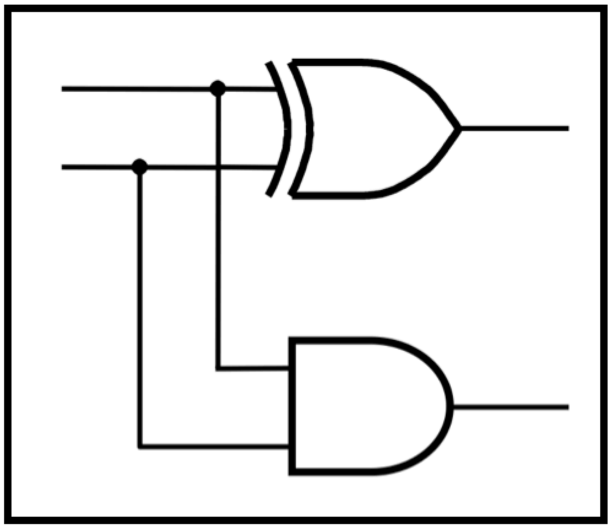


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Synthesis
Using AND, OR, and NOT Gates

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW2 is due on Monday Sep 9 @ 4pm

- Please write clearly on the first page (in block capital letters) the following three things:

  - Your First and Last Name
  - Your Student ID Number
  - Your Lab Section Letter

- If any of these are missing, then you will lose 10%
  - Staple all of your pages

- If this is not done, then you will lose 10%

Administrative Stuff

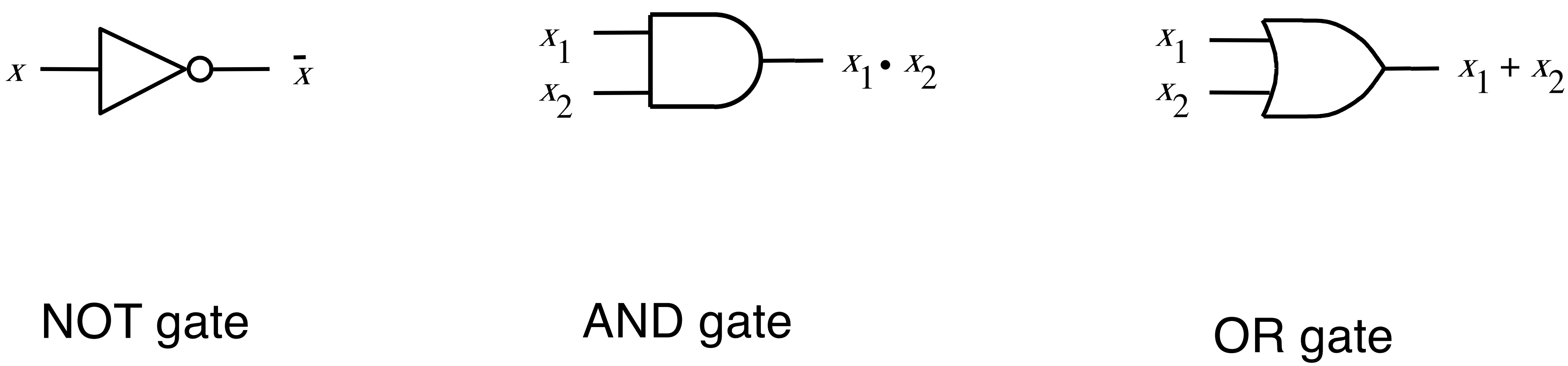
- **Next week we will start with Lab2**
- **It will be graded!**
- **Print the answer sheet for that lab and do the prelab at home. Otherwise you'll lose 20% of your grade for that lab.**

Labs Next Week

- If your lab is on Mondays, i,e.,
  - Section 13: Mondays, 12:10 - 3:00 pm
- You will have 2 labs in one on September 9.
- That is, Lab #1 and Lab #2.

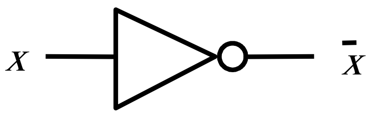
Quick Review

The Three Basic Logic Gates



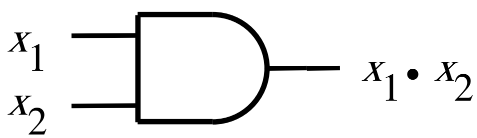
[Figure 2.8 from the textbook]

Truth Table for NOT



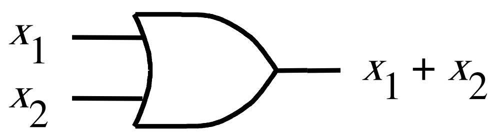
x	$\overline{x}$
0	1
1	0

Truth Table for AND



x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for OR



x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

Truth Tables for AND and OR

x_1	x_2	$x_1 \cdot x_2$	$x_1 + x_2$
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	1
		AND	OR

[Figure 2.6b from the textbook]

Operator Precedence

- In regular arithmetic and algebra, multiplication takes precedence over addition
- This is also true in Boolean algebra

Operator Precedence
(three different ways to write the same)

$$x_1 \cdot x_2 + \overline{x}_1 \cdot \overline{x}_2$$

$$(x_1 \cdot x_2) + ((\overline{x}_1) \cdot (\overline{x}_2))$$

$$x_1 x_2 + \overline{x}_1 \overline{x}_2$$

Operator Precedence

- **Negation of a single variable takes precedence over multiplication of that variable with another variable.**
- **For example,**

$\overline{A}$ B means negate A first and then multiply $\overline{A}$ by B

Operator Precedence

- However, a horizontal bar over a product of two variables means that the negation is performed after the product is computed.

- For example,

  $\overline{A\ B}$ means multiply A and B and then negate

Operator Precedence

- **Note that these two expressions are different:**

$\overline{A\,B}$ is not equal to $\overline{A}\,\overline{B}$

$\overline{A\,B}$ means multiply A and B and then negate

$\overline{A}\,\overline{B}$ means negate A and B separately and then multiply

Operator Precedence

- **Note that these two expressions are different:**

$$\overline{A\,B} \text{ is not equal to } \overline{A}\,\overline{B}$$

A	B	$\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	$\overline{A}\overline{B}$
0	0	1
0	1	0
1	0	0
1	1	0

DeMorgan's Theorem

15a. $\overline{x \bullet y} = \overline{x} + \overline{y}$

15b. $\overline{x + y} = \overline{x} \bullet \overline{y}$

Function Synthesis

Synthesize the Following Function

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

1) Split the function into a sum of 4 functions

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

1) Split the function into a sum of 4 functions

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = 1 \bullet f_{00} + 1 \bullet f_{01} + 0 \bullet f_{10} + 1 \bullet f_{11}$$

2) Write the expressions for all four

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}} + \underbrace{1 \bullet f_{01}} + \underbrace{0 \bullet f_{10}} + \underbrace{1 \bullet f_{11}}$$

2) Write the expressions for all four

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}}_{\bar{x}_1\bar{x}_2} + \underbrace{1 \bullet f_{01}}_{\bar{x}_1 x_2} + \underbrace{0 \bullet f_{10}}_{0} + \underbrace{1 \bullet f_{11}}_{x_1 x_2}$$

3) Then just add them together

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \underbrace{1 \bullet f_{00}} + \underbrace{1 \bullet f_{01}} + \underbrace{0 \bullet f_{10}} + \underbrace{1 \bullet f_{11}}$$

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

3) Then just add them together

x_1	x_2	$f(x_1, x_2)$		$f_{00}(x_1, x_2)$	$f_{01}(x_1, x_2)$	$f_{10}(x_1, x_2)$	$f_{11}(x_1, x_2)$
0	0	1		1	0	0	0
0	1	1		0	1	0	0
1	0	0		0	0	1	0
1	1	1		0	0	0	1

$$f(x_1, x_2) = \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + 0 + x_1 x_2$$

A function to be synthesized

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

[Figure 2.19 from the textbook]

Let’s look at it row by row.
How can we express the last row?

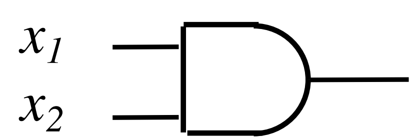
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

Let’s look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	
1	0	0	
1	1	1	$x_1 x_2$

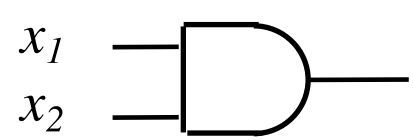
Let's look at it row by row.
How can we express the last row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



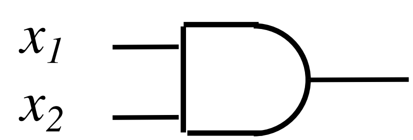
What about this row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



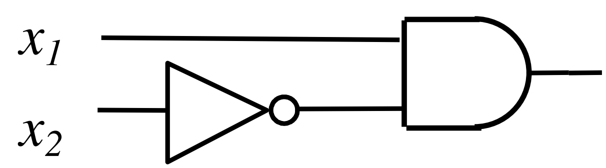
What about this row?

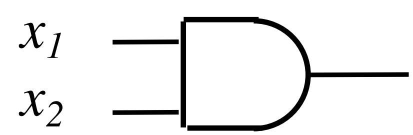
x_1	x_2	$f(x_1, x_2)$	
0	0	1	
0	1	1	$\bar{x}_1 x_2$
1	0	0	
1	1	1	



What about this row?

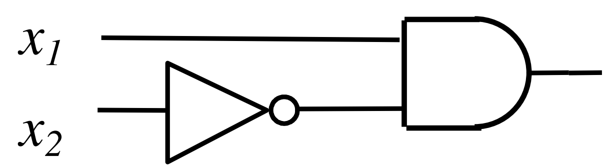
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

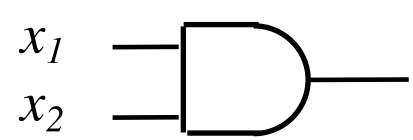




What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

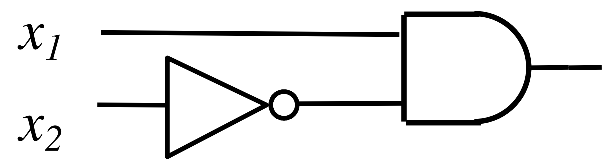


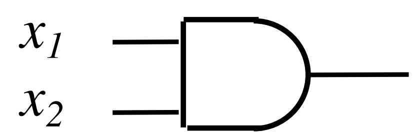


What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

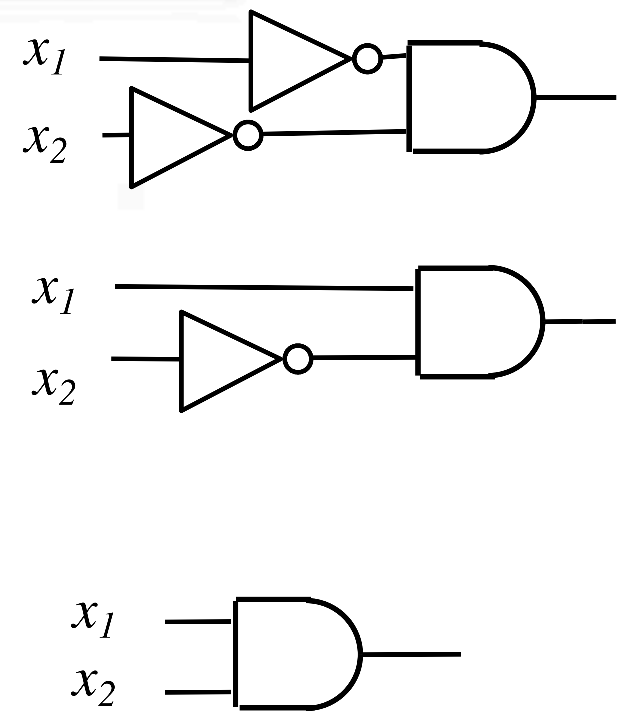
$\overline{x}_1\overline{x}_2$





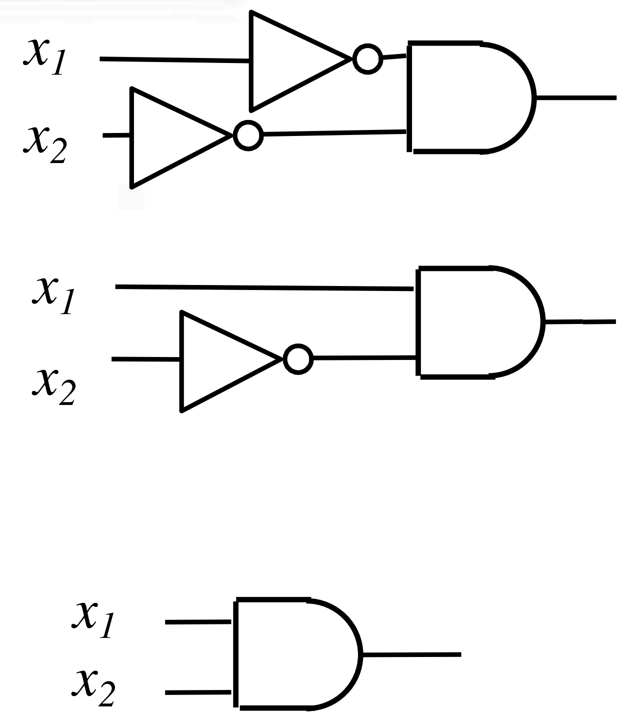
What about the first row?

x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1

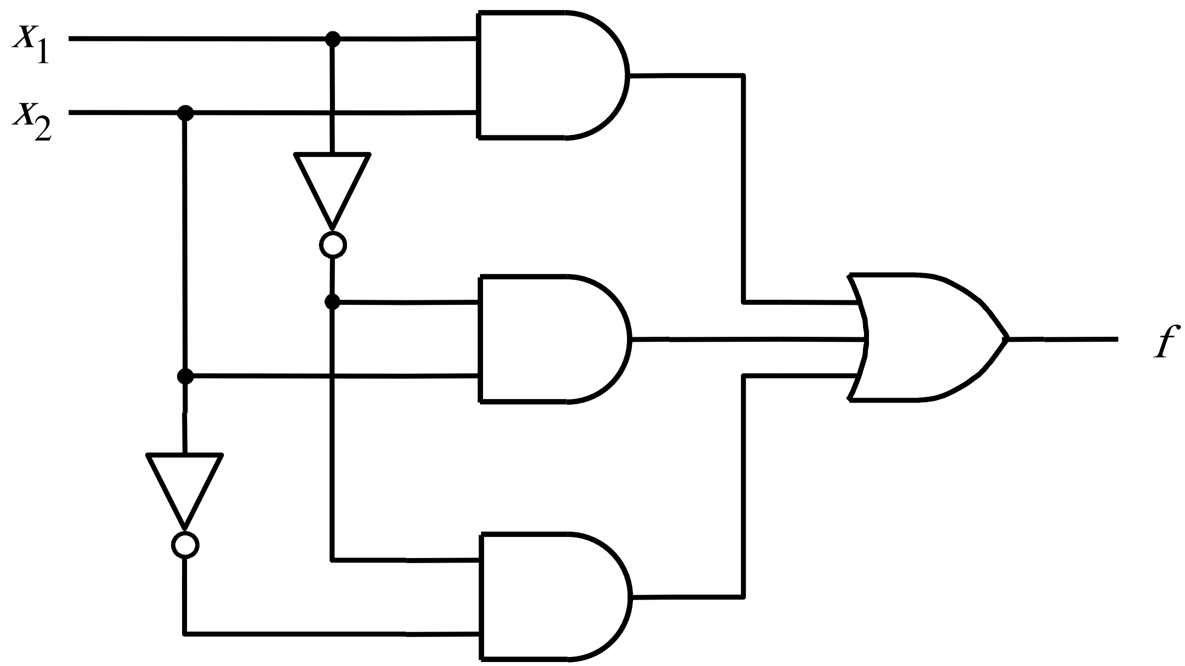


Finally, what about the zero?

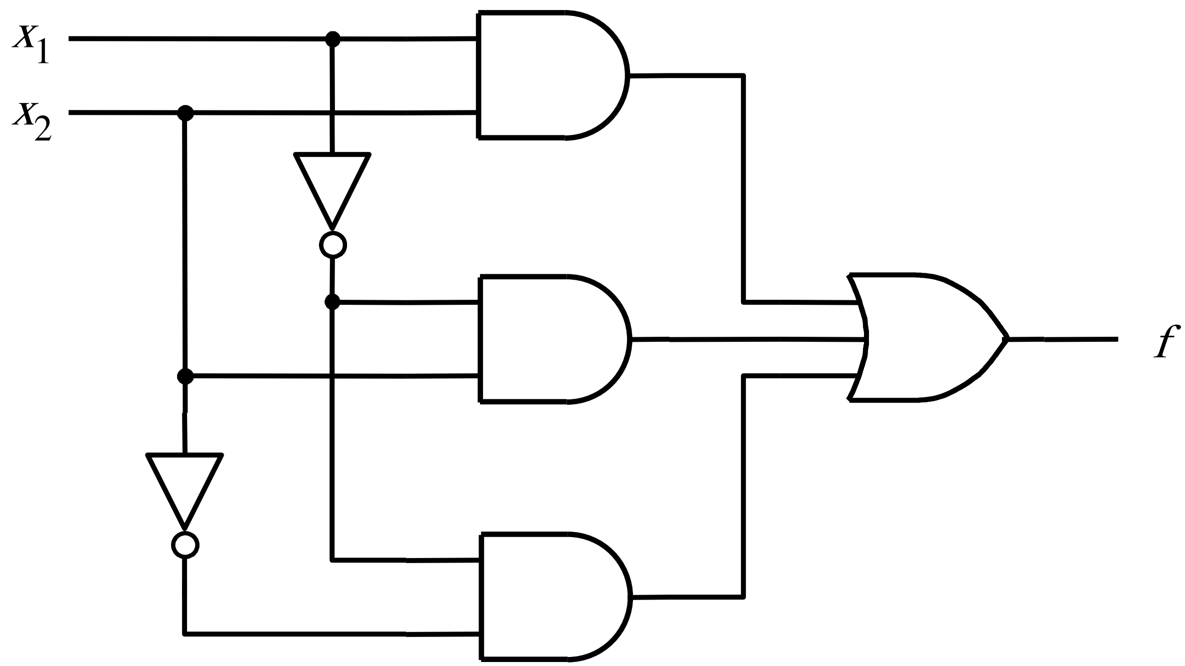
x_1	x_2	$f(x_1, x_2)$
0	0	1
0	1	1
1	0	0
1	1	1



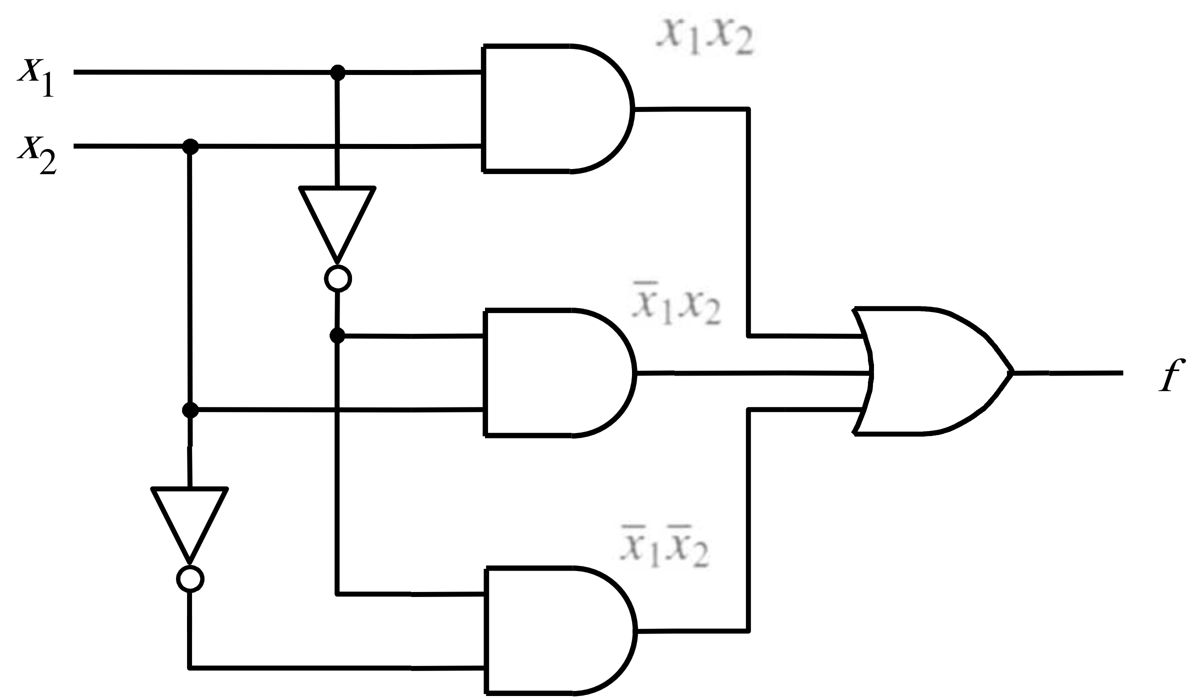
Putting it all together



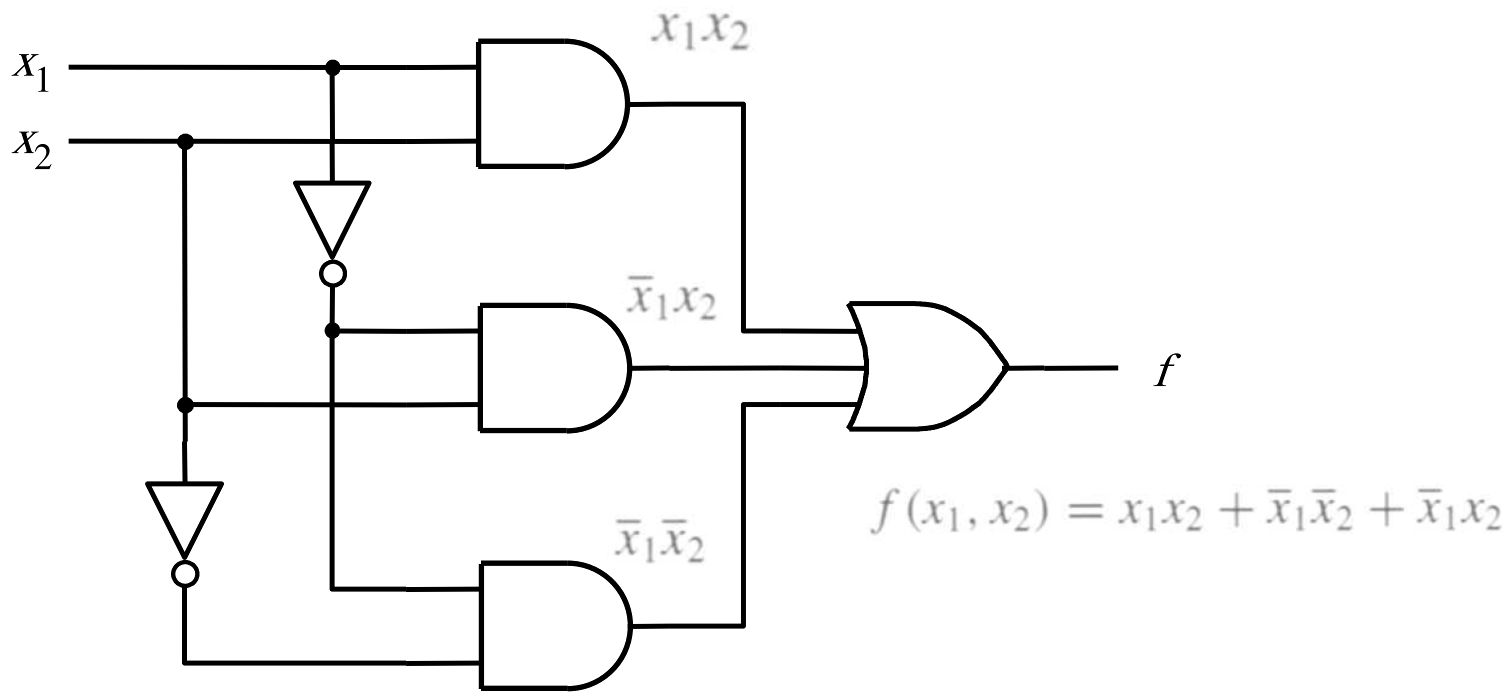
Let's verify that this circuit implements correctly the target truth table



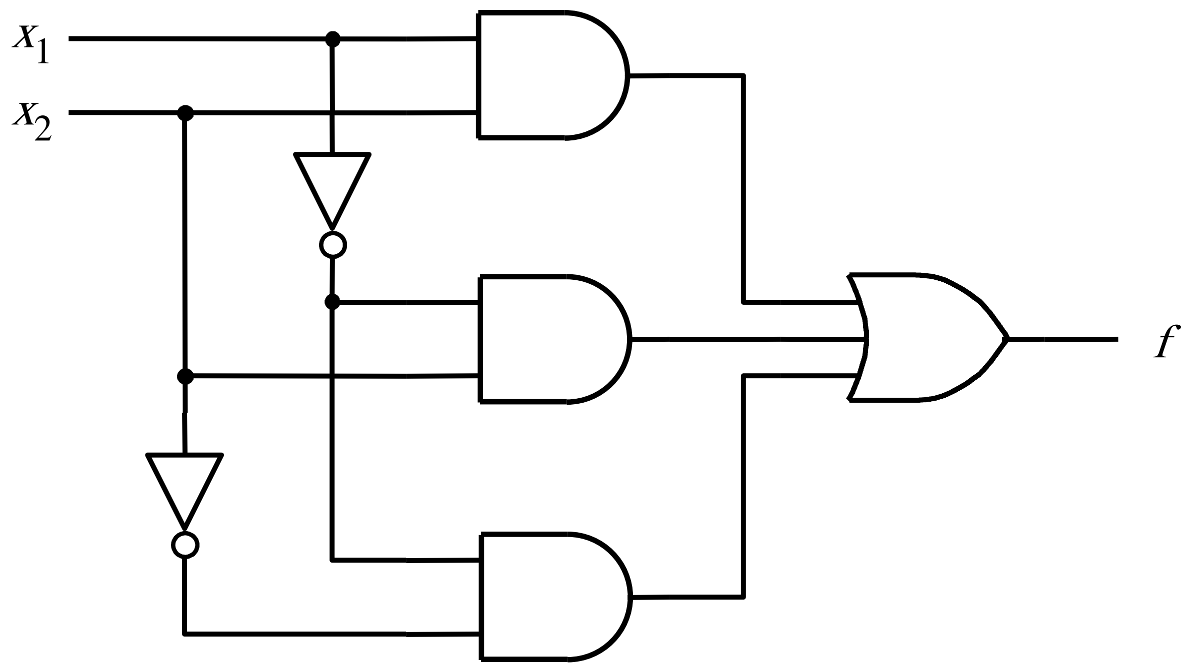
Let's verify that this circuit implements correctly the target truth table



Putting it all together



Canonical Sum-Of-Products (SOP)



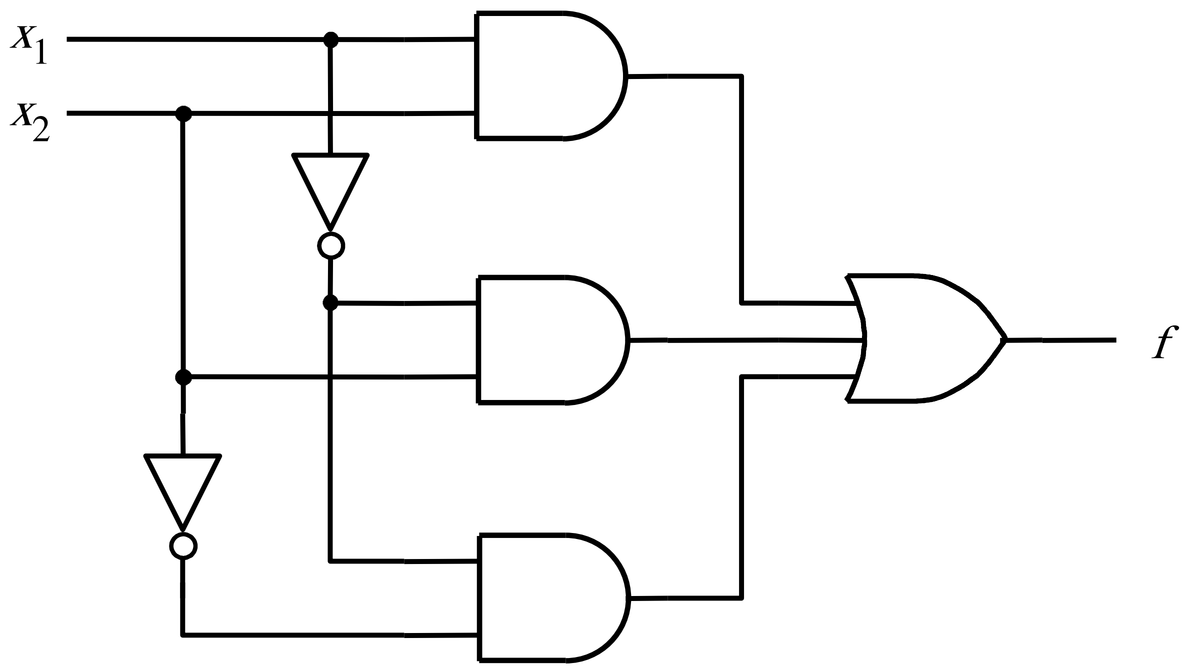
$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

[Figure 2.20a from the textbook]

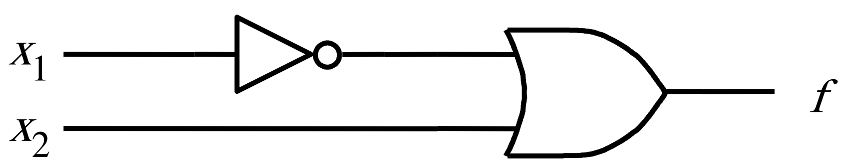
Summary of This Procedure

- Get the truth table of the function
- Form a product term (AND gate) for each row of the table for which the function is 1
- Each product term contains all input variables
- In each row, if x_i=1 enter it as x_i, otherwise use $\overline{x}_i$
- Sum all of these products (OR gate) to get the function

Two implementations for the same function



(a) Canonical sum-of-products



(b) Minimal-cost realization

[Figure 2.20 from the textbook]

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

replicate this term

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

Simplification Steps

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2$$

group
these terms

$$f(x_1, x_2) = x_1 x_2 + \bar{x}_1 \bar{x}_2 + \bar{x}_1 x_2 + \bar{x}_1 x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1) x_2 + \bar{x}_1 (\bar{x}_2 + x_2)$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

These two terms are trivially equal to 1

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

Simplification Steps

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = x_1x_2 + \bar{x}_1\bar{x}_2 + \bar{x}_1x_2 + \bar{x}_1x_2$$

$$f(x_1, x_2) = (x_1 + \bar{x}_1)x_2 + \bar{x}_1(\bar{x}_2 + x_2)$$

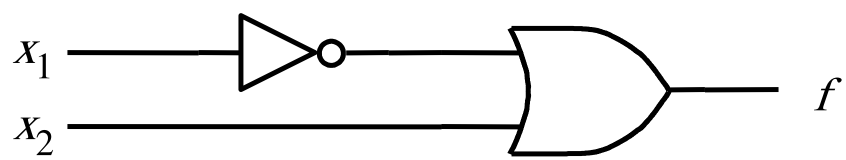
Drop the 1's

$$f(x_1, x_2) = 1 \cdot x_2 + \bar{x}_1 \cdot 1$$

$$f(x_1, x_2) = x_2 + \bar{x}_1$$

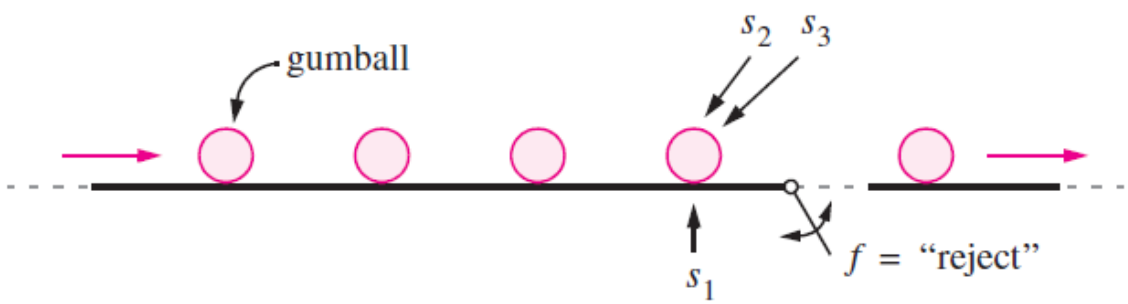
Minimal-cost realization

$$f(x_1, x_2) = x_2 + \bar{x}_1$$



[Figure 2.20b from the textbook]

Let's look at another problem



(a) Conveyor and sensors

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(b) Truth table

[Figure 2.21 from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 2.21b from the textbook]

Let's look at another problem

s_1	s_2	s_3	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1 \bar{s}_2 s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1 s_2 s_3$
1	0	0	0	
1	0	1	1	$s_1 \bar{s}_2 s_3$
1	1	0	1	$s_1 s_2 \bar{s}_3$
1	1	1	1	$s_1 s_2 s_3$

Let's look at another problem

s_1	s_2	s_3	f	
0	0	0	0	
0	0	1	1	$\bar{s}_1\bar{s}_2s_3$
0	1	0	0	
0	1	1	1	$\bar{s}_1s_2s_3$
1	0	0	0	
1	0	1	1	$s_1\bar{s}_2s_3$
1	1	0	1	$s_1s_2\bar{s}_3$
1	1	1	1	$s_1s_2s_3$

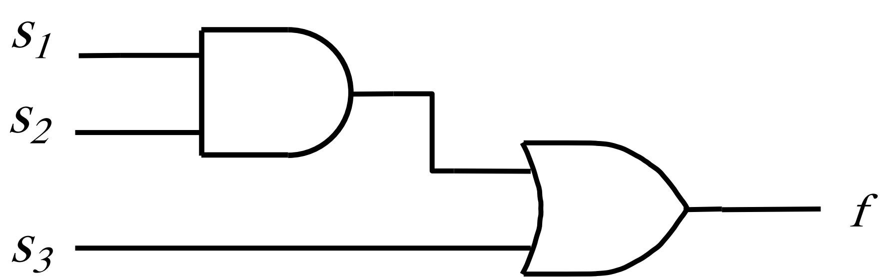
$$f = \bar{s}_1\bar{s}_2s_3 + \bar{s}_1s_2s_3 + s_1\bar{s}_2s_3 + s_1s_2\bar{s}_3 + s_1s_2s_3$$

Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3 \\
&= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1 s_3(\bar{s}_2 + s_2) + s_1 s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2 \\
&= s_3 + s_1 s_2
\end{aligned}
$$

Let's look at another problem (minimization)

$$
\begin{aligned}
f &= \bar{s}_1\bar{s}_2 s_3 + \bar{s}_1 s_2 s_3 + s_1\bar{s}_2 s_3 + s_1 s_2 s_3 + s_1 s_2 \bar{s}_3 + s_1 s_2 s_3 \\
&= \bar{s}_1 s_3(\bar{s}_2 + s_2) + s_1 s_3(\bar{s}_2 + s_2) + s_1 s_2(\bar{s}_3 + s_3) \\
&= \bar{s}_1 s_3 + s_1 s_3 + s_1 s_2 \\
&= s_3 + s_1 s_2
\end{aligned}
$$



Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Minterms and Maxterms

Row number	x_1	x_2	Minterm	Maxterm
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	$M_0 = x_1 + x_2$
1	0	1	$m_1 = \overline{x}_1 x_2$	$M_1 = x_1 + \overline{x}_2$
2	1	0	$m_2 = x_1\overline{x}_2$	$M_2 = \overline{x}_1 + x_2$
3	1	1	$m_3 = x_1 x_2$	$M_3 = \overline{x}_1 + \overline{x}_2$

Use these for
Sum-of-Products
Minimization
(1's of the function)

Use these for
Product-of-Sums
Minimization
(0's of the function)

Sum-of-Products Form

(uses the ones of the function)

Sum-of-Products Form
(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form
(for the AND logic function)

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	0
1	0	1	$m_1 = \overline{x}_1 x_2$	0
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

$$f(x_1, x_2) = m_3 = x_1\, x_2$$

(In this case there is just one product and there is no need for a sum)

Another Example

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

Sum-of-Products Form

Row number	x_1	x_2	Minterm	$f(x_1, x_2)$
0	0	0	$m_0 = \overline{x}_1\overline{x}_2$	1
1	0	1	$m_1 = \overline{x}_1 x_2$	1
2	1	0	$m_2 = x_1\overline{x}_2$	0
3	1	1	$m_3 = x_1 x_2$	1

$$
\begin{aligned}
f &= m_0 \cdot 1 + m_1 \cdot 1 + m_2 \cdot 0 + m_3 \cdot 1 \\
&= m_0 + m_1 + m_3 \\
&= \bar{x}_1\bar{x}_2 + \bar{x}_1 x_2 + x_1 x_2
\end{aligned}
$$

Product-of-Sums Form

(uses the zeros of the function)

Product-of-Sums Form
(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for the OR logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$f(x_1, x_2) = M_0 = x_1 + x_2$$

(In this case there is just one sum and there is no need for a product)

Another Example

Product-of-Sums Form
(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x_2}$	1
2	1	0	$M_2 = \overline{x_1} + x_2$	0
3	1	1	$M_3 = \overline{x_1} + \overline{x_2}$	1

Product-of-Sums Form

(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

Product-of-Sums Form
(for this logic function)

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1
2	1	0	$M_2 = \overline{x}_1 + x_2$	0
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1

$$f(x_1, x_2) = M_0 \bullet M_2 = (x_1 + x_2) \bullet (\overline{x}_1 + x_2)$$

Yet Another Example

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1
1	0	1	$M_1 = x_1 + \overline{x_2}$	1
2	1	0	$M_2 = \overline{x_1} + x_2$	0
3	1	1	$M_3 = \overline{x_1} + \overline{x_2}$	1

We need to minimize using the zeros of the function f.
But let's first minimize the inverse of f, i.e., $\overline{\text{f}}$.

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\overline{f}(x_1, x_2) = m_2$$
$$= x_1\overline{x}_2$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\overline{\overline{f}} = f = \overline{x_1 \overline{x}_2}$$
$$= \overline{x}_1 + x_2$$

$$\overline{f}(x_1, x_2) = m_2$$
$$= x_1 \overline{x}_2$$

Product-of-Sums Form

Row number	x_1	x_2	Maxterm	$f(x_1, x_2)$	$\overline{f}(x_1, x_2)$
0	0	0	$M_0 = x_1 + x_2$	1	0
1	0	1	$M_1 = x_1 + \overline{x}_2$	1	0
2	1	0	$M_2 = \overline{x}_1 + x_2$	0	1
3	1	1	$M_3 = \overline{x}_1 + \overline{x}_2$	1	0

$$\overline{\overline{f}} = f = \overline{x_1\overline{x}_2} = \overline{x}_1 + x_2$$

$$\overline{f}(x_1, x_2) = m_2 = x_1\overline{x}_2$$

$$f = \overline{m}_2 = M_2$$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

Sum-of-Products Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f(x_1, x_2, x_3) = \bar{x}_1\bar{x}_2x_3 + x_1\bar{x}_2\bar{x}_3 + x_1\bar{x}_2x_3 + x_1x_2\bar{x}_3$$

$$\begin{aligned} f &= (\bar{x}_1 + x_1)\bar{x}_2x_3 + x_1(\bar{x}_2 + x_2)\bar{x}_3 \\ &= 1 \cdot \bar{x}_2x_3 + x_1 \cdot 1 \cdot \bar{x}_3 \\ &= \bar{x}_2x_3 + x_1\bar{x}_3 \end{aligned}$$

A three-variable function

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

[Figure 2.23 from the textbook]

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$
\begin{aligned}
f &= \overline{m_0 + m_2 + m_3 + m_7} \\
&= \overline{m}_0 \cdot \overline{m}_2 \cdot \overline{m}_3 \cdot \overline{m}_7 \\
&= M_0 \cdot M_2 \cdot M_3 \cdot M_7 \\
&= (x_1 + x_2 + x_3)(x_1 + \overline{x}_2 + x_3)(x_1 + \overline{x}_2 + \overline{x}_3)(\overline{x}_1 + \overline{x}_2 + \overline{x}_3)
\end{aligned}
$$

Product-of-Sums Form

Row number	x_1	x_2	x_3	$f(x_1, x_2, x_3)$
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	0

$$f = ((x_1 + x_3) + x_2)((x_1 + x_3) + \overline{x}_2)(x_1 + (\overline{x}_2 + \overline{x}_3))(\overline{x}_1 + (\overline{x}_2 + \overline{x}_3))$$

$$f = (x_1 + x_3)(\overline{x}_2 + \overline{x}_3)$$

Shorthand Notation

- **Sum-of-Products**

$$f(x_1, x_2, x_3) = \sum(m_1, m_4, m_5, m_6)$$

or

$$f(x_1, x_2, x_3) = \sum m(1, 4, 5, 6)$$

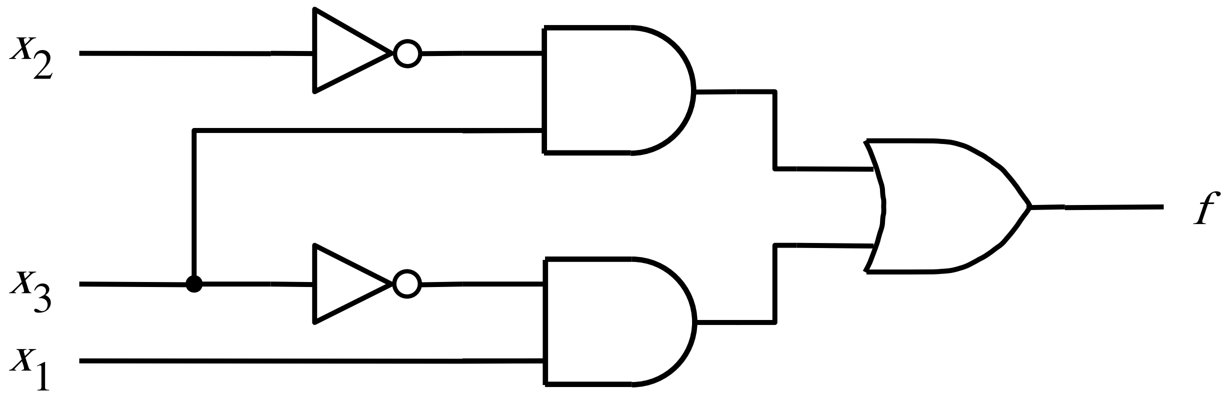
- **Product-of-sums**

$$f(x_1, x_2, x_3) = \Pi(M_0, M_2, M_3, M_7)$$

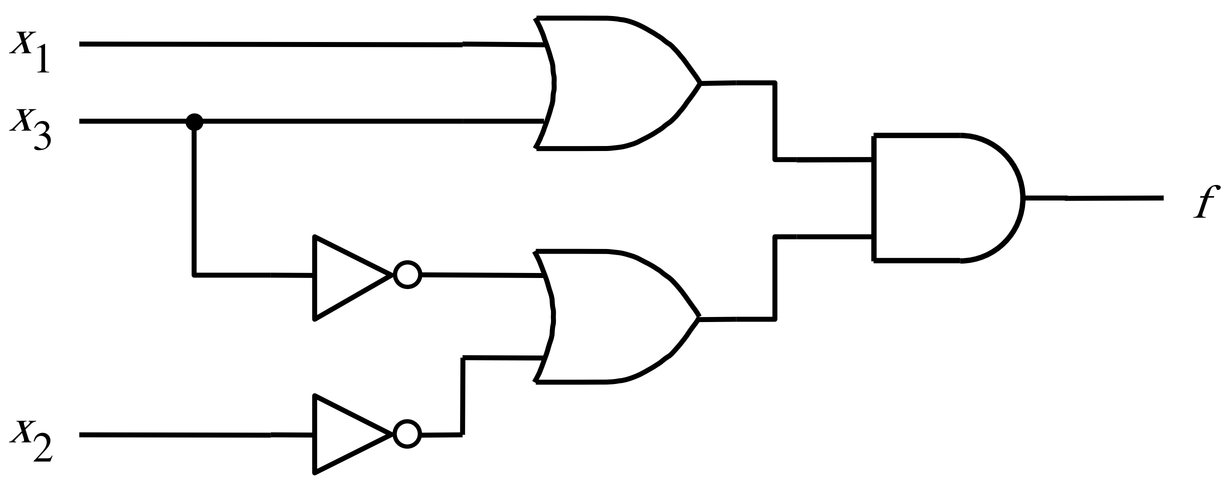
or

$$f(x_1, x_2, x_3) = \Pi M(0, 2, 3, 7)$$

Two realizations of that function



(a) A minimal sum-of-products realization



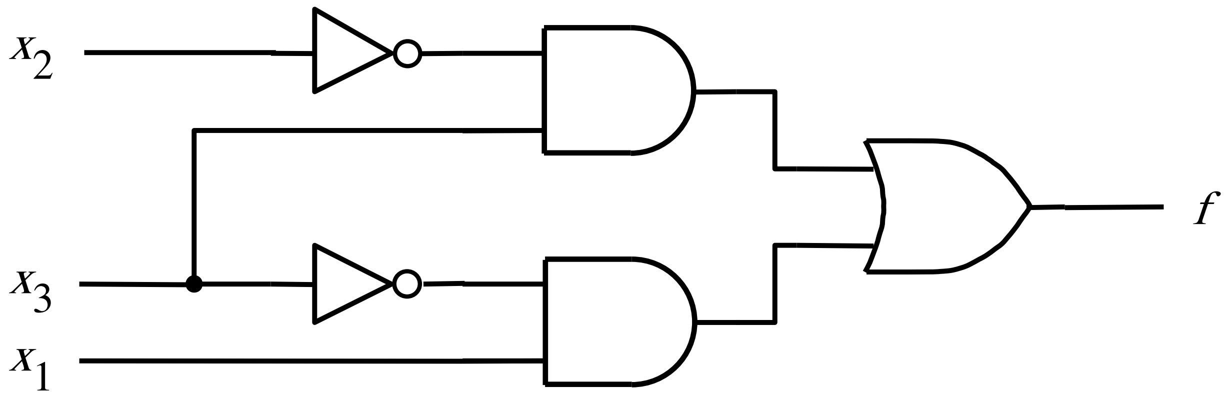
(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

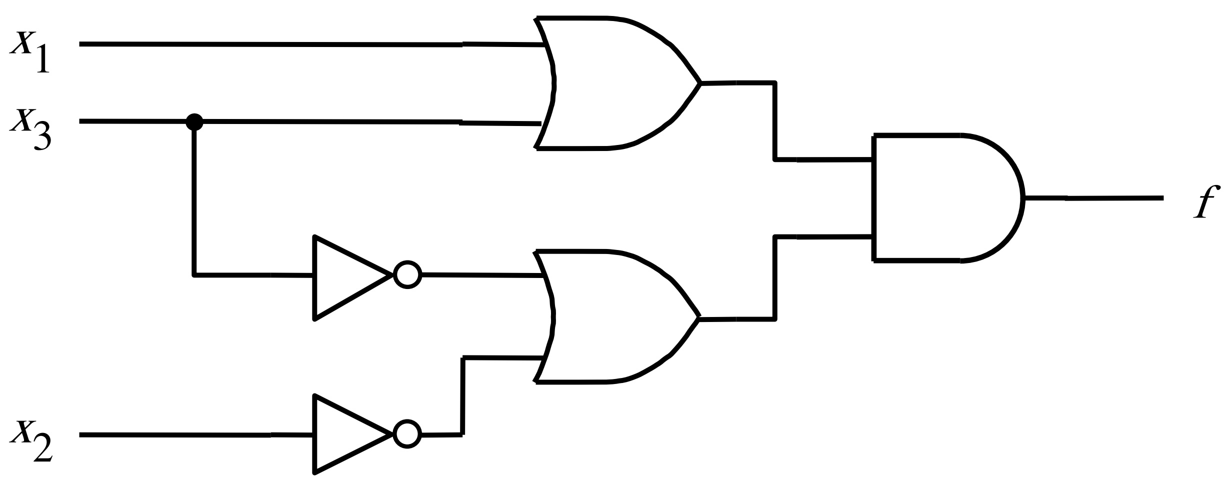
The Cost of a Circuit

- **Count all gates**
- **Count all inputs/wires to the gates**

What is the cost of each circuit?



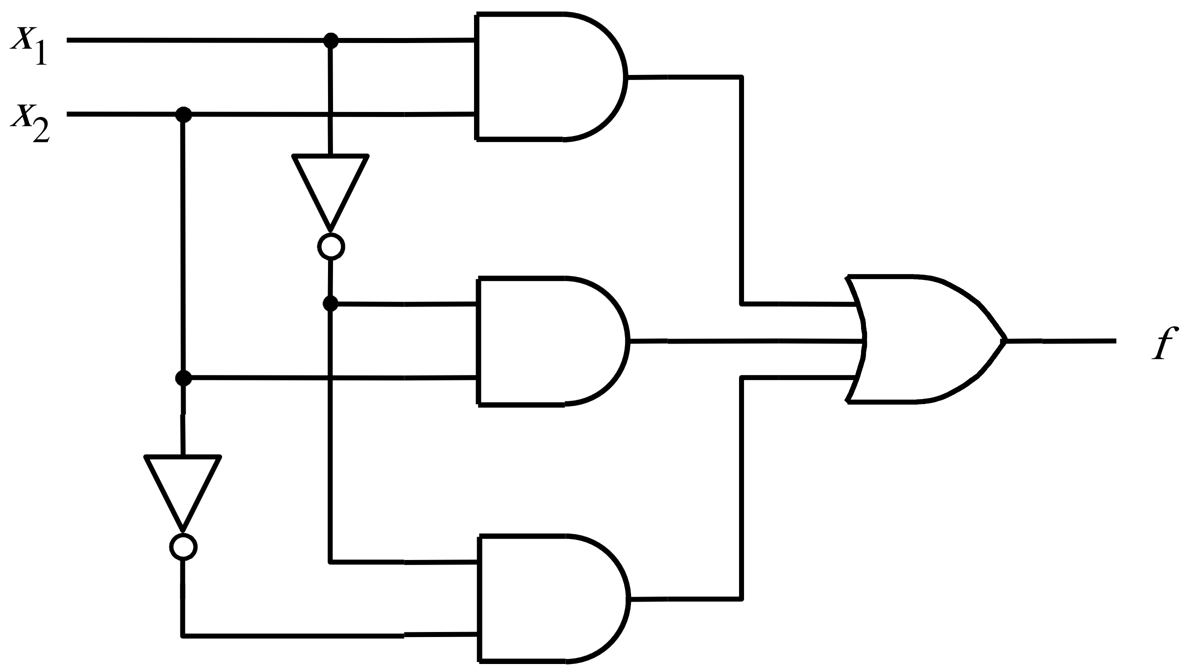
(a) A minimal sum-of-products realization



(b) A minimal product-of-sums realization

[Figure 2.24 from the textbook]

What is the cost of this circuit?



Questions?

THE END