Sample Solutions

CprE 281: Digital Logic
Midterm 1: Friday Sep 18, 2020

Name: _____ **ID Number:** _____

Lab Section:	**Tue 11-2 (#16)**	**Wed 8-11 (#8)**	**Thur 11-2 (#14)**	**Fri 11-2 (#7)**
(circle one)	**Tue 2-5 (#11)**	**Wed 11-2 (#18)**	**Thur 11-2 (#17)**	
			Thur 2-5 (#10)	

1. True/False Questions (10 x 1p each = 10p)

(a) I forgot to write down my name, student ID number, and lab section. TRUE / (FALSE)

(b) Any Boolean function can be implemented using only AND gates. TRUE / (FALSE)

(c) There are at least 4 different ways to draw a 3-variable K-Map. (TRUE) / FALSE

(d) Wampas and tauntauns are native to the ice planet Hoth. (TRUE) / FALSE

(e) The axioms of Boolean algebra can be proven with the theorems. TRUE / (FALSE)

(f) XOR(x, x) = x. TRUE / (FALSE)

(g) NAND(x, 0) = $\overline{x}$. TRUE / (FALSE)

(h) $\overline{x}(x + \overline{y})y = 0$. (TRUE) / FALSE

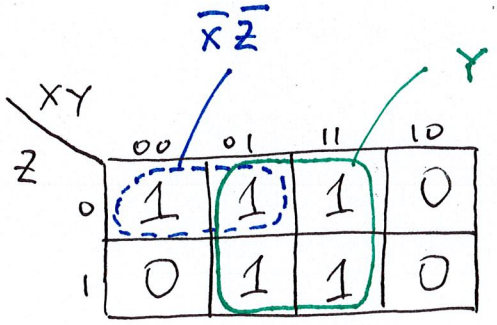
(i) $\overline{x} + xy = x + y$. TRUE / (FALSE)

(j) An SOP expression easily maps to a NOR-NOR implementation. TRUE / (FALSE)

2. Three-Variable K-Map (5p)

Use a K-map to derive the minimum SOP expression for f(x, y, z)= Π M(1, 4, 5).

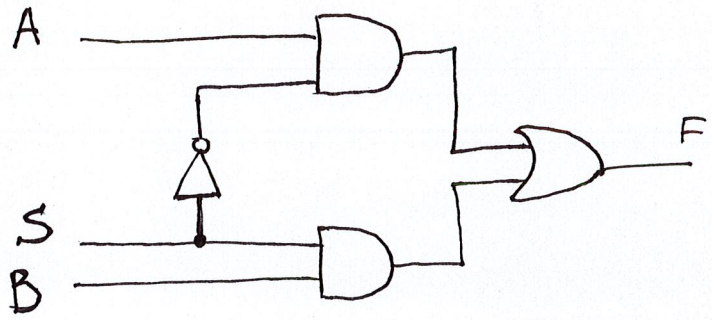
$= \Sigma m(0, 2, 3, 6, 7)$



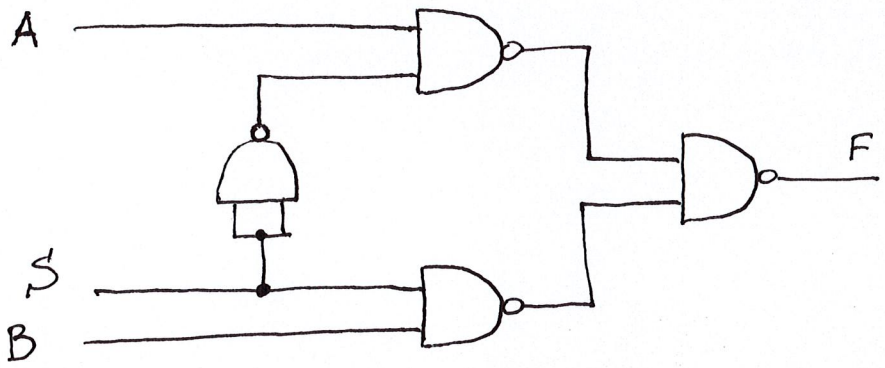
$f = Y + \overline{x}\overline{z}$

3. Multiplexer (5p + 5p = 10p)

(a) **Draw the circuit diagram for a 2-to-1 multiplexer, which has a Boolean expression $F = \overline{S}\,A + S\,B$**



(b) **Redraw your circuit form a) using only NAND gates. Clearly label all inputs and outputs of the circuit.**



4. Number Conversions (4 x 5p each = 20p)

(a) Convert 10101101_2 to decimal

$$1\times2^7+0\times2^6+1\times2^5+0\times2^4+1\times2^3+1\times2^2+0\times2^1+1\times2^0 =$$

$$= 128+0+32+0+8+4+0+1 =$$

$$= 128+40+5 = 173_{10}$$

(b) Convert 123_{10} to binary

123/2	=	61	1
61/2	=	30	1
30/2	=	15	0
15/2	=	7	1
7/2	=	3	1
3/2	=	1	1
1/2	=	0	1

1111011_2

(c) Convert 227_{10} to hexadecimal

227 / 16	=	14	3
14 / 16	=	0	14

$E3_{16}$

(d) Convert $COFFEE_{16}$ to octal.

601777756_8

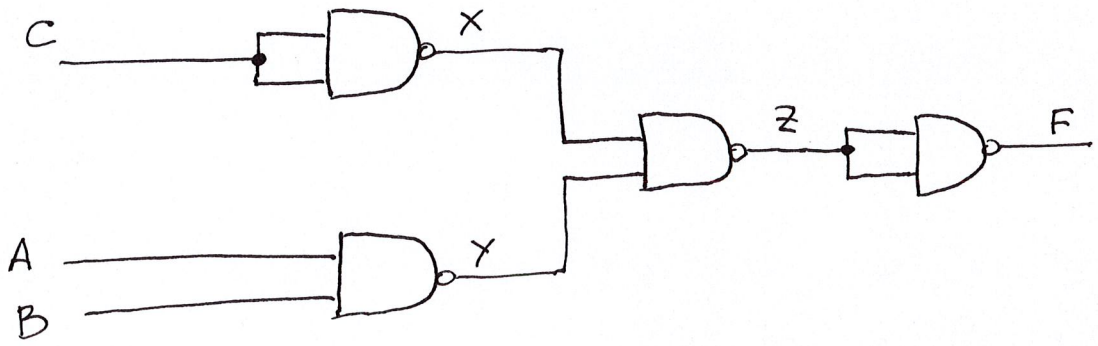
1100 0000 1111 1111 1111 1110 1110

6 0 1 7 7 7 5 6

5. From Verilog Code to Circuit (10p)

Draw the circuit diagram that corresponds to the Verilog module shown below. Clearly label all inputs, outputs and wires of your circuit.

```
module mystery (A,B,C,F);
     input A,B,C;
     output F;

     nand(X, C, C);

     nand(Y, A, B);

     nand(Z, Y, X);

     nand(F, Z, Z);

endmodule
```



6. Truth Tables (3 x 5p = 15p)

(a) Draw the truth table for the Boolean function $F(X, Y) = (X + \overline{Y})\,(\overline{X} + \overline{Y})$
Show partial results for each of the two terms.

X	Y	$(X + \overline{Y})$			$\cdot$	$(\overline{X} + \overline{Y})$			F
0	0	0	1	1		1	1	1	1
0	1	0	0	0		1	1	0	0
1	0	1	1	1		0	1	1	1
1	1	1	1	0		0	0	0	0

(b) Use a truth table to determine if the following Boolean equation is true:

$$\overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{A}\,B\,C = \overline{A}$$

They are equal.

A	B	C	$\overline{A}\,\overline{C}$	+	$\overline{A}\,\overline{B}$	+	$\overline{A}\,BC$	LHS	RHS=$\overline{A}$
0	0	0	1		1		0	1	1
0	0	1	0		1		0	1	1
0	1	0	1		0		0	1	1
0	1	1	0		0		1	1	1
1	0	0	0		0		0	0	0
1	0	1	0		0		0	0	0
1	1	0	0		0		0	0	0
1	1	1	0		0		0	0	0

(c) Draw the truth tables for the following 5 logic gates: AND, OR, XOR, NAND, NOR.
Clearly label which table corresponds to which gate.

X	Y	AND(X,Y)
0	0	0
0	1	0
1	0	0
1	1	1

X	Y	OR(X,Y)
0	0	0
0	1	1
1	0	1
1	1	1

X	Y	XOR(X,Y)
0	0	0
0	1	1
1	0	1
1	1	0

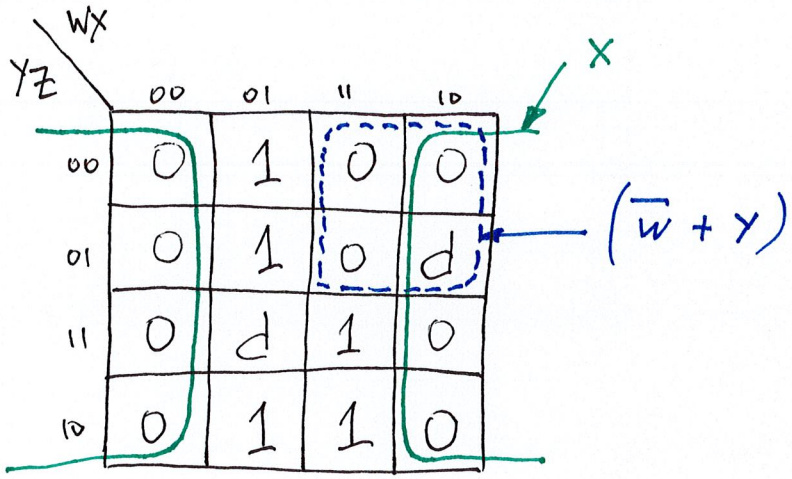
X	Y	NAND(X,Y)
0	0	1
0	1	1
1	0	1
1	1	0

X	Y	NOR(X,Y)
0	0	1
0	1	0
1	0	0
1	1	0

7. Derive the minimum <u>POS</u> expression using a K-map (10p + 5p = 15p)

(a) Use a K-map to derive the minimum-cost <u>POS</u> expression for the following function

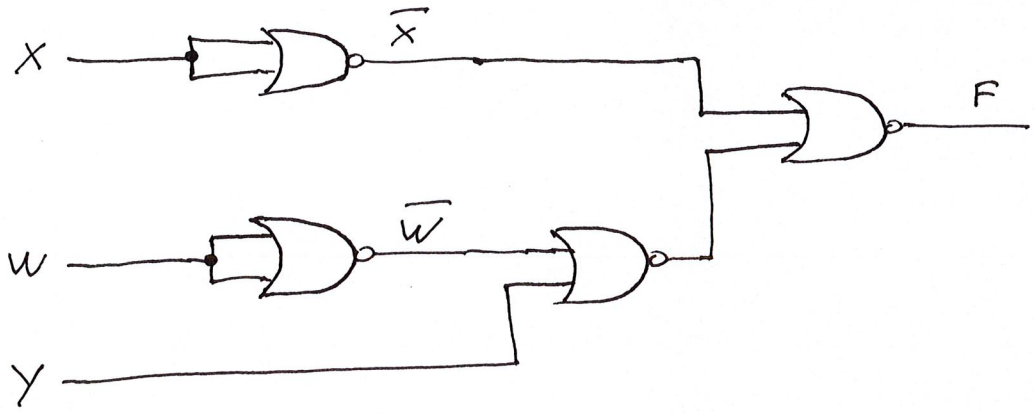
$$f(w, x, y, z) = \Sigma\, m(4, 5, 6, 14, 15) + D(7, 9)$$



$$f = X \cdot (\overline{w} + Y)$$

(b) Draw the circuit diagram for the expression derived in (a) using <u>only NOR gates</u>. Clearly label all inputs and outputs.

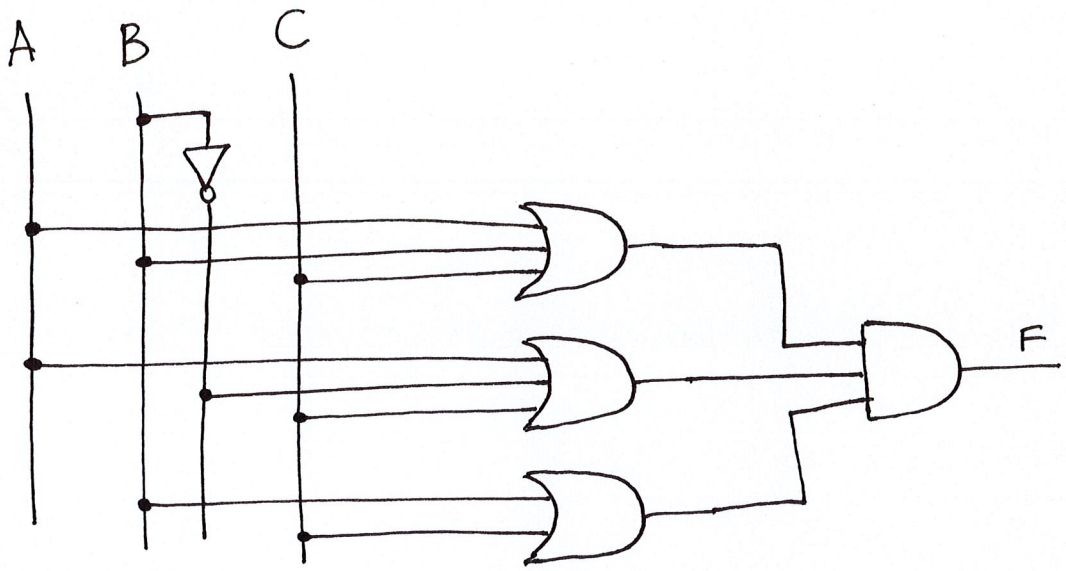
$$f = X \cdot (\overline{w} + Y) = \overline{\overline{X \cdot (\overline{w} + Y)}} = \overline{\overline{X} + \overline{(\overline{w} + Y)}}$$



8. Circuit Simplification (3 x 5p = 15p)

(a) Draw the circuit diagram for this Boolean expression (don't simplify it yet)

$$F(A, B, C) = (A + B + C)\ (A + \overline{B} + C)\ (B + C)$$



(b) Use the theorems of Boolean algebra to find a minimum-cost SOP expression for F.

$F = (A + B + C)(A + \overline{B} + C)(B + C)$ // Theorem 14.b

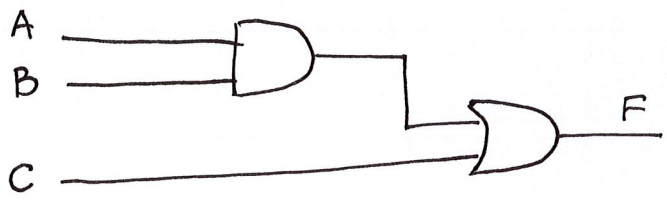
$= (A + C)(B + C)$ // expand

$= AB + AC + CB + CC$ // Theorem 7.a ($CC = C$)

$= AB + C(A + B + 1)$ // Theorem 5.b ($A + B + 1 = 1$)

$= AB + C$

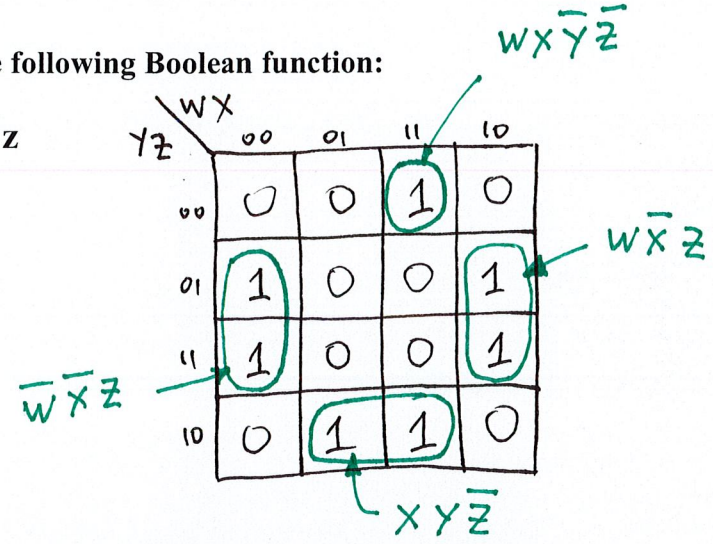
(c) Draw the circuit for the minimum-cost SOP expression. Label all inputs and outputs.



9. Minimization (3 x 5p = 15p)

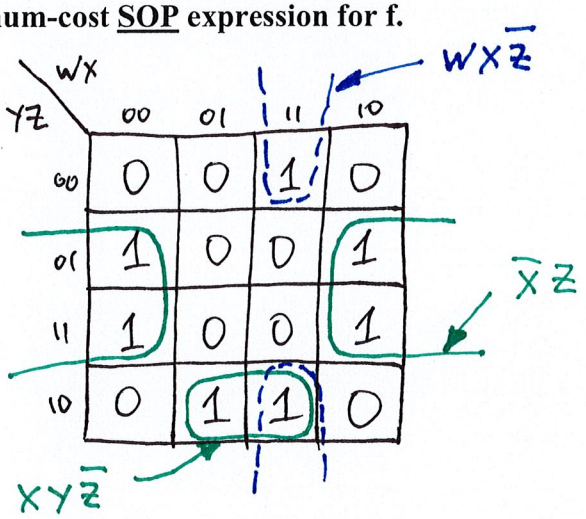
(a) Draw the K-map that corresponds to the following Boolean function:

$$f = w\bar{x}z + wx\bar{y}\bar{z} + xy\bar{z} + \bar{w}\bar{x}z$$

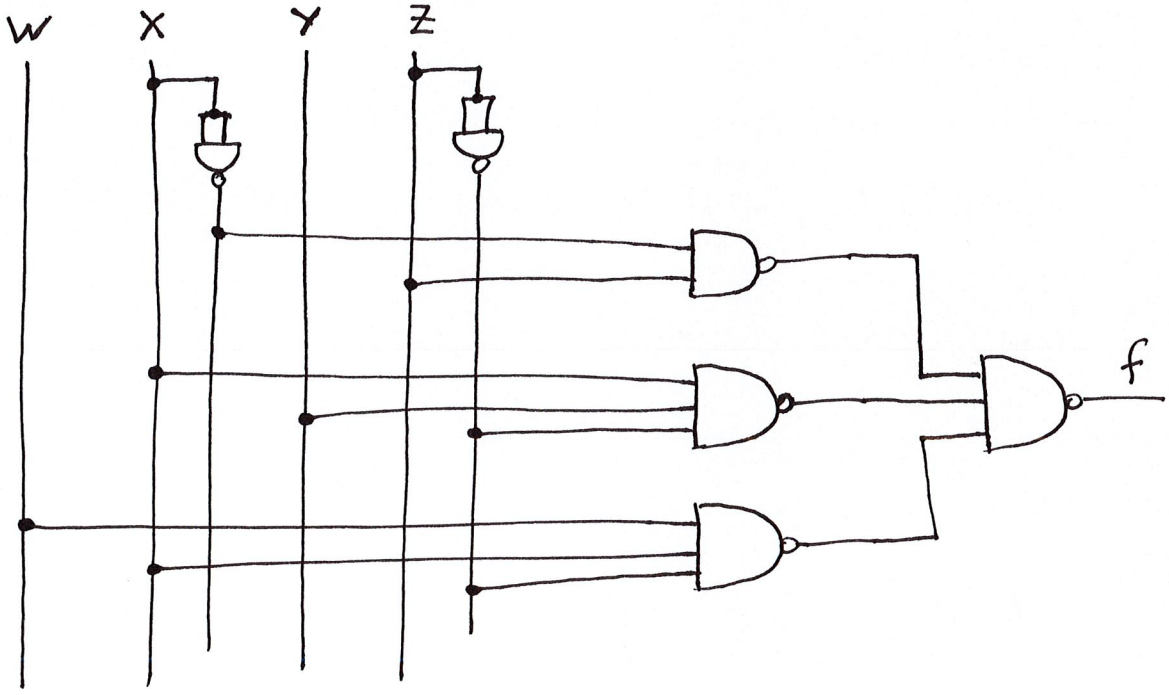


(b) Redraw the K-map from (a) and derive the minimum-cost SOP expression for f.

$$f = \bar{x}z + xy\bar{z} + wx\bar{z}$$



(c) Draw the circuit for the minimum-cost SOP expression using only NAND gates. Clearly label all inputs and outputs.



10. Boolean Algebra (10p + 5p = 15p)

(a) Use the theorems of Boolean algebra to simplify the formula given below into a minimum-cost expression.

(b) Draw the circuit diagram for the simplified expression using only NOR gates.

$$F(X, Y, Z) = \underbrace{\overline{(X + \bar{X}\bar{Y})}}_{\alpha}\ \underbrace{(X + Y + \bar{Z})}_{\beta} + \underbrace{\overline{(X + \bar{Y} + X\bar{Y})}}_{\gamma 1}\ \underbrace{(\bar{X}\,\bar{Y}\,Z)}_{\delta}$$

$$\alpha = \overline{X + \bar{X}\,\bar{Y}} = \overline{X + \bar{Y}} = \bar{X} \cdot \bar{\bar{Y}} = \bar{X}Y$$

Theorem 16.9

$$\beta = X + Y + \bar{Z}$$

// no simplification here

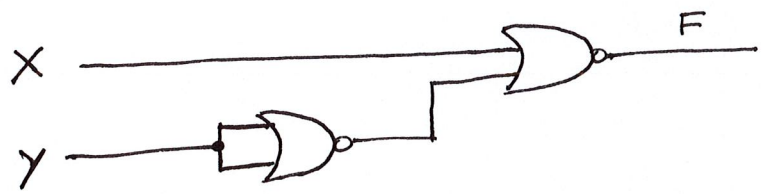
$$\gamma 1 = \overline{X + \bar{Y} + X\bar{Y}} = \overline{X + \bar{Y}} = \bar{X} \cdot \bar{\bar{Y}} = \bar{X}Y = \alpha$$

$$\delta = \bar{X}\,\bar{Y}\,Z = \overline{\overline{\bar{X}\,\bar{Y}\,Z}} = \overline{\bar{\bar{X}} + \bar{\bar{Y}} + \bar{Z}} = \overline{X + Y + \bar{Z}} = \bar{\beta}$$

$$F = \underbrace{\alpha}_{\alpha} \cdot \beta + \underbrace{\gamma 1}_{\alpha} \cdot \underbrace{\delta}_{\bar{\beta}} = \alpha\beta + \alpha\bar{\beta} = \alpha(\underbrace{\beta + \bar{\beta}}_{1}) = \alpha = \bar{X}Y$$

(b) Draw using only NOR gates.

$$F = \bar{X}Y = \overline{\overline{\bar{X}\,Y}} = \overline{\bar{\bar{X}} + \bar{Y}} = \overline{X + \bar{Y}}$$



Question	Max	Score
1. True/False	10	
2. Three-Variable K-map	5	
3. Multiplexer	10	
4. Number Conversions	20	
5. Verilog Module	10	
6. Truth Tables	15	
7. POS with K-Map	15	
8. Circuit Simplification	15	
9. Minimization	15	
10. Boolean Algebra	15	
TOTAL:	130	