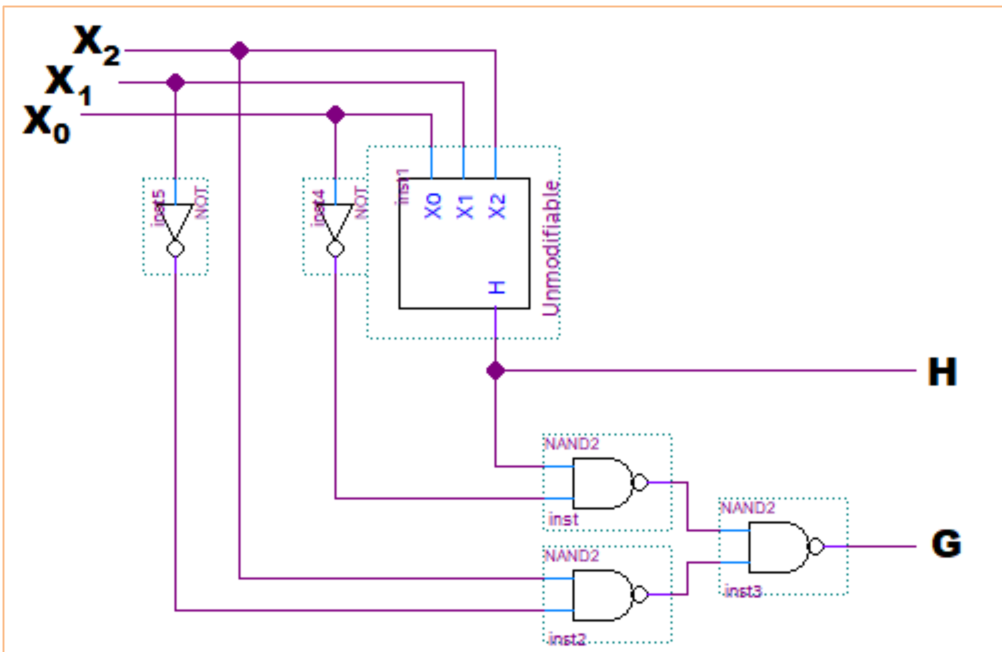
Cpr E 281 HW04

ELECTRICAL AND COMPUTER ENGINEERING

IOWA STATE UNIVERSITY

Minimization and Karnaugh Maps
Assigned: Week 4
Due Date: Sep. 23, 2019

P1 (10 points): In the circuit below, there is a block of "unmodifiable circuitry" with three inputs and one output H. The expression for H is as follows: $H = x_1x_0 + \bar{x}_2x_1 + x_2\bar{x}_1\bar{x}_0$. Note that the circuit output H and the output of the unmodifiable block have the same name. This is not strictly required in the circuits that will be analyzed in the future, but, for ease in the reader's understanding of this problem, have been given the same name.



a. Write the expression for H in shorthand.
b. Using Boolean algebra, show that the expression for G in shorthand is $G(x_2, x_1, x_0) = \sum m(2, 4, 5)$
c. The three-bit inputs to this circuit are the binary representation of a three-bit non-negative (unsigned) integer X, where (by convention) x_0 represents the least-significant bit (LSB) of the binary representation of the integer X. If X=3, what are the values of H and G, respectively?
d. What values of X will produce a result such that $\overline{H + G} = 1$?

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P2 (10 points): Produce the simplified sum-of-products (SOP) expressions for the following K-maps:

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	0	0

WX \ YZ	00	01	11	10
00	1	0	0	1
01	1	1	0	0
11	1	1	0	0
10	0	0	0	0

WX \ YZ	00	01	11	10
00	0	0	0	0
01	0	1	0	0
11	1	0	0	0
10	1	0	0	1

P3 (10 points): For each shorthand expression below, derive the simplest SOP expression:
A: $F_1(w,x,y,z) = \sum m(4,6,10,11,14,15)$
B: $F_2(x,y,z) = \sum m(2,5,6,7)$
C: $F_3(w,x,y,z) = \sum m(0,2,3,7,8,10,11,15)$

P4 (10 points): For each expression below, derive the simplest POS expression:
A: $G_1(a,b,c,d) = \prod M(0,1,2,3,4,5,10,11)$
B: $G_2(a,b,c) = \prod M(0,2,3,6)$
C: $G_3(a,b,c,d) = \prod M(0,2,5,9,13)$

P5 (10 points): Use Karnaugh Maps to convert the following expressions to <u>simplified SOP expressions</u>:
I: $H_1(A,B,C,D) = \bar{C}D + B\bar{C} + BC\bar{D} + AC\bar{D} + A\bar{B}C$
II: $H_2(A,B,C,D) = (A+C+D)(B+C+\bar{D})(\bar{A}+B+\bar{C})$
III: $H_3(A,B,C,D) = \prod M(0,2)$

P6 (15 points): Given the shorthand expression $F(W,X,Y,Z) = \sum m(4,5,7,8,10,11,12)$:
A: Use a Karnaugh map to show that F can be simplified into the expression $F = \bar{W}XZ + W\bar{X}Y + \bar{W}X\bar{Y} + W\bar{Y}\bar{Z}$

B: Another expression for F is brought to your attention as: $F_2 = \bar{W}XZ + W\bar{X}Y + \bar{Y}\bar{Z}$. What implications are there to using F2 instead of the expression for F obtained in part A?

C: It turns out that the simplification to F has more than one solution. How many complete simplifications are there to F?

P7 (20 points): The goal for this problem is to accept a four-bit integer X (x_3, x_2, x_1, x_0) and use the constituent bits to derive two expressions: P_2=1 if X is a multiple of 2 (0, 2, 4…) and P_5=1 if X is a multiple of 5 (0, 5, 10…).

A: Show the K-map for P_2 and the K-map for P_5 given the four input bits for X (x_3, x_2, x_1, x_0)

B: Write a shorthand POS expression for P_2 and show that the cost of the minimal POS circuit for P_2 is 2.

C: Write a shorthand POS expression for P_5 and show that the cost of the minimal POS circuit for P_5 is 25 (when implemented separately from P_2).

D: Let P_{10}=1 if X is a multiple of 10. Show that its expression is $P_{10} = \bar{x}_2\bar{x}_0(\bar{x}_3 + x_1)(x_3 + \bar{x}_1)$

P8 (15 points): Derive a minimal-cost circuit by producing a simplified expression for $P(W, X, Y, Z) = \sum m(0,2,3,4,6,9,10,11,13,14,15)$. Show that the minimum cost for the circuit that implements P has a cost of 22. The type of expression is not mentioned explicitly, but the correct expression will yield the aforementioned cost of 22 when converted into a circuit. Note: circuit cost is the number of gates plus the number of gate inputs.