



CprE 281: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Logic Gates

CprE 281: Digital Logic
Iowa State University, Ames, IA
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Administrative Stuff

- **HW1 is out**
- **It is due on Wednesday Sep 4 @ 4pm.**
- **Submit it on paper before the start of the lecture**
- **No late homeworks will be accepted.**
- **Staple all of your pages**
- **Please write clearly on the first page:**
 - **your name**
 - **student ID**
 - **lab section letter**

Administrative Stuff

- **HW2 is out**
- **It is due on Monday Sep 9 @ 4pm.**
- **Submit it on paper before the start of the lecture**
- **No late homeworks will be accepted.**
- **Staple all of your pages**
- **Please write clearly on the first page:**
 - **your name**
 - **student ID**
 - **lab section letter**

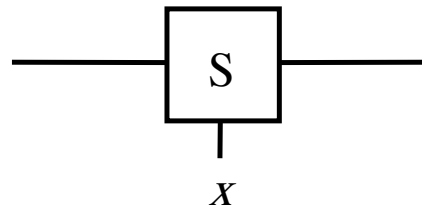
Labs Next Week

- Please download and read the lab assignment for next week before you go to your lab section.
- You must **print** the answer sheet and **do** the prelab **before** you go to the lab.
- The TAs will check your prelab answers at the **beginning of the recitation**. If you don't have it done you'll lose 20% of the lab grade for that lab.

A Binary Switch

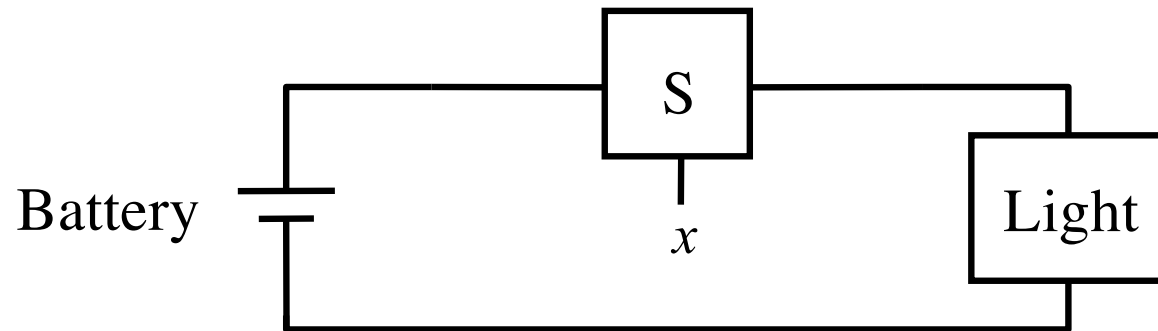


(a) Two states of a switch



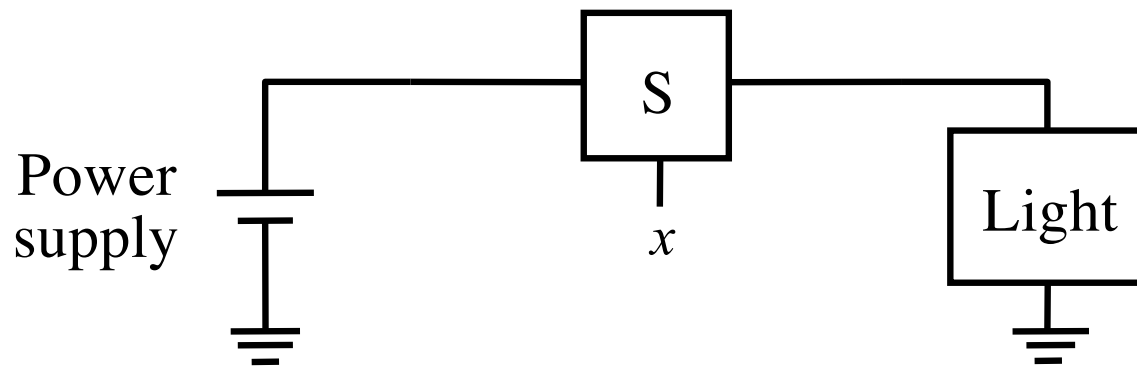
(b) Symbol for a switch

A Light Controlled by a Switch



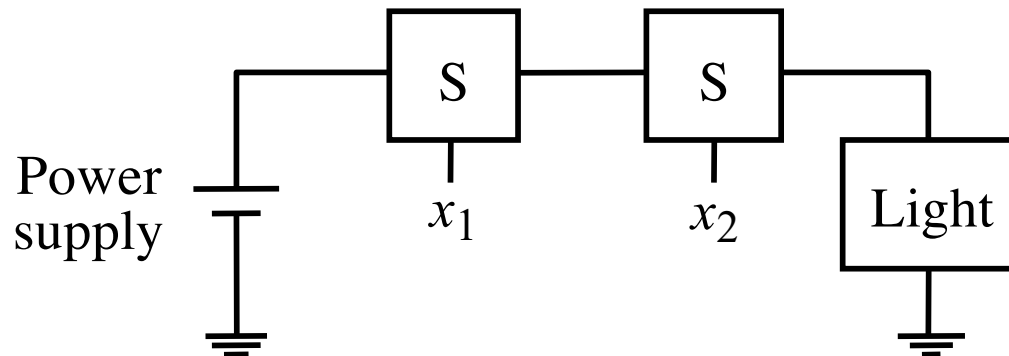
(a) Simple connection to a battery

A Light Controlled by a Switch

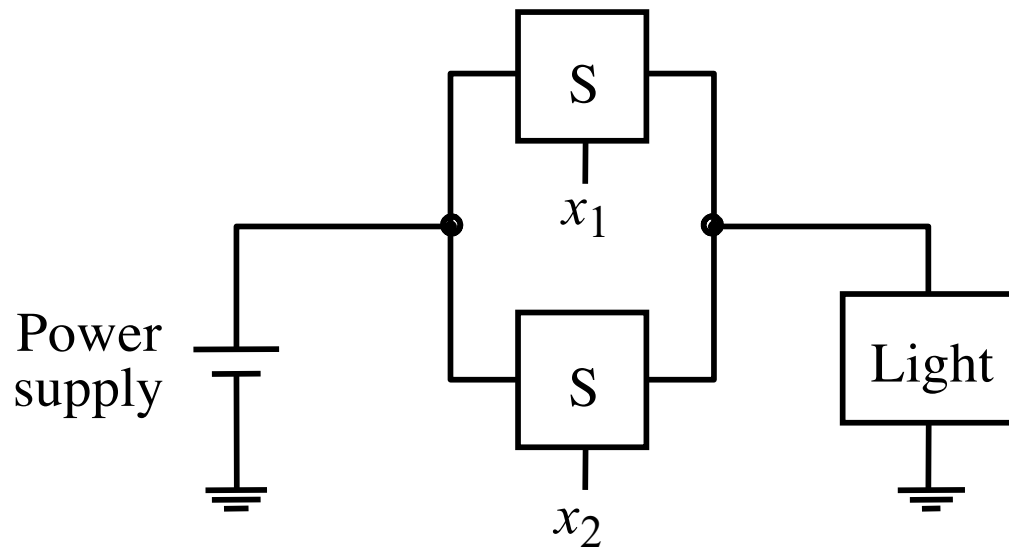


(b) Using a ground connection as the return path

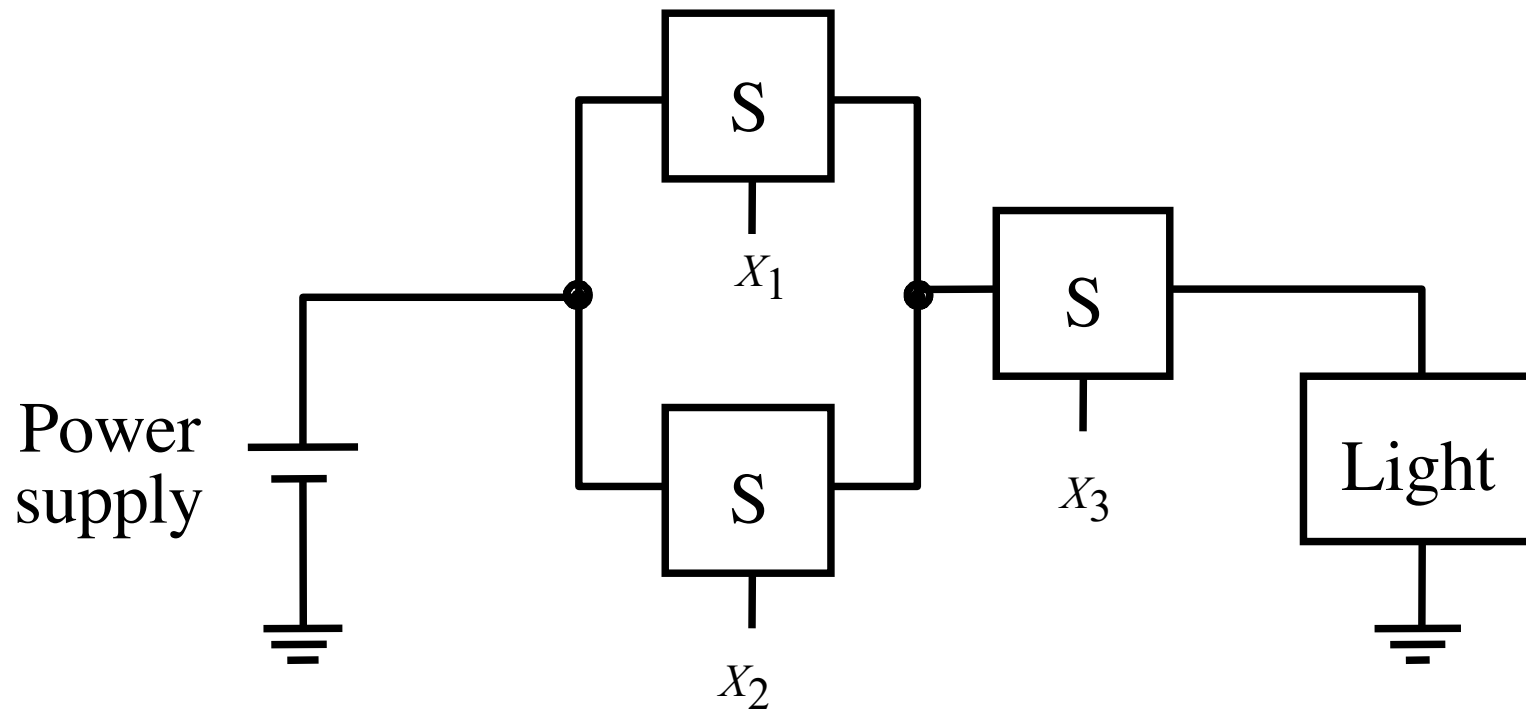
The Logical AND function (series connection of the switches)



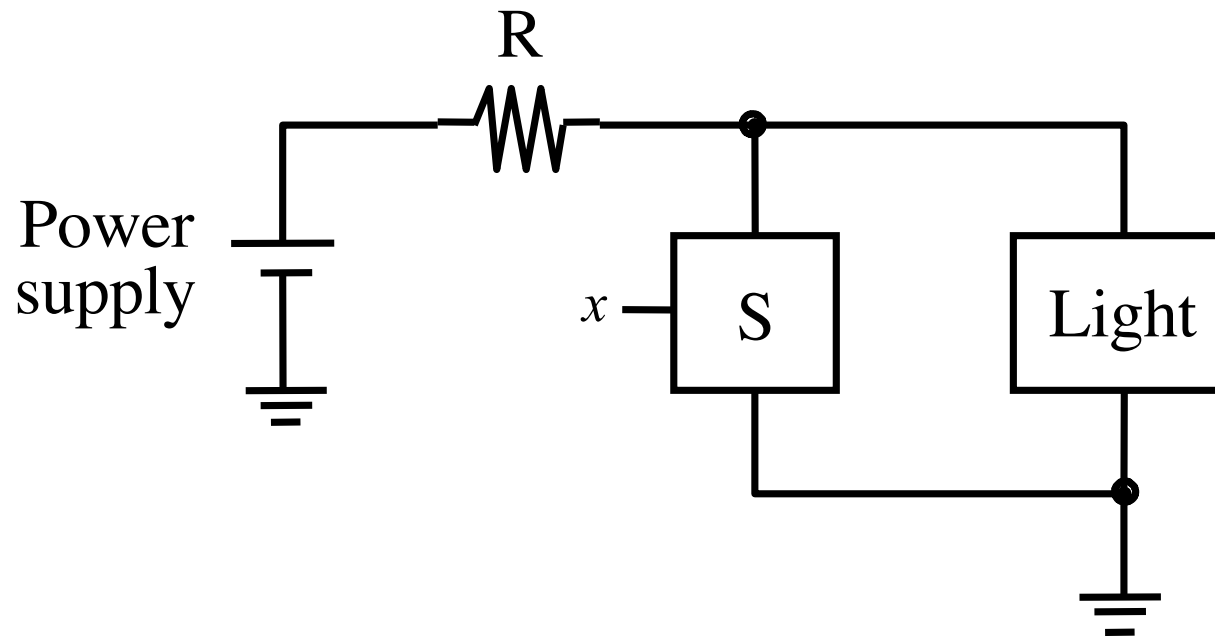
The Logical OR function (parallel connection of the switches)



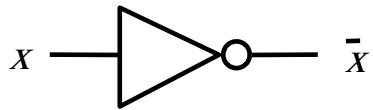
A series-parallel connection of the switches



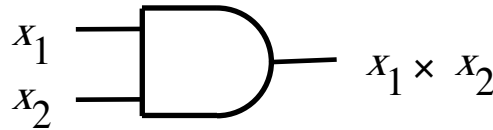
An Inverting Circuit



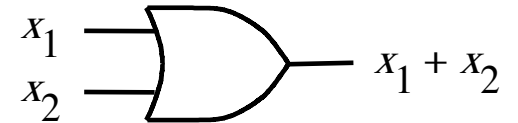
The Three Basic Logic Gates



NOT gate

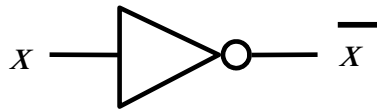


AND gate



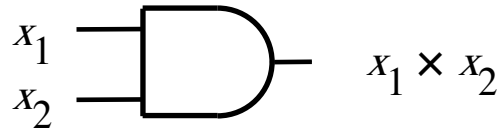
OR gate

Truth Table for NOT



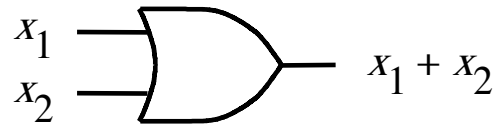
| x | \bar{x} |
|-----|-----------|
| 0 | 1 |
| 1 | 0 |

Truth Table for AND



| x_1 | x_2 | $x_1 \cdot x_2$ |
|-------|-------|-----------------|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Truth Table for OR



| x_1 | x_2 | $x_1 + x_2$ |
|-------|-------|-------------|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

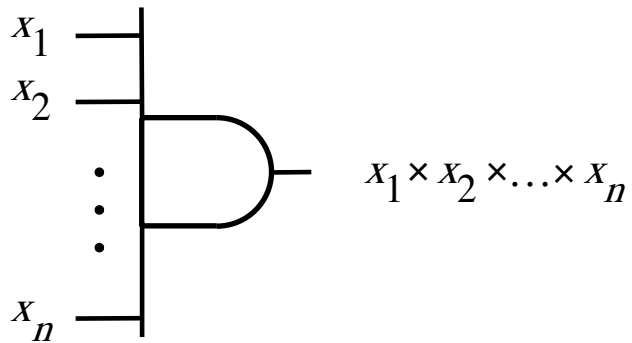
Truth Tables for AND and OR

| x_1 | x_2 | x_1 | x_2 | $x_1 + x_2$ |
|-------|-------|-------|-------|-------------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

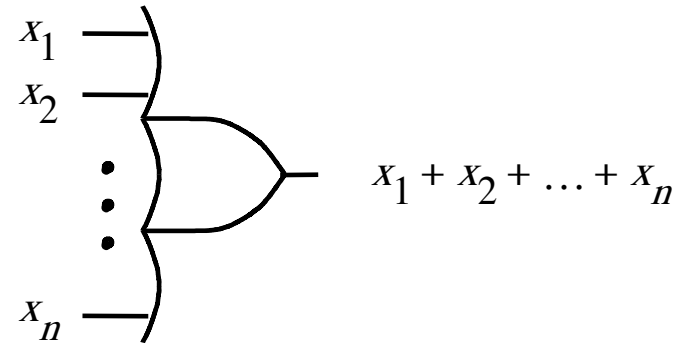
AND

OR

Logic Gates with n Inputs



AND gate

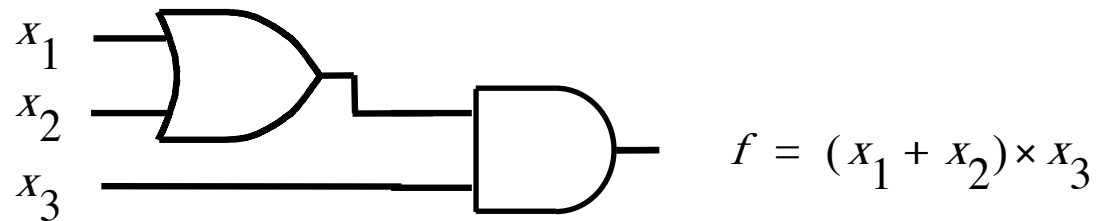


OR gate

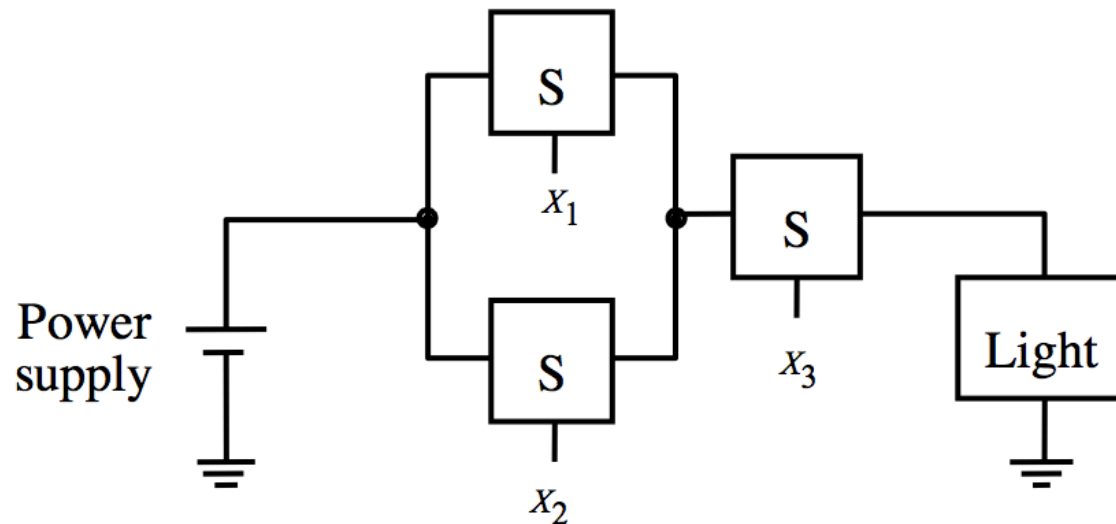
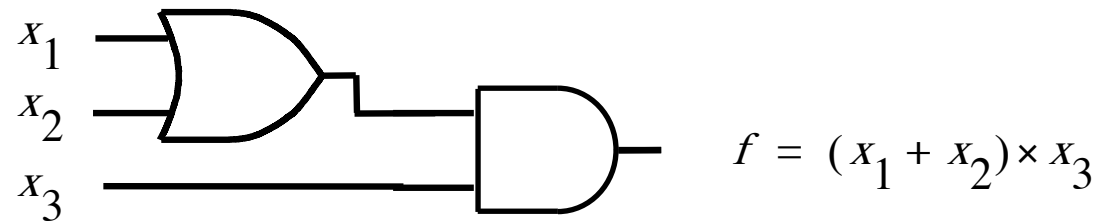
Truth Table for 3-input AND and OR

| x_1 | x_2 | x_3 | x_1 | x_2 | x_3 | $x_1 + x_2 + x_3$ |
|-------|-------|-------|-------|-------|-------|-------------------|
| 0 | 0 | 0 | | 0 | | 0 |
| 0 | 0 | 1 | | 0 | | 1 |
| 0 | 1 | 0 | | 0 | | 1 |
| 0 | 1 | 1 | | 0 | | 1 |
| 1 | 0 | 0 | | 0 | | 1 |
| 1 | 0 | 1 | | 0 | | 1 |
| 1 | 1 | 0 | | 0 | | 1 |
| 1 | 1 | 1 | | 1 | | 1 |

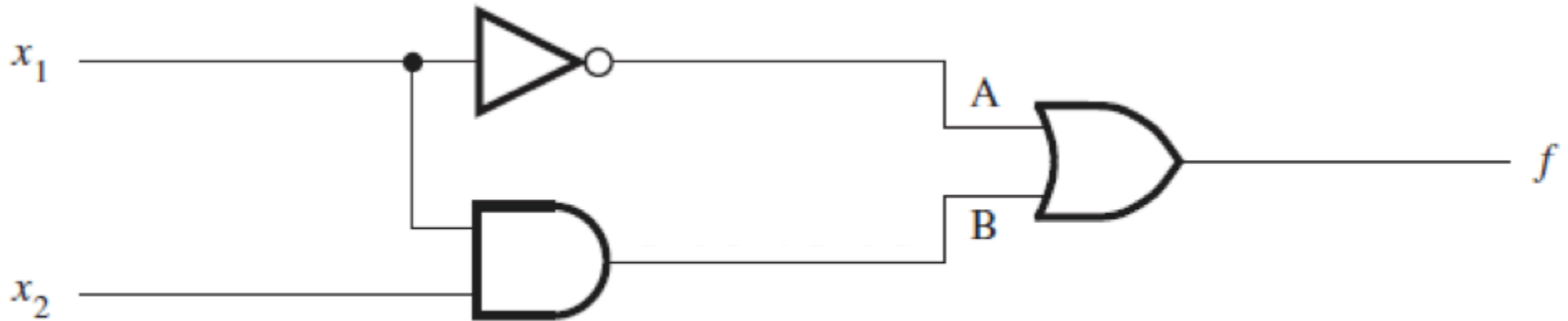
Example of a Logic Circuit Implemented with Logic Gates



Example of a Logic Circuit Implemented with Logic Gates

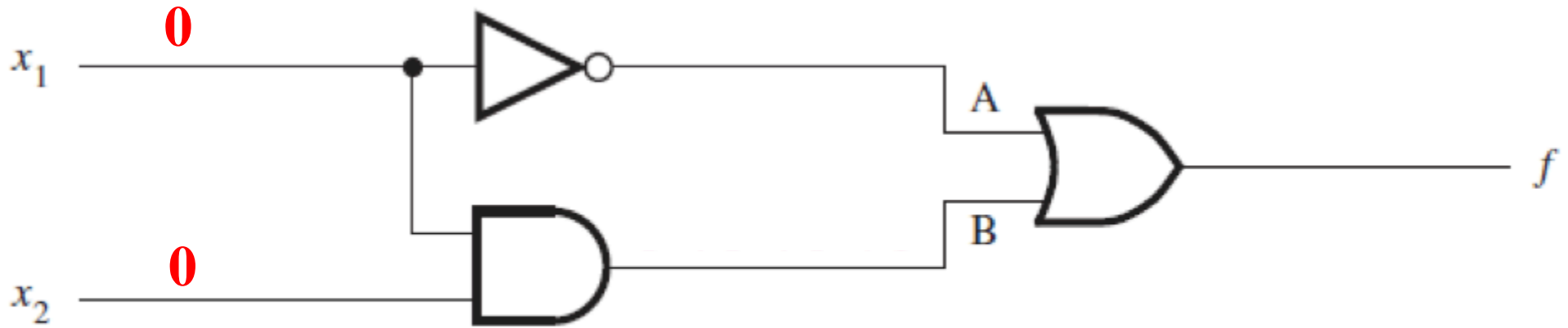


Circuit Analysis



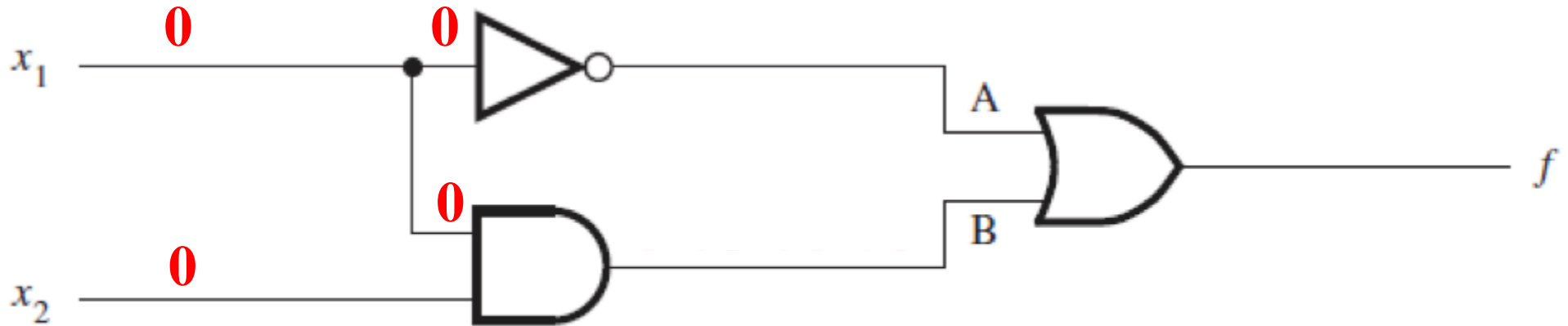
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



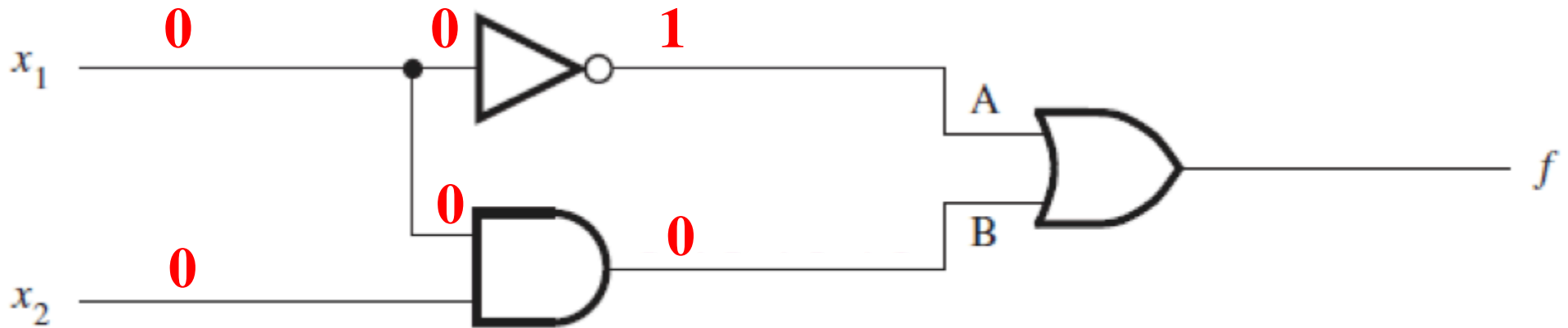
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



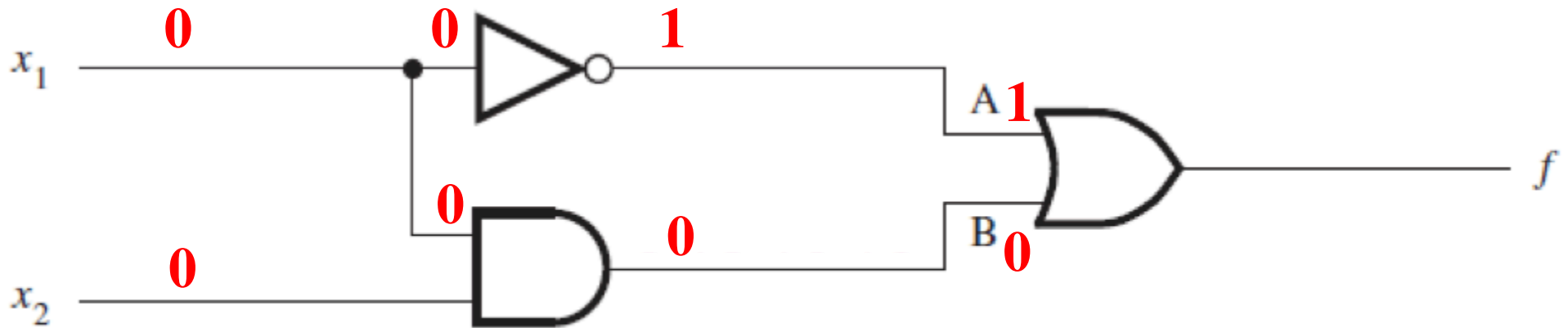
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



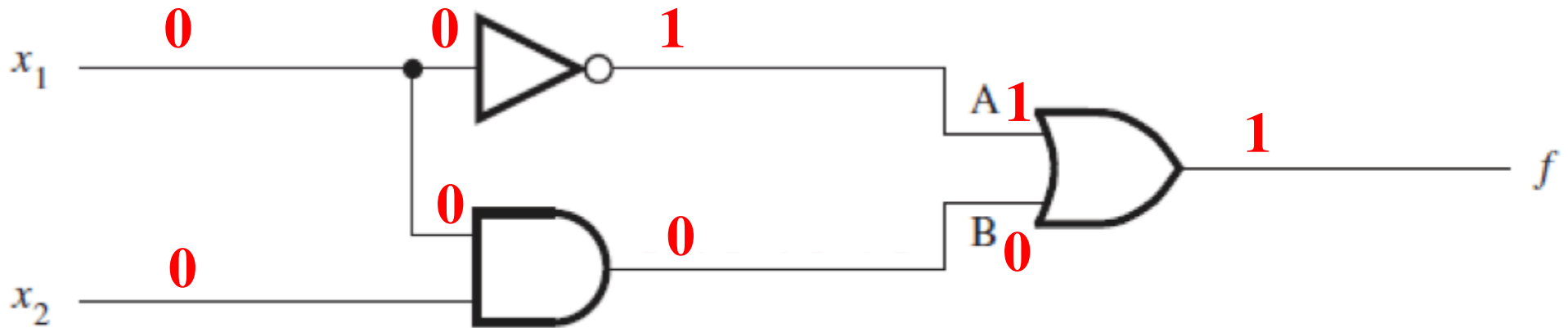
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



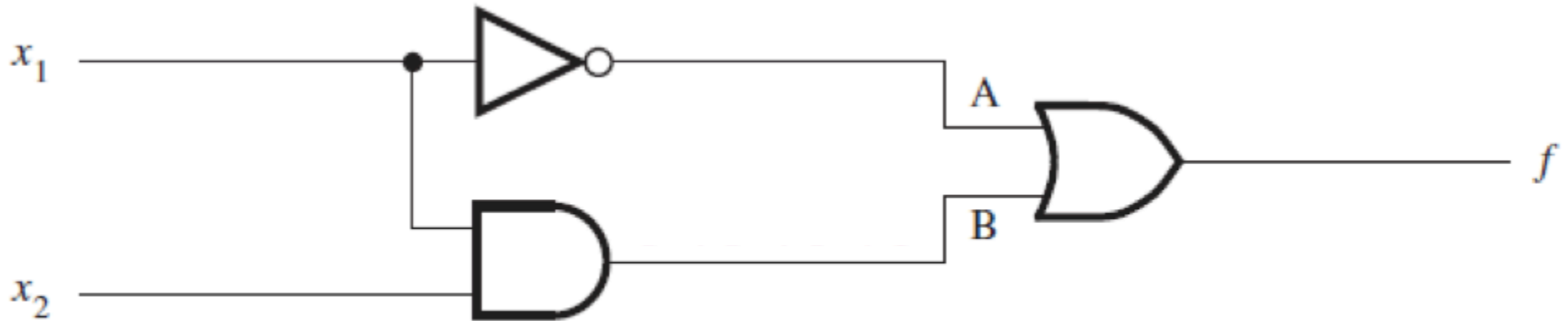
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



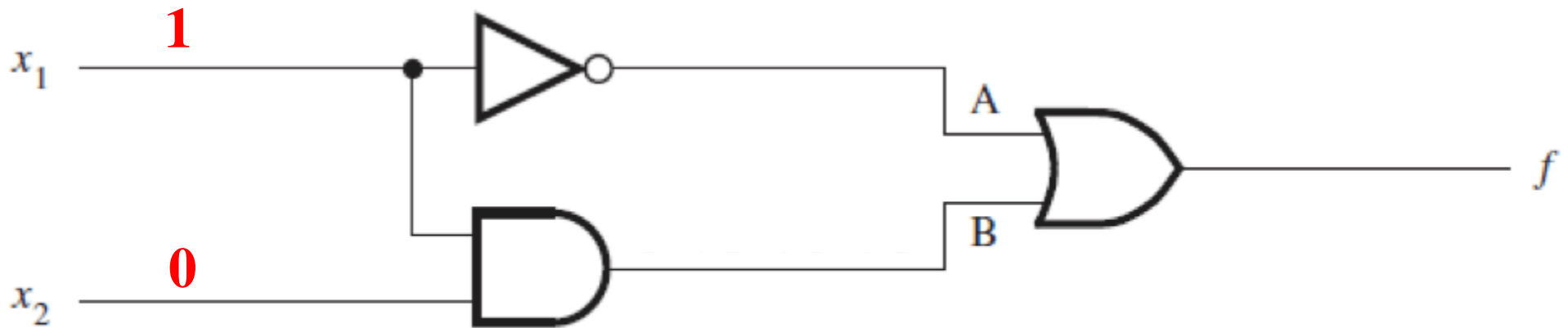
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



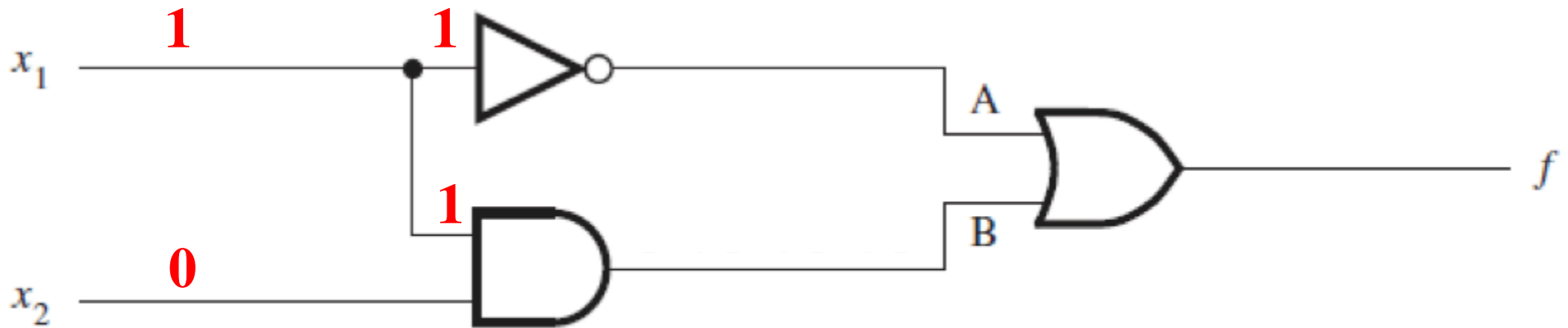
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



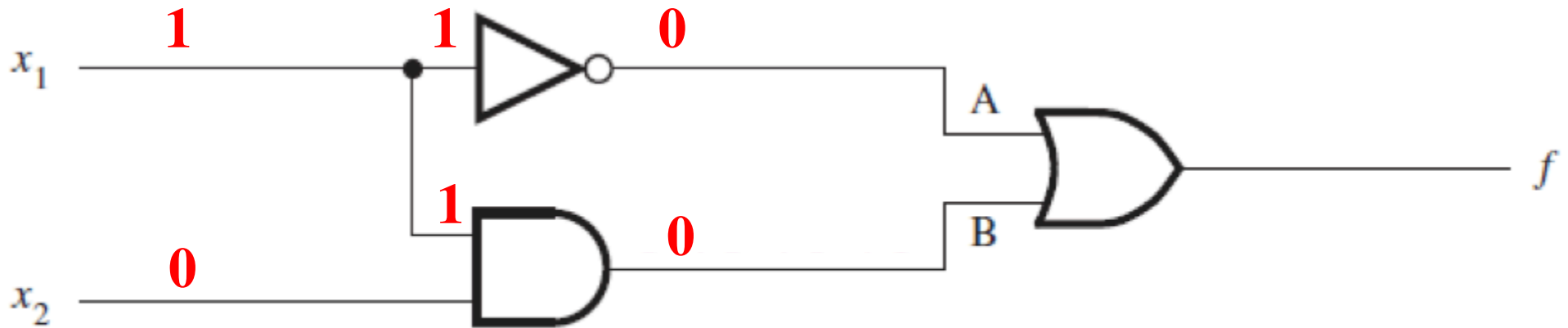
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



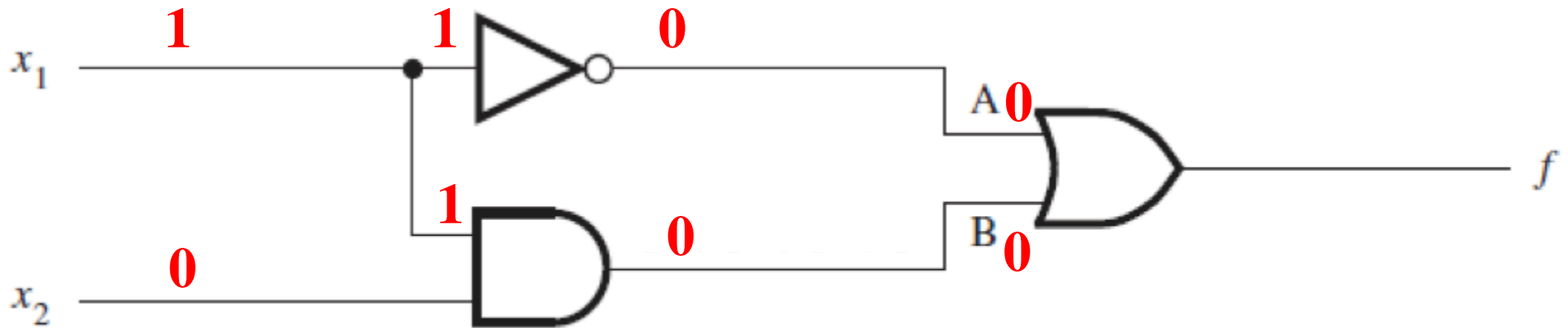
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



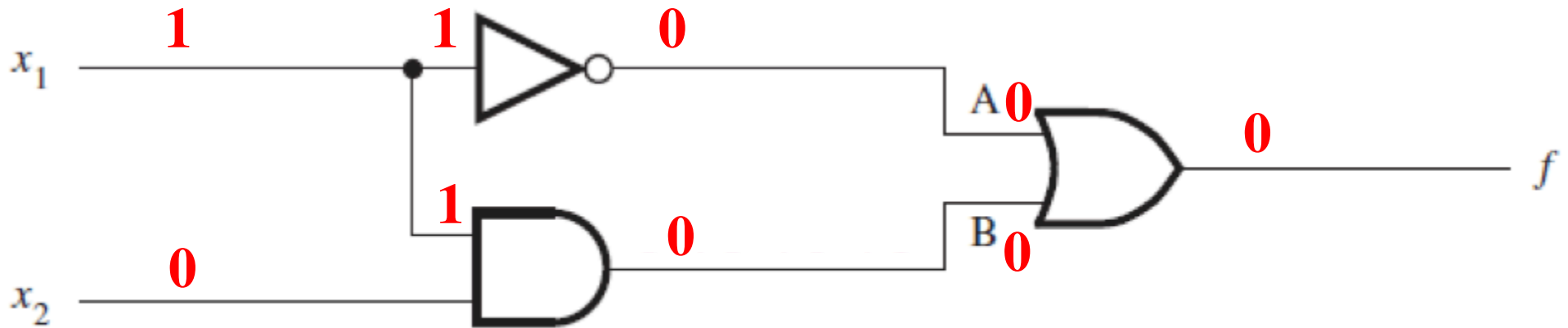
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



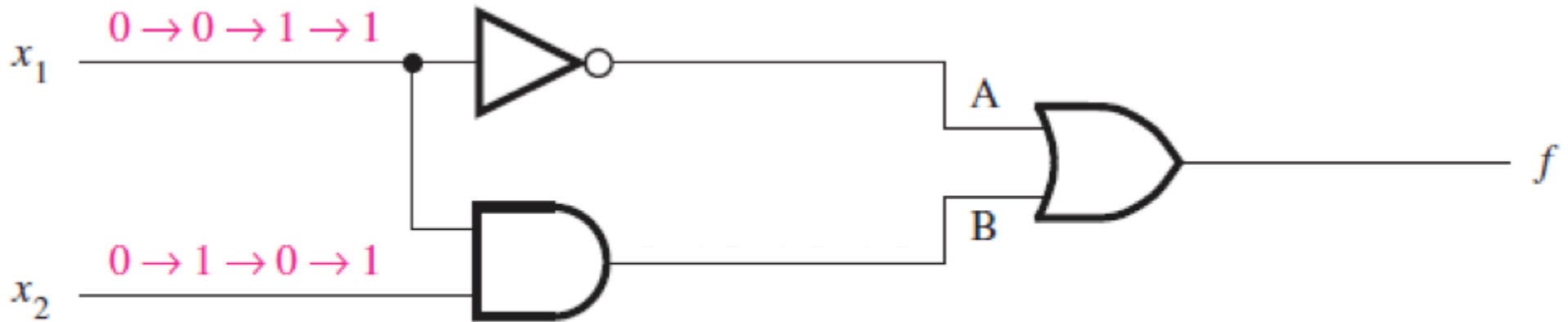
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis



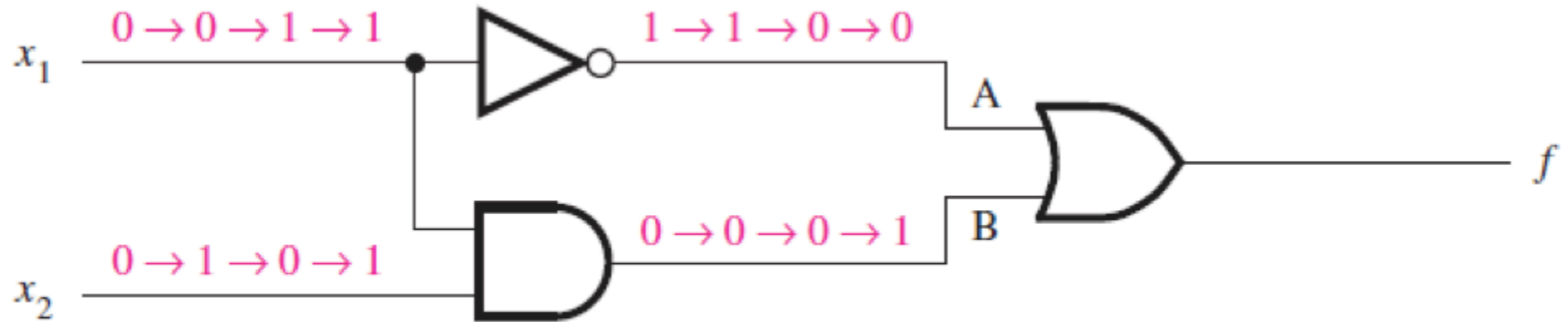
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis with Sequential Inputs



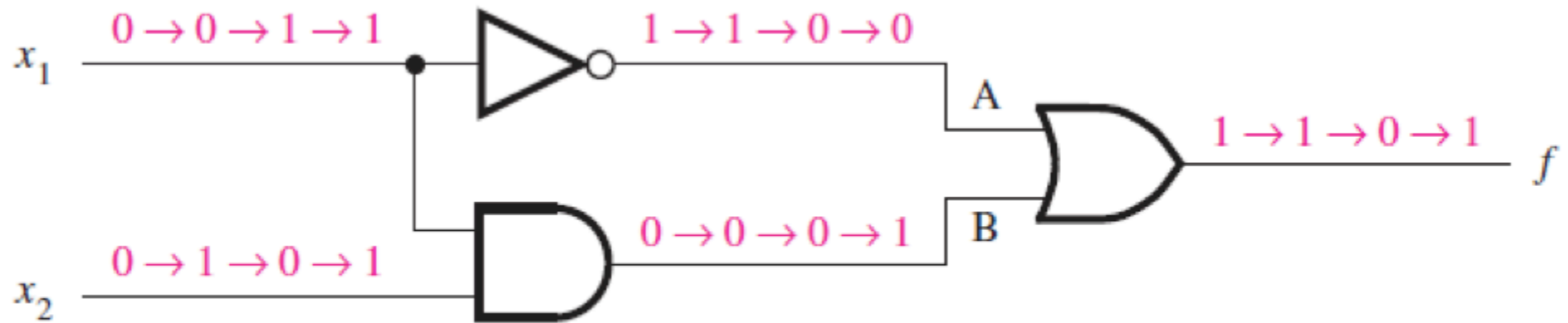
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis with Sequential Inputs



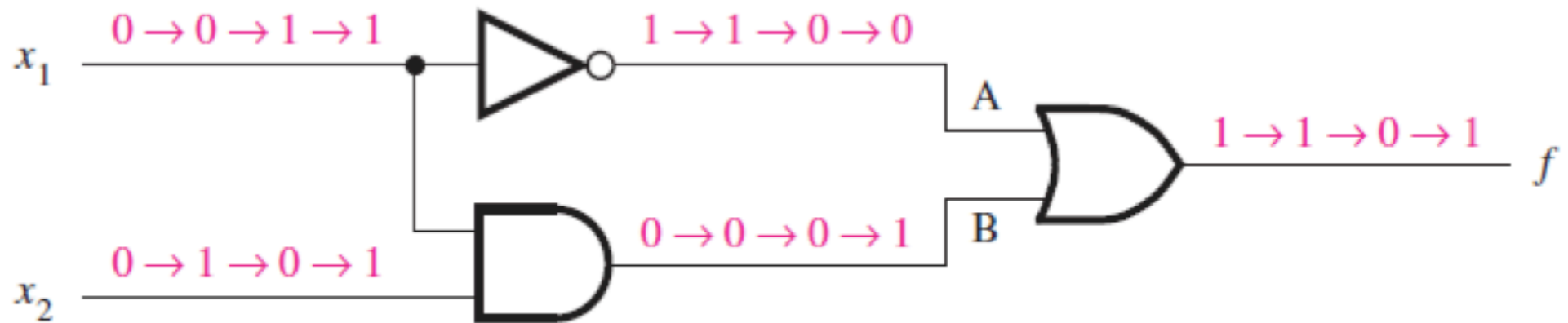
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Circuit Analysis with Sequential Inputs

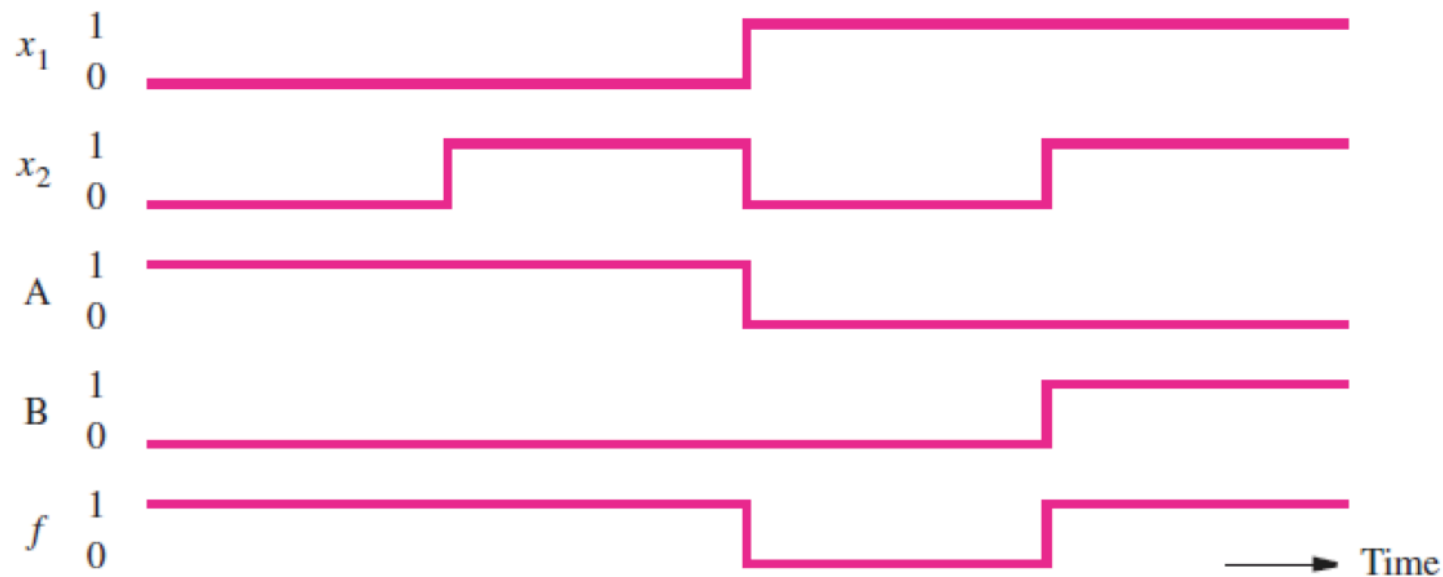


(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

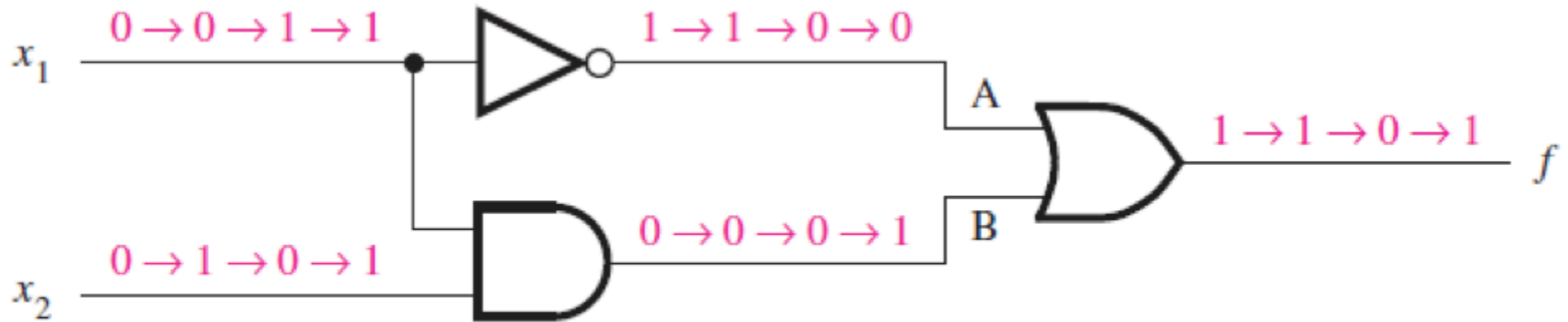
Circuit Analysis with Sequential Inputs



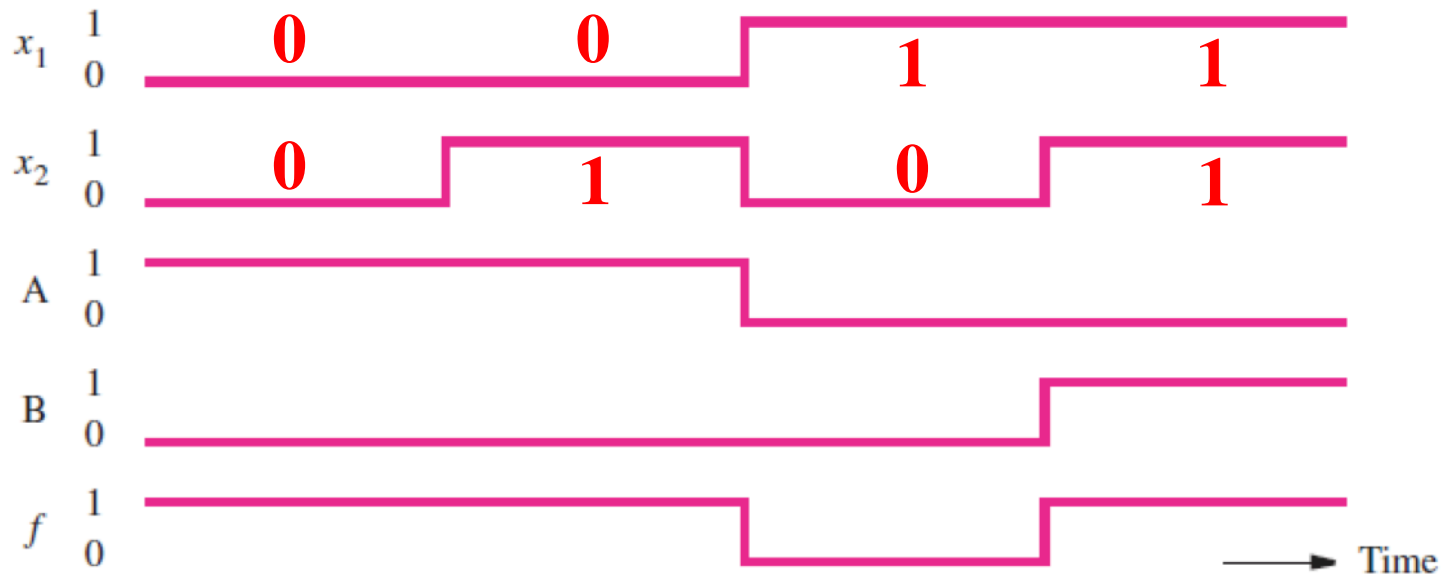
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$



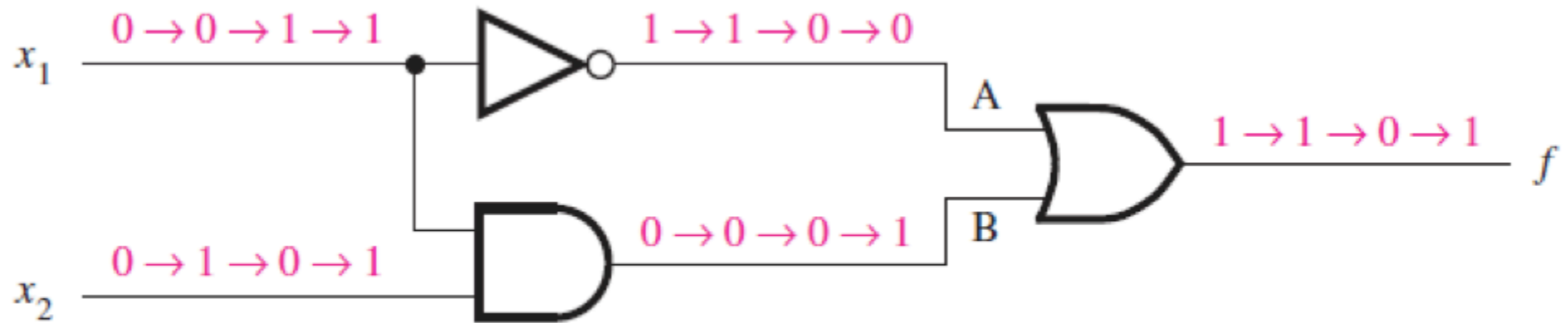
Circuit Analysis with Sequential Inputs



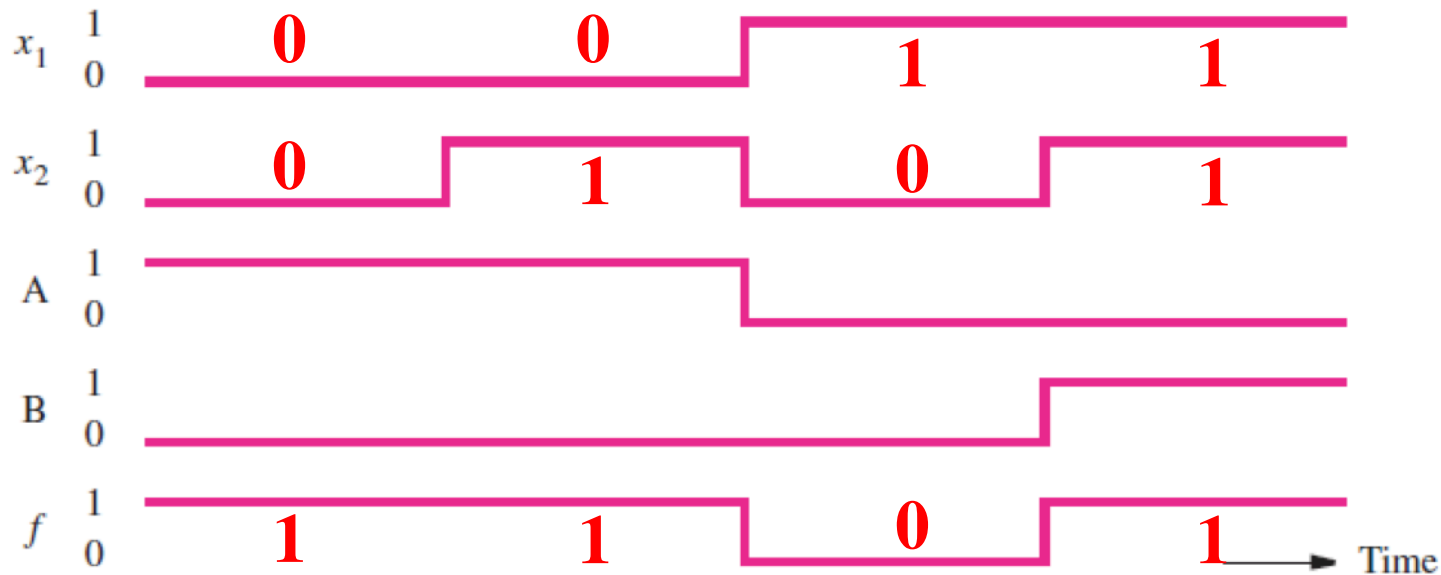
(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$



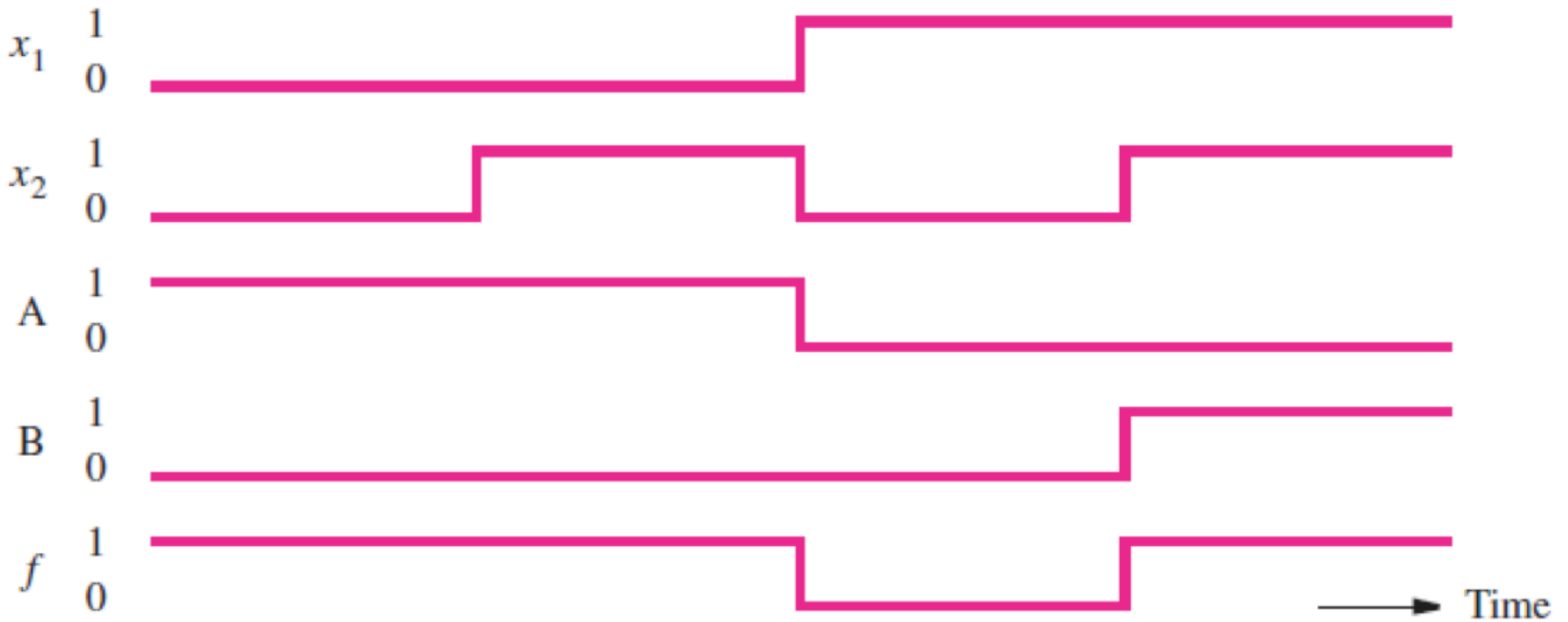
Circuit Analysis with Sequential Inputs



(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$



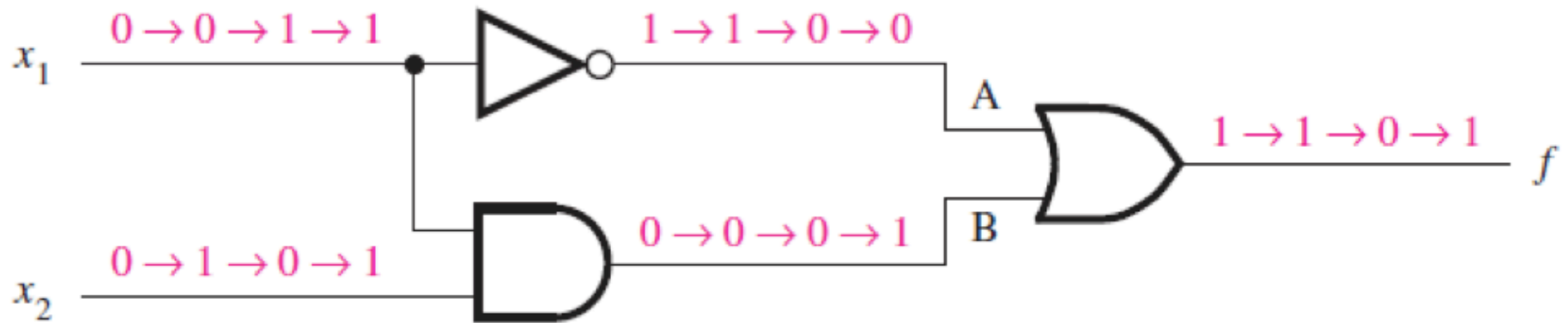
Timing Diagram



Truth Table for this Logic Circuit

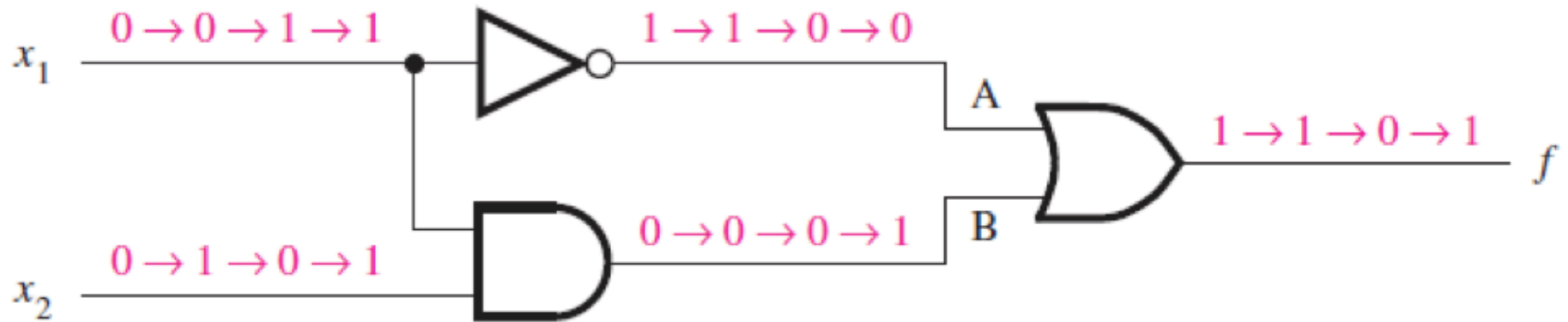
| x_1 | x_2 | $f(x_1, x_2)$ | A | B |
|-------|-------|---------------|---|---|
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

Functionally Equivalent Circuits

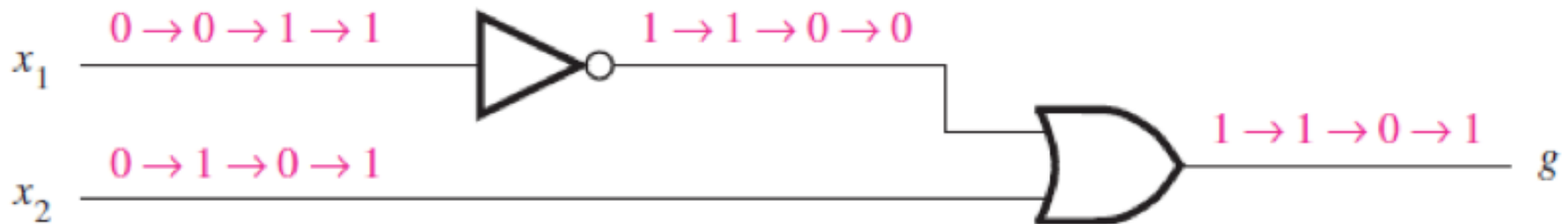


(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$

Functionally Equivalent Circuits

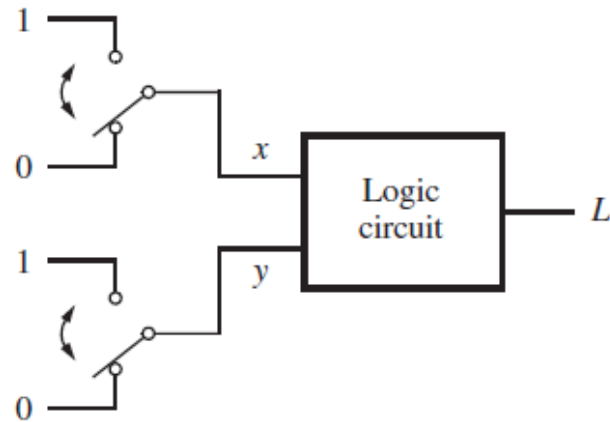


(a) Network that implements $f = \bar{x}_1 + x_1 \cdot x_2$



(d) Network that implements $g = \bar{x}_1 + x_2$

The XOR Logic Gate

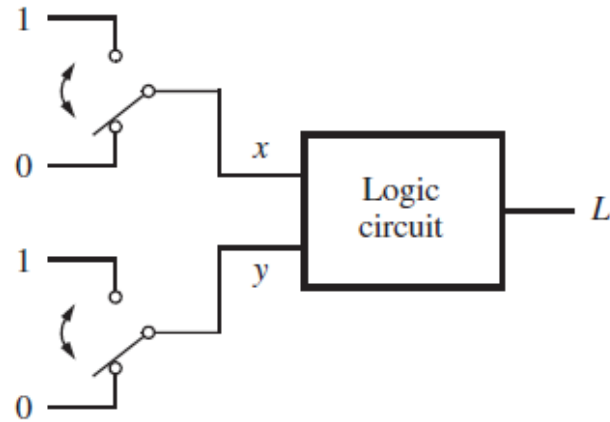


(a) Two switches that control a light

| x | y | L |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(b) Truth table

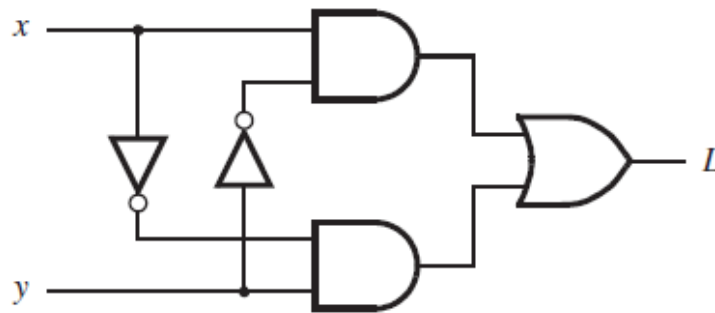
The XOR Logic Gate



(a) Two switches that control a light

| x | y | L |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(b) Truth table

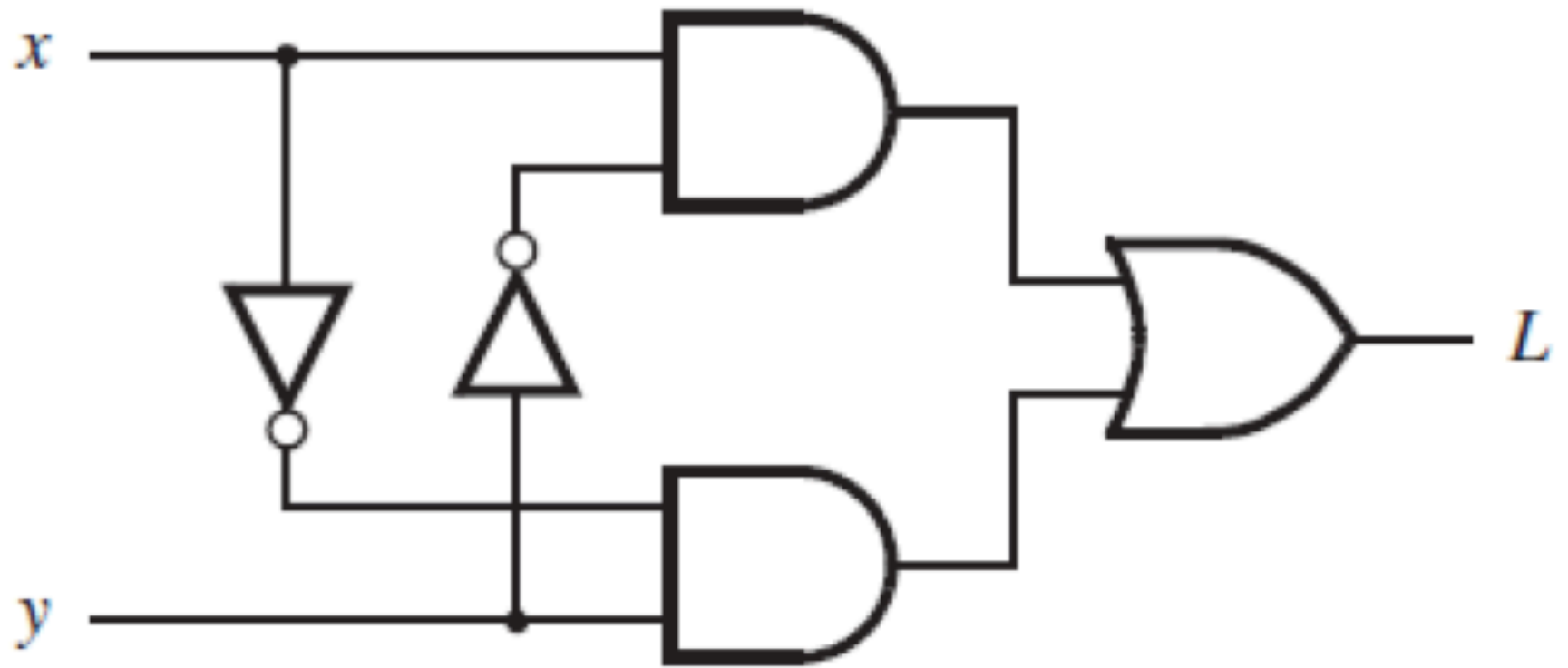


(c) Logic network



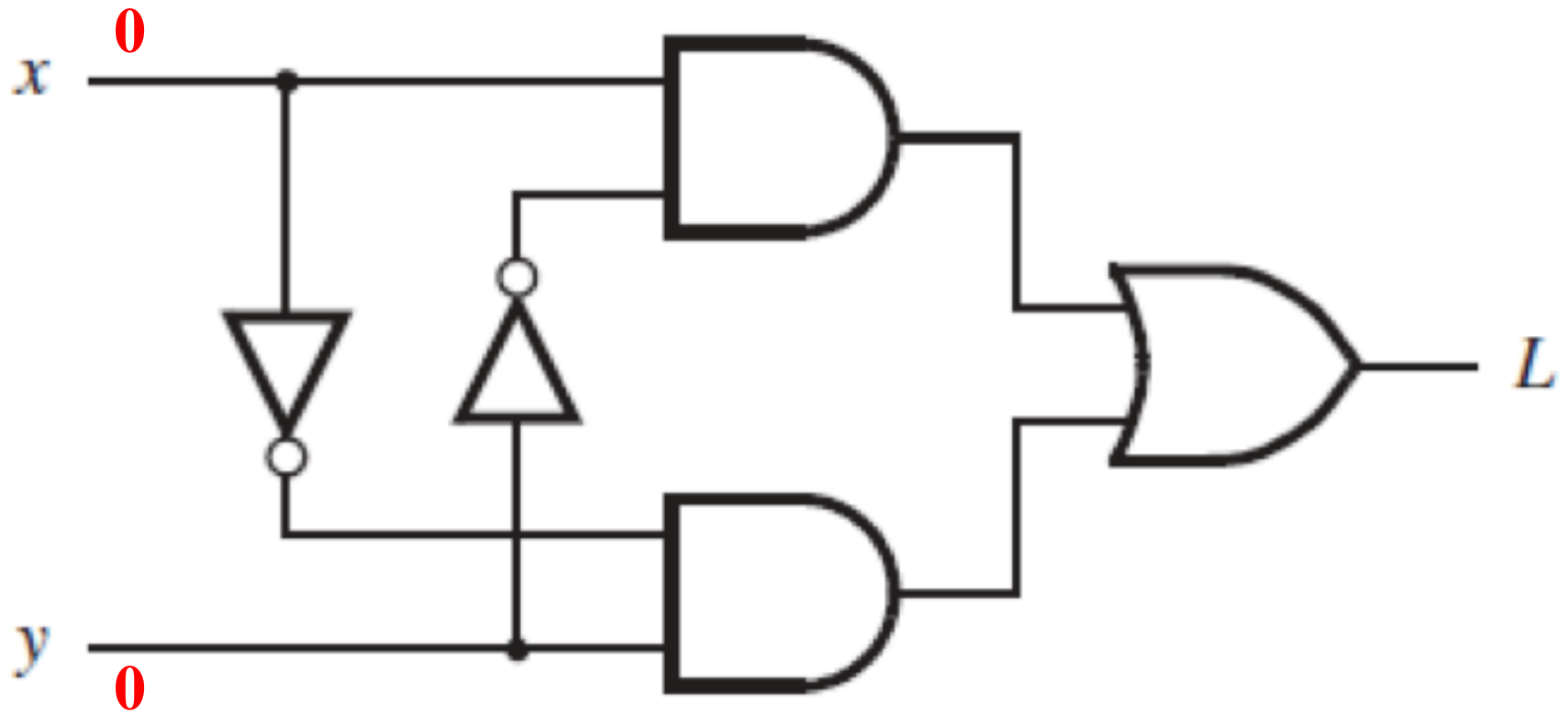
(d) XOR gate symbol

XOR Analysis

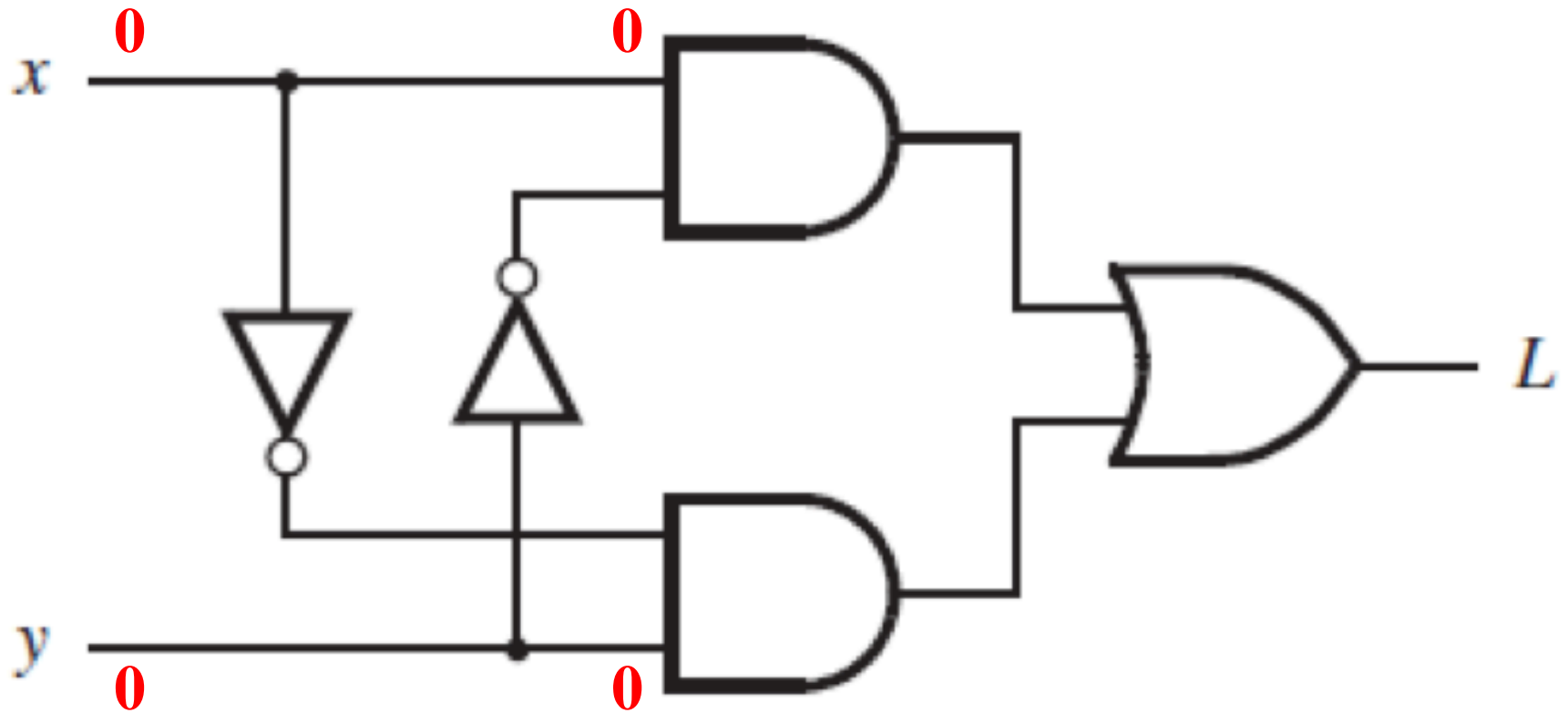


[Figure 2.11c from the textbook]

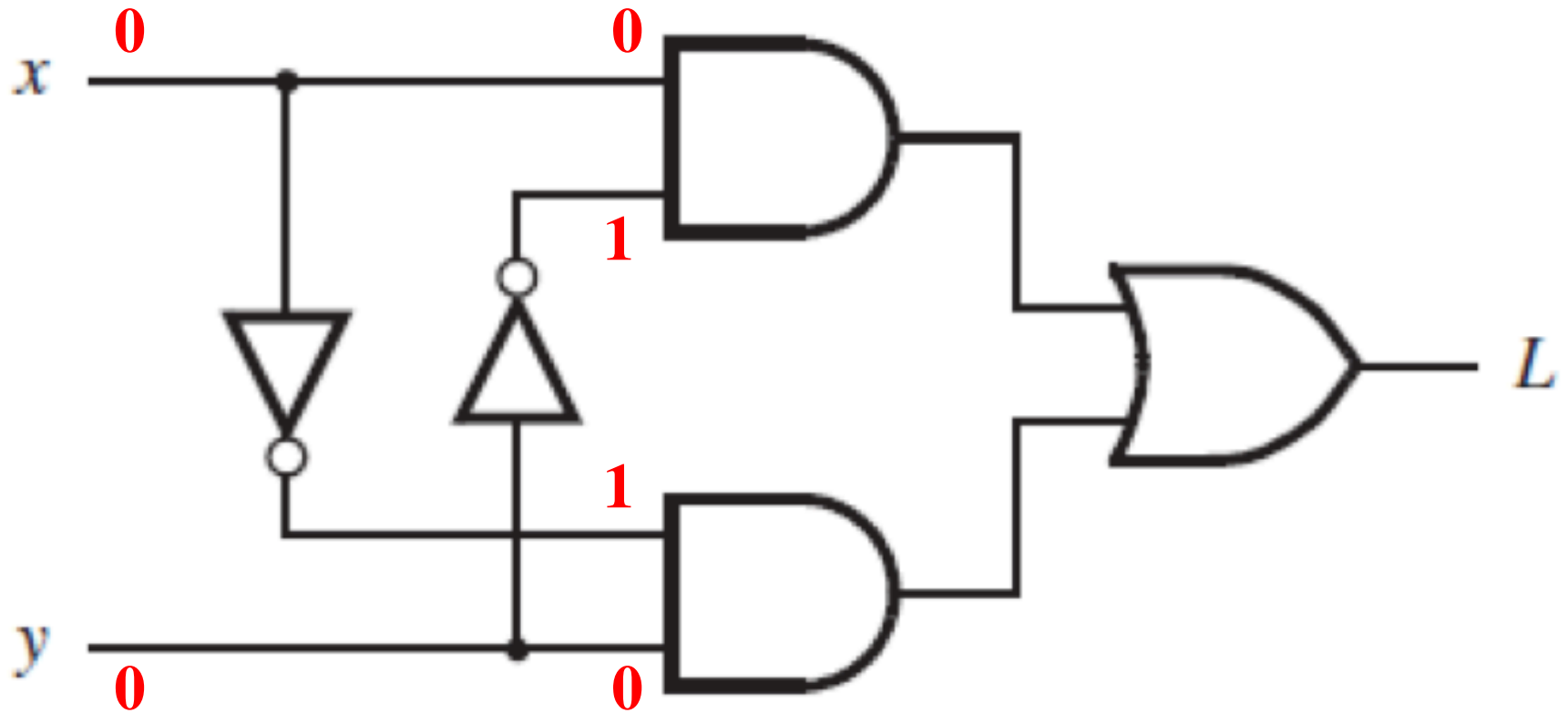
XOR Analysis (x=0, y=0)



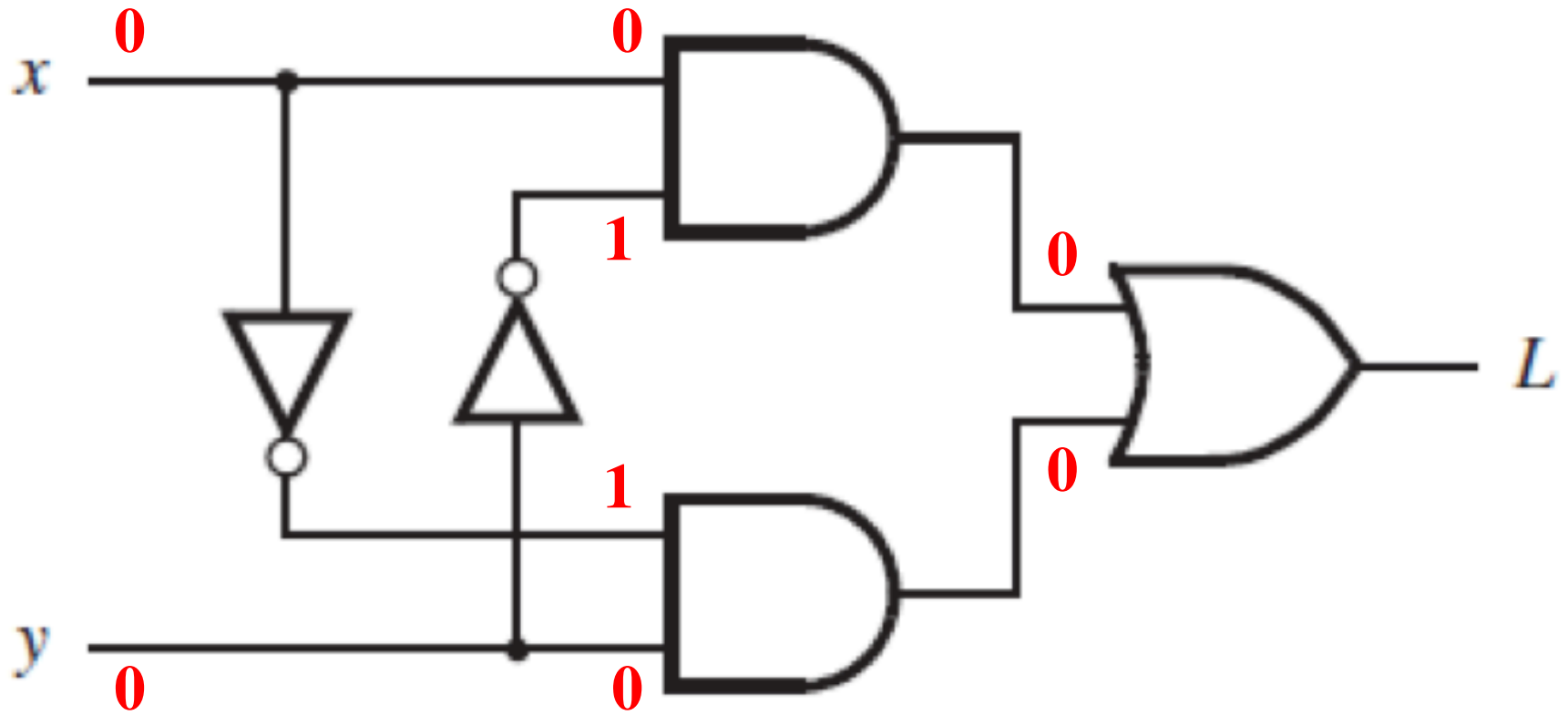
XOR Analysis (x=0, y=0)



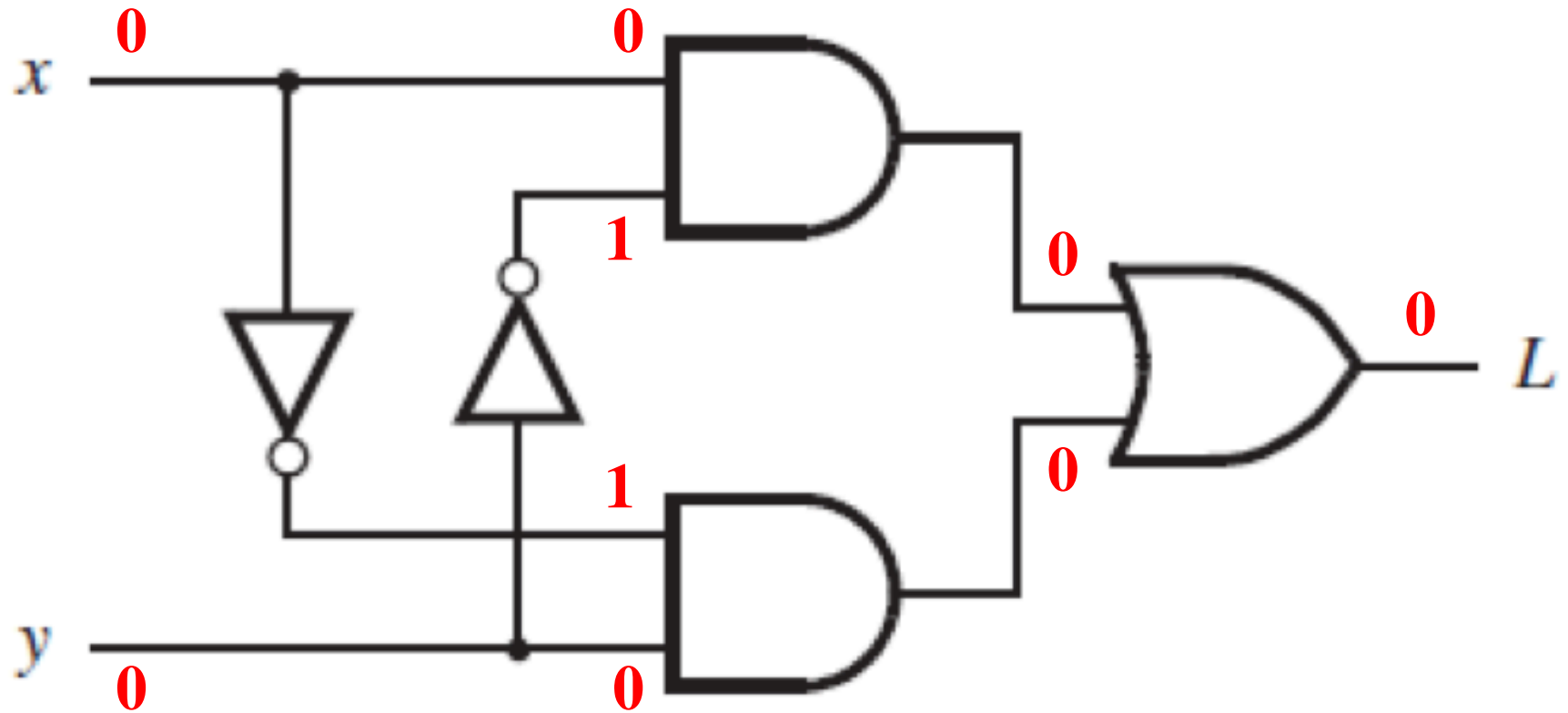
XOR Analysis (x=0, y=0)



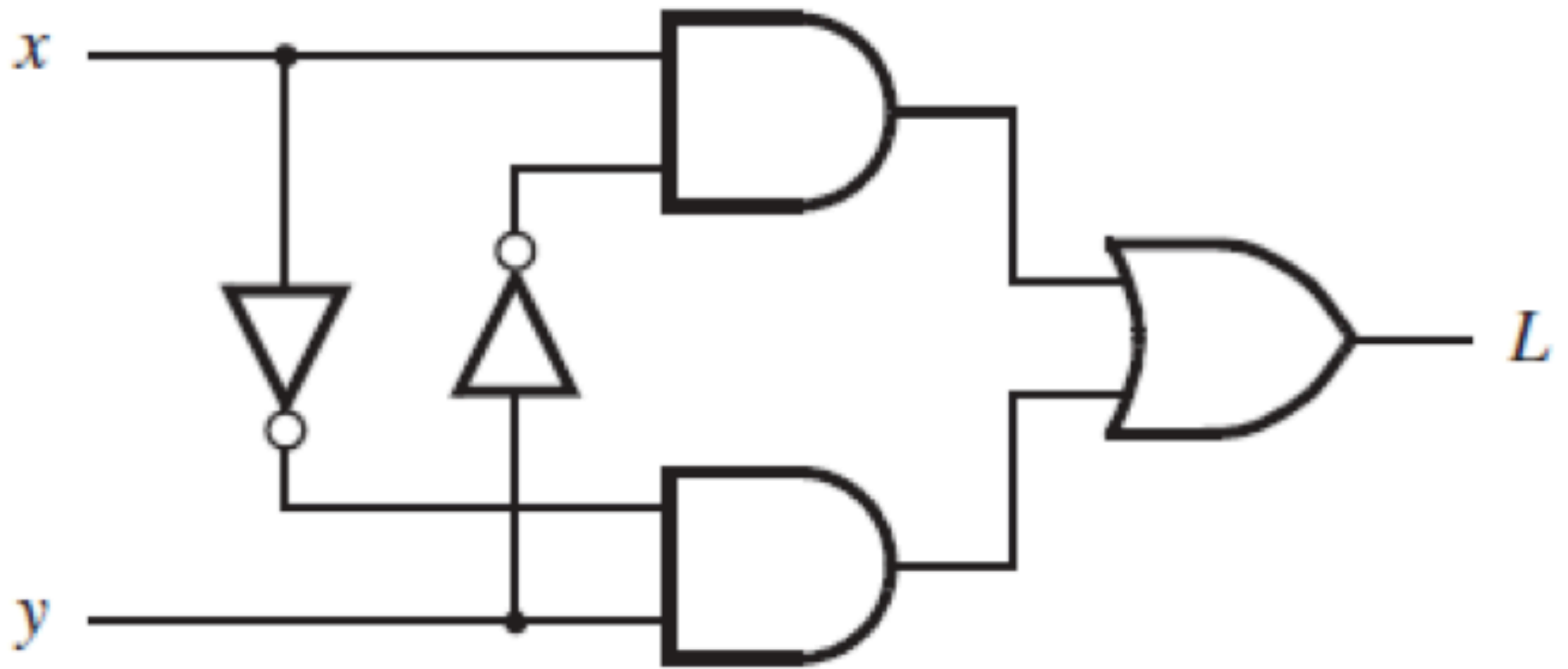
XOR Analysis (x=0, y=0)



XOR Analysis (x=0, y=0)

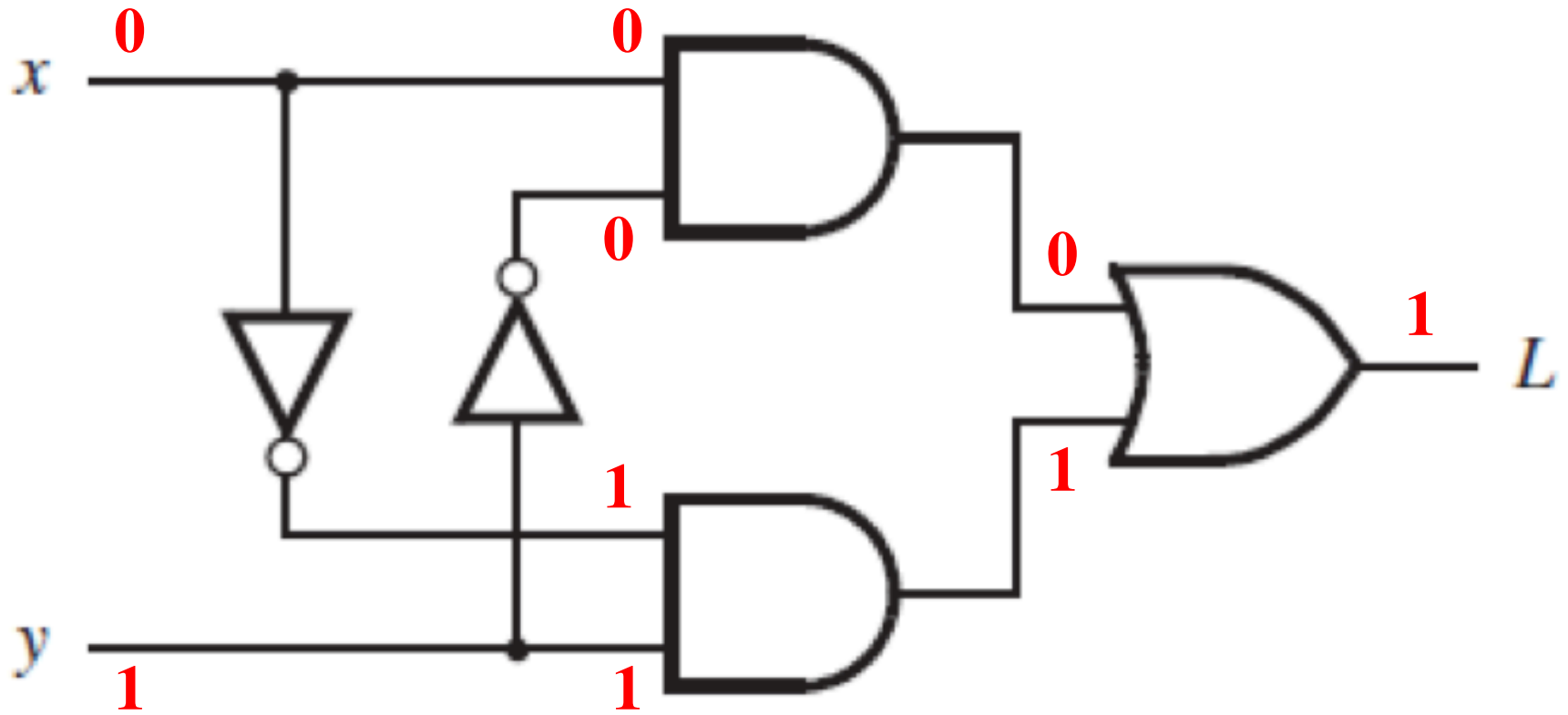


XOR Analysis

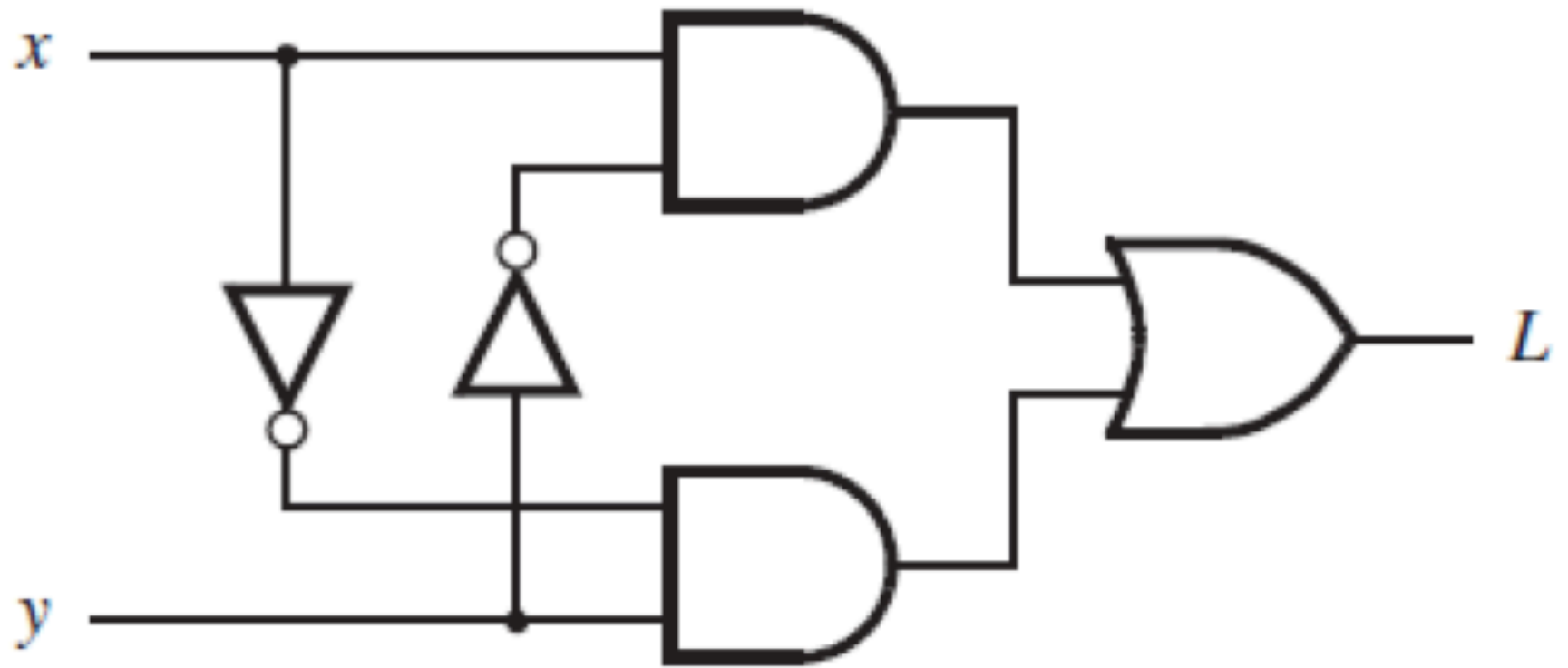


[Figure 2.11c from the textbook]

XOR Analysis (x=0, y=1)

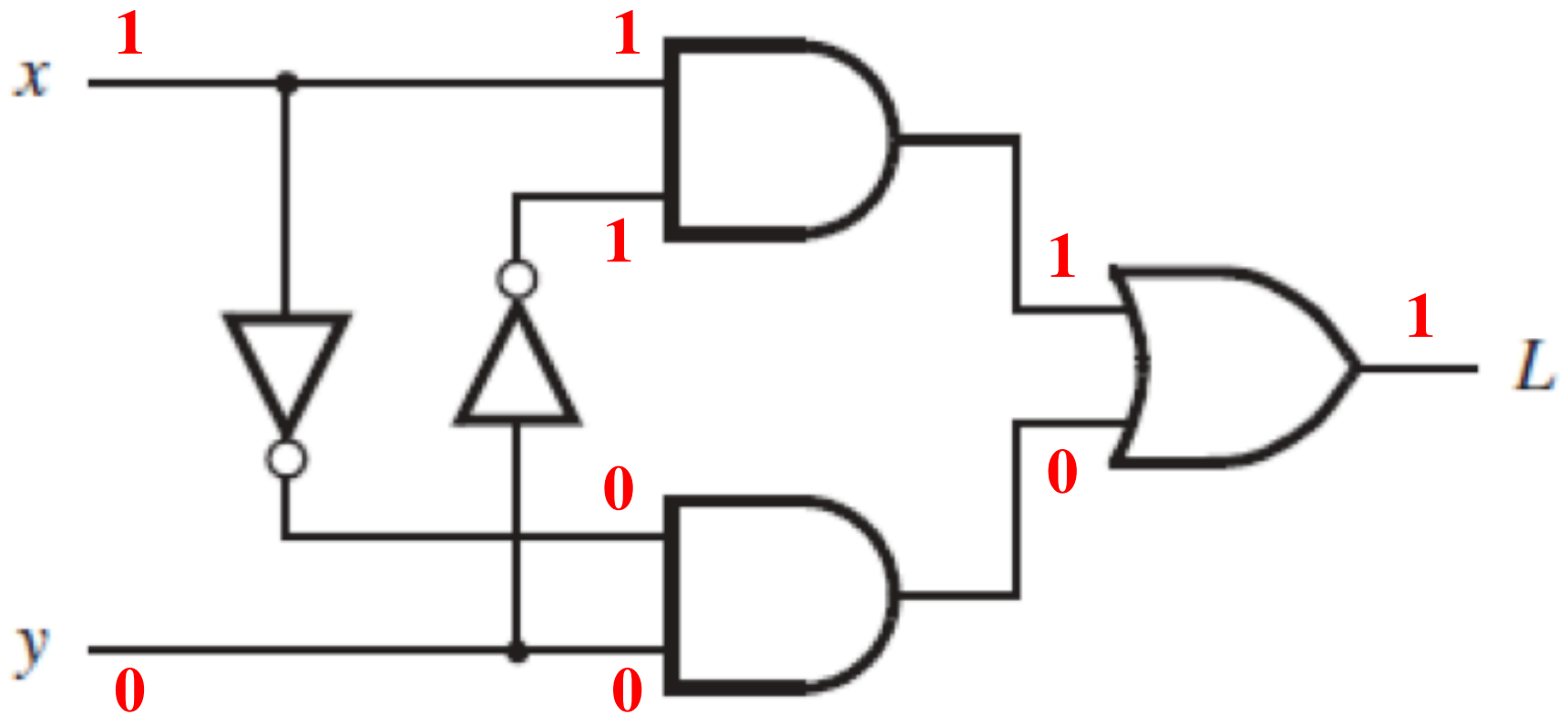


XOR Analysis

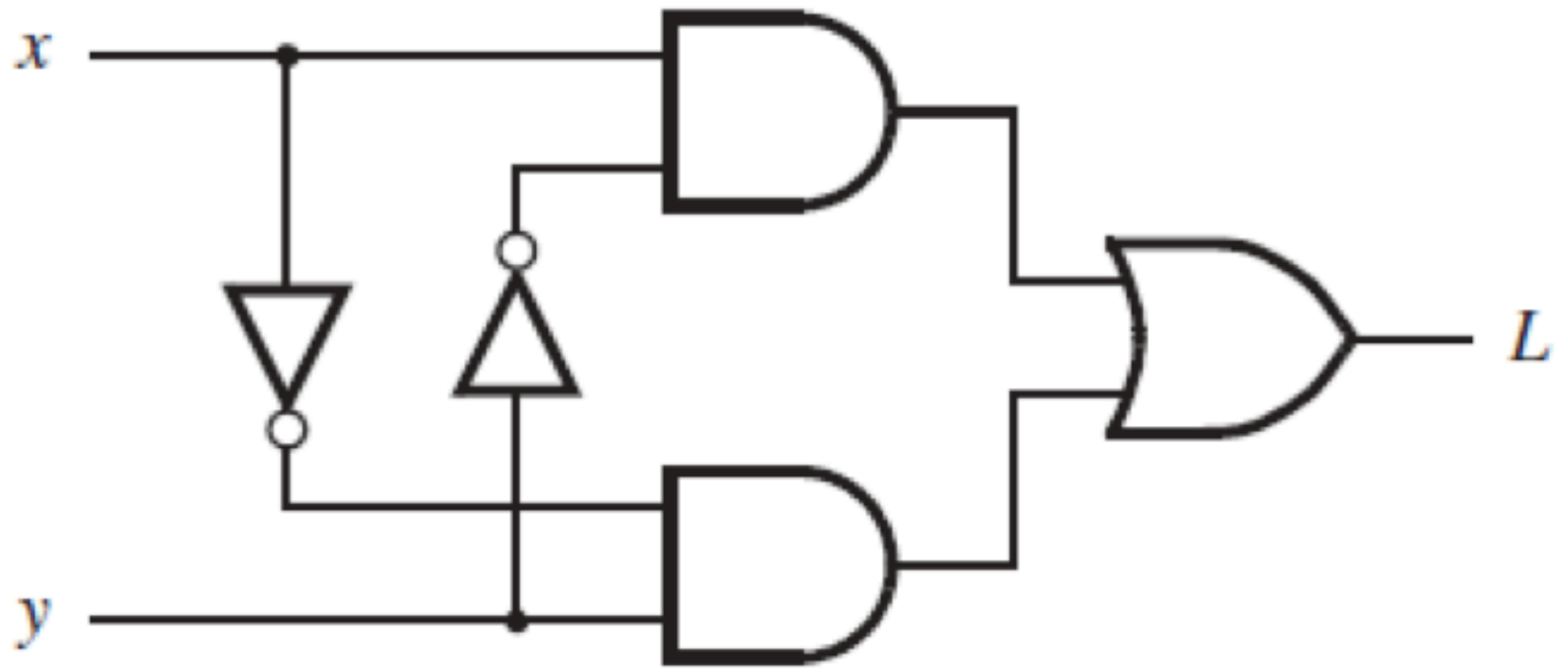


[Figure 2.11c from the textbook]

XOR Analysis (x=1, y=0)

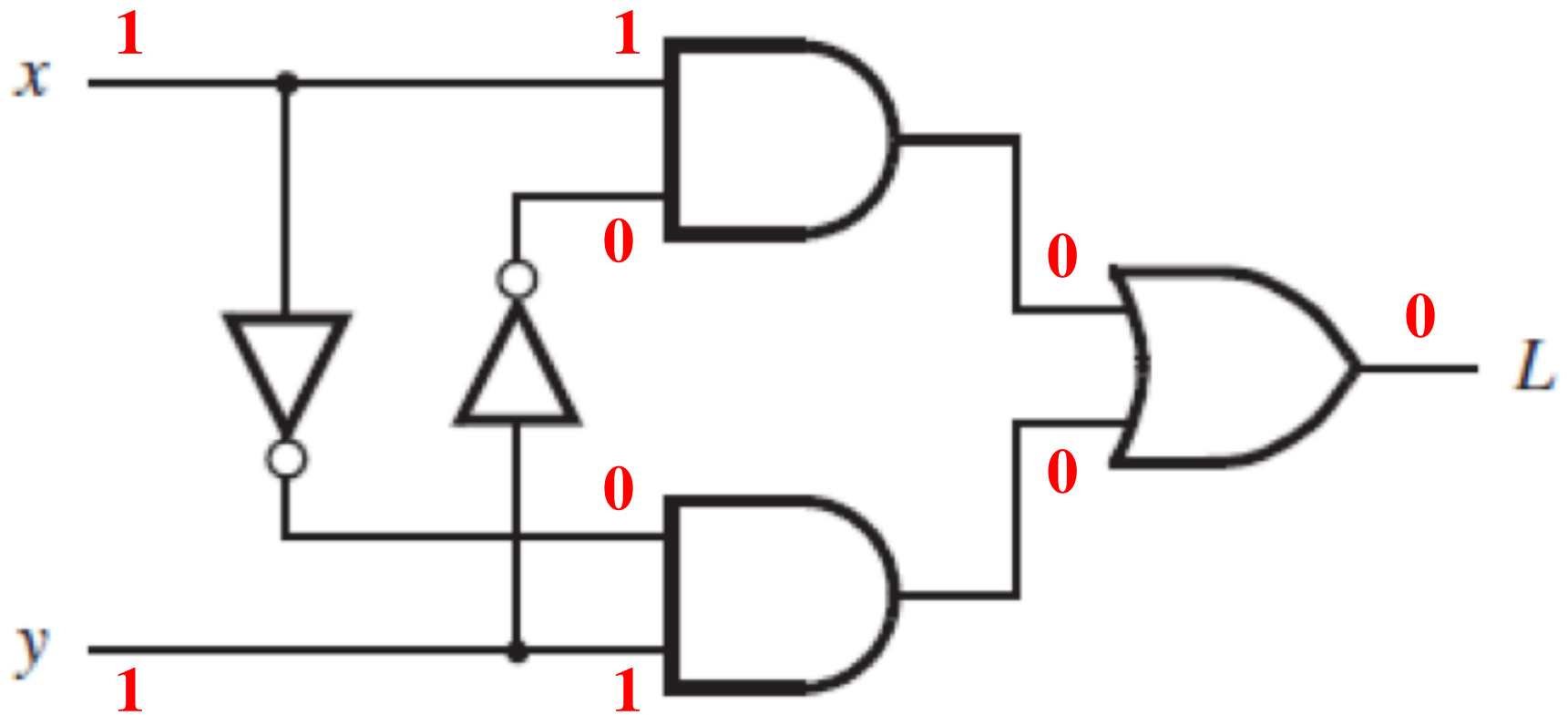


XOR Analysis



[Figure 2.11c from the textbook]

XOR Analysis (x=1, y=1)

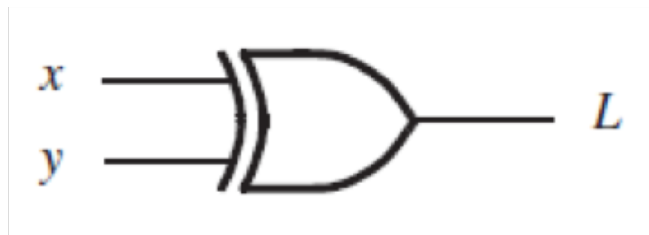


Truth Table for XOR



| x | y | L |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

Truth Table for XOR



| x | y | L |
|-----|-----|-----|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

The output is 1 only if both inputs are different.

Addition of Binary Numbers

| | | | | |
|-----------|-------|-------|-------|-------|
| a | 0 | 0 | 1 | 1 |
| $+b$ | $+0$ | $+1$ | $+0$ | $+1$ |
| <hr/> | <hr/> | <hr/> | <hr/> | <hr/> |
| $s_1 s_0$ | 0 0 | 0 1 | 0 1 | 1 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| | | | | |
|---|------------|------------|------------|------------|
| <i>a</i> | 0 | 0 | 1 | 1 |
| <u>+ <i>b</i></u> | <u>+ 0</u> | <u>+ 1</u> | <u>+ 0</u> | <u>+ 1</u> |
| <i>s</i> ₁ <i>s</i> ₀ | 0 0 | 0 1 | 0 1 | 1 0 |

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| | | | | |
|-----------|----------|----------|----------|----------|
| a | 0 | 0 | 1 | 1 |
| $+b$ | $+0$ | $+1$ | $+0$ | $+1$ |
| \hline | \hline | \hline | \hline | \hline |
| $s_1 s_0$ | 0 0 | 0 1 | 0 1 | 1 0 |

| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| | | | | |
|---|------------|------------|------------|------------|
| <i>a</i> | 0 | 0 | 1 | 1 |
| <u>+ <i>b</i></u> | <u>+ 0</u> | <u>+ 1</u> | <u>+ 0</u> | <u>+ 1</u> |
| <i>s</i> ₁ <i>s</i> ₀ | 0 0 | 0 1 | 0 1 | 1 0 |

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| | | | | |
|-----------|------|------|------|------|
| a | 0 | 0 | 1 | 1 |
| $+b$ | $+0$ | $+1$ | $+0$ | $+1$ |
| $s_1 s_0$ | 0 0 | 0 1 | 0 1 | 1 0 |

| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 \ 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 \ 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 \ 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 \ 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 \boxed{s_0} \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 \boxed{0} \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 \boxed{1} \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 \boxed{1} \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 \boxed{0} \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

$$\begin{array}{r} a \\ + b \\ \hline s_1 s_0 \end{array} \quad \begin{array}{r} 0 \\ + 0 \\ \hline 0 0 \end{array} \quad \begin{array}{r} 0 \\ + 1 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 0 \\ \hline 0 1 \end{array} \quad \begin{array}{r} 1 \\ + 1 \\ \hline 1 0 \end{array}$$

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

?

| <i>a</i> | <i>b</i> | | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|--|-----------------------|-----------------------|
| 0 | 0 | | 0 | 0 |
| 0 | 1 | | 0 | 1 |
| 1 | 0 | | 0 | 1 |
| 1 | 1 | | 1 | 0 |

Addition of Binary Numbers

AND

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

?

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

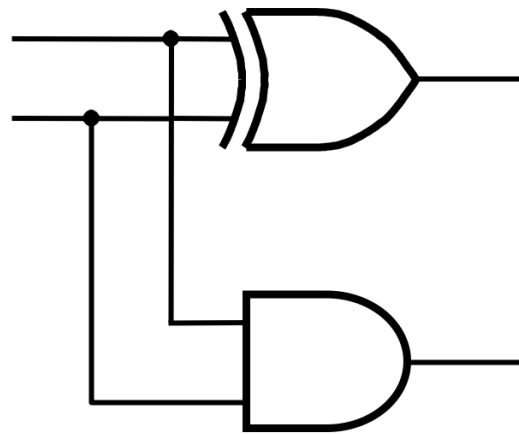
XOR

| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers

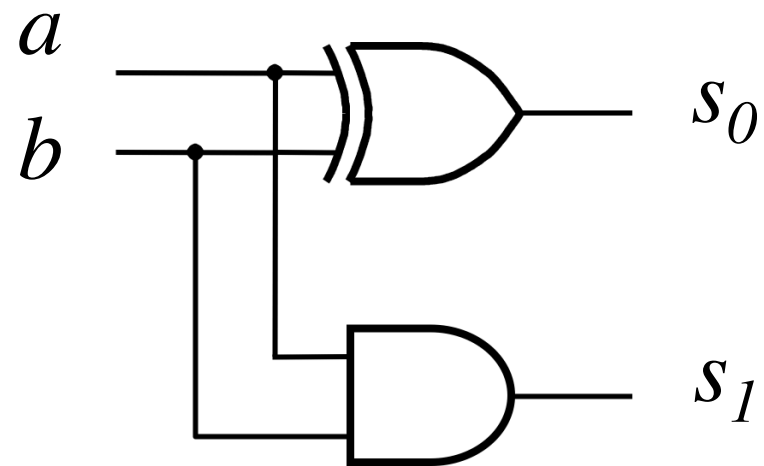
| <i>a</i> | <i>b</i> | <i>s</i> ₁ | <i>s</i> ₀ |
|----------|----------|-----------------------|-----------------------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

Addition of Binary Numbers



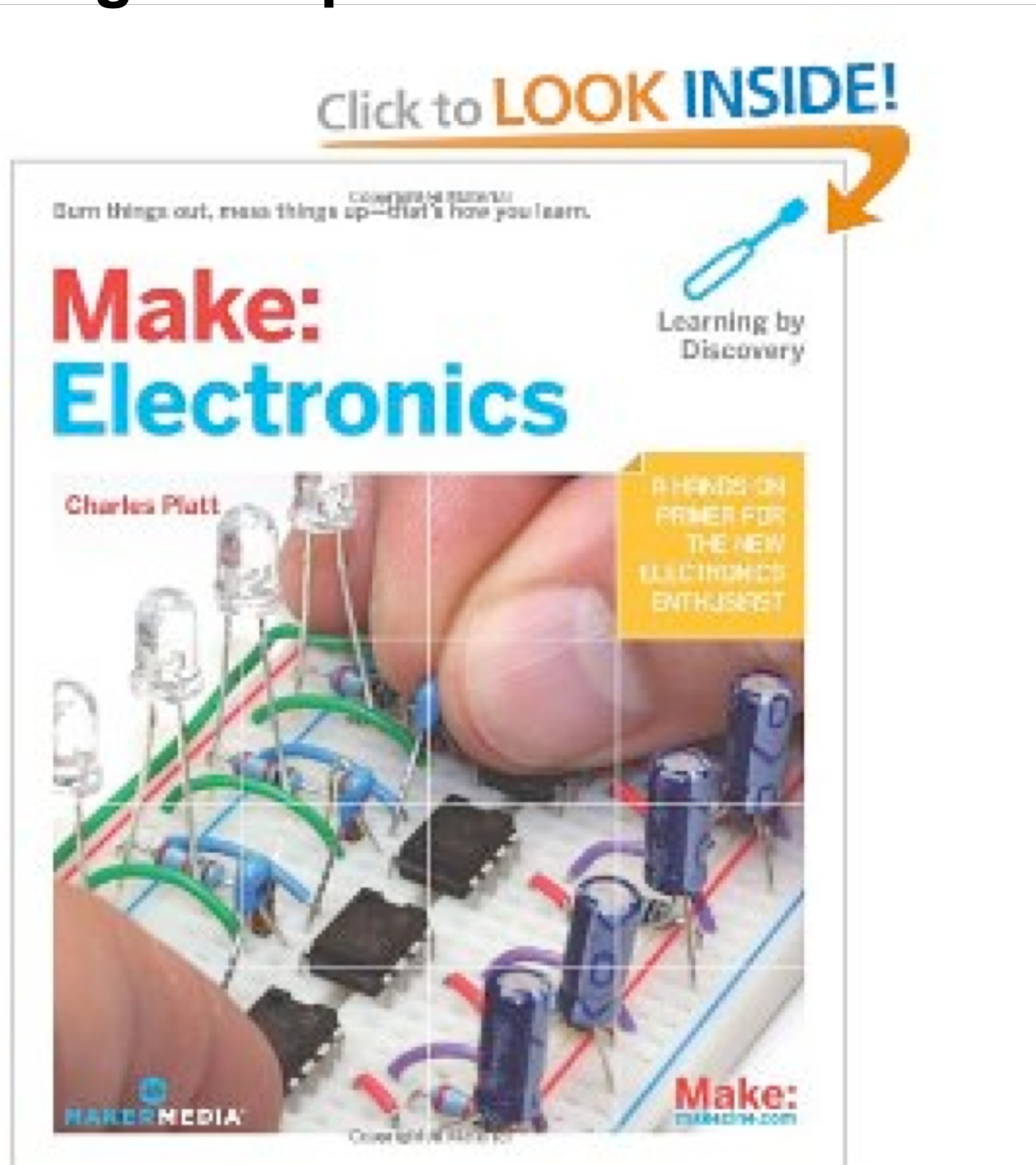
| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

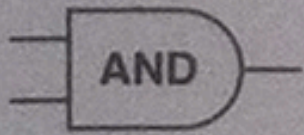
Addition of Binary Numbers



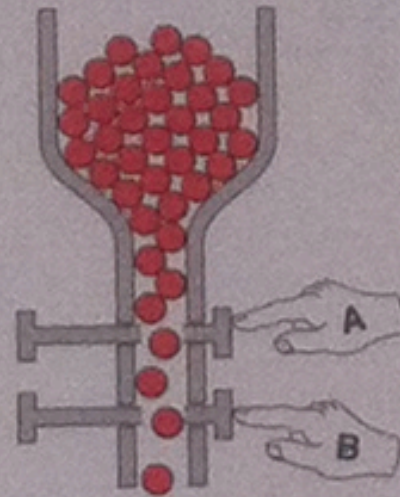
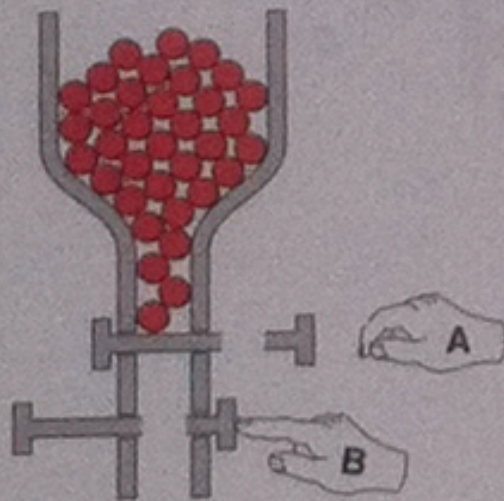
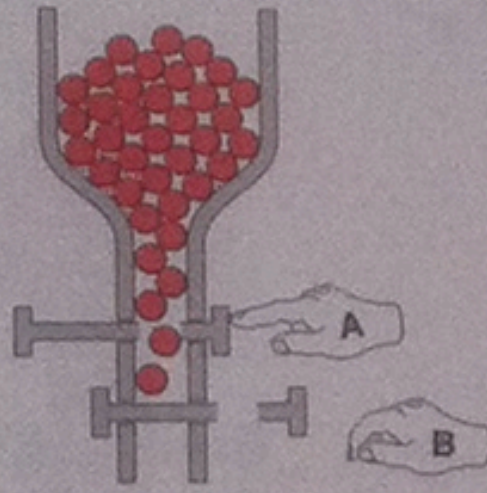
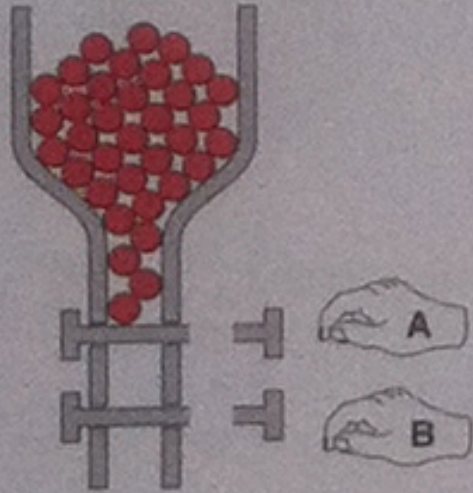
| a | b | s_1 | s_0 |
|-----|-----|-------|-------|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 |

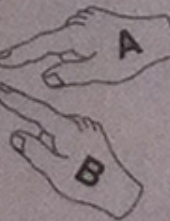
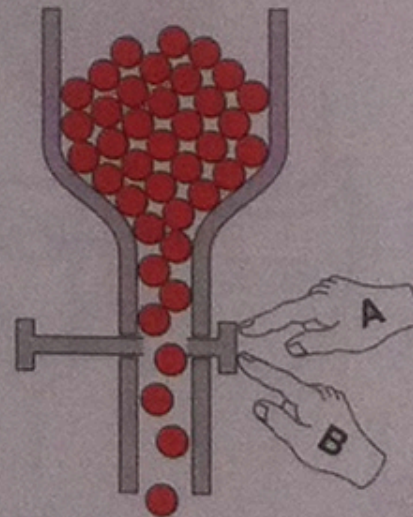
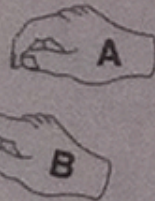
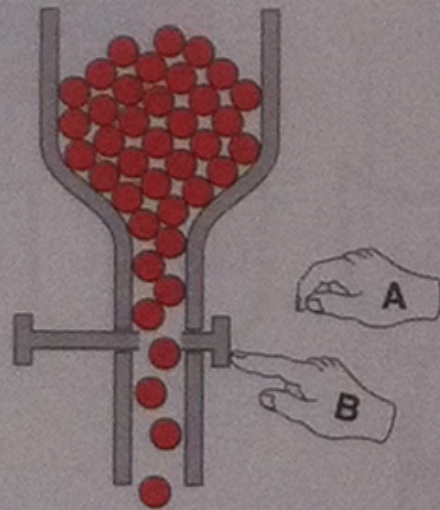
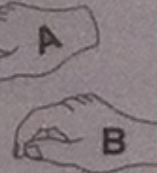
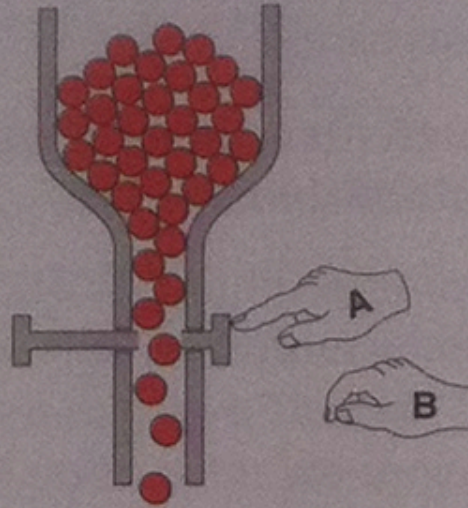
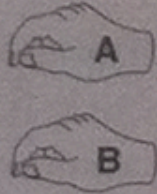
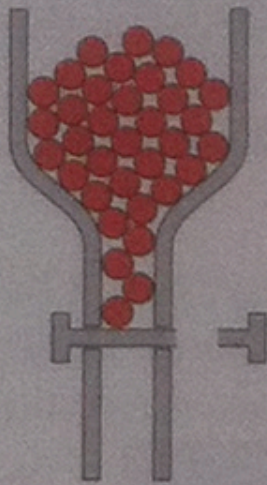
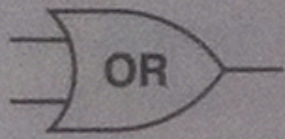
The following examples came from this book



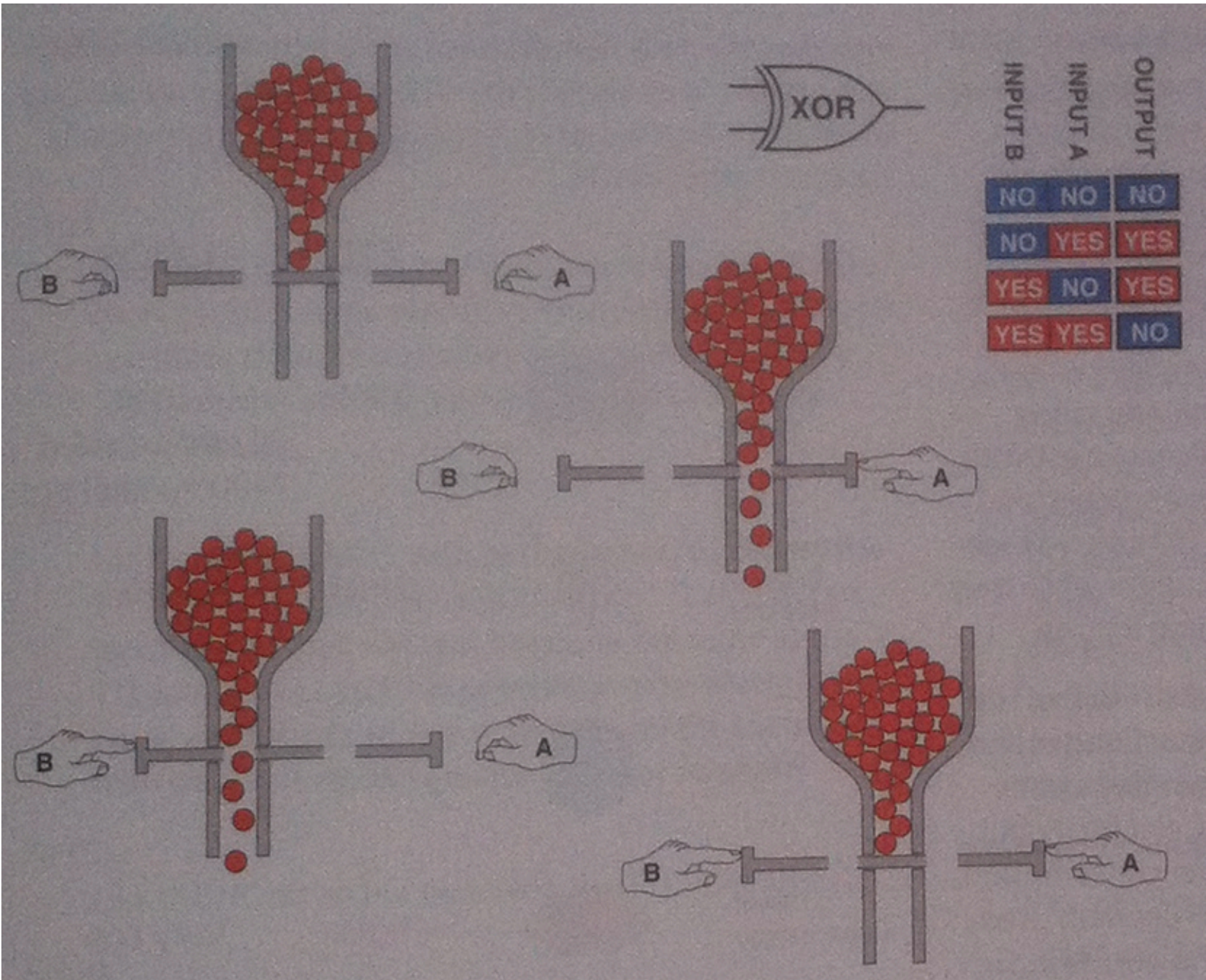


| INPUT B | INPUT A | OUTPUT |
|---------|---------|--------|
| NO | NO | NO |
| NO | YES | NO |
| YES | NO | NO |
| YES | YES | YES |





| INPUT B | INPUT A | OUTPUT |
|---------|---------|--------|
| NO | NO | NO |
| NO | YES | YES |
| YES | NO | YES |
| YES | YES | YES |



Questions?

THE END