



# **CprE 281: Digital Logic**

**Instructor: Alexander Stoytchev**

**<http://www.ece.iastate.edu/~alexs/classes/>**

# Design Examples

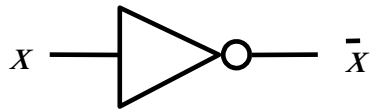
*CprE 281: Digital Logic*  
*Iowa State University, Ames, IA*  
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# Administrative Stuff

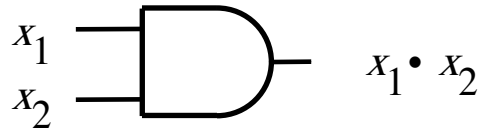
- **HW3 is out**
- **It is due on Monday Sep 16 @ 4pm.**
- **Please write clearly on the first page (in BLOCK CAPITAL letters) the following three things:**
  - **Your First and Last Name**
  - **Your Student ID Number**
  - **Your Lab Section Letter**
- **Also, please**
  - **Staple your pages**

# Quick Review

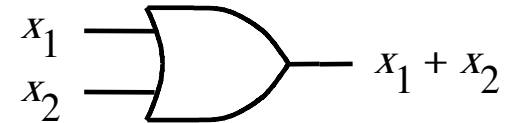
# The Three Basic Logic Gates



NOT gate

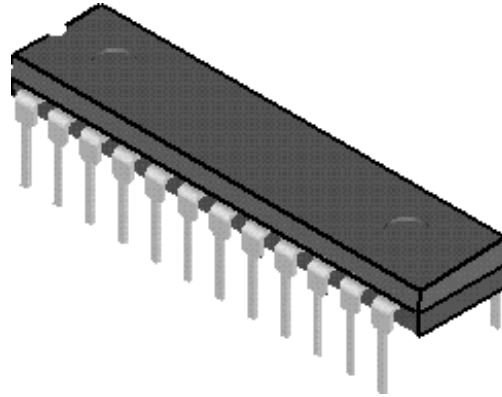


AND gate

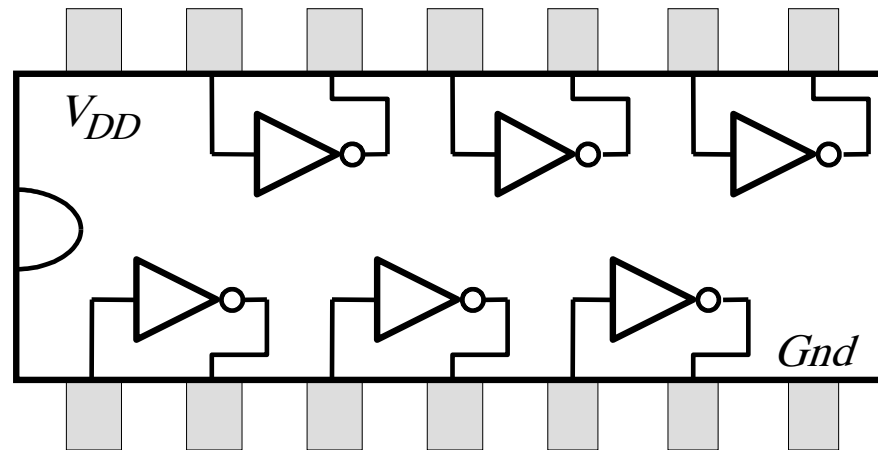


OR gate

You can build any circuit using only these three gates



(a) Dual-inline package



(b) Structure of 7404 chip

Figure B.21. A 7400-series chip.

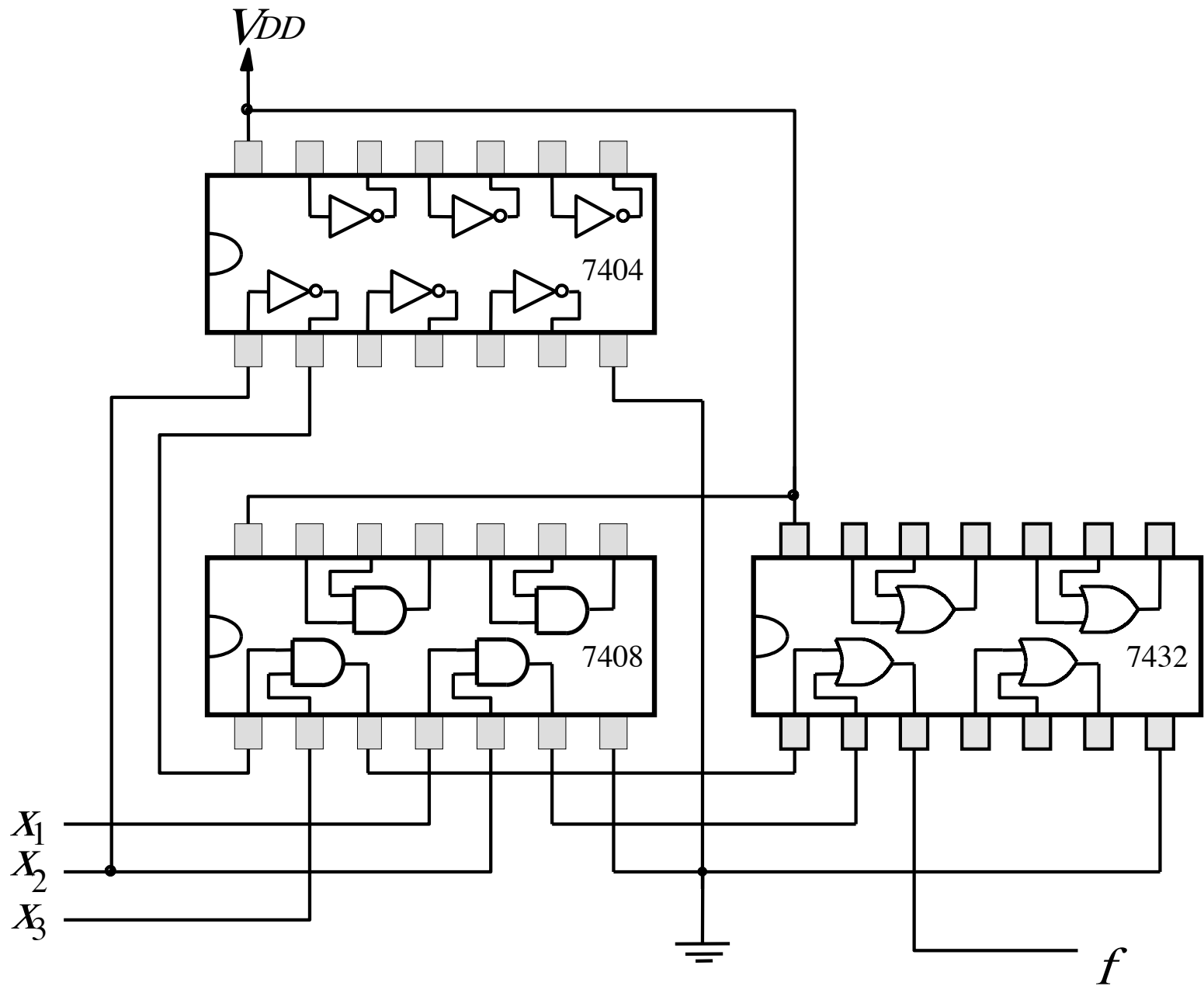
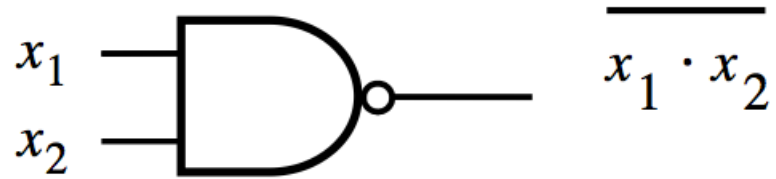


Figure B.22. An implementation of  $f = x_1x_2 + \overline{x_2}x_3$ .

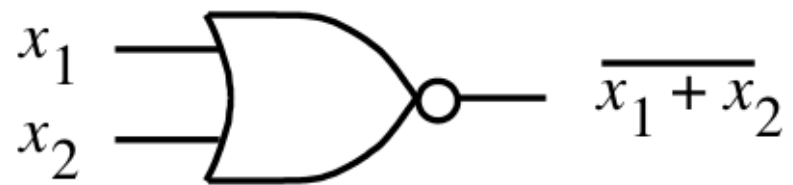
# NAND Gate



$x_1$	$x_2$	f
0	0	1
0	1	1
1	0	1
1	1	0



# NOR Gate



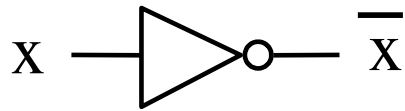
$x_1$	$x_2$	f
0	0	1
0	1	0
1	0	0
1	1	0

# **Why do we need two more gates?**

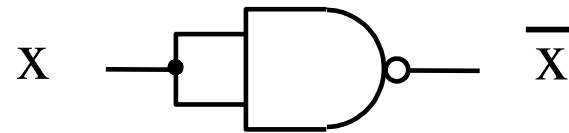
**They can be implemented with fewer transistors.**

**(more about this later)**

# Building a NOT Gate with NAND



$x$	$\bar{x}$
0	1
1	0



$x$	$x$	$f$
0	0	1
[Redacted]		
1	1	0

impossible combinations

Thus, the two truth tables are equal!

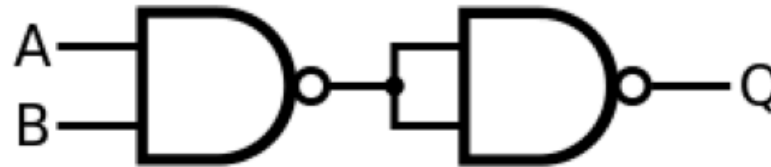
# Building an AND gate with NAND gates

**Desired AND Gate**



$$Q = A \text{ AND } B$$

**NAND Construction**



$$= \text{NOT}(\text{NOT}(A \text{ AND } B))$$

**Truth Table**

Input A	Input B	Output Q
0	0	0
0	1	0
1	0	0
1	1	1

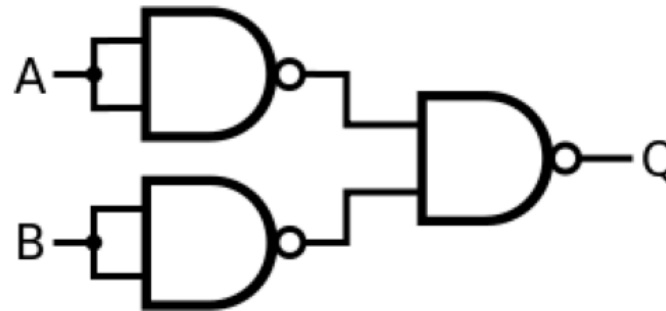
# Building an OR gate with NAND gates

Desired OR Gate



$$Q = A \text{ OR } B$$

NAND Construction



$$= \text{NOT} [ \text{NOT}( A \text{ AND } A ) \text{ AND } \text{NOT}( B \text{ AND } B ) ]$$

Truth Table

Input A	Input B	Output Q
0	0	0
0	1	1
1	0	1
1	1	1

# Implications

**Any Boolean function can be implemented  
with only NAND gates!**

# Implications

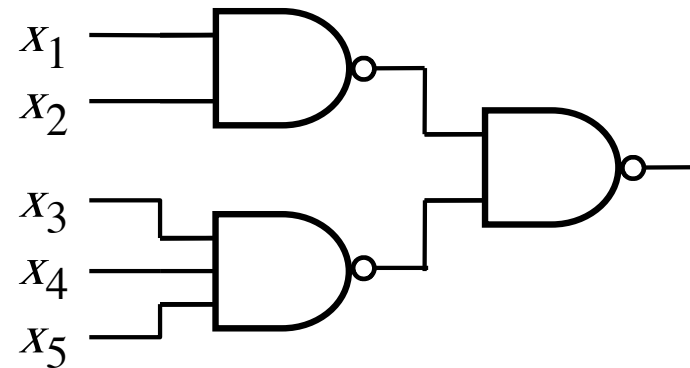
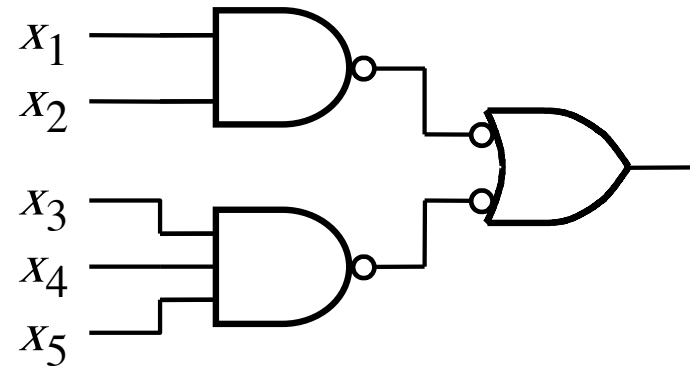
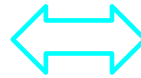
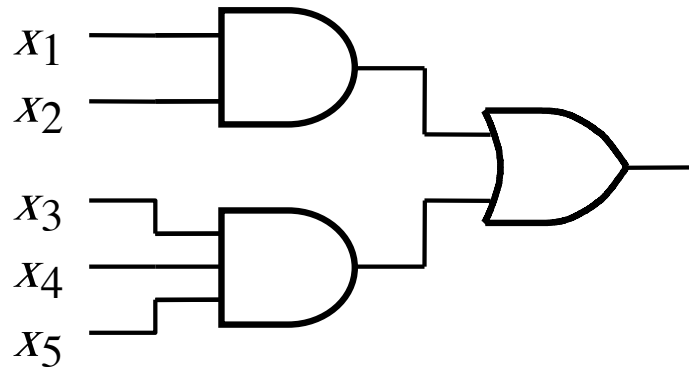
**Any Boolean function can be implemented  
with only NAND gates!**

**The same is also true for NOR gates!**

# **NAND-NAND Implementation of Sum-of-Products Expressions**



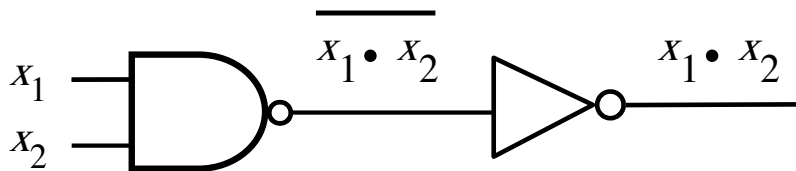
# Sum-Of-Products



This circuit uses ANDs & OR

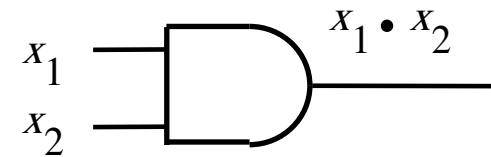
This circuit uses only NANDs

# NAND followed by NOT = AND



$x_1$	$x_2$	f
0	0	1
0	1	1
1	0	1
1	1	0

f
0
0
0
1



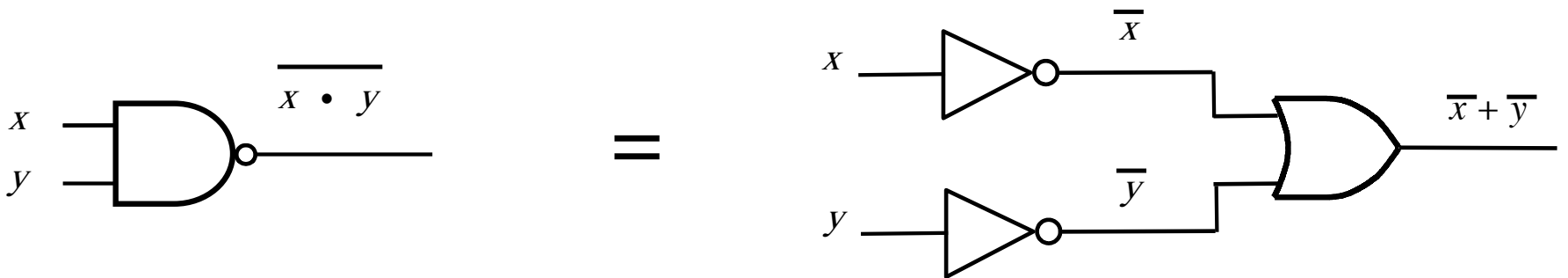
$x_1$	$x_2$	f
0	0	0
0	1	0
1	0	0
1	1	1

# DeMorgan's Theorem

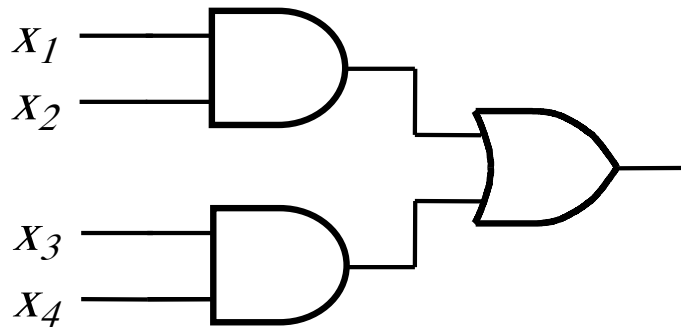
**15a.**  $\overline{\mathbf{x} \cdot \mathbf{y}} = \overline{\mathbf{x}} + \overline{\mathbf{y}}$

# DeMorgan's Theorem

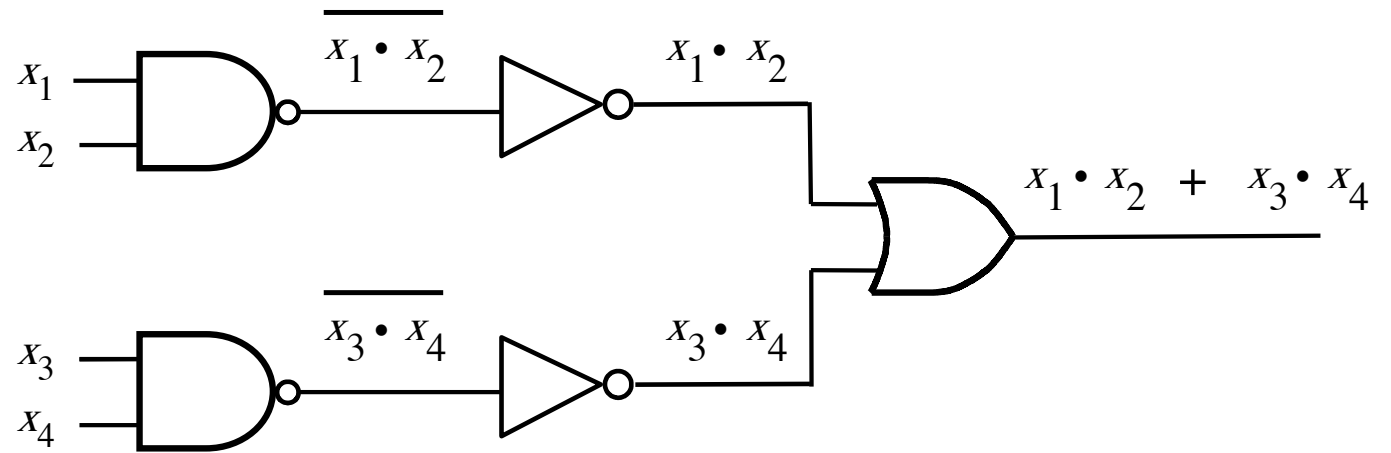
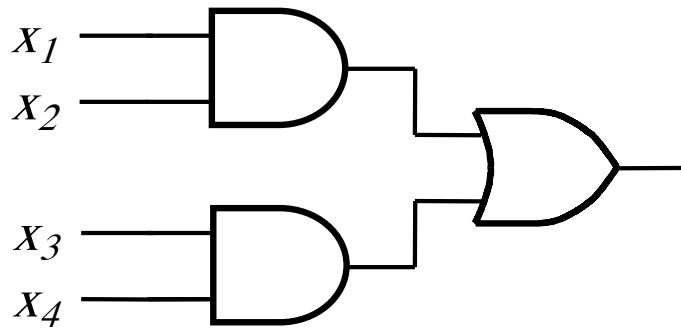
15a.  $\overline{x \cdot y} = \bar{x} + \bar{y}$



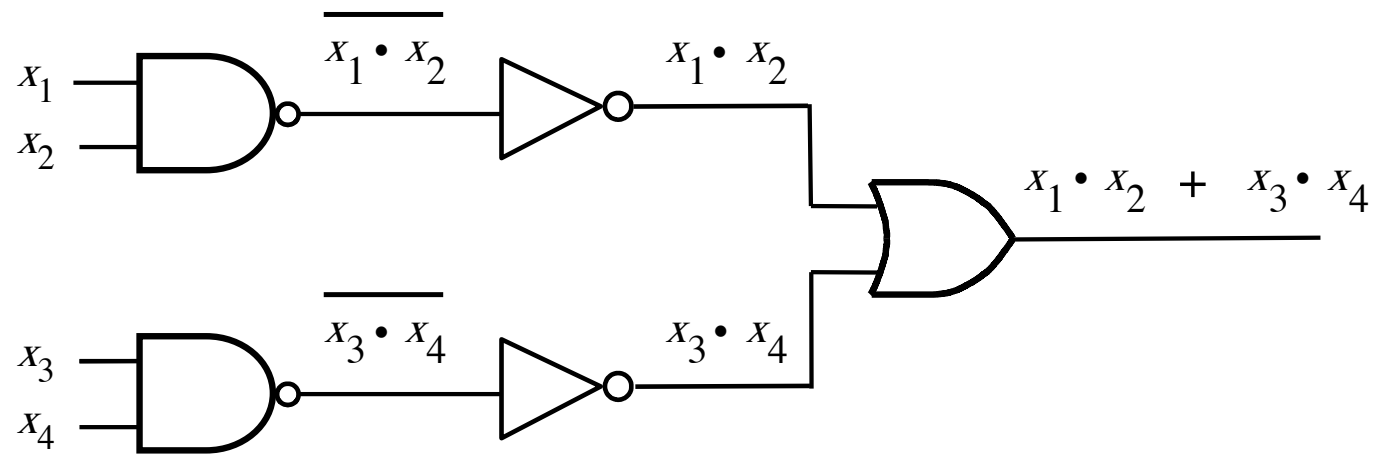
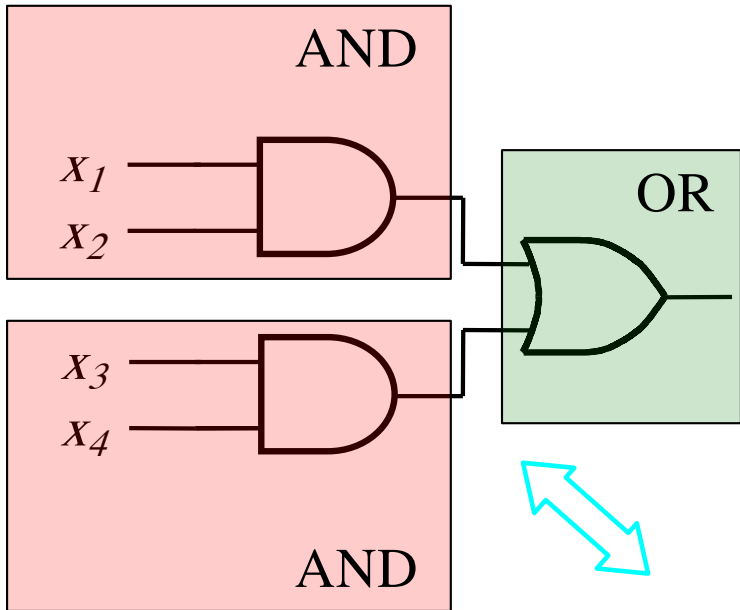
# Sum-Of-Products



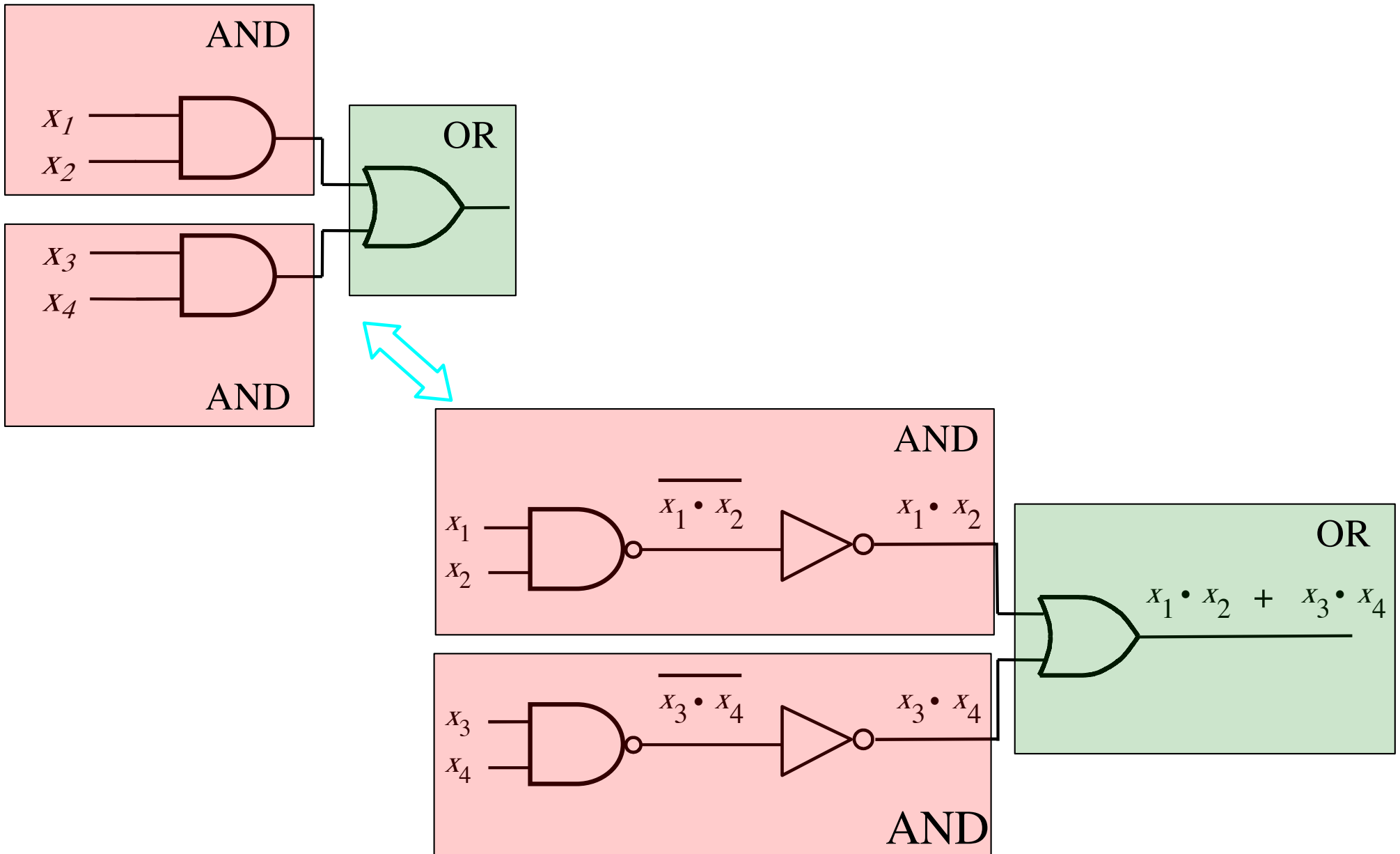
# Sum-Of-Products



# Sum-Of-Products

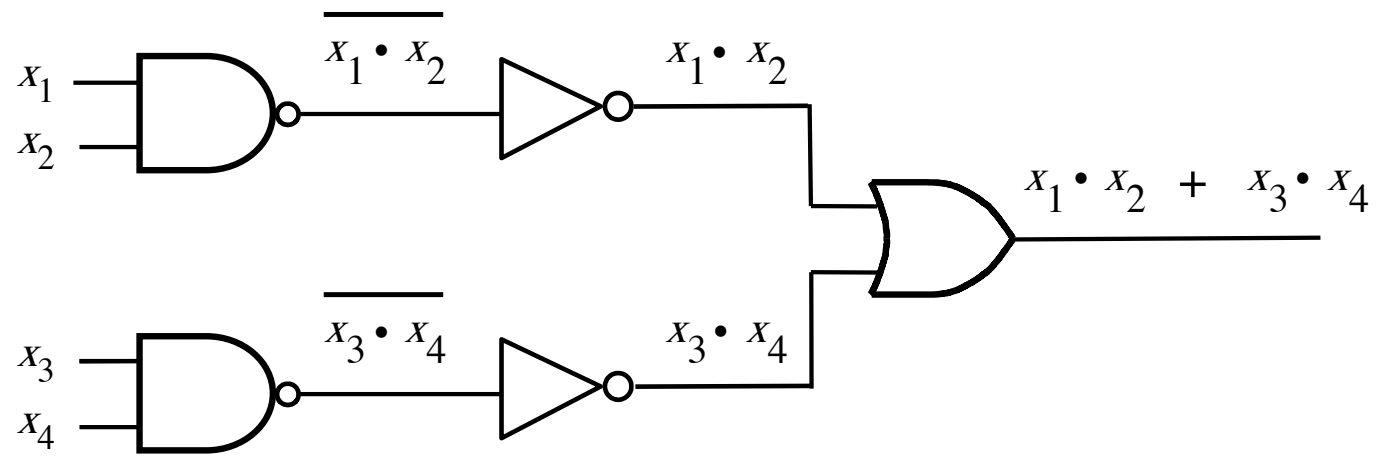
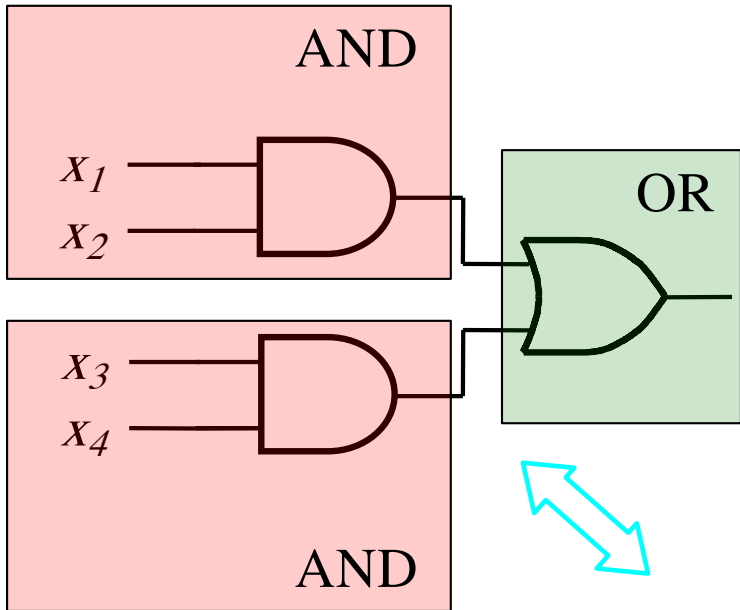


# Sum-Of-Products

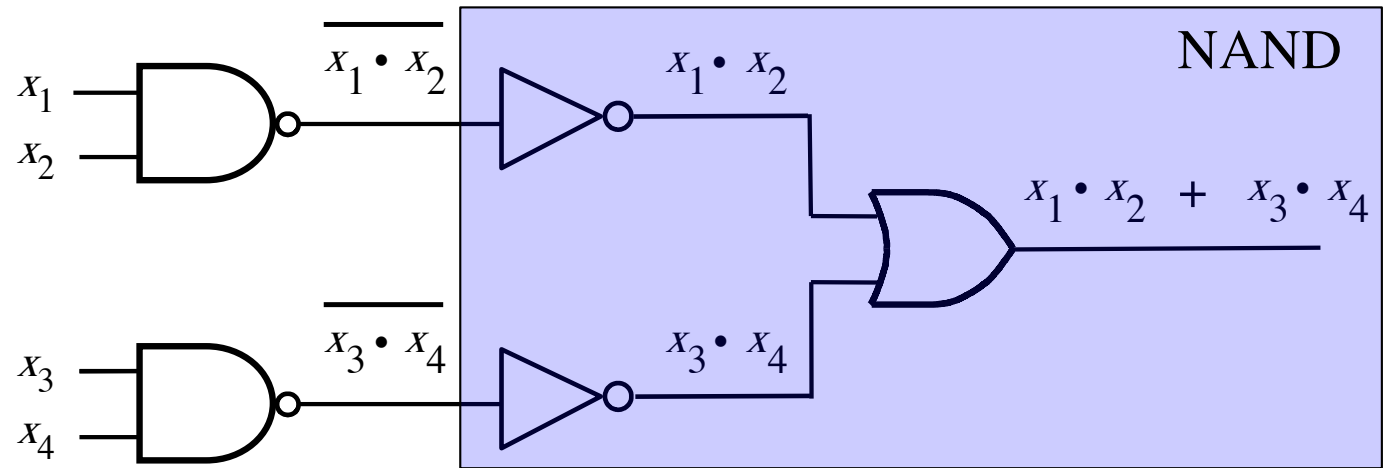
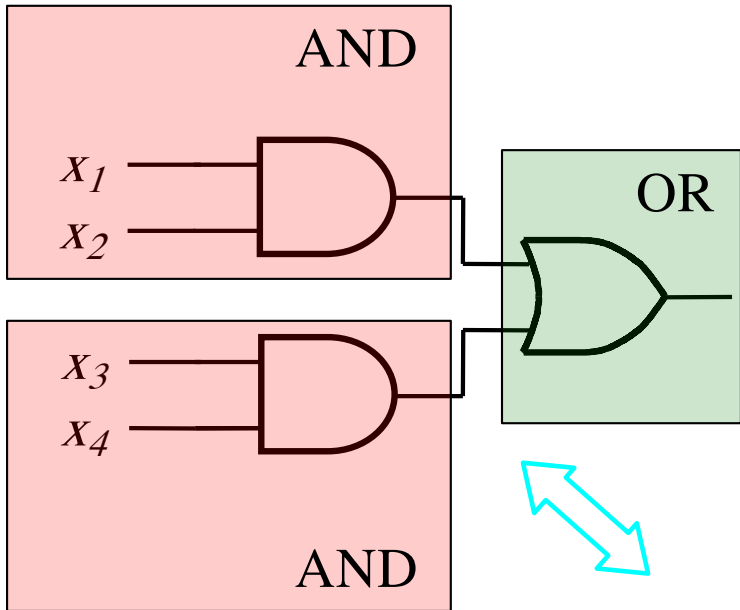




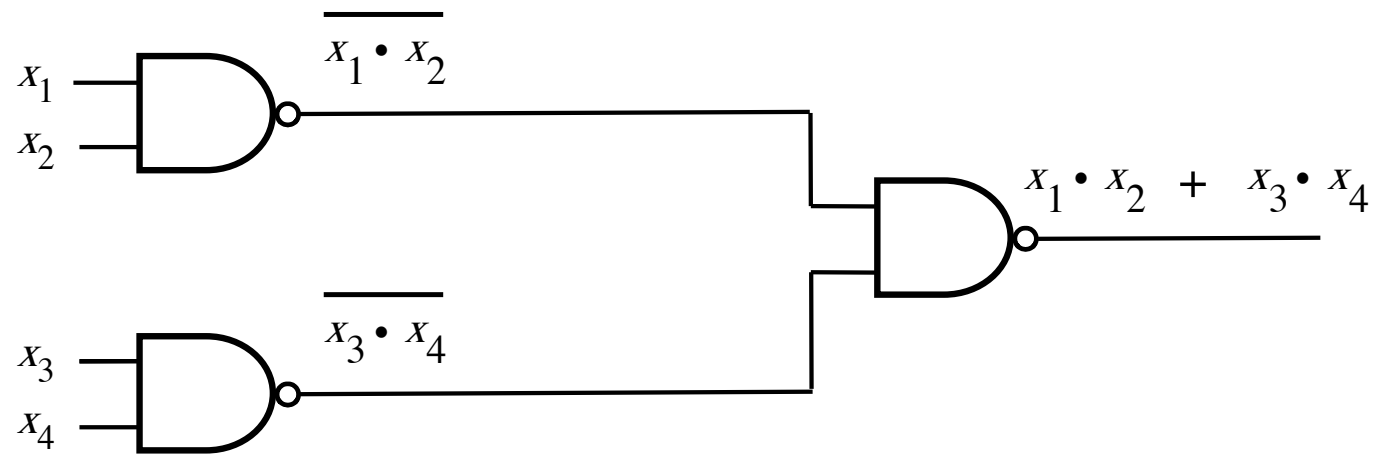
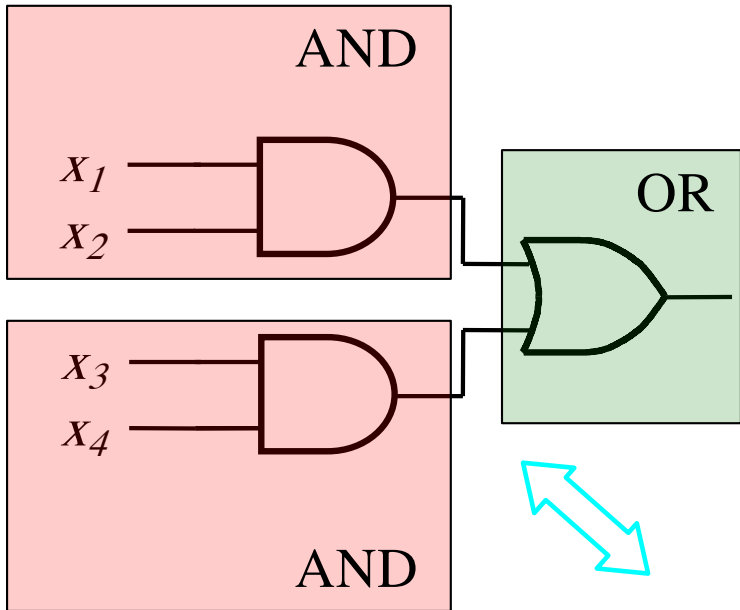
# Sum-Of-Products



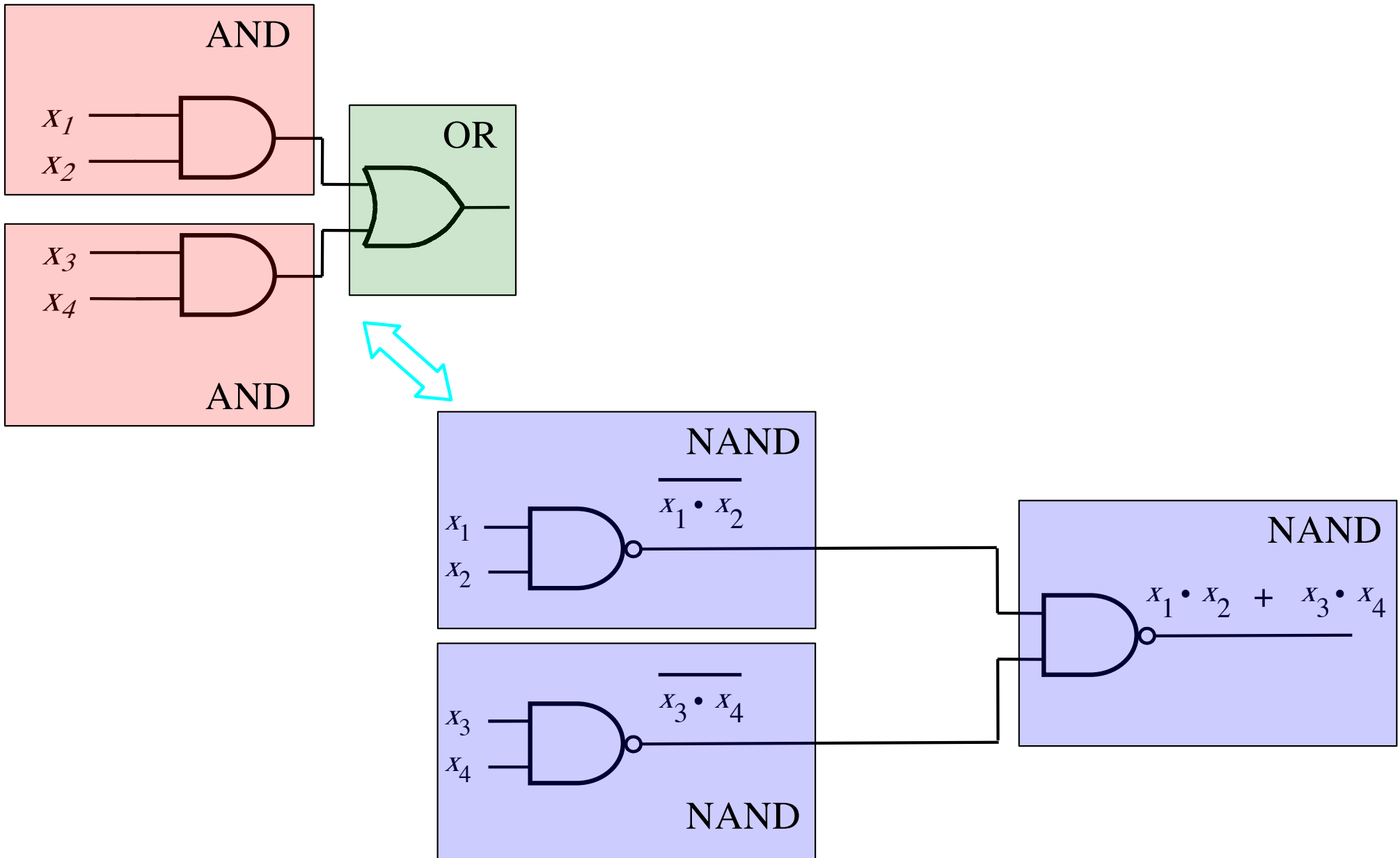
# Sum-Of-Products



# Sum-Of-Products

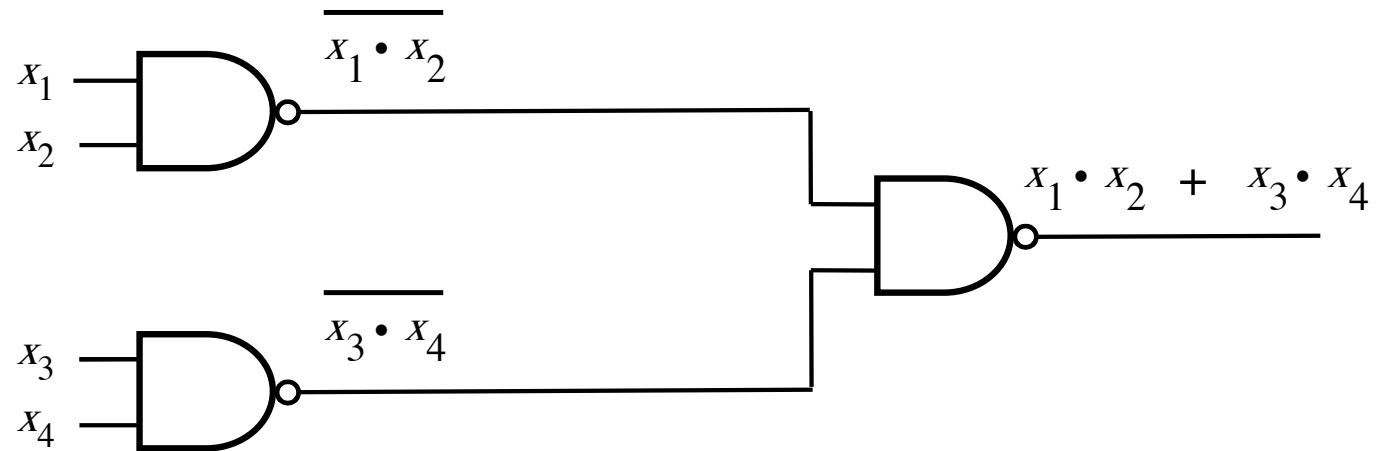
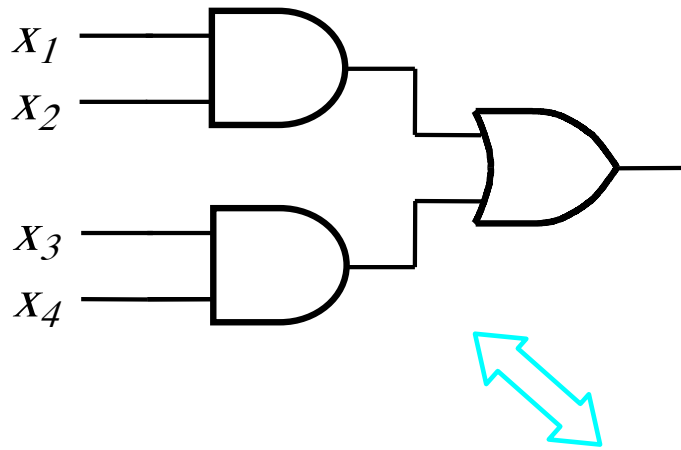


# Sum-Of-Products



This circuit uses only NANDs

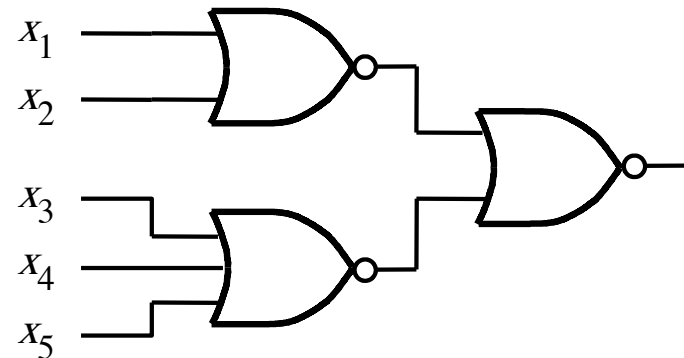
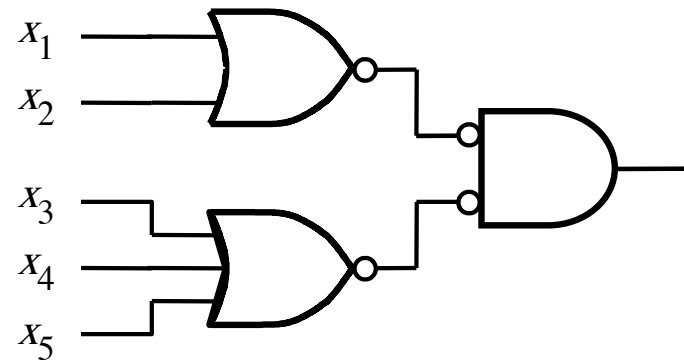
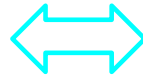
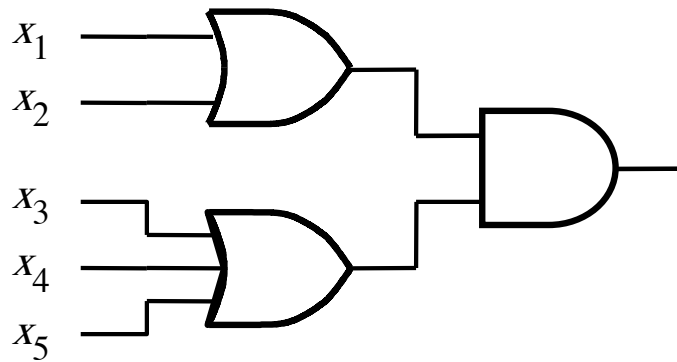
# Sum-Of-Products



This circuit uses only NANDs

# **NOR-NOR Implementation of Product-of-Sums Expressions**

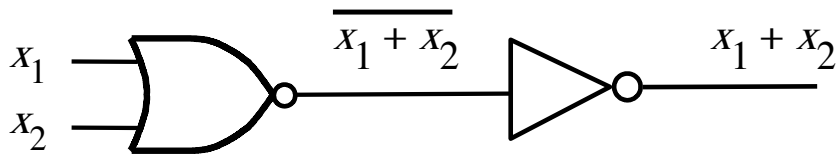
# Product-Of-Sums



This circuit uses ORs & AND

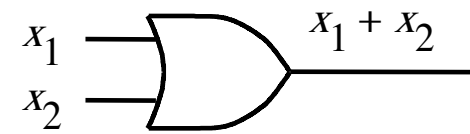
This circuit uses only NORs

# NOR followed by NOT = OR



$x_1$	$x_2$	f
0	0	1
0	1	0
1	0	0
1	1	0

f
0
1
1
1



$x_1$	$x_2$	f
0	0	0
0	1	1
1	0	1
1	1	1

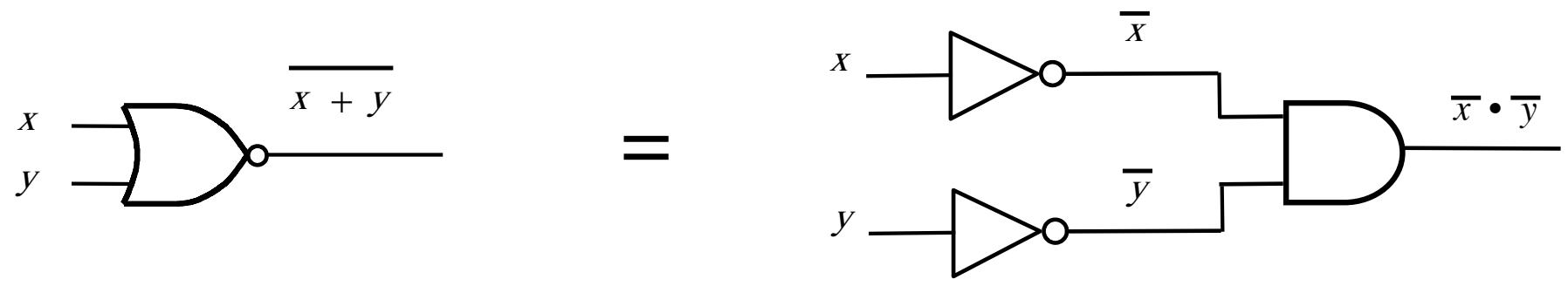


# DeMorgan's Theorem

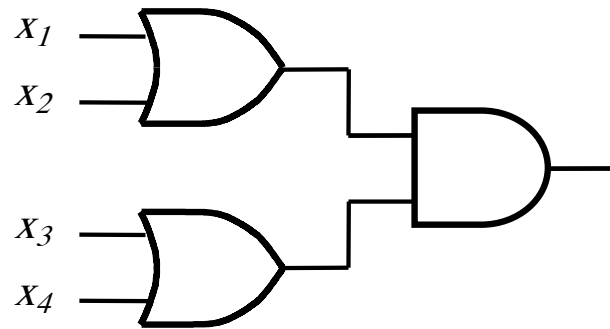
$$15b. \quad \overline{x + y} = \bar{x} \cdot \bar{y}$$

# DeMorgan's Theorem

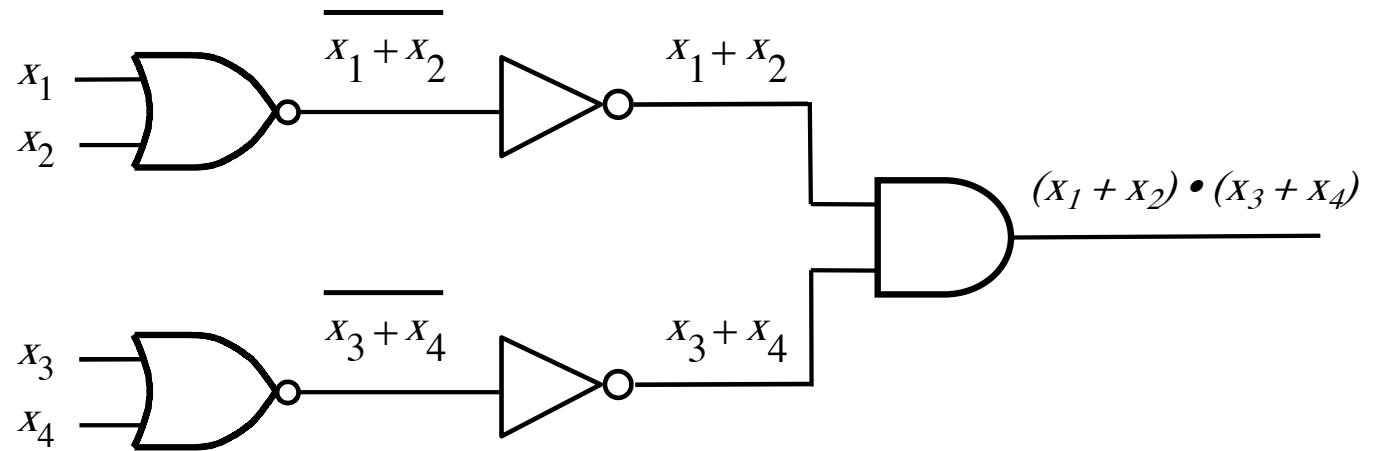
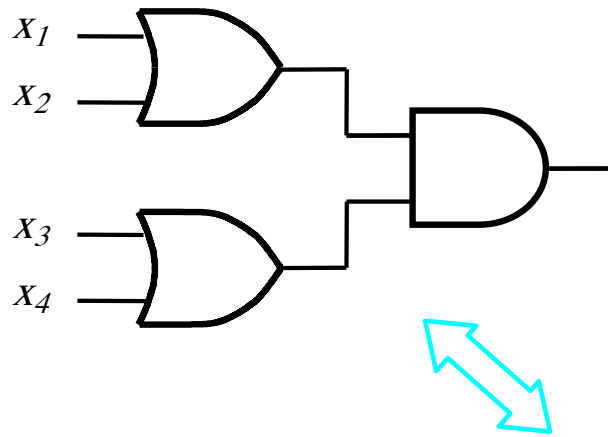
**15b.**       $\overline{x + y} = \bar{x} \cdot \bar{y}$



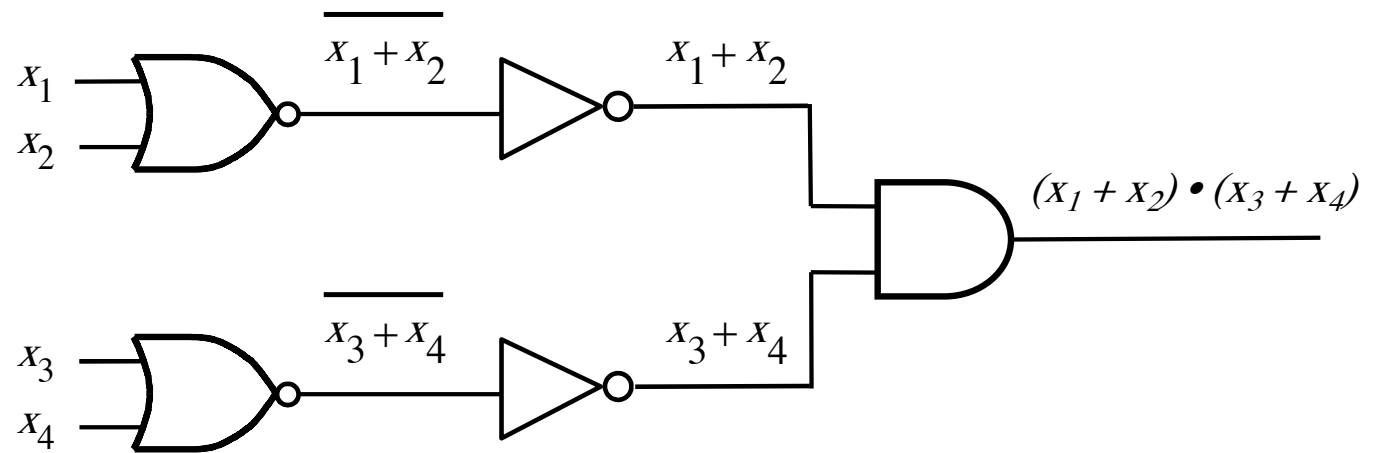
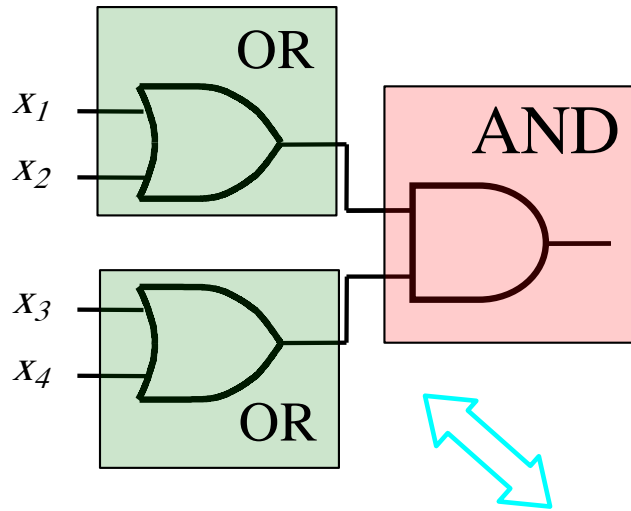
# Product-Of-Sums



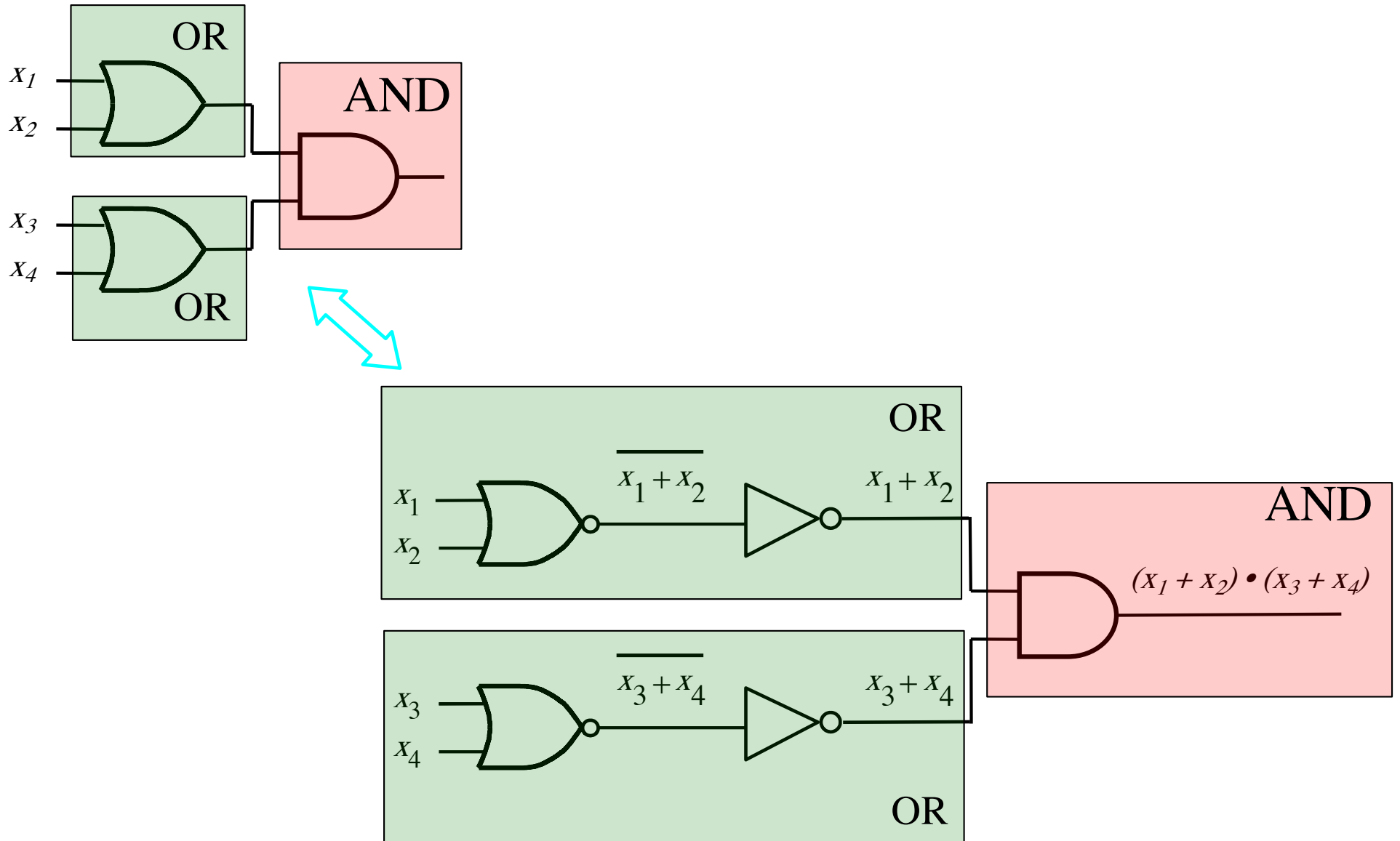
# Product-Of-Sums



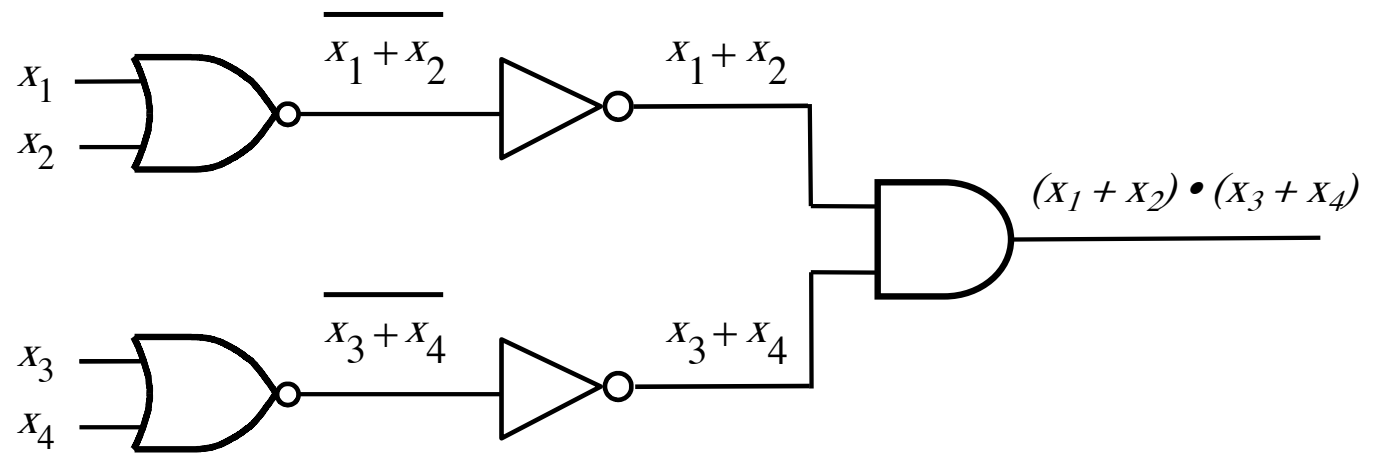
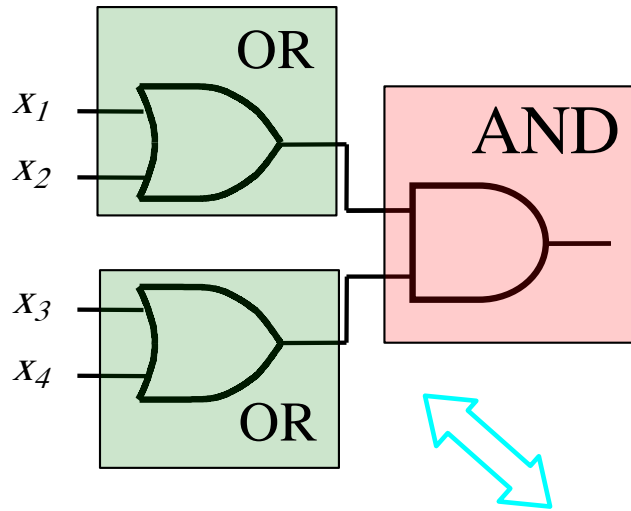
# Product-Of-Sums



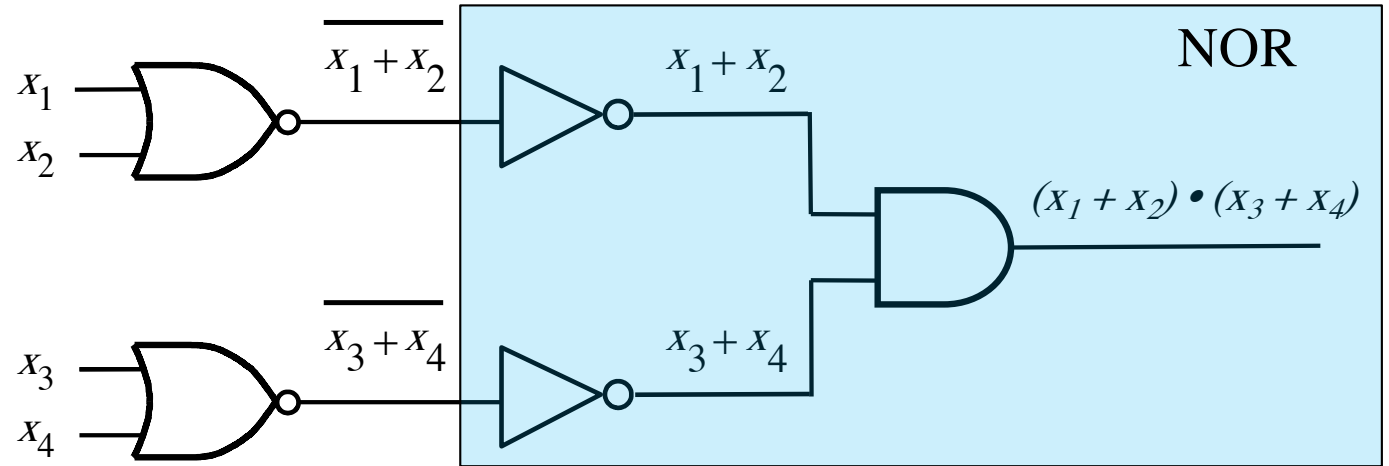
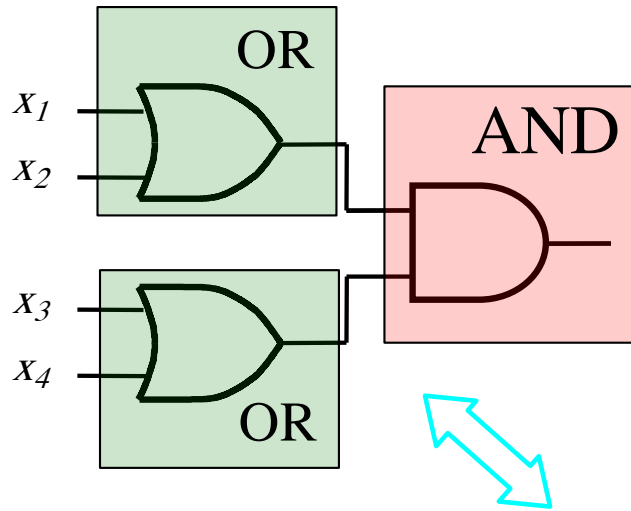
# Product-Of-Sums



# Product-Of-Sums

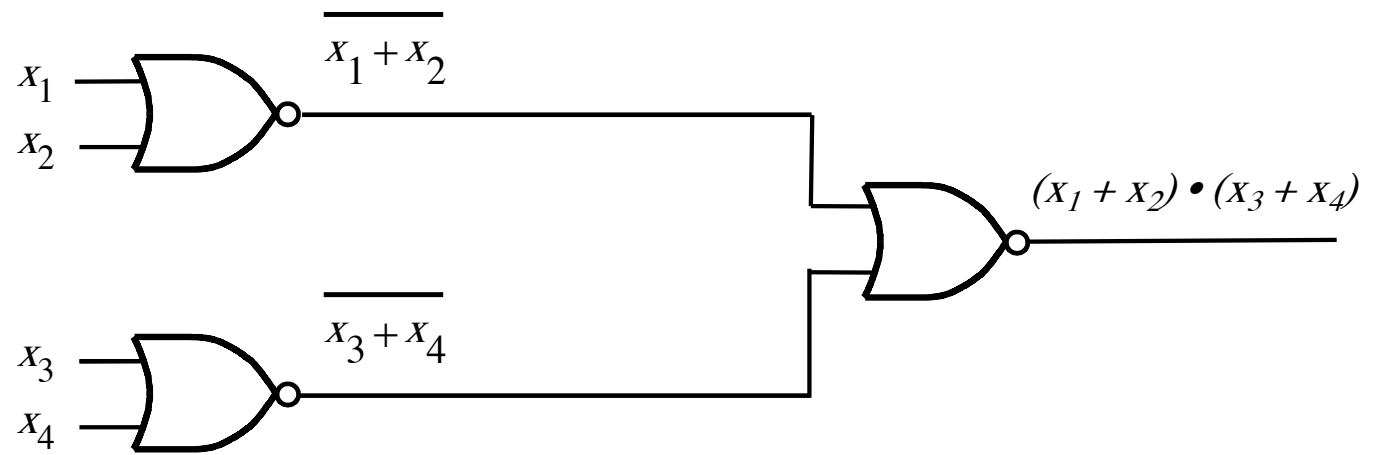
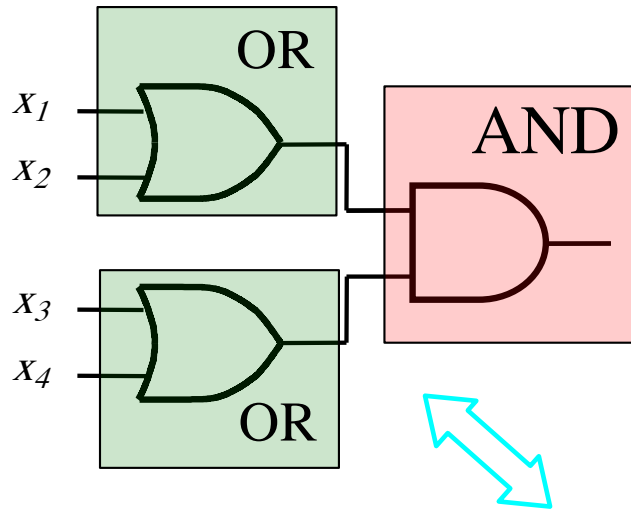


# Product-Of-Sums

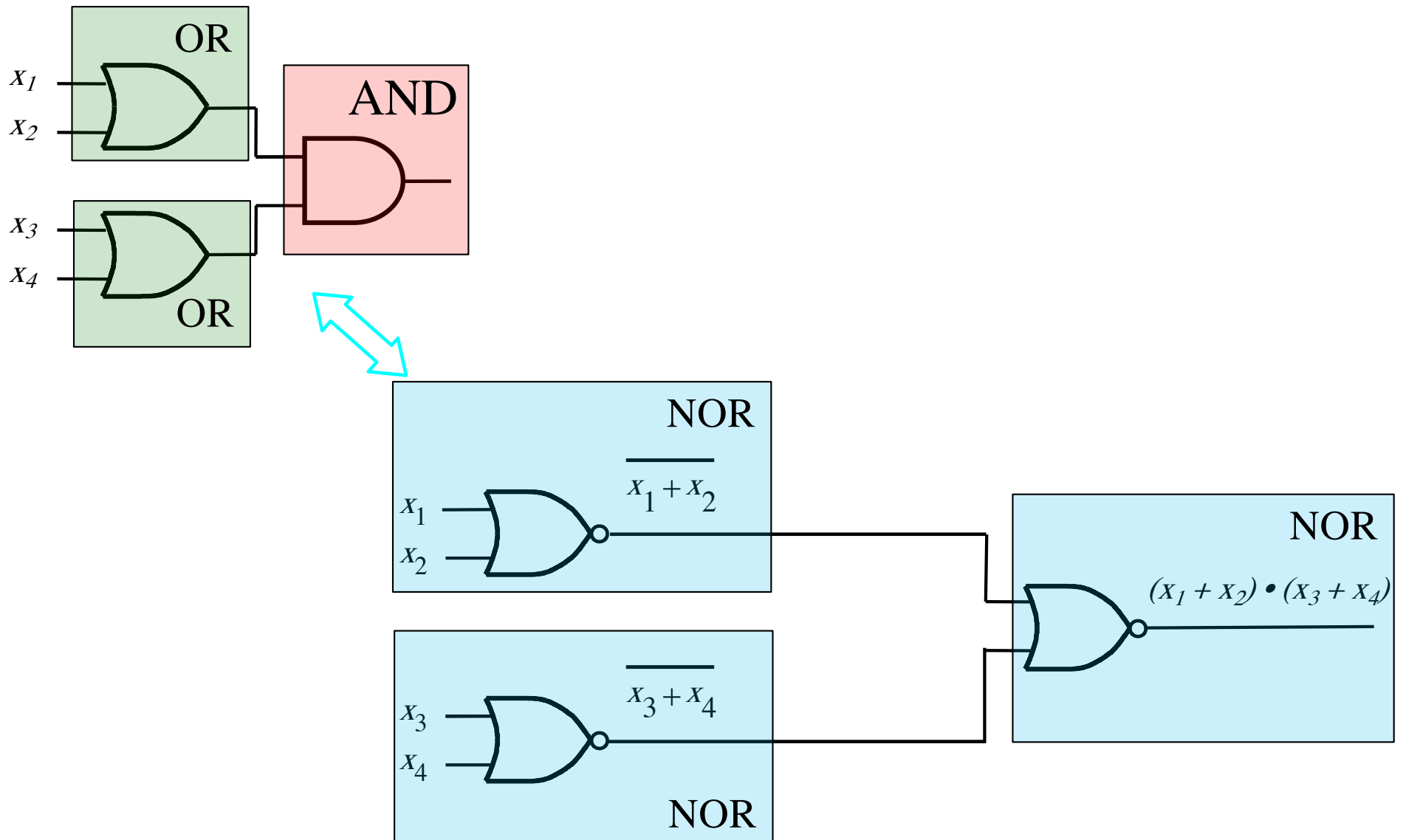




# Product-Of-Sums

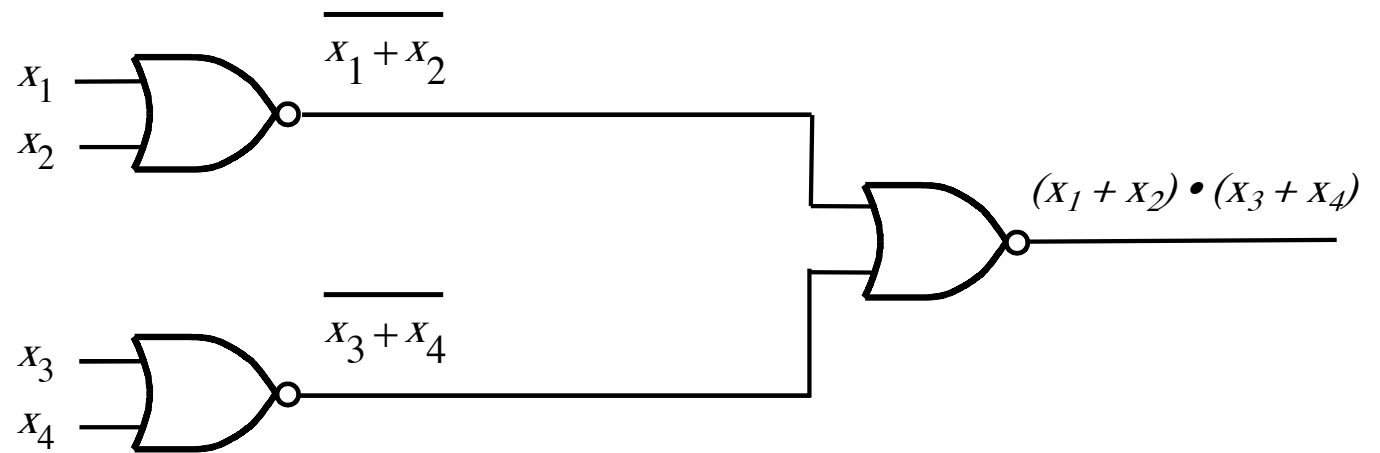
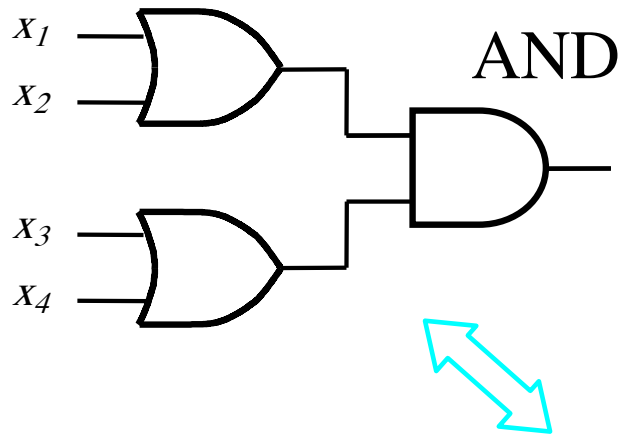


# Product-Of-Sums



This circuit uses only NORs

# Product-Of-Sums



This circuit uses only NORs

# **Another Synthesis Example**

# Truth table for a three-way light control

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

# Let's Derive the SOP form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

# Let's Derive the SOP form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



# Let's Derive the SOP form

$x_1$	$x_2$	$x_3$	$f$	
0	0	0	0	
0	0	1	1	$x_1 x_2 x_3$
0	1	0	1	$x_1 x_2 x_3$
0	1	1	0	
1	0	0	1	$x_1 x_2 x_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1 x_2 x_3$

# Let's Derive the SOP form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$\overline{x}_1 \overline{x}_2 x_3$$

$$\overline{x}_1 x_2 \overline{x}_3$$

$$x_1 \overline{x}_2 \overline{x}_3$$

$$x_1 x_2 x_3$$

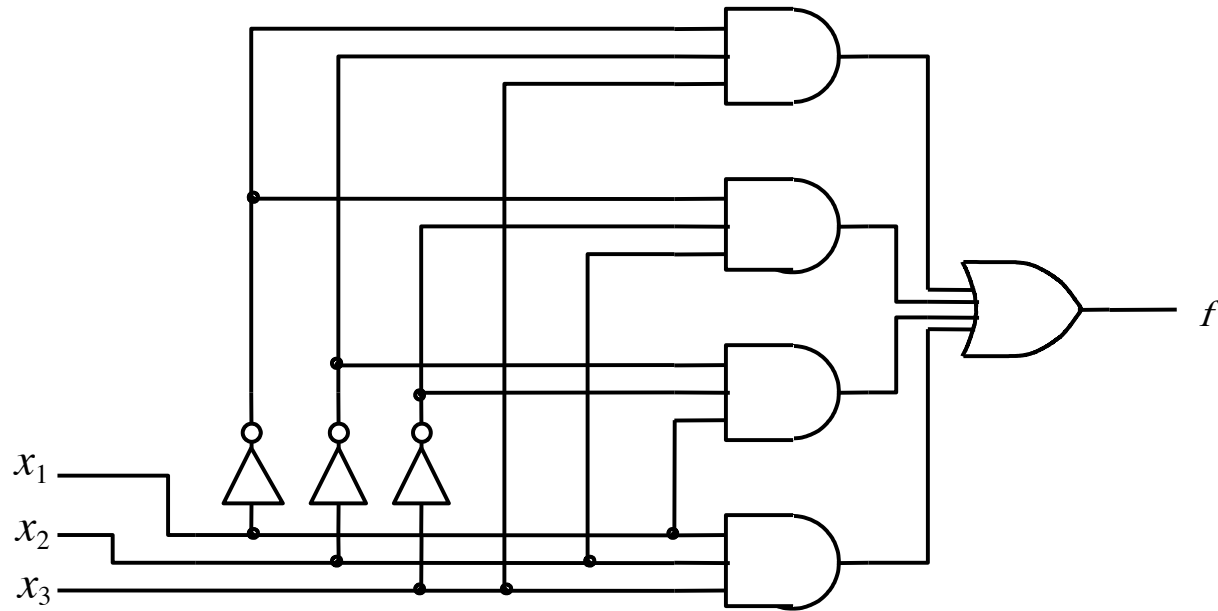
# Let's Derive the SOP form

$x_1$	$x_2$	$x_3$	$f$	
0	0	0	0	
0	0	1	1	$\bar{x}_1 \bar{x}_2 x_3$
0	1	0	1	$\bar{x}_1 x_2 \bar{x}_3$
0	1	1	0	
1	0	0	1	$x_1 \bar{x}_2 \bar{x}_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1 x_2 x_3$

$$f = m_1 + m_2 + m_4 + m_7$$

$$= \bar{x}_1 \bar{x}_2 x_3 + \bar{x}_1 x_2 \bar{x}_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 x_3$$

# Sum-of-products realization



[ Figure 2.32a from the textbook ]

# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

# Minterms and Maxterms (with three variables)

Row number	$x_1$	$x_2$	$x_3$	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + x_2 + x_3)$
1	0	0	1	
1	0	1	0	$(x_1 + x_2 + x_3)$
1	1	0	0	$(x_1 + x_2 + x_3)$
1	1	1	1	

# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \bar{x}_2 + \bar{x}_3)$
1	0	0	1	
1	0	1	0	$(\bar{x}_1 + x_2 + \bar{x}_3)$
1	1	0	0	$(\bar{x}_1 + \bar{x}_2 + x_3)$
1	1	1	1	

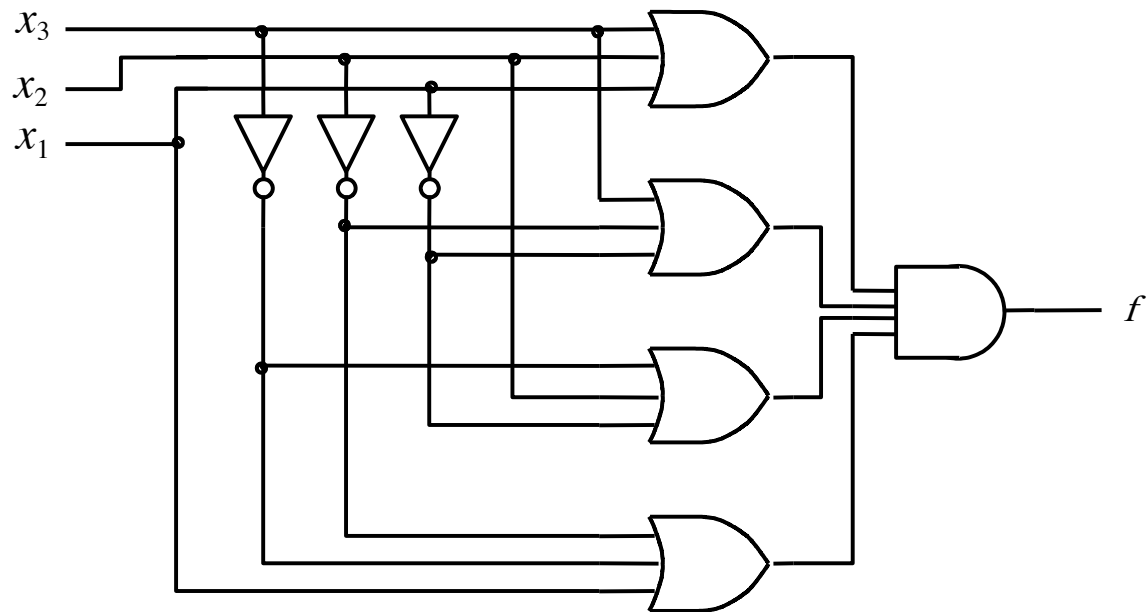
# Let's Derive the POS form

$x_1$	$x_2$	$x_3$	$f$	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \bar{x}_2 + \bar{x}_3)$
1	0	0	1	
1	0	1	0	$(\bar{x}_1 + x_2 + \bar{x}_3)$
1	1	0	0	$(\bar{x}_1 + \bar{x}_2 + x_3)$
1	1	1	1	

$$f = M_0 \cdot M_3 \cdot M_5 \cdot M_6$$

$$= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3)$$

# Product-of-sums realization

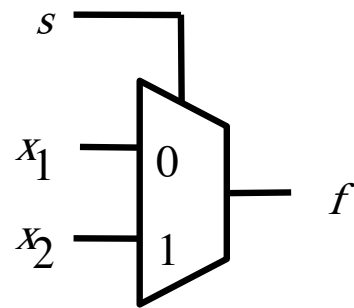


# Multiplexers

## **2-1 Multiplexer (Definition)**

- **Has two inputs:  $x_1$  and  $x_2$**
- **Also has another input line  $s$**
- **If  $s=0$ , then the output is equal to  $x_1$**
- **If  $s=1$ , then the output is equal to  $x_2$**

# Graphical Symbol for a 2-1 Multiplexer

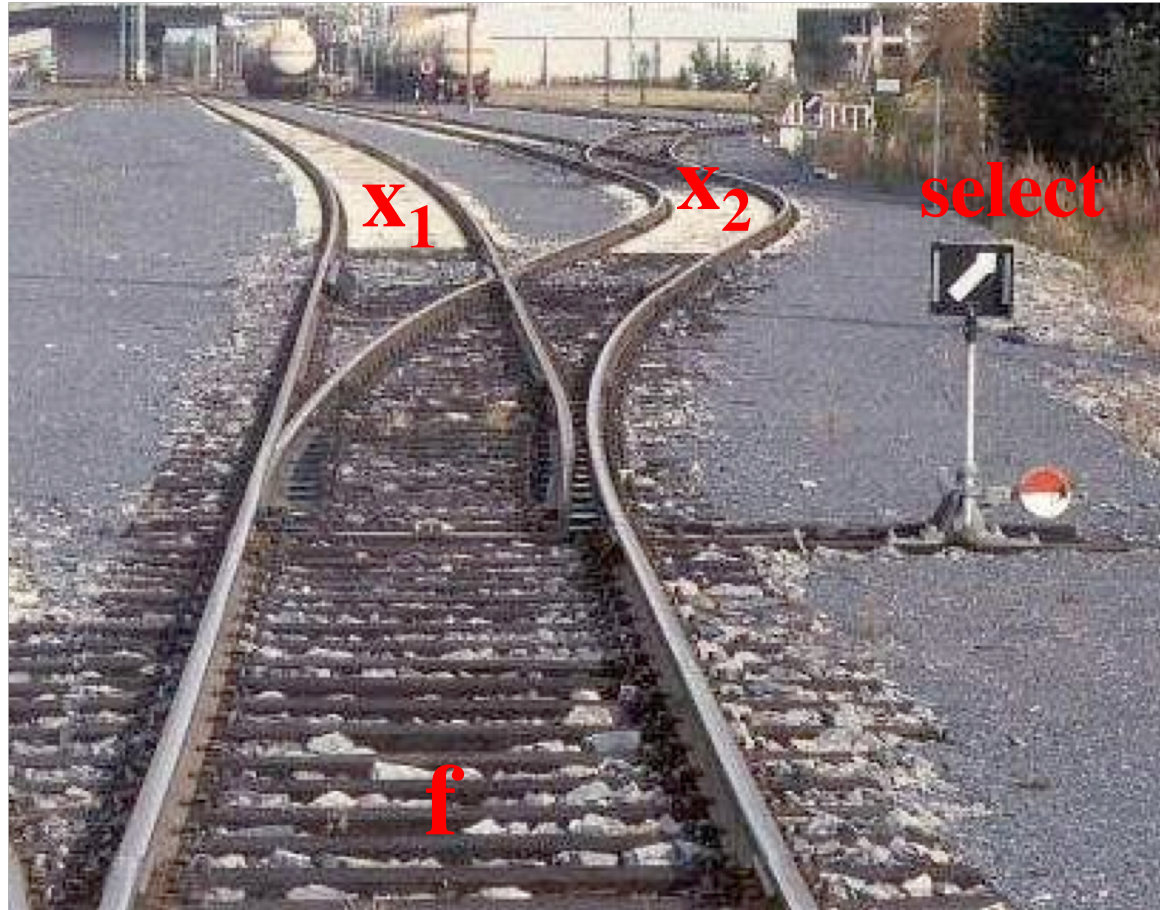


# Analogy: Railroad Switch

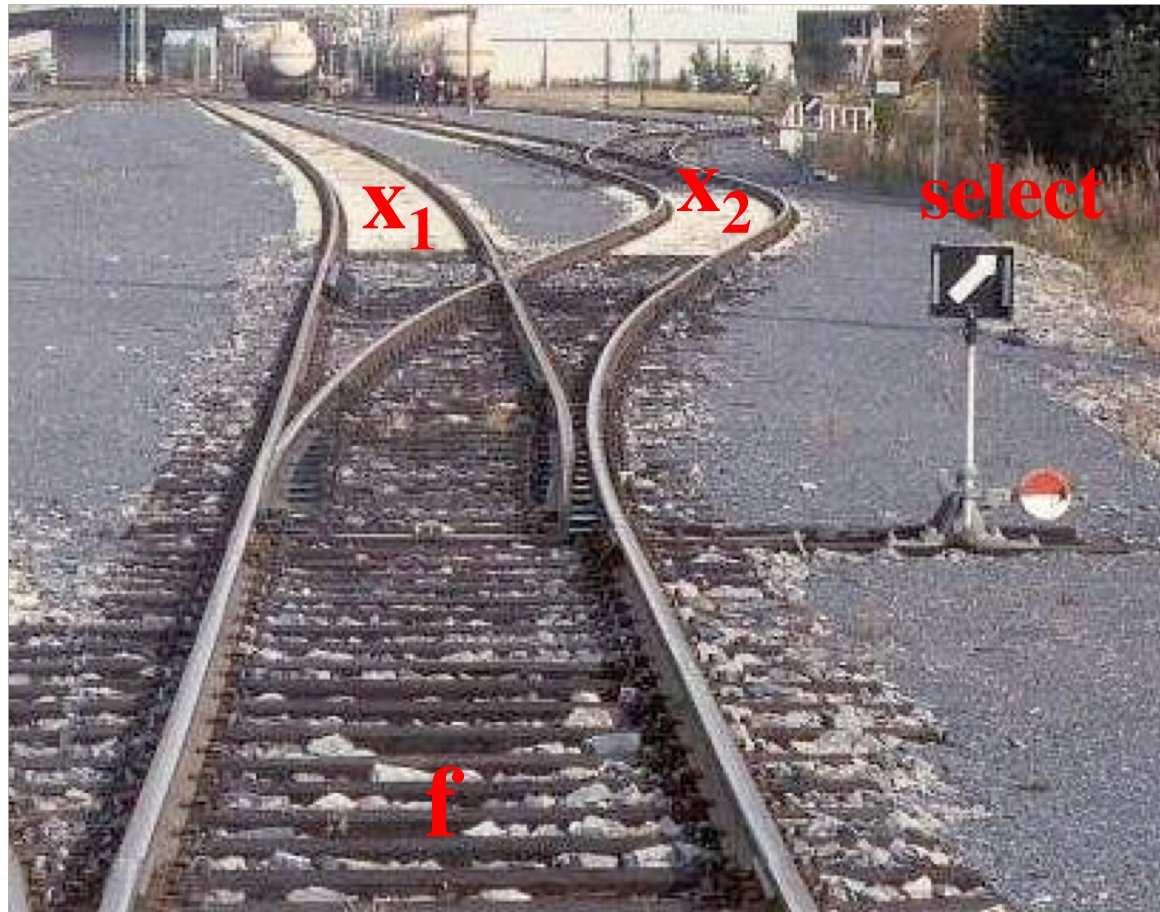




# Analogy: Railroad Switch



# Analogy: Railroad Switch



This is not a perfect analogy because the trains can go in either direction, while the multiplexer would only allow them to go from top to bottom.

# Truth Table for a 2-1 Multiplexer

$s$ $x_1$ $x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

# Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

# Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

# Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Where should we  
put the negation signs?

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

$s \ x_1 \ x_2$

# Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\bar{s} \ x_1 \ \bar{x}_2$
0 1 1	1	$\bar{s} \ x_1 \ x_2$
1 0 0	0	
1 0 1	1	$s \ \bar{x}_1 \ x_2$
1 1 0	0	
1 1 1	1	$s \ x_1 \ x_2$

# Let's Derive the SOP form

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\bar{s} x_1 \bar{x}_2$
0 1 1	1	$\bar{s} x_1 x_2$
1 0 0	0	
1 0 1	1	$s \bar{x}_1 x_2$
1 1 0	0	
1 1 1	1	$s x_1 x_2$

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$



**Let's simplify this expression**

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

# Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

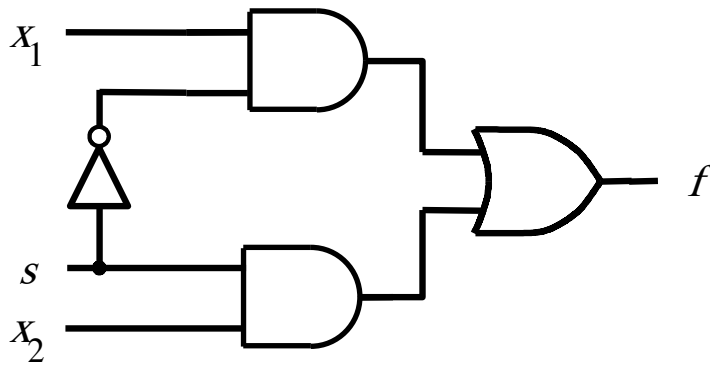
# Let's simplify this expression

$$f(s, x_1, x_2) = \bar{s} x_1 \bar{x}_2 + \bar{s} x_1 x_2 + s \bar{x}_1 x_2 + s x_1 x_2$$

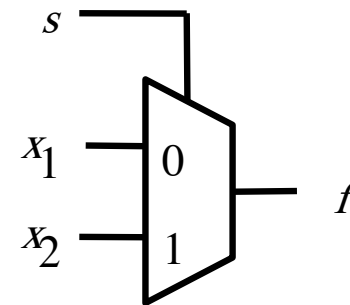
$$f(s, x_1, x_2) = \bar{s} x_1 (\bar{x}_2 + x_2) + s (\bar{x}_1 + x_1) x_2$$

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

# Circuit for 2-1 Multiplexer



(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s}x_1 + sx_2$$

# More Compact Truth-Table Representation

$s \ x_1 \ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

(a) Truth table

$s$	$f(s, x_1, x_2)$
0	$x_1$
1	$x_2$

# 4-1 Multiplexer (Definition)

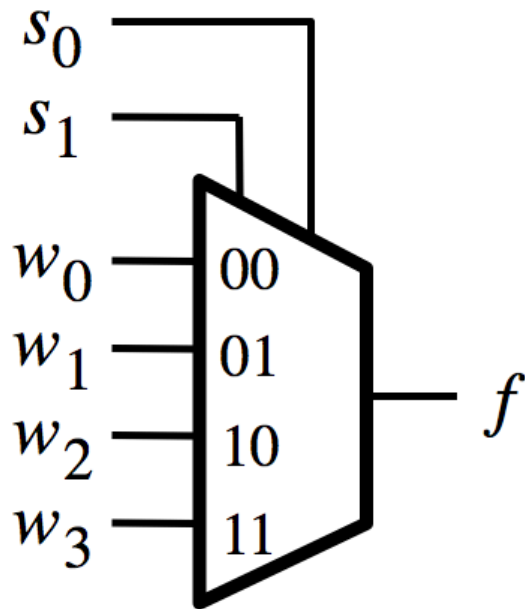
- Has four inputs:  $w_0$ ,  $w_1$ ,  $w_2$ ,  $w_3$
- Also has two select lines:  $s_1$  and  $s_0$
- If  $s_1=0$  and  $s_0=0$ , then the output  $f$  is equal to  $w_0$
- If  $s_1=0$  and  $s_0=1$ , then the output  $f$  is equal to  $w_1$
- If  $s_1=1$  and  $s_0=0$ , then the output  $f$  is equal to  $w_2$
- If  $s_1=1$  and  $s_0=1$ , then the output  $f$  is equal to  $w_3$

# 4-1 Multiplexer (Definition)

- Has four inputs:  $w_0$ ,  $w_1$ ,  $w_2$ ,  $w_3$
- Also has two select lines:  $s_1$  and  $s_0$
- If  $s_1=0$  and  $s_0=0$ , then the output  $f$  is equal to  $w_0$
- If  $s_1=0$  and  $s_0=1$ , then the output  $f$  is equal to  $w_1$
- If  $s_1=1$  and  $s_0=0$ , then the output  $f$  is equal to  $w_2$
- If  $s_1=1$  and  $s_0=1$ , then the output  $f$  is equal to  $w_3$

We'll talk more about this when we get to chapter 4, but here is a quick preview.

# Graphical Symbol and Truth Table



(a) Graphic symbol

$s_1$	$s_0$	$f$
0	0	$w_0$
0	1	$w_1$
1	0	$w_2$
1	1	$w_3$

(b) Truth table



# The long-form truth table

# The long-form truth table

$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F
0 0	0 0 0 0	0	0 1	0 0 0 0	0	1 0	0 0 0 0	0	1 1	0 0 0 0	0
	0 0 0 1	1		0 0 0 1	0		0 0 0 1	0		0 0 0 1	0
	0 0 1 0	0		0 0 1 0	1		0 0 1 0	0		0 0 1 0	0
	0 0 1 1	1		0 0 1 1	1		0 0 1 1	0		0 0 1 1	0
	0 1 0 0	0		0 1 0 0	0		0 1 0 0	1		0 1 0 0	0
	0 1 0 1	1		0 1 0 1	0		0 1 0 1	1		0 1 0 1	0
	0 1 1 0	0		0 1 1 0	1		0 1 1 0	1		0 1 1 0	0
	0 1 1 1	1		0 1 1 1	1		0 1 1 1	1		0 1 1 1	0
	1 0 0 0	0		1 0 0 0	0		1 0 0 0	0		1 0 0 0	1
	1 0 0 1	1		1 0 0 1	0		1 0 0 1	0		1 0 0 1	1
	1 0 1 0	0		1 0 1 0	1		1 0 1 0	0		1 0 1 0	1
	1 0 1 1	1		1 0 1 1	1		1 0 1 1	0		1 0 1 1	1
	1 1 0 0	0		1 1 0 0	0		1 1 0 0	1		1 1 0 0	1
	1 1 0 1	1		1 1 0 1	0		1 1 0 1	1		1 1 0 1	1
	1 1 1 0	0		1 1 1 0	1		1 1 1 0	1		1 1 1 0	1
	1 1 1 1	1		1 1 1 1	1		1 1 1 1	1		1 1 1 1	1

# The long-form truth table

$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F
0 0	0	0	0	0	0	0 1	0	0	0	0	0	1 0	0	0	0	0	0	1 1	0	0	0	0	0
0 0	0	0	0	1	1	0 1	0	0	0	1	0	1 0	0	0	0	1	0	1 1	0	0	0	1	0
0 0	0	0	1	0	0	0 1	0	0	1	0	1	1 0	0	0	1	0	0	1 1	0	0	1	0	0
0 0	0	0	1	1	1	0 1	0	0	1	1	1	1 0	0	0	1	1	0	1 1	0	0	1	1	0
0 0	0	1	0	0	0	0 1	0	1	0	0	0	1 0	0	1	0	0	0	1 1	0	1	0	0	0
0 0	0	1	0	1	1	0 1	0	1	0	1	0	1 0	0	1	0	1	0	1 1	0	1	0	1	0
0 0	0	1	1	0	0	0 1	0	1	1	0	1	1 0	0	1	1	0	0	1 1	0	1	1	0	0
0 0	0	1	1	1	1	0 1	0	1	1	1	1	1 0	0	1	1	1	0	1 1	0	1	1	1	0
0 0	1	0	0	0	0	0 1	1	0	0	0	0	1 0	1	0	0	0	0	1 1	1	0	0	0	1
0 0	1	0	0	1	1	0 1	1	0	0	1	0	1 0	1	0	0	1	0	1 1	1	0	0	1	1
0 0	1	0	1	0	0	0 1	1	0	1	0	1	1 0	1	0	1	0	0	1 1	1	0	1	0	1
0 0	1	0	1	1	1	0 1	1	0	1	1	1	1 0	1	0	1	1	0	1 1	1	0	1	1	1
0 0	1	1	0	0	0	0 1	1	1	0	0	0	1 0	1	1	0	0	0	1 1	1	1	0	0	1
0 0	1	1	0	1	1	0 1	1	1	0	1	0	1 0	1	1	0	1	0	1 1	1	1	0	1	1
0 0	1	1	1	0	0	0 1	1	1	1	0	1	1 0	1	1	1	0	0	1 1	1	1	1	0	1
0 0	1	1	1	1	1	0 1	1	1	1	1	1	1 0	1	1	1	1	0	1 1	1	1	1	1	1

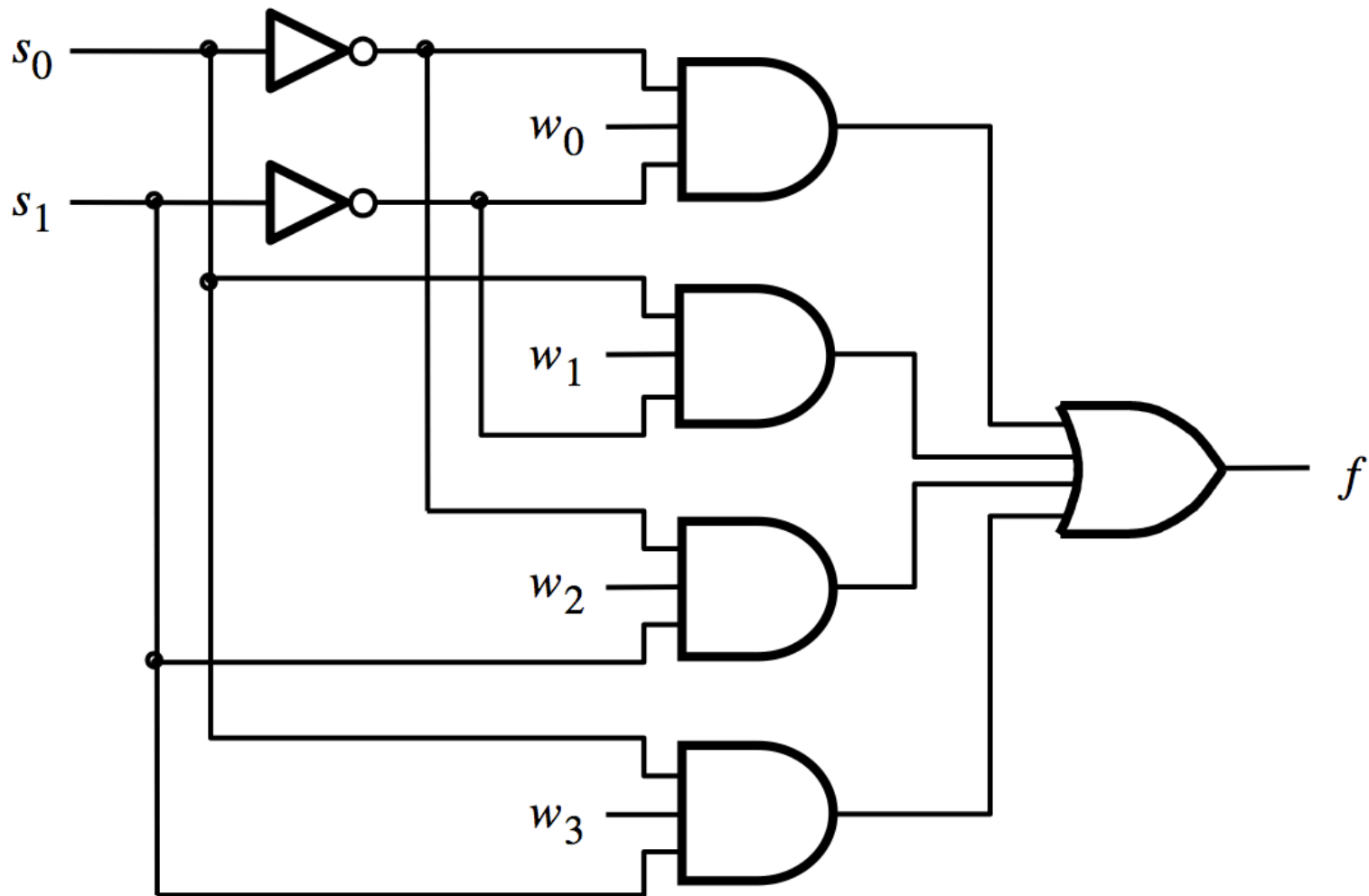
# The long-form truth table

$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F	$S_1 S_0$	$I_3$	$I_2$	$I_1$	$I_0$	F
0 0	0	0	0	0	0	0 1	0	0	0	0	0	1 0	0	0	0	0	0	1 1	0	0	0	0	0
	0	0	0	1	1		0	0	0	1	0		0	0	0	1	0		0	0	0	1	0
	0	0	1	0	0		0	0	1	0	1		0	0	1	0	0		0	0	1	0	0
	0	0	1	1	1		0	0	1	1	1		0	0	1	1	0		0	0	1	1	0
	0	1	0	0	0		0	1	0	0	0		0	1	0	0	1		0	1	0	0	0
	0	1	0	1	1		0	1	0	1	0		0	1	0	1	1		0	1	0	1	0
	0	1	1	0	0		0	1	1	0	1		0	1	1	0	1		0	1	1	0	0
	0	1	1	1	1		0	1	1	1	1		0	1	1	1	1		0	1	1	1	0
	1	0	0	0	0		1	0	0	0	0		1	0	0	0	0		1	0	0	0	1
	1	0	0	1	1		1	0	0	1	0		1	0	0	1	0		1	0	0	1	1
	1	0	1	0	0		1	0	1	0	1		1	0	1	0	0		1	0	1	0	1
	1	0	1	1	1		1	0	1	1	1		1	0	1	1	0		1	0	1	1	1
	1	1	0	0	0		1	1	0	0	0		1	1	0	0	1		1	1	0	0	1
	1	1	0	1	1		1	1	0	1	0		1	1	0	1	1		1	1	0	1	1
	1	1	1	0	0		1	1	1	0	1		1	1	1	0	1		1	1	1	0	1
	1	1	1	1	1		1	1	1	1	1		1	1	1	1	1		1	1	1	1	1

# The long-form truth table

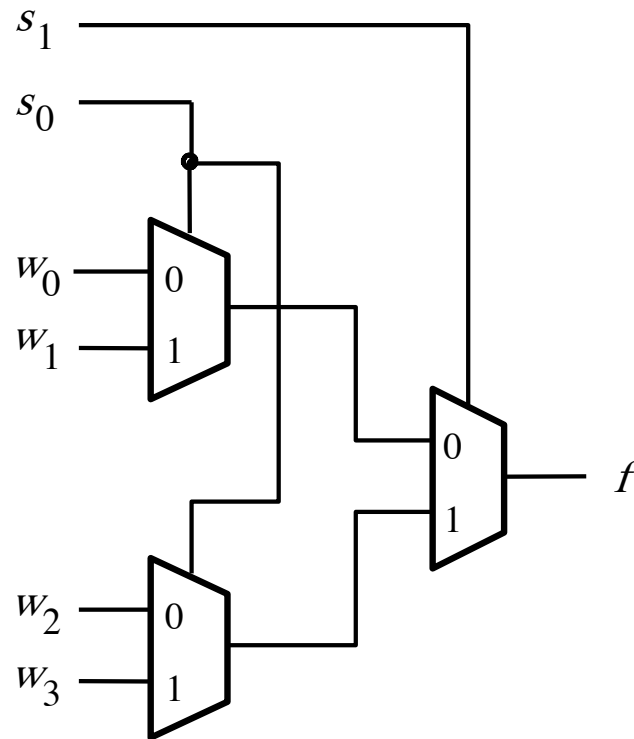
$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F	$S_1 S_0$	$I_3 I_2 I_1 I_0$	F
0 0	0 0 0 0	0	0 1	0 0 0 0	0	1 0	0 0 0 0	0	1 1	0 0 0 0	0
	0 0 0 1	1		0 0 0 1	0		0 0 0 1	0		0 0 0 1	0
	0 0 1 0	0		0 0 1 0	1		0 0 1 0	0		0 0 1 0	0
	0 0 1 1	1		0 0 1 1	1		0 0 1 1	0		0 0 1 1	0
	0 1 0 0	0		0 1 0 0	0		0 1 0 0	1		0 1 0 0	0
	0 1 0 1	1		0 1 0 1	0		0 1 0 1	1		0 1 0 1	0
	0 1 1 0	0		0 1 1 0	1		0 1 1 0	1		0 1 1 0	0
	0 1 1 1	1		0 1 1 1	1		0 1 1 1	1		0 1 1 1	0
	1 0 0 0	0		1 0 0 0	0		1 0 0 0	0		1 0 0 0	1
	1 0 0 1	1		1 0 0 1	0		1 0 0 1	0		1 0 0 1	1
	1 0 1 0	0		1 0 1 0	1		1 0 1 0	0		1 0 1 0	1
	1 0 1 1	1		1 0 1 1	1		1 0 1 1	0		1 0 1 1	1
	1 1 0 0	0		1 1 0 0	0		1 1 0 0	1		1 1 0 0	1
	1 1 0 1	1		1 1 0 1	0		1 1 0 1	1		1 1 0 1	1
	1 1 1 0	0		1 1 1 0	1		1 1 1 0	1		1 1 1 0	1
	1 1 1 1	1		1 1 1 1	1		1 1 1 1	1		1 1 1 1	1

# 4-1 Multiplexer (SOP circuit)



[ Figure 4.2c from the textbook ]

# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



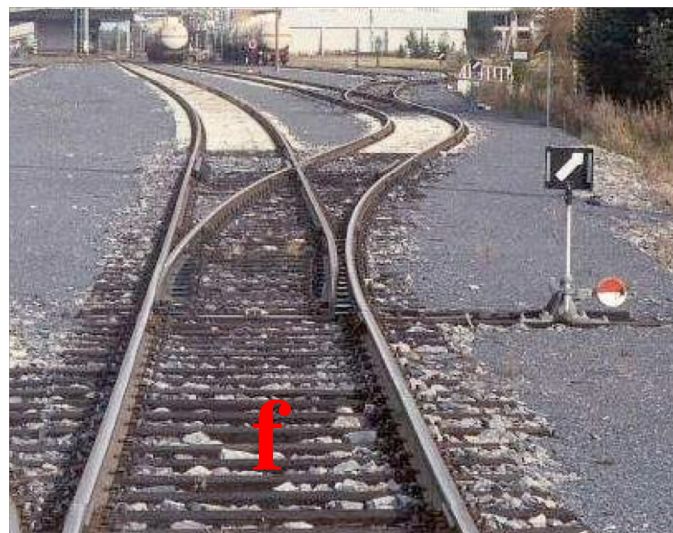
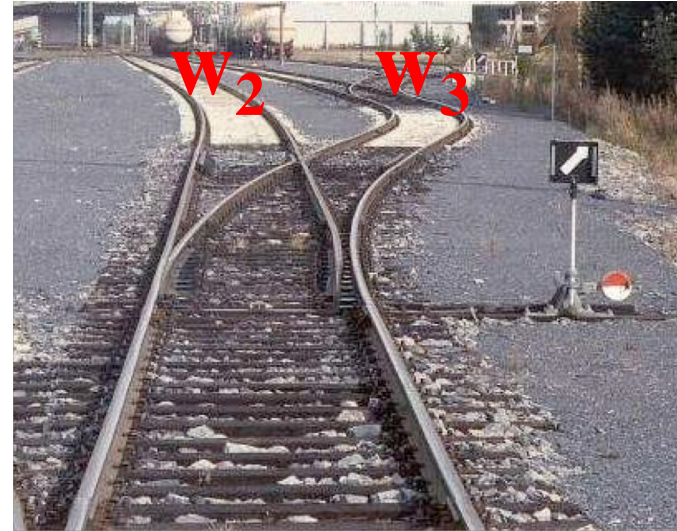
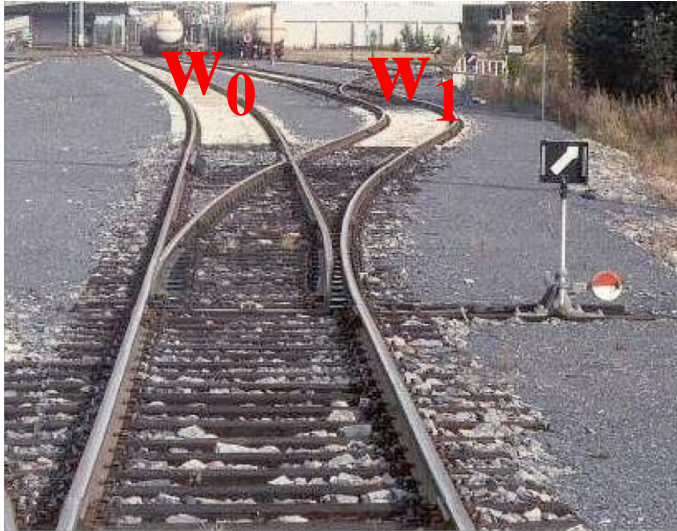
# Analogy: Railroad Switches



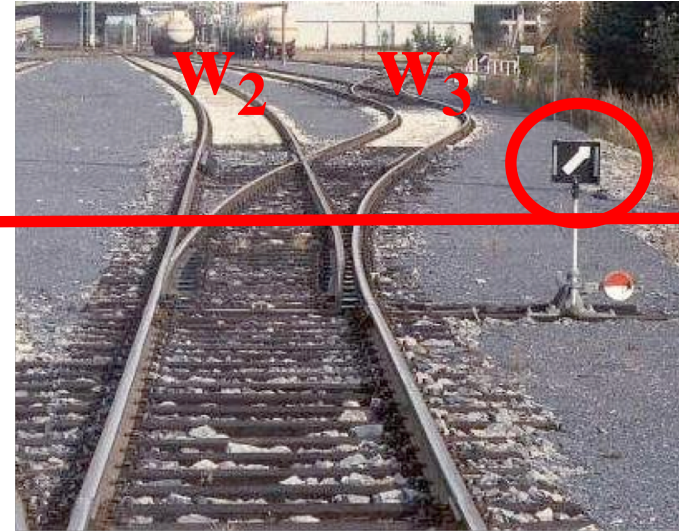
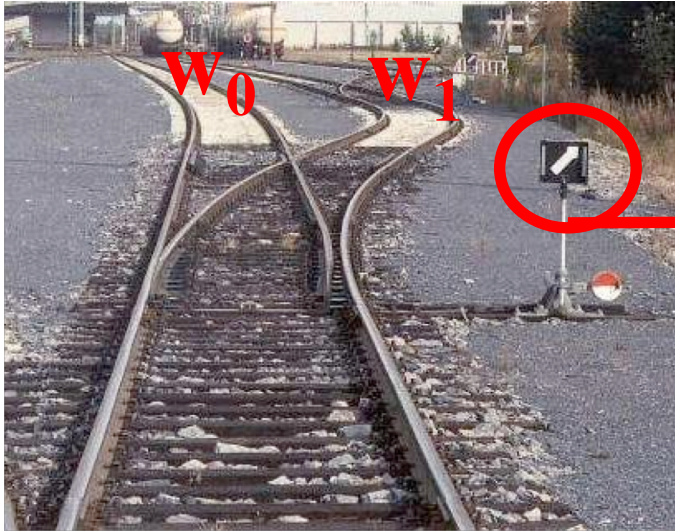
[http://en.wikipedia.org/wiki/Railroad\\_switch](http://en.wikipedia.org/wiki/Railroad_switch)]



# Analogy: Railroad Switches

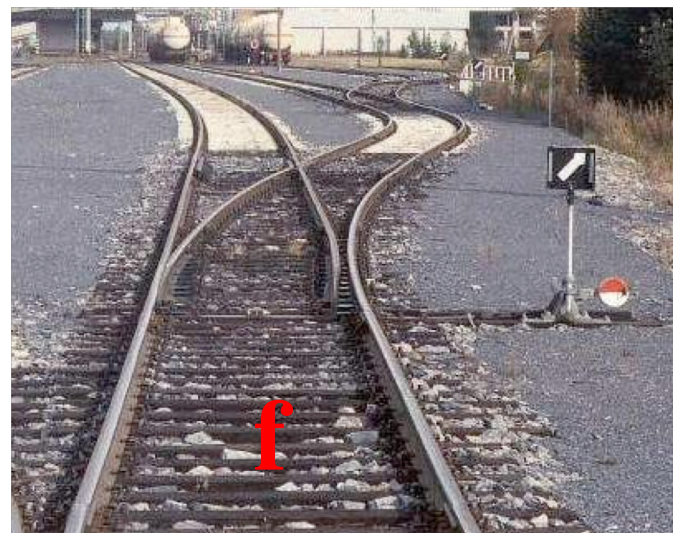


# Analogy: Railroad Switches



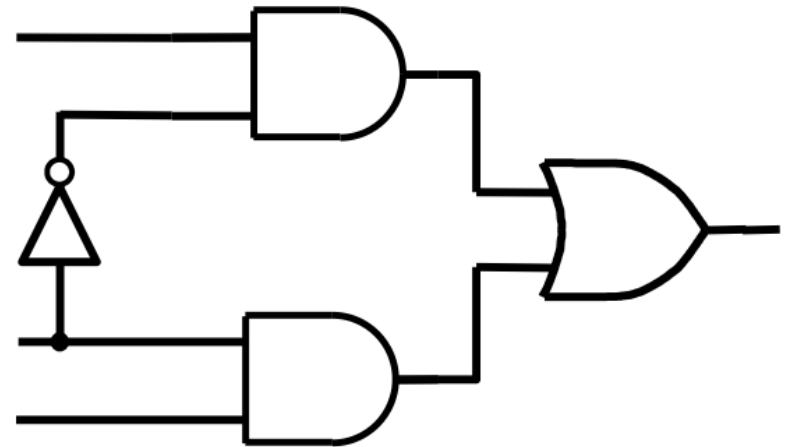
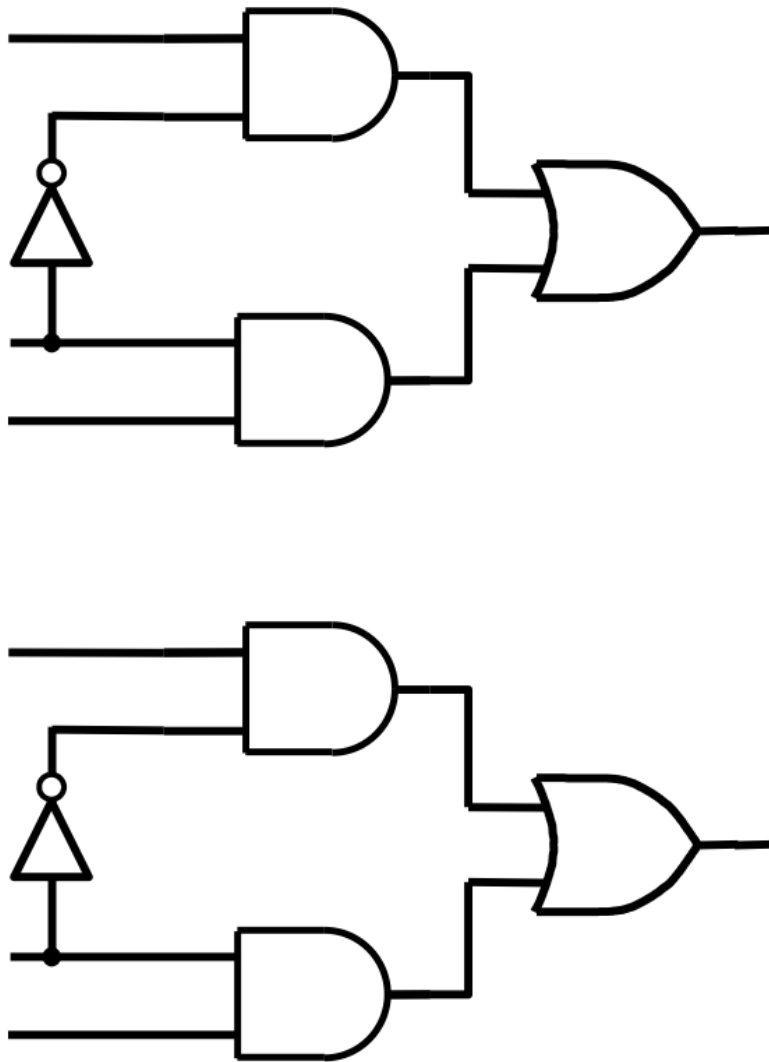
$S_0$

these two  
switches are  
controlled  
together

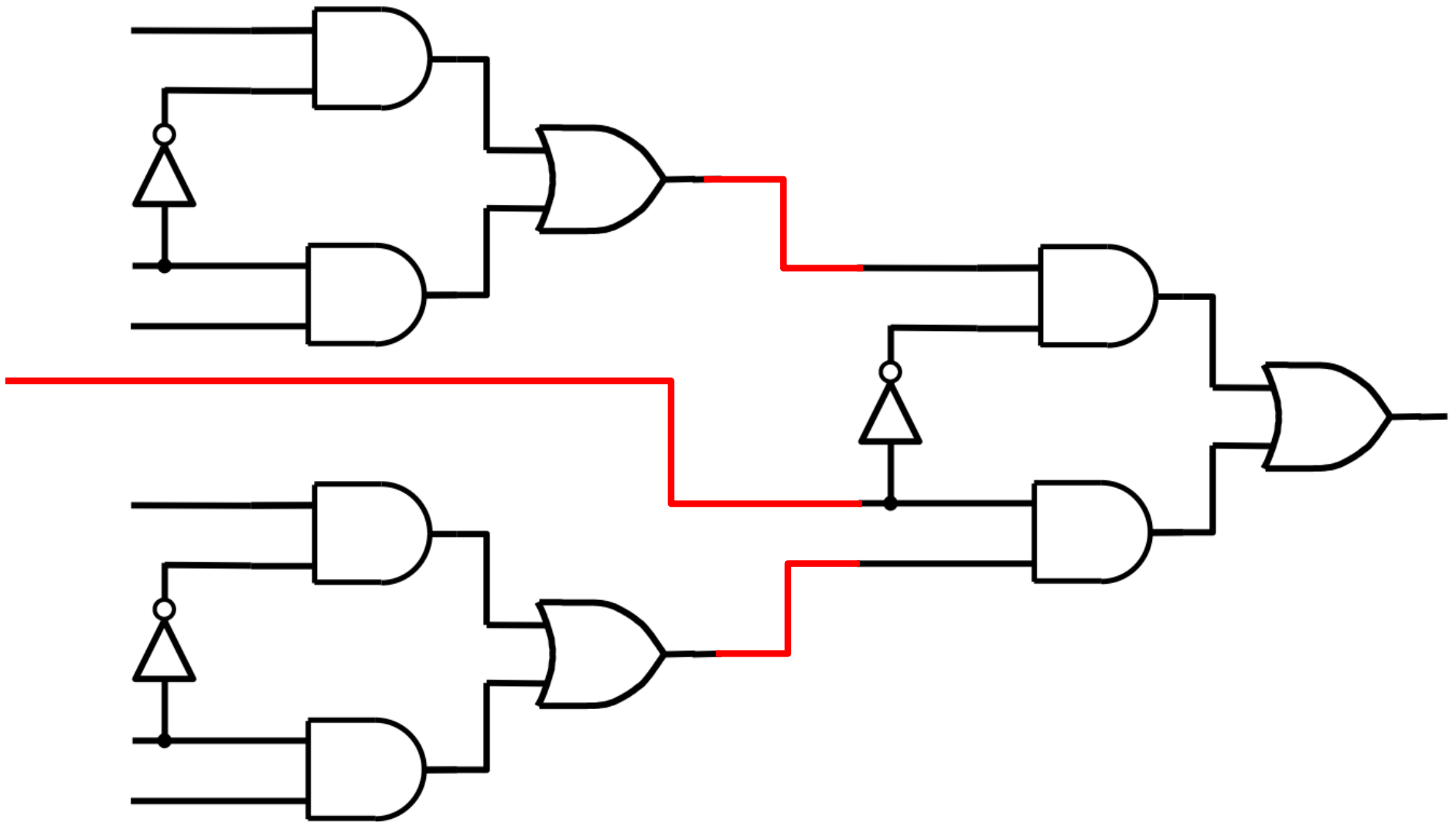


$S_1$

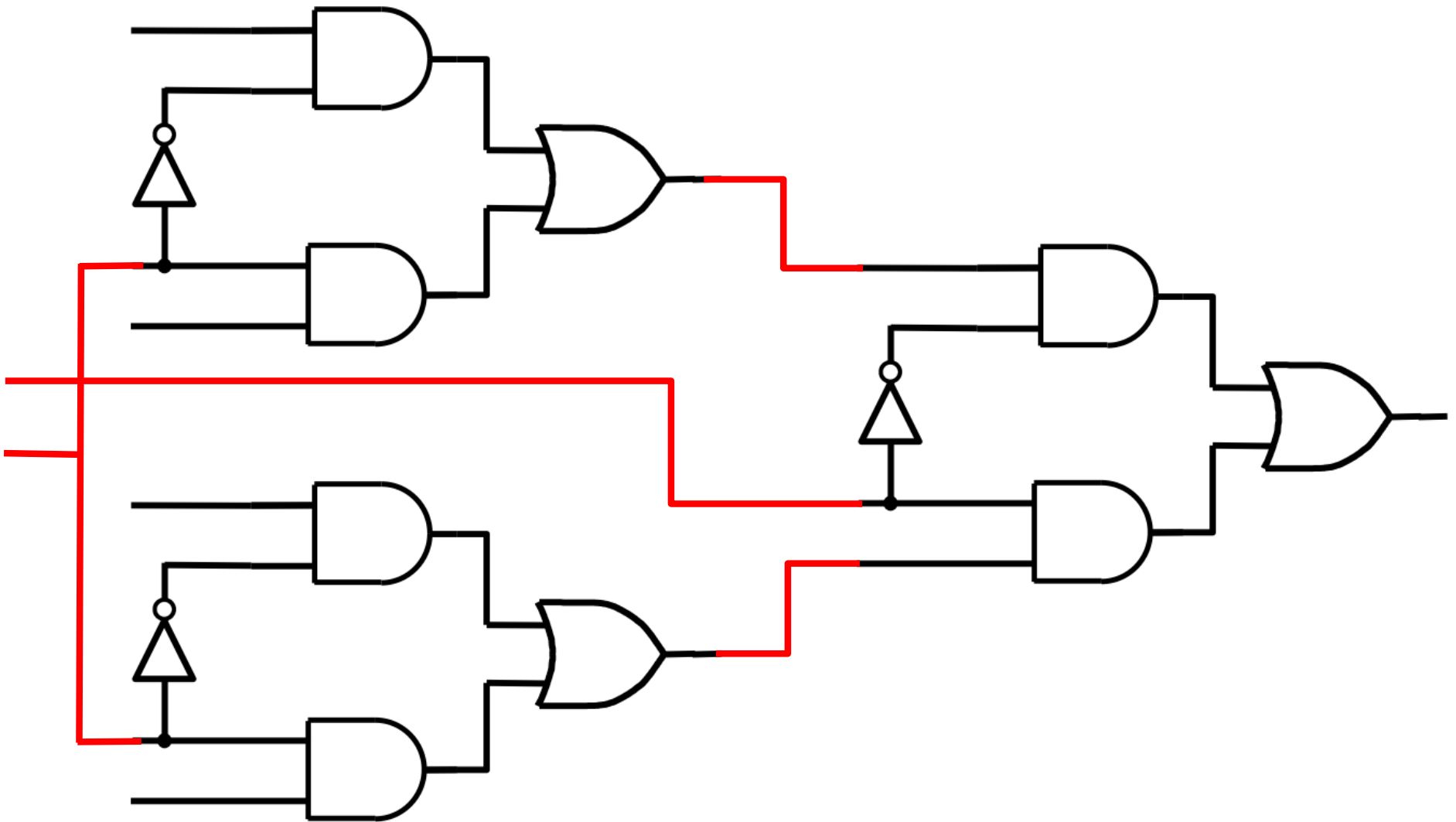
# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



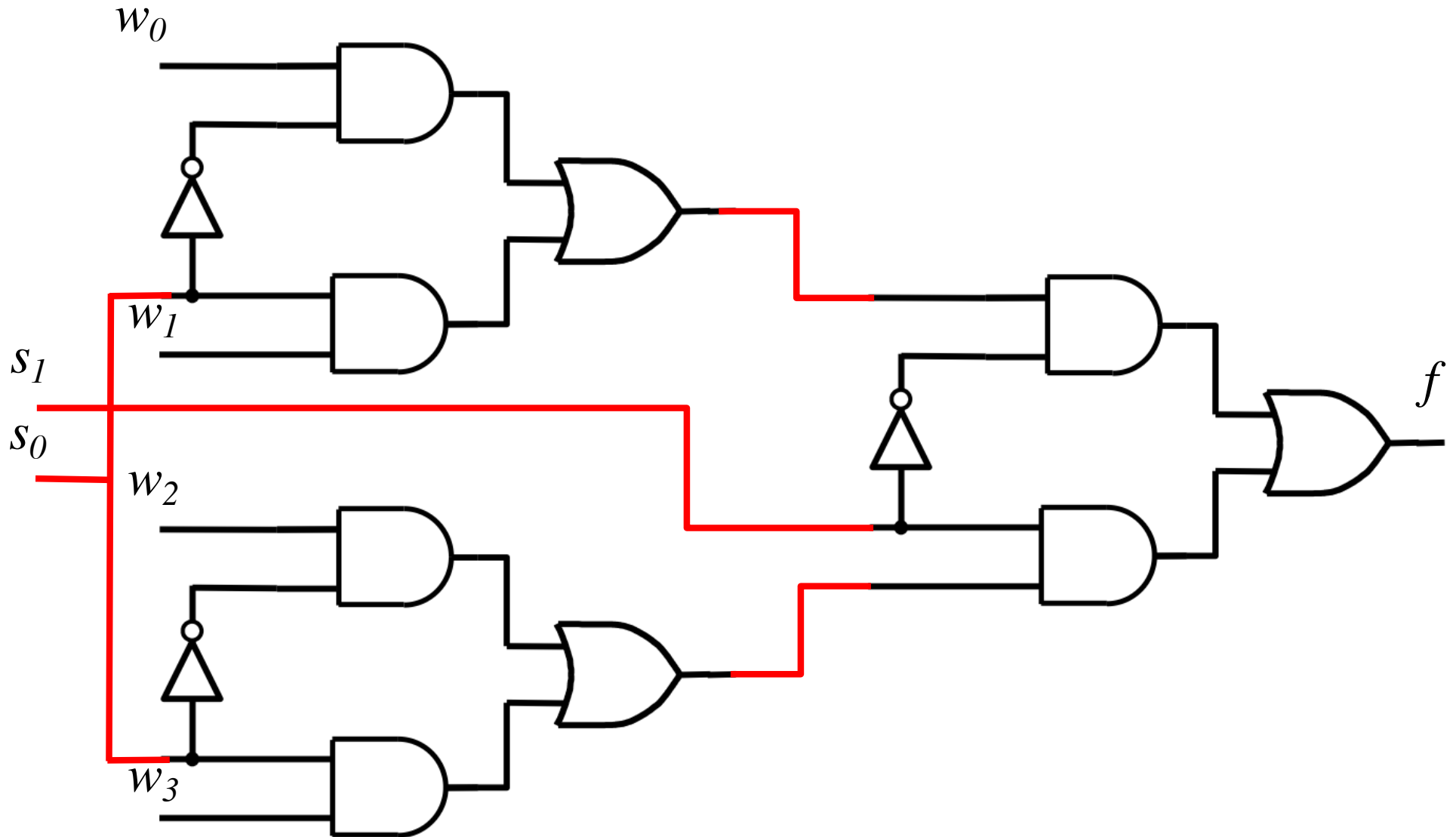
# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



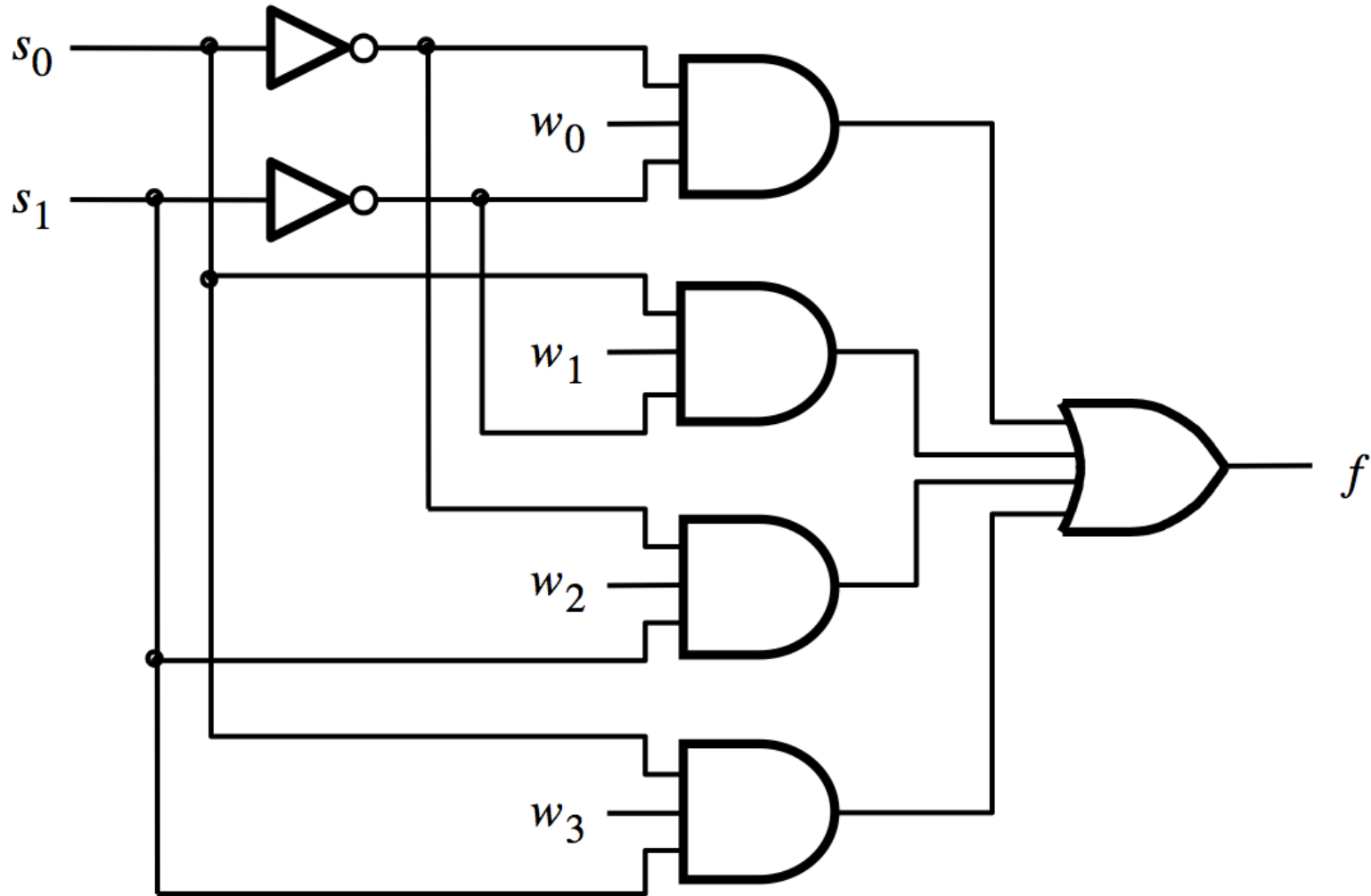
# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



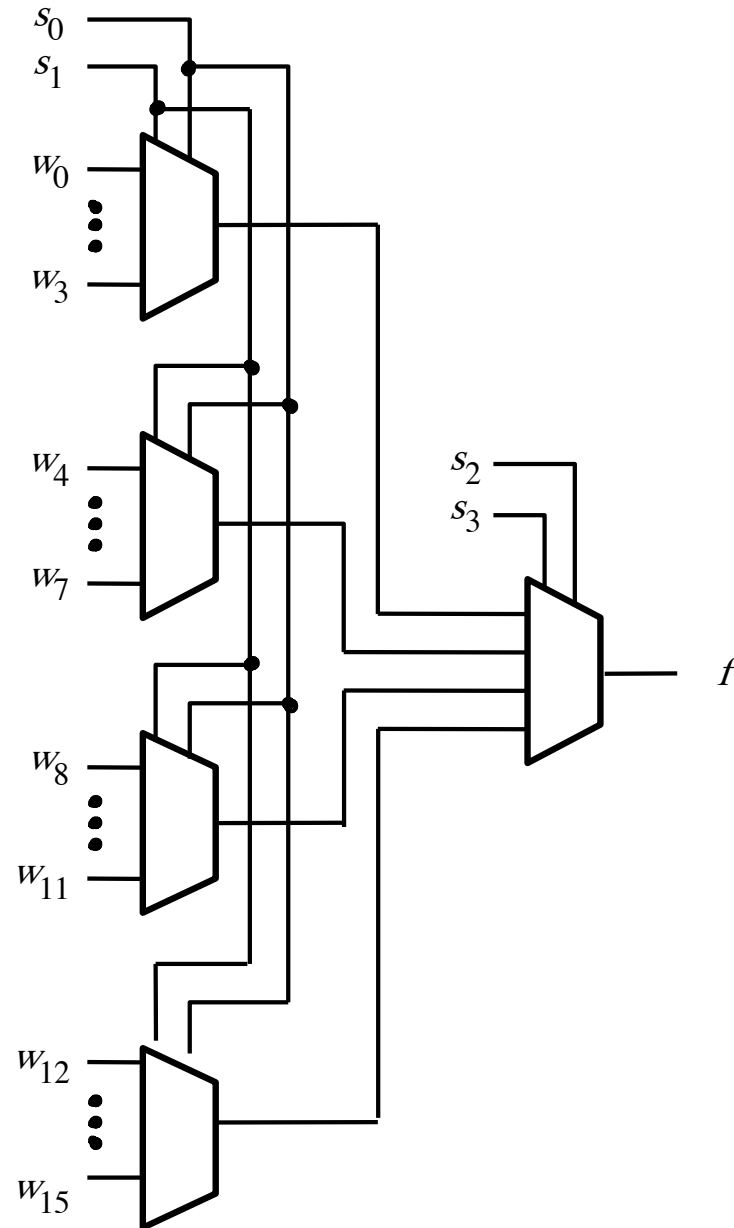
# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



**That is different from the SOP form of the 4-1 multiplexer shown below, which uses fewer gates**



# 16-1 Multiplexer



[ Figure 4.4 from the textbook ]





[<http://upload.wikimedia.org/wikipedia/commons/2/26/SunsetTracksCrop.JPG>]

**Questions?**

**THE END**