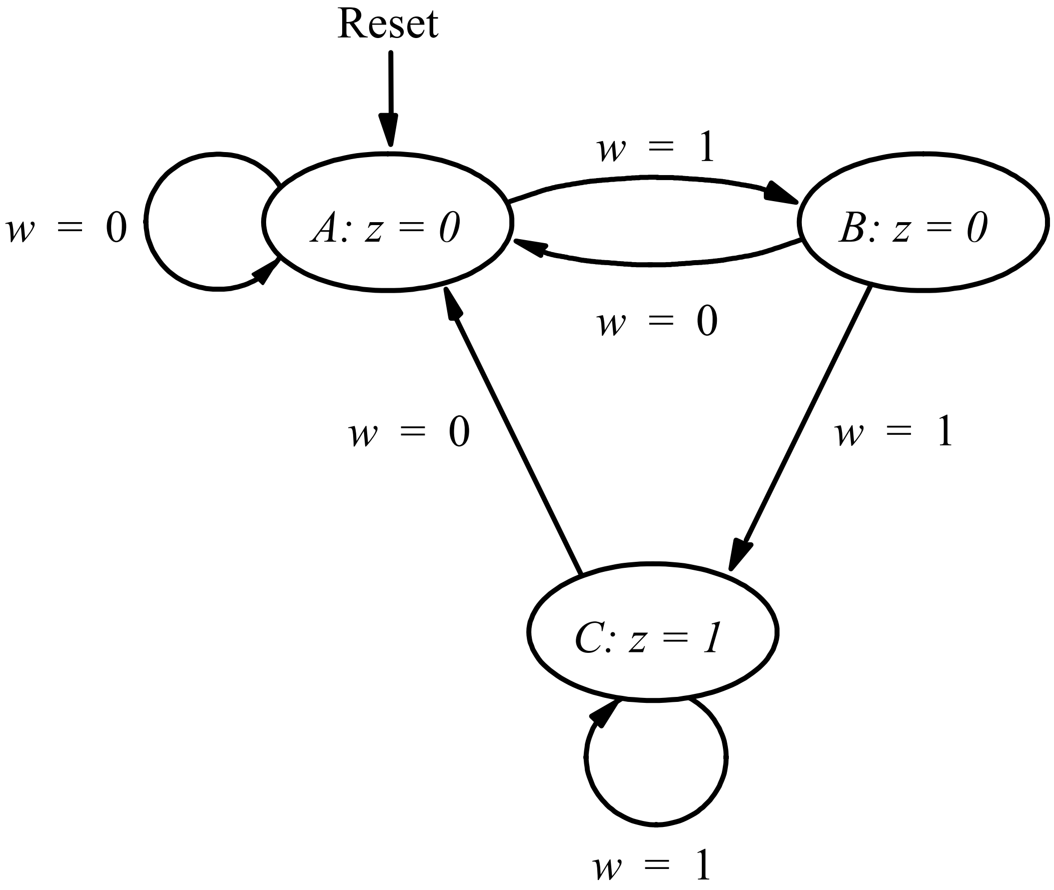
- Design an FSM that detects if the previous two values of the input w were equal to 00 or 11.
- But do this with two different FSMs. The first one detects two consecutive 1's. The second one detects two consecutive 0's.
- If either condition (i.e., output of FSM) is true then the output z should be set to 1; otherwise to 0.

Example 6.13

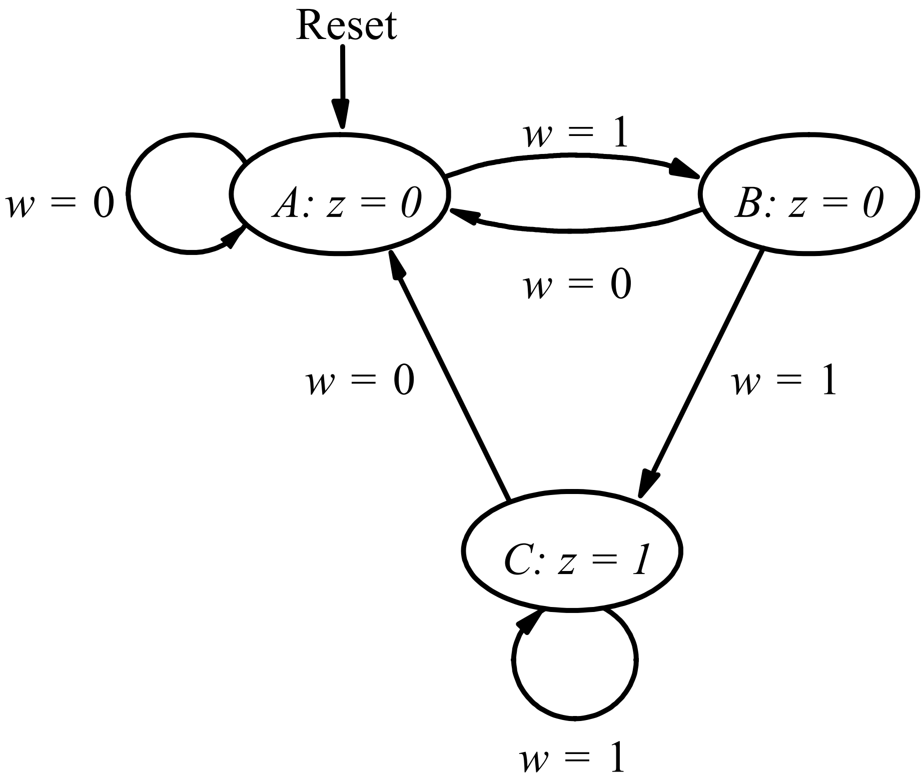
(Construct the first FSM)

FSM to detect two consecutive 1's

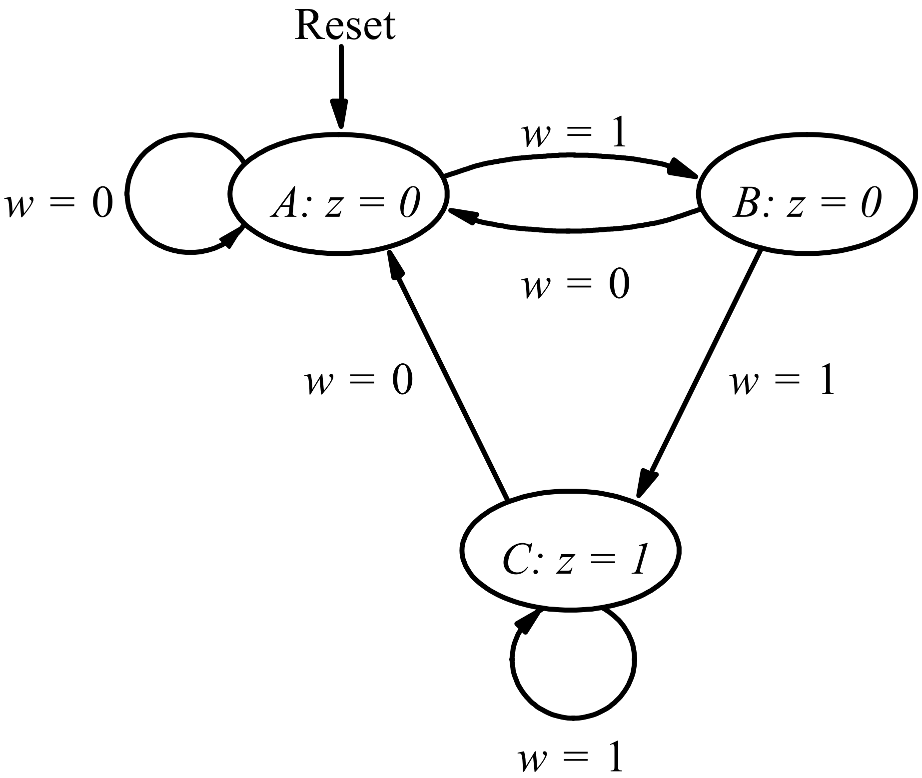
(this was the first example in Chapter 6)



[Figure 6.3 from the textbook]



Present state	Next state		Output z
	w = 0	w = 1	
A B C			



Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

[Figure 6.4 from the textbook]

A Better State Encoding

Present state	Next state		Output z
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

Suppose we encoded our states another way:

A ~ 00

B ~ 01

C ~ 11

A Better State Encoding

Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	A	B	0
B	A	C	0
C	A	C	1

Present state	Next state		Output z
	$w = 0$	$w = 1$	

A ~ 00

B ~ 01

C ~ 11

A Better State Encoding

Present state	Next state		Output
	$w = 0$	$w = 1$	z
A	A	B	0
B	A	C	0
C	A	C	1

	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_2 y_1$	$Y_2 Y_1$	$Y_2 Y_1$	z
A	00	00	01	0
B	01	00	11	0
C	11	00	11	1
	10	*dd*	*dd*	*d*

Let's Derive the Logic Expressions

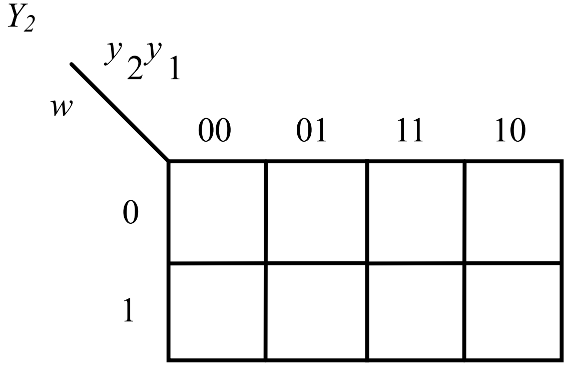
	Present state	Next state		Output
		$w = 0$	$w = 1$	
	y_2y_1	Y_2Y_1	Y_2Y_1	z
A	00	00	01	0
B	01	00	11	0
C	11	00	11	1
	10	*dd*	*dd*	*d*

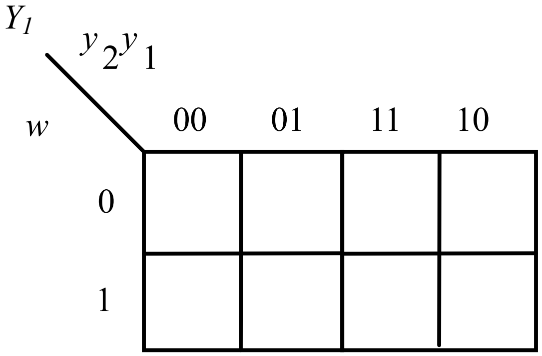
[Figure 6.16 from the textbook]

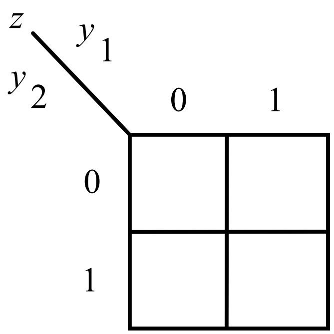
Let's Derive the Logic Expressions

Warning:
This table does not enumerate y_2y_1, in the standard way, so be careful when filling out the K-Map.

	Present state	Next state		Output
		$w = 0$	$w = 1$	
	y_2y_1	Y_2Y_1	Y_2Y_1	z
A	00	00	01	0
B	01	00	11	0
C	11	00	11	1
	10	*dd*	*dd*	*d*



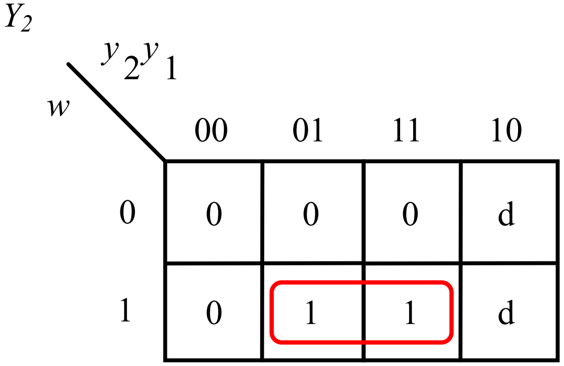




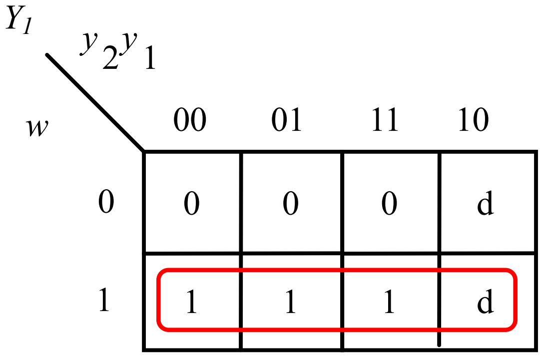
Let's Derive the Logic Expressions

Warning:
This table does not enumerate y_2y_1, in the standard way, so be careful when filling out the K-Map.

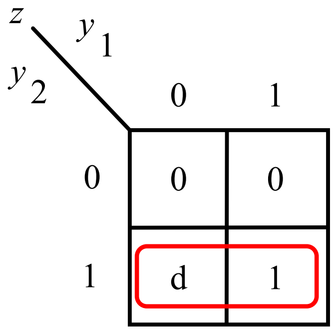
	Present state	Next state		Output
		$w = 0$	$w = 1$	z
	y_2y_1	Y_2Y_1	Y_2Y_1	
A	00	00	01	0
B	01	00	11	0
C	11	00	11	1
	10	*dd*	*dd*	*d*



$$Y_2(w, y_2, y_1) = wy_1$$

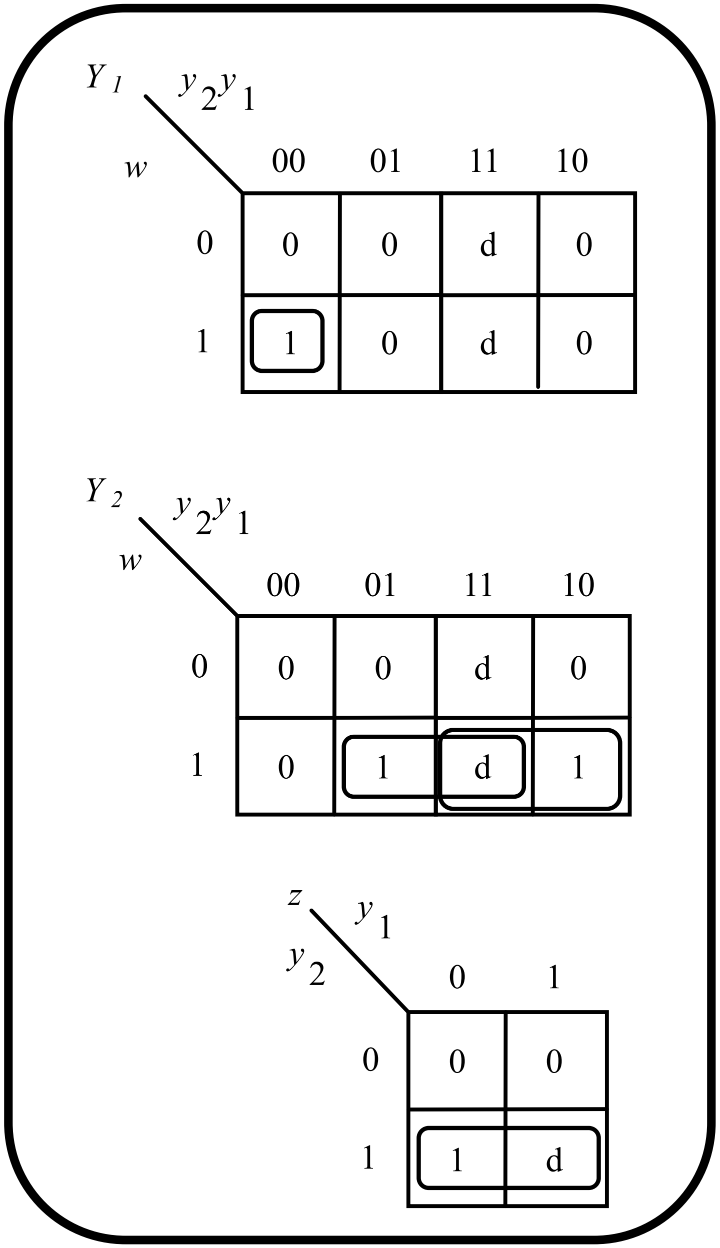


$$Y_1(w, y_2, y_1) = w$$



$$z(y_2, y_1) = y_2$$

Original State Encodings



Y_1

w \ y_2y_1	00	01	11	10
0	0	0	d	0
1	1	0	d	0

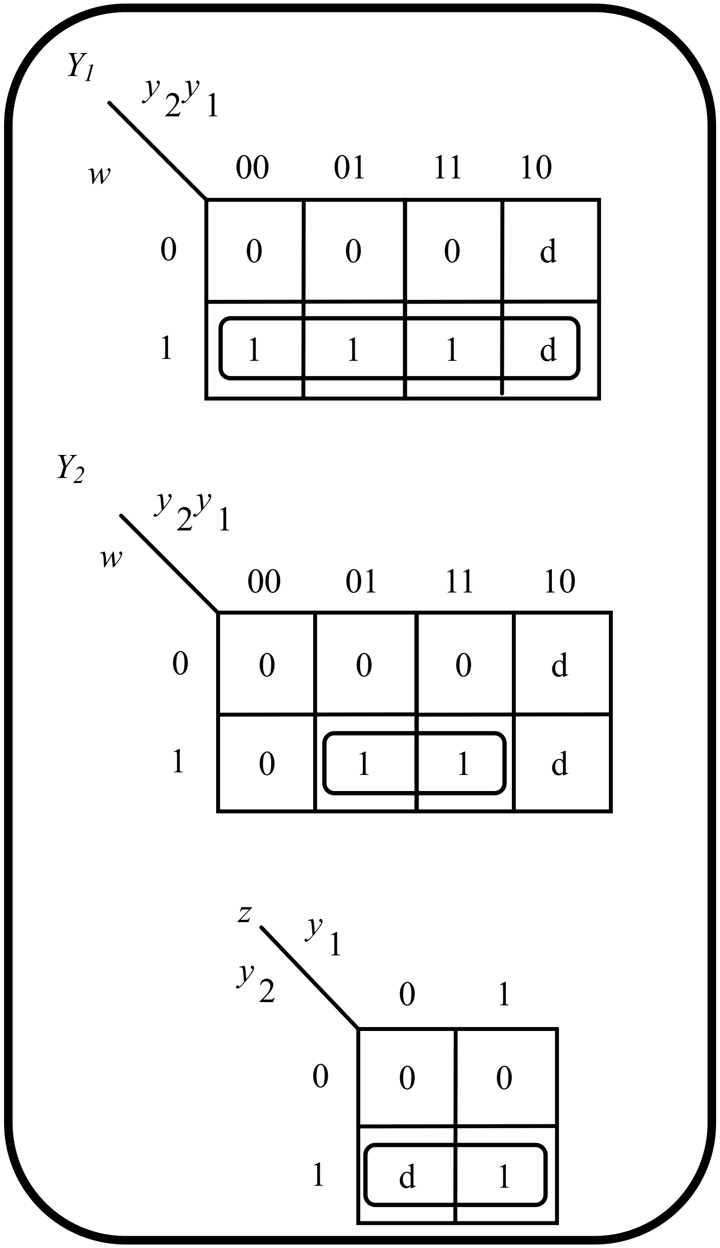
Y_2

w \ y_2y_1	00	01	11	10
0	0	0	d	0
1	0	1	d	1

z

y_2 \ y_1	0	1
0	0	0
1	1	d

New State Encodings



Y_1

w \ y_2y_1	00	01	11	10
0	0	0	0	d
1	1	1	1	d

Y_2

w \ y_2y_1	00	01	11	10
0	0	0	0	d
1	0	1	1	d

z

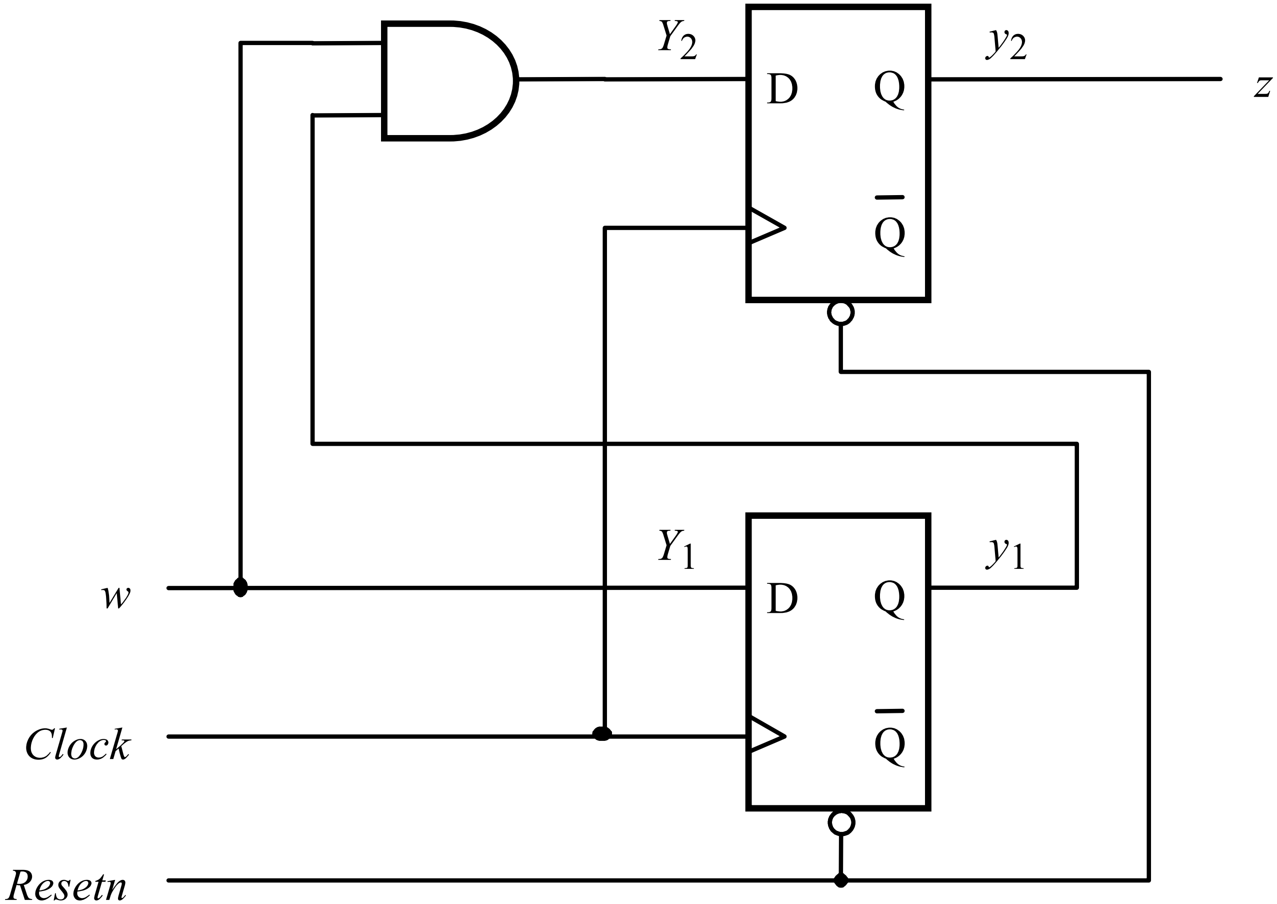
y_2 \ y_1	0	1
0	0	0
1	d	1

The Circuit Diagram

$$Y_1(w, y_2, y_1) = w$$

$$Y_2(w, y_2, y_1) = wy_1$$

$$z(y_2, y_1) = y_2$$

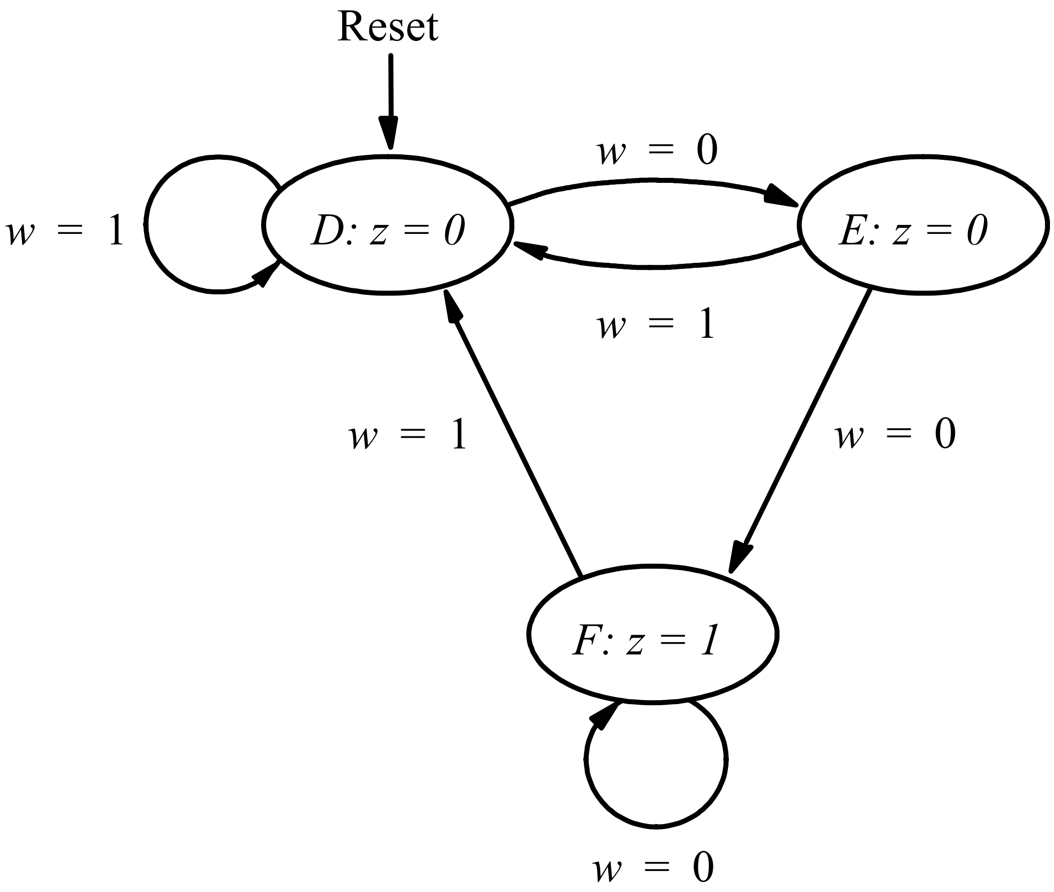


[Figure 6.17 from the textbook]

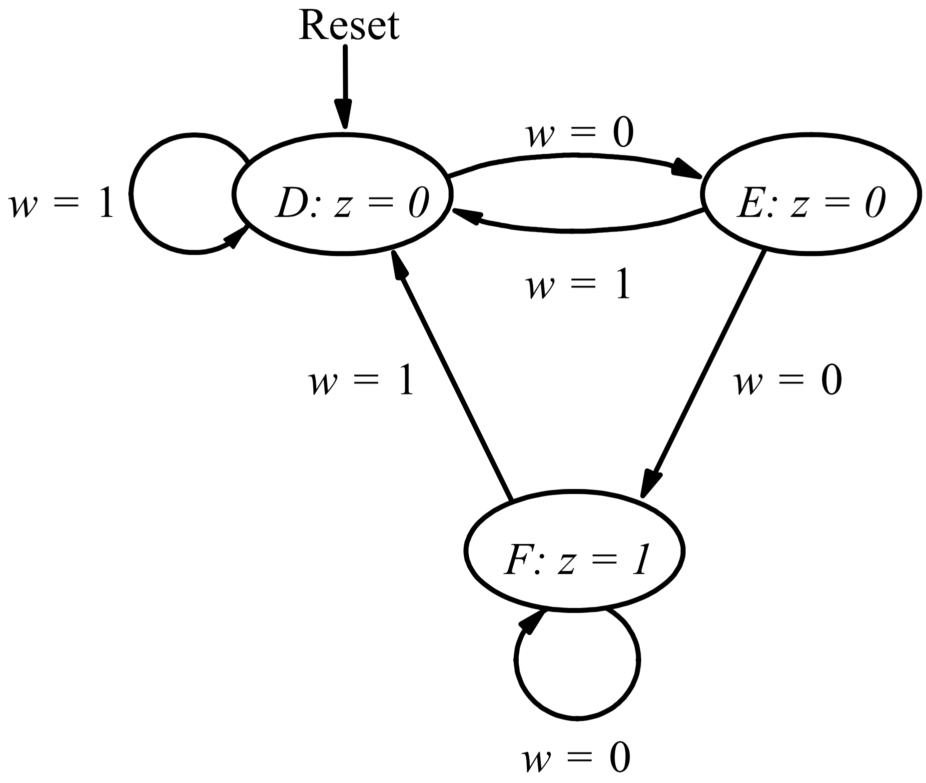
Example 6.13

(Construct the second FSM)

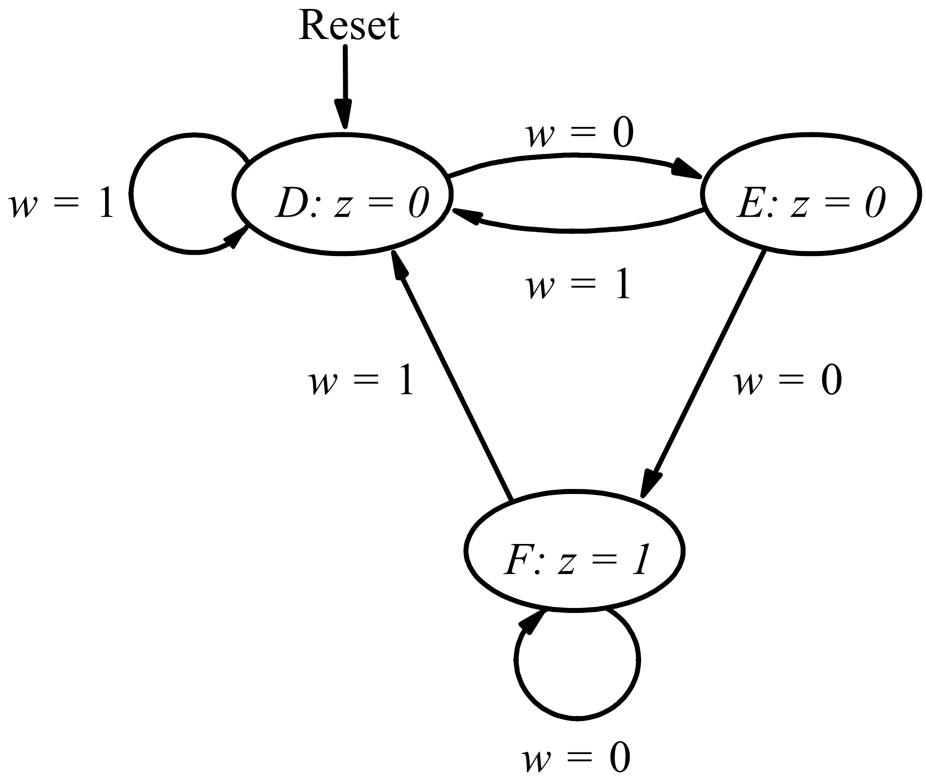
FSM to detect two consecutive 0's



This is similar to the previous one. Just invert the w's and relabel the states to D,E,F.

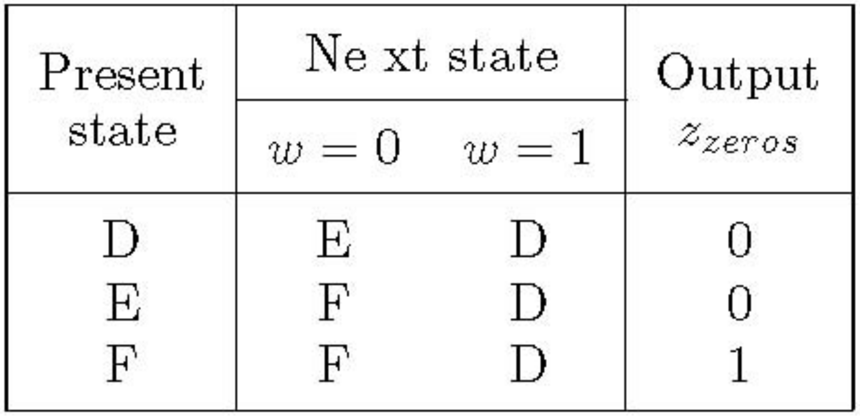


Present state	Next state		Output z
	$w = 0$	$w = 1$	
D			
E			
F			



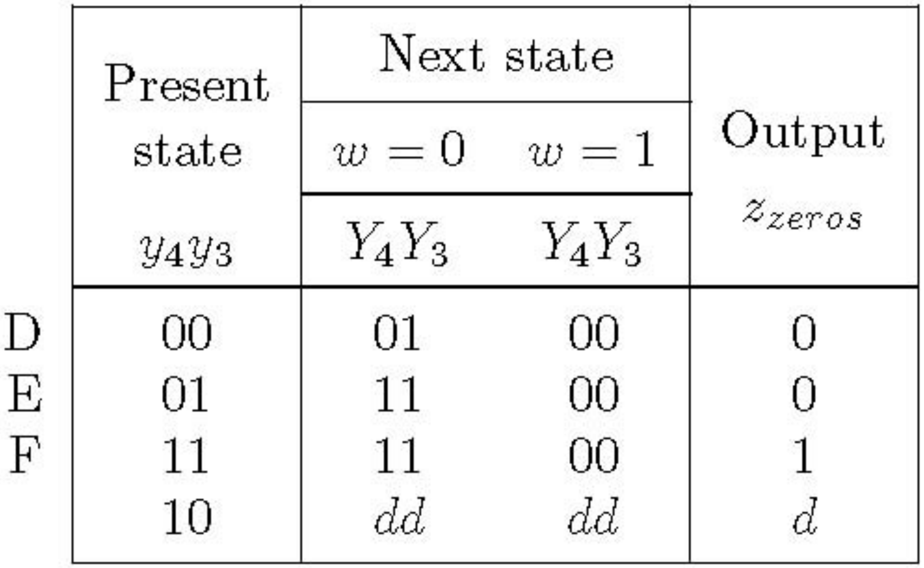
Present state	Next state		Output z
	$w = 0$	$w = 1$	
D	E	D	0
E	F	D	0
F	F	D	1

FSM that detects a sequence of two zeros



Present state	Next state		Output
	$w = 0$	$w = 1$	z_{zeros}
D	E	D	0
E	F	D	0
F	F	D	1

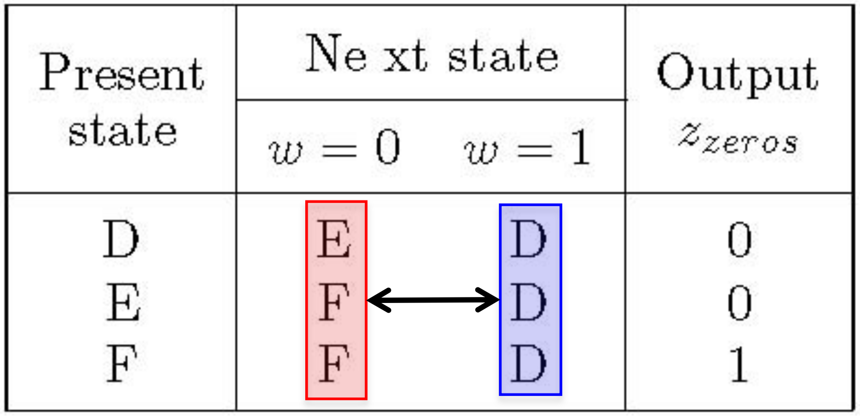
(a) State table



	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_4 y_3$	$Y_4 Y_3$	$Y_4 Y_3$	z_{zeros}
D	00	01	00	0
E	01	11	00	0
F	11	11	00	1
	10	*dd*	*dd*	*d*

[Figure 6.90 from the textbook]

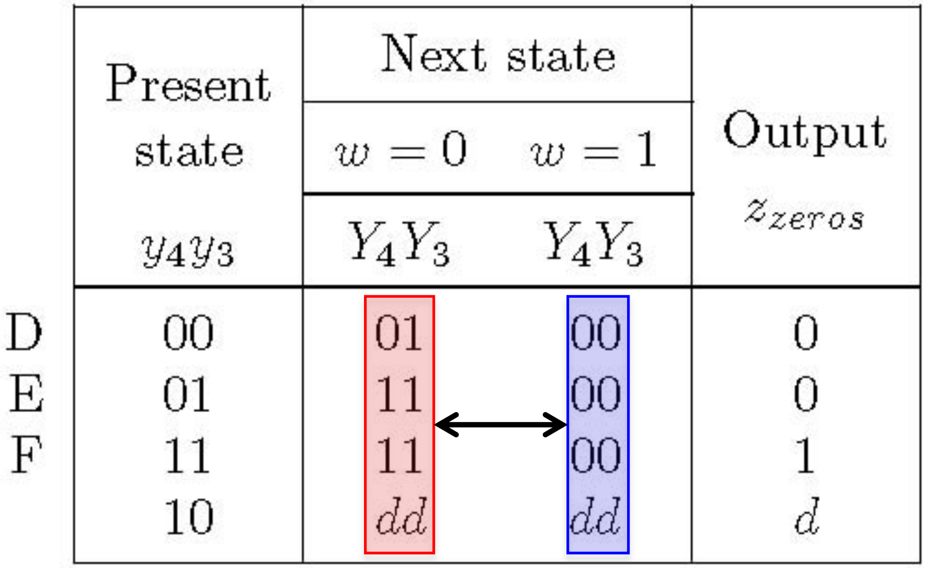
FSM that detects a sequence of two zeros



Present state	Next state		Output
	$w=0$	$w=1$	z_{zeros}
D	E	D	0
E	F	D	0
F	F	D	1

Only these two columns are swapped relative to the first FSM. And the states have different names now.

(a) State table



	Present state	Next state		Output
		$w=0$	$w=1$	
	y_4y_3	Y_4Y_3	Y_4Y_3	z_{zeros}
D	00	01	00	0
E	01	11	00	0
F	11	11	00	1
	10	dd	dd	d

Only these two columns are swapped relative to the first FSM.

[Figure 6.90 from the textbook]

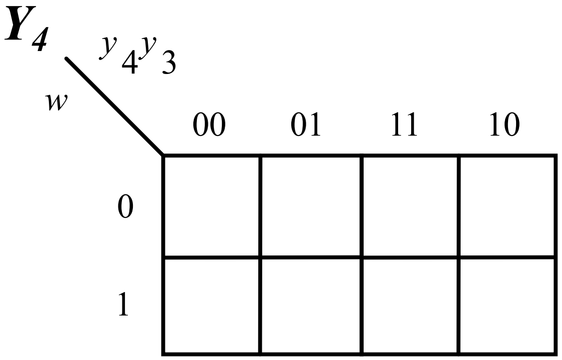
Let's Derive the Logic Expressions

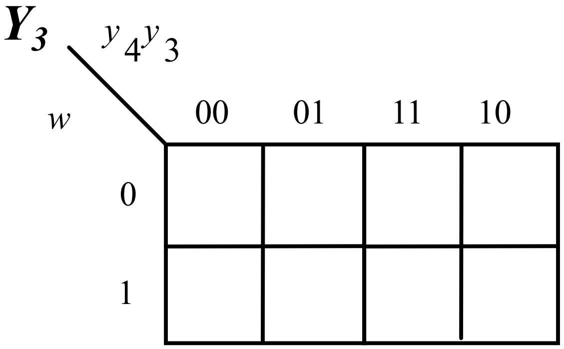
	Present state	Next state		Output
		$w = 0$	$w = 1$	
	y_4y_3	Y_4Y_3	Y_4Y_3	z
D	00	01	00	0
E	01	11	00	0
F	11	11	00	1
	10	*dd*	*dd*	*d*

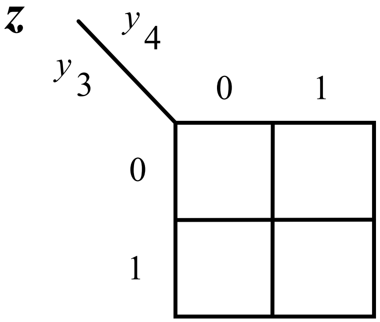
[Figure 6.90 from the textbook]

Let's Derive the Logic Expressions

	Present state	Next state		Output
		$w = 0$	$w = 1$	
	$y_4 y_3$	$Y_4 Y_3$	$Y_4 Y_3$	z
D	00	01	00	0
E	01	11	00	0
F	11	11	00	1
	10	*dd*	*dd*	*d*

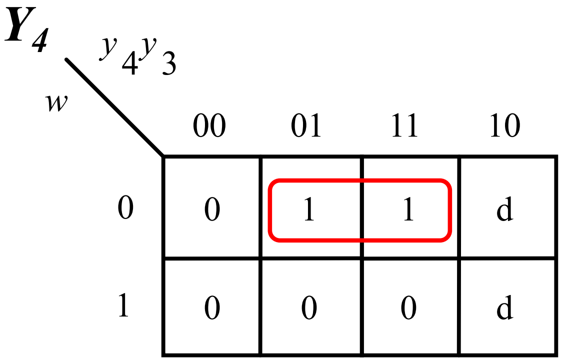




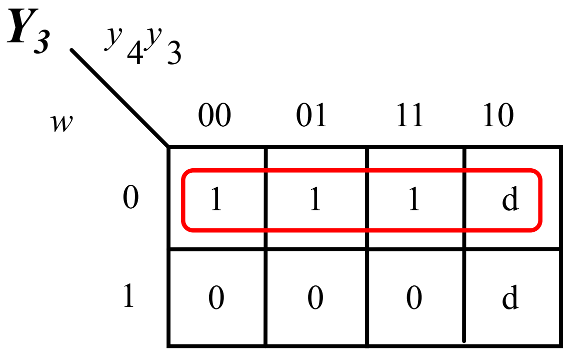


Let's Derive the Logic Expressions

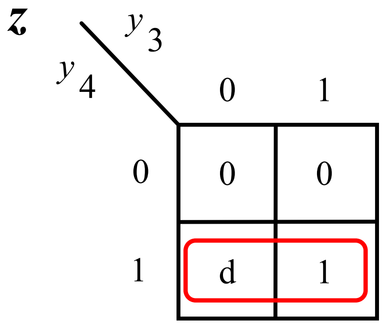
	Present state	Next state		Output
		$w = 0$	$w = 1$	
	y_4y_3	Y_4Y_3	Y_4Y_3	z
D	00	01	00	0
E	01	11	00	0
F	11	11	00	1
	10	*dd*	*dd*	*d*



$$Y_4(w, y_4, y_3) = \overline{w}\, y_3$$



$$Y_3(w, y_4, y_3) = \overline{w}$$



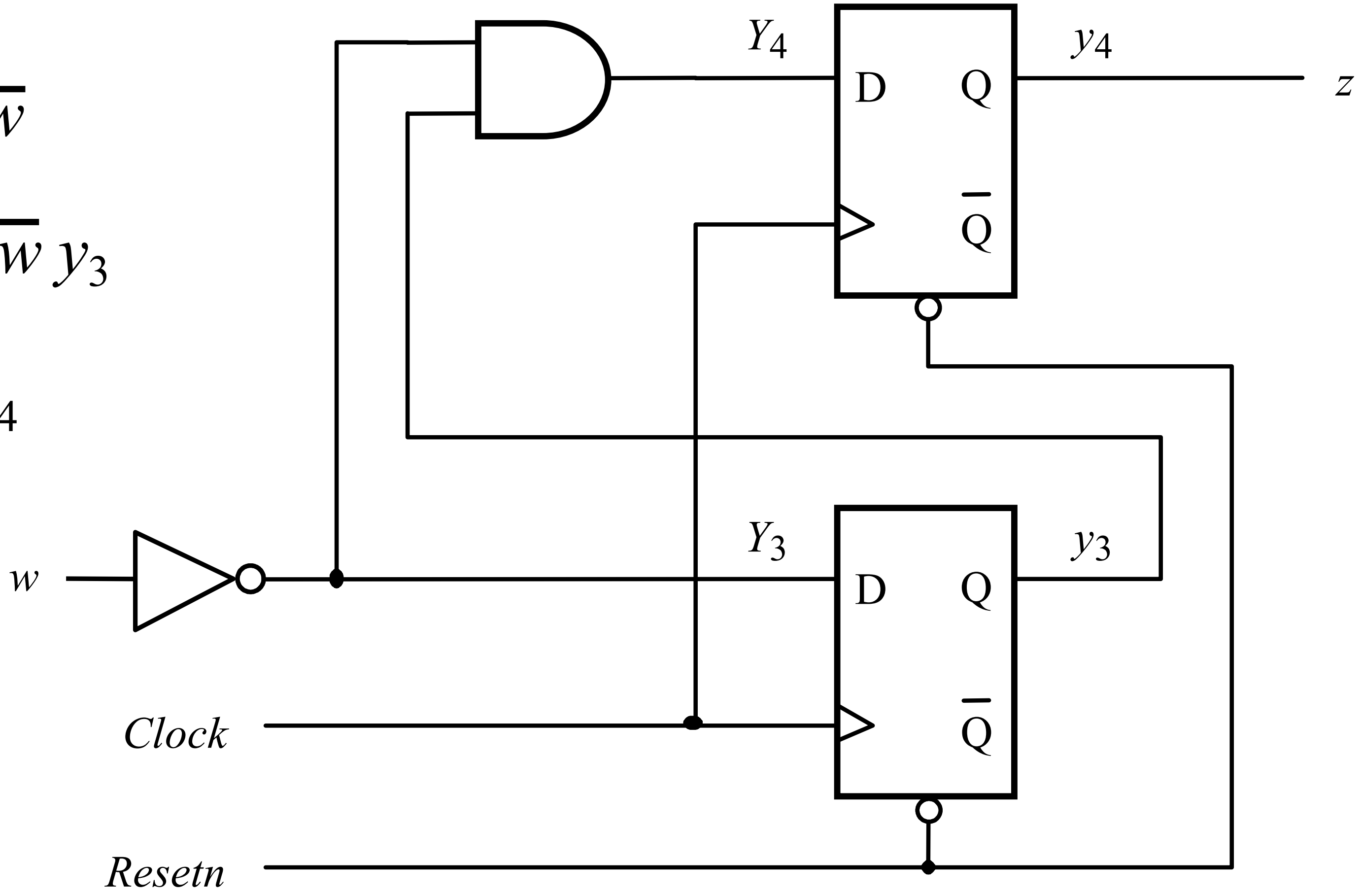
$$z(y_4, y_3) = y_4$$

The Circuit Diagram

$$Y_3(w, y_4, y_3) = \overline{w}$$

$$Y_4(w, y_4, y_3) = \overline{w}\, y_3$$

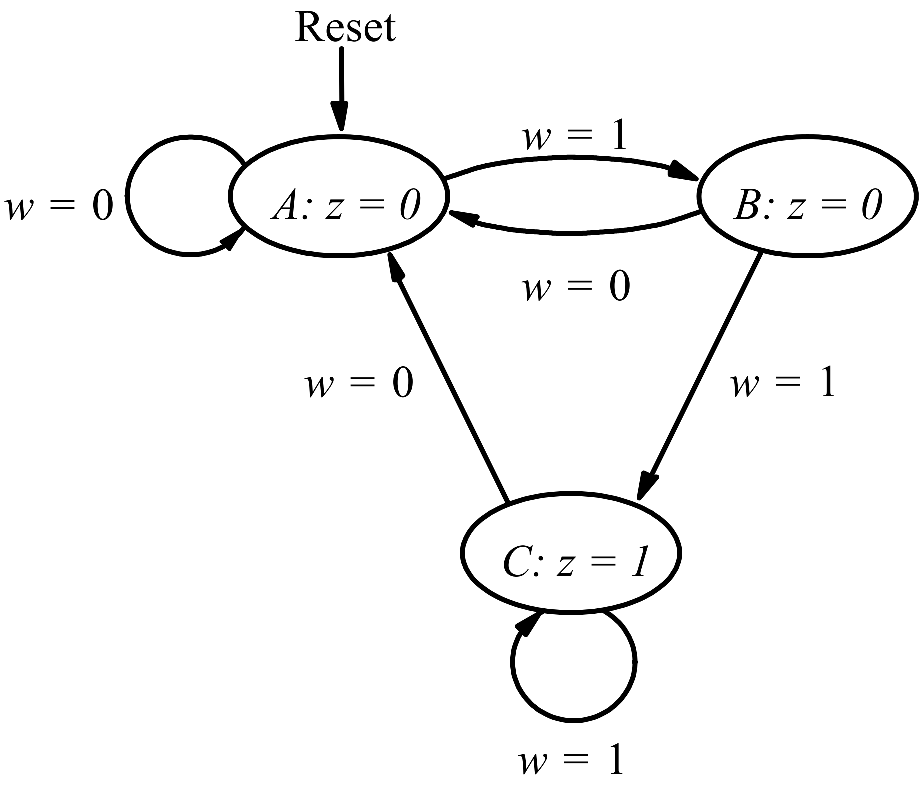
$$z(y_2, y_1) = y_4$$



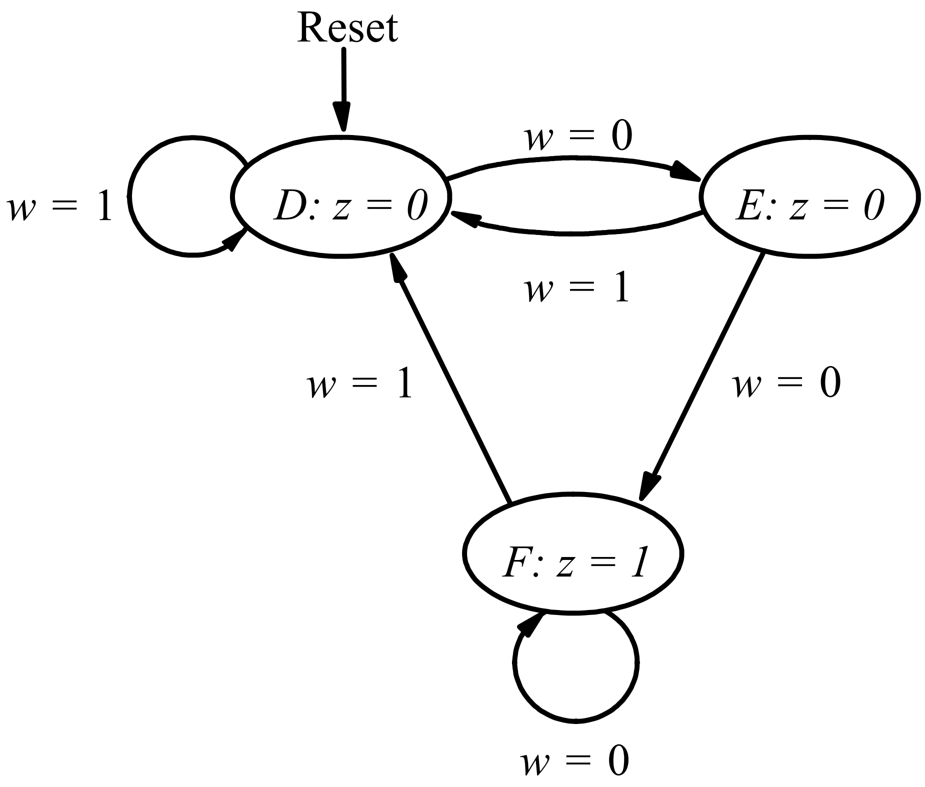
Example 6.13

(Combine the two FSMs)

The Two FSMs

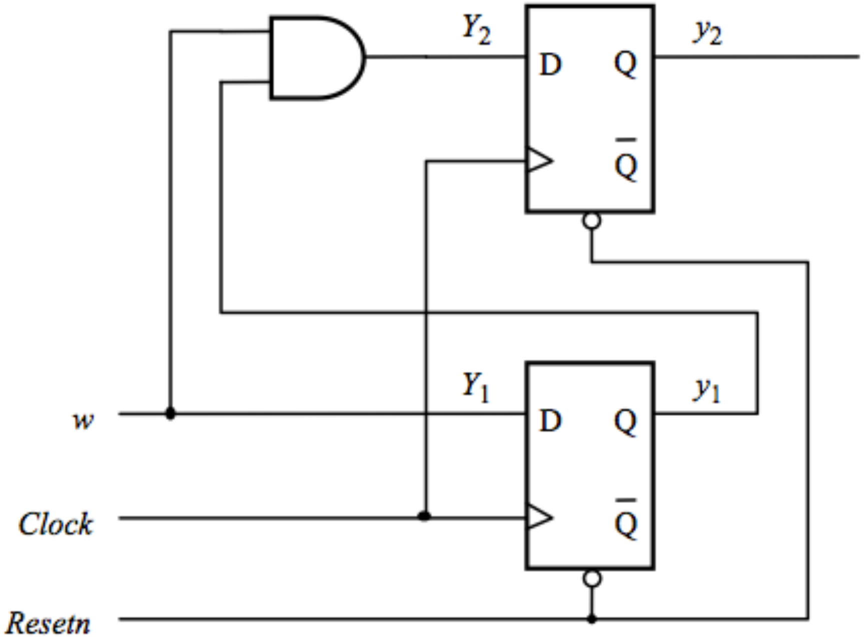


Detect two consecutive 1's

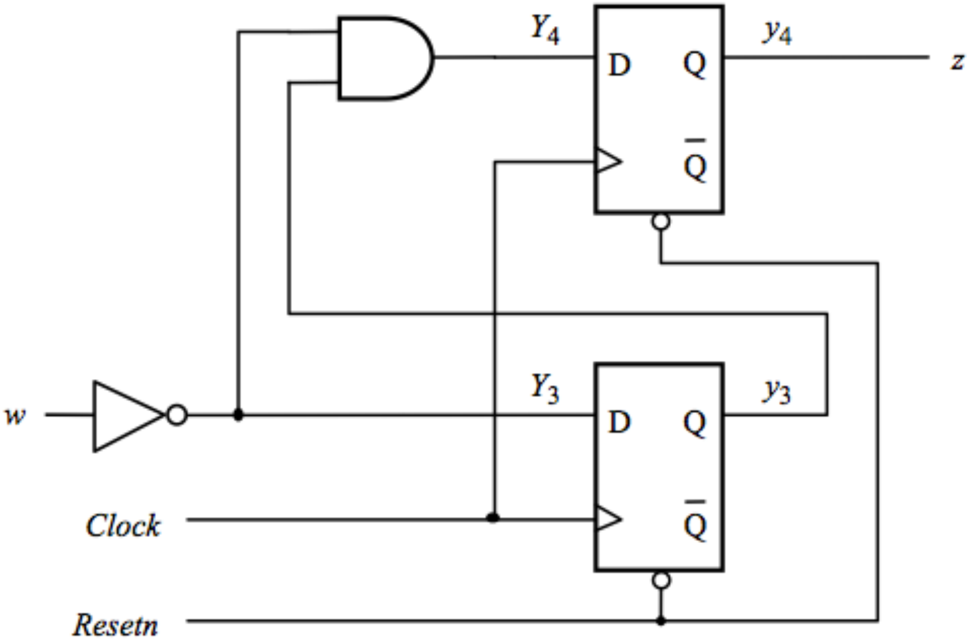


Detect two consecutive 0's

The Two Circuit Diagrams

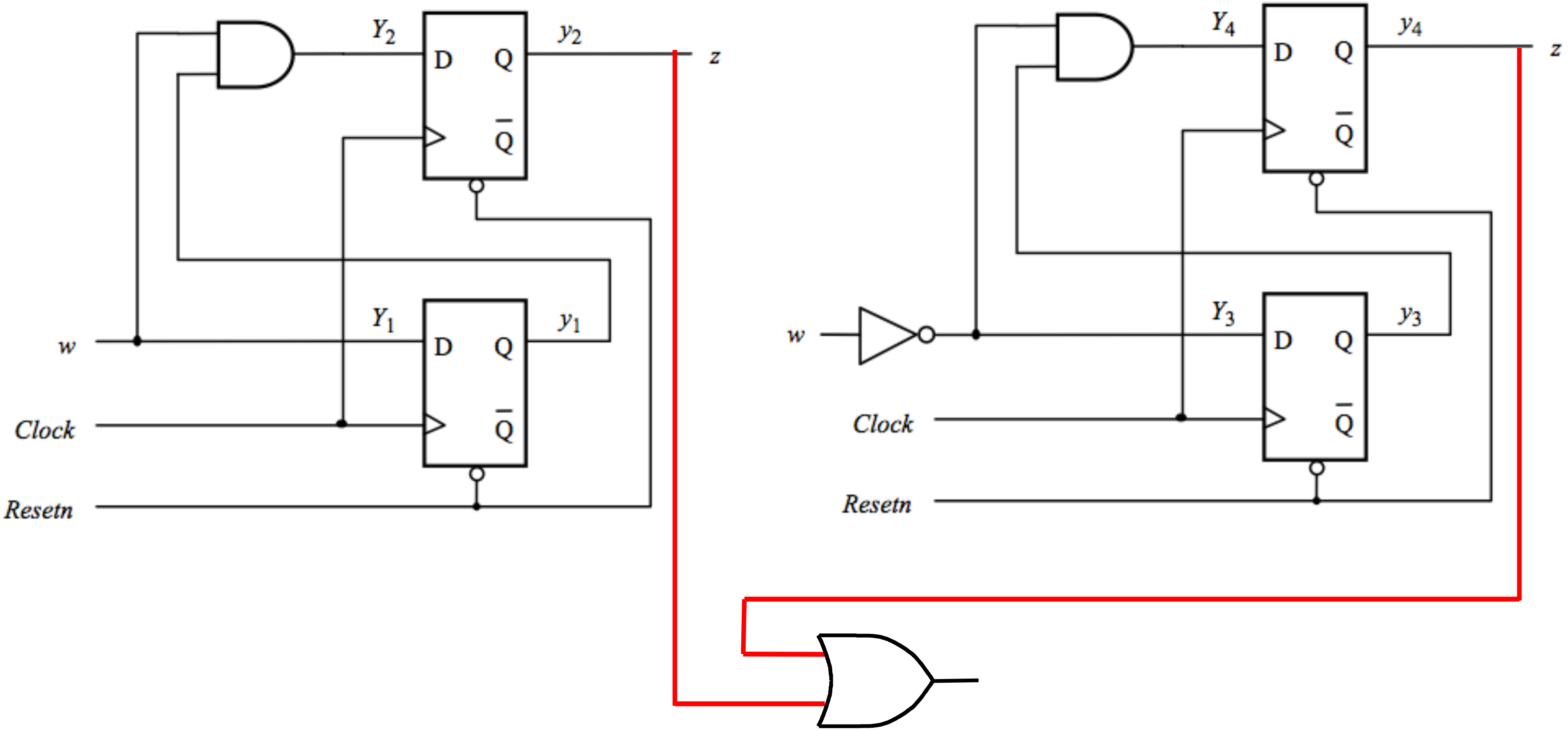


Detect two consecutive 1's



Detect two consecutive 0's

The Combined Circuit Diagram



Detect two consecutive 1's or two consecutive 0's

Example 6.14

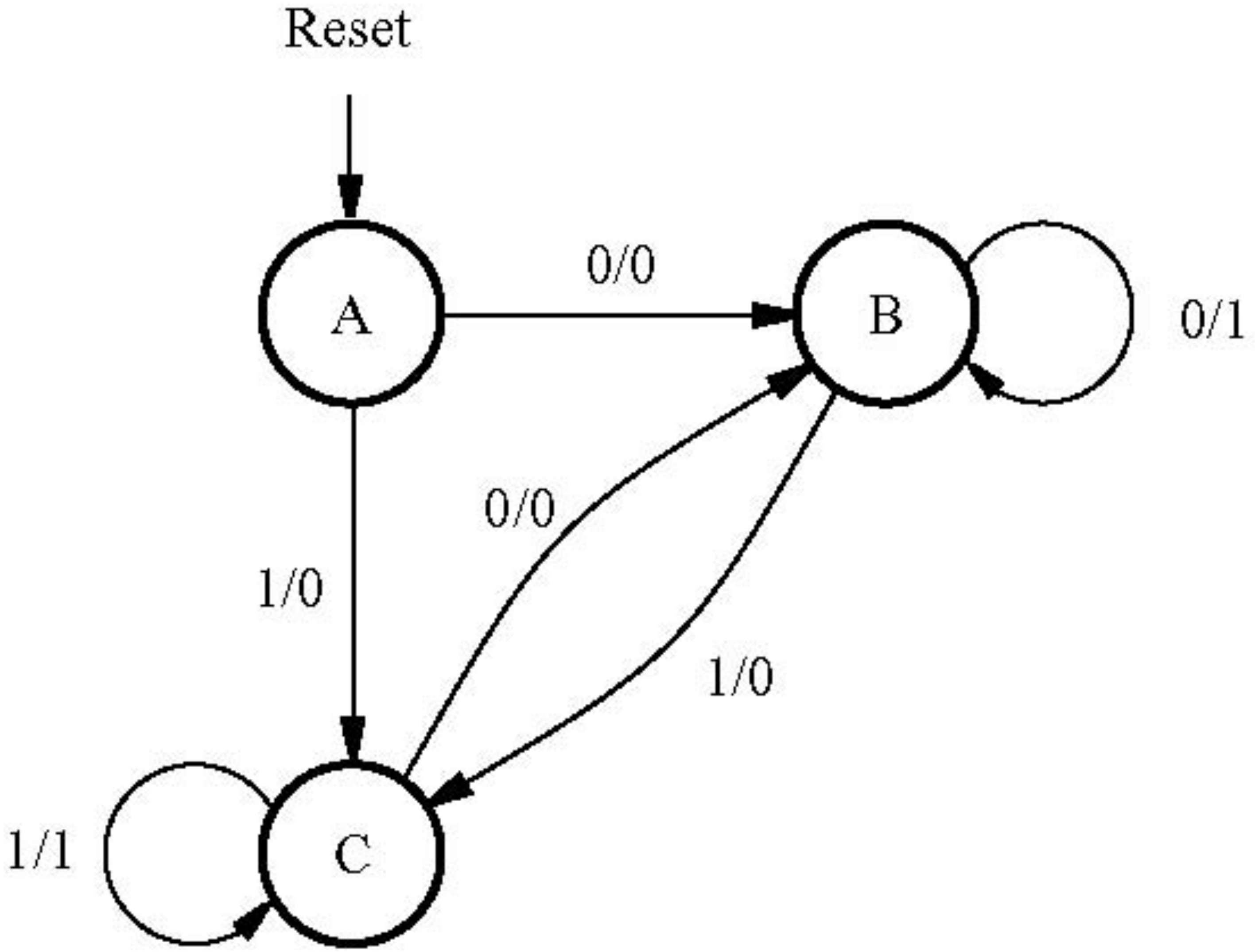
Goal

- **Design an FSM that detects if the previous two values of the input w were equal to 00 or 11.**

- **If either condition is true then the output z should be set to 1; otherwise to 0.**

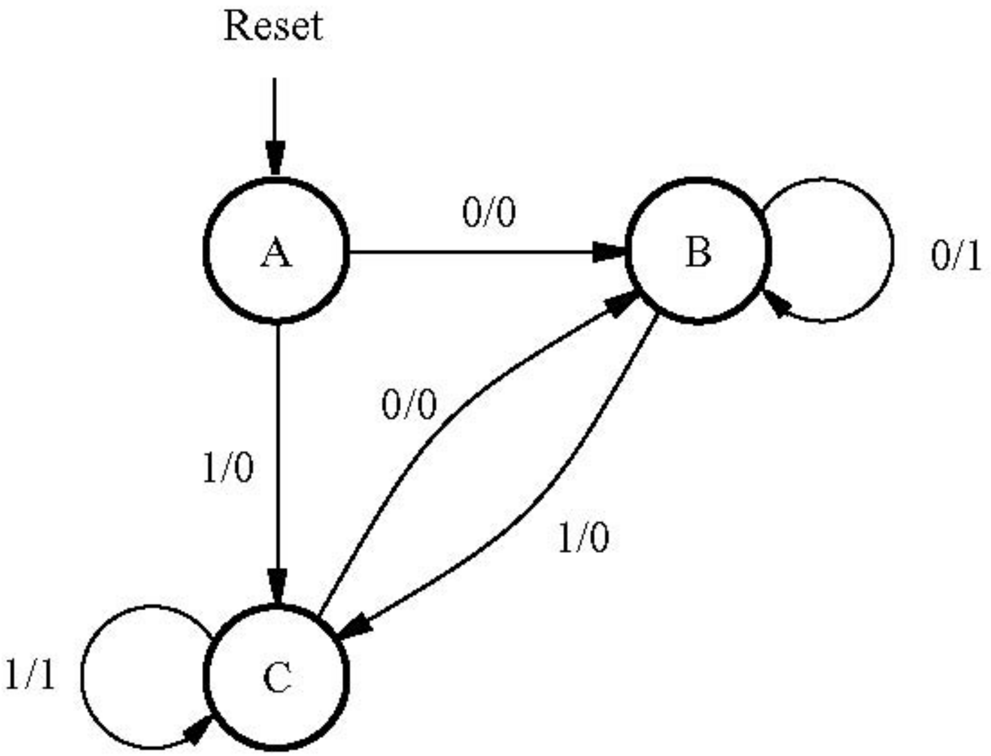
- **Implement this as a Mealy-type machine**

State Diagram



[Figure 6.91 from the textbook]

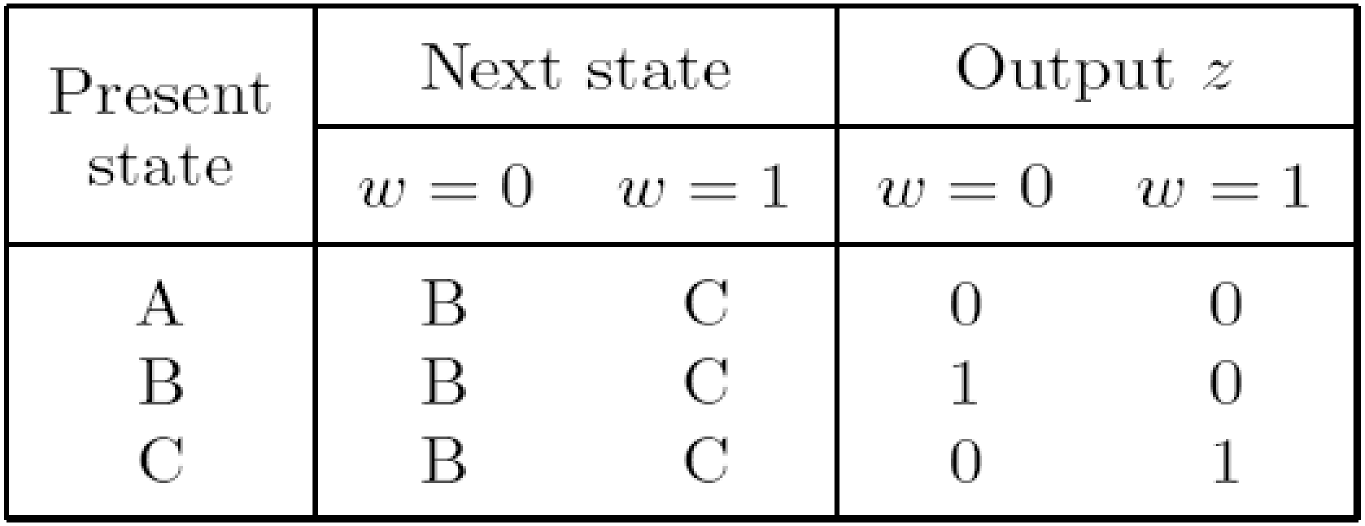
Building the State Table



Present state	Next state		Output z	
	$w=0$	$w=1$	$w=0$	$w=1$
A	B	C	0	0
B	B	C	1	0
C	B	C	0	1

[Figure 6.92 from the textbook]

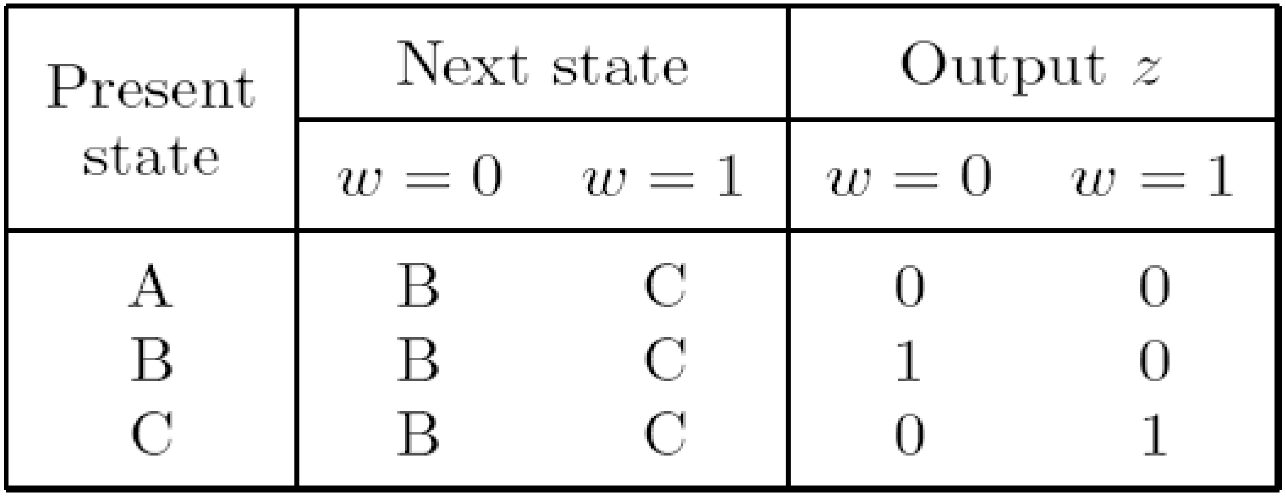
State Table



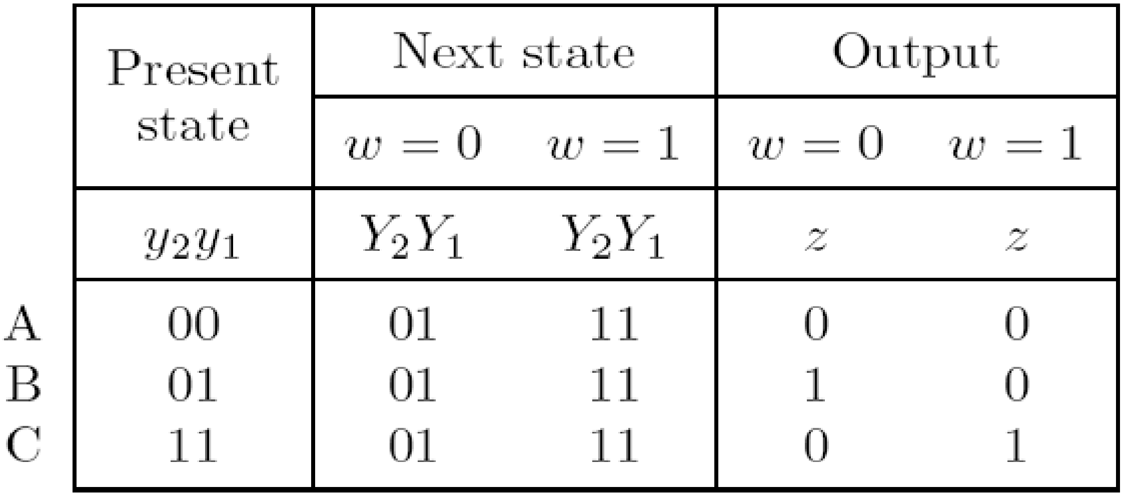
Present state	Next state		Output z	
	$w = 0$	$w = 1$	$w = 0$	$w = 1$
A	B	C	0	0
B	B	C	1	0
C	B	C	0	1

[Figure 6.92 from the textbook]

Building the State-Assigned Table



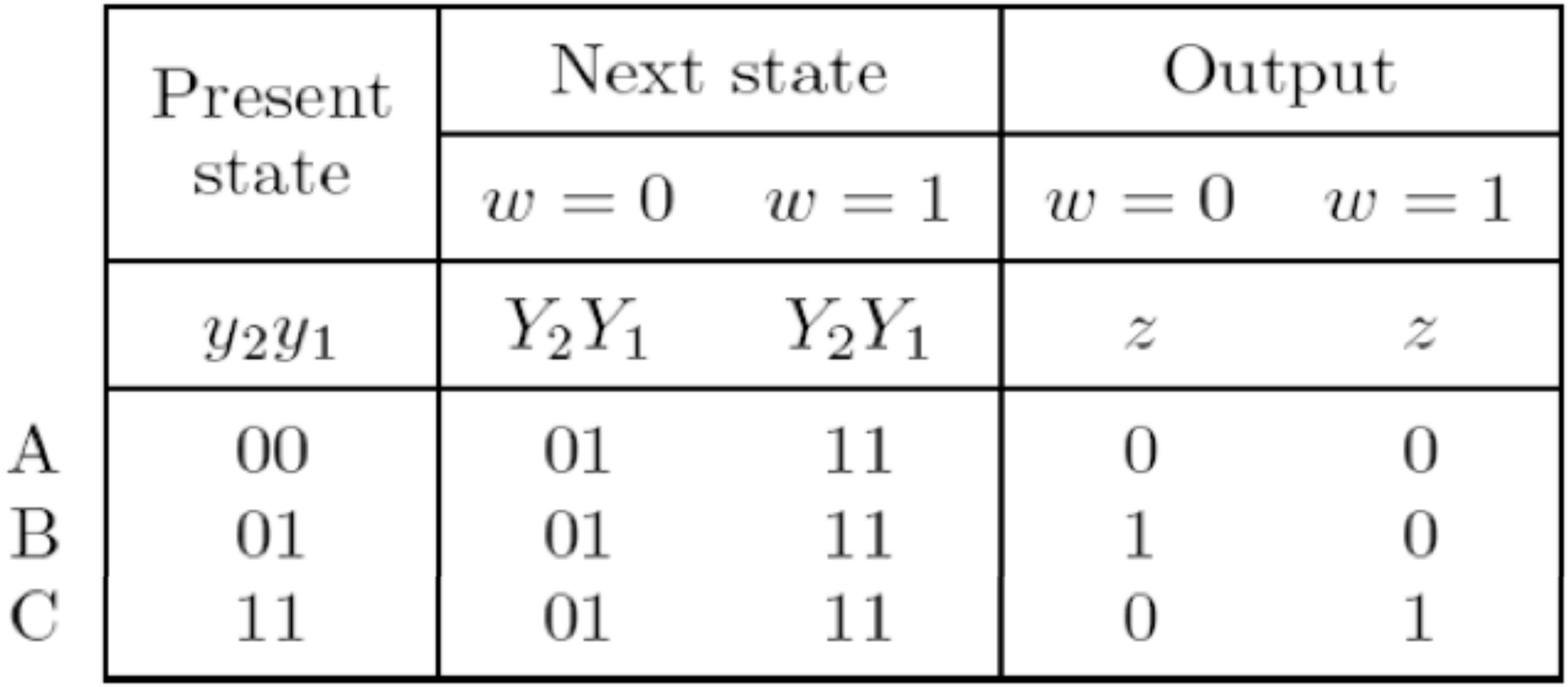
Present state	Next state		Output z	
	$w = 0$	$w = 1$	$w = 0$	$w = 1$
A	B	C	0	0
B	B	C	1	0
C	B	C	0	1



	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	$y_2 y_1$	$Y_2 Y_1$	$Y_2 Y_1$	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

[Figure 6.93 from the textbook]

State-Assigned Table



	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

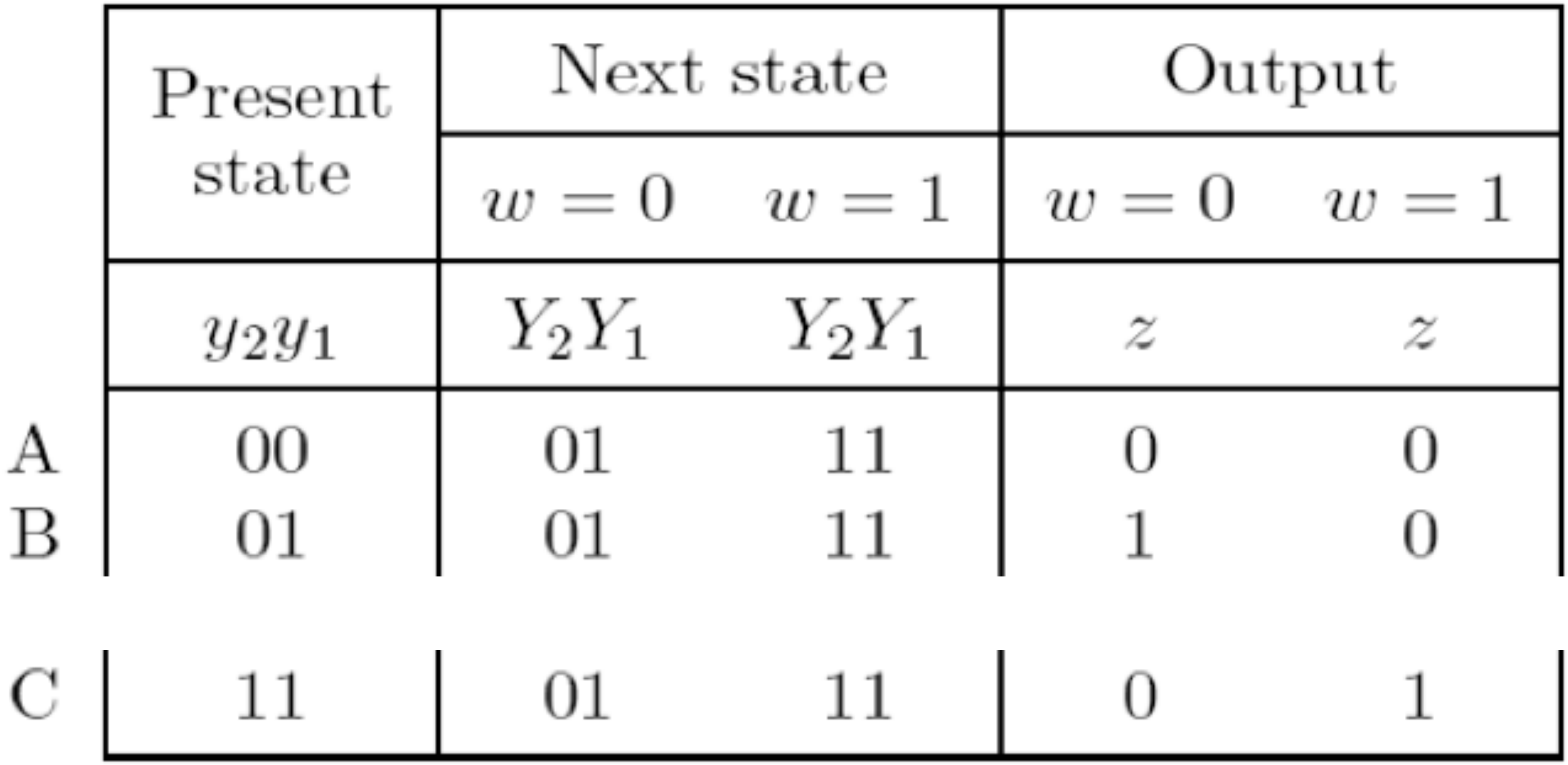
[Figure 6.93 from the textbook]

State-Assigned Table

	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

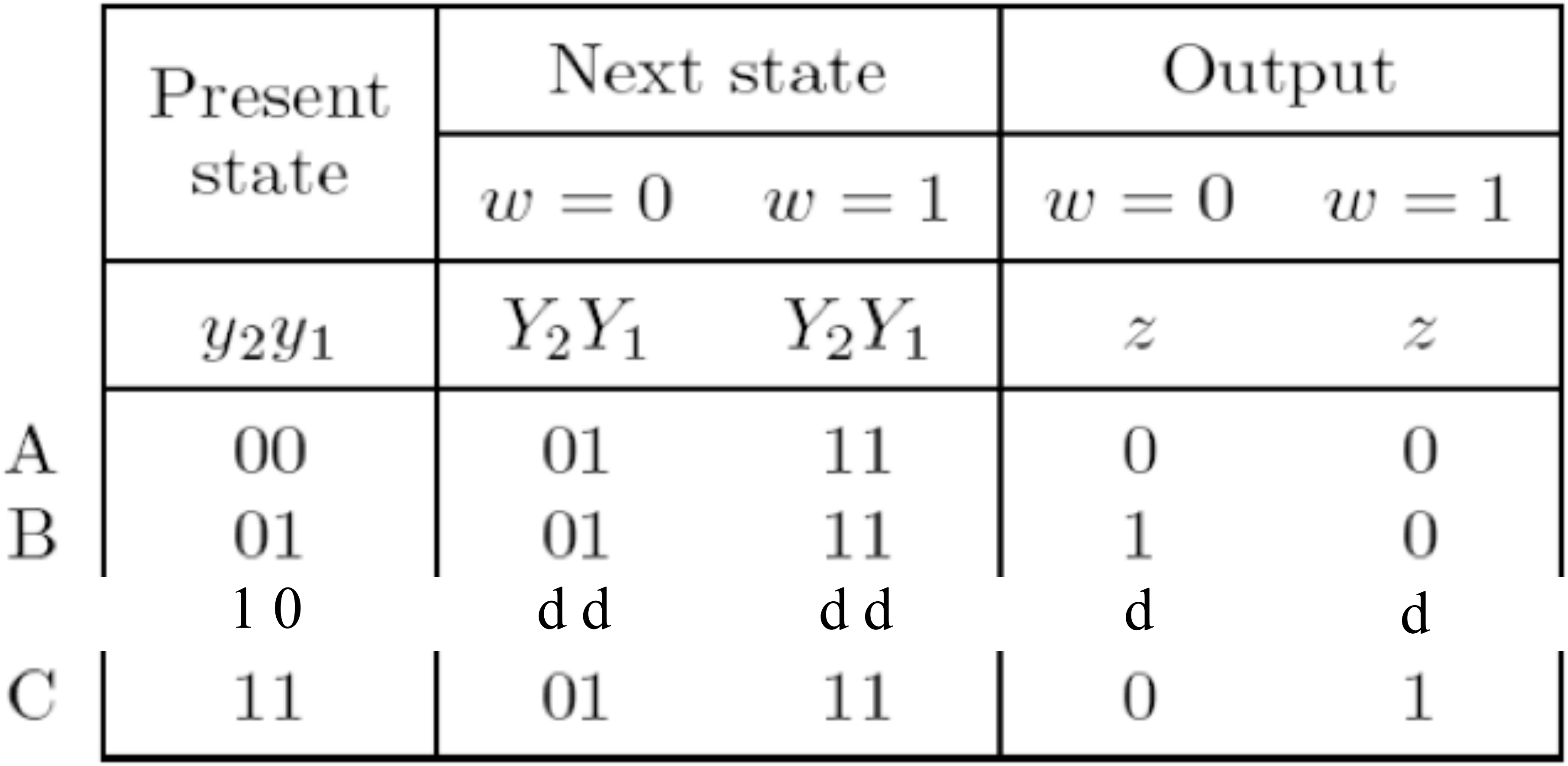
cut here

State-Assigned Table



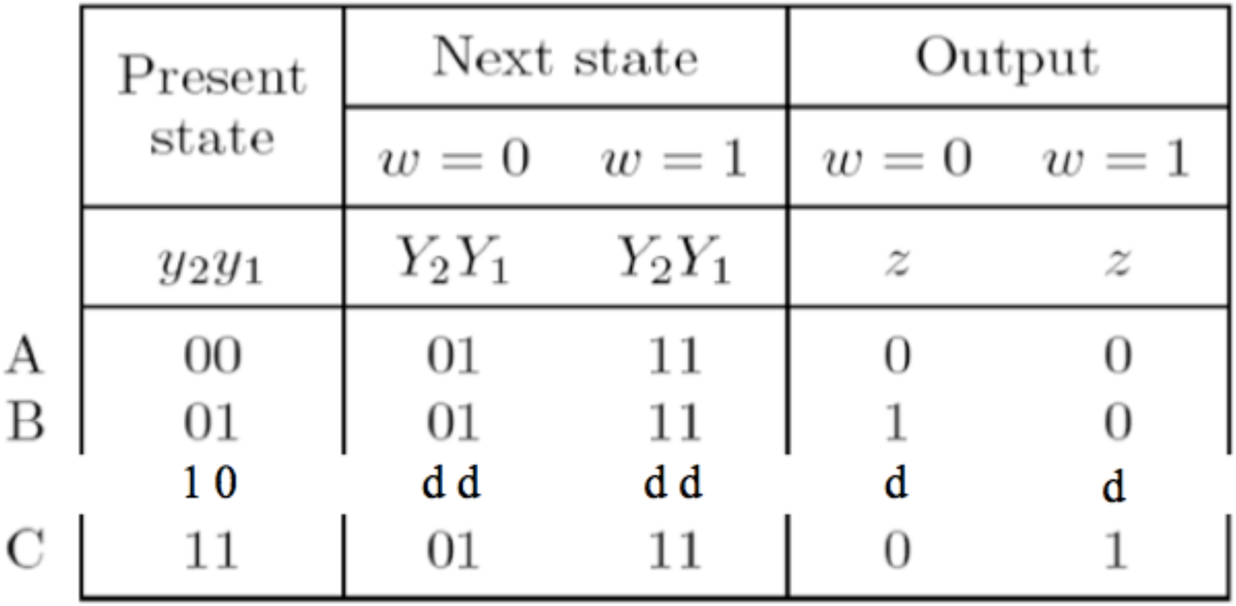
	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

State-Assigned Table



	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
	1 0	d d	d d	d	d
C	11	01	11	0	1

Truth Table for Y_2, Y_1, and z



	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
	1 0	d d	d d	d	d
C	11	01	11	0	1

w	y_2	y_1	Y_2	Y_1	z
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	d	d	d
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	0
1	1	0	d	1	d
1	1	1	1	1	1

K-Maps for Y_2, Y_1, and z

Y_2

$w\ y_2$

y_1

	00	01	11	10
0	0	d	d	1
1	0	0	1	1

Y_1

$w\ y_2$

y_1

	00	01	11	10
0	1	d	1	1
1	1	1	1	1

z

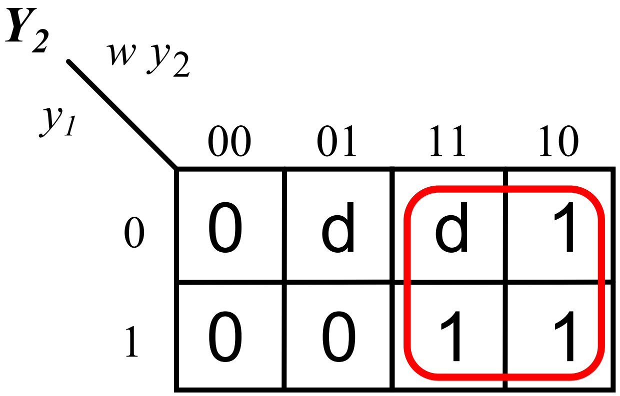
$w\ y_2$

y_1

	00	01	11	10
0	0	d	d	0
1	1	0	1	0

w	y_2	y_1	Y_2	Y_1	z
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	d	d	d
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	0
1	1	0	d	1	d
1	1	1	1	1	1

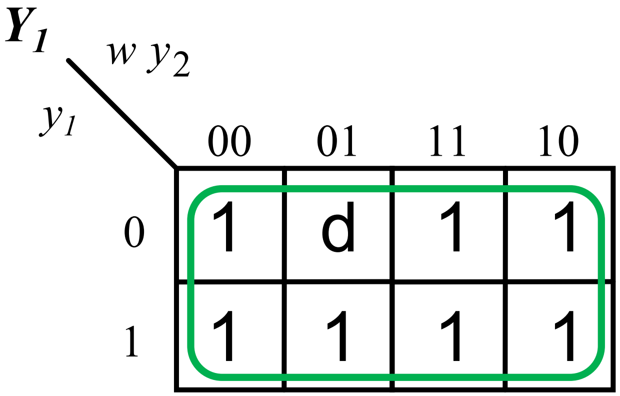
K-Maps for Y_2, Y_1, and z



Y_2

y_1 \ $w\,y_2$	00	01	11	10
0	0	d	d	1
1	0	0	1	1

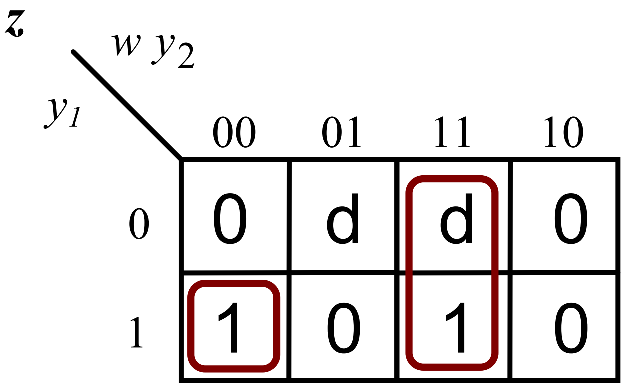
$Y_2 = w$



Y_1

y_1 \ $w\,y_2$	00	01	11	10
0	1	d	1	1
1	1	1	1	1

$Y_1 = 1$



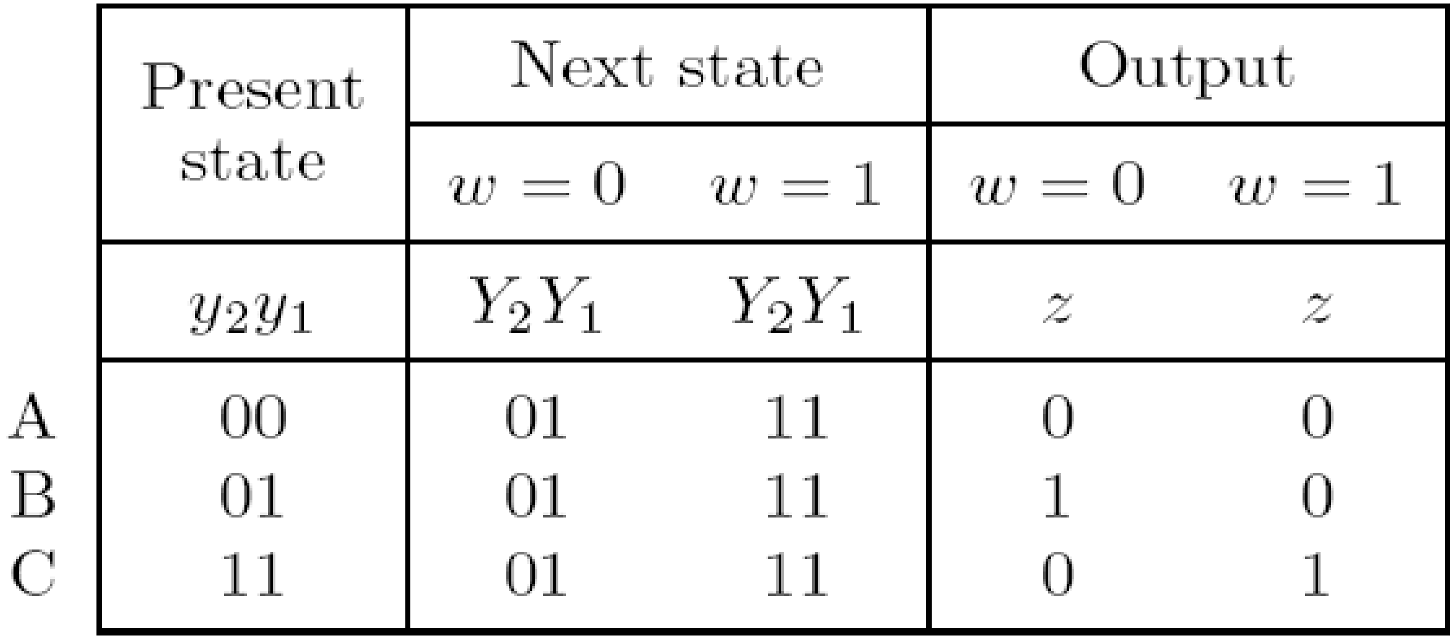
z

y_1 \ $w\,y_2$	00	01	11	10
0	0	d	d	0
1	1	0	1	0

$z = \overline{w}\, y_1\, \overline{y_2} + w\, y_2$

w	y_2	y_1	Y_2	Y_1	z
0	0	0	0	1	0
0	0	1	0	1	1
0	1	0	d	d	d
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	1	1	0
1	1	0	d	1	d
1	1	1	1	1	1

State-Assigned Table



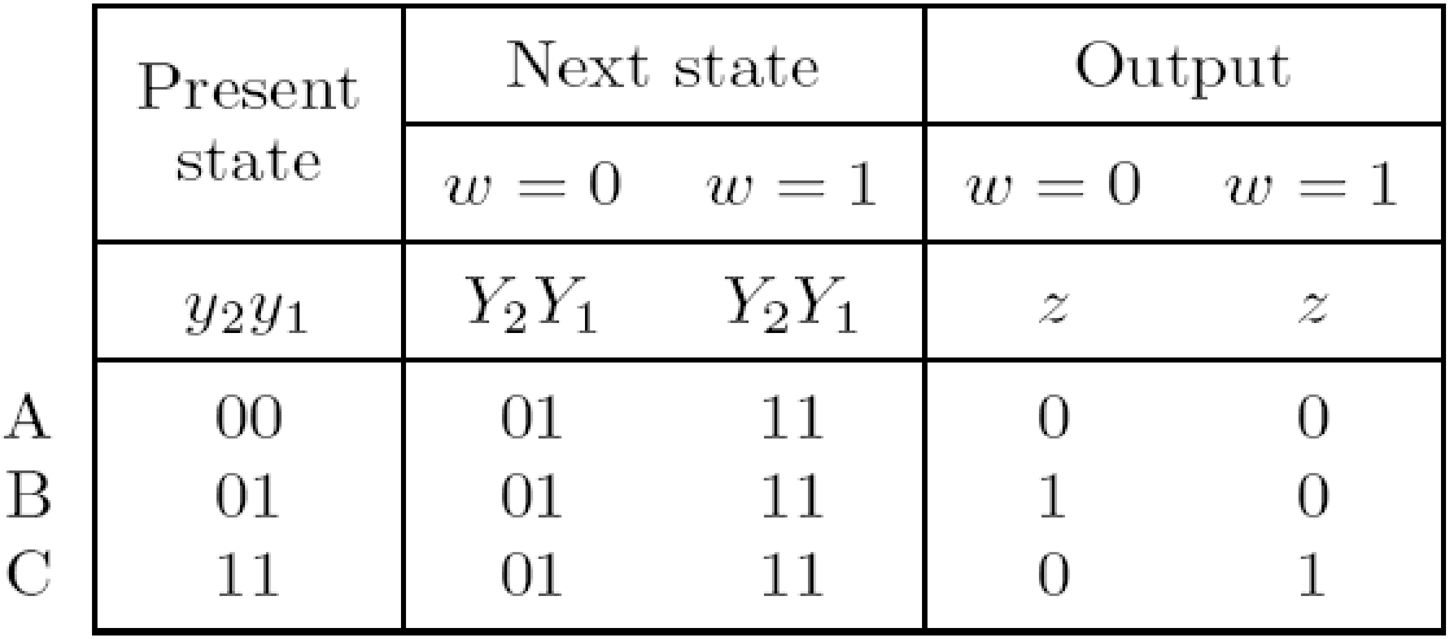
	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	$y_2 y_1$	$Y_2 Y_1$	$Y_2 Y_1$	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

$Y_1 = 1$

$Y_2 = w$

$z = \overline{w}\, y_1 \overline{y_2} + w\, y_2$

State-Assigned Table



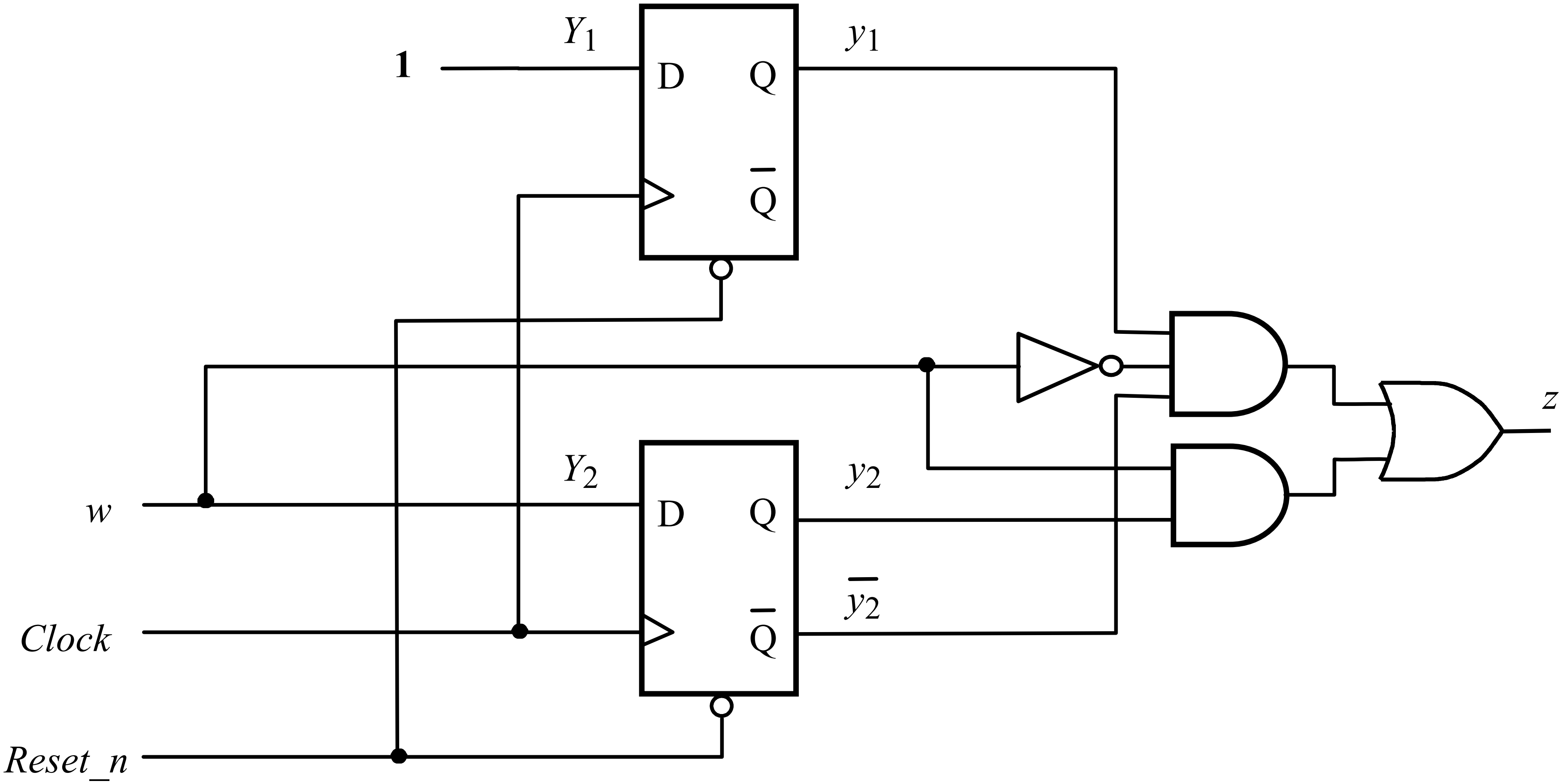
	Present state	Next state		Output	
		$w = 0$	$w = 1$	$w = 0$	$w = 1$
	y_2y_1	Y_2Y_1	Y_2Y_1	z	z
A	00	01	11	0	0
B	01	01	11	1	0
C	11	01	11	0	1

~~$Y_1 = 1$~~

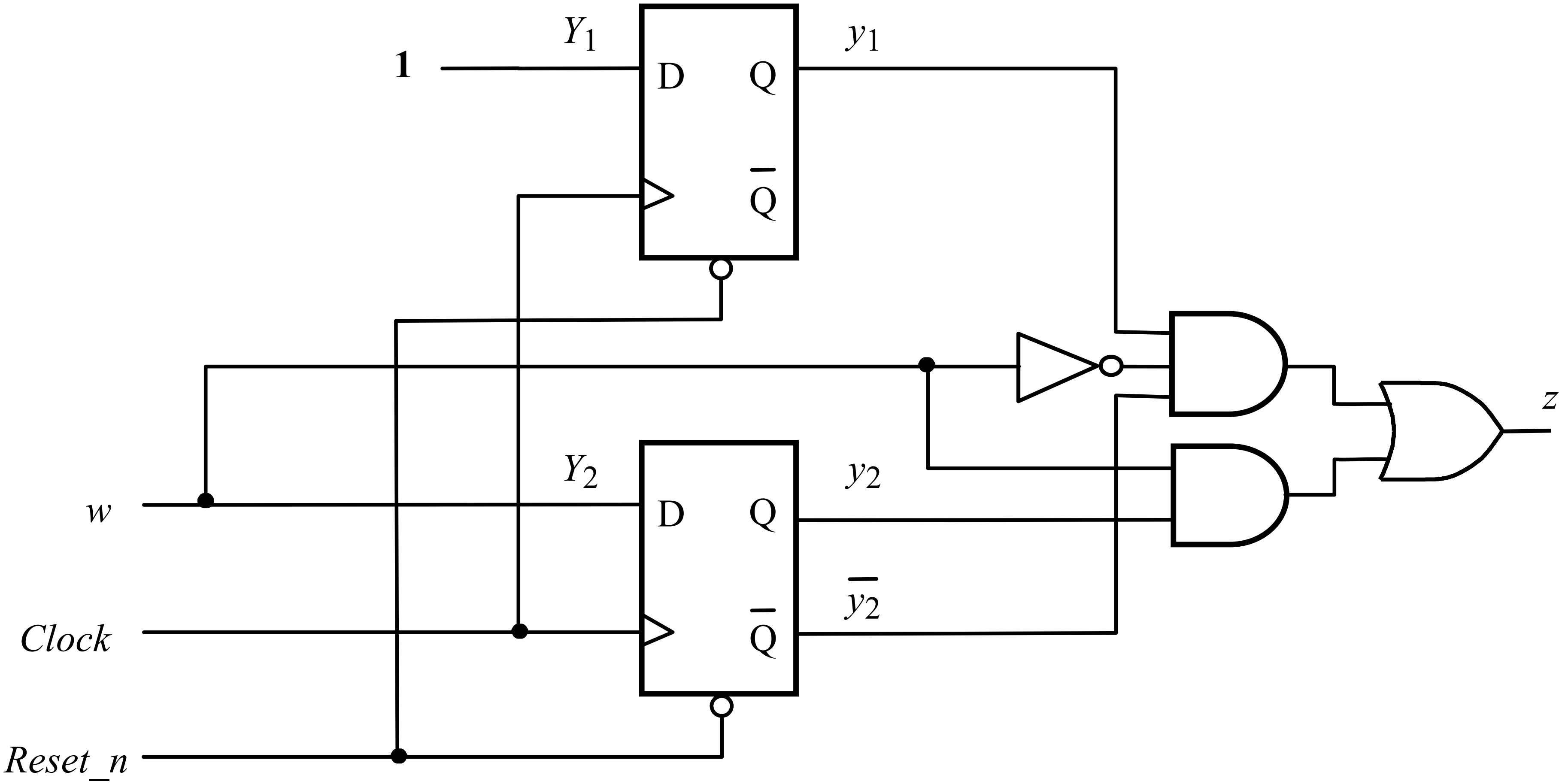
$Y_2 = w$

$z = \overline{w}\,\cancel{y_1}\,\overline{y}_2 + w\,y_2$

The Circuit Diagram



The Circuit Diagram

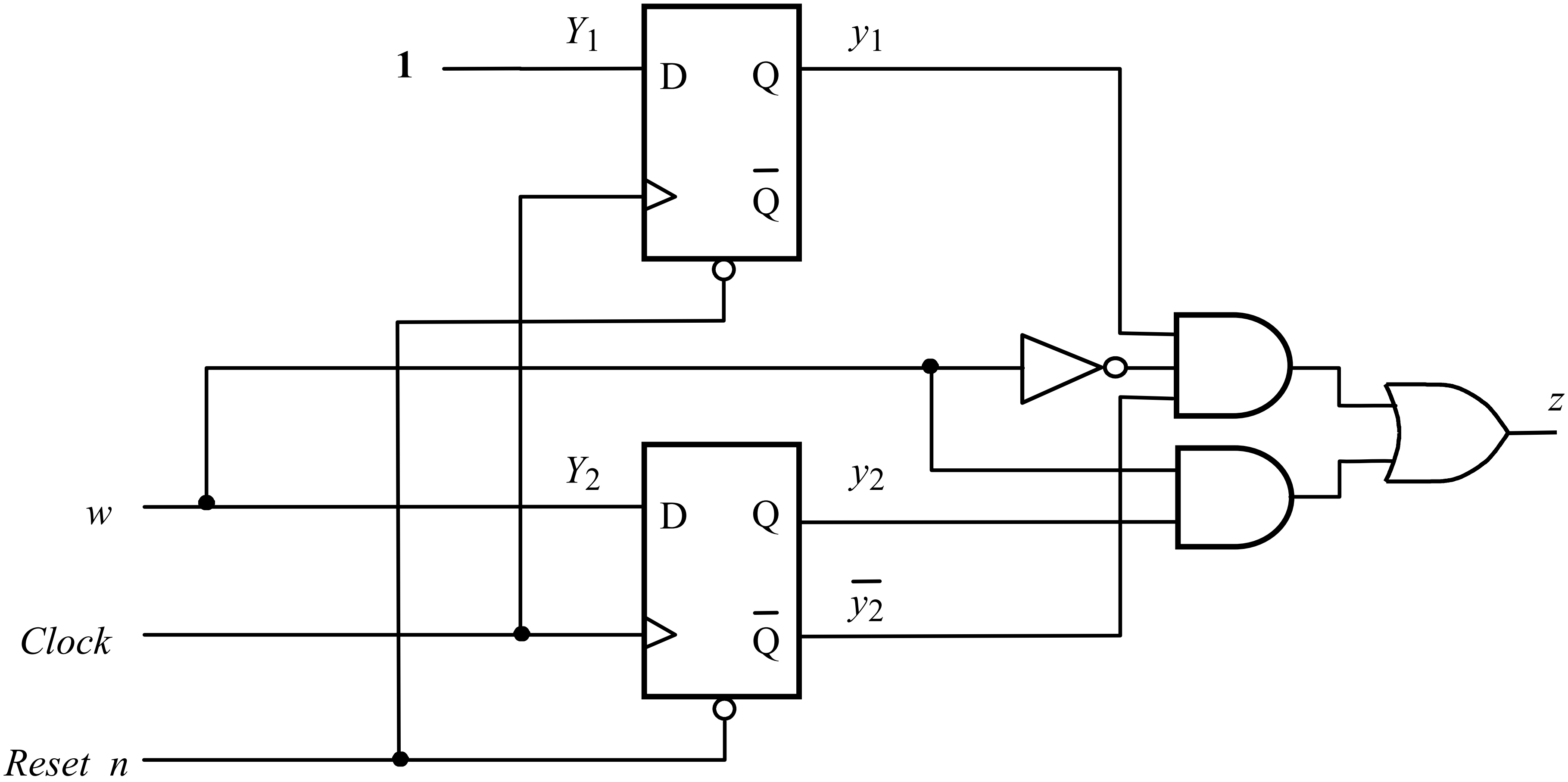


$Y_1 = 1$

$Y_2 = w$

$z = \overline{w}\, y_1\, \overline{y_2} + w\, y_2$

The Circuit Diagram

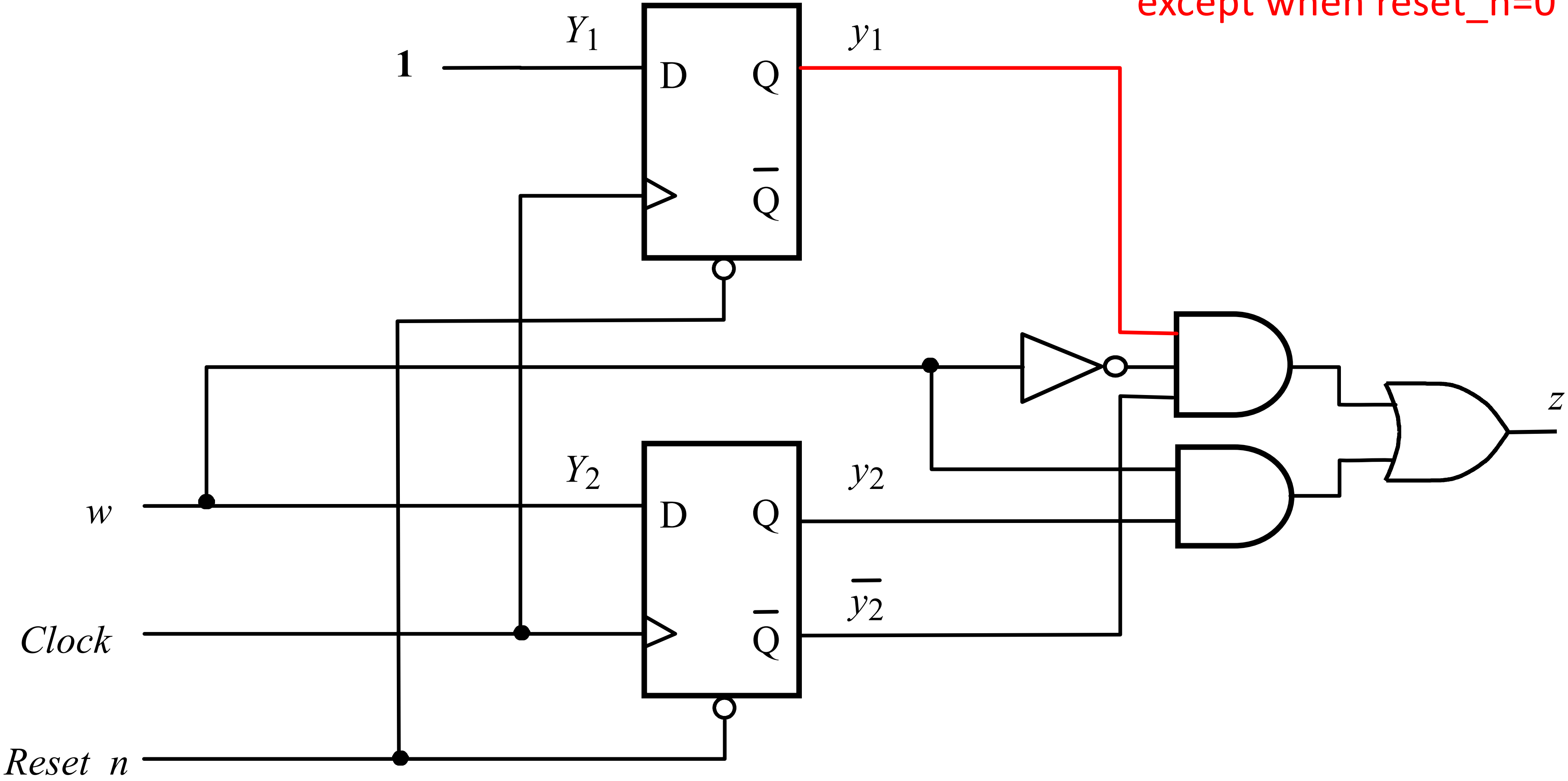


~~$Y_1 = 1$~~

$Y_2 = w$

$z = \overline{w}\,\cancel{y_1}\,\overline{y_2} + w\,y_2$

The Circuit Diagram



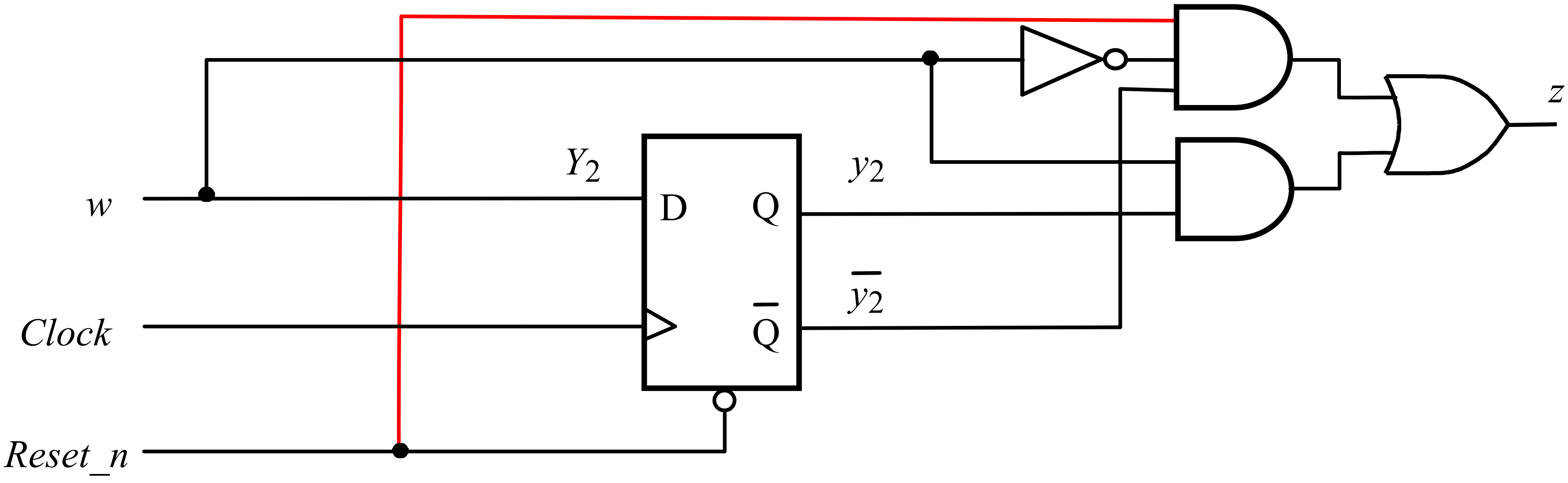
this is always 1,
except when reset_n=0

$\cancel{Y_1 = 1}$

$Y_2 = w$

$z \;= \overline{w}\,\cancel{y_1}\,\overline{y_2} + w\,y_2$

The Simplified Circuit Diagram



$Y_2 = w$

$z = \overline{w}\ \overline{y_2} + w\, y_2$

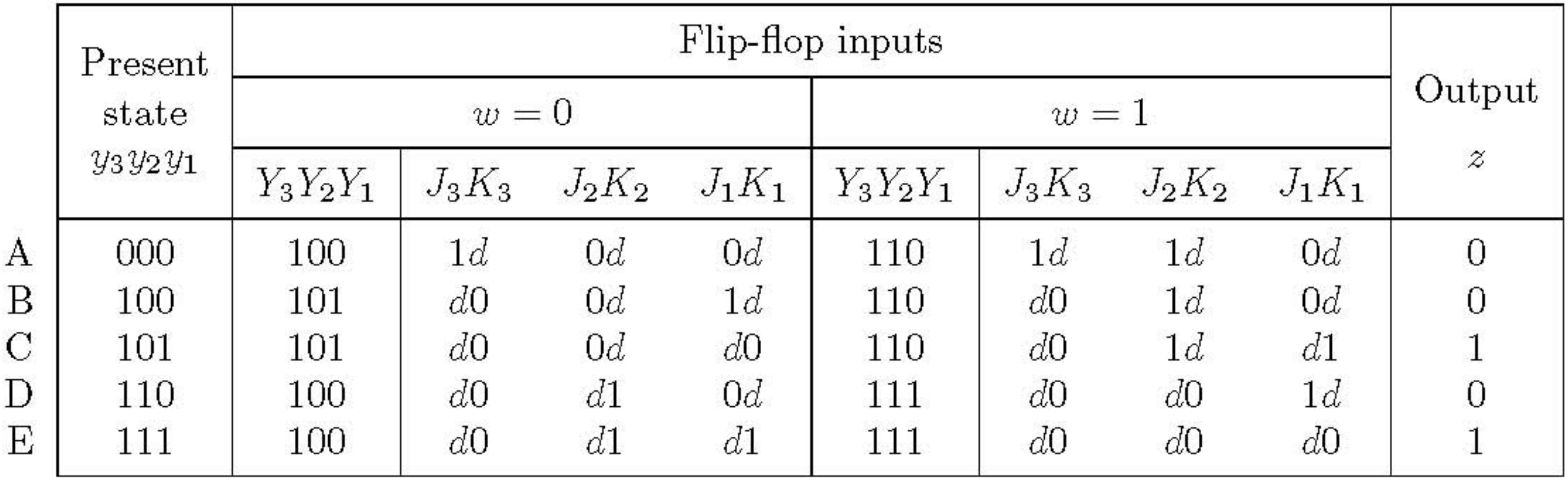
Example 6.15

Goal

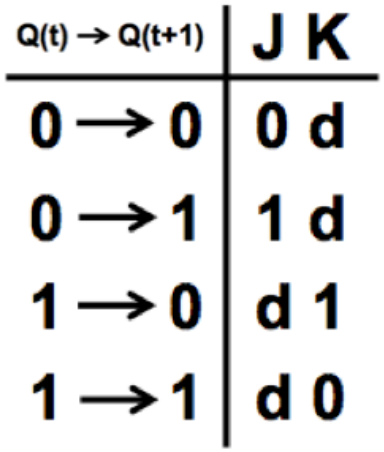
Implement this state-assigned Table using JK flip-flops

	Present state $y_3y_2y_1$	Next state		Output z
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

Excitation table with JK flip-flops



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1



Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

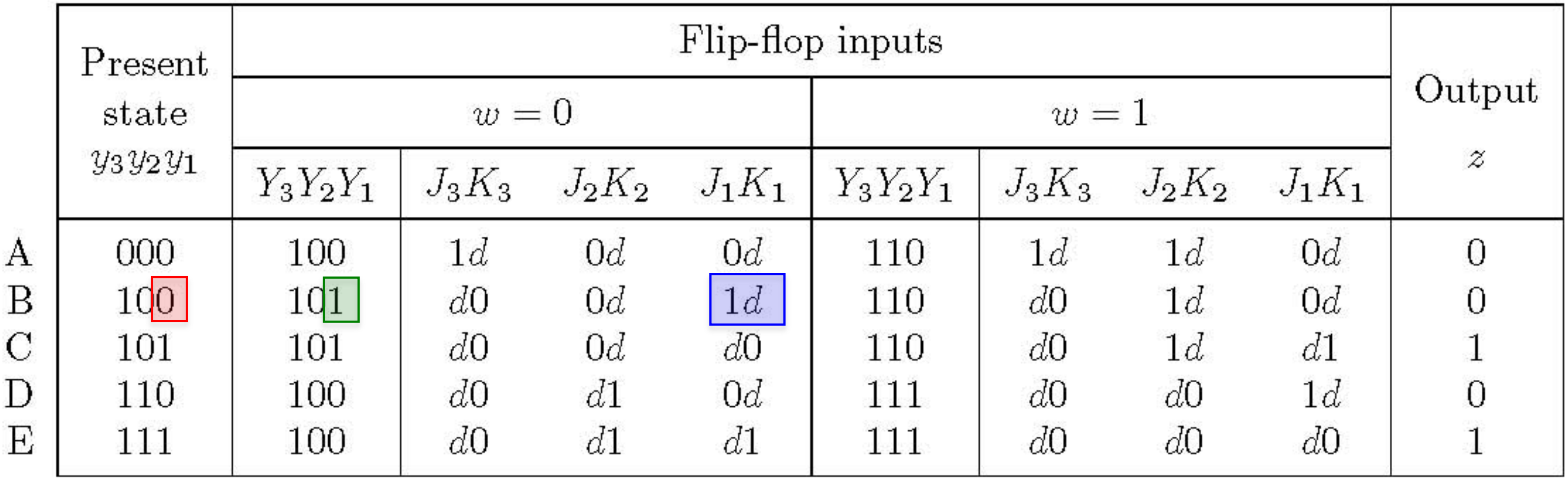
[Figure 6.94 from the textbook]

Excitation table with JK flip-flops

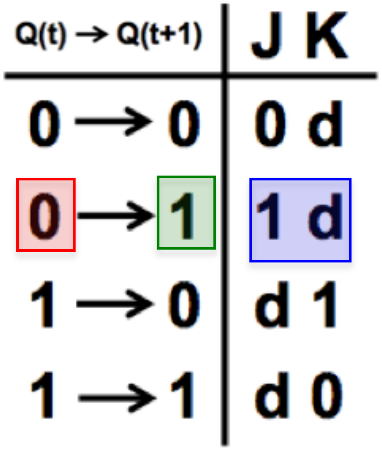
	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w = 0$				$w = 1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops



	Present state	Flip-flop inputs								Output
		$w = 0$				$w = 1$				
	$y_3y_2y_1$	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	z
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1



Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops

	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w = 0$				$w = 1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

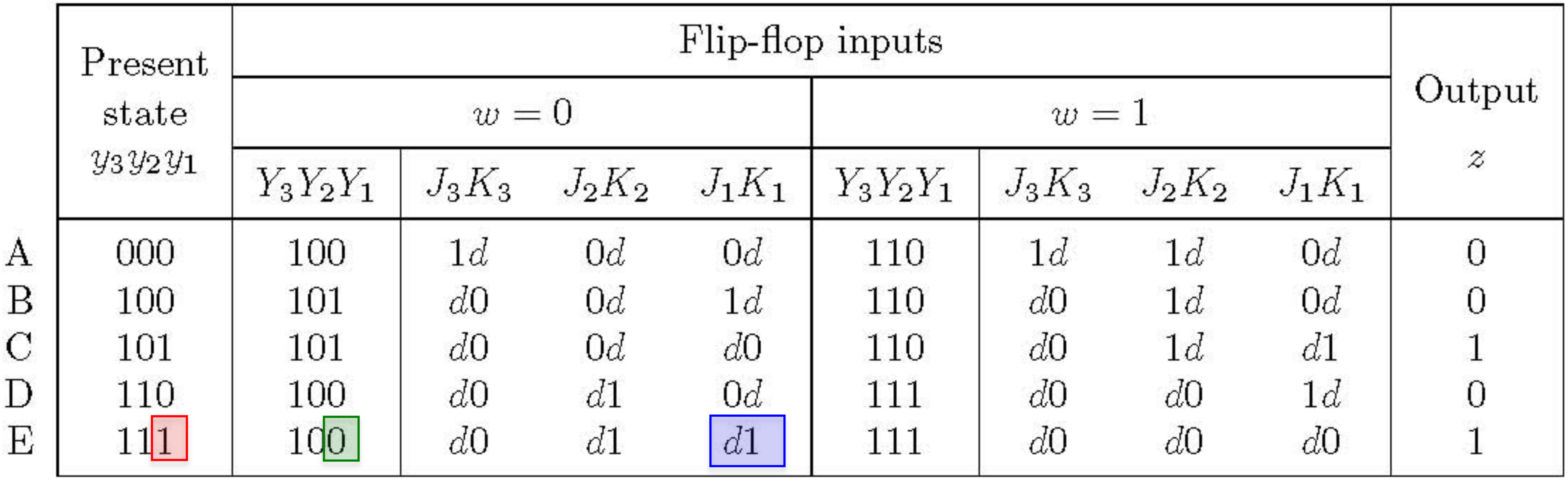
Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops

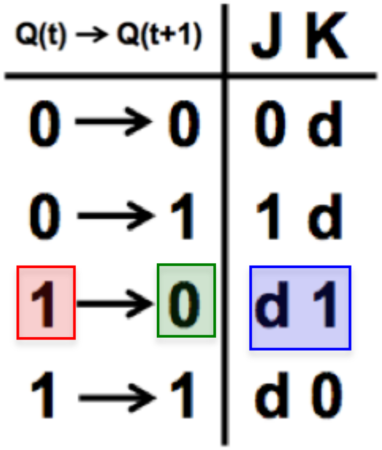
	Present state	Flip-flop inputs								Output
		$w = 0$				$w = 1$				
	$y_3y_2y_1$	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	z
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1



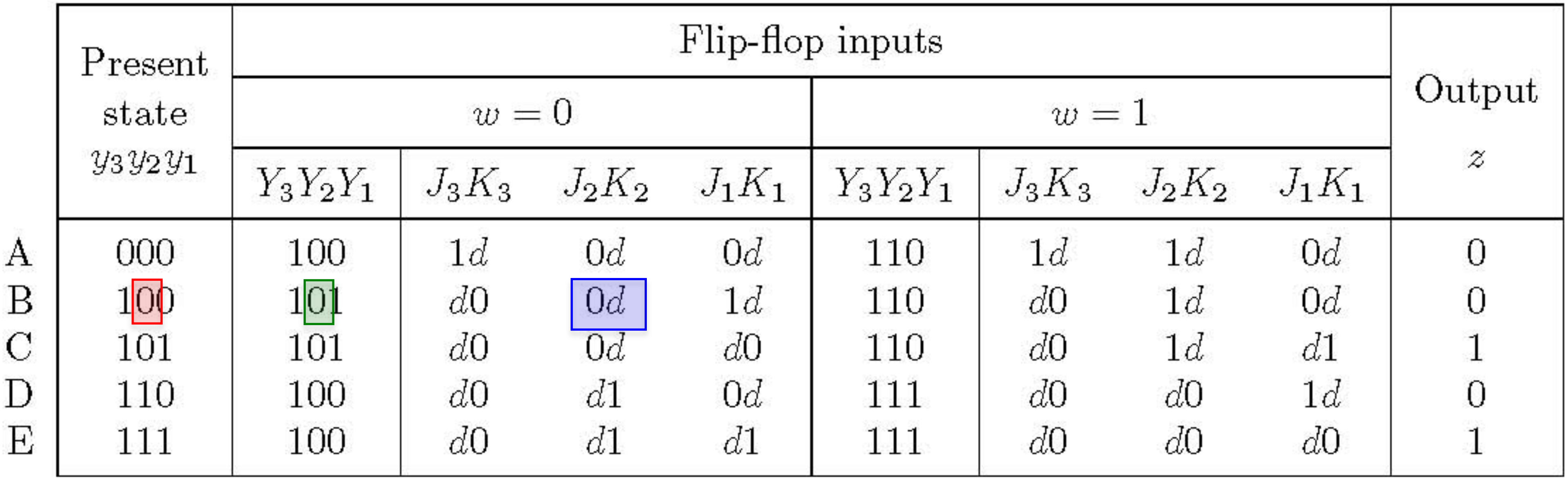
Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops

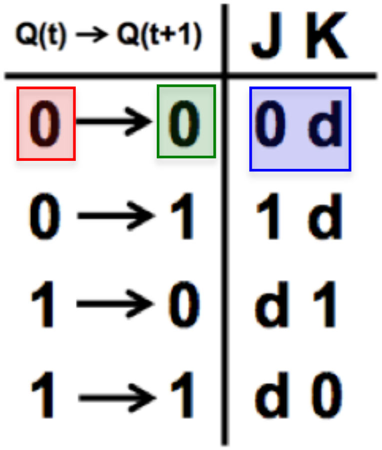
	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w = 0$				$w = 1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

Excitation table with JK flip-flops



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1



Q(t) → Q(t+1)	J K
0 → 0	0 d
0 → 1	1 d
1 → 0	d 1
1 → 1	d 0

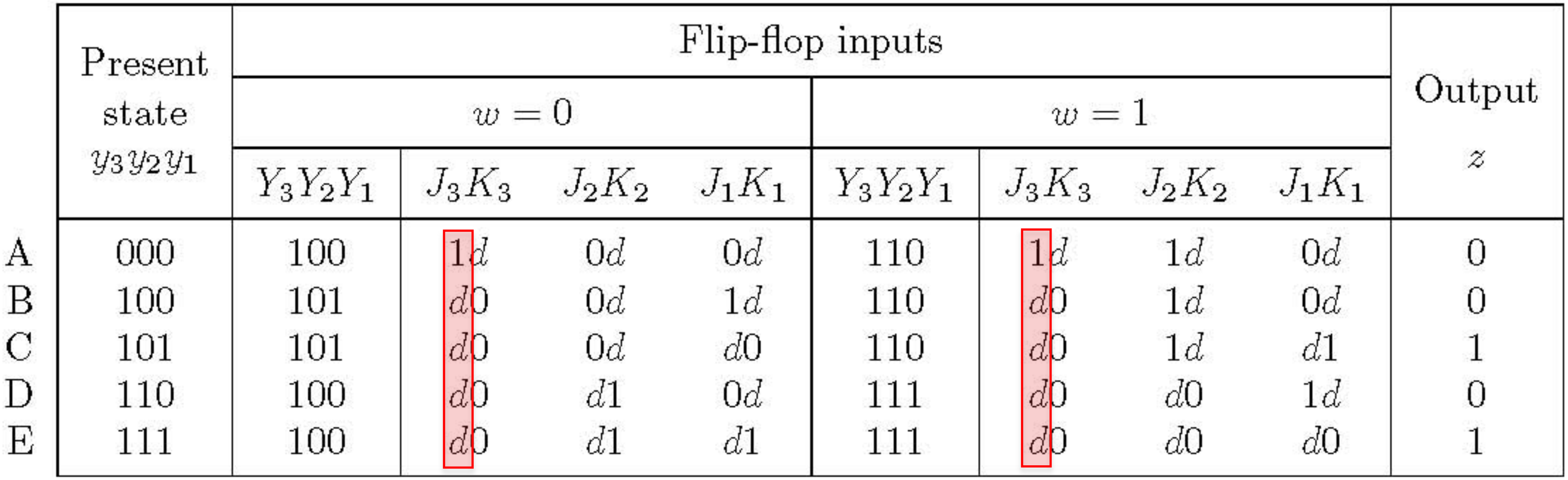
And so on...

The Expression for z

	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	1d	0d	0d	110	1d	1d	0d	0
B	100	101	d0	0d	1d	110	d0	1d	0d	0
C	101	101	d0	0d	d0	110	d0	1d	d1	1
D	110	100	d0	d1	0d	111	d0	d0	1d	0
E	111	100	d0	d1	d1	111	d0	d0	d0	1

z is equal to y_1

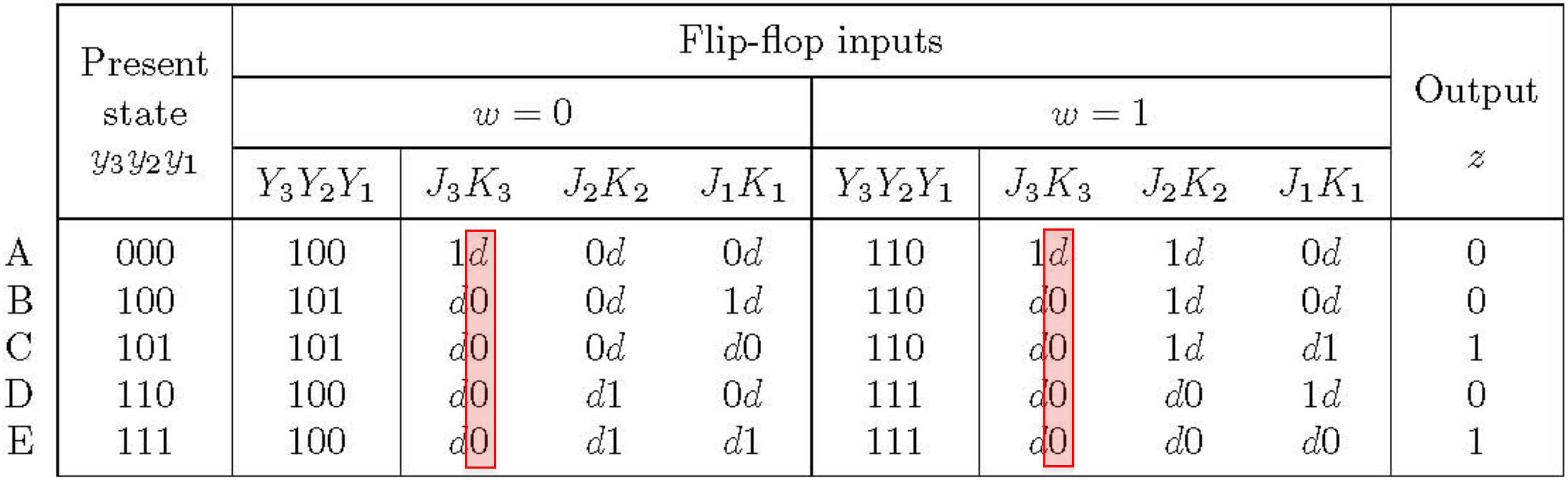
The Expression for J_3



	Present state	Flip-flop inputs								Output
		$w = 0$				$w = 1$				
	$y_3y_2y_1$	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	z
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

J_3 is equal to 1

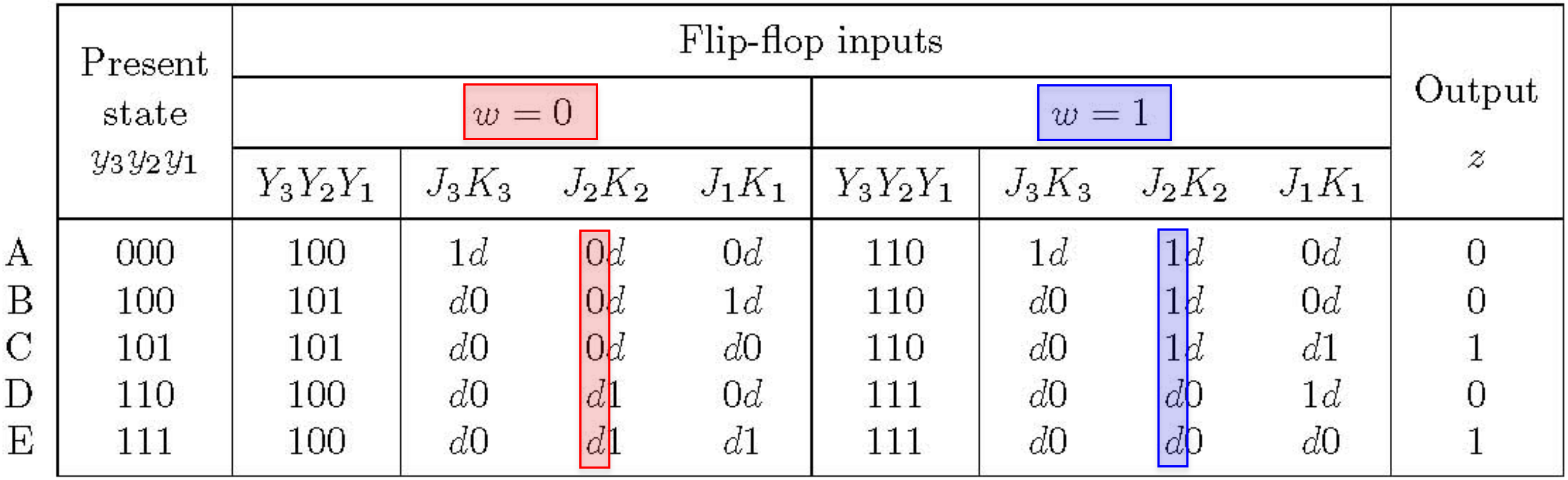
The Expression for K_3



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	1d	0d	0d	110	1d	1d	0d	0
B	100	101	d0	0d	1d	110	d0	1d	0d	0
C	101	101	d0	0d	d0	110	d0	1d	d1	1
D	110	100	d0	d1	0d	111	d0	d0	1d	0
E	111	100	d0	d1	d1	111	d0	d0	d0	1

K_3 is equal to 0

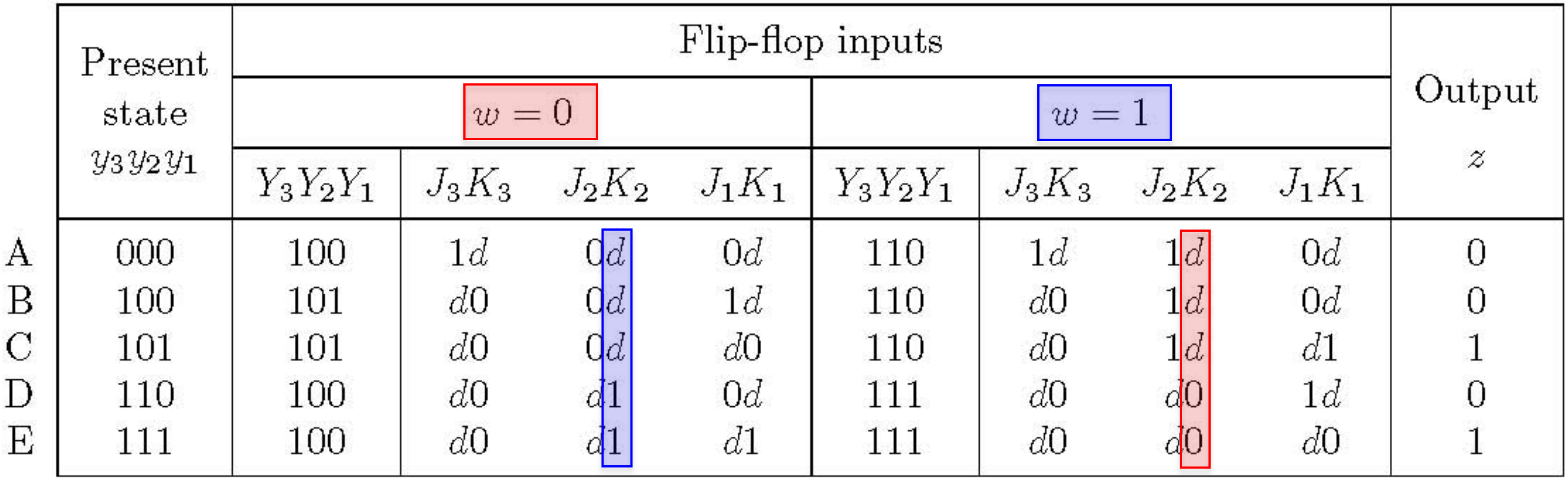
The Expression for J_2



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w = 0$				$w = 1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	1d	0d	0d	110	1d	1d	0d	0
B	100	101	d0	0d	1d	110	d0	1d	0d	0
C	101	101	d0	0d	d0	110	d0	1d	d1	1
D	110	100	d0	d1	0d	111	d0	d0	1d	0
E	111	100	d0	d1	d1	111	d0	d0	d0	1

J_2 is equal to w

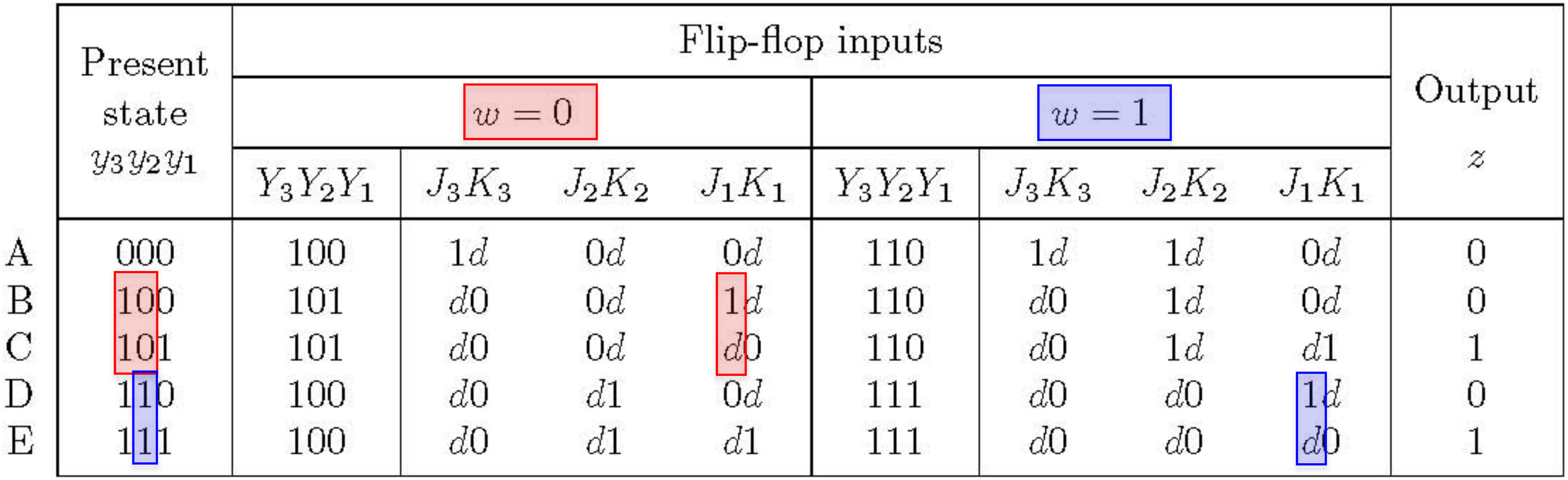
The Expression for K_2



	Present state	Flip-flop inputs								Output
		$w = 0$				$w = 1$				
	$y_3y_2y_1$	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	z
A	000	100	1d	0d	0d	110	1d	1d	0d	0
B	100	101	d0	0d	1d	110	d0	1d	0d	0
C	101	101	d0	0d	d0	110	d0	1d	d1	1
D	110	100	d0	d1	0d	111	d0	d0	1d	0
E	111	100	d0	d1	d1	111	d0	d0	d0	1

K_2 is equal to $\overline{w}$

The Expression for J_1



	Present state $y_3y_2y_1$	Flip-flop inputs								Output z
		$w=0$				$w=1$				
		$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

J_1 is equal to $w\,y_2 + \overline{w}\,y_3\,\overline{y_2}$

The Expression for K_1

	Present state	Flip-flop inputs								Output
		$w = 0$				$w = 1$				
	$y_3y_2y_1$	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	$Y_3Y_2Y_1$	J_3K_3	J_2K_2	J_1K_1	z
A	000	100	1d	0d	0d	110	1d	1d	0d	0
B	100	101	d0	0d	1d	110	d0	1d	0d	0
C	101	101	d0	0d	d0	110	d0	1d	d1	1
D	110	100	d0	d1	0d	111	d0	d0	1d	0
E	111	100	d0	d1	d1	111	d0	d0	d0	1

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K_1 is equal to $\overline{w}\, y_2 + w\, \overline{y_2}\, y_1$

All Logic Expressions

$$J_1 = wy_2 + \overline{w}y_3\overline{y}_2$$

$$K_1 = \overline{w}y_2 + wy_1\overline{y}_2$$

$$J_2 = w$$

$$K_2 = \overline{w}$$

$$J_3 = 1$$

$$K_3 = 0$$

$$z = y_1$$

Questions?

THE END