

CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Counters & Solved Problems

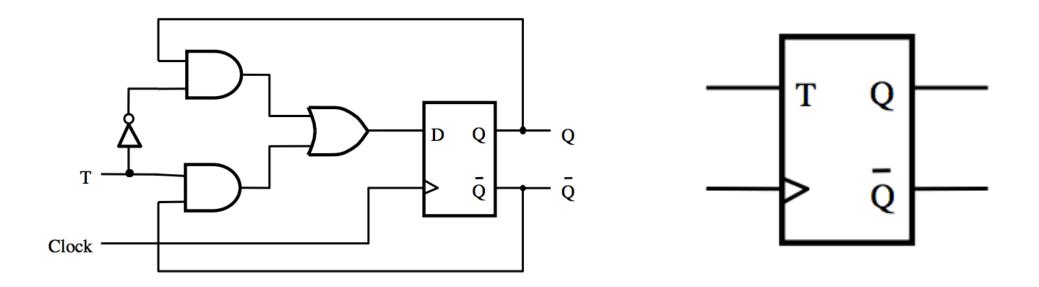
CprE 281: Digital Logic Iowa State University, Ames, IA Copyright © 2013

Administrative Stuff

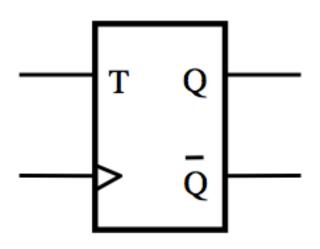
Homework 9 is due today

Counters

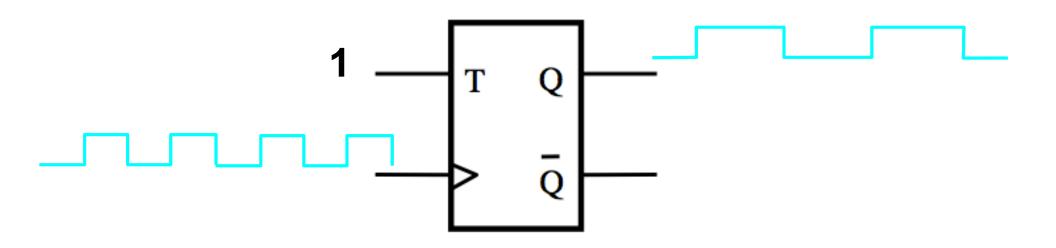
T Flip-Flop (circuit and graphical symbol)

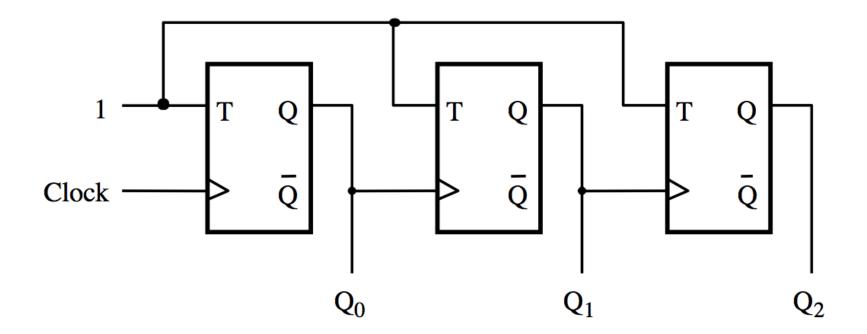


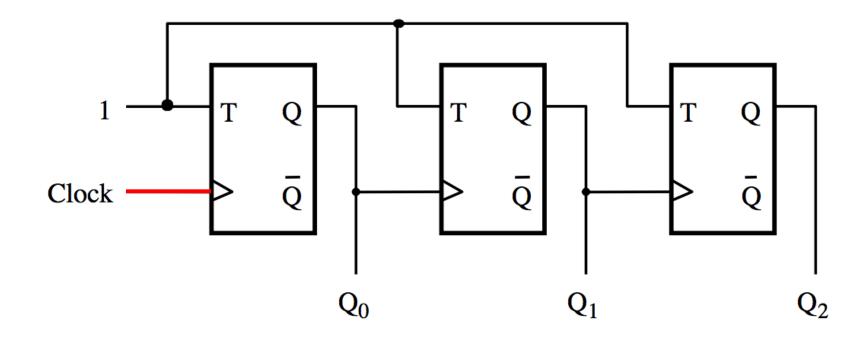
The output of the T Flip-Flop divides the frequency of the clock by 2



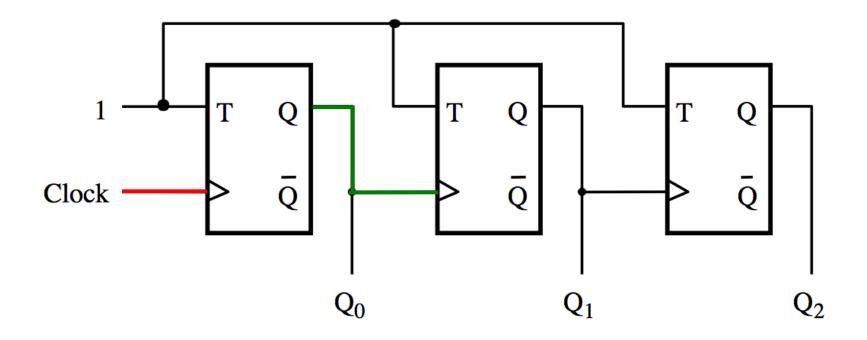
The output of the T Flip-Flop divides the frequency of the clock by 2





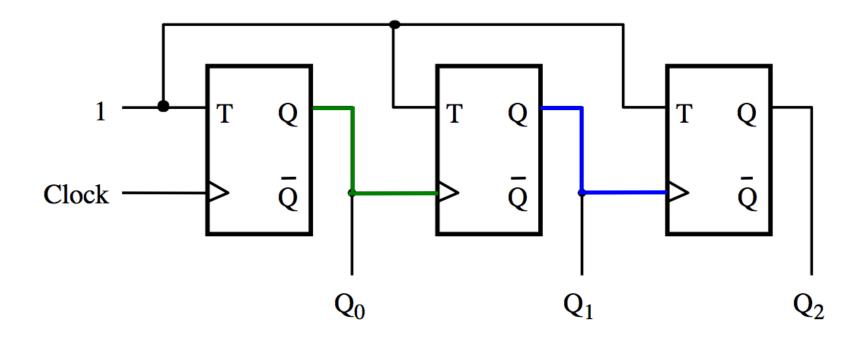


The first flip-flop changes on the positive edge of the clock



The first flip-flop changes on the positive edge of the clock

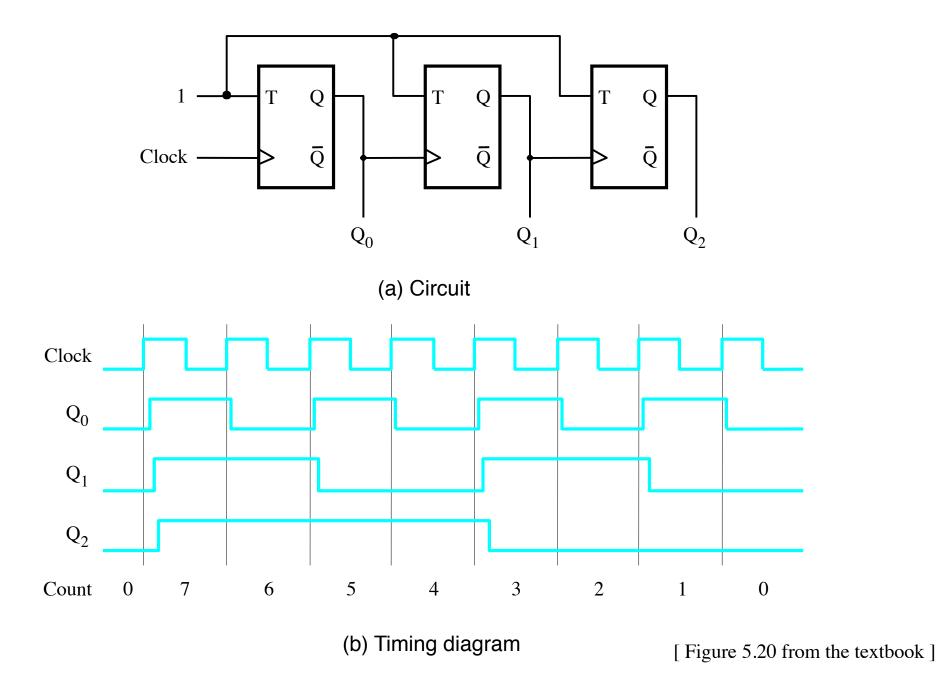
The second flip-flop changes on the positive edge of Q_0

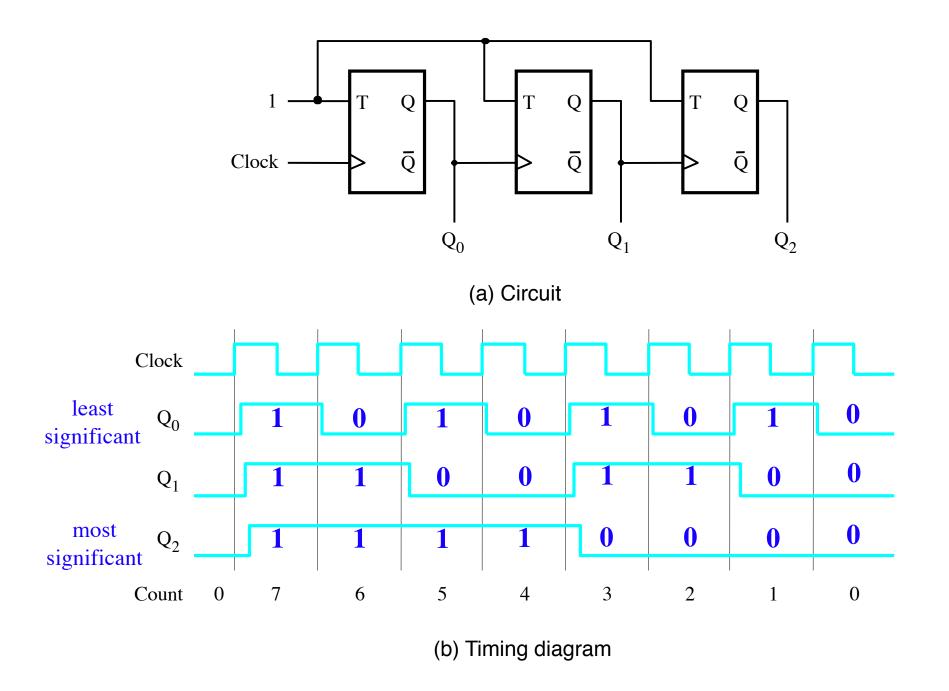


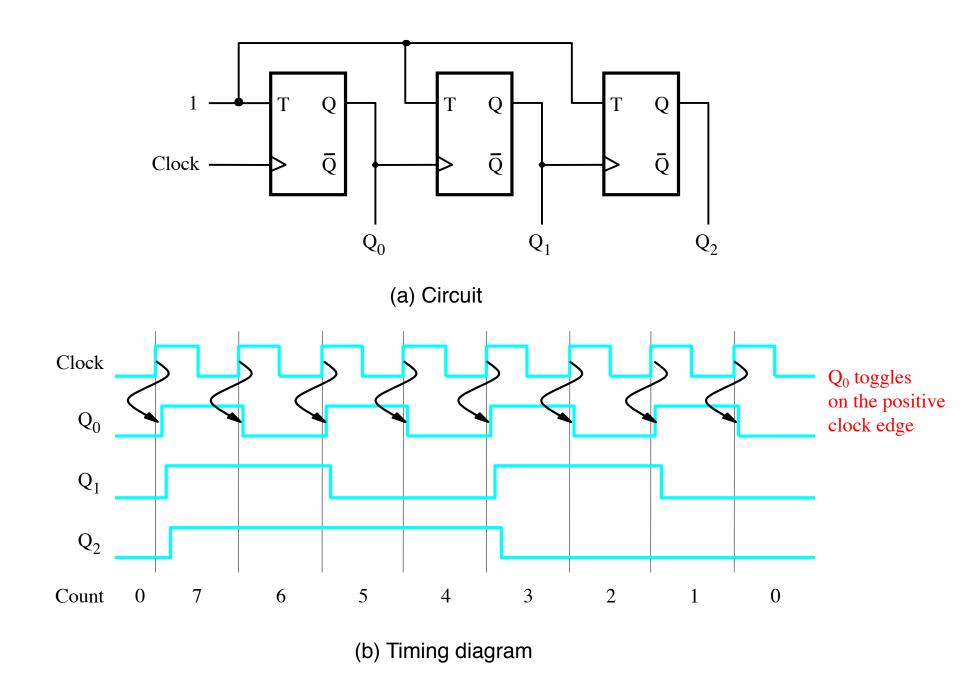
The first flip-flop changes on the positive edge of the clock

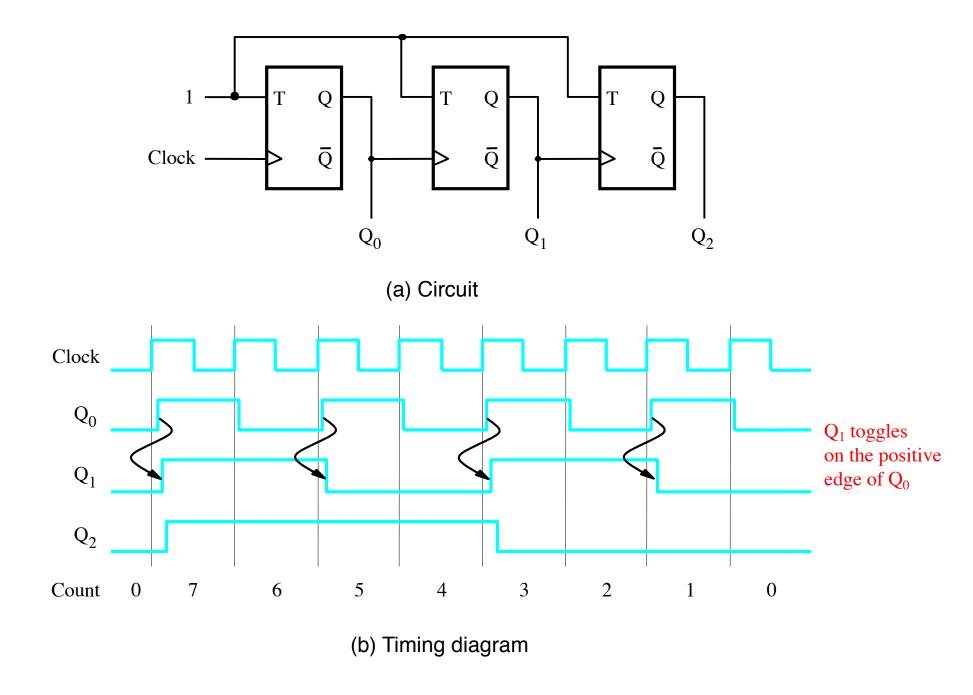
The second flip-flop changes on the positive edge of Q_0

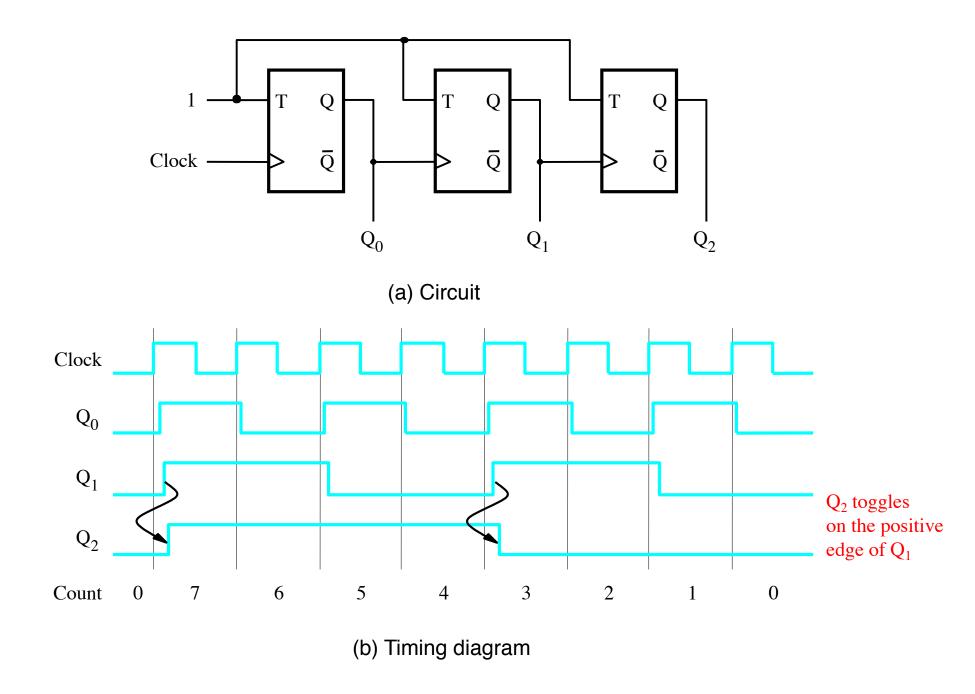
The third flip-flop changes on the positive edge of Q_1

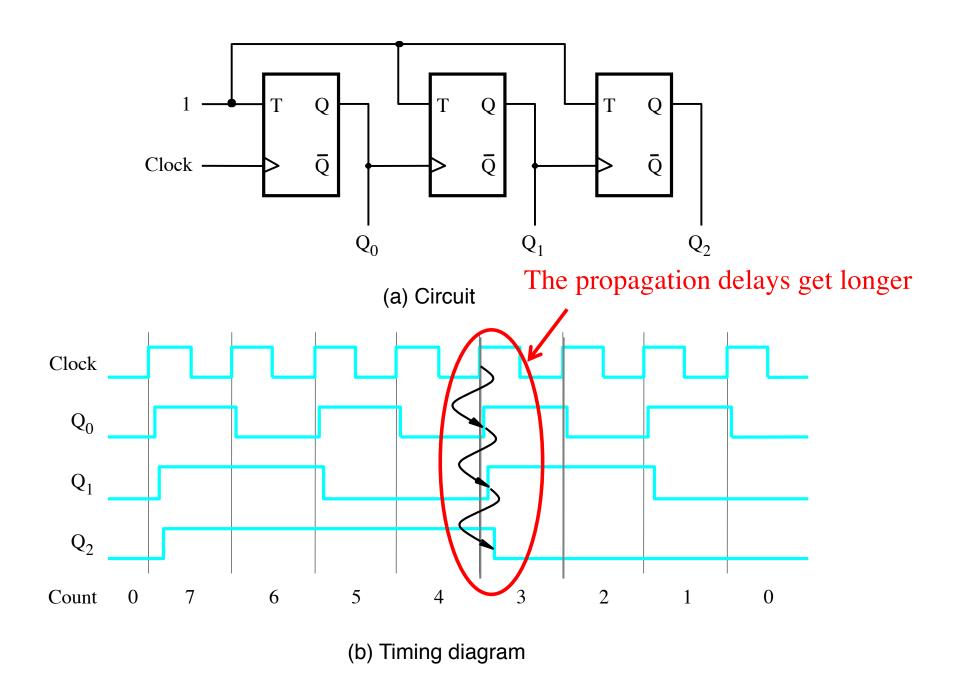


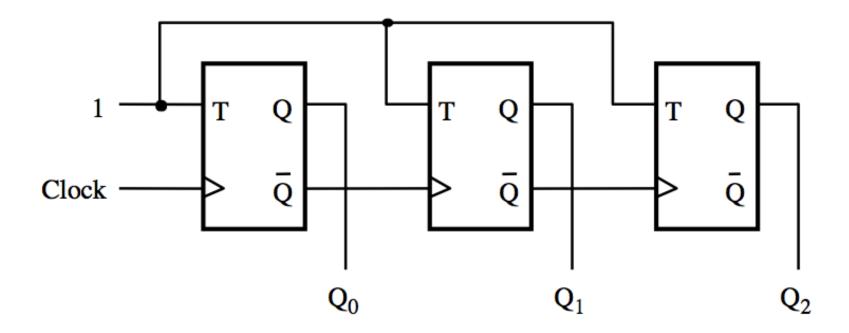


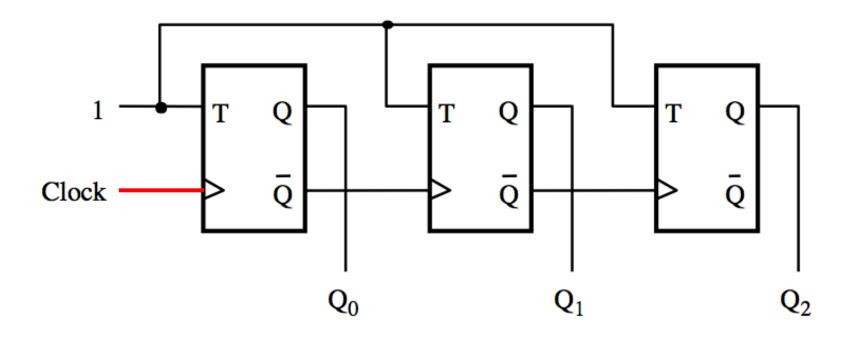




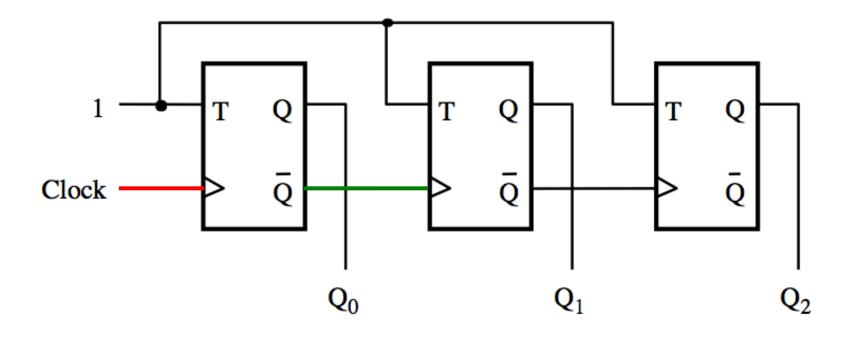






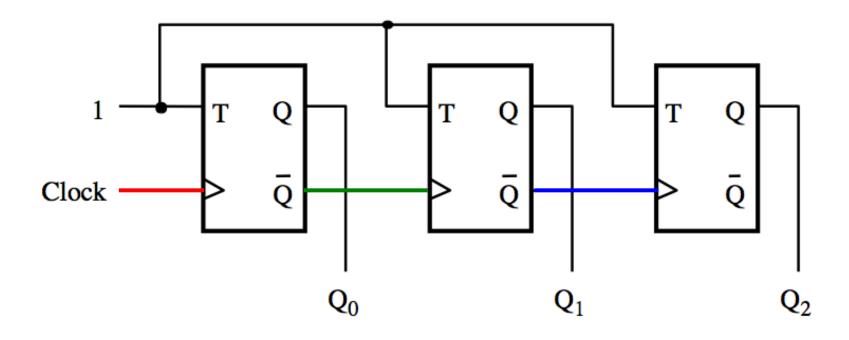


The first flip-flop changes on the positive edge of the clock



The first flip-flop changes on the positive edge of the clock

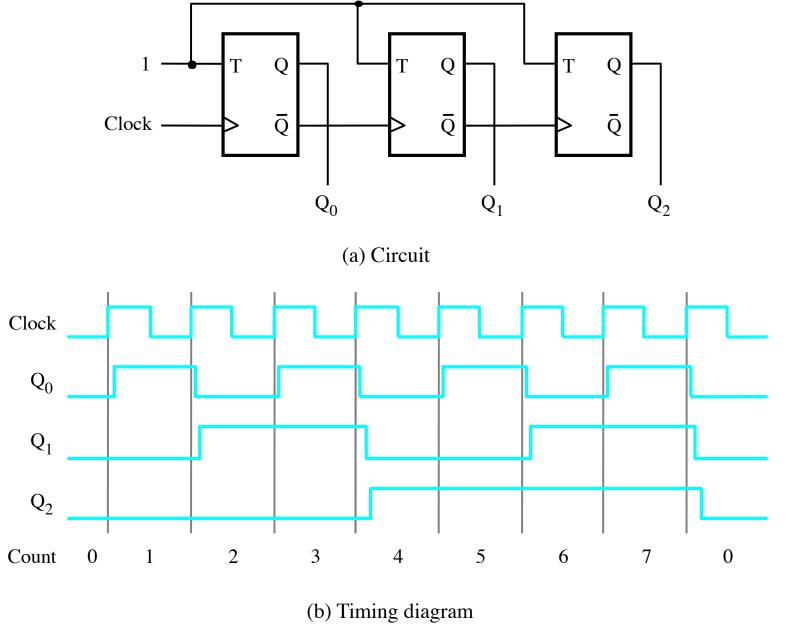
The second flip-flop changes on the positive edge of $\overline{\mathbb{Q}}_0$



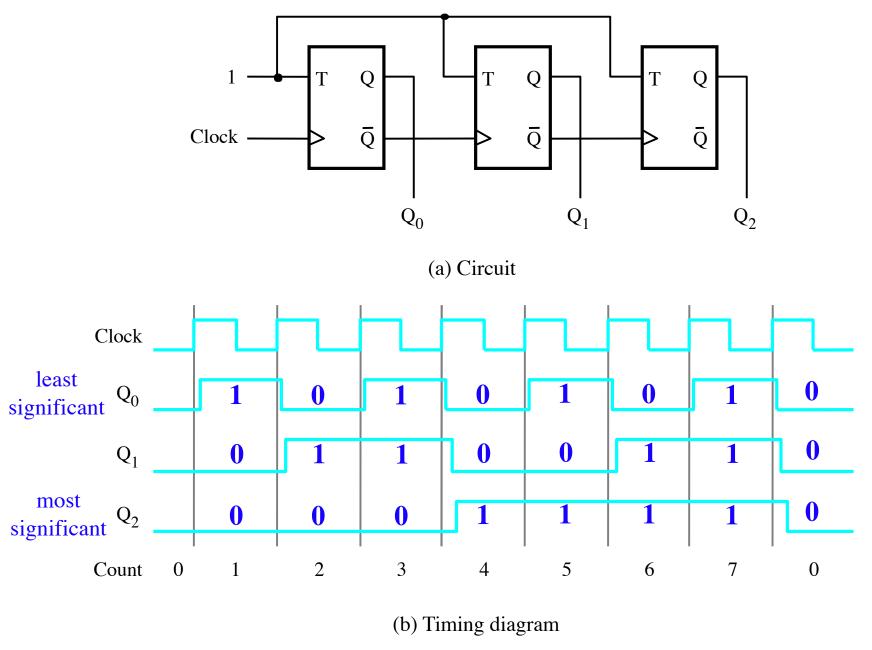
The first flip-flop changes on the positive edge of the clock

The second flip-flop changes The third flip-flop changes on the positive edge of $\overline{\mathbb{Q}}_0$

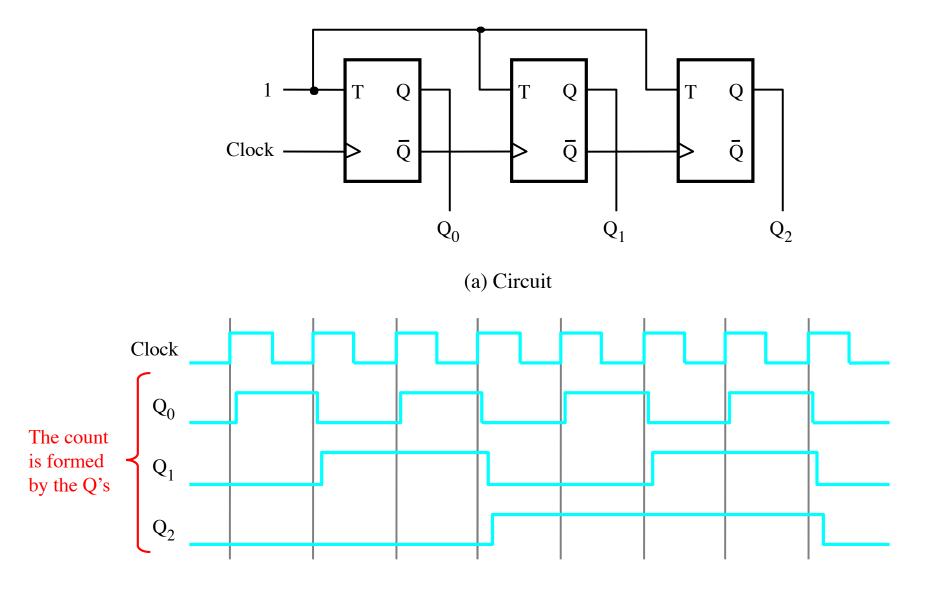
on the positive edge of \overline{Q}_1



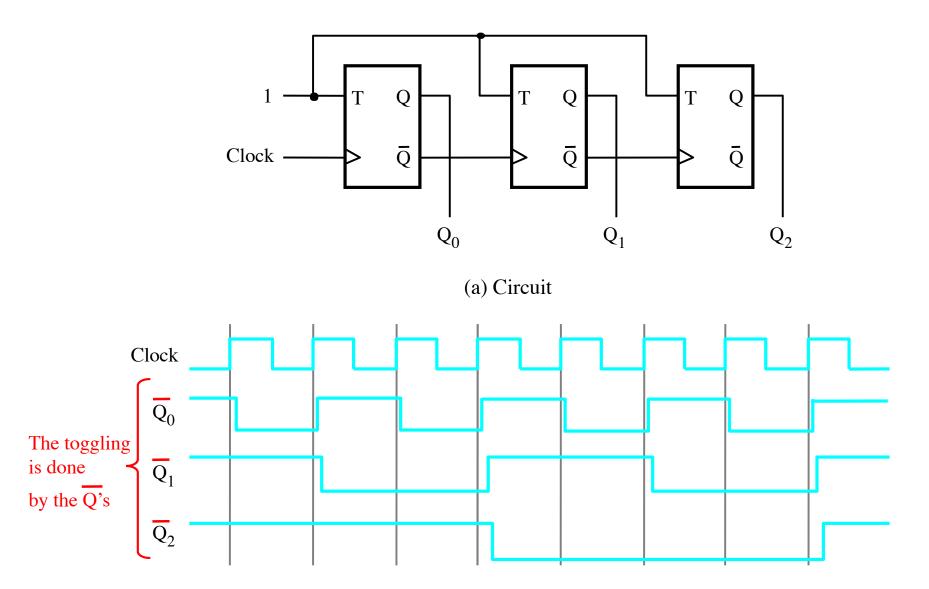
[Figure 5.19 from the textbook]

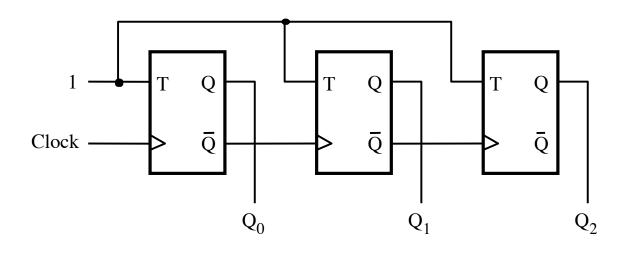


[Figure 5.19 from the textbook]

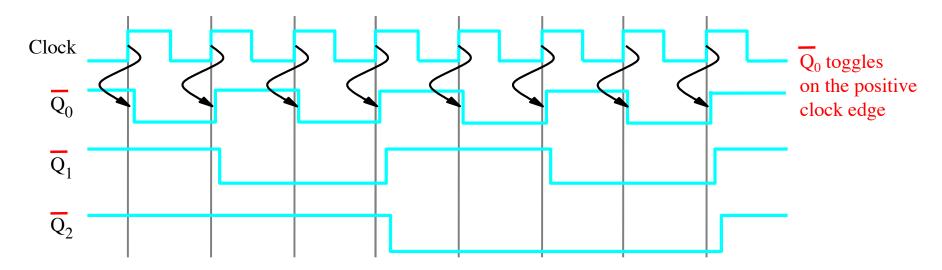


(b) Timing diagram

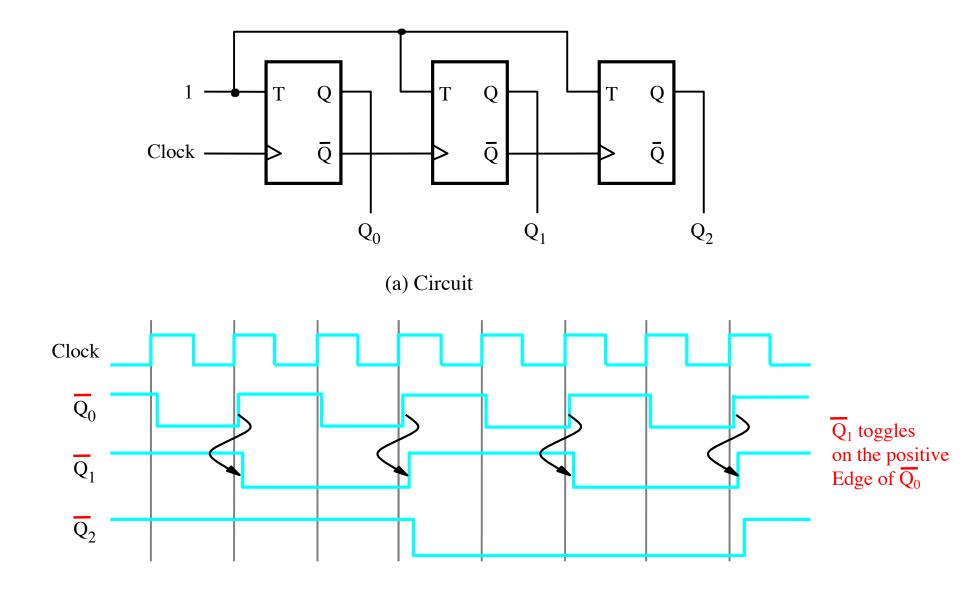




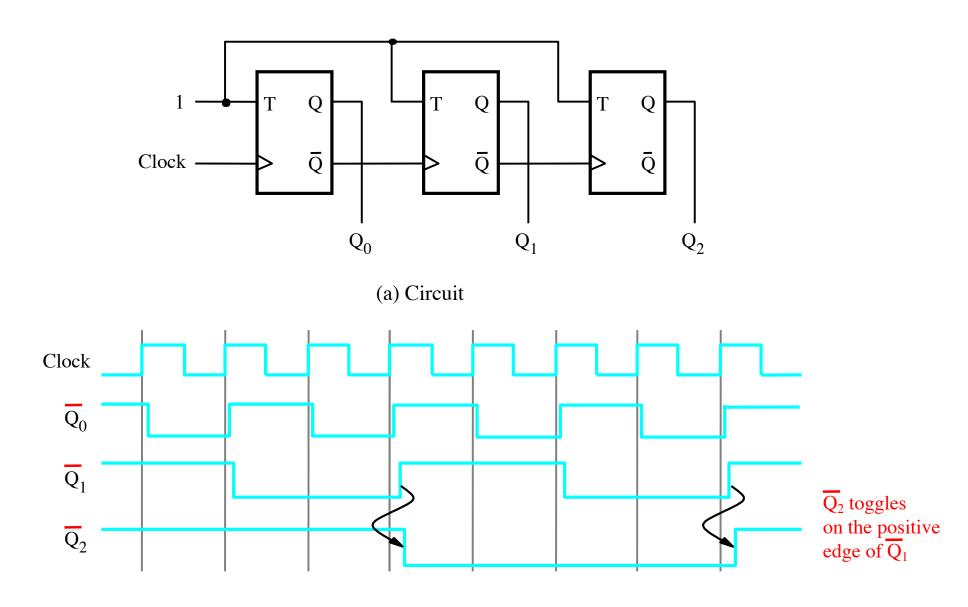
(a) Circuit



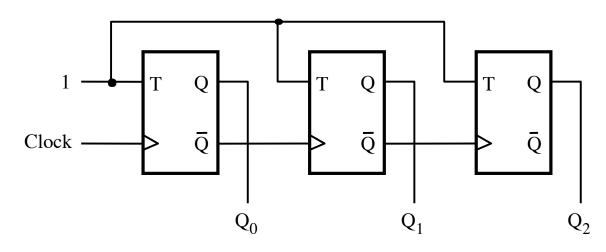
(b) Timing diagram

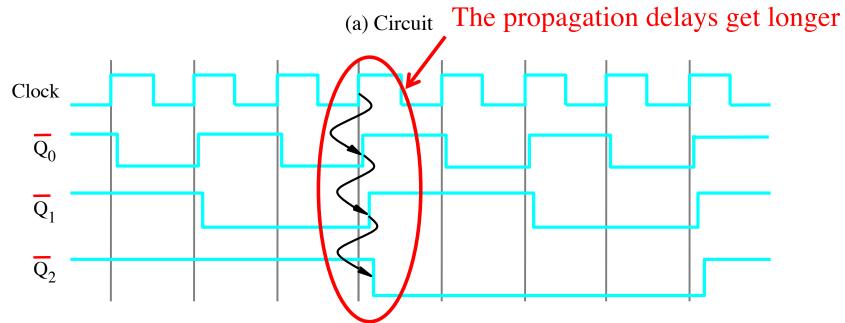


(b) Timing diagram

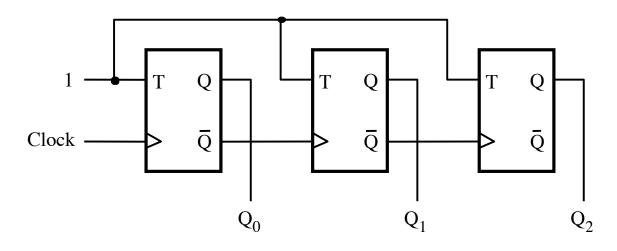


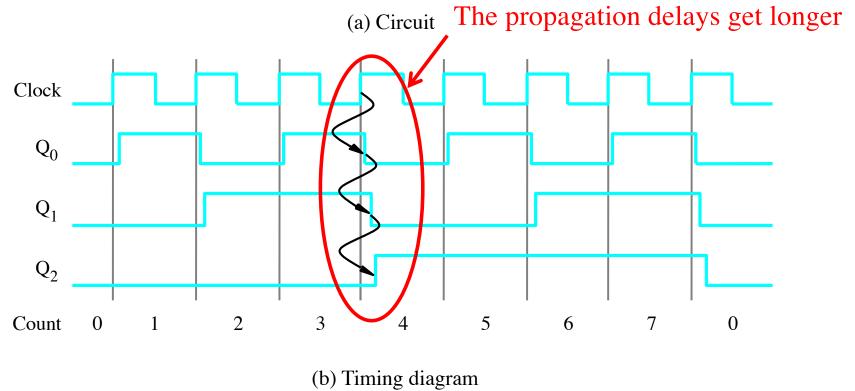
(b) Timing diagram





(b) Timing diagram

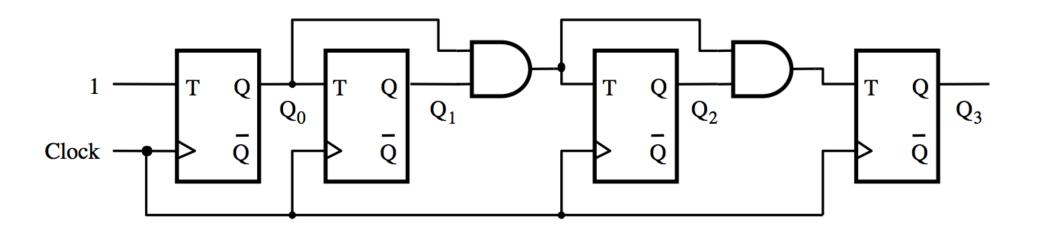




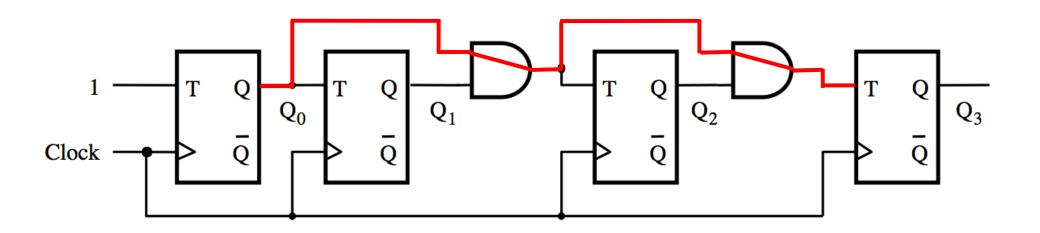
[Figure 5.19 from the textbook]

Synchronous Counters

A four-bit synchronous up-counter

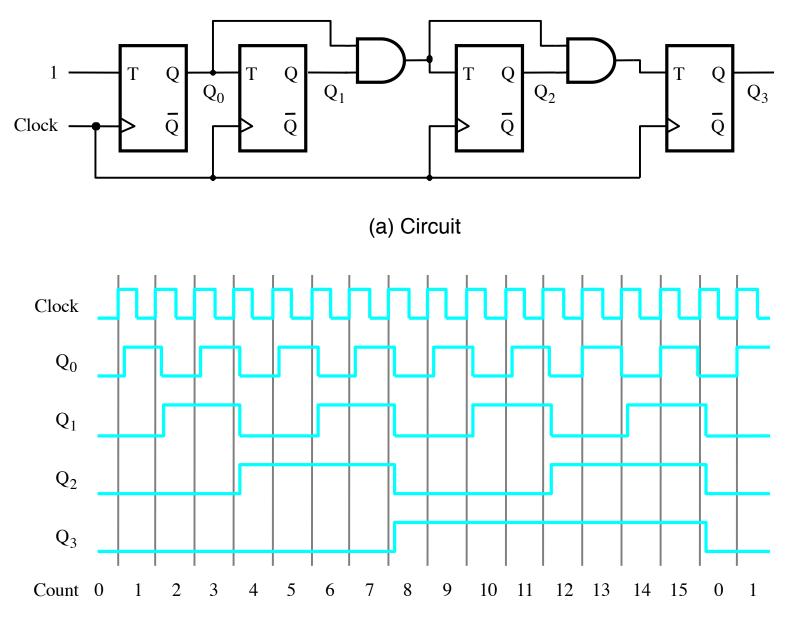


A four-bit synchronous up-counter

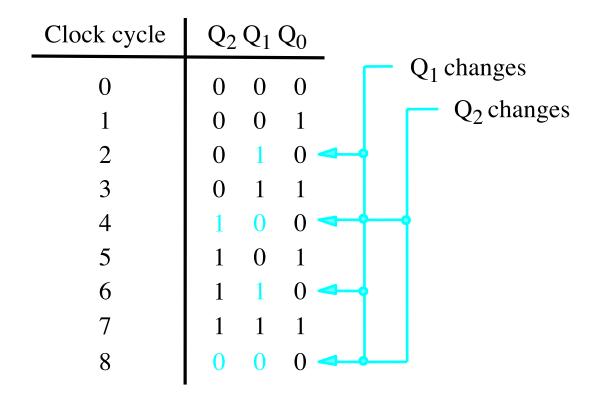


The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

A four-bit synchronous up-counter



Derivation of the synchronous up-counter



Derivation of the synchronous up-counter

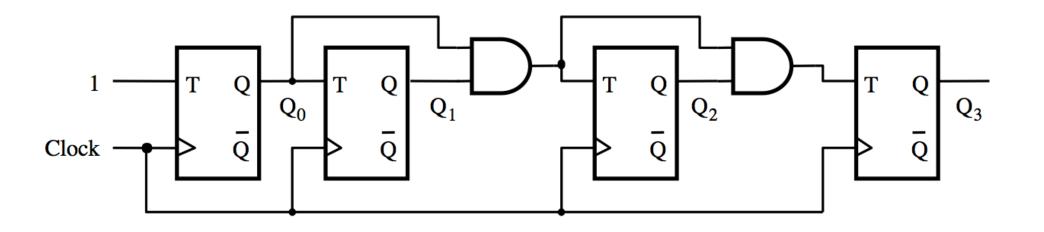
Clock cycle	$Q_2 Q_1 Q_0$	
0 1 2 3 4 5		Q ₁ changes — Q ₂ changes
7	1 1 1	
8	0 0 0	

$$T_0=1$$

$$T_1=Q_0$$

$$T_2=Q_0 Q_1$$

A four-bit synchronous up-counter



$$T_0=1$$

$$T_1=Q_0$$

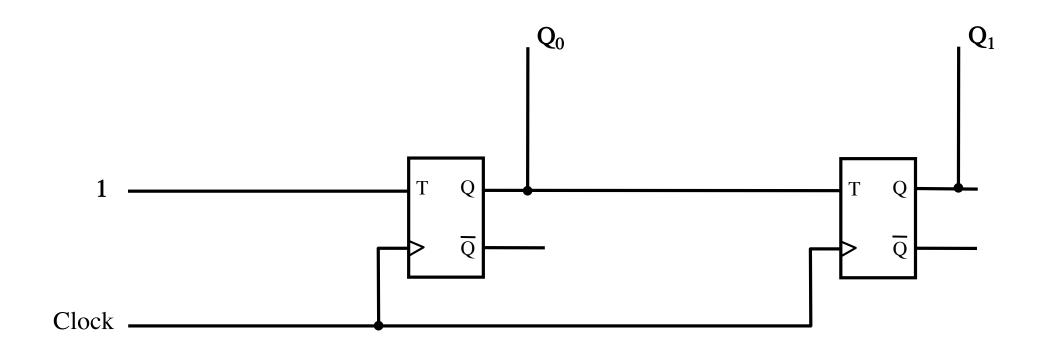
$$T_2=Q_0 Q_1$$

In general we have

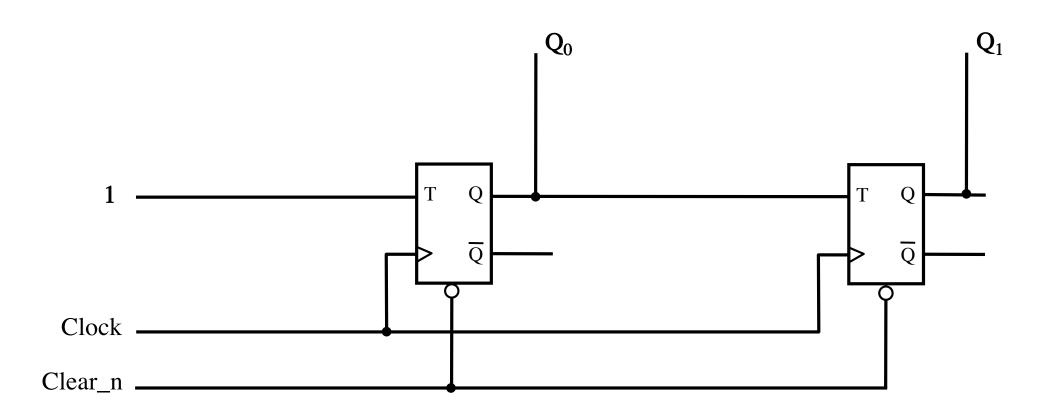
$$T_0 = 1$$
 $T_1 = Q_0$
 $T_2 = Q_0 Q_1$
 $T_3 = Q_0 Q_1 Q_2$
...
 $T_n = Q_0 Q_1 Q_2 ... Q_{n-1}$



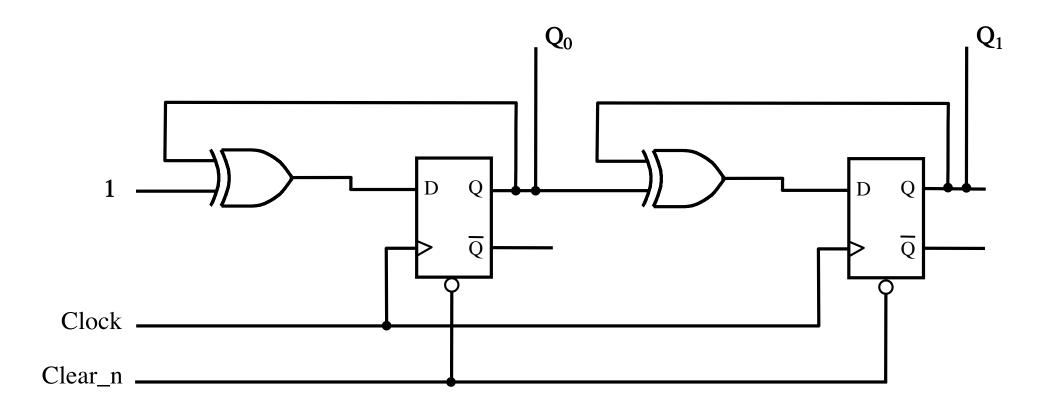
2-Bit Synchronous Up-Counter (without clear capability)



2-Bit Synchronous Up-Counter (with asynchronous clear)

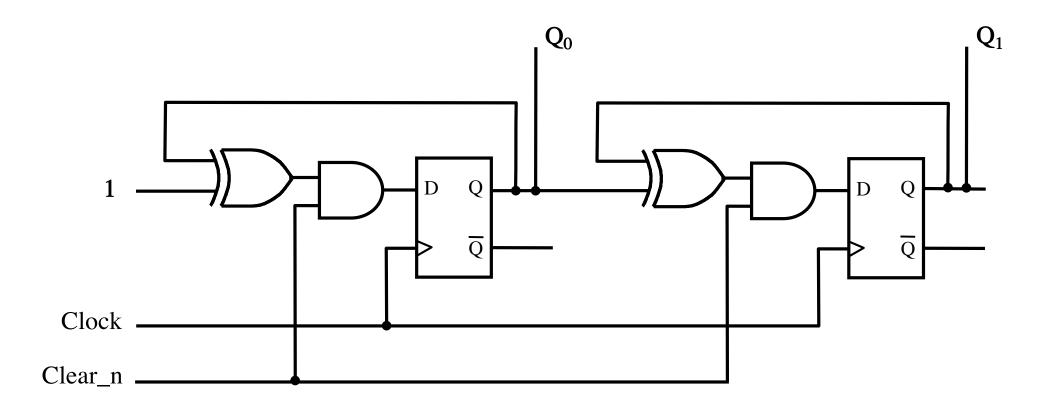


2-Bit Synchronous Up-Counter (with asynchronous clear)



This is the same circuit but uses D Flip-Flops.

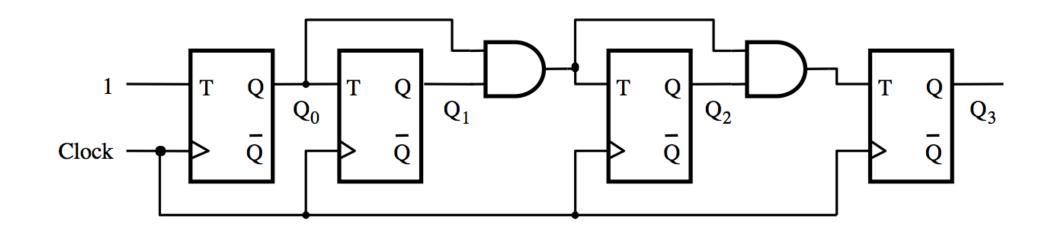
2-Bit Synchronous Up-Counter (with synchronous clear)



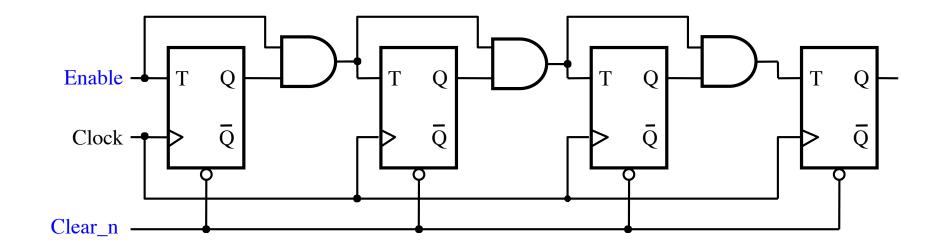
This counter can be cleared only on the positive clock edge.

Adding Enable Capability

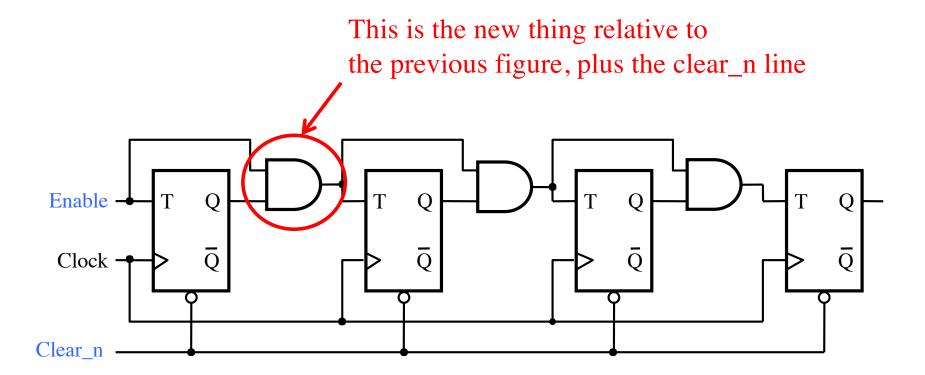
A four-bit synchronous up-counter



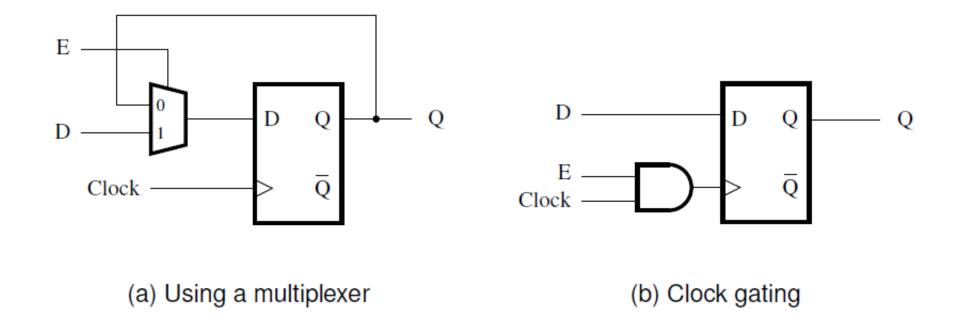
Inclusion of Enable and Clear Capability



Inclusion of Enable and Clear Capability

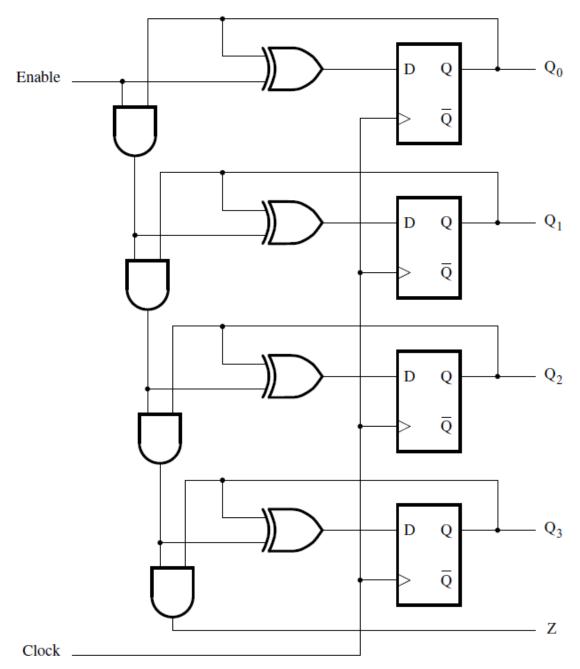


Providing an enable input for a D flip-flop

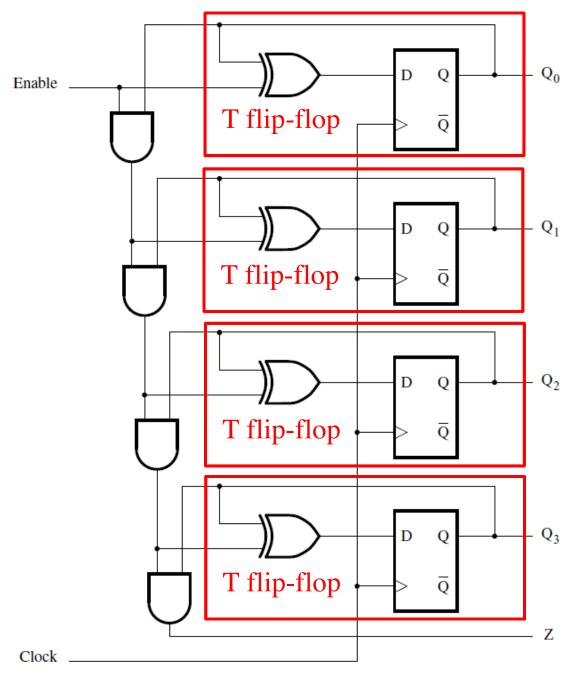


Synchronous Counter (with D Flip-Flops)

A 4-bit up-counter with D flip-flops

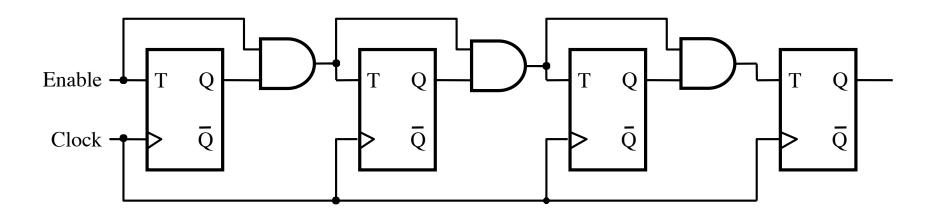


A 4-bit up-counter with D flip-flops

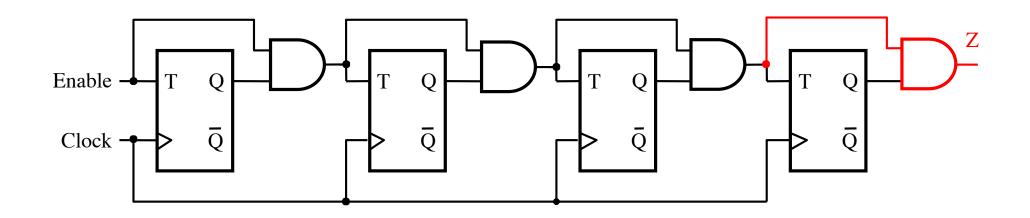


[Figure 5.23 from the textbook]

Equivalent to this circuit with T flip-flops



Equivalent to this circuit with T flip-flops

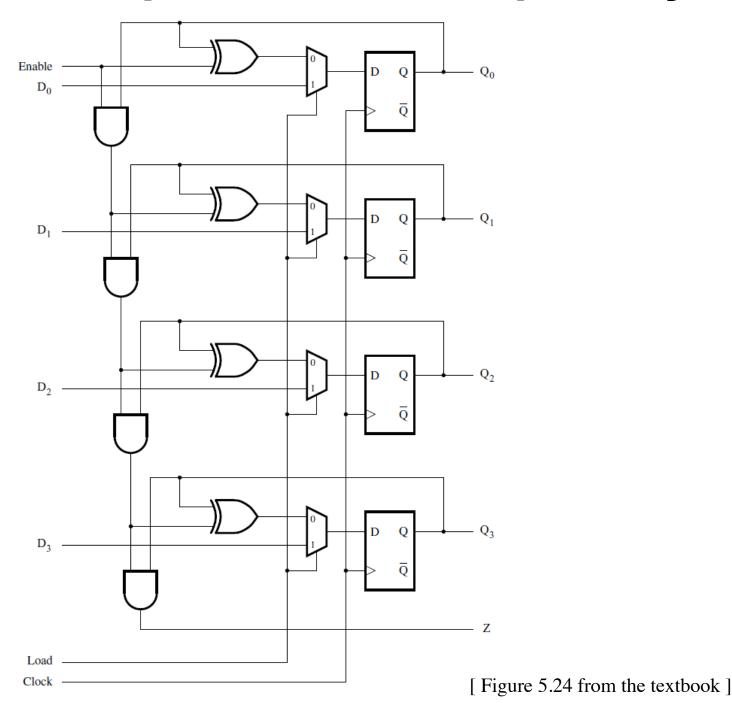


But has one extra output called Z, which can be used to connect two 4-bit counters to make an 8-bit counter.

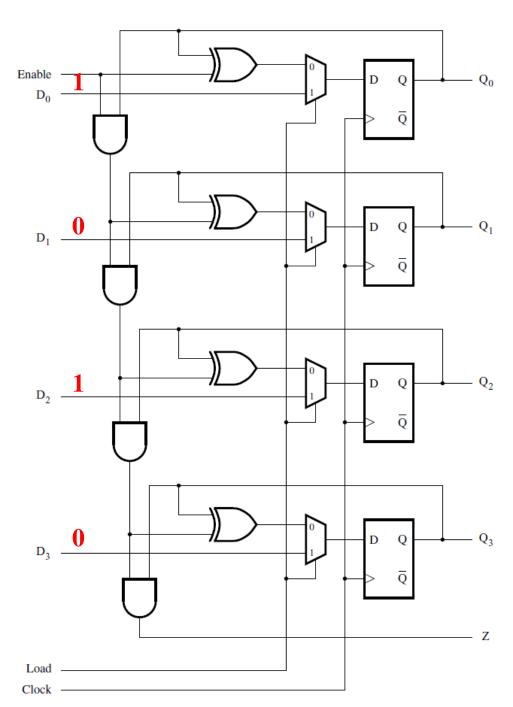
When Z=1 the counter will go 0000 on the next clock edge, i.e., the outputs of all flip-flops are currently 1 (maximum count value).

Counters with Parallel Load

A counter with parallel-load capability

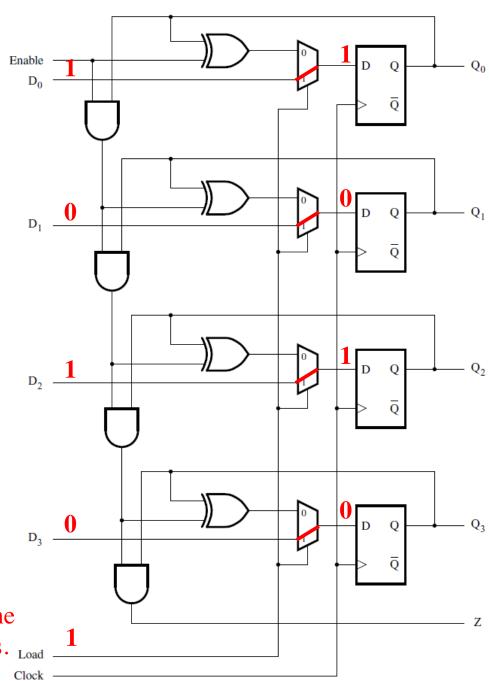


How to load the initial count value



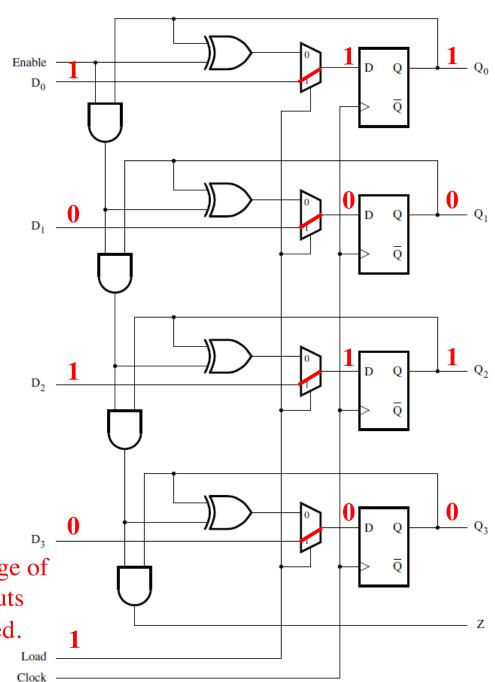
Set the initial count on the parallel load lines (in this case 5).

How to zero a counter



Set "Load" to 1, to open the "1" line of the multiplexers. Load

How to zero a counter



When the next positive edge of the clock arrives, the outputs of the flip-flops are updated.

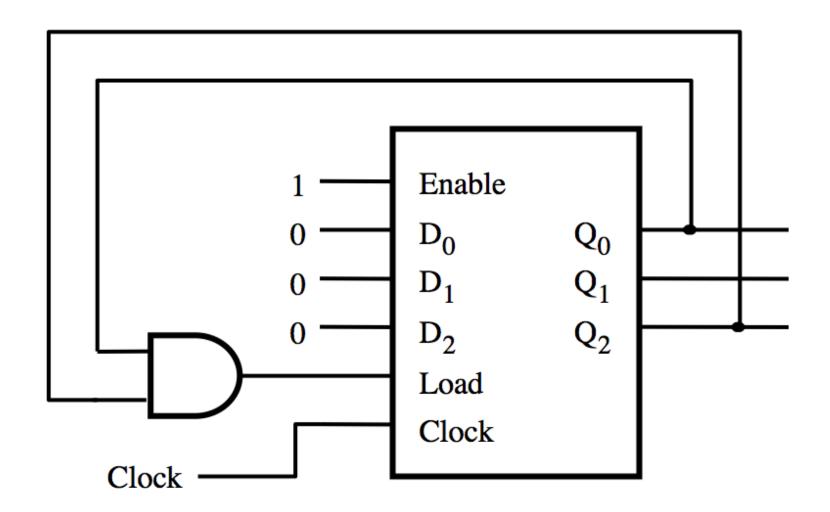
Reset Synchronization

Motivation

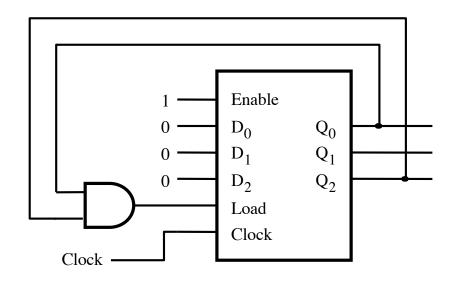
- An n-bit counter counts from 0, 1, ..., 2ⁿ-1
- For example a 3-bit counter counts up as follow
 - **0**, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, ...

- What if we want it to count like this
 - **0**, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0, 1, ...
- In other words, what is the cycle is not a power of 2?

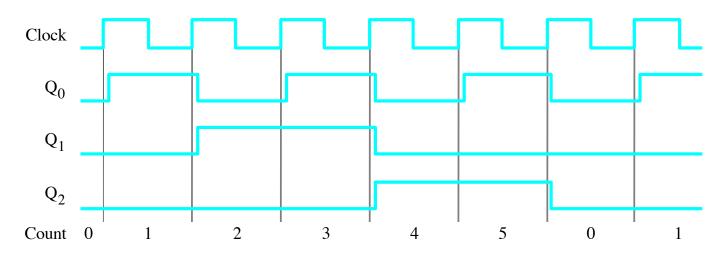
What does this circuit do?



A modulo-6 counter with synchronous reset

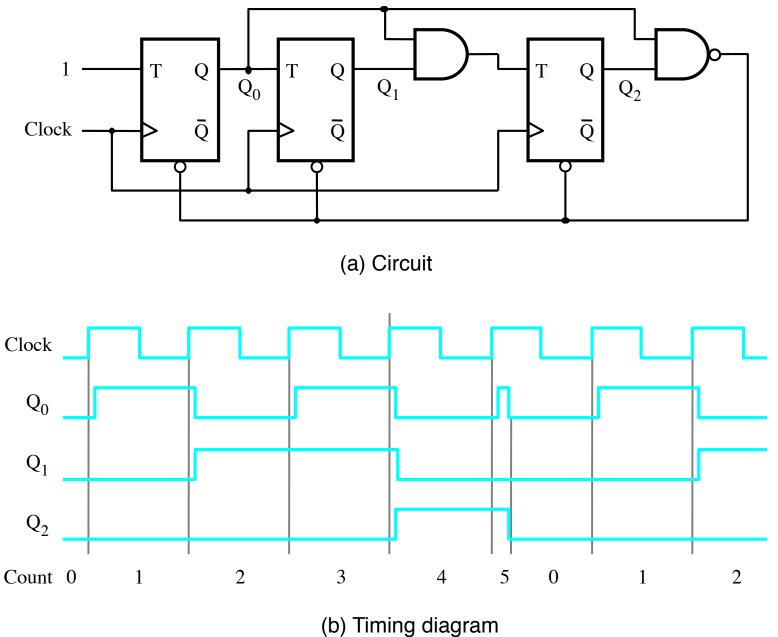


(a) Circuit



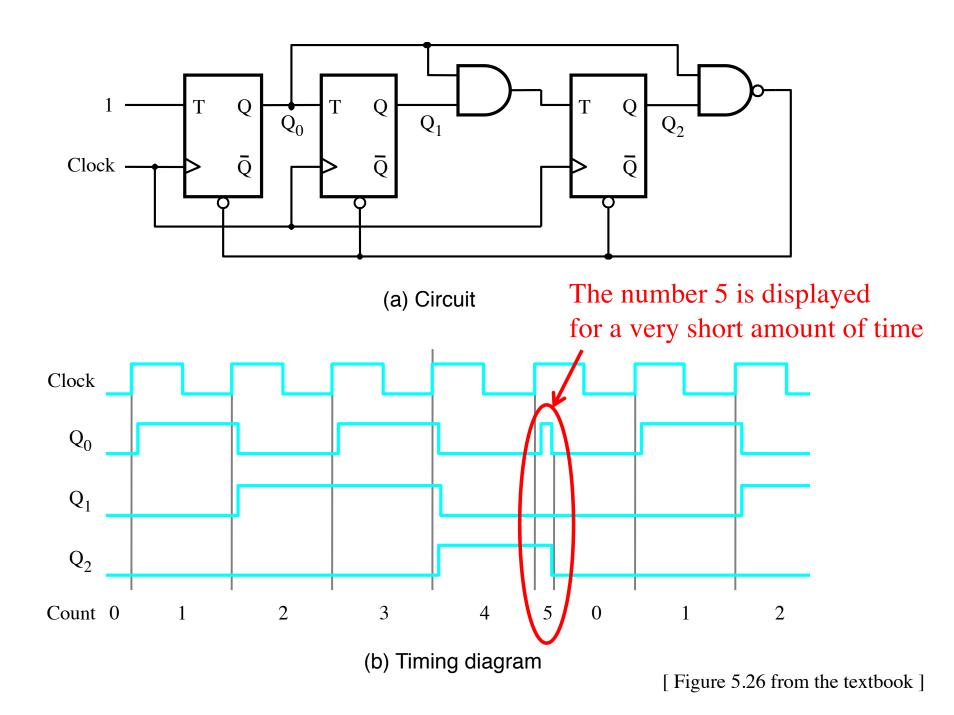
(b) Timing diagram

A modulo-6 counter with asynchronous reset



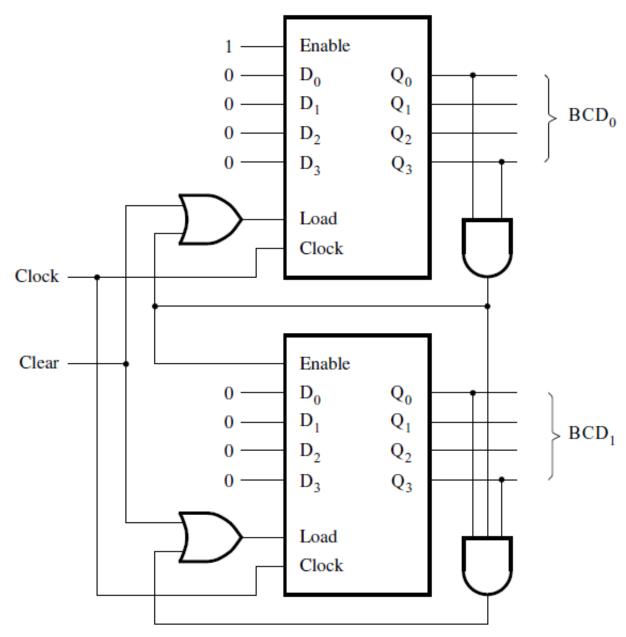
[Figure 5.26 from the textbook]

A modulo-6 counter with asynchronous reset

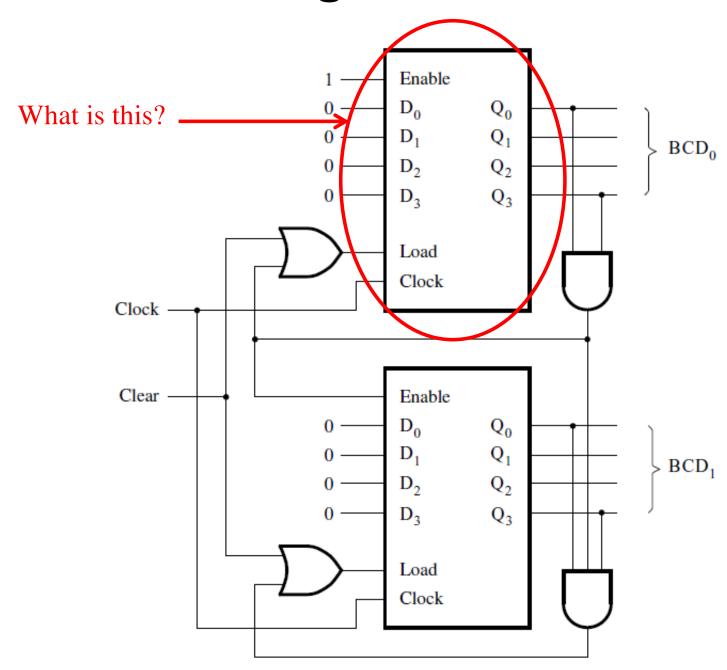


Other Types of Counters (Section 5.11)

- 2: Parallel-load four-bit counter
 - Figure 5.24
- Each counts in binary
 - **0-9**
- Resets generated on 9
 - Reset by loading 0's
- Second digit enabled by a 9 on first counter

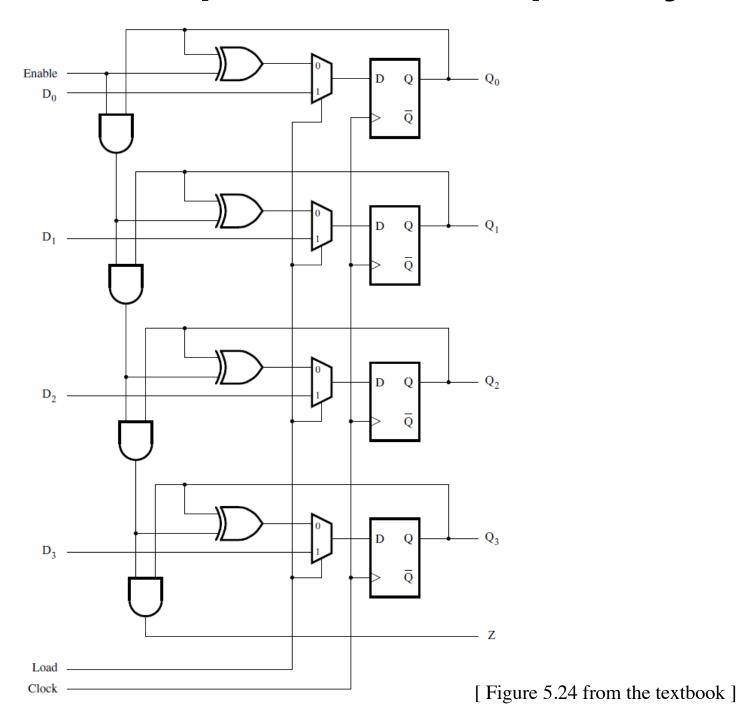


[Figure 5.27 from the textbook]

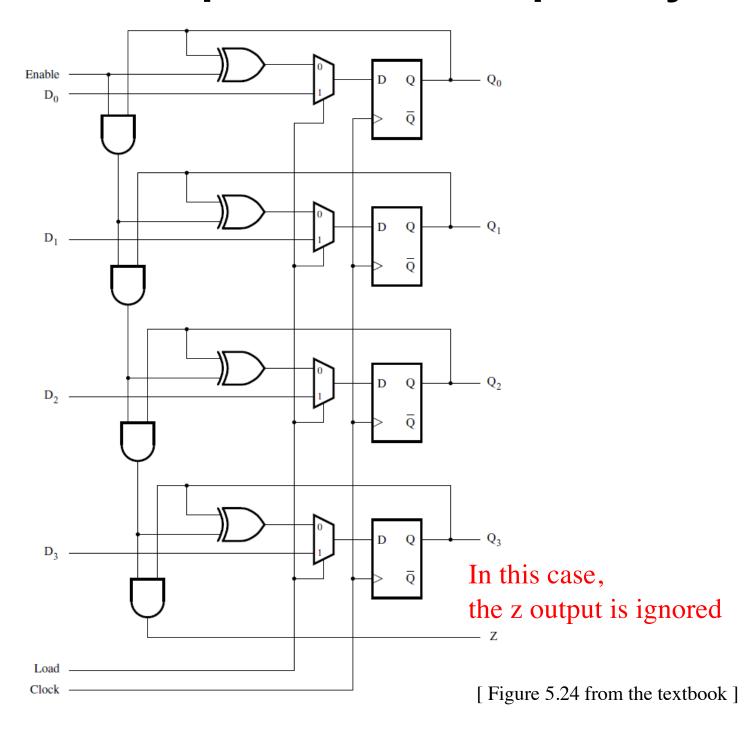


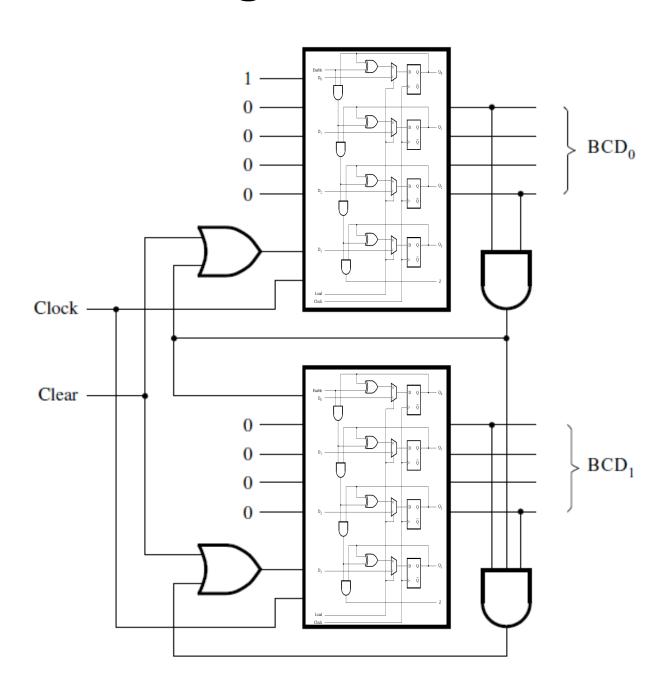
[Figure 5.27 from the textbook]

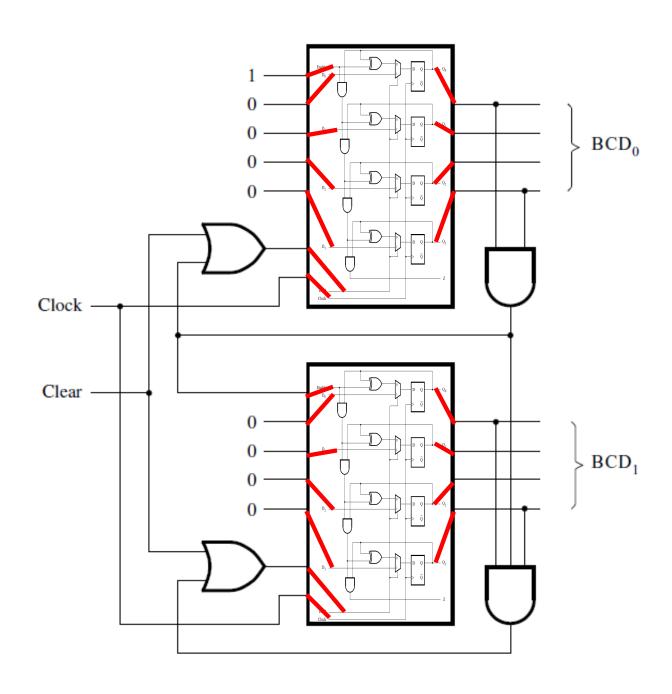
It is a counter with parallel-load capability



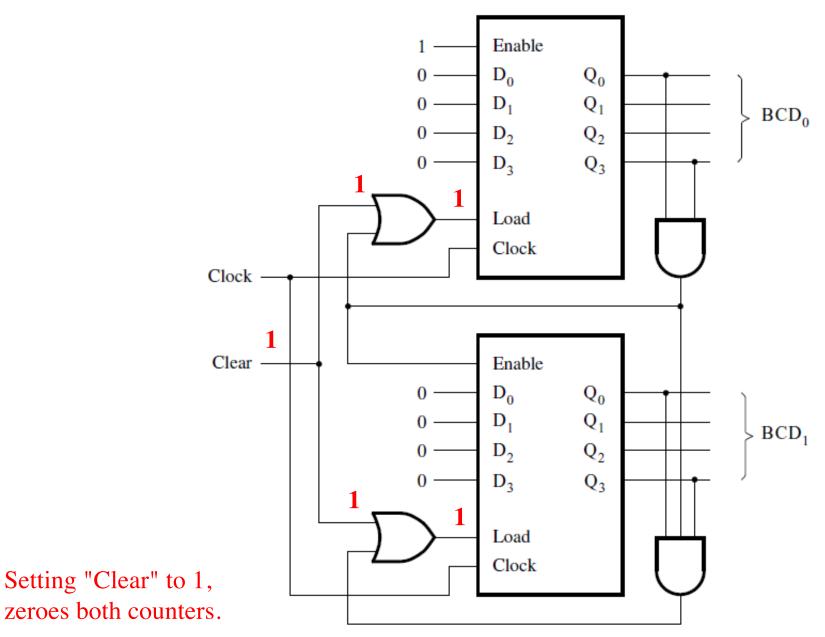
It is a counter with parallel-load capability





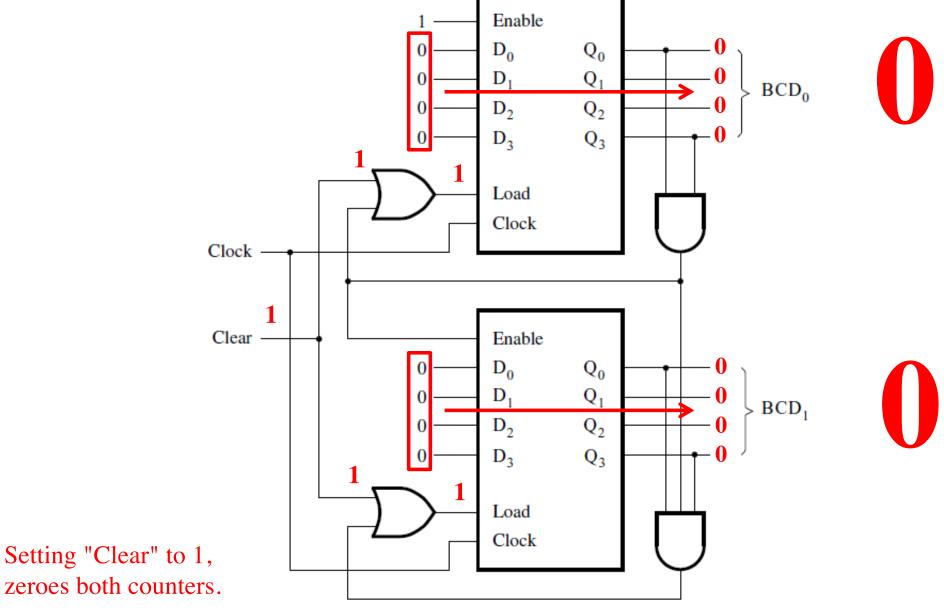


Zeroing the BCD counter



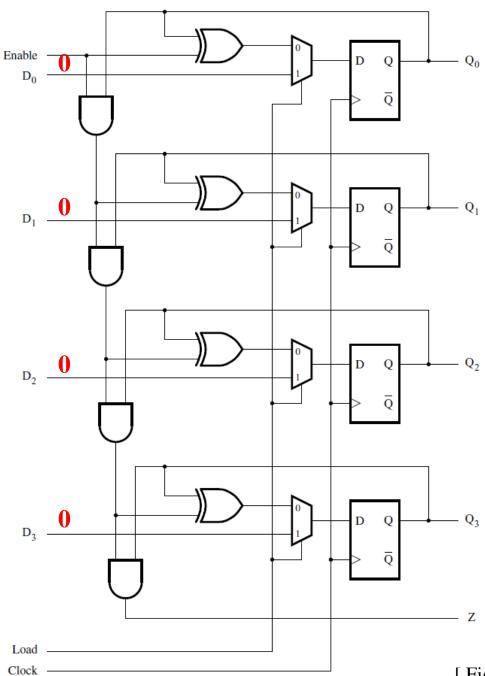
[Figure 5.27 from the textbook]

Zeroing the BCD counter



[Figure 5.27 from the textbook]

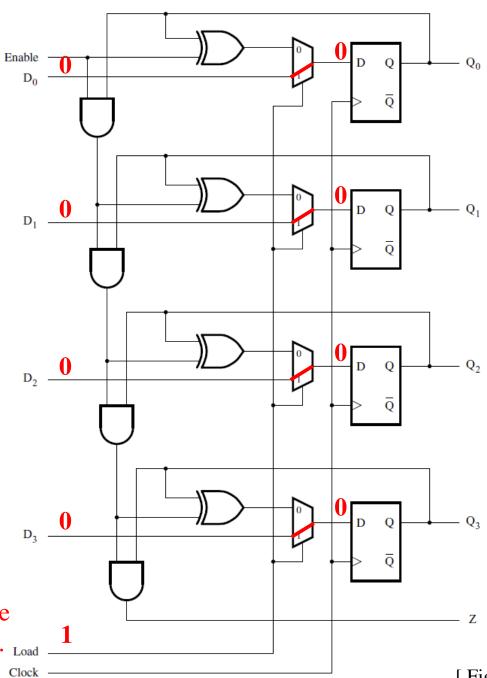
How to zero a counter



Set all parallel load input lines to zero.

[Figure 5.24 from the textbook]

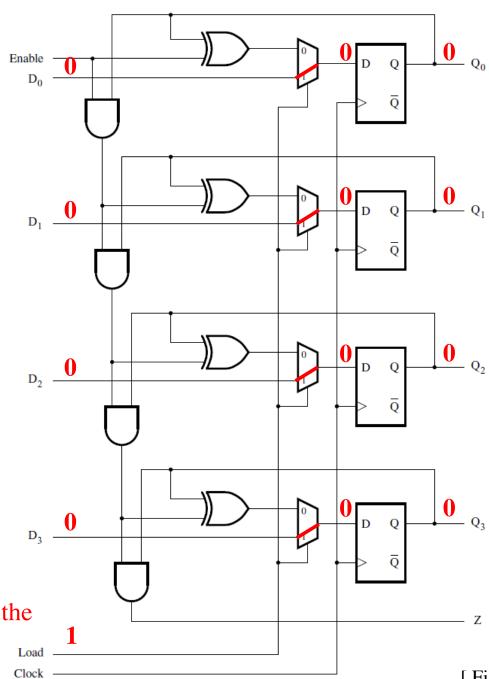
How to zero a counter



Set "Load" to 1, to open the "1" line of the multiplexers. Load .

[Figure 5.24 from the textbook]

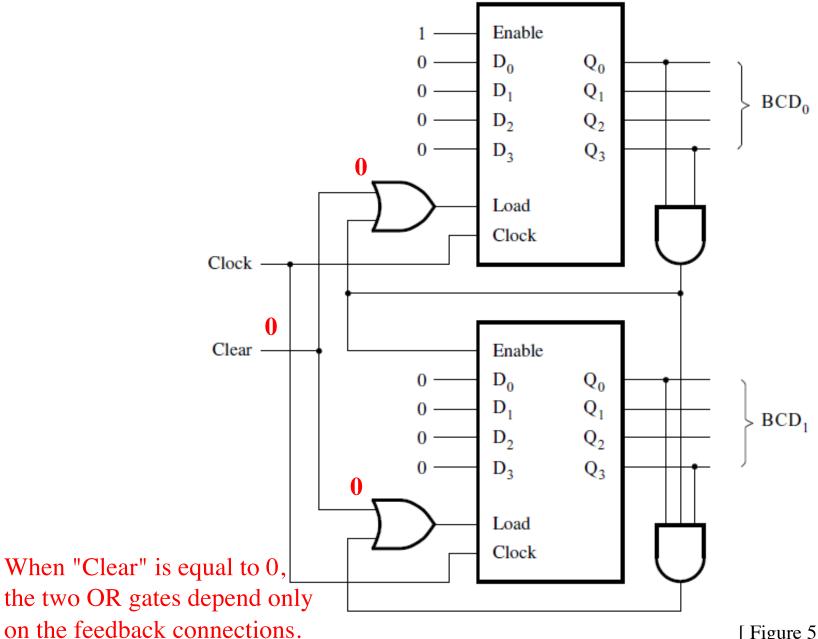
How to zero a counter



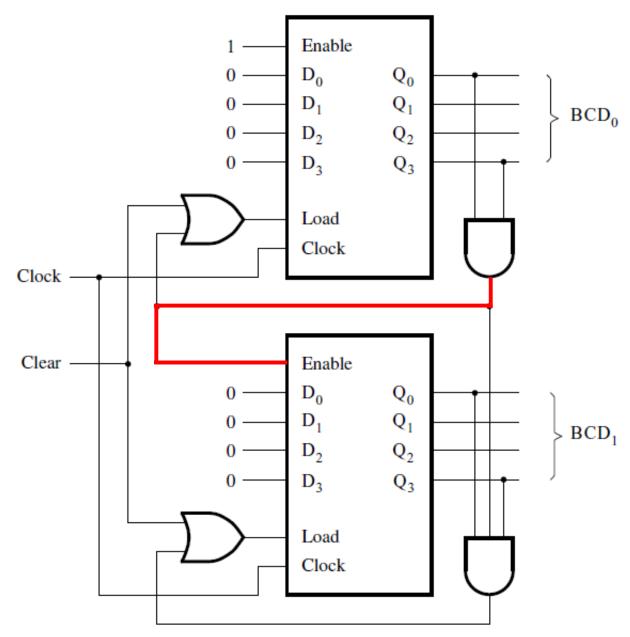
When the positive edge of the clock arrives, all outputs are set to zero together.

[Figure 5.24 from the textbook]

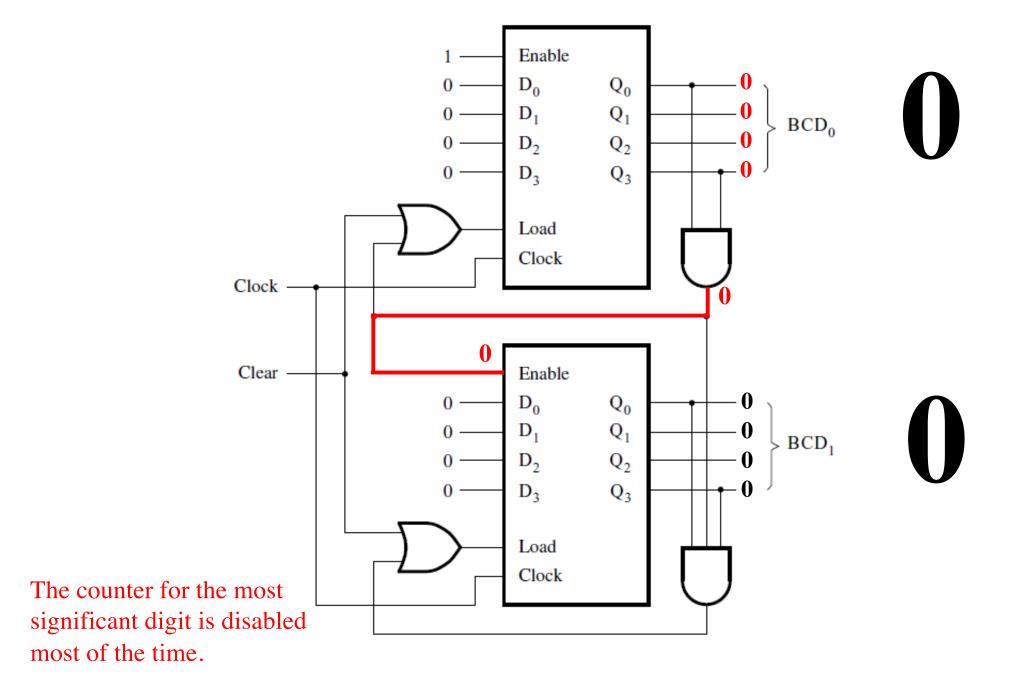
When Clear = 0

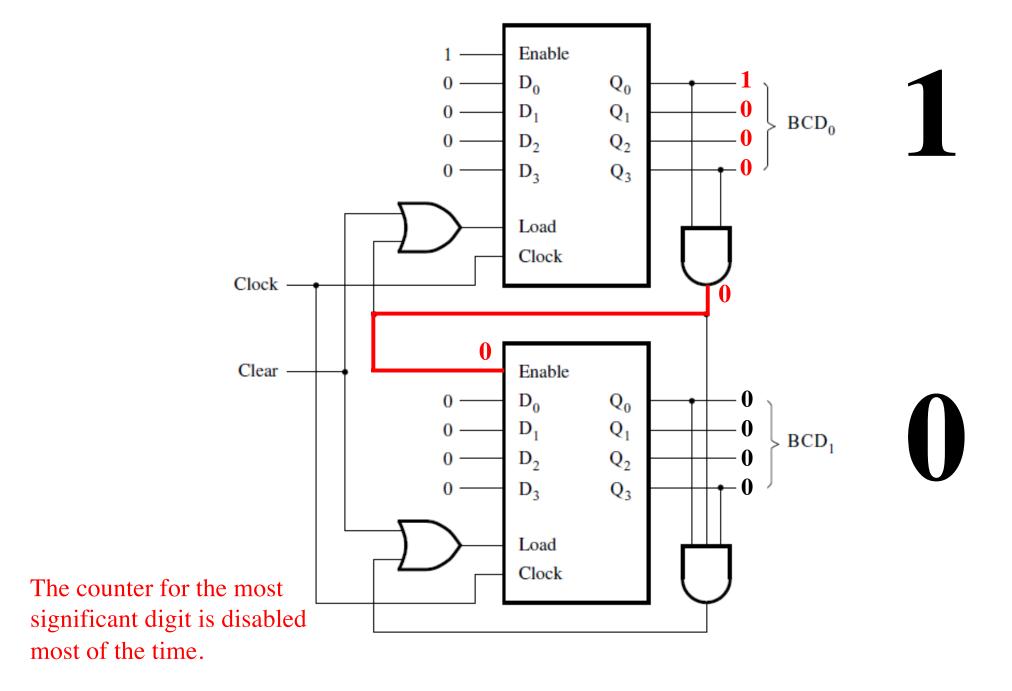


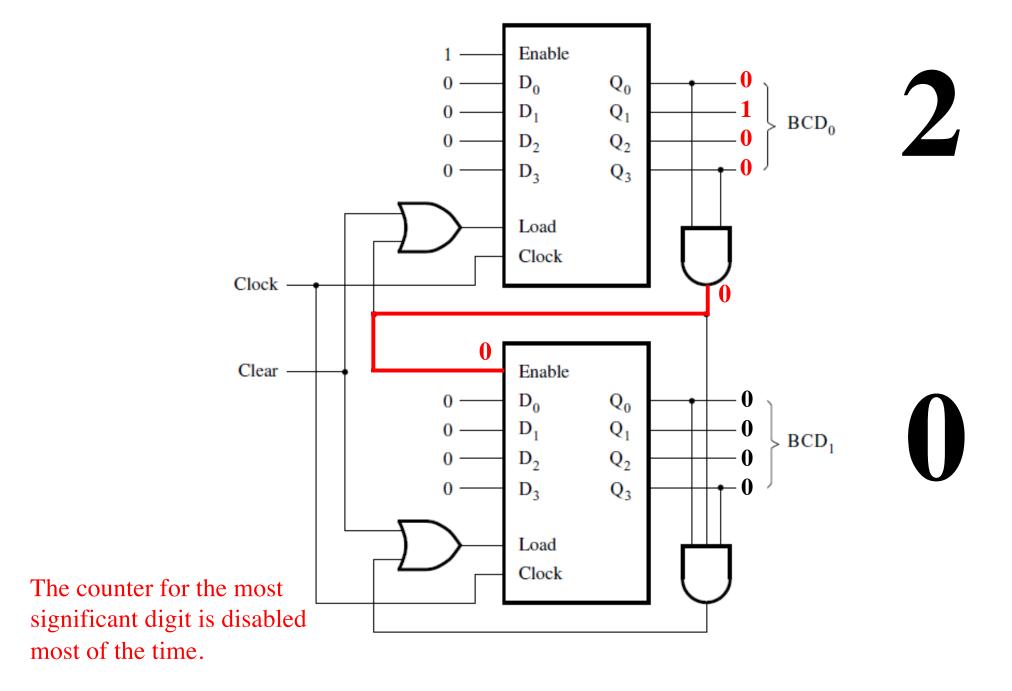
[Figure 5.27 from the textbook]

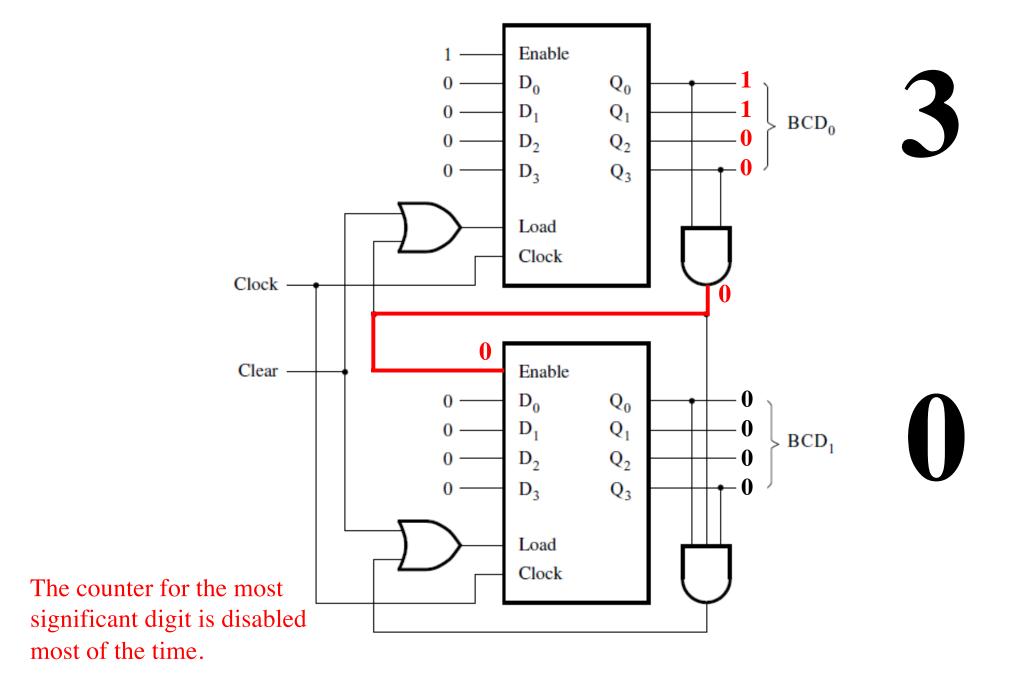


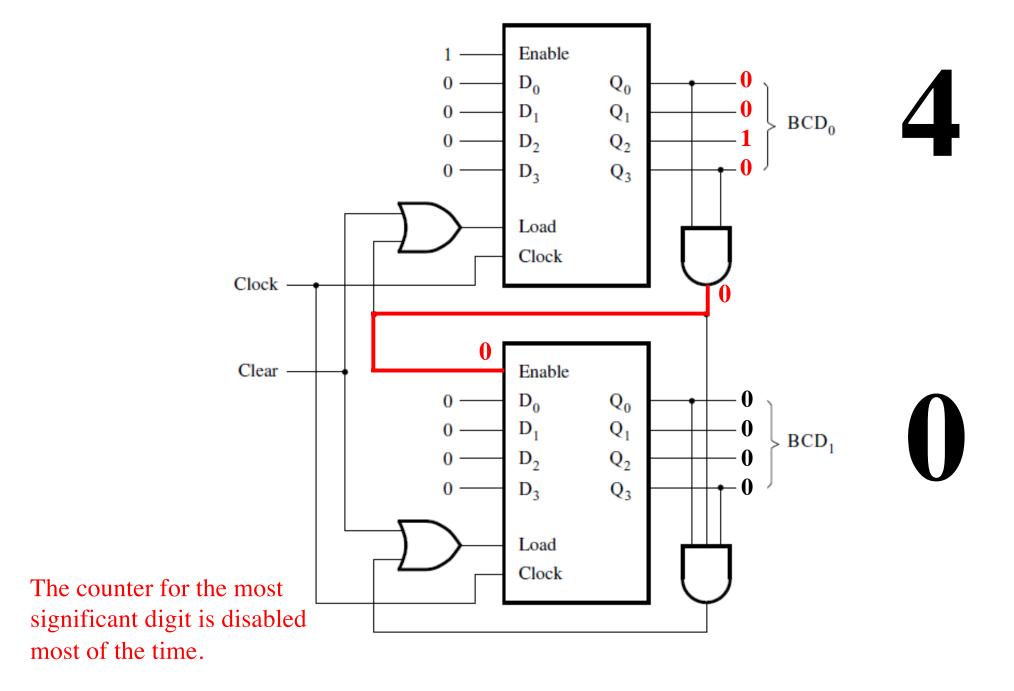
[Figure 5.27 from the textbook]

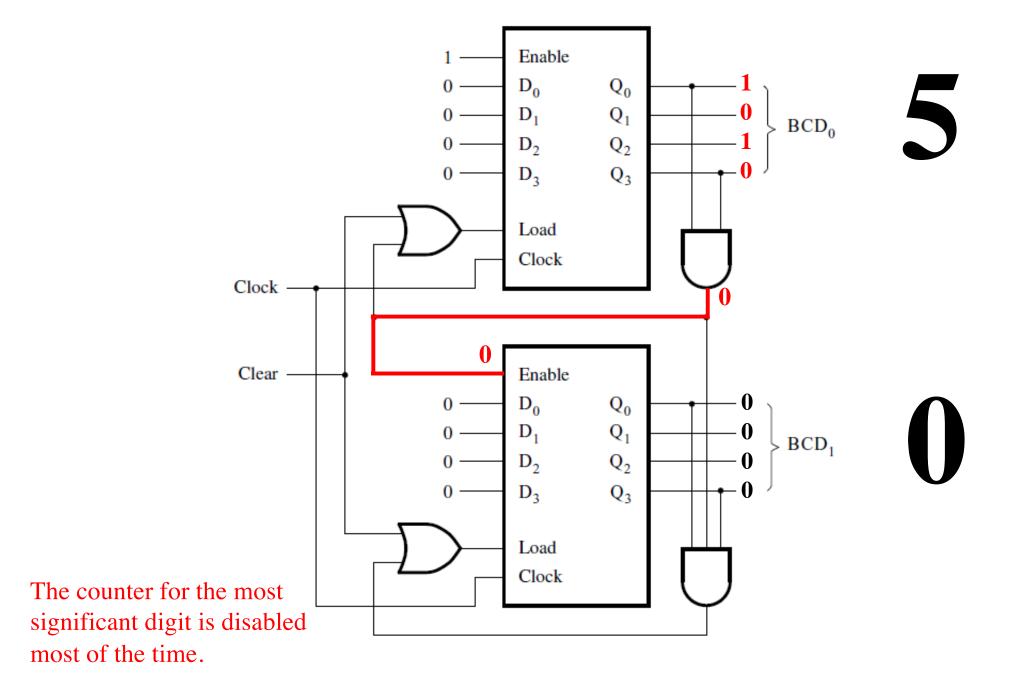


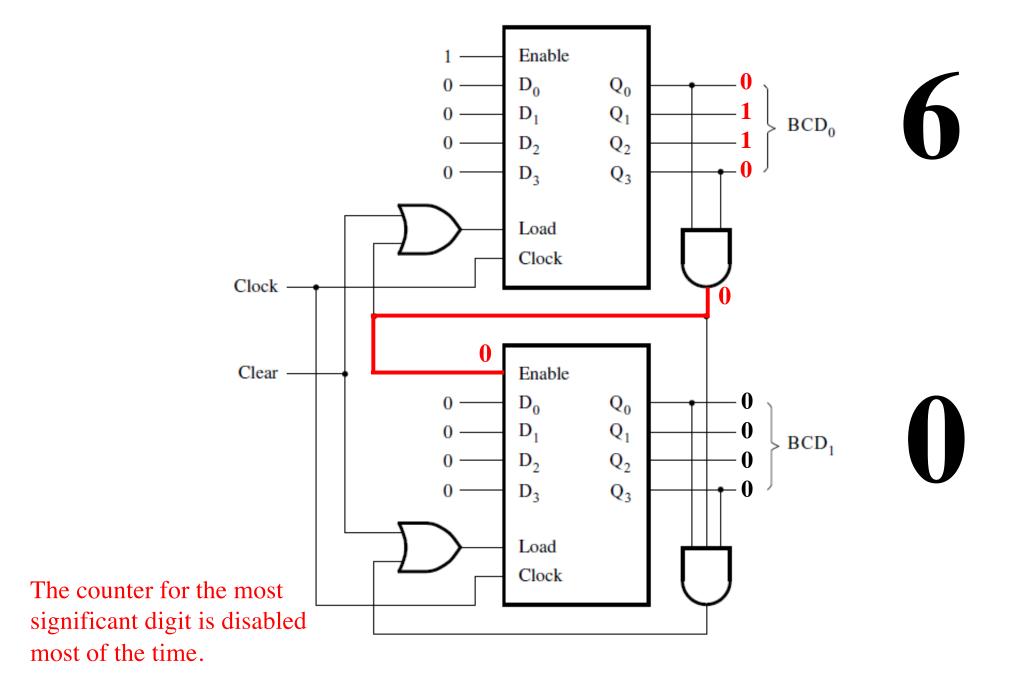


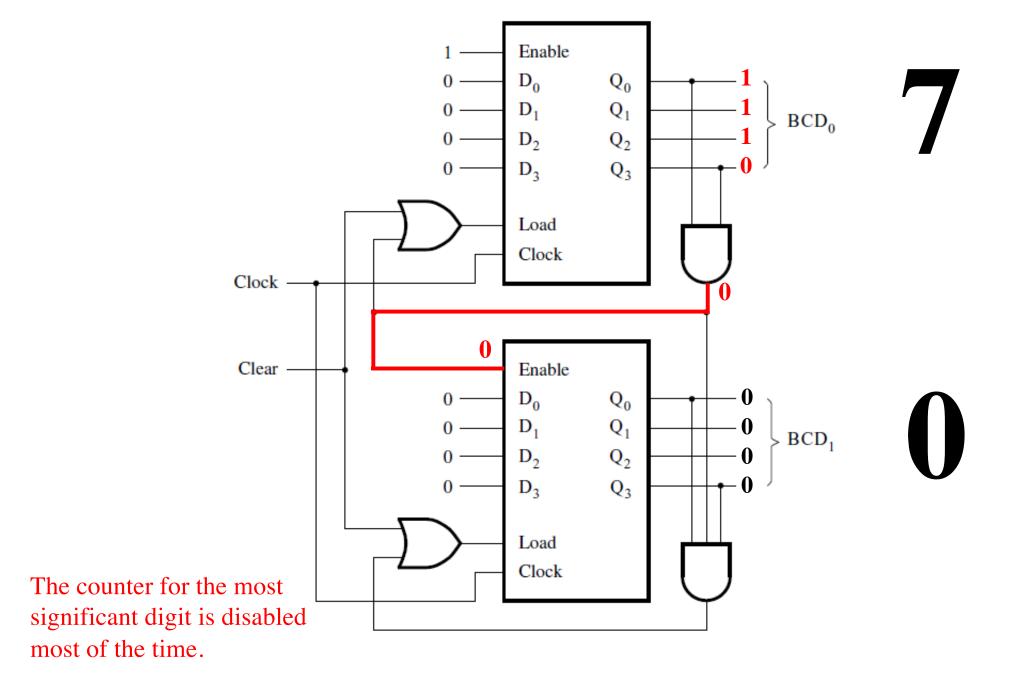


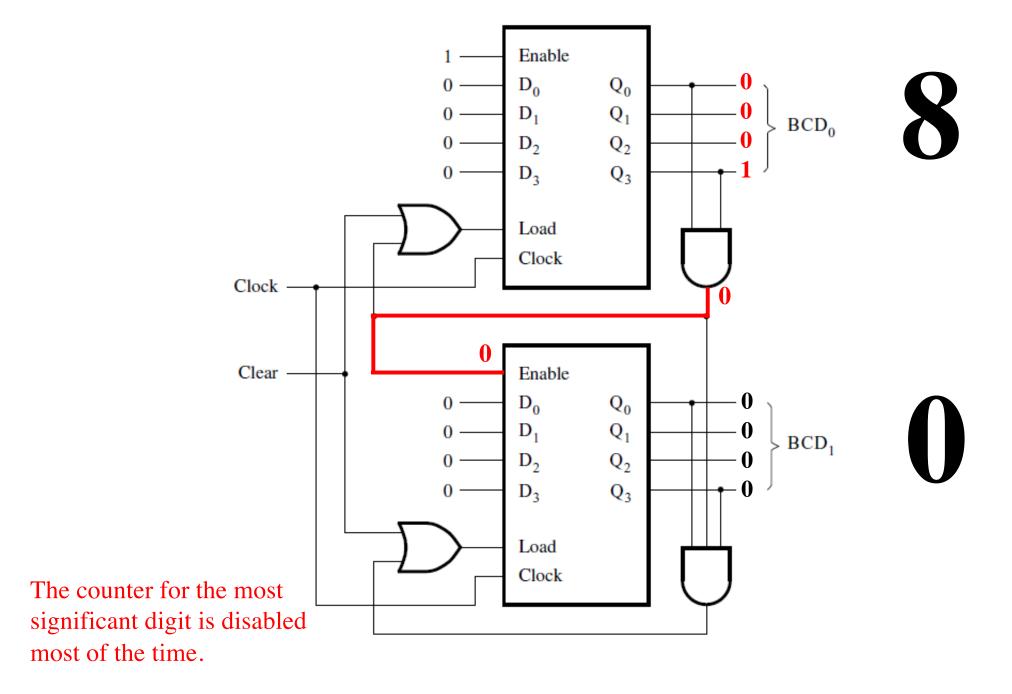


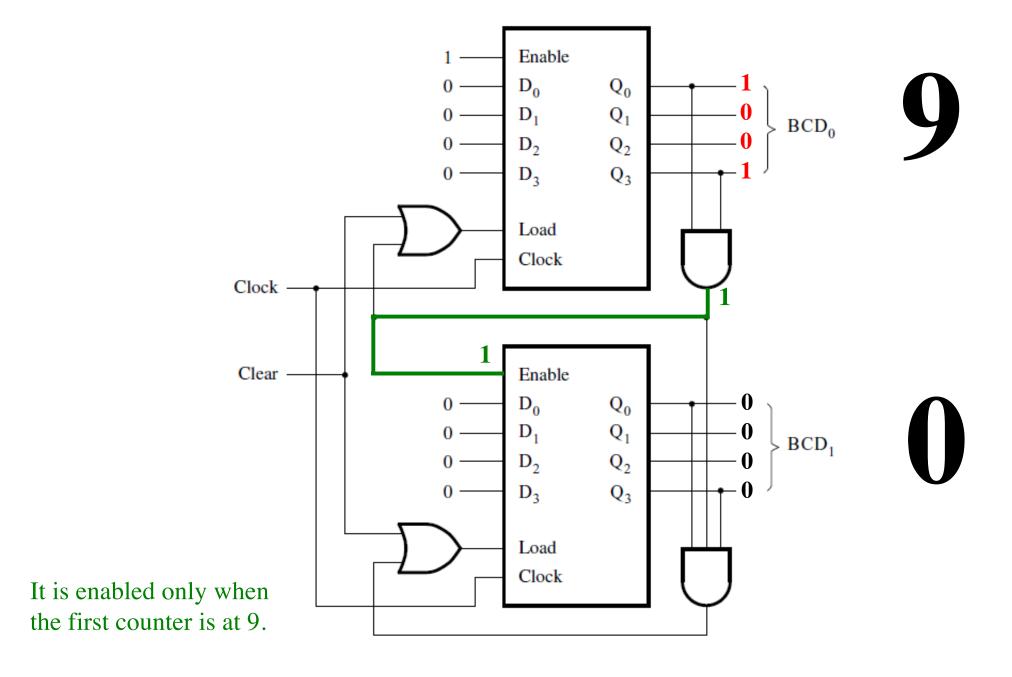


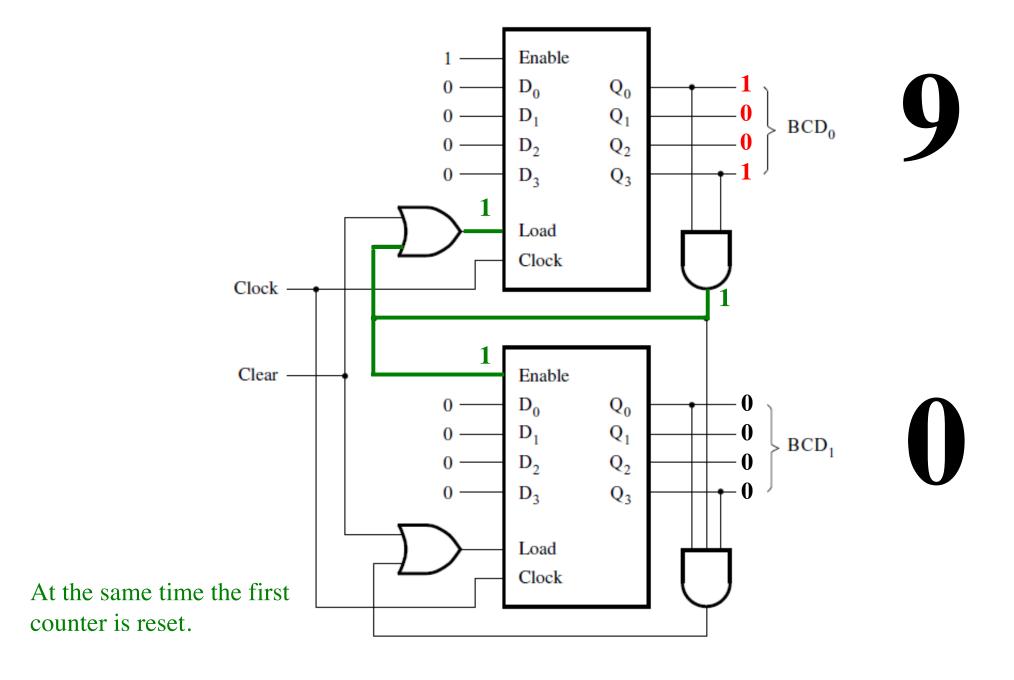


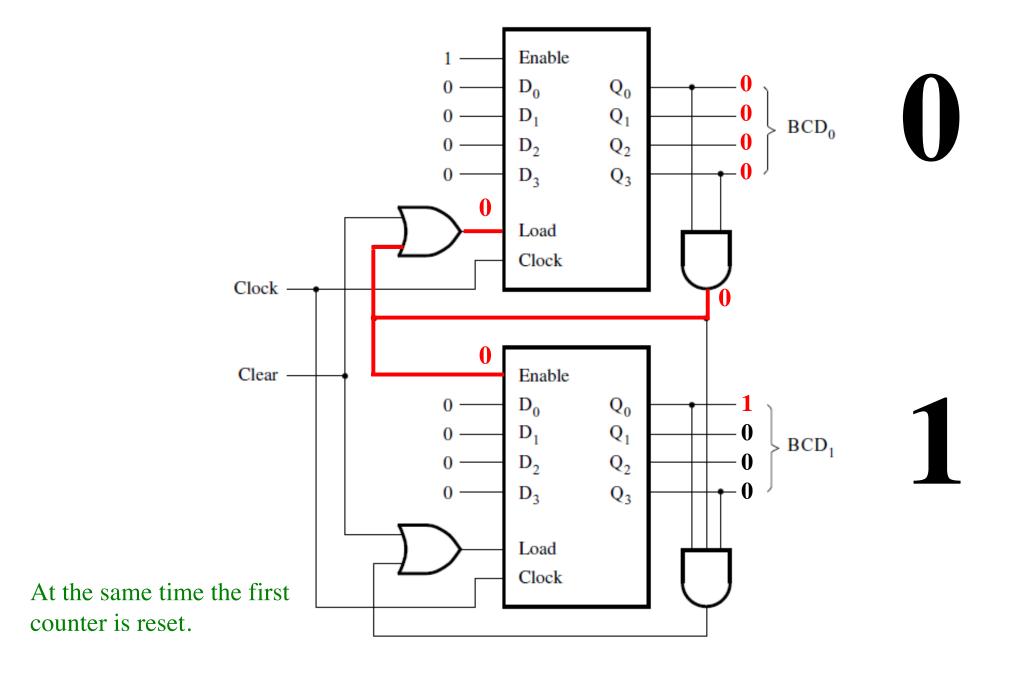


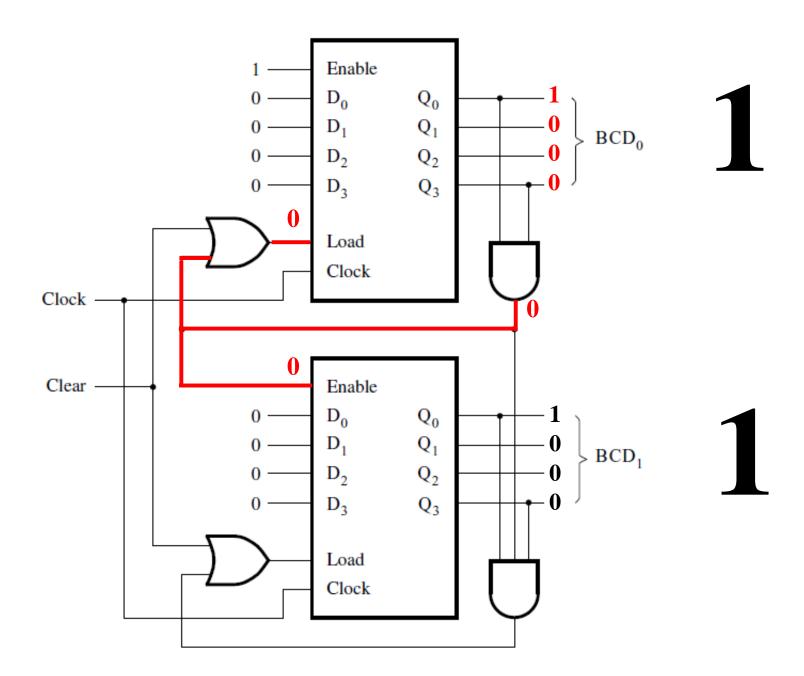


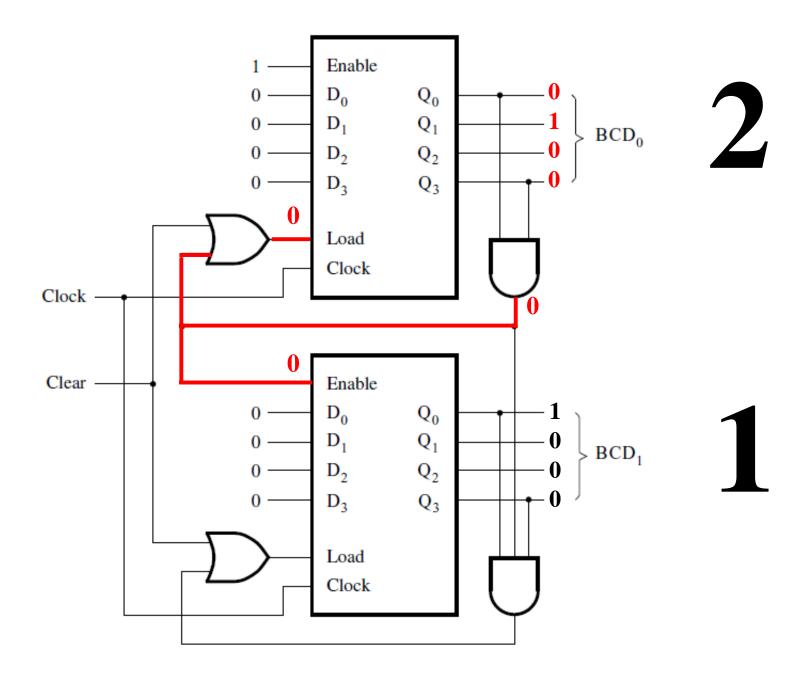




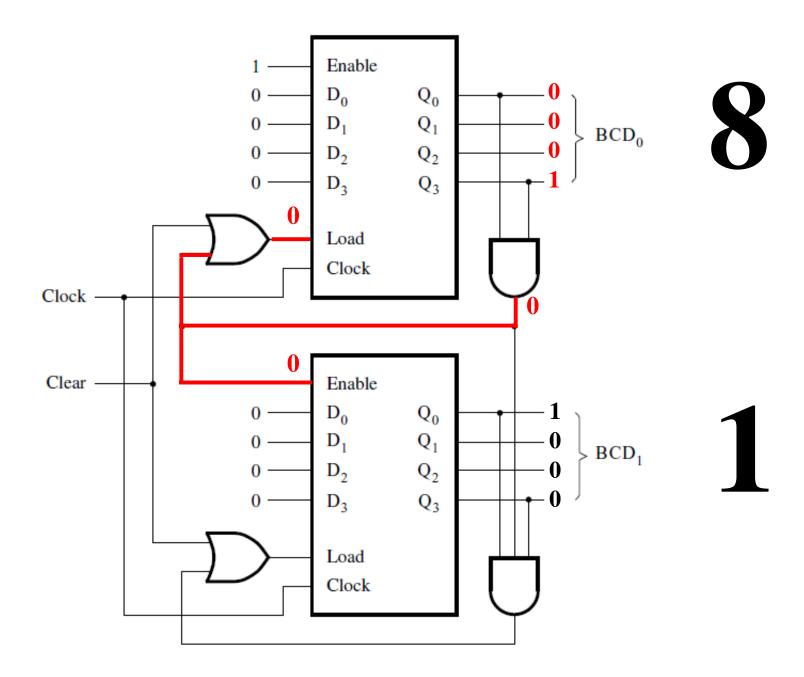


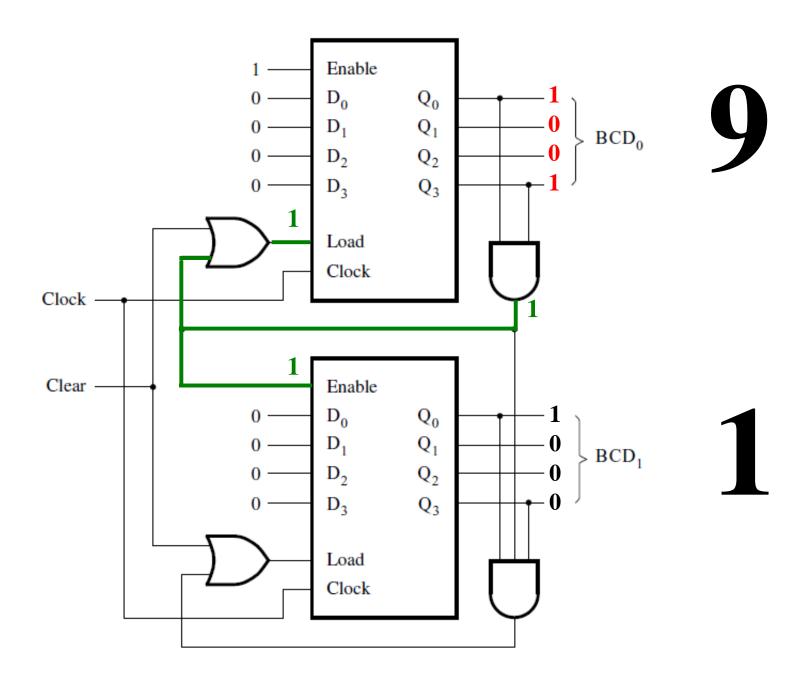


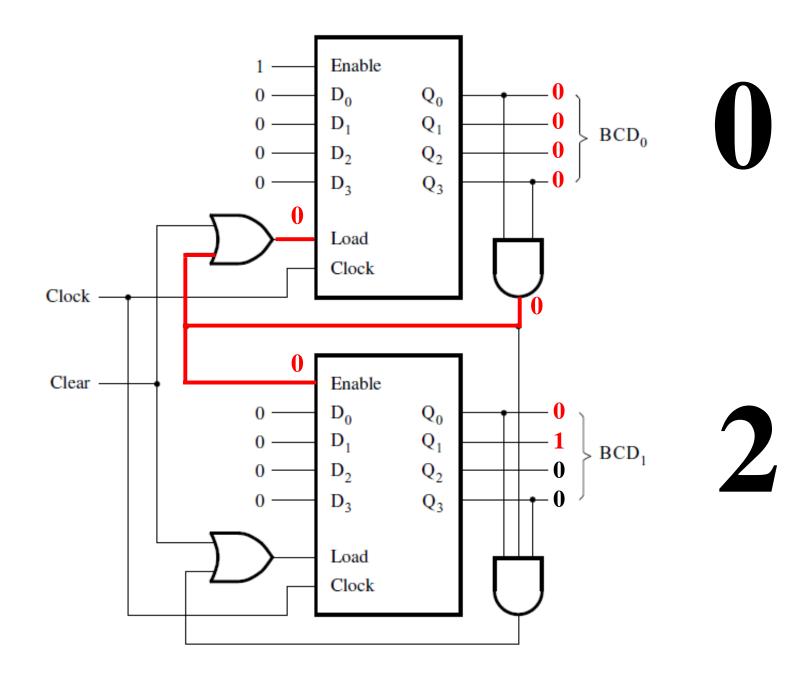


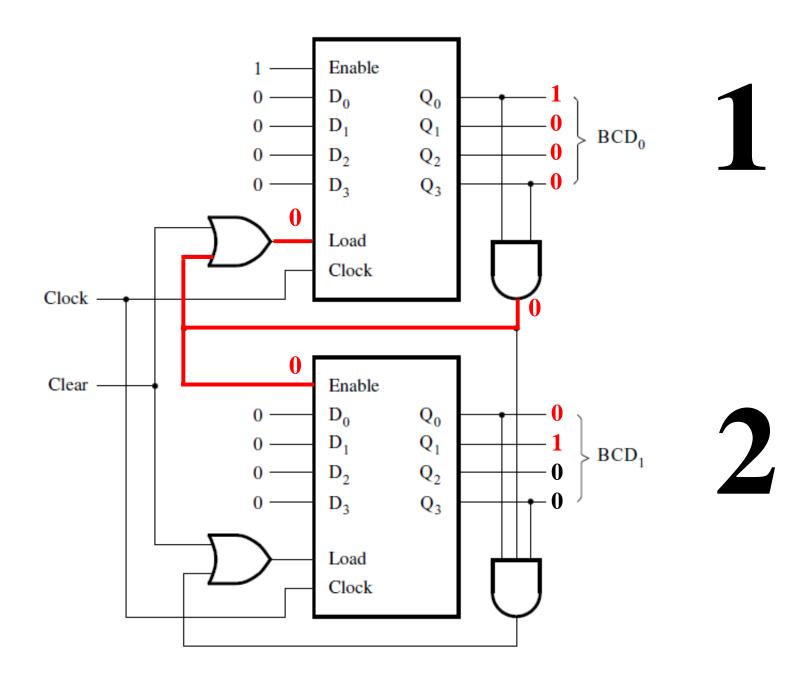




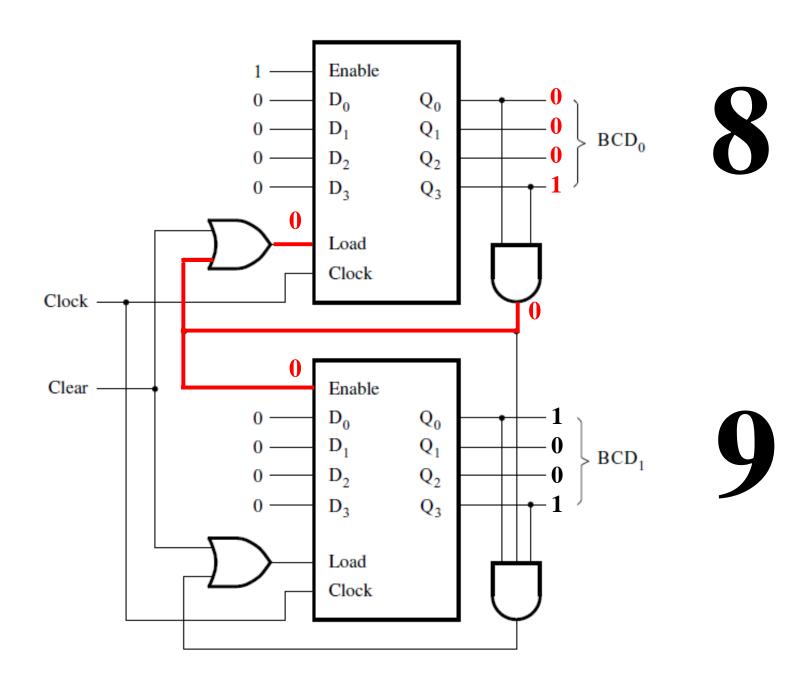


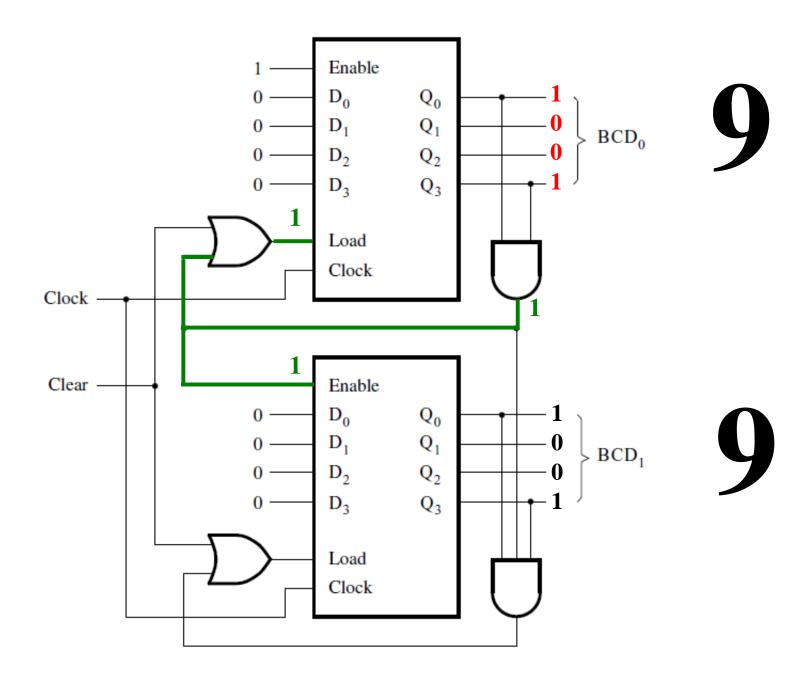


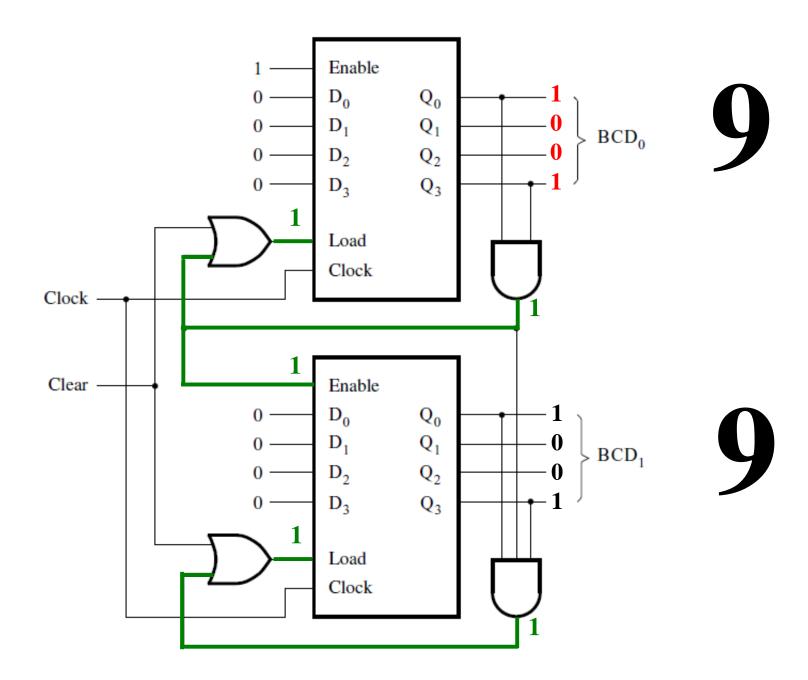


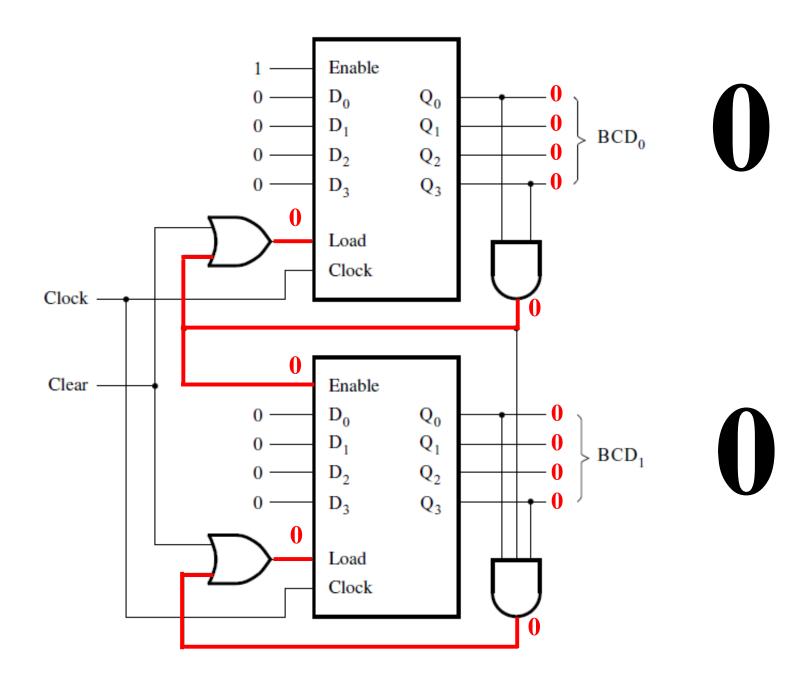


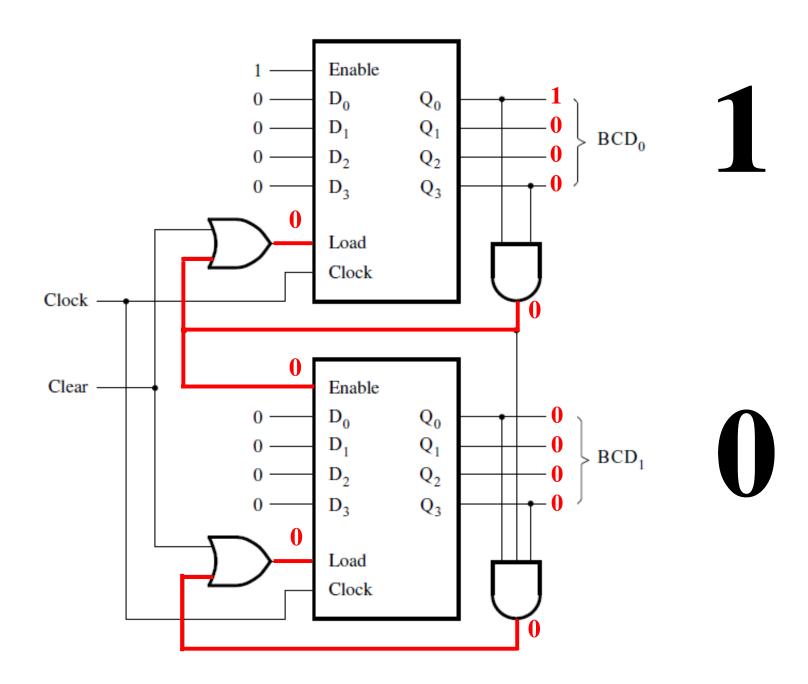






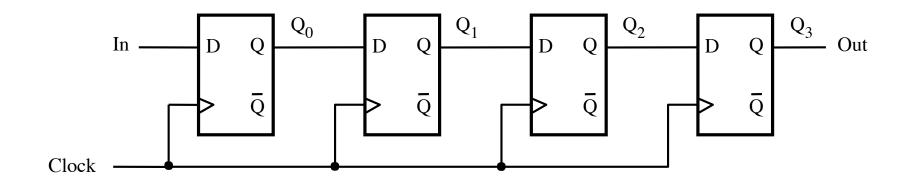






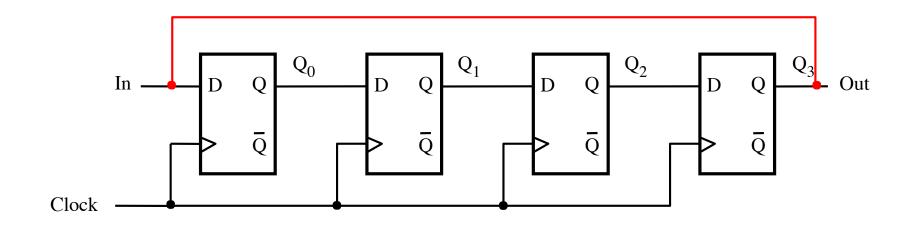
Ring Counter

How to build a 4-bit ring counter



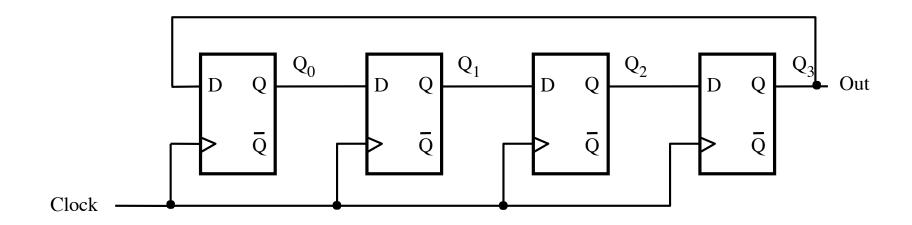
To build a ring counter we start with a shift register.

How to build a 4-bit ring counter



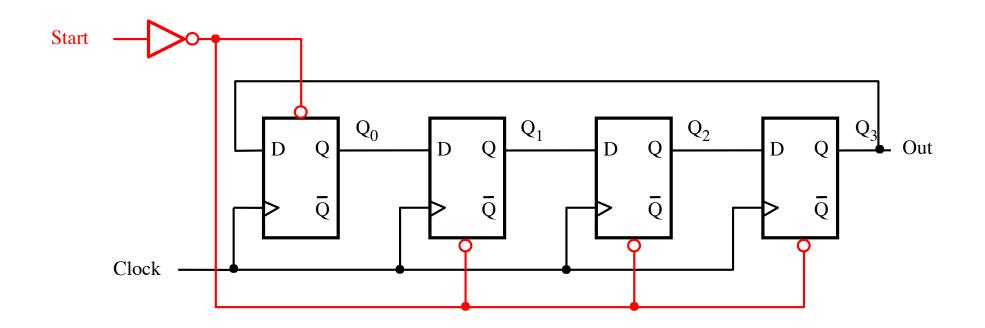
Next, add a loop from the last flip-flop to the first...

How to build a 4-bit ring counter



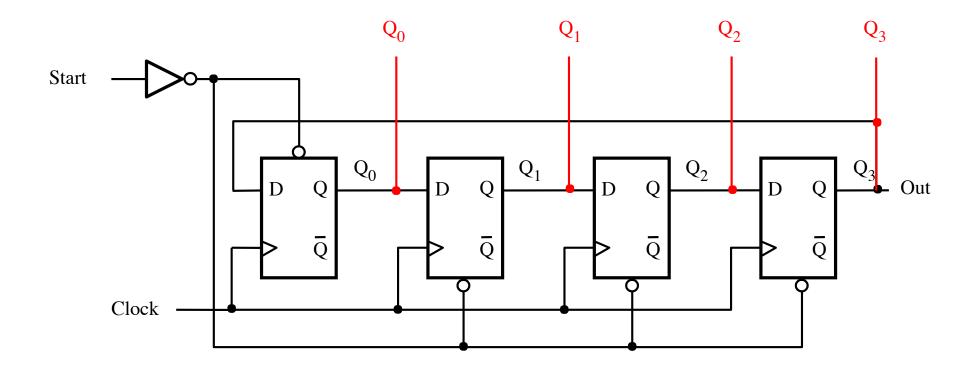
... and remove the In input line.

How to build a 4-bit ring counter



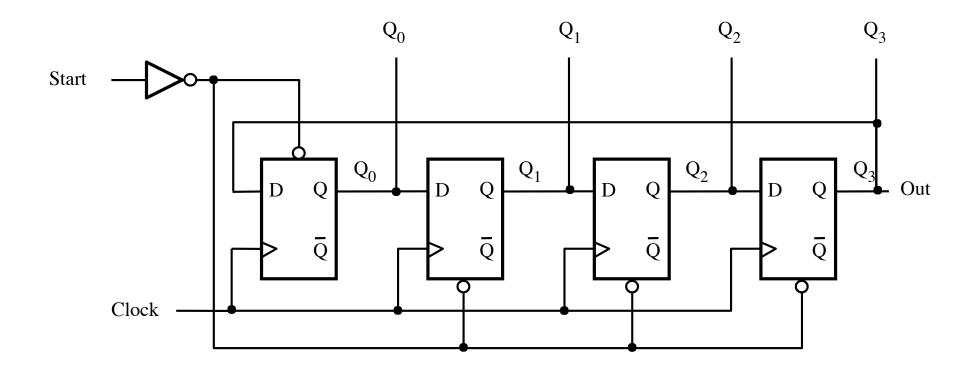
Also, add a start input that goes inverted to preset_n of the first flip=flop and to clear_n of all remaining flip-flops.

How to build a 4-bit ring counter



Finally, extend the output lines that form the count number.

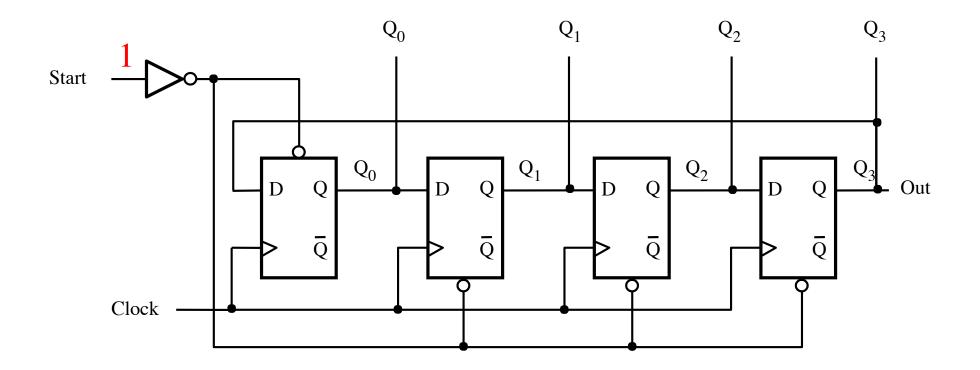
How to build a 4-bit ring counter



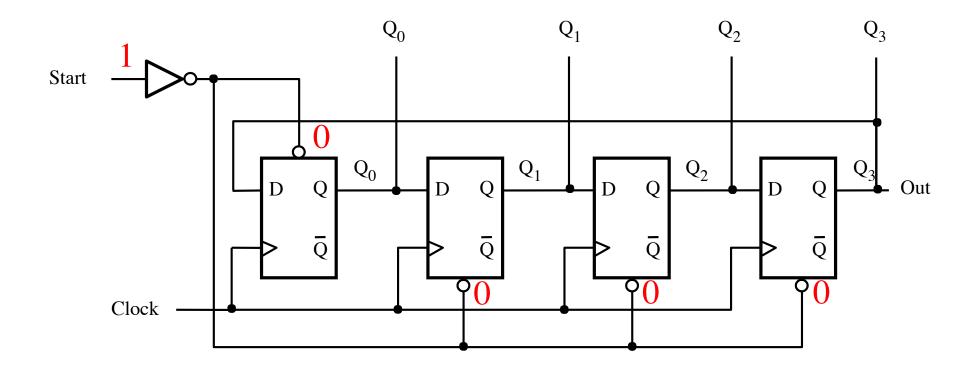
This is the final circuit diagram.

4-bit ring counter

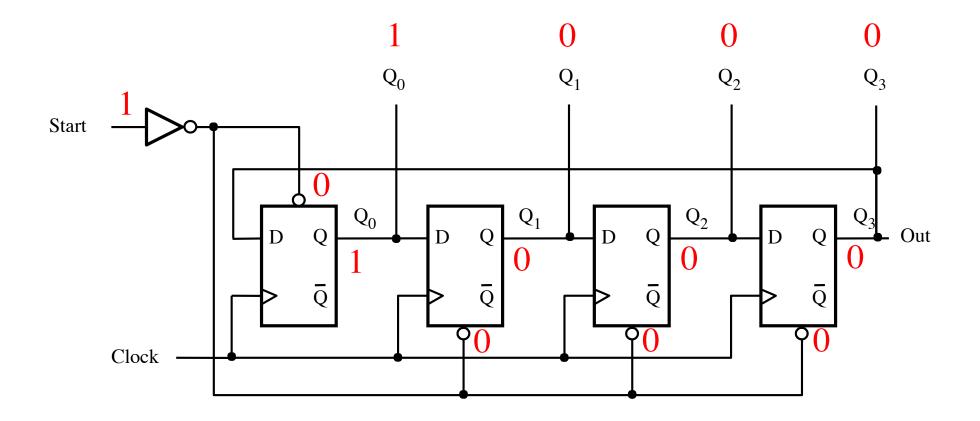
- There is only one 1 on the outputs of the four flip-flops
- The counting sequence is: 1000, 0100, 0010, 0001, 1000, ...
- To reset the counter
 - set start to 1 for a short period of time
 - This sets the four outputs to 1000



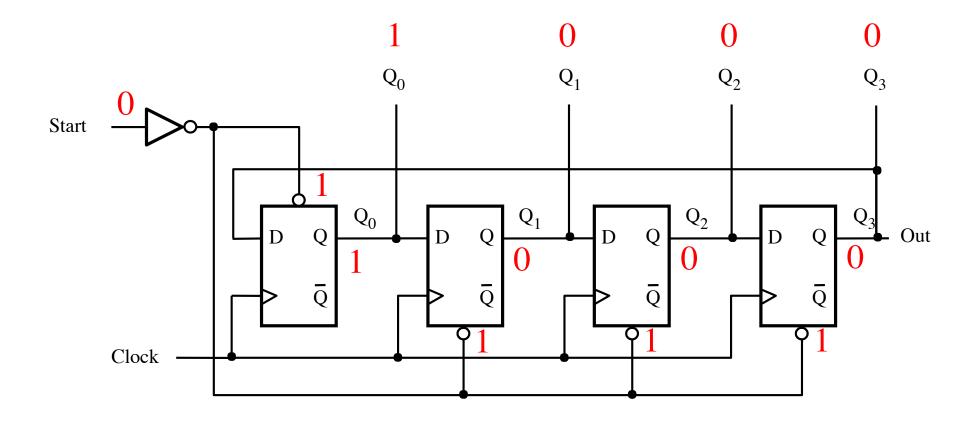
To initialize the counter set Start to 1.



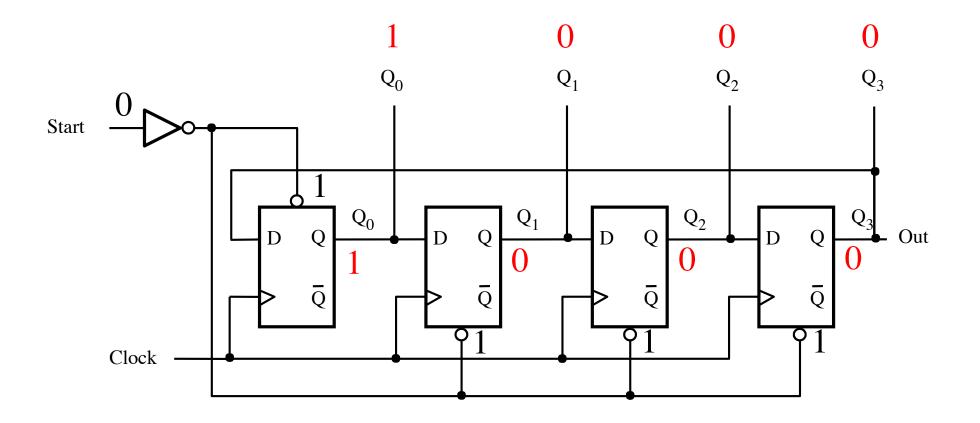
After the NOT gate, this 1 goes as 0 to preset_n of the first flip-flop and to clear_n of all remaining flip-flops.



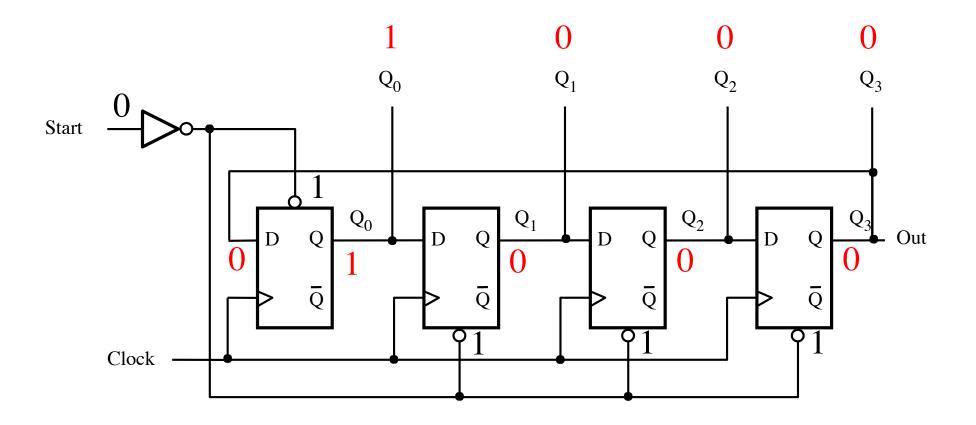
This sets the output pattern to 1000, i.e., only the first bit is one and the rest are zeros.



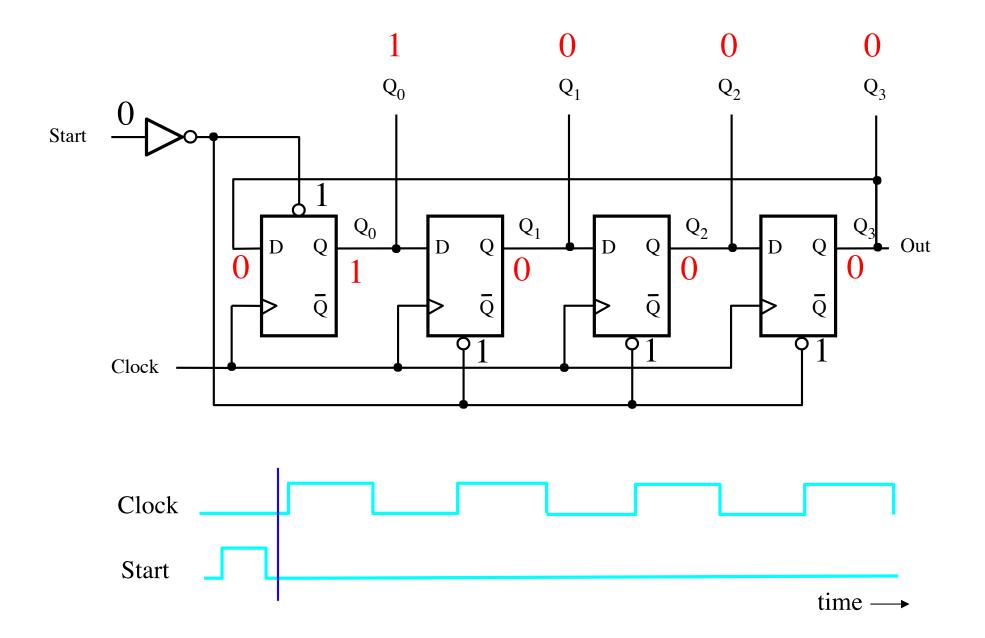
Setting Start to 0 has no effect on the outputs, because both preset_n and clear_n are sensitive only to 0.

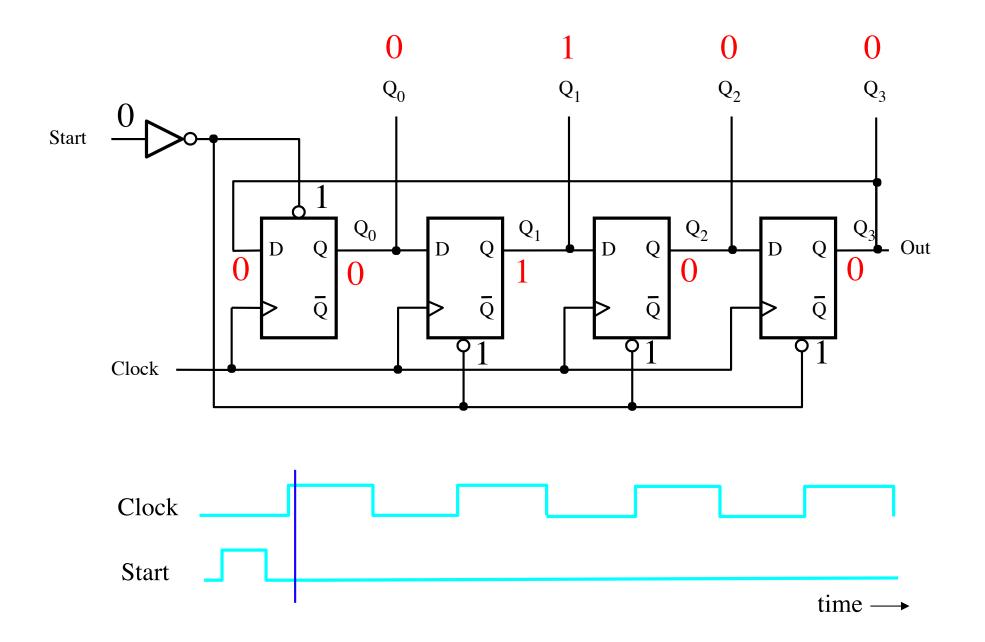


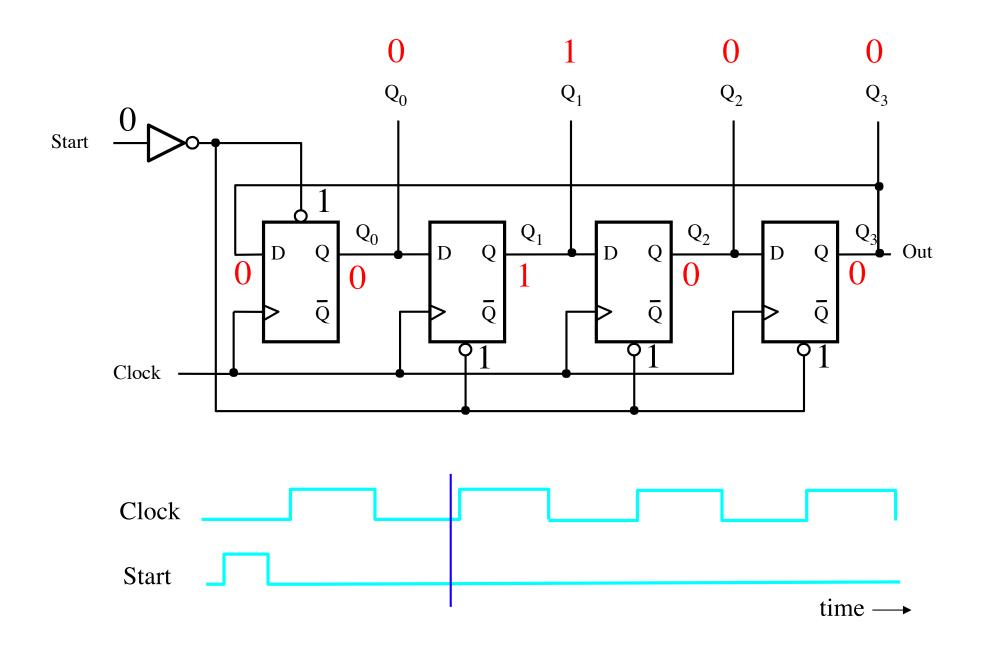
The initialization does not depend on the clock since both preset_n and clear_n bypass the gates of the latches in the flip-flops.

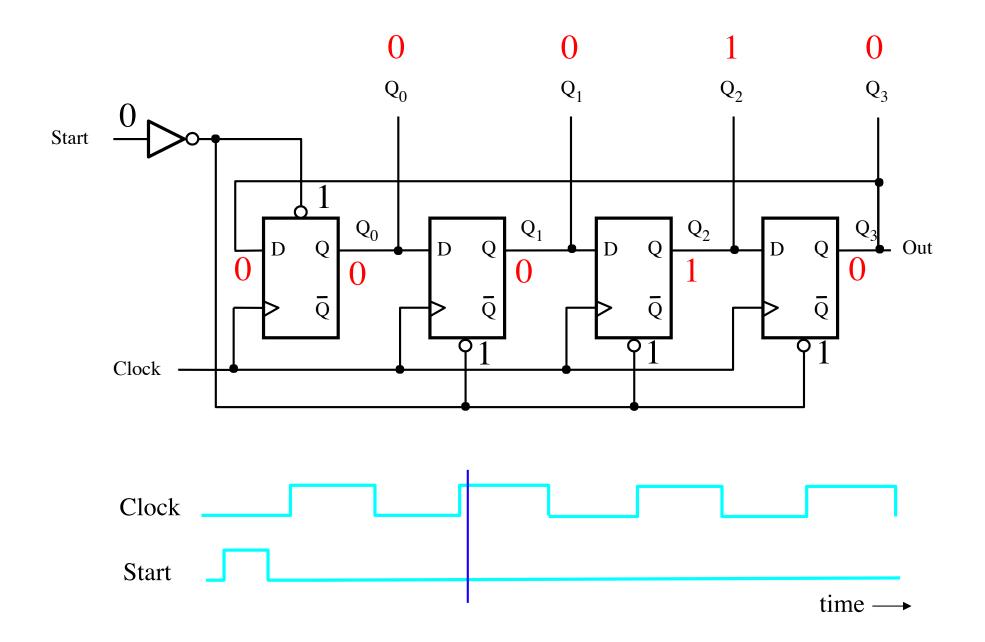


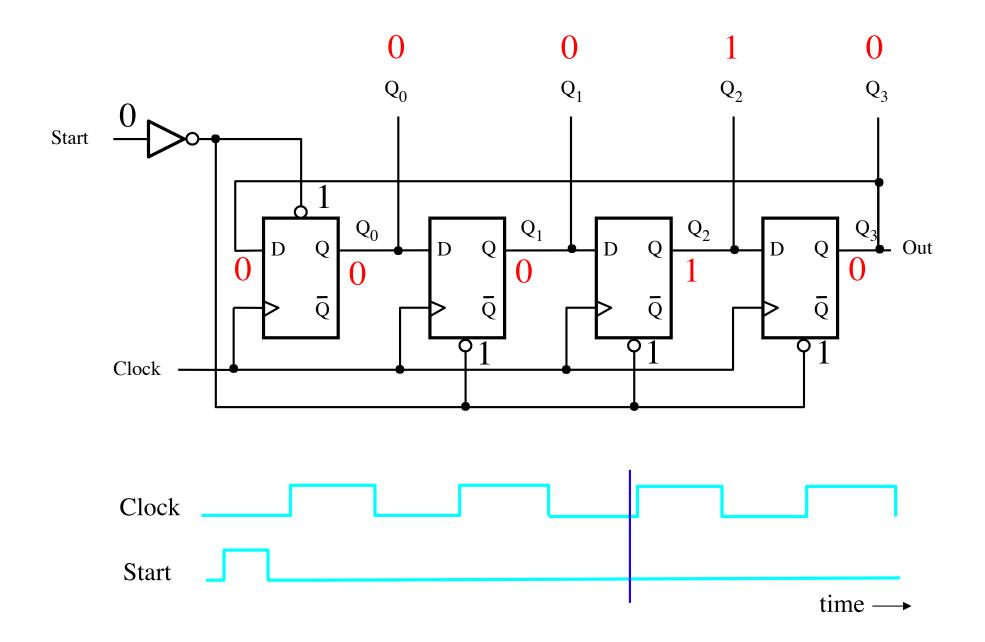
That last 0 loops back to the D input of the first flip-flop.

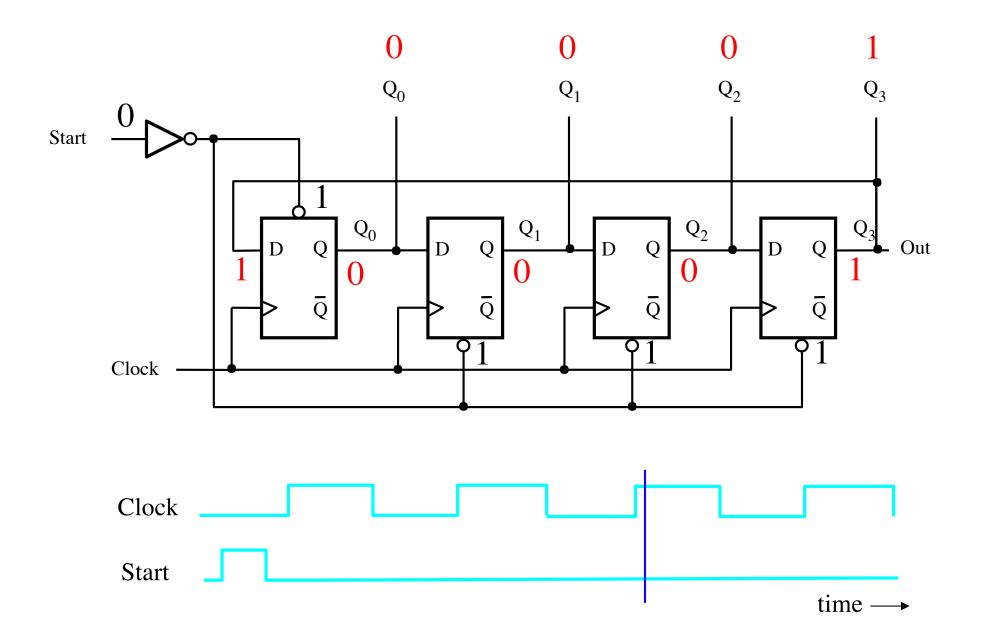


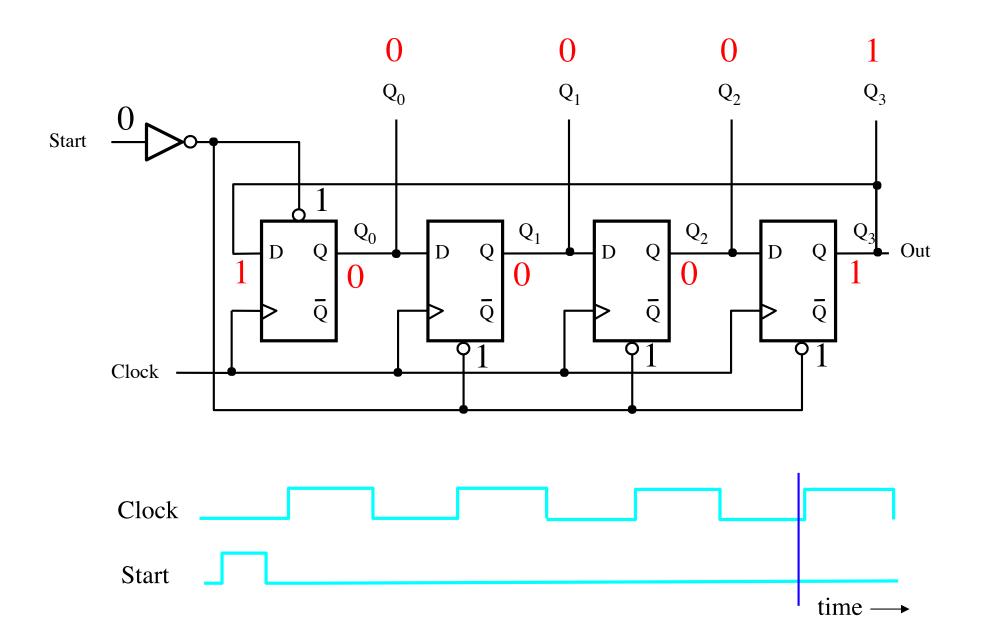


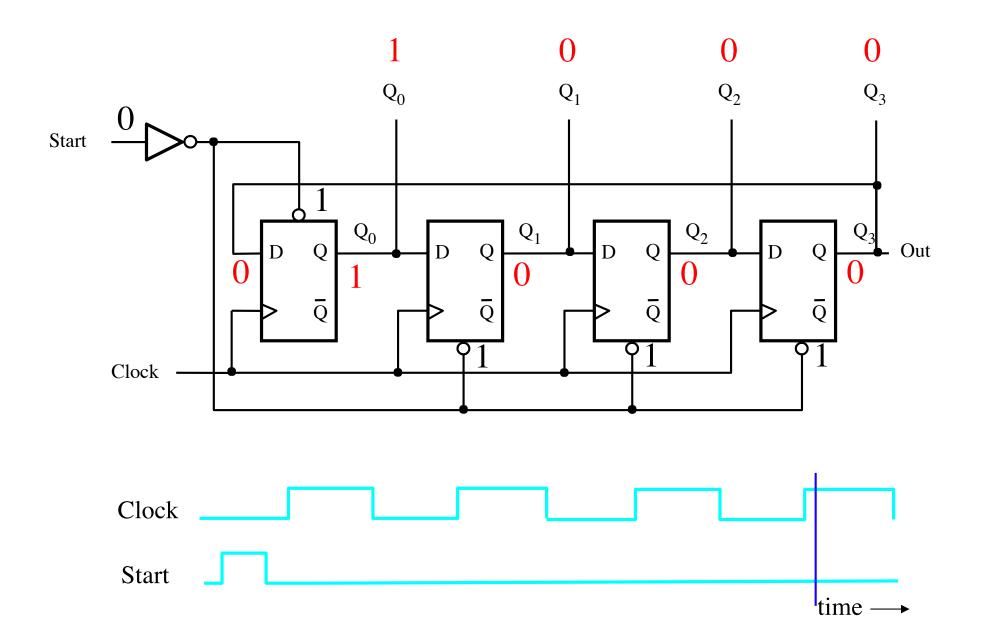


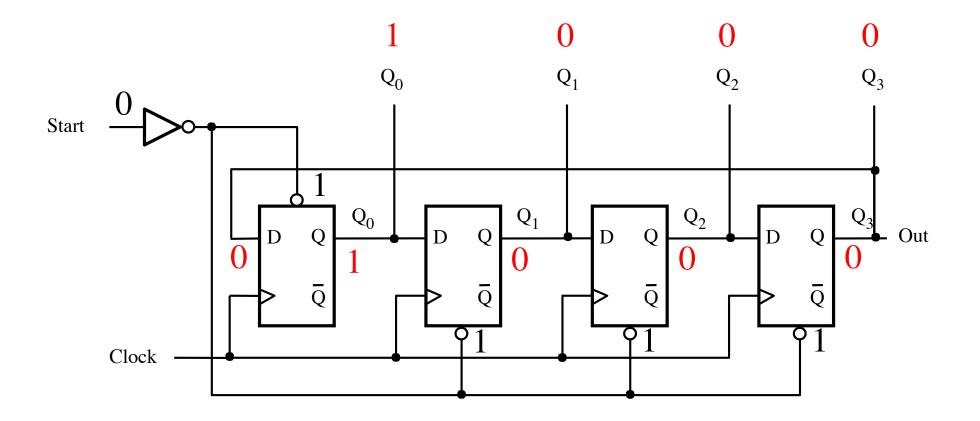






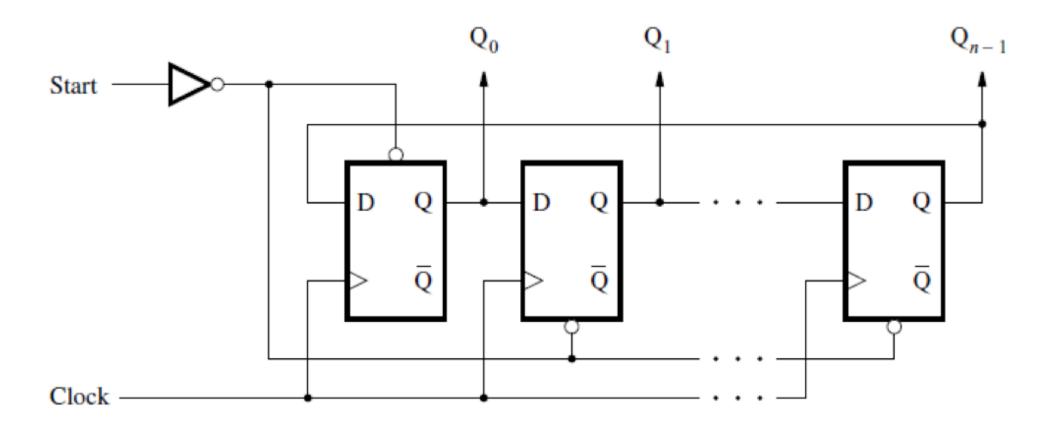






It is back to the start of the counting sequence, which is: 1000, 0100, 0010, 0001.

n-bit ring counter

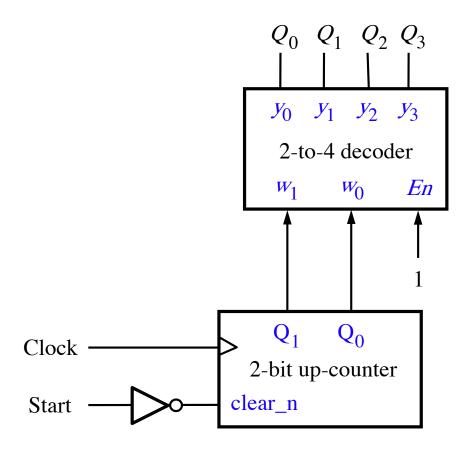


Ring Counter (alternative implementation)

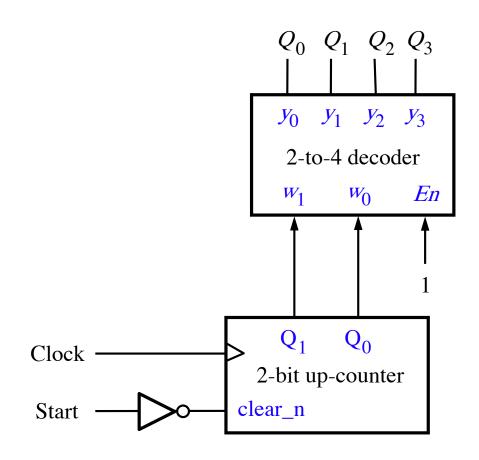
Alternative version of a 4-bit ring counter

- This implementation uses 2-bit up-counter followed by a 2-to-4 decoder.
- The counter cycles through 00, 01, 10, 11, 00, ...
- Recall that the outputs of the decoder are one-hot encoded. Thus, there is only one 1 on its outputs.
- Because the output of the counter is the input to the decoder, the outputs of the decoder cycle through: 1000, 0100, 0010, 0001, 1000, ...
- This is the counting sequence for a ring counter.

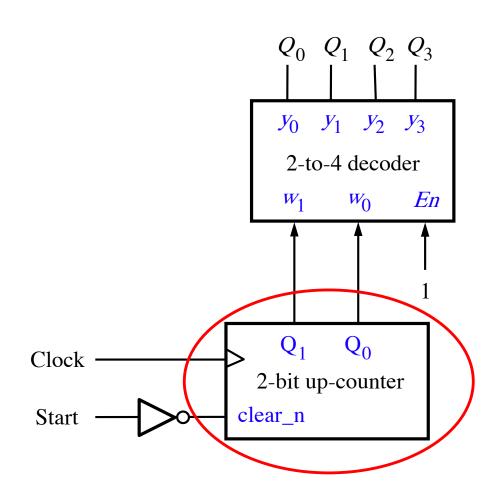
Alternative version of a 4-bit ring counter



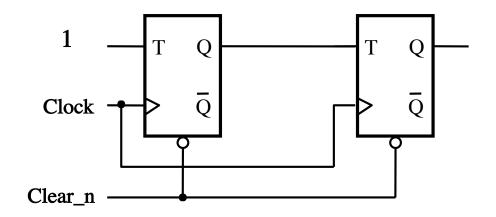
What are the components?



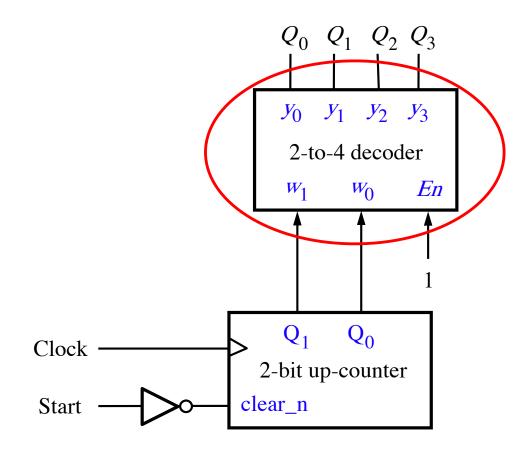
2-Bit Synchronous Up-Counter



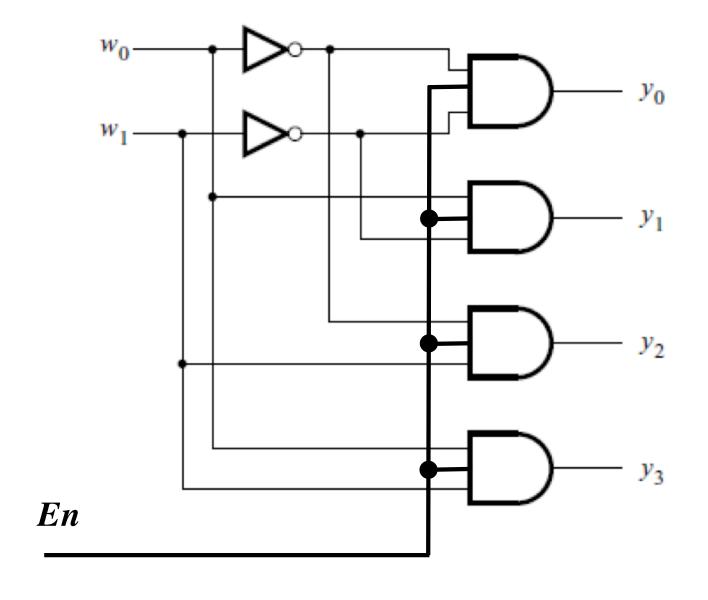
2-Bit Synchronous Up-Counter



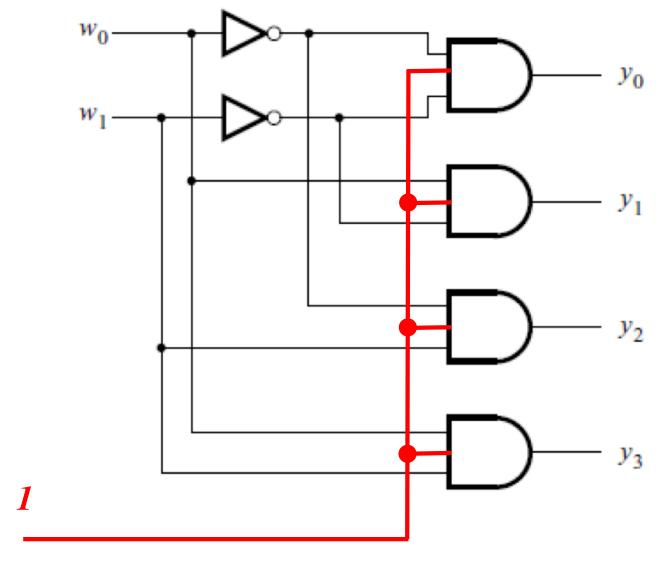
2-to-4 Decoder with Enable Input



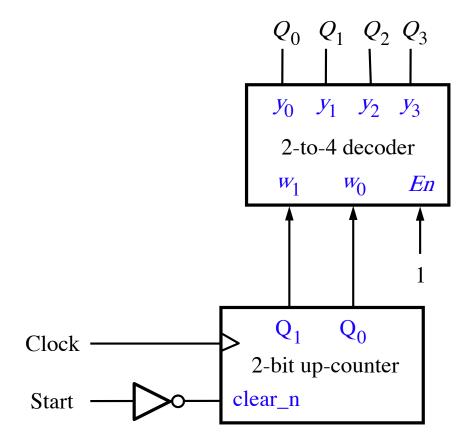
2-to-4 Decoder with Enable Input

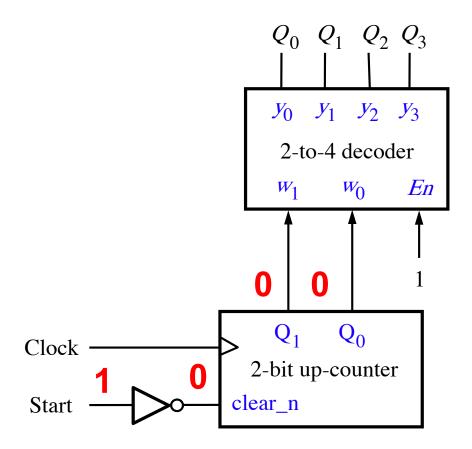


2-to-4 Decoder with Enable Input

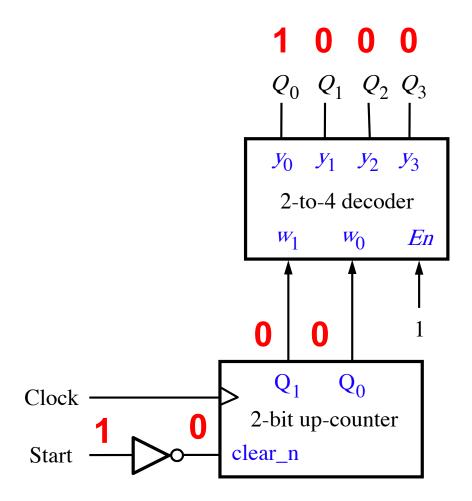


(always enabled in this example)

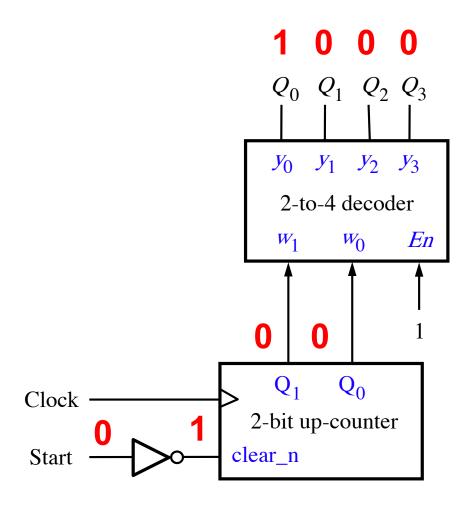




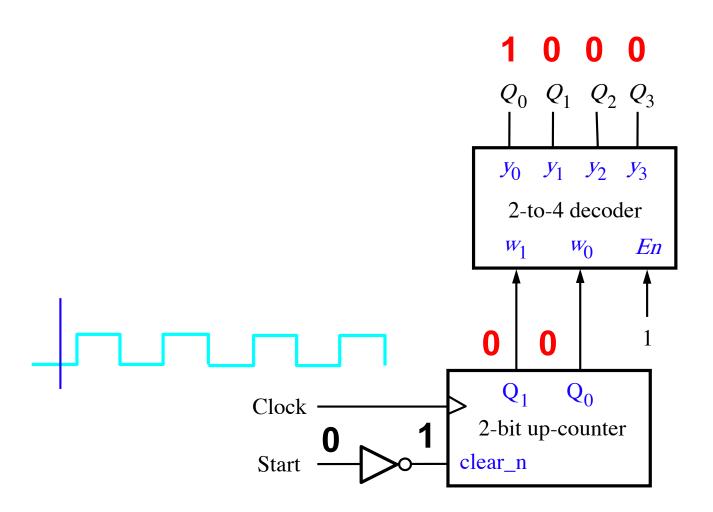
To initialize this circuit set Start to 1, which sets the counter to 00.

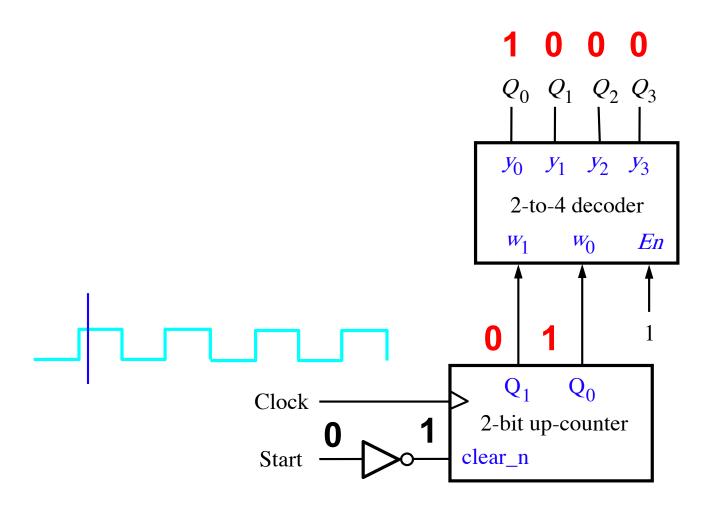


This sets the outputs of the decoder to the start of the counting sequence.

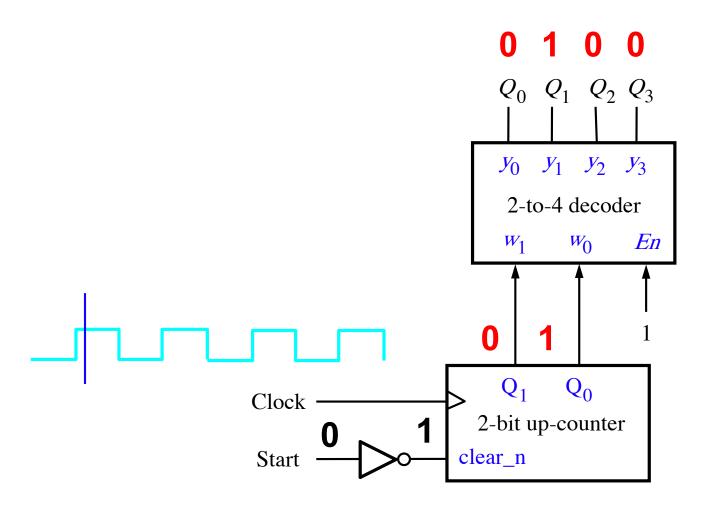


When Start is equal to 0, clear_n has no effect.

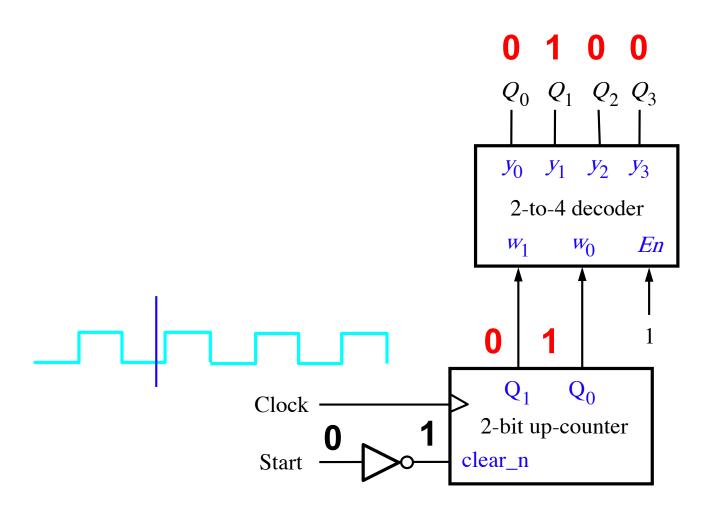


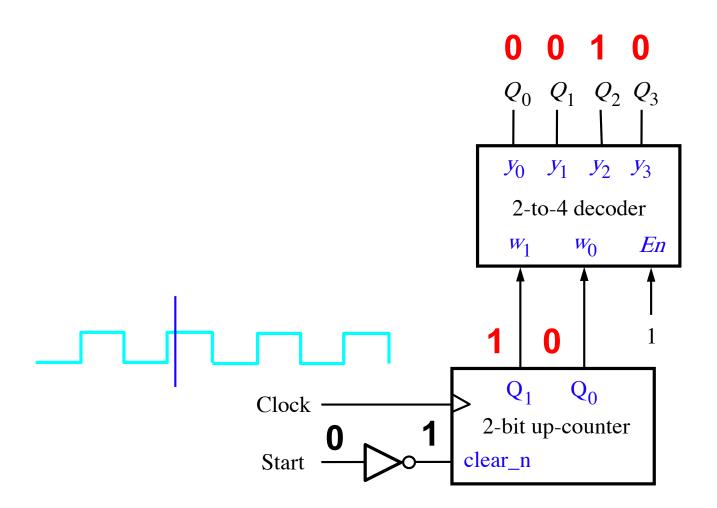


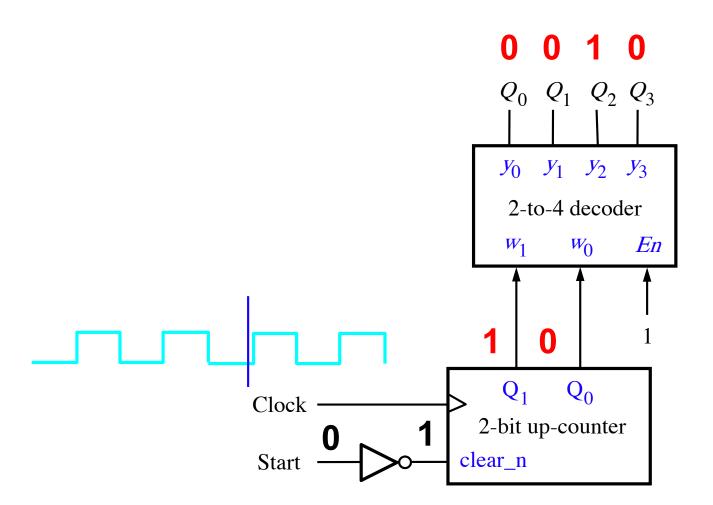
The counter increments the count on the positive clock edge ...

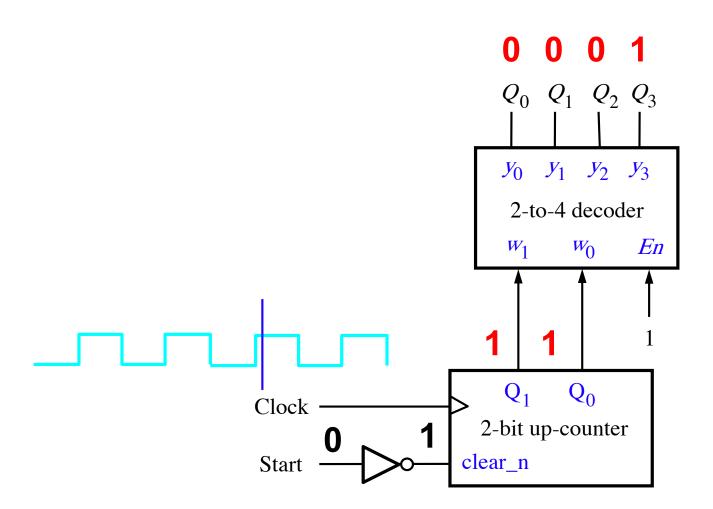


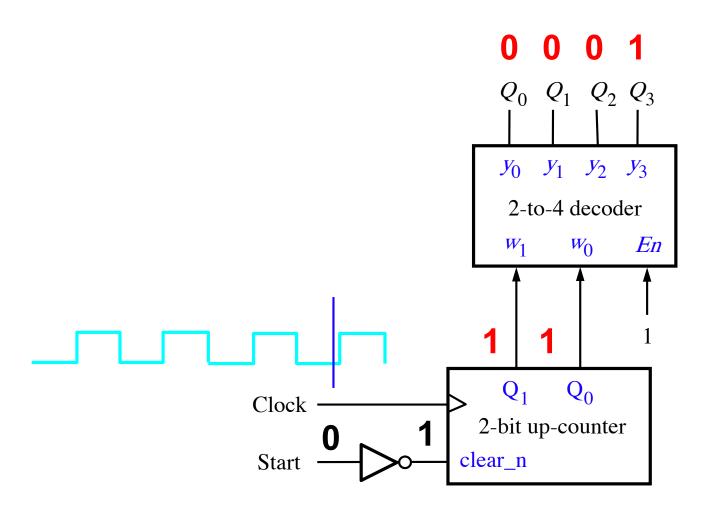
... this updates the outputs of the decoder (which are one hot encoded).

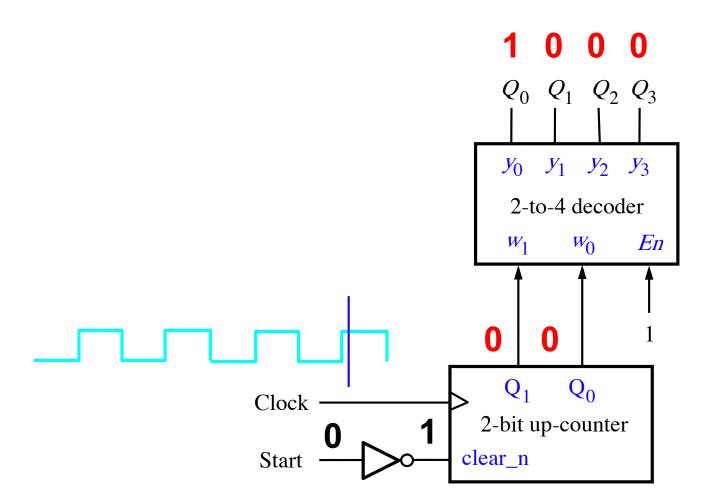






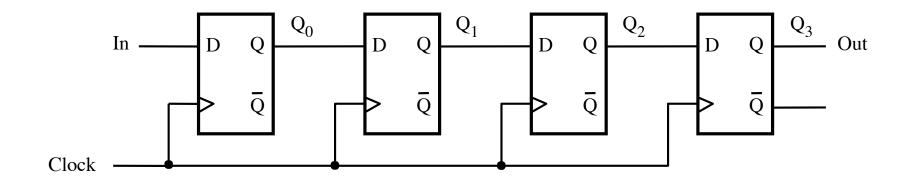




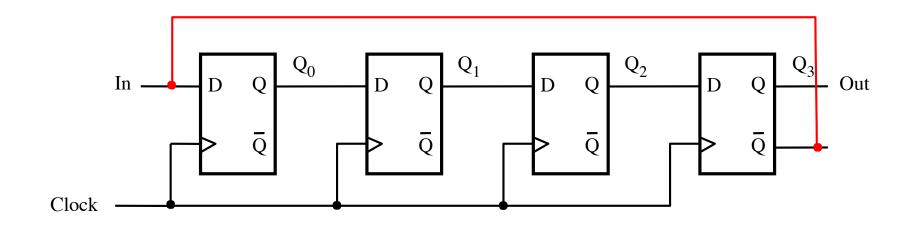


It is back to the start of the counting sequence, which is: 1000, 0100, 0010, 0001.

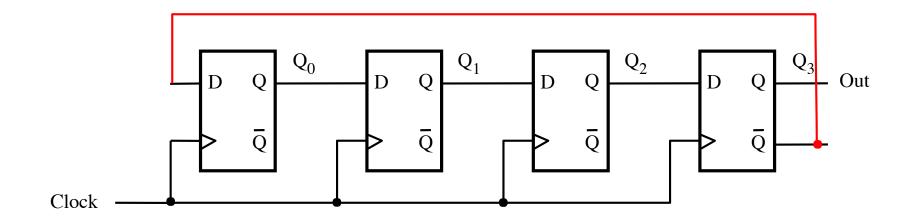
Johnson Counter



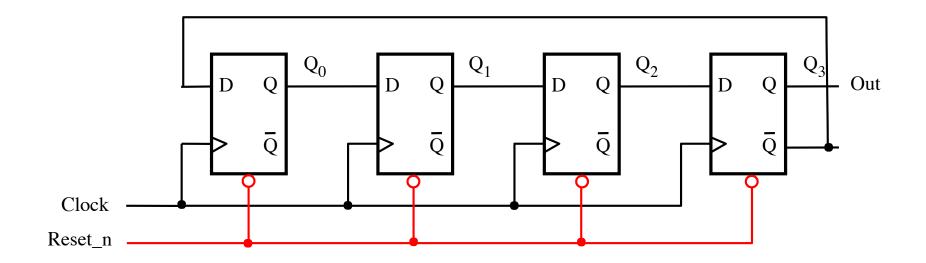
To build a Johnson counter start with a shift register.



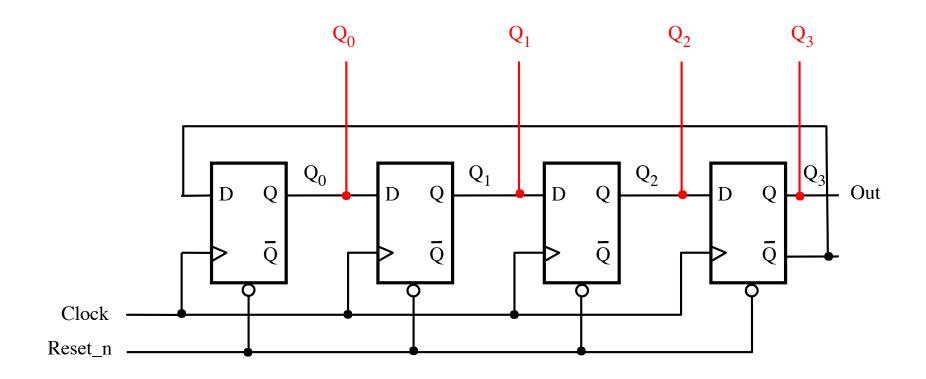
Next, add a loop from the \overline{Q} output of the last flip-flop to the first...



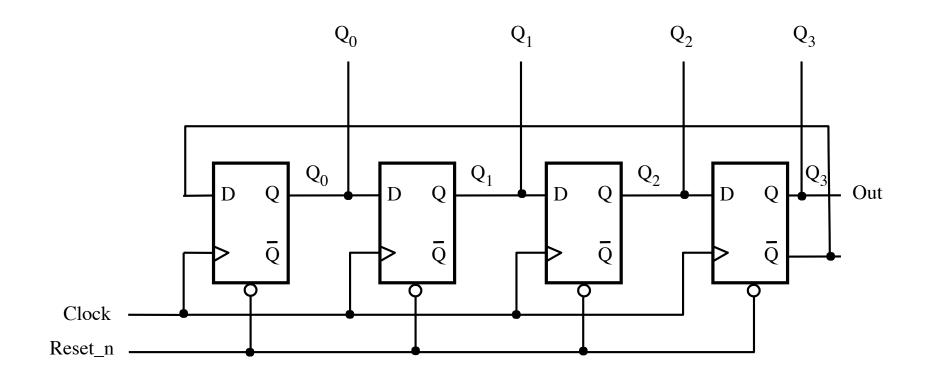
... and remove the In input line.



Also, add a Reset_n line that goes to clear_n of all flip-flops.



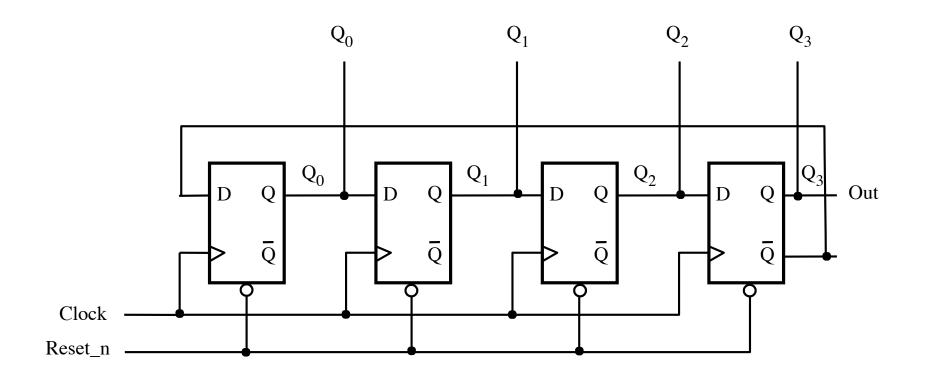
Finally, extend the output lines that form the count number.

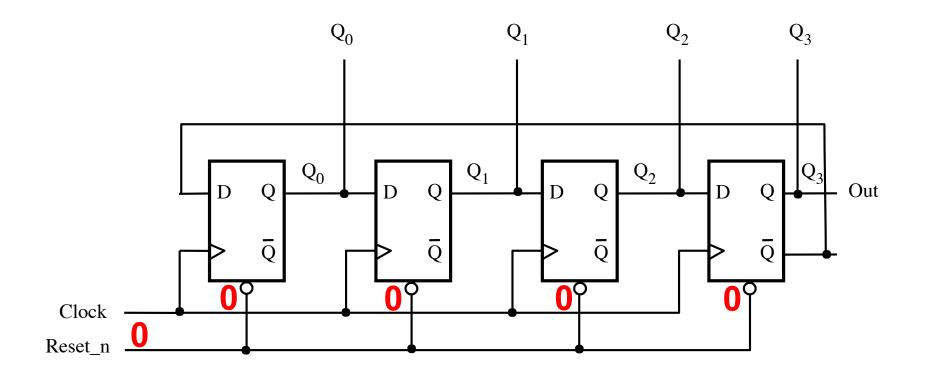


This is the final circuit diagram.

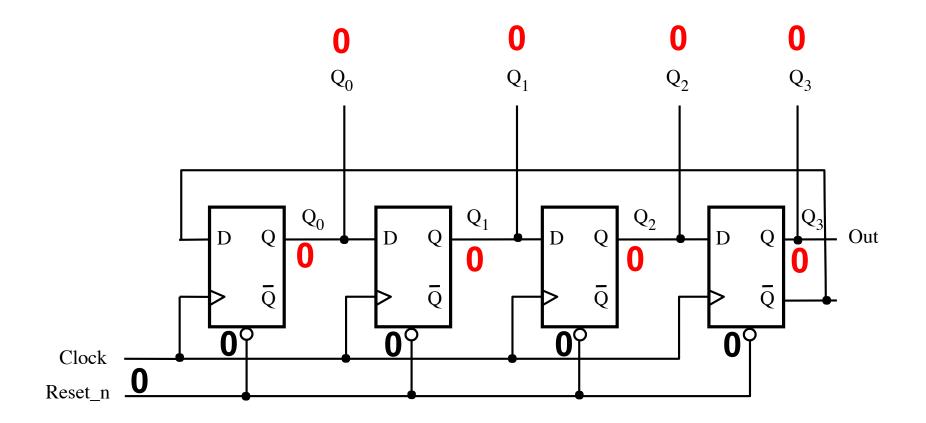
4-Bit Johnson Counter

- Only 1-bit changes at a time
- Start with a reset of all flip-flops
- The counting sequence is:
 0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001, 0000
- An n-bit Johnson counter has a counting sequence of length 2n

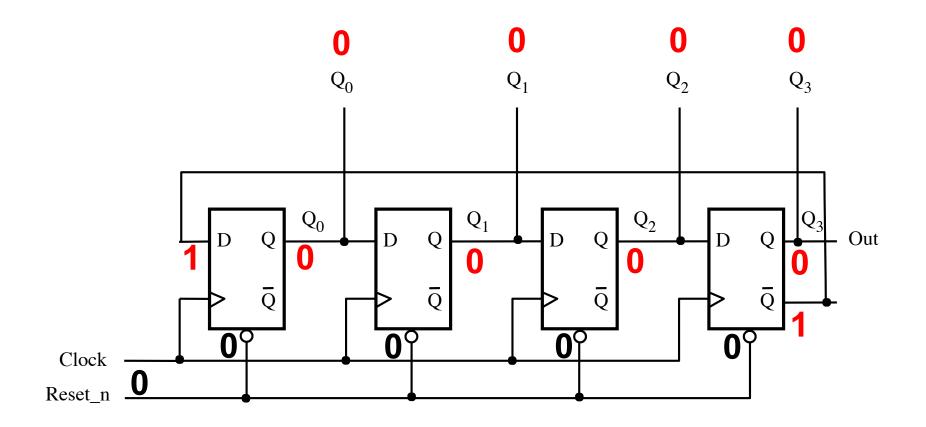




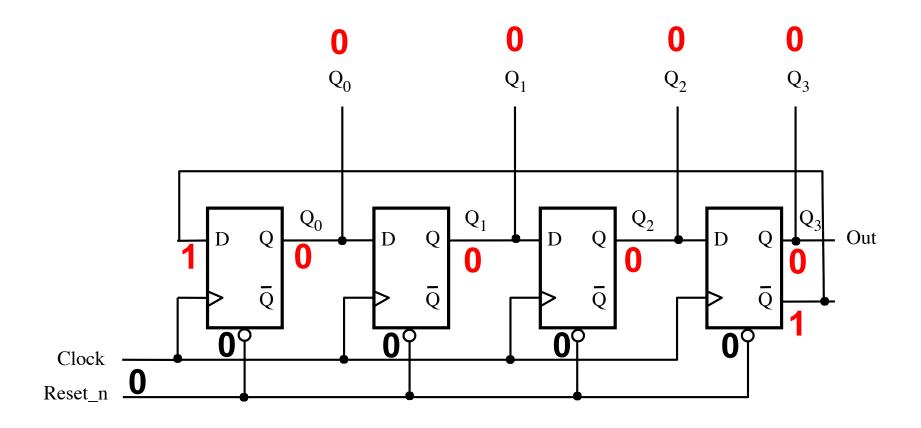
To initialize the Johnson counter set Reset_n to 0 ...

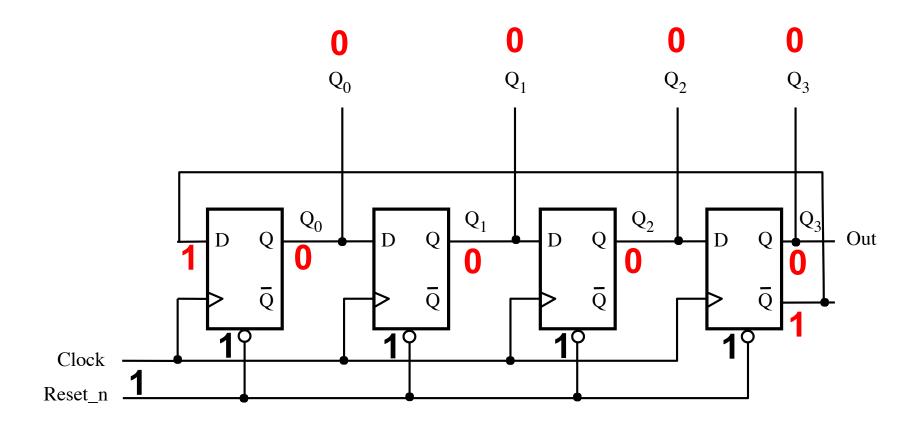


... this zeros the outputs of all flip-flops ...

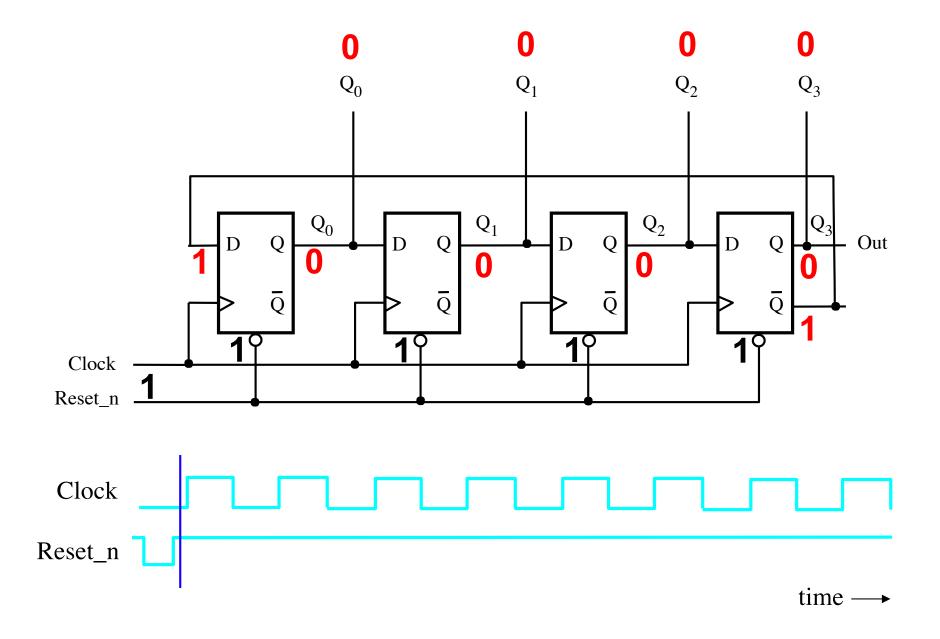


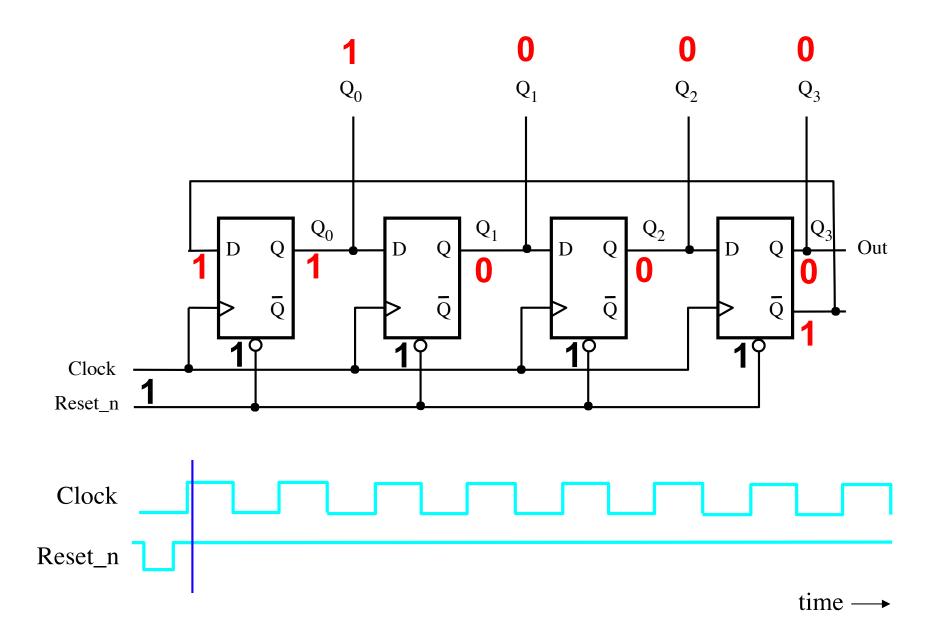
... and also sets the \overline{Q} output of the last flip-flop to 1.

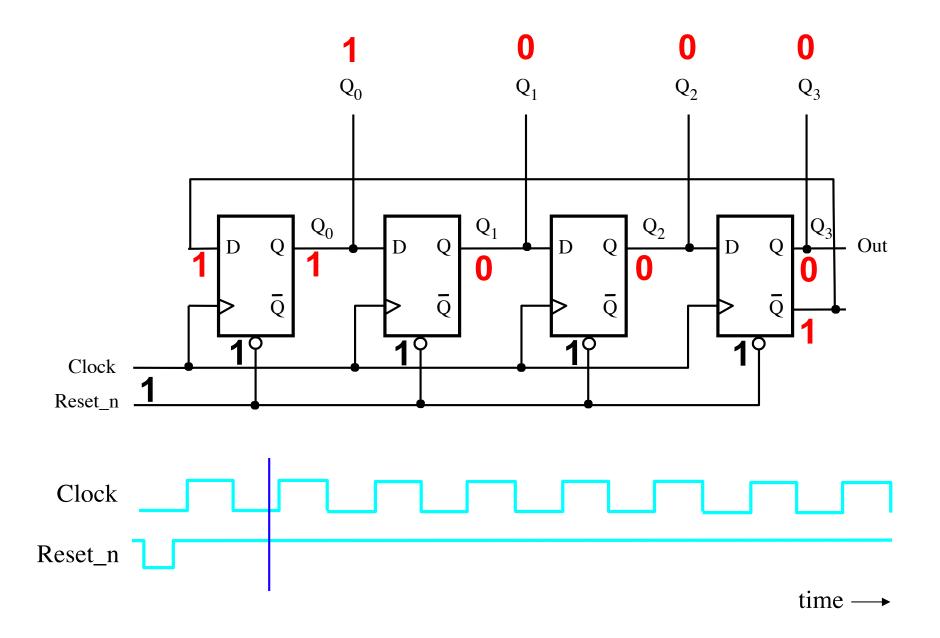


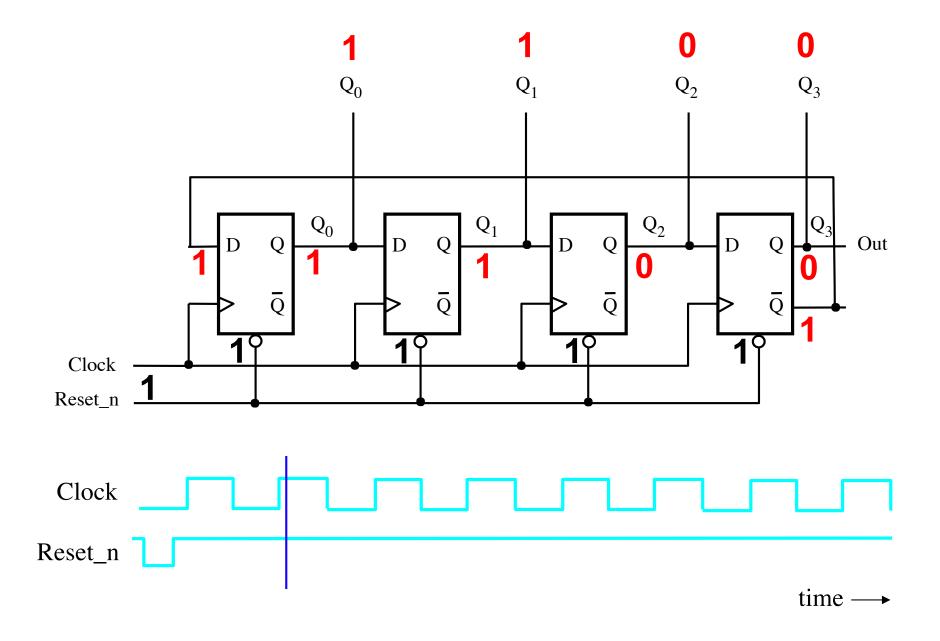


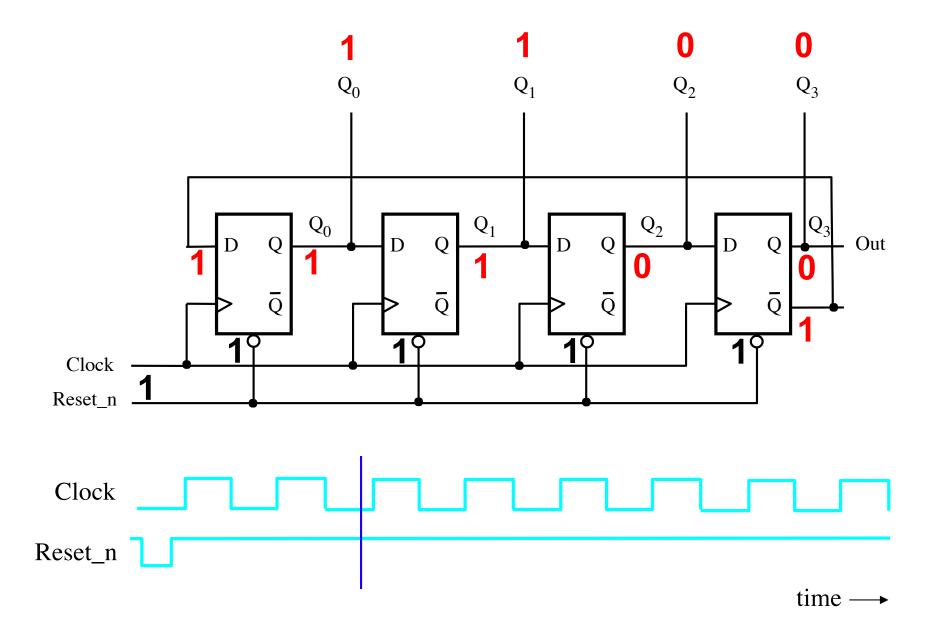
To start counting, Reset_n needs to be set to 1.

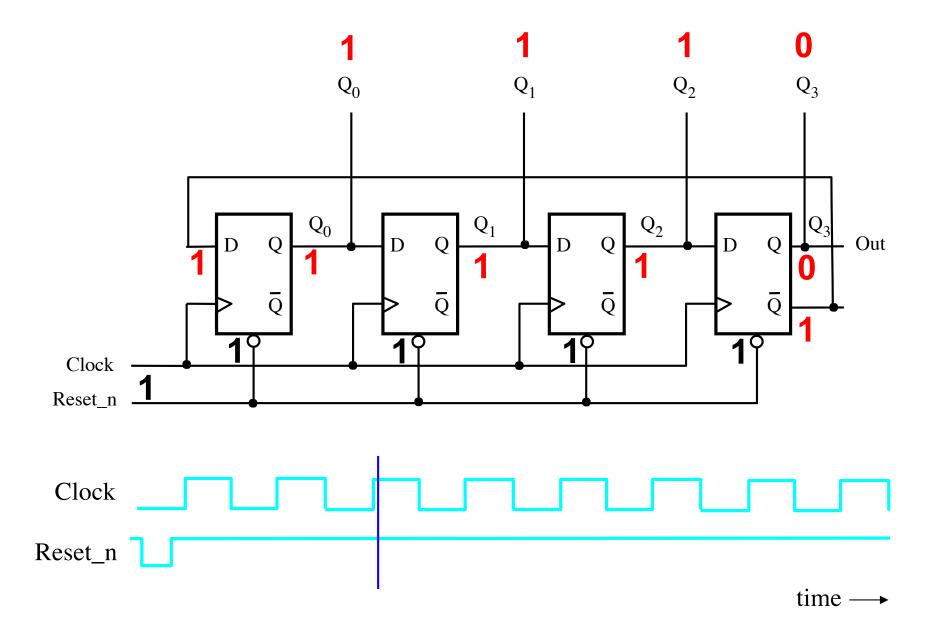


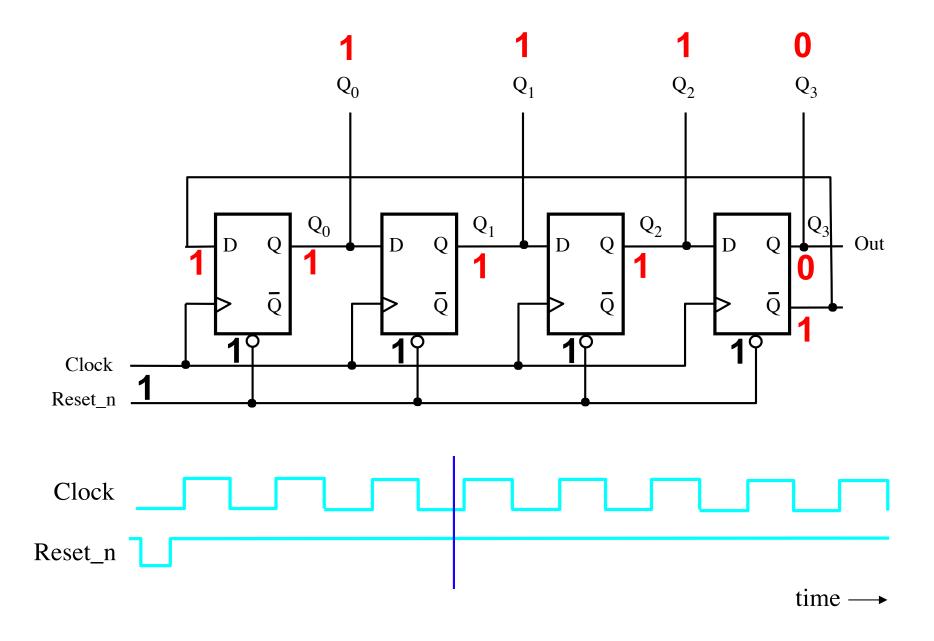


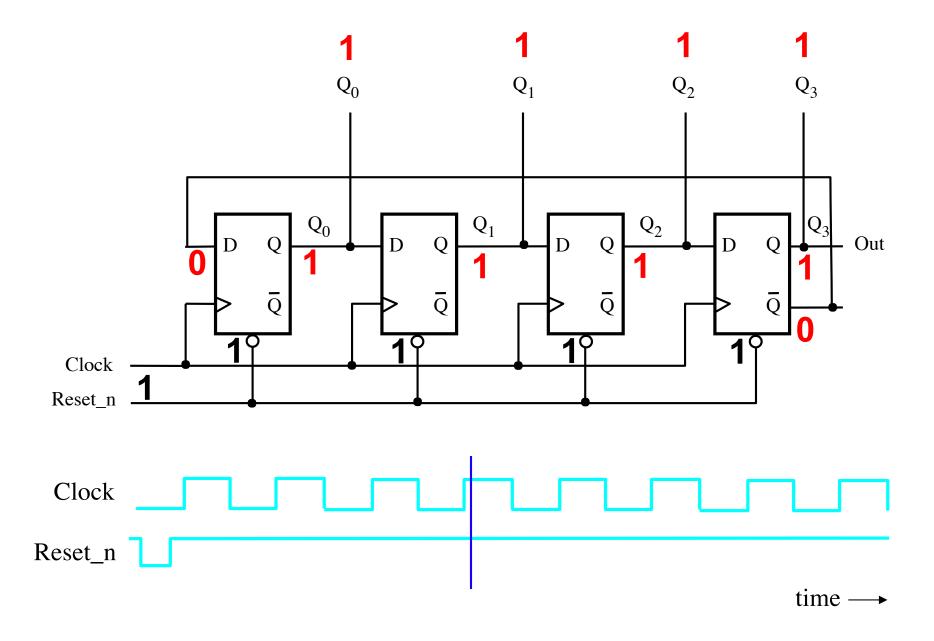


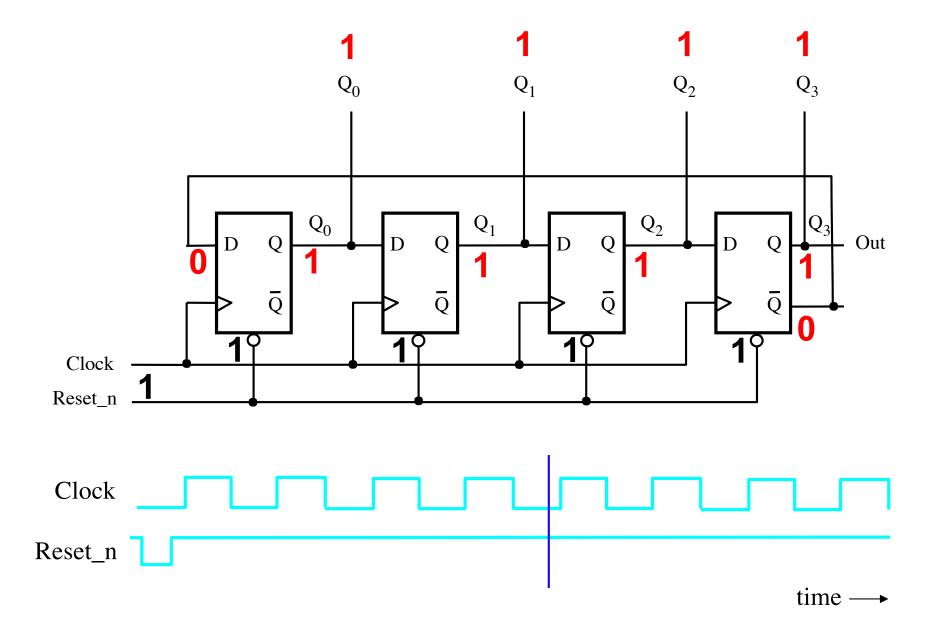


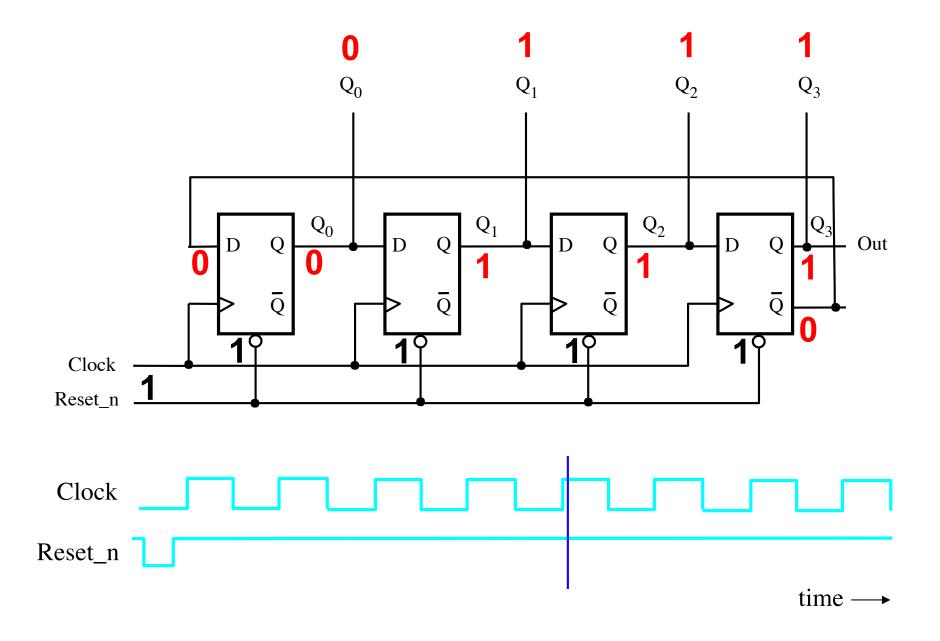


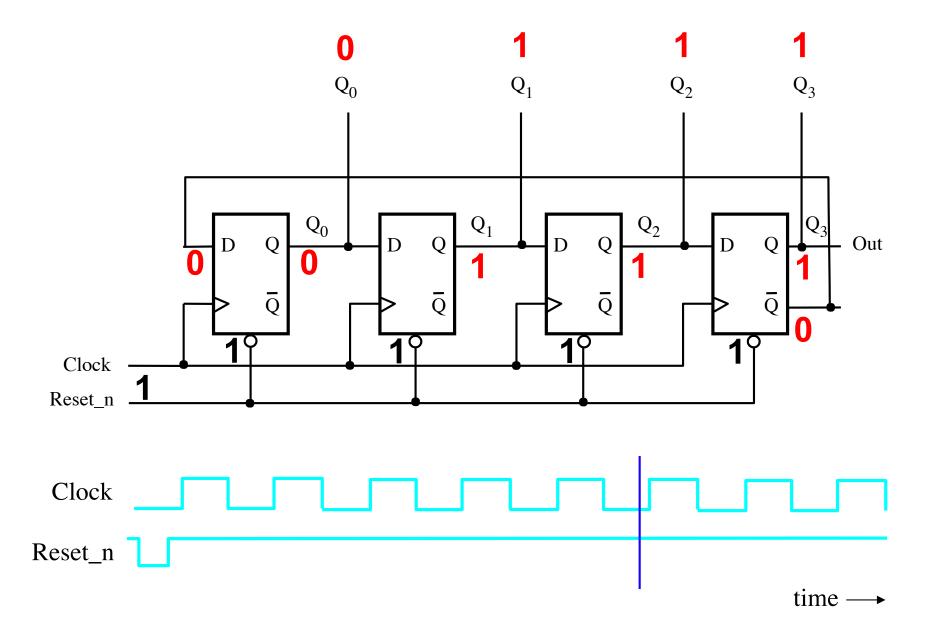


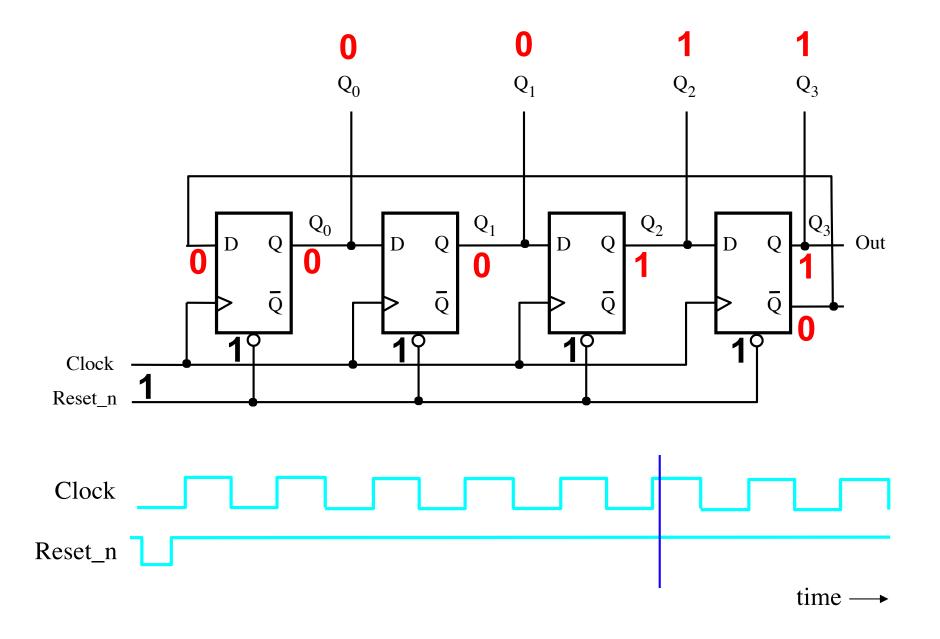


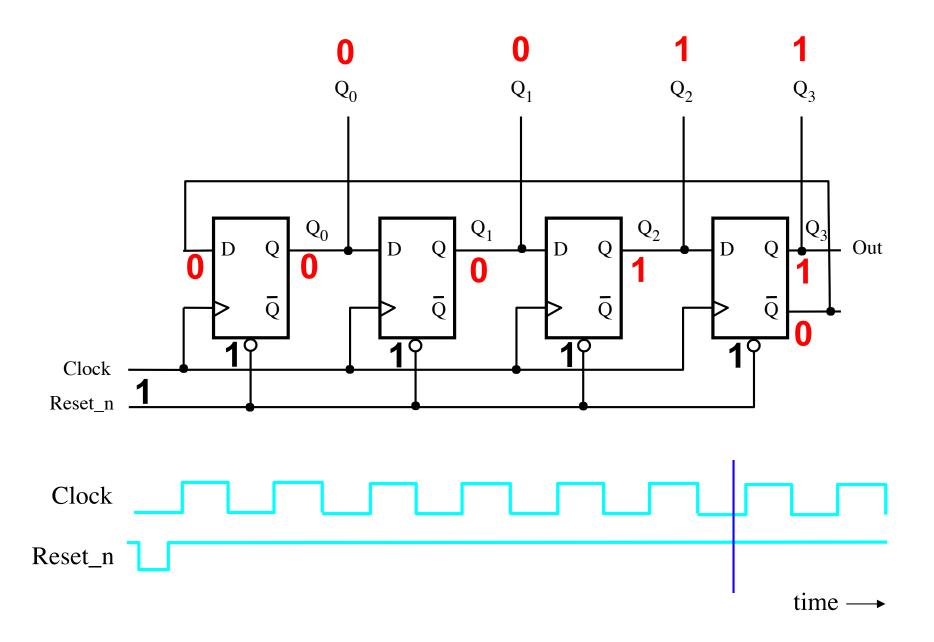


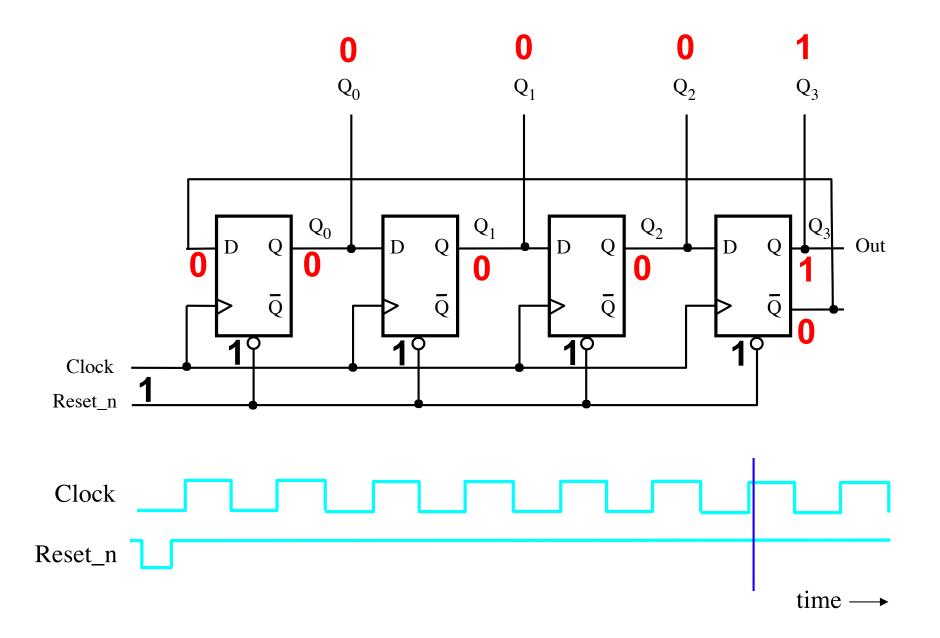


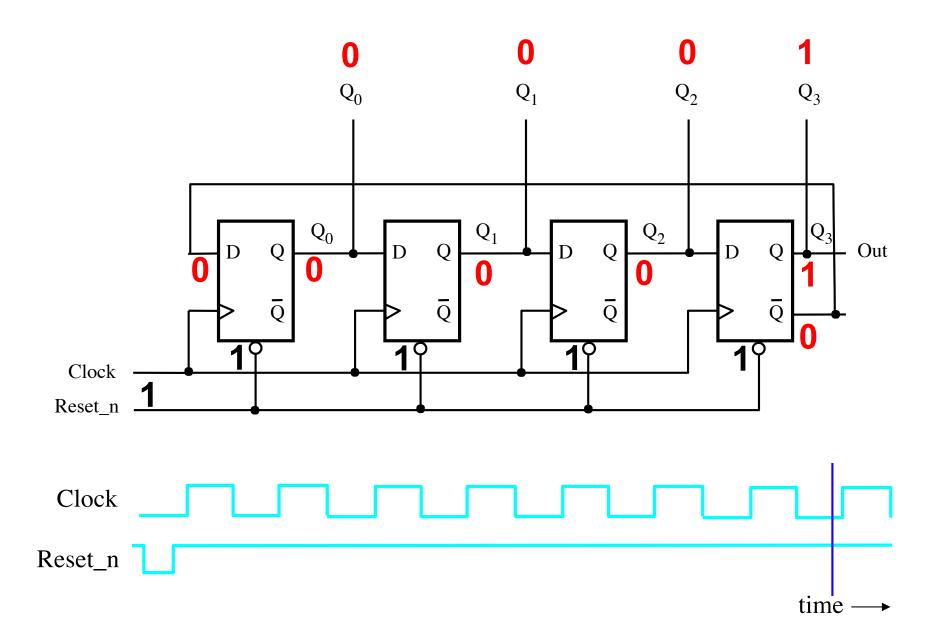


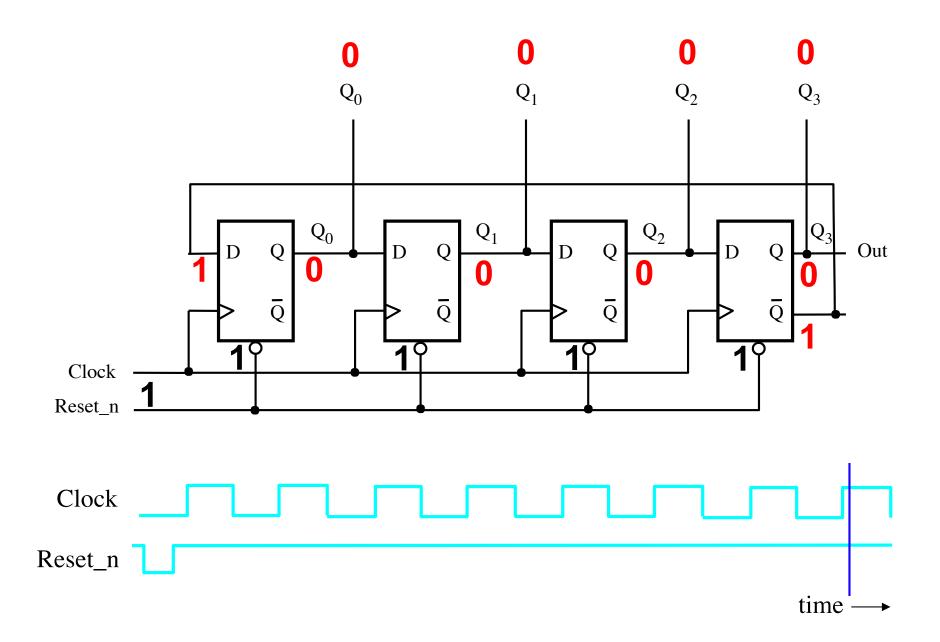




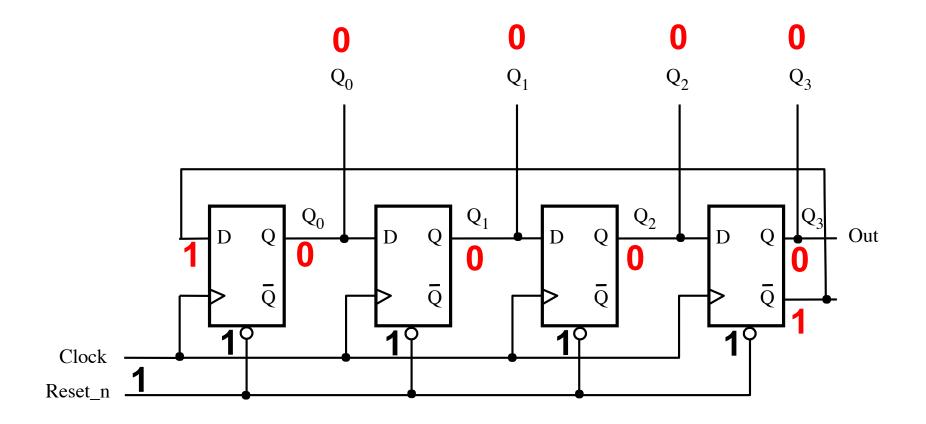






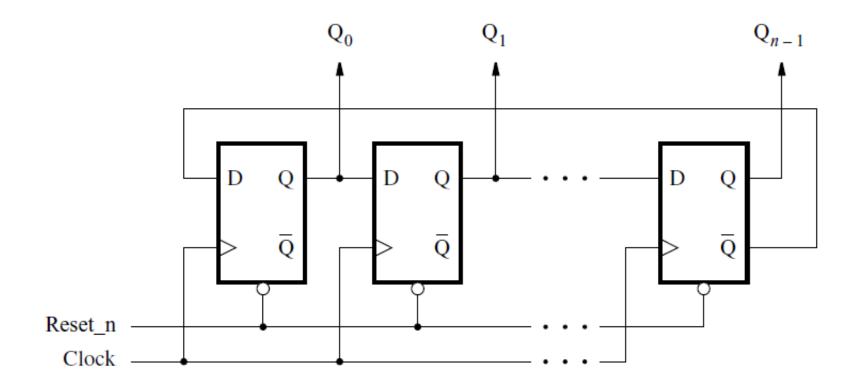


Counting: How does it work?



It is back to the start of the counting sequence, which is: 0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001.

n-bit Johnson Counter



Timing Analysis of Flip-Flop Circuits (Section 5.15)

Timing Review

t_{su}: setup time

t_h: hold time

t_{cQ}: propagation delay

Timing Example

- t_{su}: 0.6ns
- t_h: 0.4ns
- t_{cO} : 0.8ns to 1.0ns
 - Which value to use?

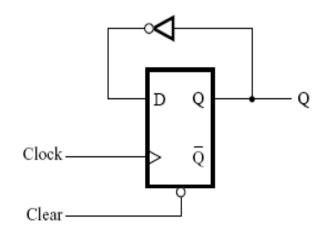


k is equal to the number of inputs

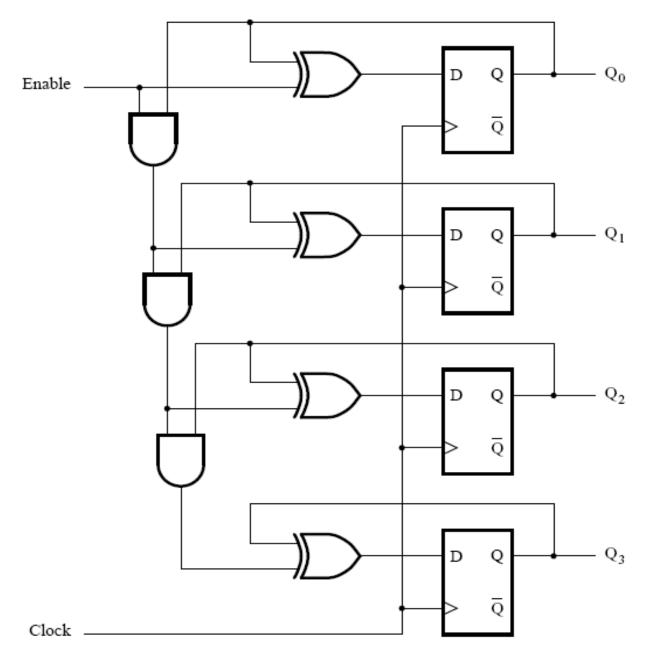
•
$$T_{min} = t_{su} + t_{cQ} + t_{not} = 0.6 + 1.0 + 1.1 = 2.7$$
ns

•
$$F_{max} = 1/T_{min} = 370.37 \text{ MHz}$$

- Check for hold violations
 - Fastest Q can change = $t_{cQ} + t_{not} = 0.8 + 1.1 = 1.9$ ns
 - 1.9ns > 0.4ns therefore no hold violations



Timing Example: 4-bit counter



Timing Example: 4-bit counter

- Look for longest path
 - Q0 to Q3
- Propagation delay of Q0
- 3 AND propagation delays
- 1 XOR propagation delay
- Setup delay for Q3
- $T_{min} = 1.0 + 3(1.2) + 1.2 + 0.6 = 6.4$ ns
- $F_{max} = 1/6.4$ ns = 156.25MHz
- Check for hold violations
 - Fastest Q can change = $t_{cQ} + t_{XOR} = 0.8 + 1.2 = 2$ ns
 - 2.0ns > 0.4ns therefore no hold violations

Timing Example: Clock Skew

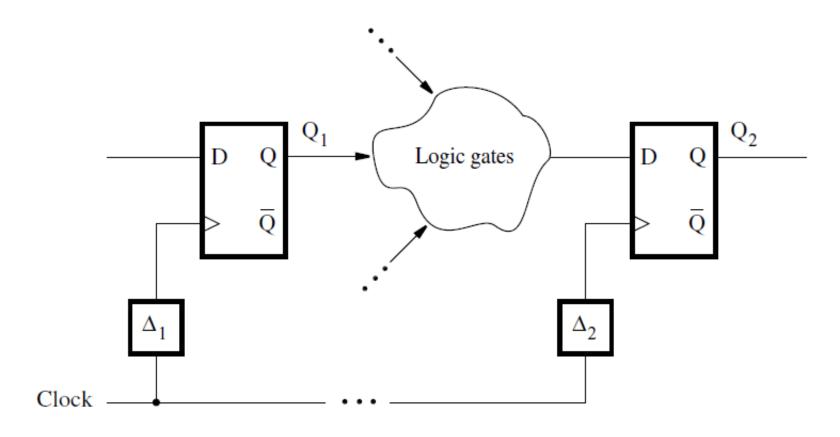


Figure 5.68. A general example of clock skew.

Skew Timing Example: 4-bit counter

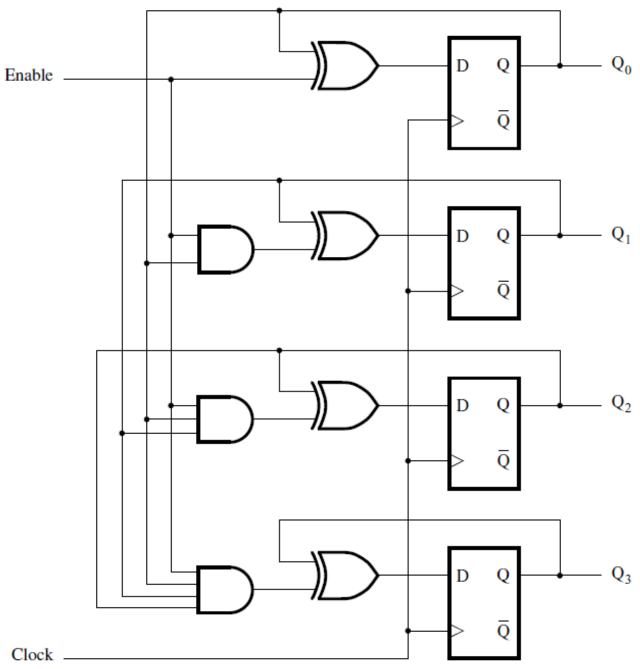
- Q3 now has a clock slew delay: 1.5ns
 - T = 1.0 + 3(1.2) + 1.2 + 0.6 1.5 = 4.9ns
- Now might not be the longest path
- Check Q0 to Q2
 - T = 1.0 + 2(1.2) + 1.2 + 0.6 = 5.2ns
- $F_{max} = 1/5.2ns = 192.31 MHz$

Example 5.22

Faster 4-bit Counter

- Want to increase the speed of the 4-bit counter
- Use a similar method as the one used in 4-bit adder
- Replace the series of 2-input AND gates with AND gates with more input lines.

A faster 4-bit counter



[Figure 5.75 from the textbook]

Faster 4-bit Counter

Longest path: Q0 to Q3

•
$$T_{min} = t_{cQ0} + t_{AND} + t_{XOR} + t_{su}$$

= 1.0 + 1.4 + 1.2 + 0.6 = 4.2ns

• $F_{max} = 1/4.2$ ns = 238.1 MHz > 156.25 MHz

Reaction Timer Circuit (Section 5.14)

Problem Statement

- Want to design a reaction timer
- The circuit turns on a light (LED)
- A person then presses a switch
- Measure the elapsed time from when the LED is turned on until the switch is pressed

Clock Divider

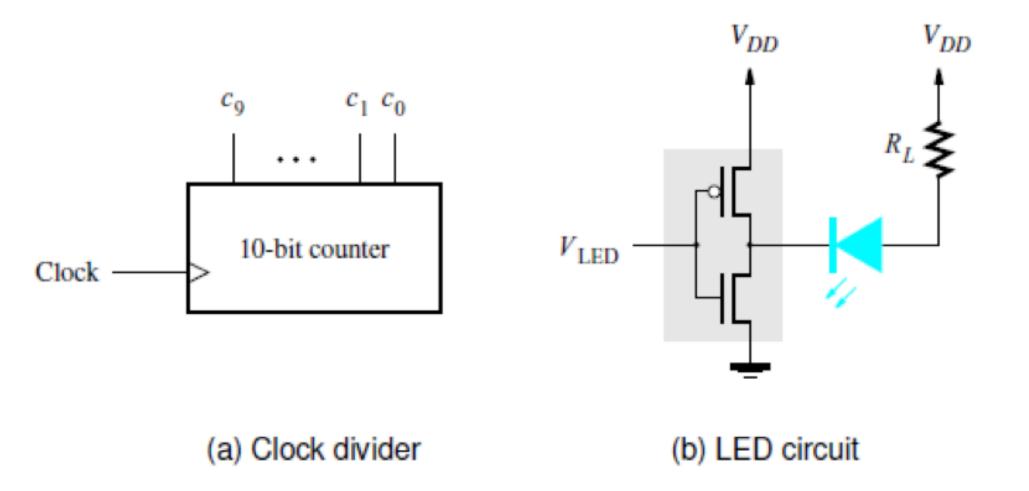
• Input: 102.4kHz

Output: 100Hz

10-bit Counter to divide

• Output Frequency = $102.4k / 2^{10} = 100Hz$

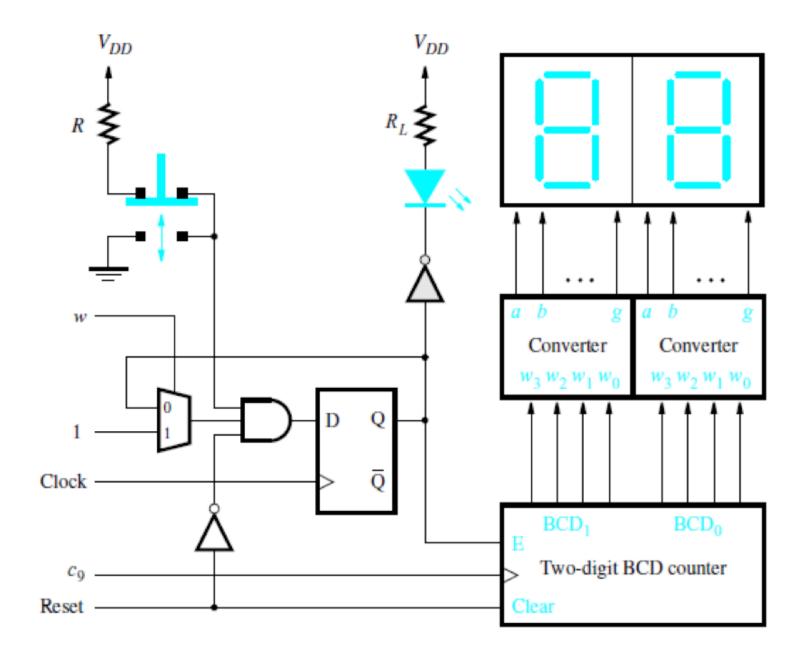
A reaction-timer circuit



Functionality of circuit

- Push switch
 - Nominally 1
- DFF to keep track of the state
- Two-digit BCD counter
 - Output goes to converters to a 7-segment display
- Start-up
 - Assert the Reset signal
 - Clears counter
 - Clears flip-flop
 - Assert w=1 for one cycle
 - Once switch is hit
 - Clears flip-flop
 - Stops counting

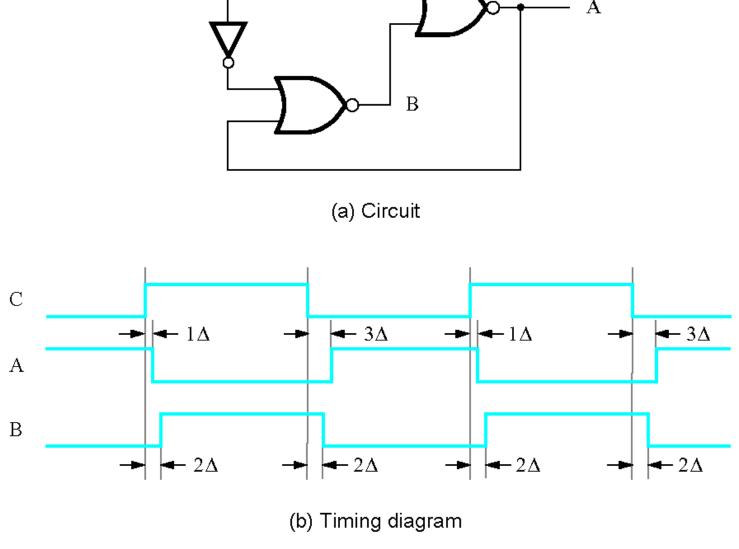
Push-button switch, LED, and 7-segment displays



Examples of Solved Problems (Section 5.17)

Example 5.18

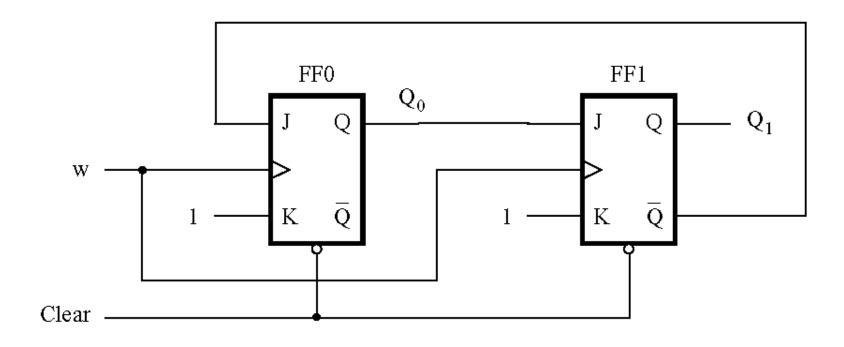
Circuit for Example 5.18



[Figure 5.70 from the textbook]

Example 5.19

Circuit for Example 5.19



	Q(t+1)	K	J
No Change	Q(t)	0	0 0 1
Reset	0	1 0	
Set	1		
Complement	Q'(t)	1	1

Summary of the behavior of the circuit

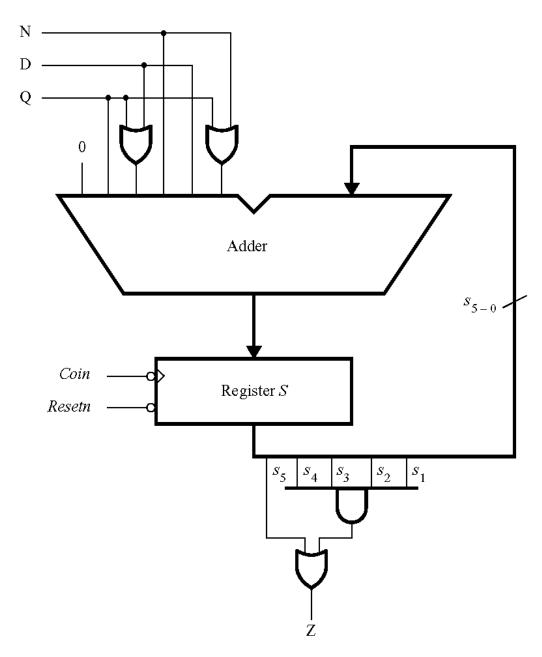
Time	FF0			FF1		
interval	J_0	K_0	Q_0	J_1	K_1	Q_1
Clear	1	1	0	0	1	0
t_1	1	1	1	1	1	0
t_2	0	1	0	0	1	1
t_3	1	1	0	0	1	0
t_4	1	1	1	1	1	0

Example 5.20

Vending Machine Example

- Inputs N, D, Q, Coin, Resetn
 - N, D, Q: nickel, dime, quarter
 - Coin: pulsed when a coin is entered
 - Used to store values into register
 - Resetn: resets the register value to zero
- Add up new coin with old value
 - Store new sum into old value register
- See if total is above thirty cents
 - If so output Z goes high

Circuit for Example 5.20



Questions?

THE END