Sample Solutions

CprE 281: Digital Logic
Midterm 1: Friday Sep 27, 2019

Name: _____   ID Number:_____

Lab Section:	Mon 12-3 (#13)	Tue 11-2 (#16)	Wed 8-11 (#8)	Thur 11-2 (#14)	Fri 11-2 (#7)
(circle one)		Tue 2-5 (#11)	Wed 11-2 (#18)	Thur 11-2 (#17)	
		Tue 2-5 (#20)		Thur 2-5 (#10)	
				Thur 5-8 (#9)	

1. **True/False Questions (10 x 1p each = 10p)**

(a) I forgot to write down my name, student ID number, and lab section.   TRUE / **(FALSE)**

(b) Any Boolean function can be implemented using only NOT gates.   TRUE / **(FALSE)**

(c) In a 3-variable K-map the positions of m_4 and M_4 are the same.   **(TRUE)** / FALSE

(d) In a 4-variable K-map the positions of m_{13} and M_{13} are not the same.   TRUE / **(FALSE)**

(e) Refined coaxium is stored in the Imperial Depository on Vandor.   **(TRUE)** / FALSE

(f) An XOR gate can be implemented with two XNOR gates.   **(TRUE)** / FALSE

(g) A NAND gate can be implemented with three NOR gates.   TRUE / **(FALSE)**

(h) $(a + \overline{b\,c})(\overline{a\,b} + c) = a$ TRUE / **(FALSE)**

(i) $\overline{x\,y} + x\,y = 0$ TRUE / **(FALSE)**

(j) Calculus 101 in binary is Calculus 5 in decimal.   **(TRUE)** / FALSE

2. **Three-Variable K-Map (5p)**

**Use a K-map to derive the minimum <u>POS</u> expression for f(x, y, z)= Π M(0, 1, 4, 7).**

$$f = (x+y)\cdot(\overline{x}+\overline{y}+\overline{z})\cdot(y+z)$$
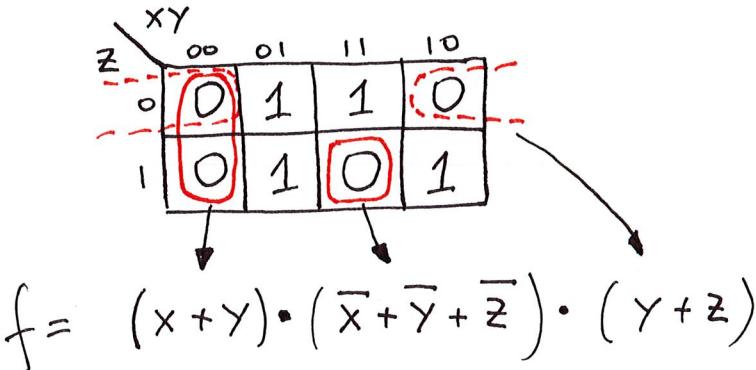
3. Venn Diagram (4p + 3p + 3p = 10p)

(a) Draw the truth table for the following Venn diagram.
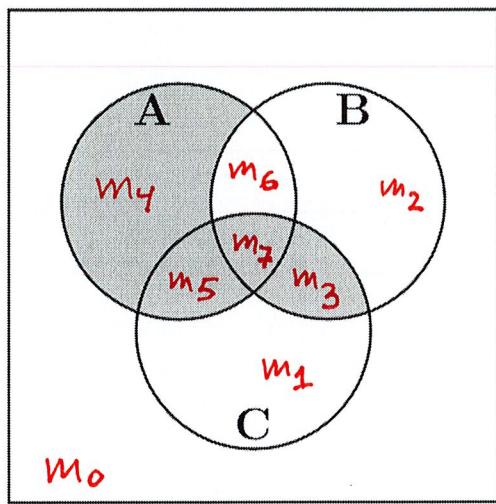
	A	B	C	F
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1

(b) Use a K-map to derive a minimum-cost product-of-sums (POS) expression for the function show in the Venn diagram above.
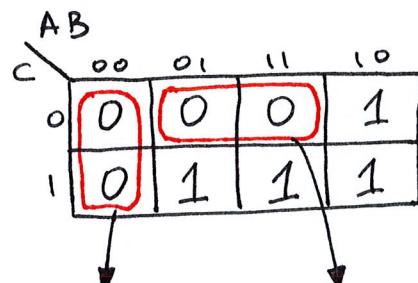
$$F = (A + B) \cdot (\bar{B} + C)$$

(c) Draw the circuit diagram for the minimum-cost <u>POS</u> expression.
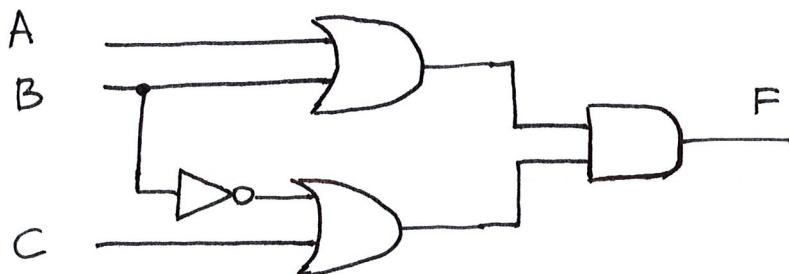
4. Number Conversions (5 x 4p each = 20p)

    (a) Convert 10111001_2 to decimal

$$\underbrace{1 \times 2^7}_{128} + 0 \times 2^6 + \underbrace{1 \times 2^5}_{32} + \underbrace{1 \times 2^4}_{16} + \underbrace{1 \times 2^3}_{8} + 0 \times 2^2 + 0 \times 2^1 + \underbrace{1 \times 2^0}_{1} = \boxed{185}$$

    (b) Convert 135_{10} to binary

                             remainder

$135 / 2$	$=$	67	1
$67 / 2$	$=$	33	1
$33 / 2$	$=$	16	1
$16 / 2$	$=$	8	0
$8 / 2$	$=$	4	0
$4 / 2$	$=$	2	0
$2 / 2$	$=$	1	0
$1 / 2$	$=$	0	1

$$\boxed{10000111_2}$$

first convert to binary   **(c) Convert 751_8 to hexadecimal**

$$111 \vdots 101 \vdots 001$$

pad

$$\underbrace{0001}_{1} \vdots \underbrace{1110}_{E} \underbrace{1001}_{9}$$

then convert from binary to hexadecimal

$$\boxed{1E9_{16}}$$

    (d) Convert 219_{10} to hexadecimal

                        remainder

$219 / 16$	$=$	13	11
$13 / 16$	$=$	0	13

$$\boxed{DB_{16}}$$

$A=10$, $B=11$, $C=12$, $D=13$, $E=14$, $F=15$

    (e) Convert 134_6 to binary

First convert to decimal

$$\underbrace{1 \times 6^2}_{36} + \underbrace{3 \times 6^1}_{18} + \underbrace{4 \times 6^0}_{4} = 58_{10}$$

                          remainder

$58 / 2$	$=$	29	0
$29 / 2$	$=$	14	1
$14 / 2$	$=$	7	0
$7 / 2$	$=$	3	1
$3 / 2$	$=$	1	1
$1 / 2$	$=$	0	1

Convert from decimal to binary

$$\boxed{111010_2}$$

5. From Circuit to Verilog Code (10p)

Write a behavioral Verilog module that corresponds to the following circuit.

$\bar{x}\bar{y}a$

$\bar{x}yb$

$x\bar{y}c$

xyd

This is a 4-to-1 multiplexer.
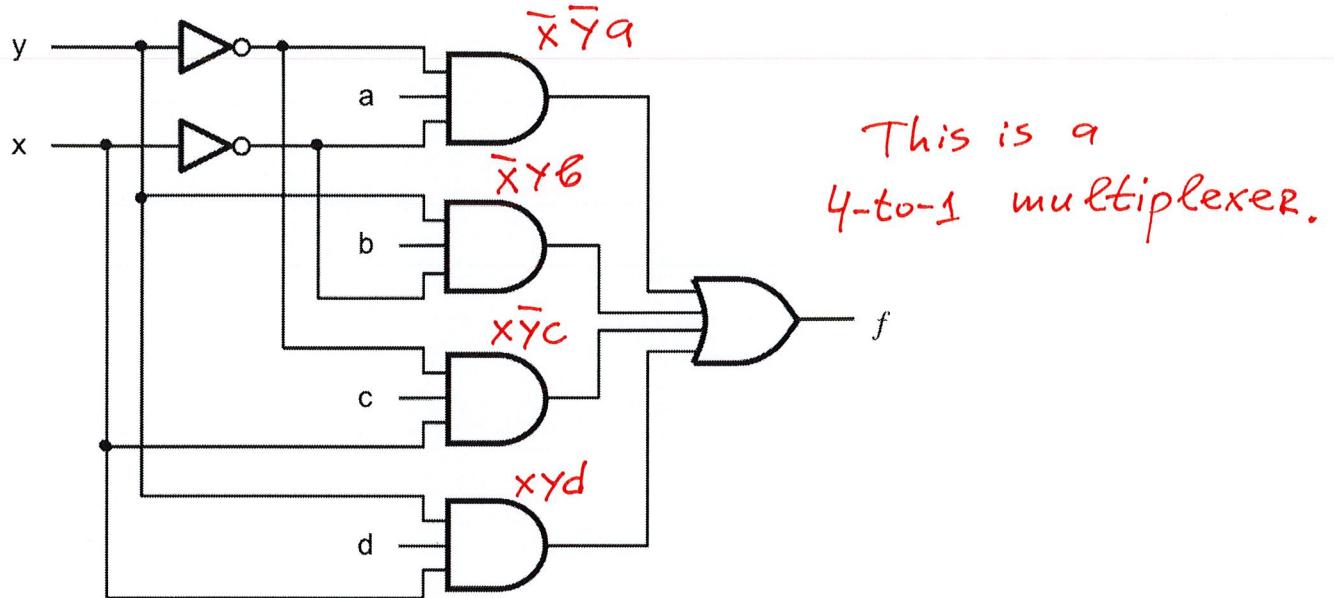
```
module   mux ( x, y, a, b, c, d, f );
   input  x, y, a, b, c, d;
   output f;

   assign f = (~x & ~y & a) | (~x & y & b) | (x & ~y & c) | (x & y & d);
endmodule
```

6. Minimization (3 x 5p = 15p)

(a) Draw the K-map that corresponds to the following Boolean function:

$$f = \overline{a}\,c\,\overline{d} + \overline{a}\,\overline{b}\,c\,\overline{d} + a\,b\,d + a\,\overline{b}$$

$$\overline{a}\,\overline{b}\,c\,\overline{d}$$

$$\overline{a}\,c\,\overline{d}$$

$$a\,b\,d \qquad a\,\overline{b}$$
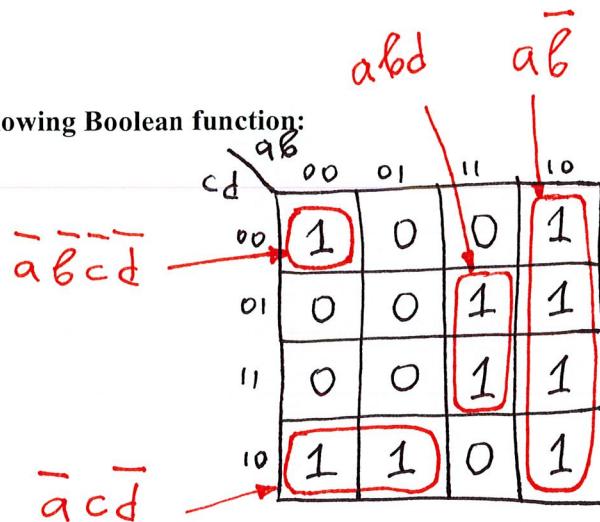
(b) Redraw the K-map from (a) and derive the minimum-cost Sum-of-Products (SOP) expression for f.

$$f = \overline{b}\,\overline{d} + a\,d + \overline{a}\,c\,\overline{d}$$

$$\overline{b}\,\overline{d}$$

$$a\,d$$

$$\overline{a}\,c\,\overline{d}$$
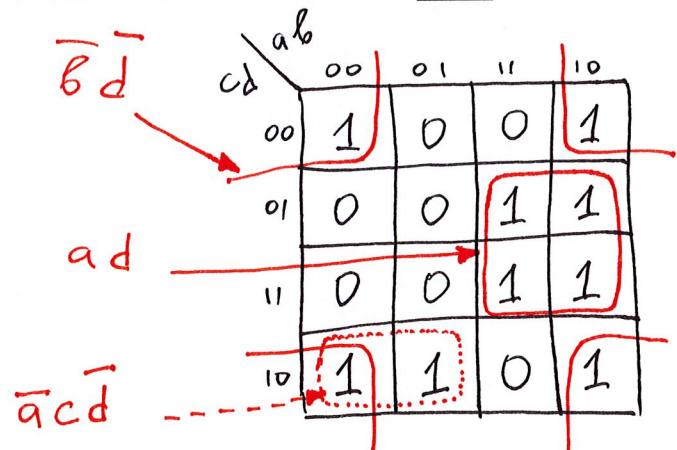
(c) Draw the circuit for the minimum-cost SOP expression using only NAND gates. Clearly label all inputs and outputs.
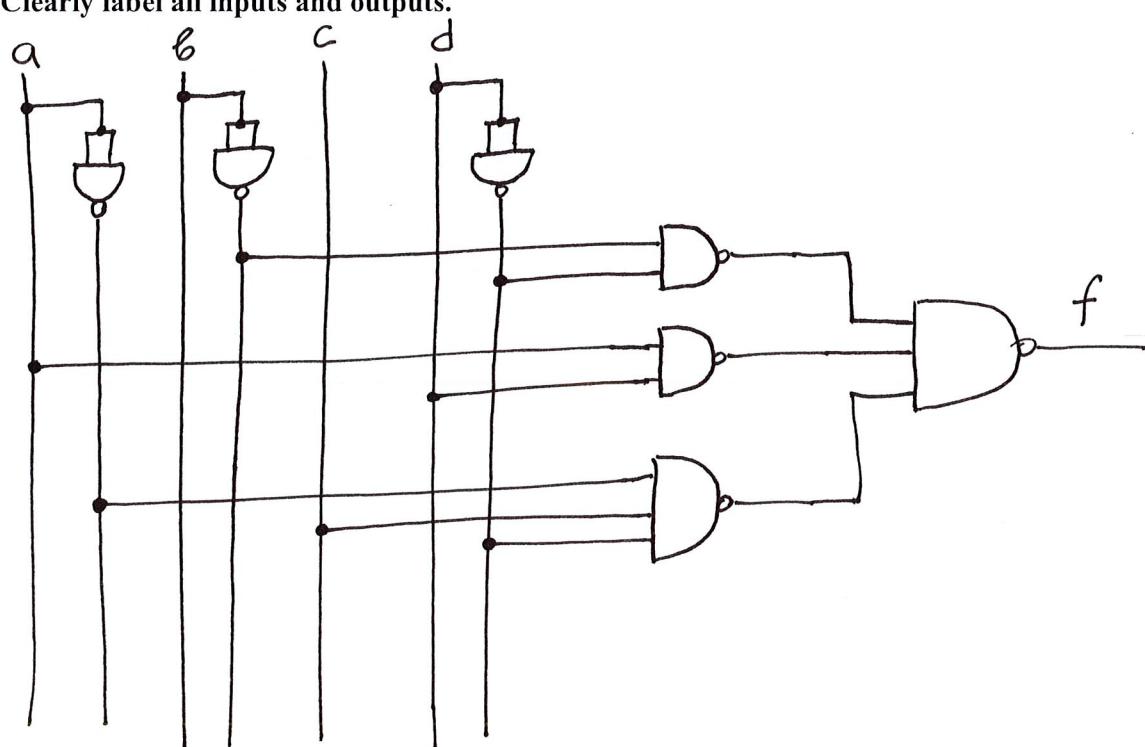
**7. Derive the minimum <u>POS</u> expression using a K-map (10p + 5p = 15p)**

(a) Use a K-map to derive the minimum-cost <u>POS</u> expression for the following function

$$f(a,b,c,d) = \Pi M(2, 6, 7, 11, 12, 13) + D(1, 3, 5, 10, 14)$$
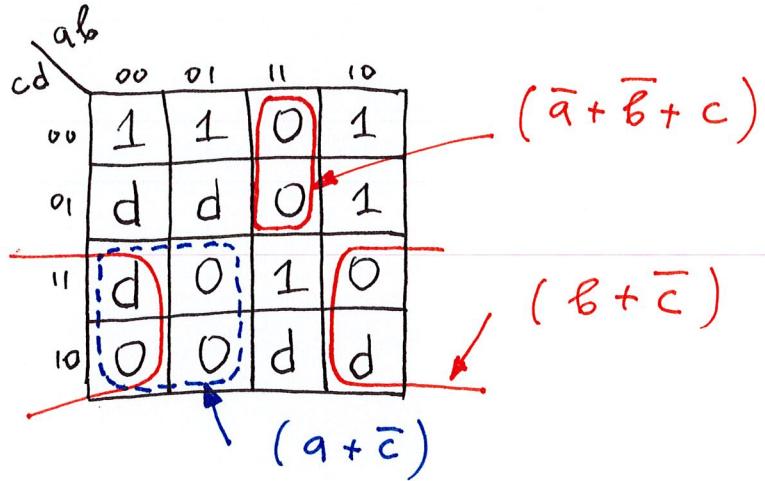
$$f = (a + \bar{c}) \cdot (b + \bar{c}) \cdot (\bar{a} + \bar{b} + c)$$

(b) Draw the circuit diagram for the expression derived in (a) using <u>only NOR gates</u>. Clearly label all inputs and outputs.
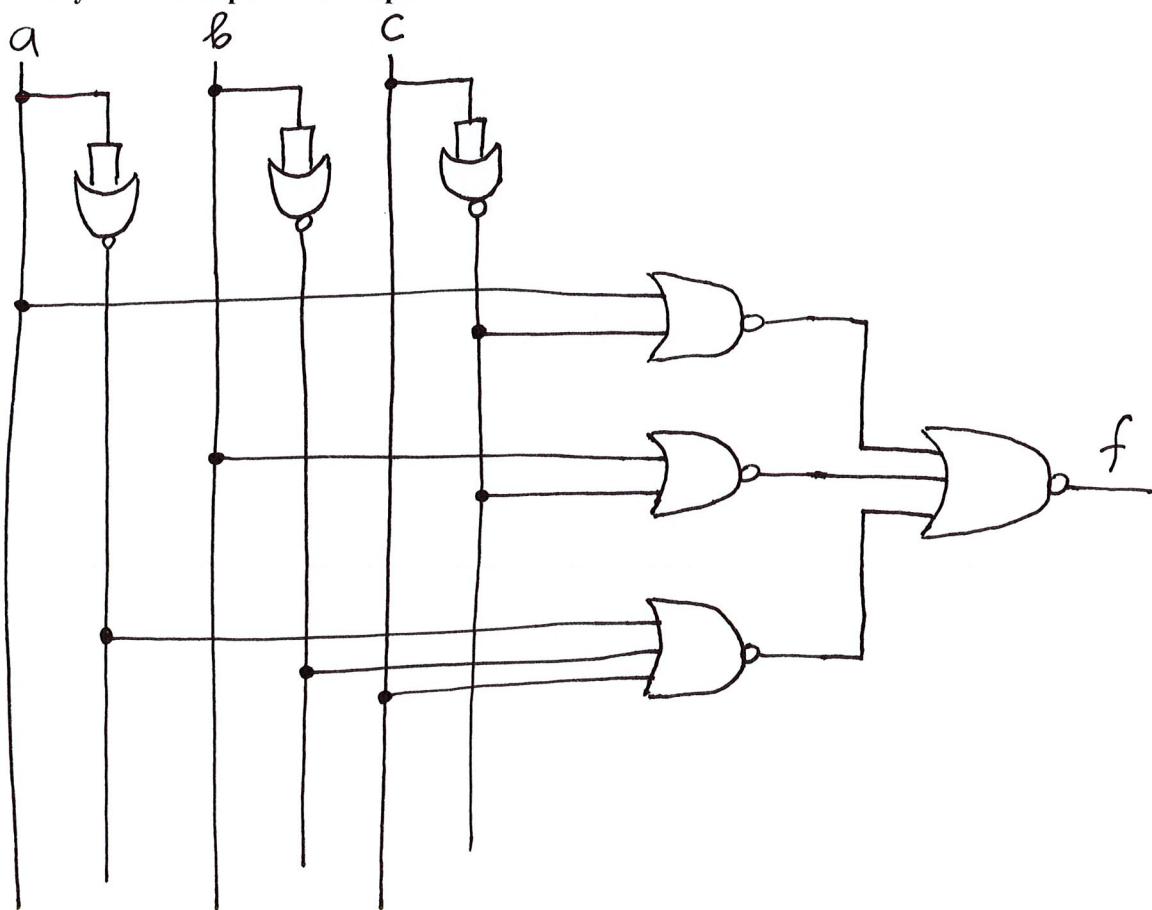
8. Circuit Simplification (3 x 5p = 15p)

**(a) Write the Boolean expression for the function F(A, B, C) that is given by this circuit.**
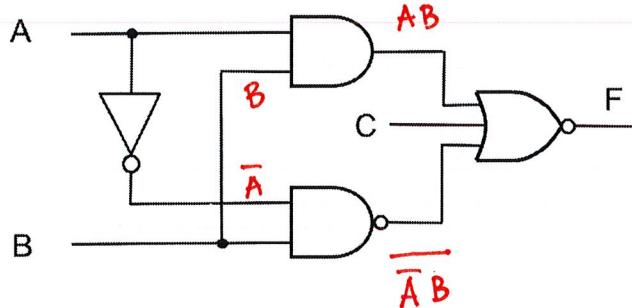
$$F = \overline{AB + C + \overline{A}\,B}$$

(b) Use the theorems of Boolean algebra to find the simplest form of the expression for F.

$$F = \overline{AB + C + \overline{A}\,B}$$

$$= \overline{AB + C + \overline{\overline{A} + \overline{B}}}$$

$$= \overline{AB + C + A + \overline{B}}$$

$$= \overline{A(B+1) + C + \overline{B}}$$

$$= \overline{A + \overline{B} + C}$$

$$= \overline{A}\, B\, \overline{C}$$

(c) Draw the circuit for the simplest expression.
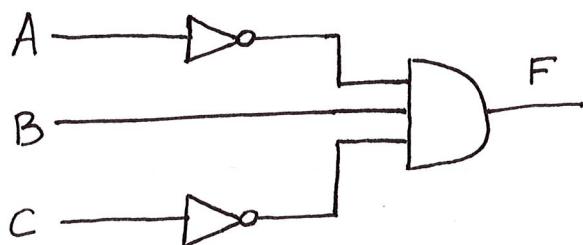
9. Truth Table for a Pet Shelter (15p)

A pet shelter has taken in a new kitten. Two staff members, Ashley and Greg, have had trouble feeding her and have taken notes on when she won't eat. The kitten won't eat when neither Ashley nor Greg feed her, and she has had more than an hour of play time, and she gets fed Food-B (a type of cat food). When only Greg feeds the kitten, she won't eat unless she has an hour of play time and gets Food-B. When only Ashley feeds her, the kitten only eats if she has gotten an hour of play time, or she gets Food-B, but won't eat if both occur. When both Ashley and Greg feed her together, the kitten won't eat if she gets Food-B.

Use this information to fill out a truth table for the eating habits of this kitten. Clearly label all variables that your solution uses. The output function should be called **E** (short for Eat) and it should be equal to 1 it the cat will eat and 0 if it won't.

A	G	P	B	E
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

Variables

A - Ashley feeds her

G - Greg feeds her

P - ≥ 1 hour of play

B - served Food-B

E - the cat Eats

10. Boolean Algebra (10p + 5p = 15p)

(a) Use the theorems of Boolean algebra to simplify the formula given below into a minimum-cost sum-of-products (SOP) expression.

(b) Draw the Venn diagram for the minimum-cost SOP expression.

$$F(A,B,C) = \underbrace{\bar{A} * (\bar{A} + \bar{C})}_{\alpha} * \underbrace{\overline{(A\bar{B} + \bar{A}\bar{B} + \bar{C})}}_{\beta}\underbrace{\overline{(B + \bar{B}C)}}_{\gamma} = \alpha\,\overline{\beta}\,\overline{\gamma}$$

$$\alpha = \bar{A}\,(\bar{A} + \bar{C}) = \underbrace{\bar{A}\,\bar{A}}_{\bar{A}} + \bar{A}\,\bar{C} = \bar{A} + \bar{A}\,\bar{C} = \bar{A}\,\underbrace{(1 + \bar{C})}_{1} = \bar{A}$$

$$\beta = A\bar{B} + \bar{A}\,\bar{B} + \bar{C} = \underbrace{(A + \bar{A})}_{1}\bar{B} + \bar{C} = \bar{B} + \bar{C}$$

$$\gamma = B + \bar{B}C = B + C \qquad (\text{Theorem } 16.a)$$

$$F = \alpha\,\overline{\beta}\,\overline{\gamma}$$

$$= \bar{A}\;\overline{(\bar{B} + \bar{C})}\,\overline{(B + C)}$$

$$= \bar{A}\,\left(\overline{\bar{B} + \bar{C}} + \overline{B + C}\right)$$

$$= \bar{A}\,\left(\overline{\bar{B}}\,\overline{\bar{C}} + \bar{B}\,\bar{C}\right)$$

$$= \bar{A}\,\left(BC + \bar{B}\,\bar{C}\right)$$

$$= \underbrace{\bar{A}\,BC}_{m_4} + \underbrace{\bar{A}\,\bar{B}\,\bar{C}}_{m_0}$$
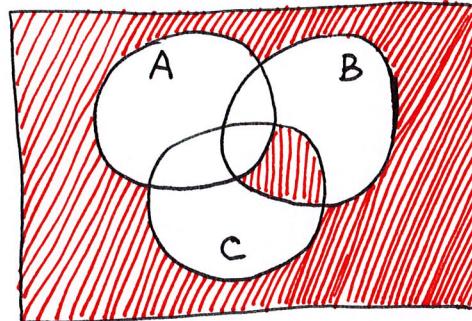
Question	Max	Score
1. True/False	10	
2. Three-Variable K-map	5	
3. Venn Diagram	10	
4. Number Conversions	20	
5. Verilog Module	10	
6. Minimization	15	
7. POS with K-Map	15	
8. Circuit Simplification	15	
9. Pet Shelter	15	
10. Boolean Algebra	15	
TOTAL:	130	