# Synchronous Sequential Circuits <br> Assigned: Week 12 <br> Due Date: Nov. 18, 2019 

P1 (20 points): Create a Moore FSM for a circuit that outputs $z=1$ if input $\mathrm{w}=1$ for the last three clock cycles. The states should be as follows: $\mathrm{SA}=00, \mathrm{SB}=10, \mathrm{SC}=11$, and $\mathrm{SD}=01$ are the states encountered in order from reset to completion of the three clock cycles.
A: Draw a state diagram for this FSM.
B: Create a state-assigned table for this FSM.
C: Use a K-map to show that the following circuit implements this FSM. Note the expressions should be in POS form, in accordance with the circuit below.


D: Let us instead create an FSM for a circuit with three inputs $\left(x_{2}, x_{1}\right.$, and $x_{0}$ ) and outputs $z=1$ if at least two of the three inputs of $x$ are one. The states should be set in the same order that they were in P1. Draw a state diagram for this FSM.

P2 (10 points): Design a Moore FSM that has one-bit input A and onebit output $B$, where $B=1$ if the last six bits of $A$ are 110100 (from earliest to latest). Draw the state diagram for this FSM.

P3 (10 points): Design a Mealy FSM that has one-bit input A and one-bit output B, where $\mathrm{B}=1$ if the last six bits of A are 101001 (from earliest to latest). Draw the state diagram for this FSM.

P4 (10 points): Design a Mealy FSM with the following specifications:

- There is a two-bit input X ( $\mathrm{x}_{1} \mathrm{x}_{0}$ )
- There is a one-bit output $Z$
- Each cycle, the input X is added to the value in memory. Let us refer to this sum as V.


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- The output $Z=1$ if $V$ is greater than 5 but less than 10 . Draw the state table for this Mealy FSM.

P5 (20 points): A FSM has five states, five state variables, one-bit input W, one-bit reset R, and one-bit output M. The FSM has an output that corresponds with the timing diagram shown below. Answer the following questions.


A: The five states encountered in this timing diagram, in order, are $\underline{A, B}$, $\mathrm{C}, \mathrm{D}$, and then E . What are the state encodings for these states?
B: Draw a state diagram for this FSM.
C: Draw a state-assigned table for this FSM.
D: This circuit is implemented using five DFFs. Derive expressions for the next state variables and the output M .
E: In addition to the five DFFs needed to implement this FSM, if no other gates are used, how many 2-to-1 MUXes are necessary to implement this circuit?
F: Explain why this particular circuit has the same number of states as it does state variables and why this isn't necessarily true of other circuits?

P6 (15 points): The FSM state diagram below has two inputs $\mathrm{x}_{1}$ and $\mathrm{x}_{0}$. In addition, it has two DFFs, three 4-to-1 MUXes, a single XOR gate, a single AND gate, and a single output bit $Z$. Answer the following questions about this FSM.

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A: Is this a Moore FSM or a Mealy FSM?
$B$ : The state encodings are $A=00, B=01, C=10$, and $D=11$. Write a stateassigned table for this state diagram.
C: Use K-maps to show that the expressions for the next-state variables and the output are as follows:

$$
\begin{gathered}
Y_{1}^{\text {new }}=\bar{x}_{1} y_{1} y_{0}+\bar{x}_{1} x_{0} y_{1}+x_{1} \bar{x}_{0} \bar{y}_{1} y_{0}+x_{1} \bar{x}_{0} y_{1} \bar{y}_{0} \\
Y_{0}^{\text {new }}=\bar{x}_{1} y_{1} y_{0}+\bar{x}_{1} x_{0} y_{1}+x_{1} x_{0} \bar{y}_{1} \\
Z=\bar{x}_{1} \bar{x}_{0} y_{1} y_{0}
\end{gathered}
$$

D: Draw a circuit which implements this FSM as mentioned above: two DFFs, three 4-to-1 MUXes, one AND gate, and one XOR gate. Use the two-bit input X for the select lines.

P7 (15 points): This problem concerns the circuit diagram shown below, containing three FSMs connected together.


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A: Design a circuit that has nine-bit input ( $\mathrm{L} \mathrm{X}_{7} \mathrm{X}_{6} \mathrm{X}_{5} \mathrm{X}_{4} \mathrm{X}_{3} \mathrm{X}_{2} \mathrm{X}_{1} \mathrm{X}_{0}$ ) and one-bit output $Z$ such that if $L=0$, the value in $X$ is stored into memory (hereafter referred to as Y ) on the clock signal and, if $\mathrm{L}=1$, the value in memory $(\mathrm{Y})$ is shifted right. The output $Z$ is the least significant bit of the value Y in memory. It is not necessary to create an FSM to implement this circuit as it is a circuit that you have seen before. What circuit meets the above criteria (describe the width and input/output method)? B: As this circuit is an FSM, how many states would it have?
C: Next, design a Moore FSM with two inputs (G and Z) and one output EN. If $G=0$ at any point on this $F S M$, the circuit always returns to the beginning and outputs 0 . If $G=1$, then the circuit looks at $Z$ from this circuit as an input and outputs $\mathrm{EN}=1$ until the second 1 is observed on the input $Z$, at which point the circuit will always output EN=0 until G is set to zero and the cycle continues again. Draw the state diagram for this FSM. Note: G is not an asynchronous reset and should also be included within the state diagram.
D: The output of the previous FSM (EN) will be used as an input to the following state diagram with input EN and four-bit output F. What type of circuit does this Moore FSM state diagram implement?


E: If the user inputs the value $\mathrm{X}=11101000$ to the first FSM (by setting $\mathrm{L}=0$ and $\mathrm{G}=0$ and then advancing the clock one cycle), sets $\mathrm{L}=1$ and $\mathrm{G}=1$, and then starts the counter, what will be the final value of $F$ ?

