



CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Incompletely Specified Functions & Multiple-Output Circuits

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW4 is due on Monday Sep 14 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 18.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **More details to follow.**

Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**

- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**

- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**

- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**

- **Something from Star Wars**

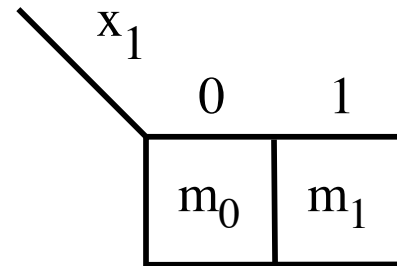
Quick Review

One-Variable K-Map

One-Variable K-map

x_1	f
0	m_0
1	m_1

(a) Truth table

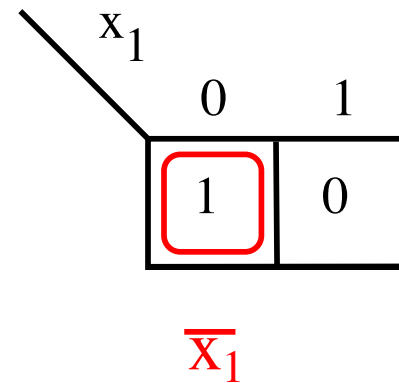


(b) Karnaugh map

One-Variable K-map

x_1	f
0	1
1	0

(a) Truth table



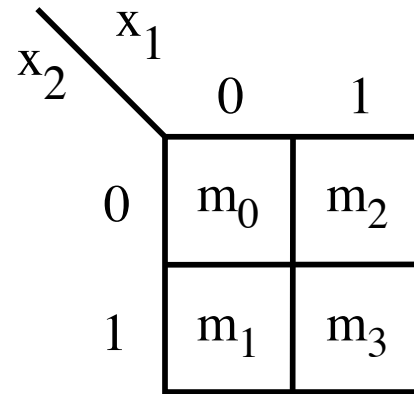
(b) Karnaugh map

Two-Variable K-Map

Two-Variable K-map

x_1	x_2	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3

(a) Truth table



(b) Karnaugh map

These are all valid groupings

	x_1	0	1
x_2	0	1	0
	1	0	0

$$\overline{x_1} \overline{x_2}$$

	x_1	0	1
x_2	0	0	0
	1	1	0

$$\overline{x_1} x_2$$

	x_1	0	1
x_2	0	0	1
	1	0	0

$$x_1 \overline{x_2}$$

	x_1	0	1
x_2	0	0	0
	1	0	1

$$x_1 x_2$$

These are all valid groupings

	x_1	0	1
x_2	0	1	0
	1	1	0

\bar{x}_1

	x_1	0	1
x_2	0	0	1
	1	0	1

x_1

	x_1	0	1
x_2	0	1	1
	1	0	0

\bar{x}_2

	x_1	0	1
x_2	0	0	0
	1	1	1

x_2

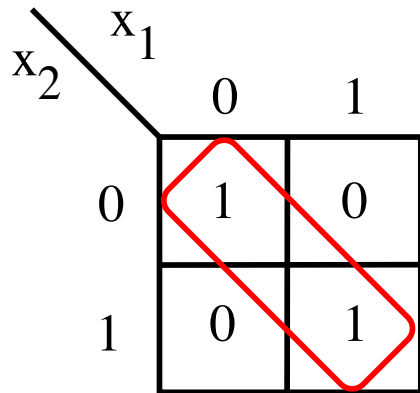
This one is valid too

$x_2 \backslash x_1$	0	1
0	1	1
1	1	1

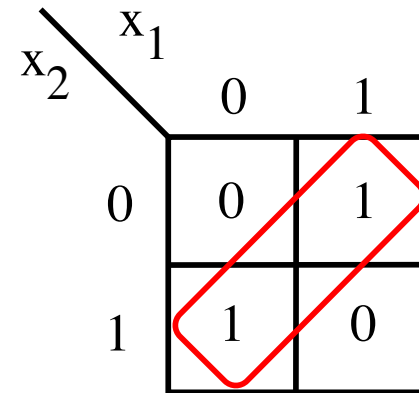
In this case the result is the constant function 1.

Why are these two not valid?

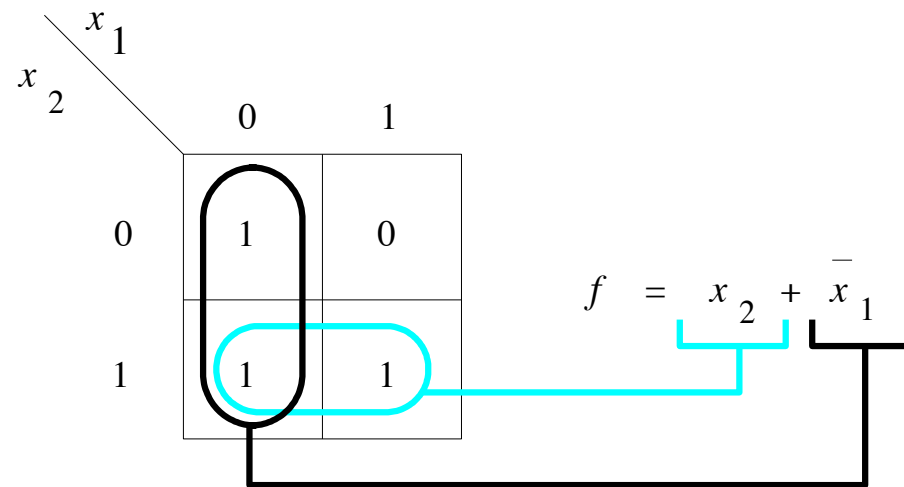
$x_2 \backslash x_1$	0	1
0	1	0
1	0	1



$x_2 \backslash x_1$	0	1
0	0	1
1	1	0



Minimization Example with a two-variable K-map



Three-Variable K-Map

Three-Variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Location of three-variable minterms

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
<hr/>			
0	1	0	m_2
0	1	1	m_3
<hr/>			
1	0	0	m_4
1	0	1	m_5
<hr/>			
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3	$x_1 x_2$	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

(b) Karnaugh map

Notice the placement of

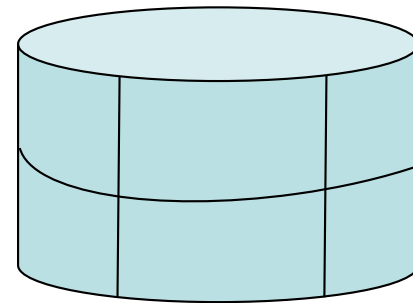
- **Variables**
- **Binary pair values**
- **Minterms**

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns



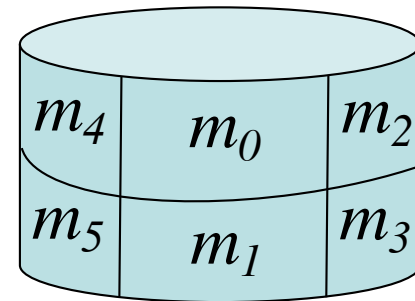
As if the K-map were
drawn on a cylinder

Adjacency Rules

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5



adjacent
columns



As if the K-map were
drawn on a cylinder

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	0	1	0	0
	1	0	0	1	0

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	1	0	0

Can't group diagonally.

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	0
	1	0	0	0	0

		x_1x_2			
		00	01	11	10
x_3	0	0	0	0	0
	1	0	1	1	1

Can't group three in a row.
Each side must be a power of 2.

Some **Invalid** Groupings

		x_1x_2			
		00	01	11	10
x_3	0	1	0	1	1
	1	0	0	0	0

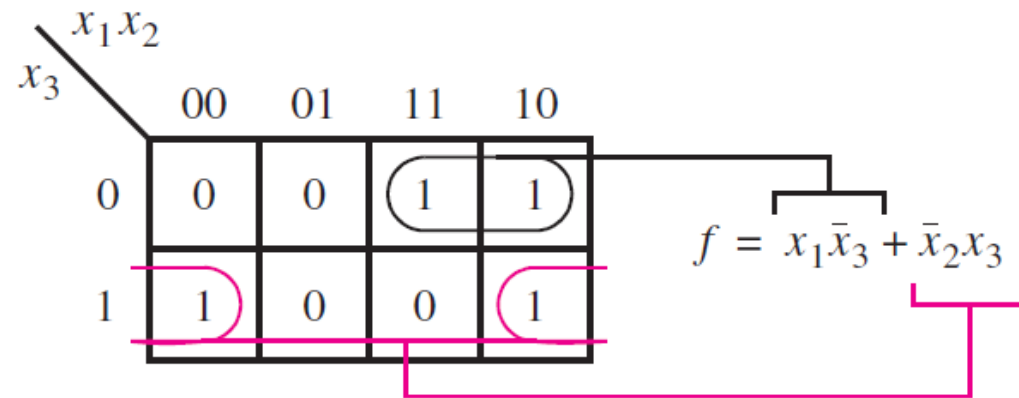
A 2x4 Karnaugh map for variables x_1, x_2, x_3 . The columns are labeled x_1x_2 with values 00, 01, 11, 10. The rows are labeled x_3 with values 0 and 1. The cell at $(x_3=0, x_1x_2=01)$ contains a red **0**. A red rounded rectangle groups the four cells in the $x_3=0$ row: (0,00) with value 1, (0,01) with value **0**, (0,11) with value 1, and (0,10) with value 1.

		x_1x_2			
		00	01	11	10
x_3	0	0	0	1	0
	1	0	1	1	0

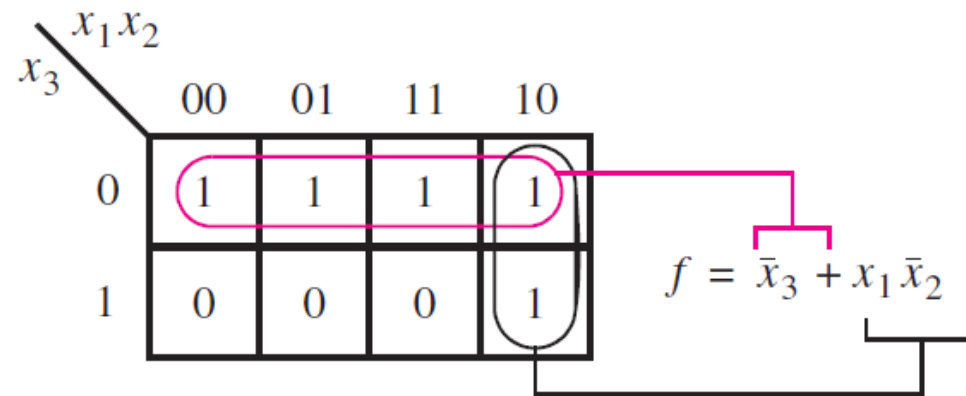
A 2x4 Karnaugh map for variables x_1, x_2, x_3 . The columns are labeled x_1x_2 with values 00, 01, 11, 10. The rows are labeled x_3 with values 0 and 1. The cell at $(x_3=0, x_1x_2=01)$ contains a red **0**. A red rounded rectangle groups the four cells in the $x_1x_2 \in \{01, 11\}$ columns: (0,01) with value **0**, (0,11) with value 1, (1,01) with value 1, and (1,11) with value 1.

Can't group zeros and ones together.

Three-Variable K-map



(a) The function of Figure 2.23



(b) The function of Figure 2.48

Two Different Ways to Draw the K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		$x_2 x_3$			
		00	01	11	10
x_1	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

Another Way to Draw 3-variable K-map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

		x_1	
		0	1
$x_2 x_3$	00	m_0	m_4
	01	m_1	m_5
	11	m_3	m_7
	10	m_2	m_6

There are 4 different versions!

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

		x_2x_3			
		00	01	11	10
x_1	0	m_0	m_1	m_3	m_2
	1	m_4	m_5	m_7	m_6

		x_3	
		0	1
x_1x_2	00	m_0	m_1
	01	m_2	m_3
	11	m_6	m_7
	10	m_4	m_5

		x_1	
		0	1
x_2x_3	00	m_0	m_4
	01	m_1	m_5
	11	m_3	m_7
	10	m_2	m_6

Gray Code

- **Sequence of binary codes**
- **Consecutive lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the LAST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These two neighbors
differ only in the FIRST bit

Gray Code & K-map

	s	x_1	x_2
m_0	0	0	0
m_1	0	0	1
m_2	0	1	0
m_3	0	1	1
m_4	1	0	0
m_5	1	0	1
m_6	1	1	0
m_7	1	1	1

		$s \ x_1$			
		00	01	11	10
x_2	0	000	010	110	100
	1	001	011	111	101

These four neighbors
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

**Why is it OK to combine
a group of four ones?**

The K-Map theory uses the combining theorems of Boolean algebra

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

The K-Map theory uses the combining theorems of Boolean algebra

optimization by 1's

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

**The K-Map theory uses the
combining theorems of Boolean algebra**

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

optimization by 0's

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Theorem 14a is behind the K-Map theory.
But that theorem is just for two variables.
Why is this grouping of four ones possible?

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + x y \bar{z} + \bar{x} y z + x y z$$

Why can we group these four ones?

		x y			
	z	00	01	11	10
0		0	1	1	0
1		0	1	1	0

$$f = \underbrace{\bar{x} y \bar{z} + x y \bar{z}}_{(\bar{x} y + x y) \bar{z}} + \underbrace{\bar{x} y z + x y z}_{(\bar{x} y + x y) z}$$

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = (\bar{x}y + xy)\bar{z} + (\bar{x}y + xy)z$$

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = (\bar{x}y + xy)\bar{z} + (\bar{x}y + xy)z$$

$y\bar{z}$ (by 14a) yz (by 14a)

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y \bar{z} + y z$$

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{y \bar{z} + y z}_{y \text{ (by 14a)}}$$

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y$$

Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + x y \bar{z} + \bar{x} y z + x y z$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + \bar{x} y z + x y \bar{z} + x y z$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{\bar{x} y \bar{z} + \bar{x} y z}_{\bar{x} (y \bar{z} + y z)} + \underbrace{x y \bar{z} + x y z}_{x (y \bar{z} + y z)}$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} (y \bar{z} + y z) + x (y \bar{z} + y z)$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{\bar{x} (y \bar{z} + y z)}_{\bar{x} y \text{ (by 14a)}} + \underbrace{x (y \bar{z} + y z)}_{x y \text{ (by 14a)}}$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x}y + xy$$

Alternative Derivation

		x y			
	z	00	01	11	10
0		0	1	1	0
1		0	1	1	0

$$f = \underbrace{\bar{x}y + xy}_{y \text{ (by 14a)}}$$

Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y$$

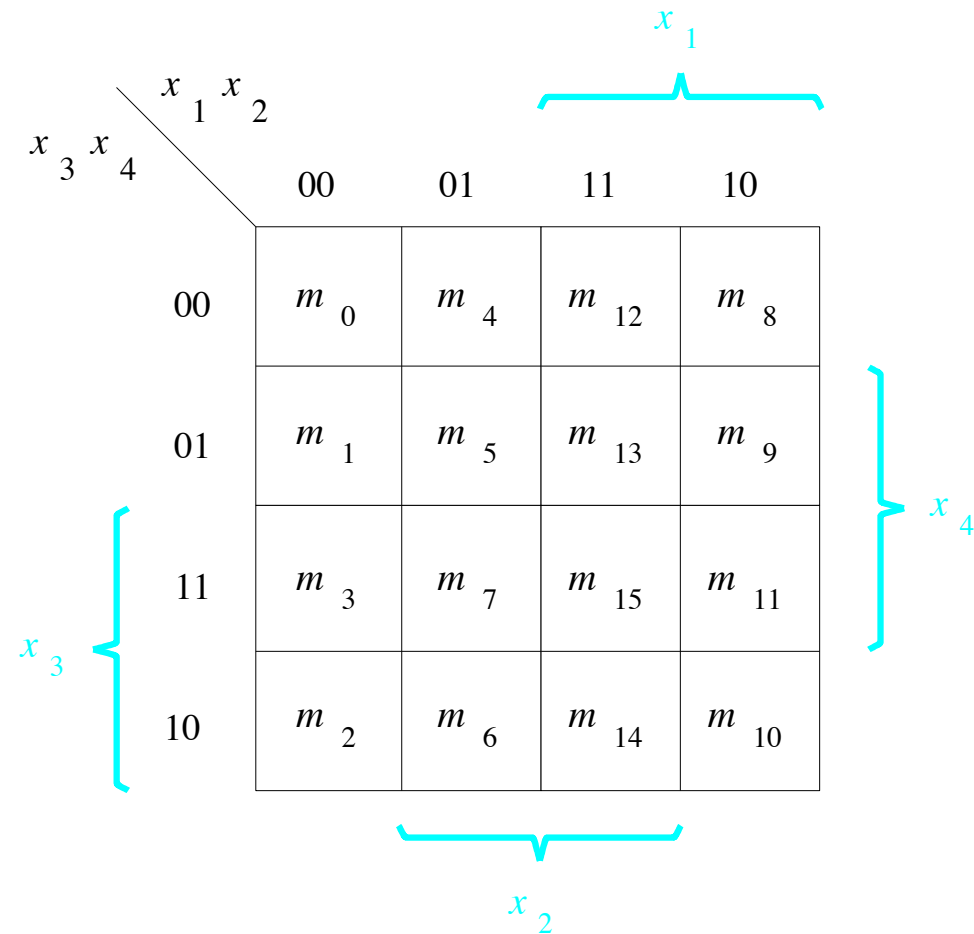
Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

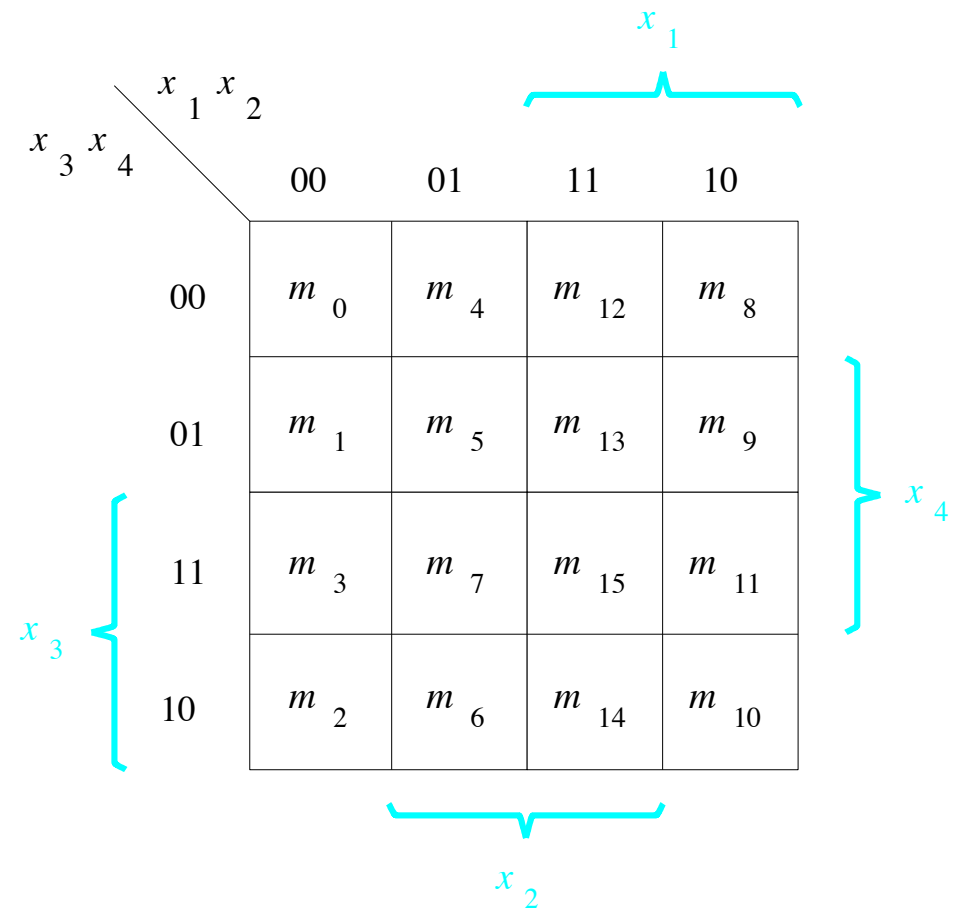
Four-Variable K-Map

A four-variable Karnaugh map



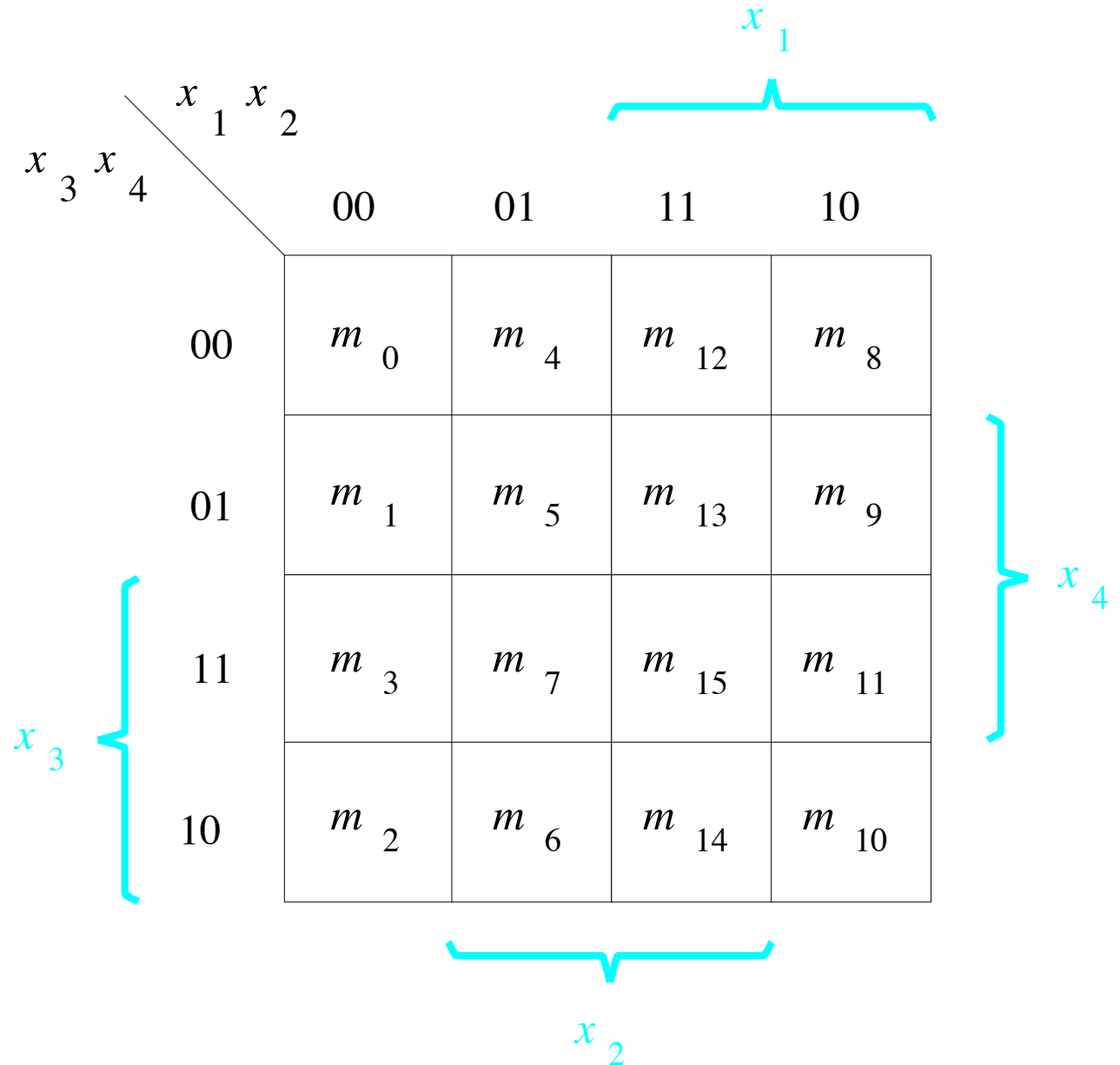
A four-variable Karnaugh map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



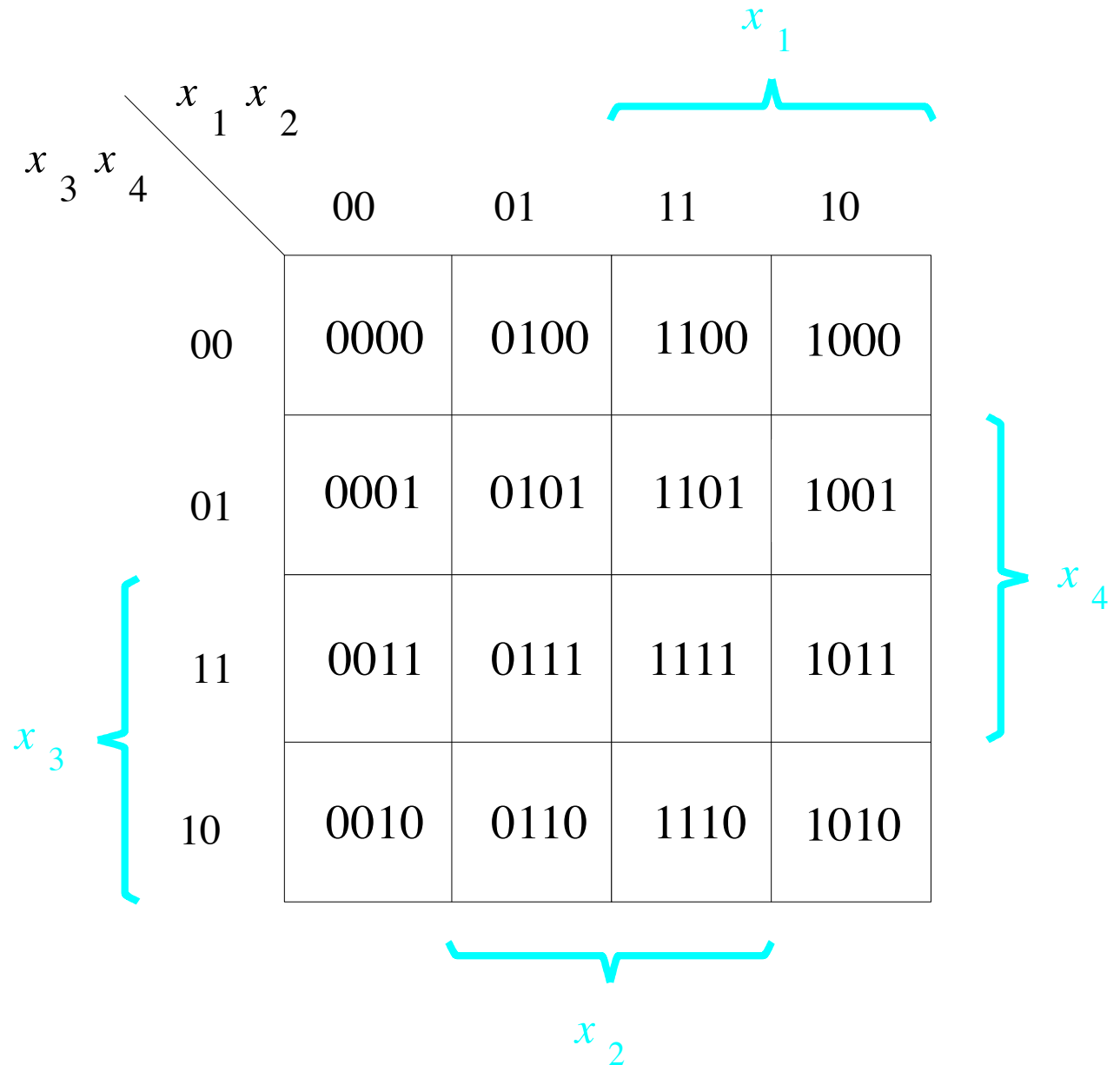
Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



Adjacency Rules

x_3	x_1x_2	00	01	11	10
0		m_0	m_2	m_6	m_4
1		m_1	m_3	m_7	m_5

adjacent
columns

x_3x_4	x_1x_2	00	01	11	10
00		m_0	m_4	m_{12}	m_8
01		m_1	m_5	m_{13}	m_9
11		m_3	m_7	m_{15}	m_{11}
10		m_2	m_6	m_{14}	m_{10}

adjacent
columns

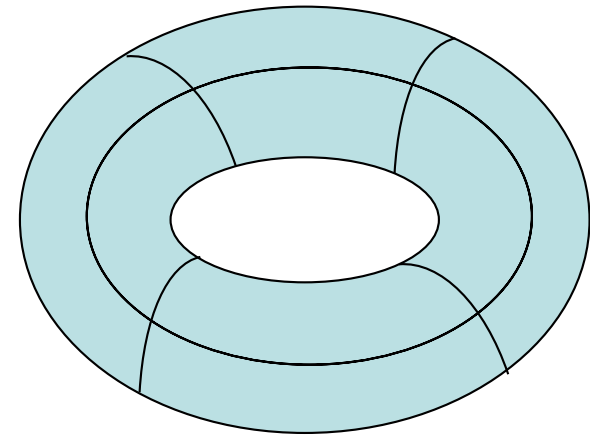
adjacent
rows

Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

adjacent
rows

adjacent
columns



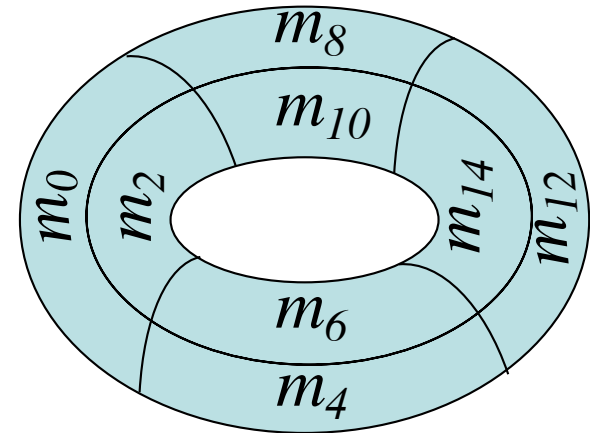
As if the K-map were
drawn on a torus

Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

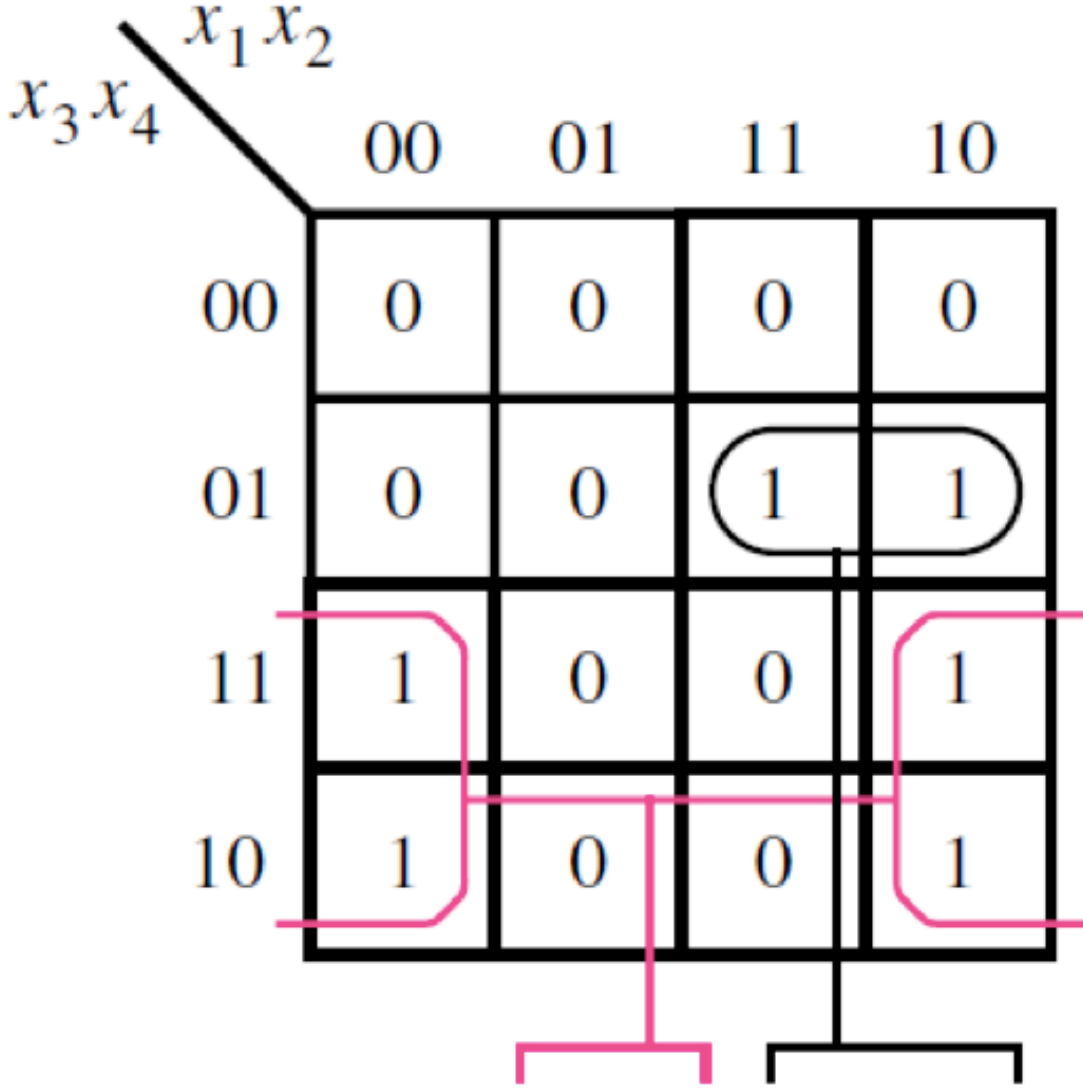
adjacent
rows

adjacent
columns



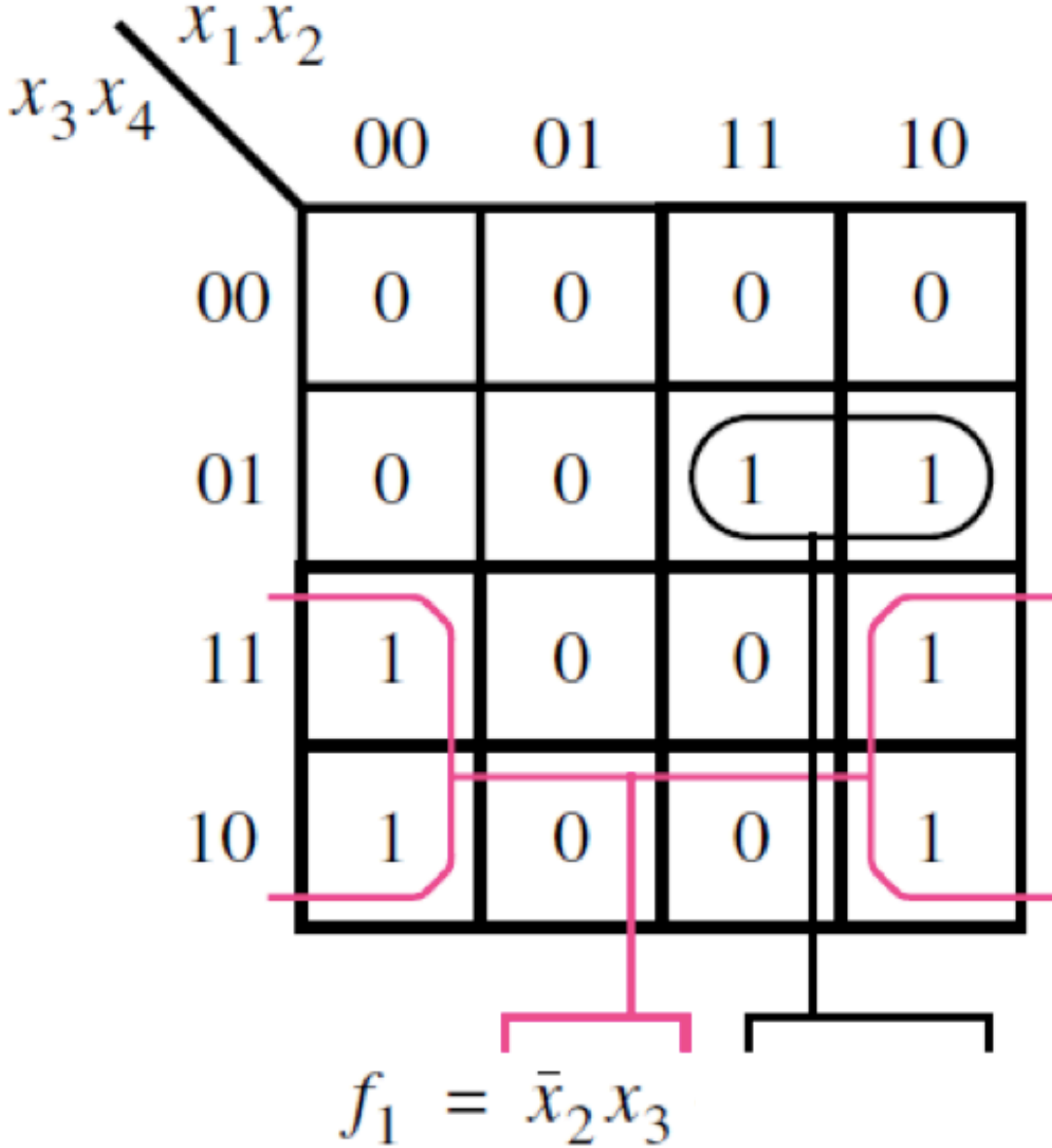
As if the K-map were
drawn on a torus

Example of a four-variable Karnaugh map



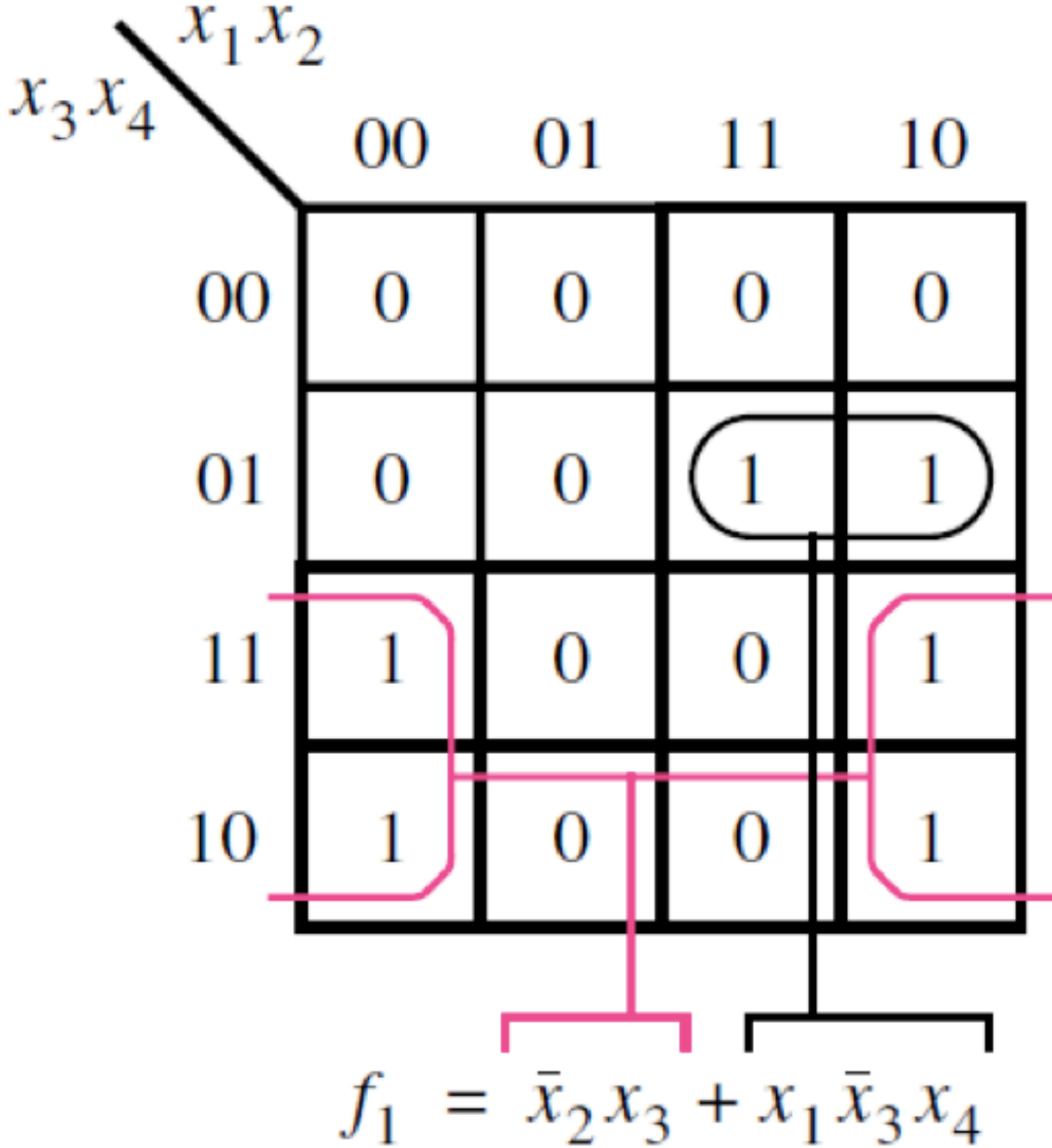
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



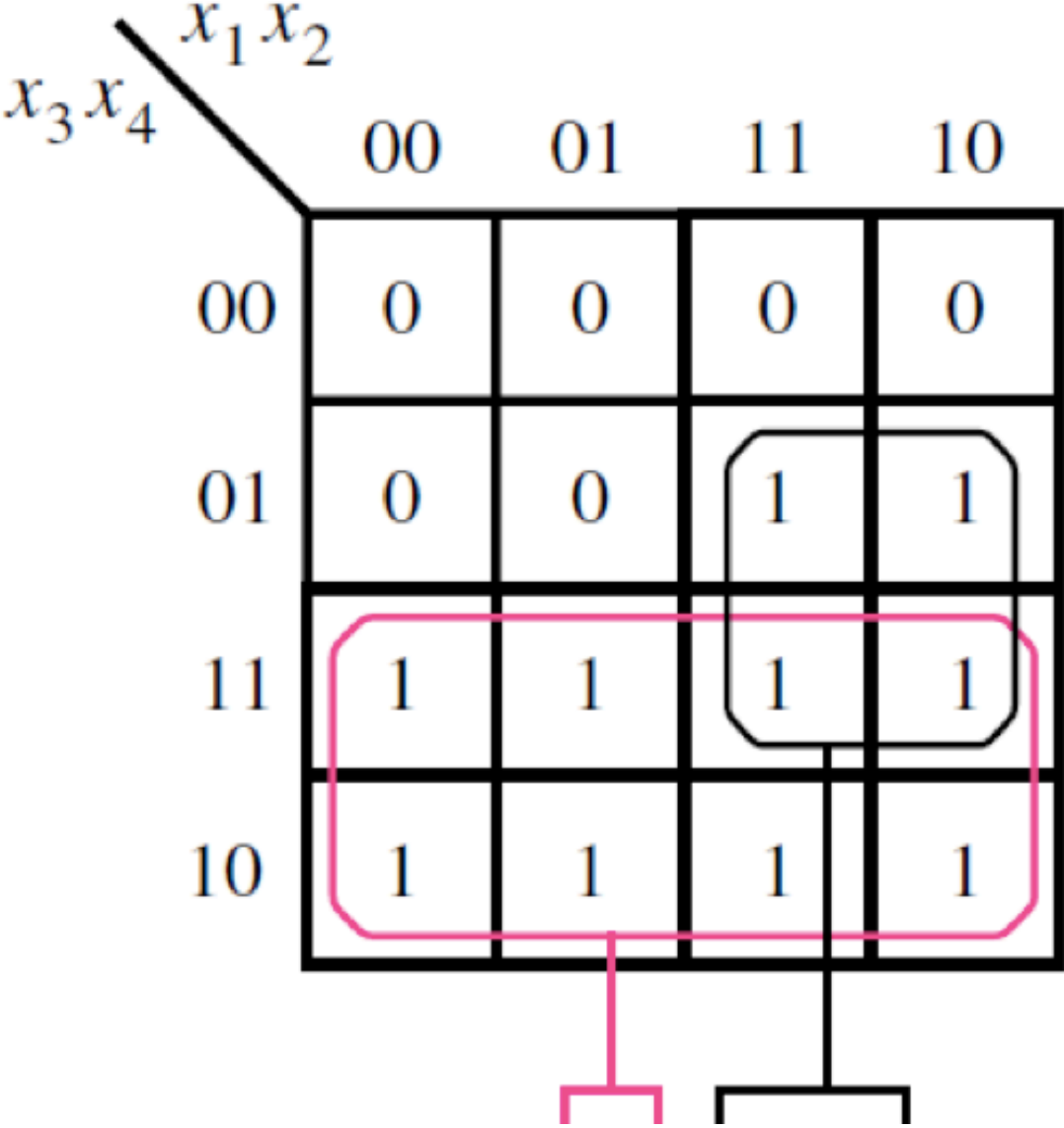
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



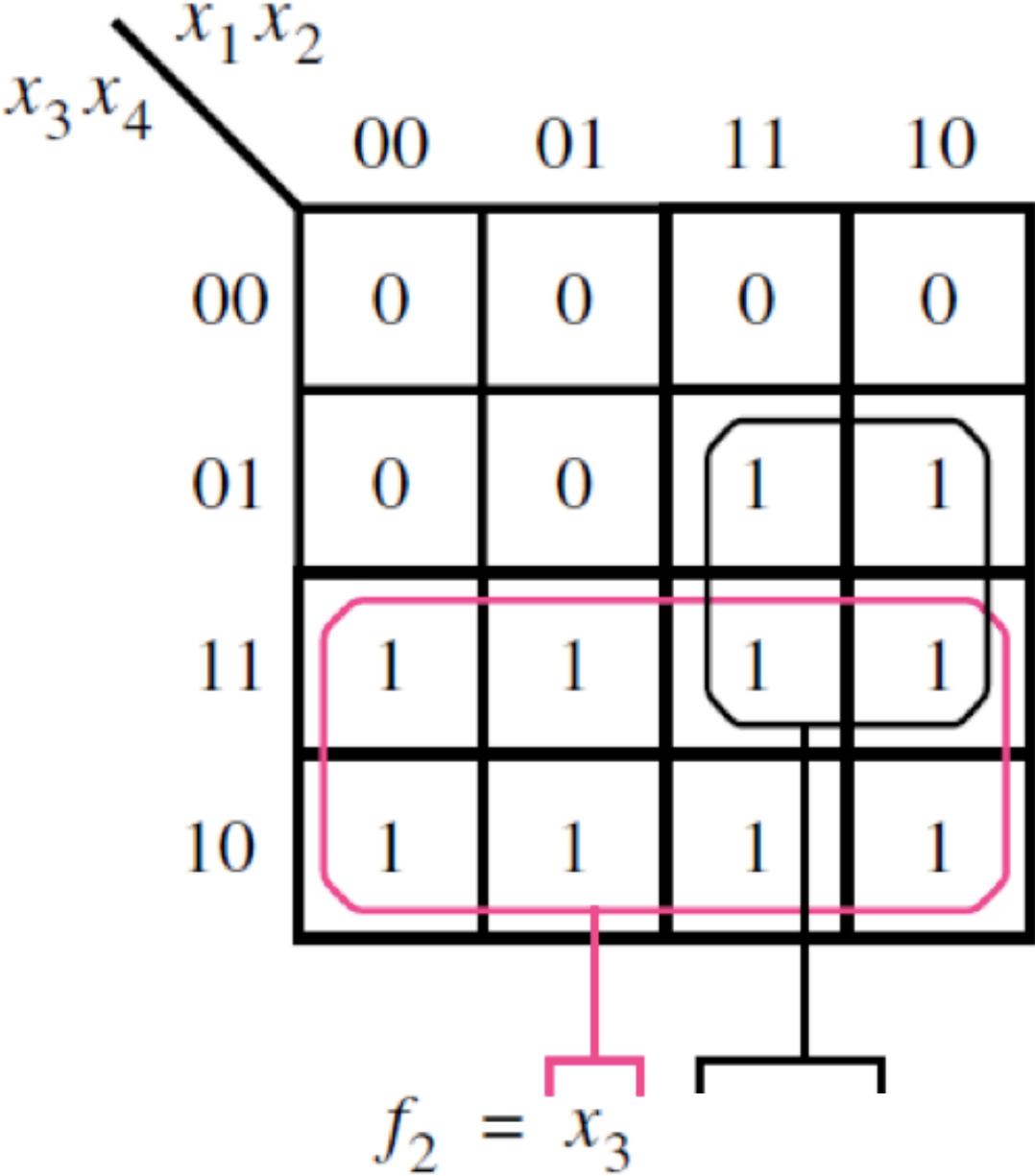
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



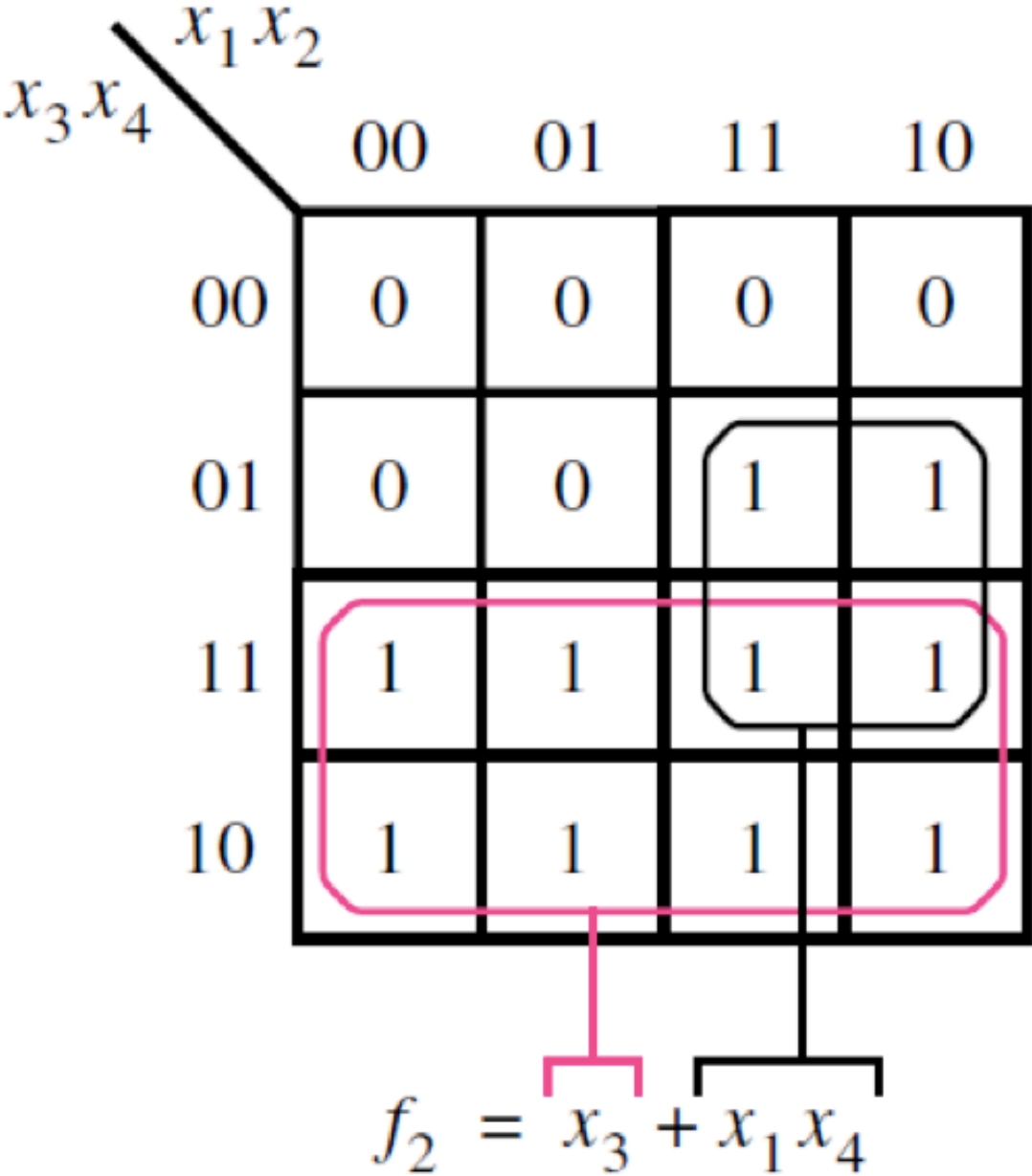
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



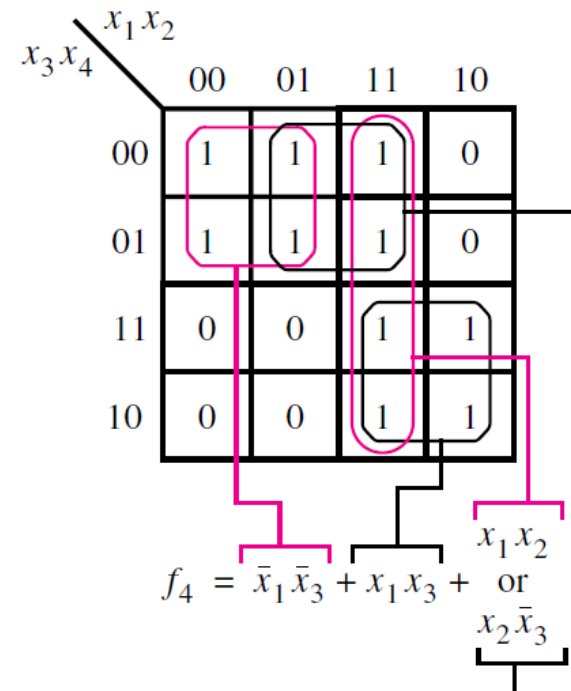
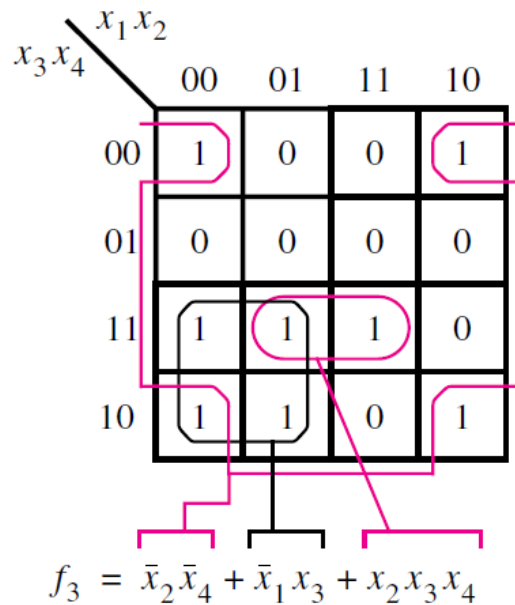
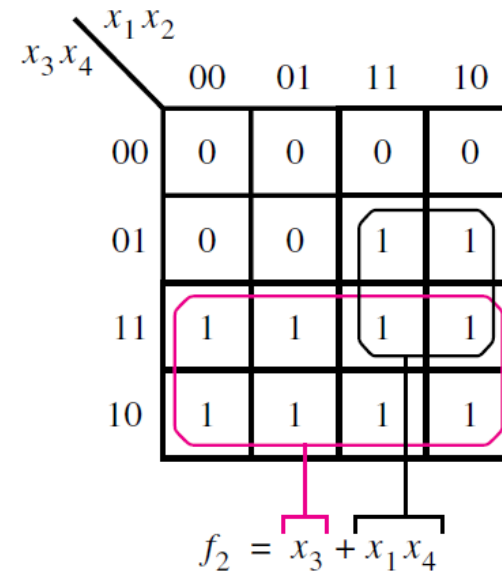
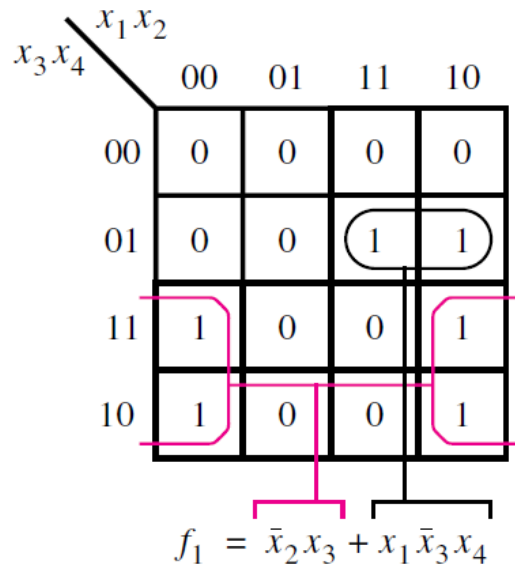
[Figure 2.54 from the textbook]

Example of a four-variable Karnaugh map



[Figure 2.54 from the textbook]

Other Four-Variable K-map Examples

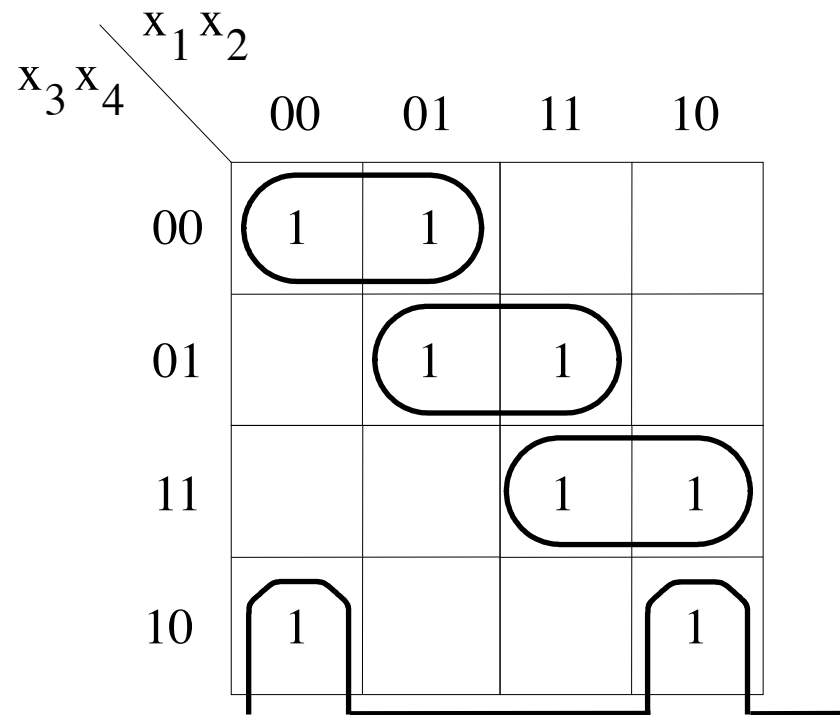


[Figure 2.54 from the textbook]

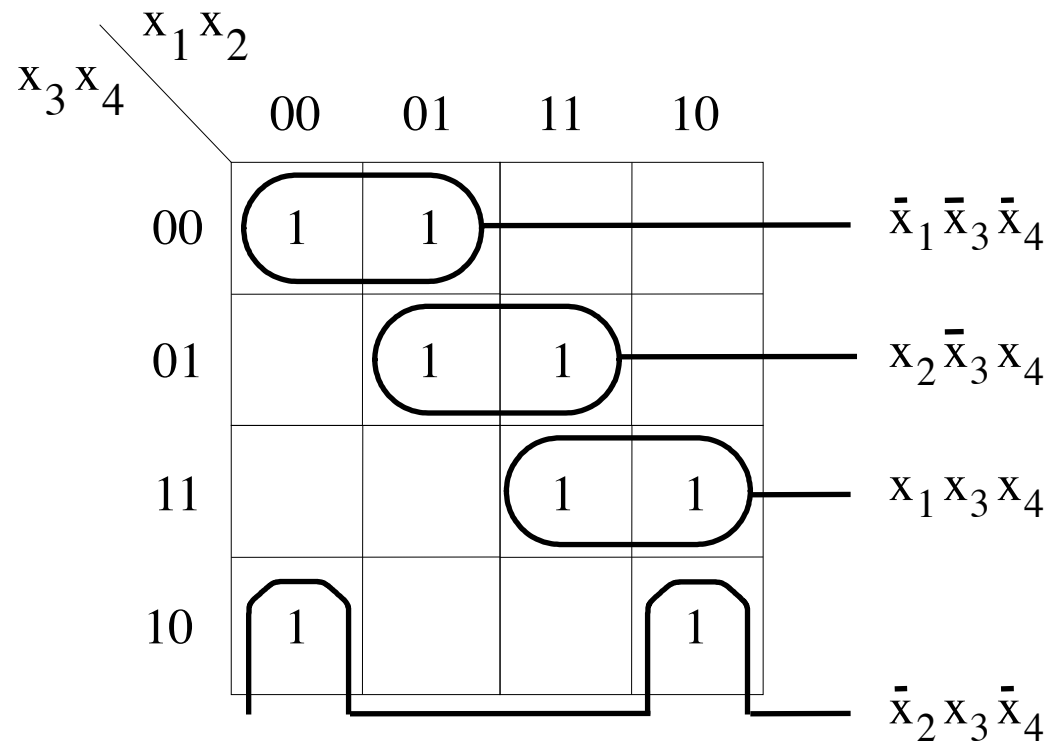
Example

$x_3 x_4$		$x_1 x_2$			
		00	01	11	10
00	1	1			
01		1	1		
11			1	1	
10	1			1	

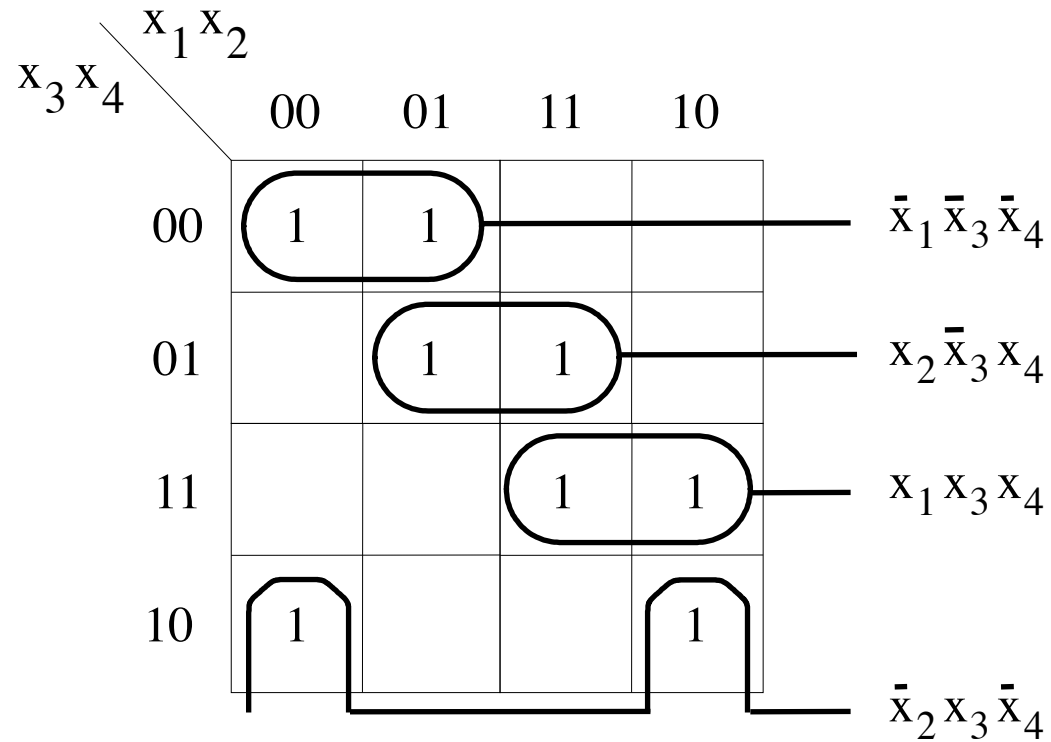
Example



Example



Example

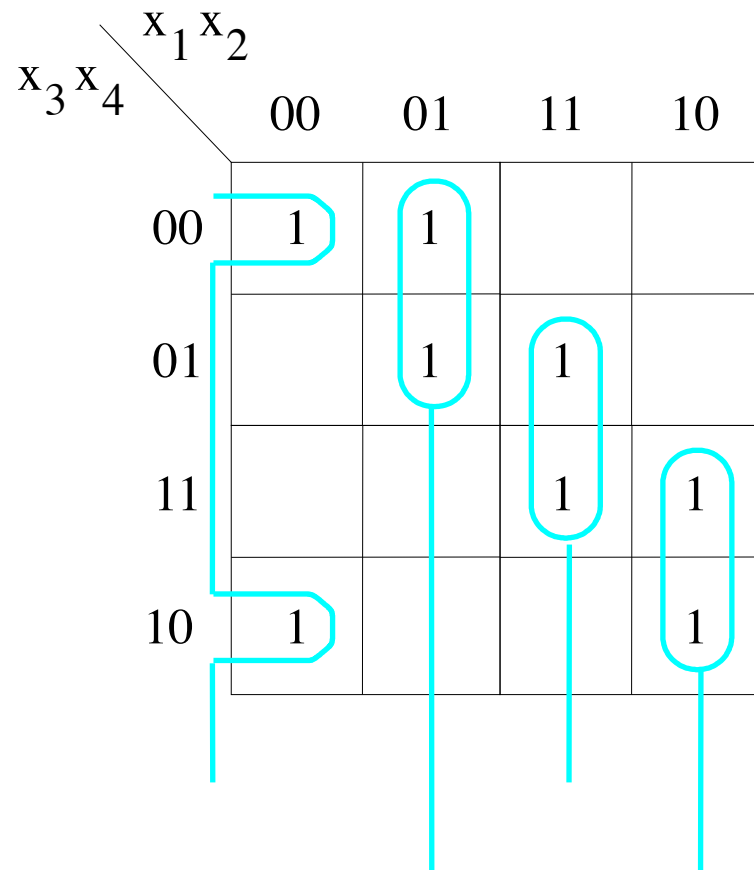


$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

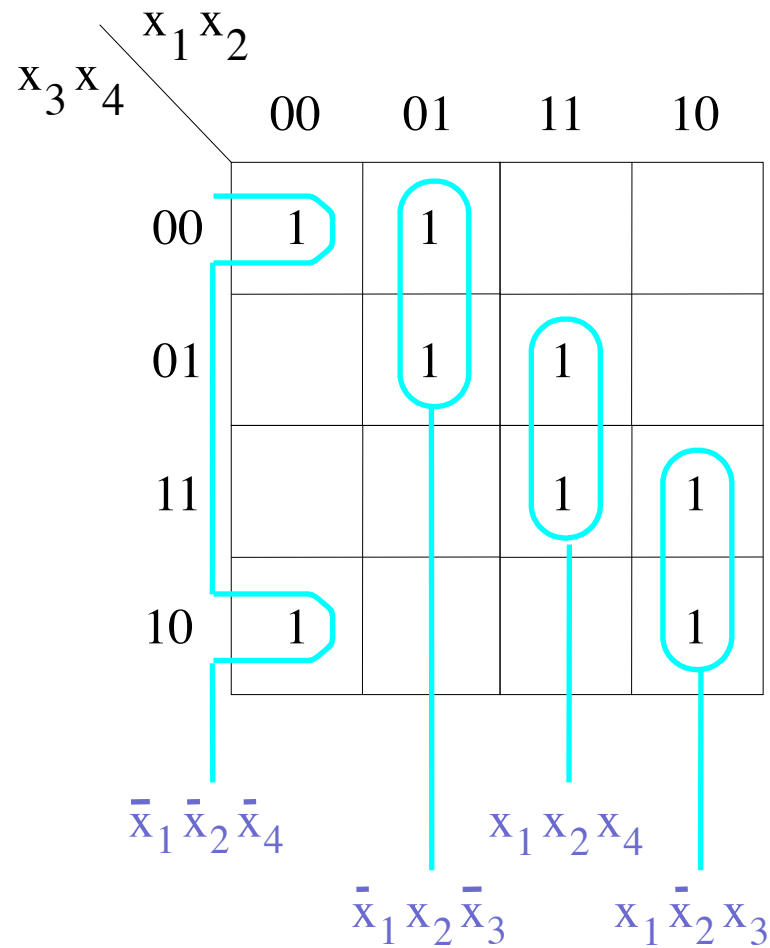
Example: Another Solution

$x_3 x_4$		$x_1 x_2$			
		00	01	11	10
00	1	1			
01		1	1		
11			1	1	
10	1			1	

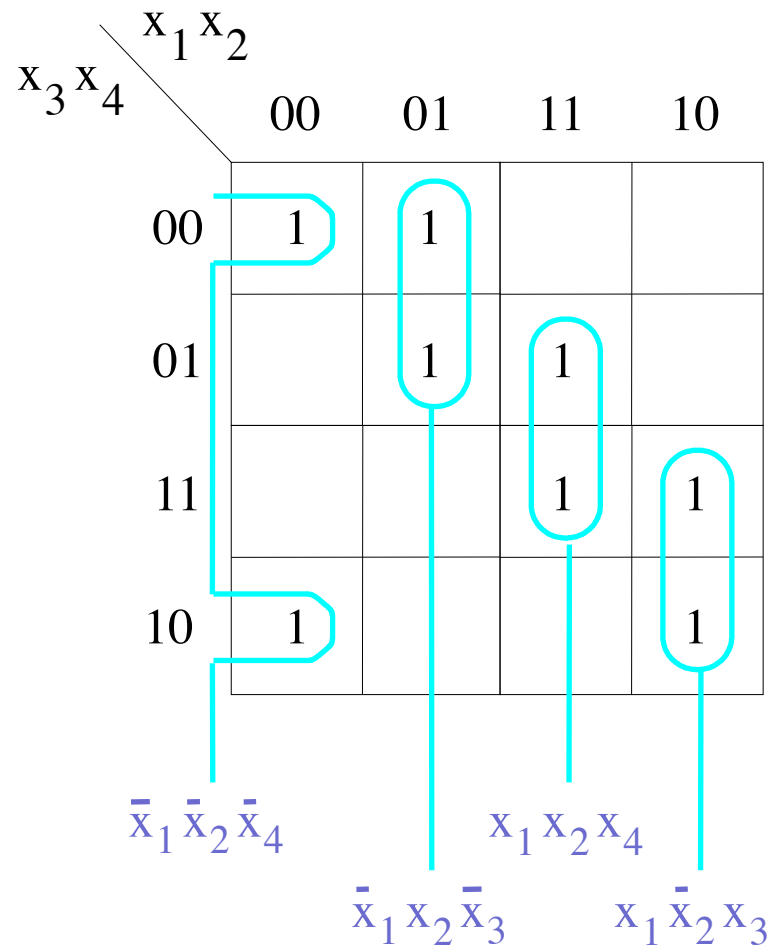
Example: Another Solution



Example: Another Solution

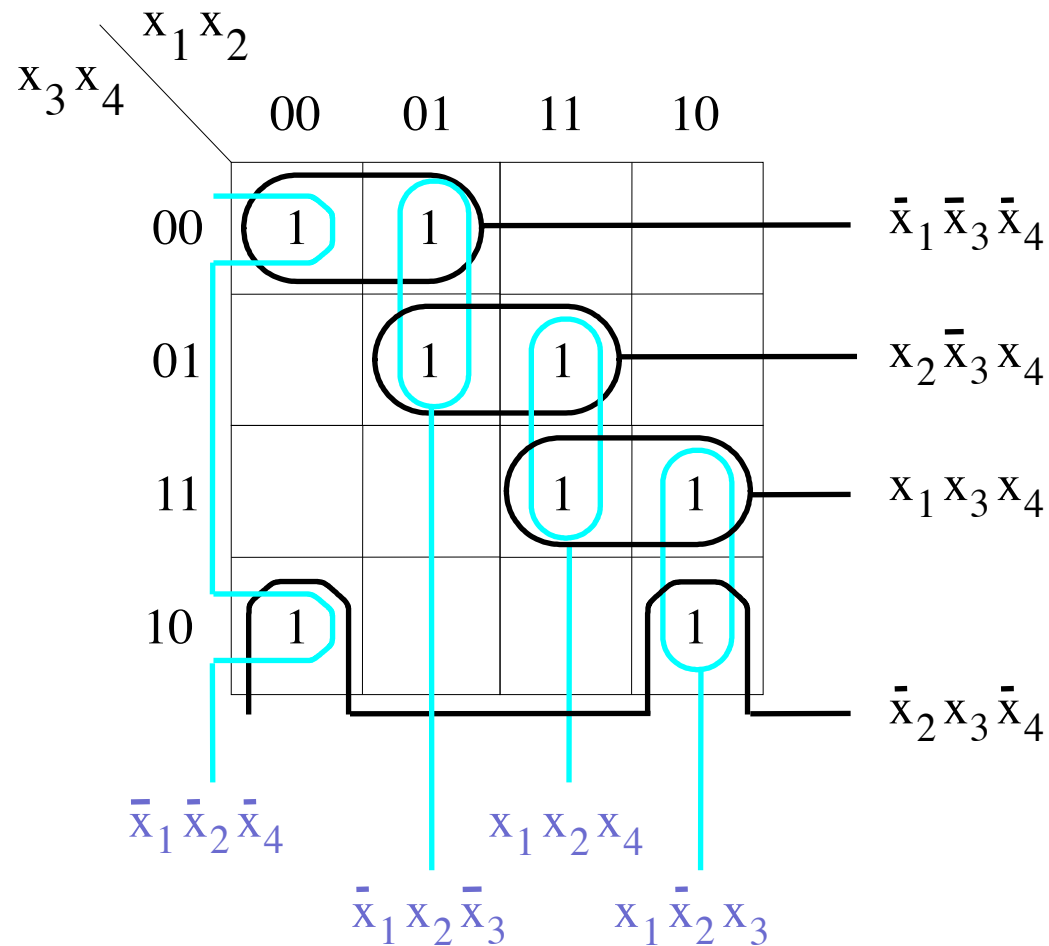


Example: Another Solution

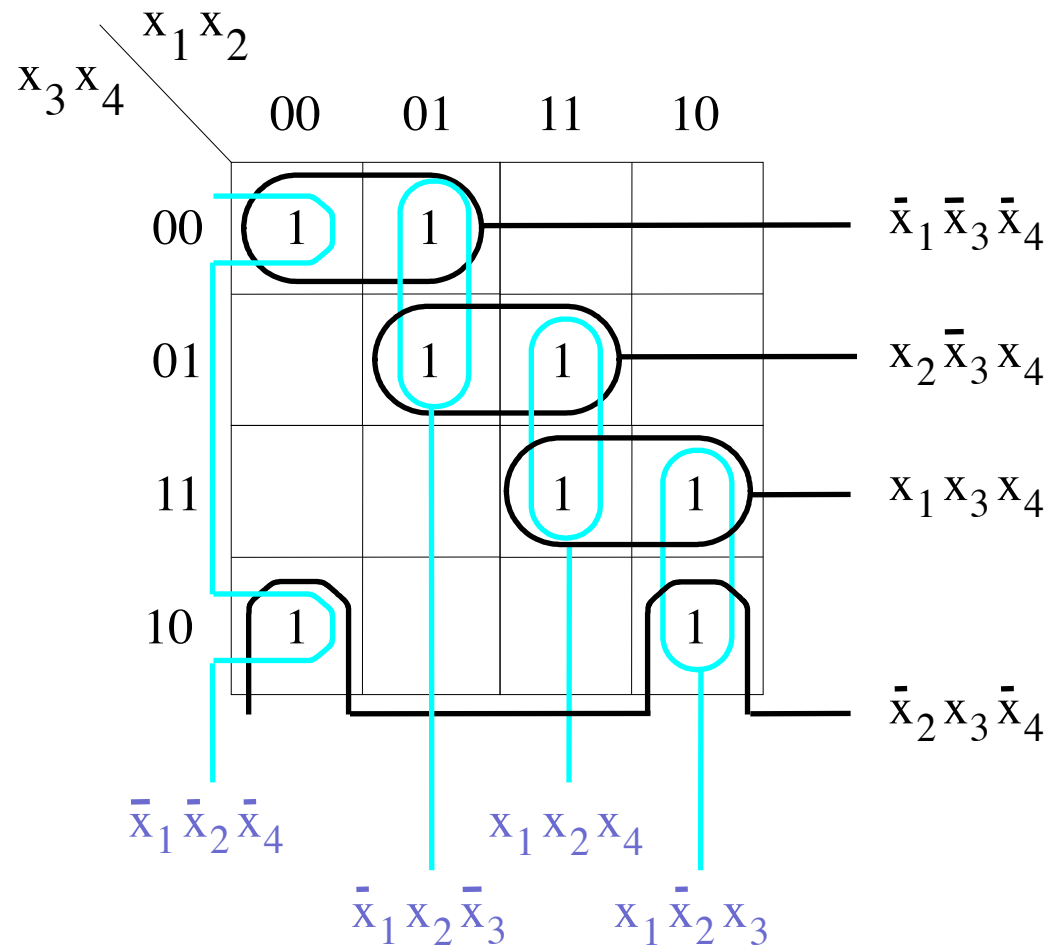


$$f = \bar{x}_1 \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

Example: Both Are Valid Solutions



Example: Both Are Valid Solutions



$$f = \bar{x}_1 \bar{x}_3 \bar{x}_4 + x_2 \bar{x}_3 x_4 + x_1 x_3 x_4 + \bar{x}_2 x_3 \bar{x}_4$$

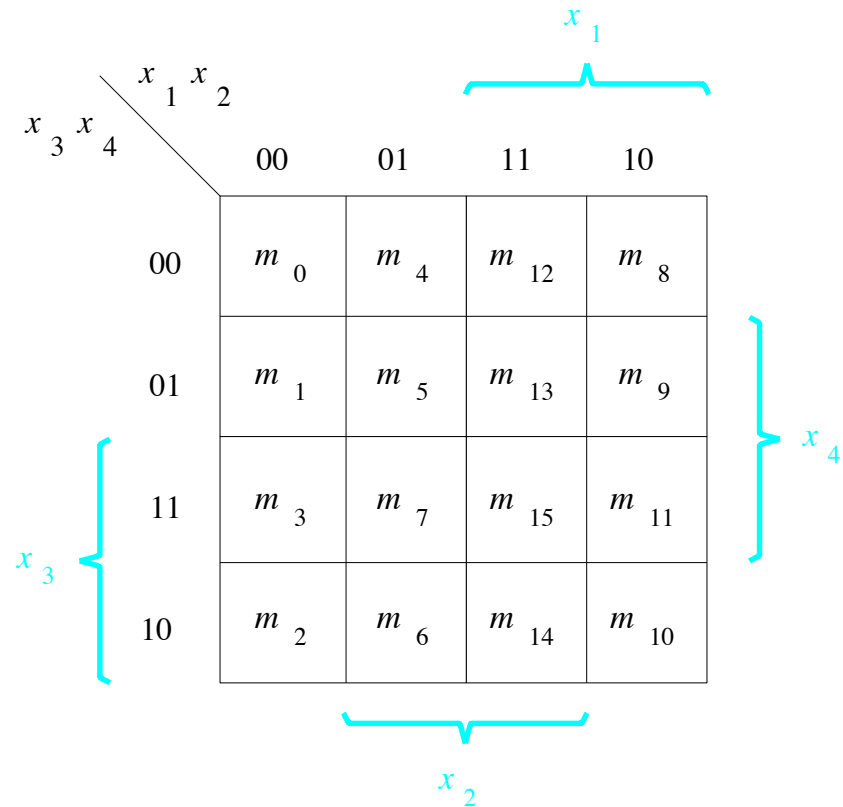
$$f = \bar{x}_1 \bar{x}_2 \bar{x}_4 + \bar{x}_1 x_2 \bar{x}_3 + x_1 x_2 x_4 + x_1 \bar{x}_2 x_3$$

Example:
Incompletely Specified Function

Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1	x_2	x_3	x_4	f
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
0	1	0	0	m_4
0	1	0	1	m_5
0	1	1	0	m_6
0	1	1	1	m_7
1	0	0	0	m_8
1	0	0	1	m_9
1	0	1	0	m_{10}
1	0	1	1	m_{11}
1	1	0	0	m_{12}
1	1	0	1	m_{13}
1	1	1	0	m_{14}
1	1	1	1	m_{15}



Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

x_1	x_2	x_3	x_4	f
0	0	0	0	m_0 0
0	0	0	1	m_1 0
0	0	1	0	m_2 1
0	0	1	1	m_3 0
0	1	0	0	m_4 1
0	1	0	1	m_5 1
0	1	1	0	m_6 1
0	1	1	1	m_7 1
1	0	0	0	m_8 0
1	0	0	1	m_9 0
1	0	1	0	m_{10} 1
1	0	1	1	m_{11} 0
1	1	0	0	m_{12} d
1	1	0	1	m_{13} d
1	1	1	0	m_{14} d
1	1	1	1	m_{15} d

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	0	1	d	0
	01	0	1	d	0
	11	0	0	d	0
	10	1	1	d	1

SOP implementation

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

$x_2\bar{x}_3$

$x_3\bar{x}_4$

(a) SOP implementation

POS implementation

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

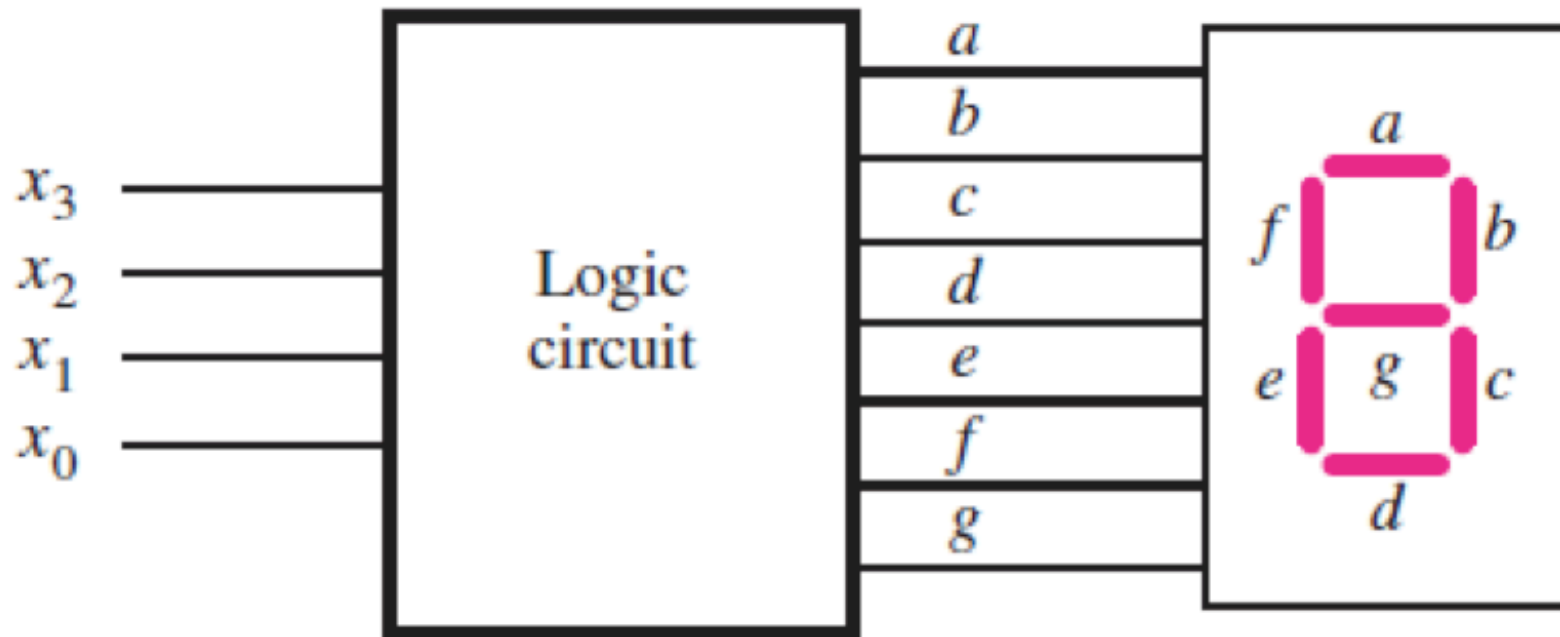
$(x_2 + x_3)$

$(\bar{x}_3 + \bar{x}_4)$

(b) POS implementation

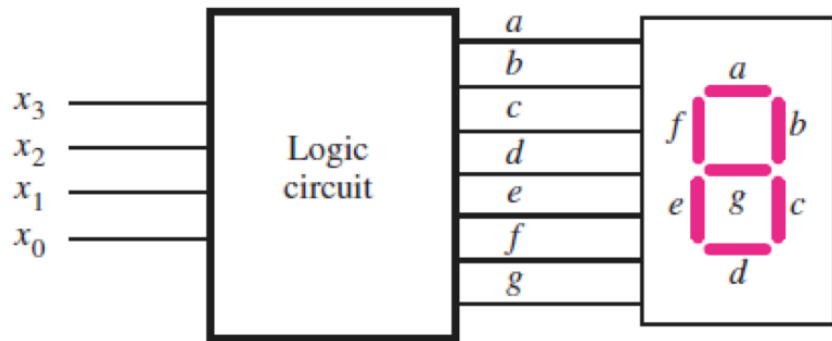
Example:
A circuit with multiple outputs

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

Seven-Segment Indicator



(a) Logic circuit and 7-segment display

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0							
1011	1	0	1	1							
1100	1	1	0	0							
1101	1	1	0	1							
1110	1	1	1	0							
1111	1	1	1	1							

Seven-Segment Indicator

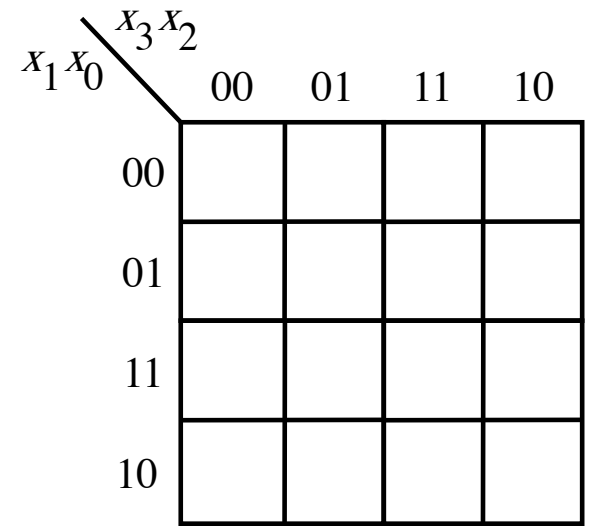
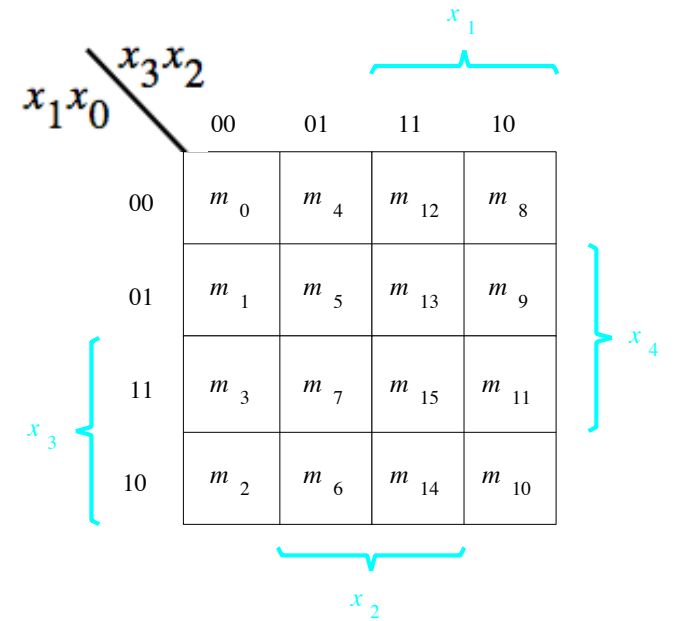
	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0000-1011	0	0	0	0	1	1	1	1	1	1	0
	0	0	0	1	0	1	1	0	0	0	0
	0	0	1	0	1	1	0	1	1	0	1
	0	0	1	1	1	1	1	1	0	0	1
	0	1	0	0	0	1	1	0	0	1	1
	0	1	0	1	1	0	1	1	0	1	1
	0	1	1	0	1	1	0	1	1	1	1
	0	1	1	1	1	1	1	0	0	0	0
	1	0	0	0	1	1	1	1	1	1	1
	1	0	0	1	1	1	1	1	0	1	1
	1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d	
1	1	0	0	d	d	d	d	d	d	d	
1	1	0	1	d	d	d	d	d	d	d	
1	1	1	0	d	d	d	d	d	d	d	
1	1	1	1	d	d	d	d	d	d	d	

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d

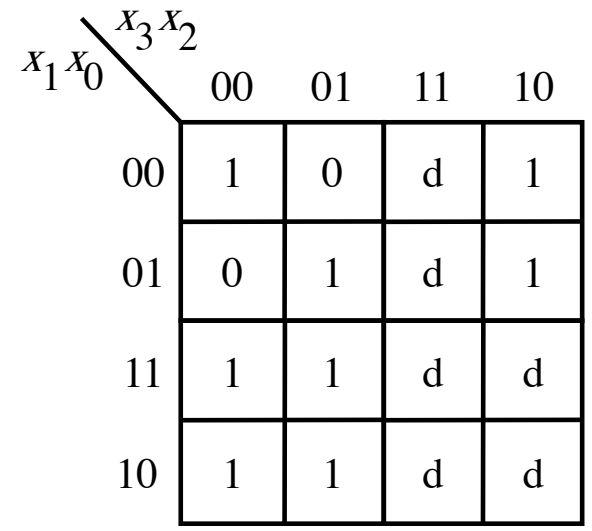
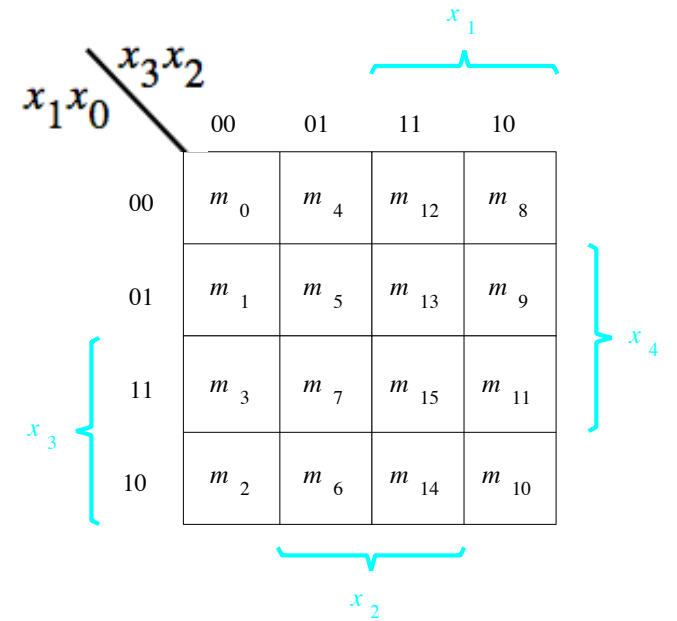
Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d



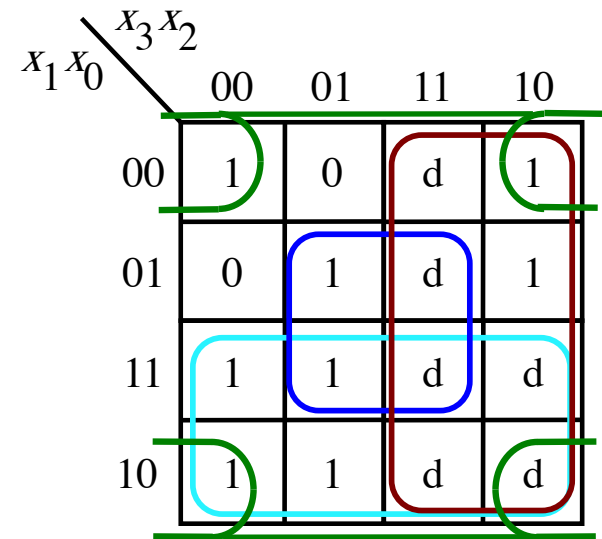
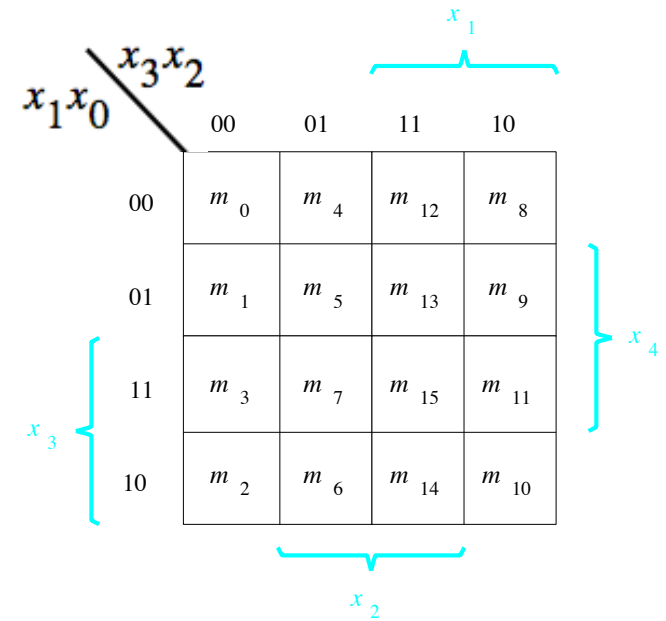
Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0	0	0	0	0	1	1	1	1	1	1	0
0	0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	1	0	0	1
0	1	0	0	0	0	1	1	0	0	1	1
0	1	0	1	1	1	0	1	1	0	1	1
0	1	1	0	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1	0	0	0	0
1	0	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	1	1
1	0	1	0	0	d	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d	d
1	1	0	0	0	d	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d	d
1	1	1	0	0	d	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d	d



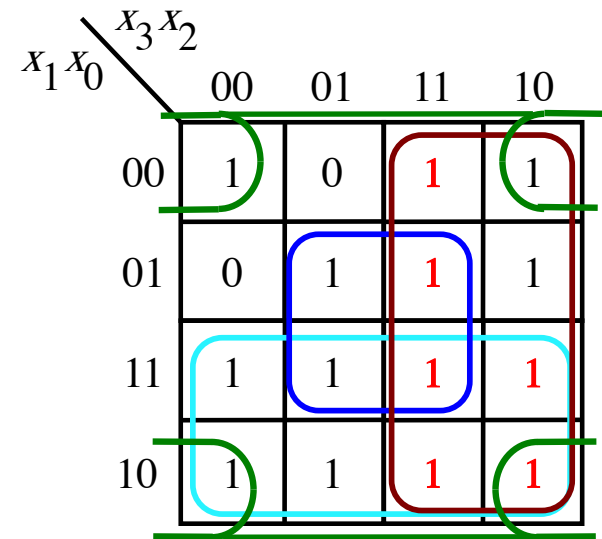
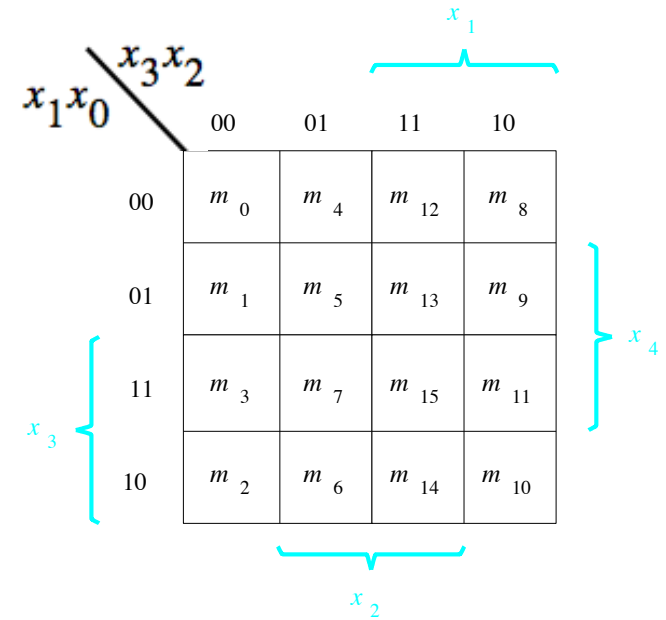
Seven-Segment Indicator

x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d



Seven-Segment Indicator

	x_3	x_2	x_1	x_0	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d



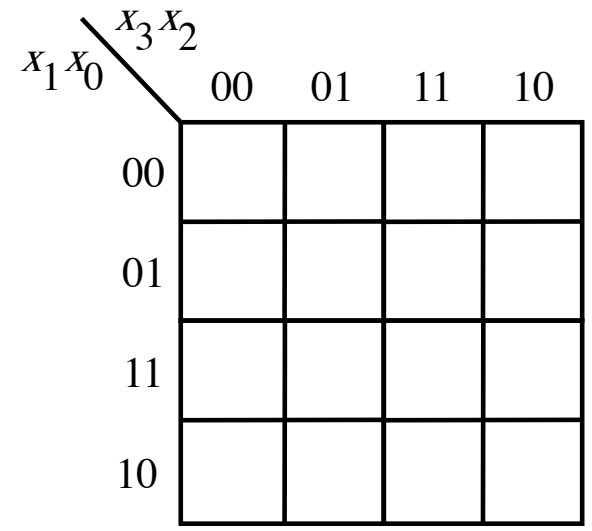
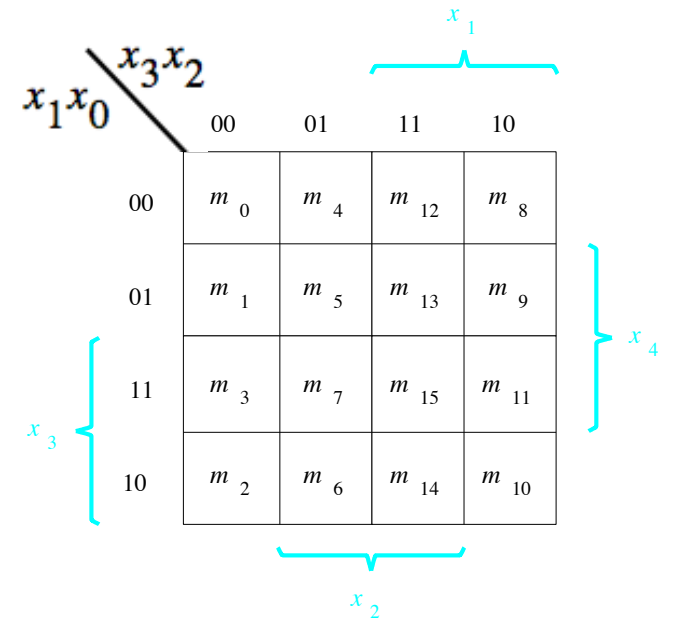
In this case all d's were treated as 1's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
d	1	0	1	0	1	d	d	d	d	d	d
d	1	0	1	1	1	d	d	d	d	d	d
d	1	1	0	0	1	d	d	d	d	d	d
d	1	1	0	1	1	d	d	d	d	d	d
d	1	1	1	0	1	d	d	d	d	d	d
d	1	1	1	1	1	d	d	d	d	d	d

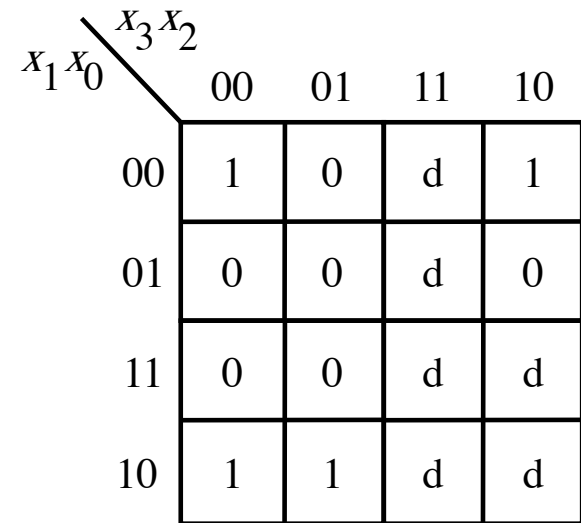
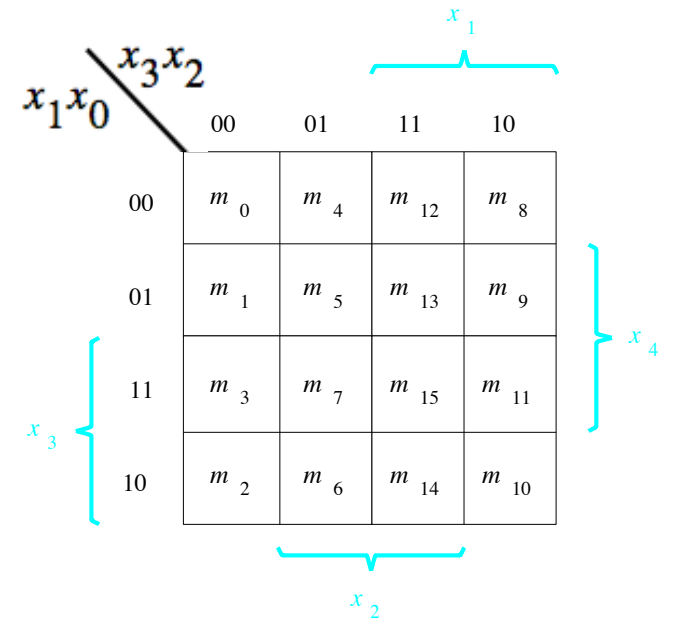
Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d



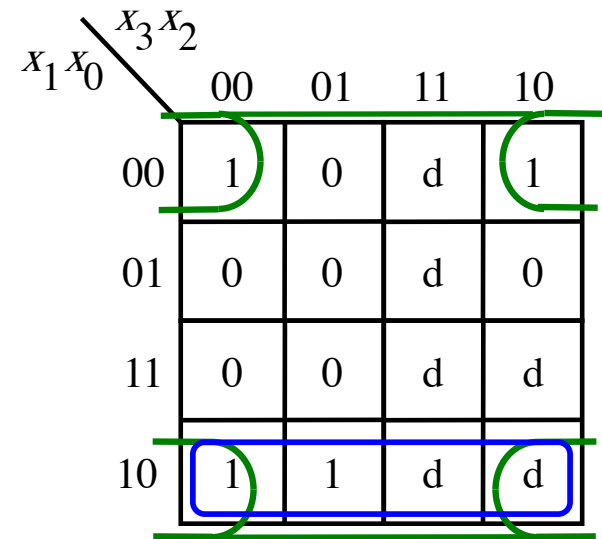
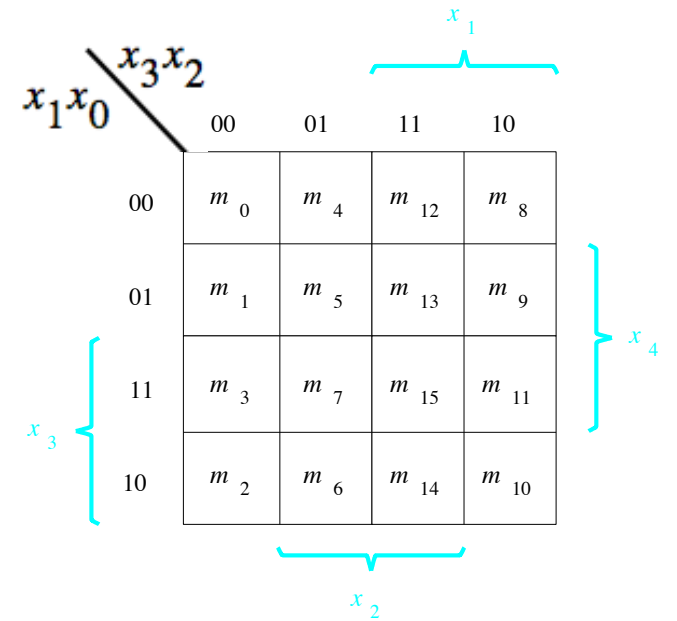
Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d



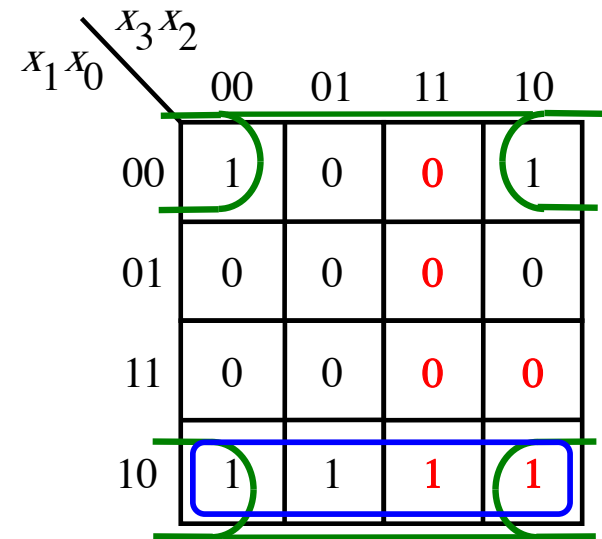
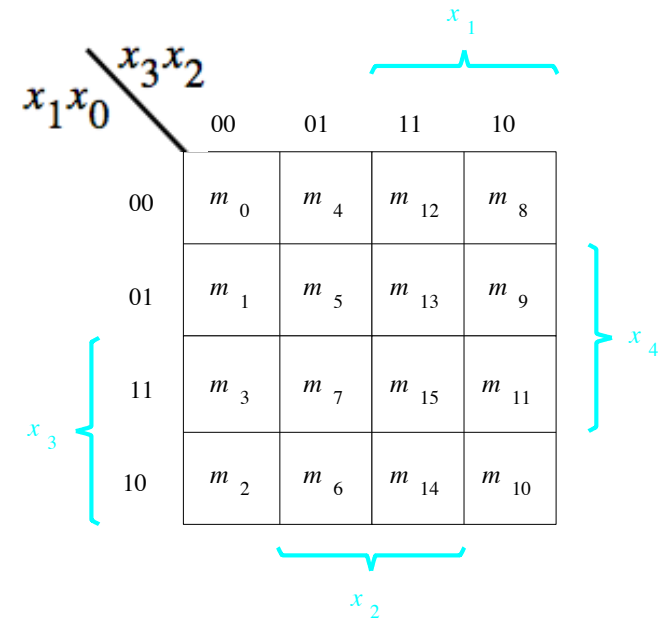
Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d



Seven-Segment Indicator

x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	1	d	d
1	0	1	1	1	d	d	d	0	d	d
1	1	0	0	1	d	d	d	0	d	d
1	1	0	1	1	d	d	d	0	d	d
1	1	1	0	1	d	d	d	1	d	d
1	1	1	1	1	d	d	d	0	d	d

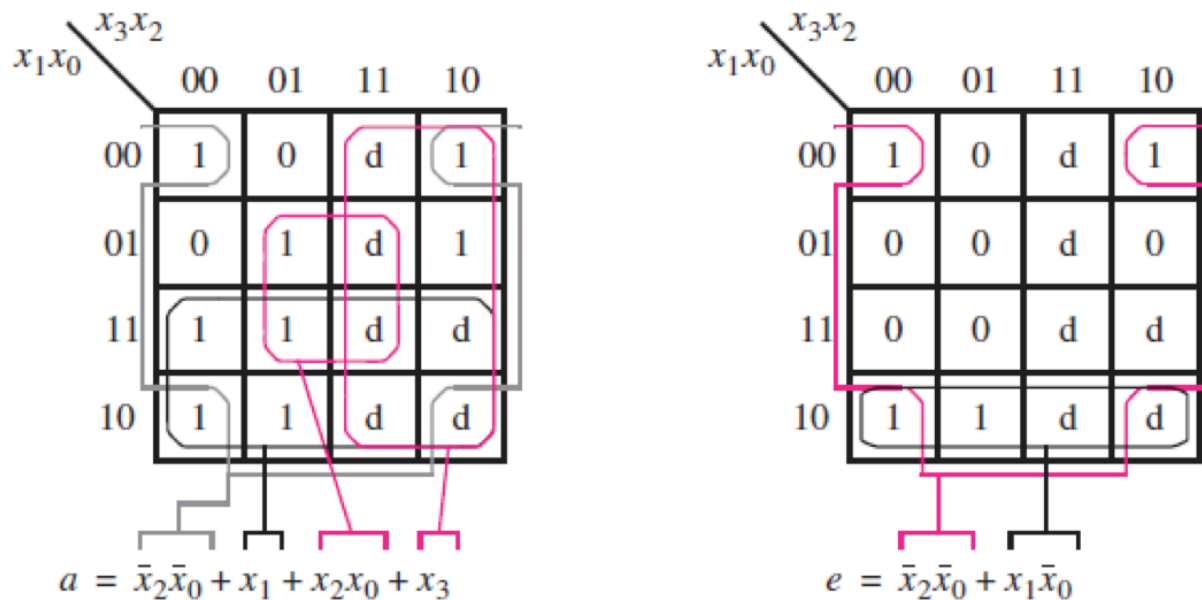


In this case some d's were treated as 1's, others as 0's.

Seven-Segment Indicator

	x_3	x_2	x_1	x_0	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table



(c) The Karnaugh maps for outputs a and e .

Another Example

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

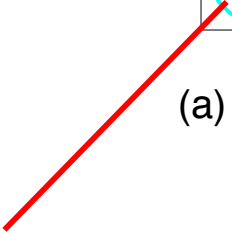
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function f_2

$\bar{x}_1 x_3$



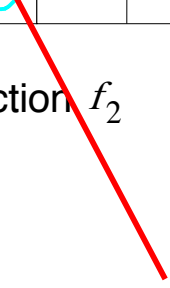
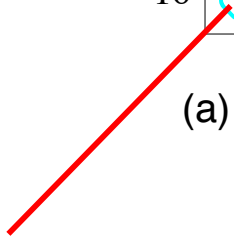
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$\bar{x}_1 x_3$



$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{X}_1 \bar{X}_3$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$\bar{X}_1 X_3$

$\bar{X}_1 X_3$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{x}_1 x_3$

$x_1 \bar{x}_3$

$x_2 \bar{x}_3 x_4$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$x_2 x_3 x_4$

$\bar{x}_1 x_3$

	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{X}_1 \bar{X}_3$

$\bar{X}_2 \bar{X}_3 X_4$

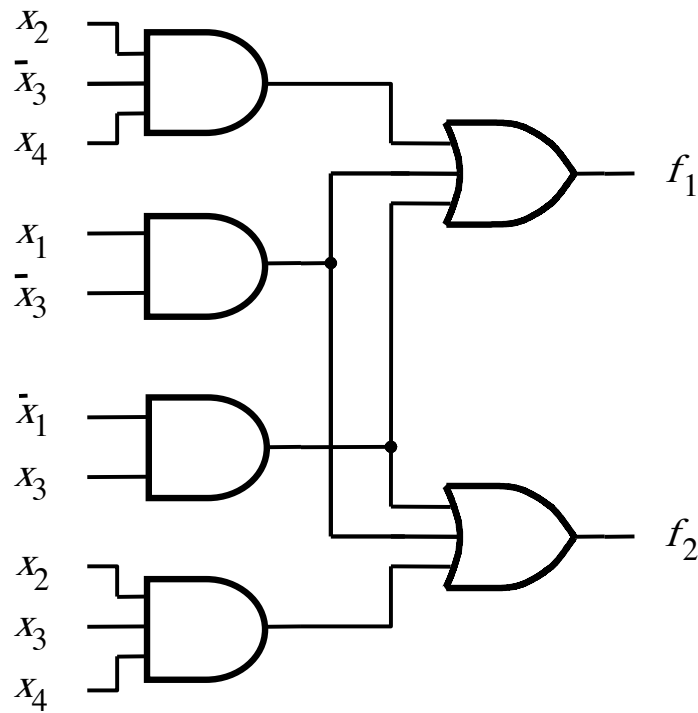
	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$X_2 X_3 X_4$

$\bar{X}_1 X_3$

$\bar{X}_1 X_3$



(c) Combined circuit for f_1 and f_2

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$\bar{x}_1 \bar{x}_3$

$\bar{x}_2 \bar{x}_3 x_4$

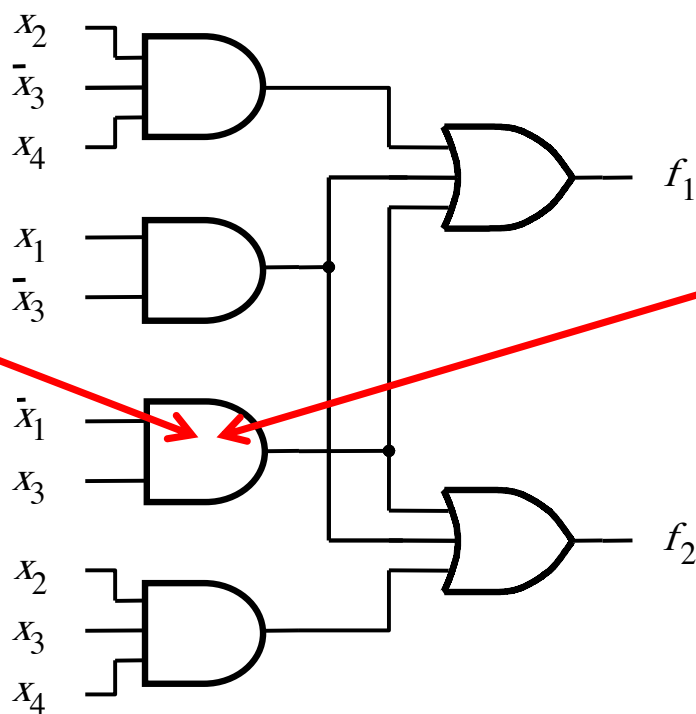
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2

$x_2 x_3 x_4$

$\bar{x}_1 \bar{x}_3$

$\bar{x}_1 \bar{x}_3$



(c) Combined circuit for f_1 and f_2

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

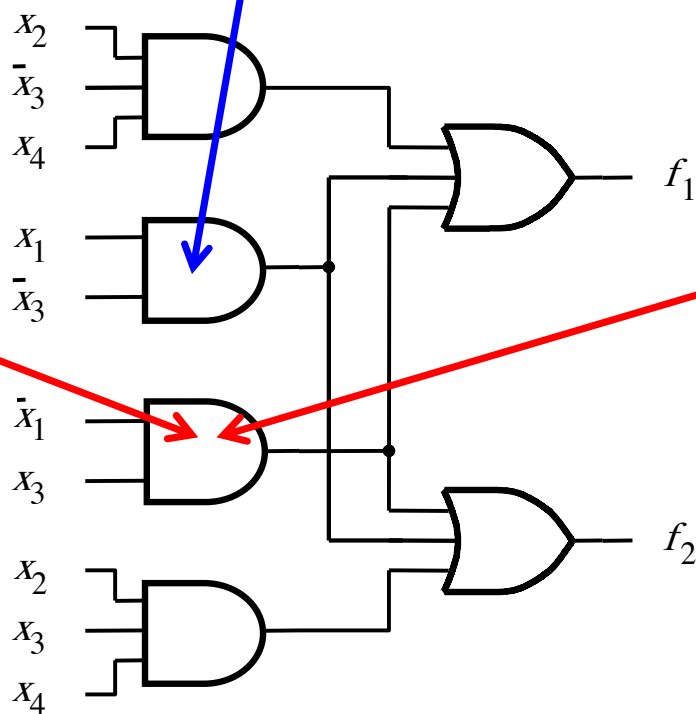
(b) Function f_2

$$\bar{x}_2 \bar{x}_3 x_4$$

$$x_2 x_3 x_4$$

$$\bar{x}_1 x_3$$

$$\bar{x}_1 x_3$$



(c) Combined circuit for f_1 and f_2

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

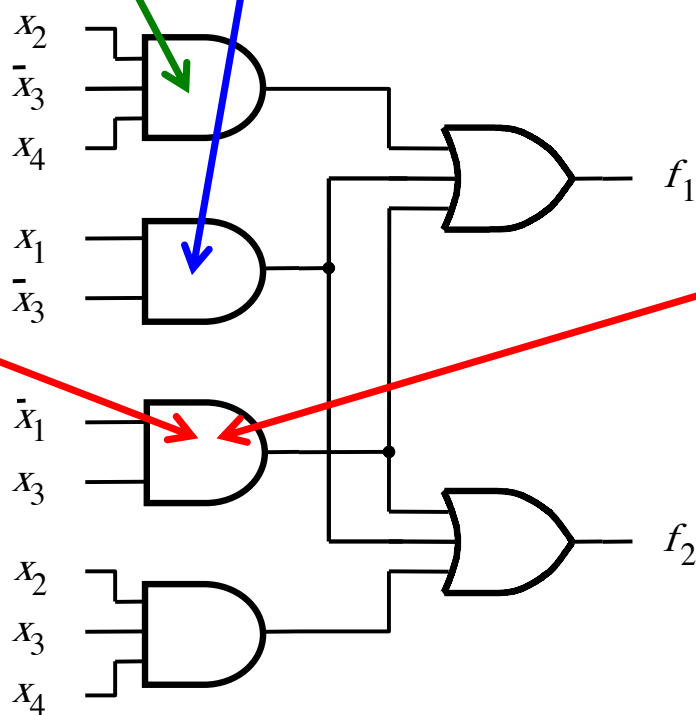
(b) Function f_2

$\bar{x}_2 \bar{x}_3 x_4$

$x_2 x_3 x_4$

$\bar{x}_1 \bar{x}_3$

$\bar{x}_1 x_3$



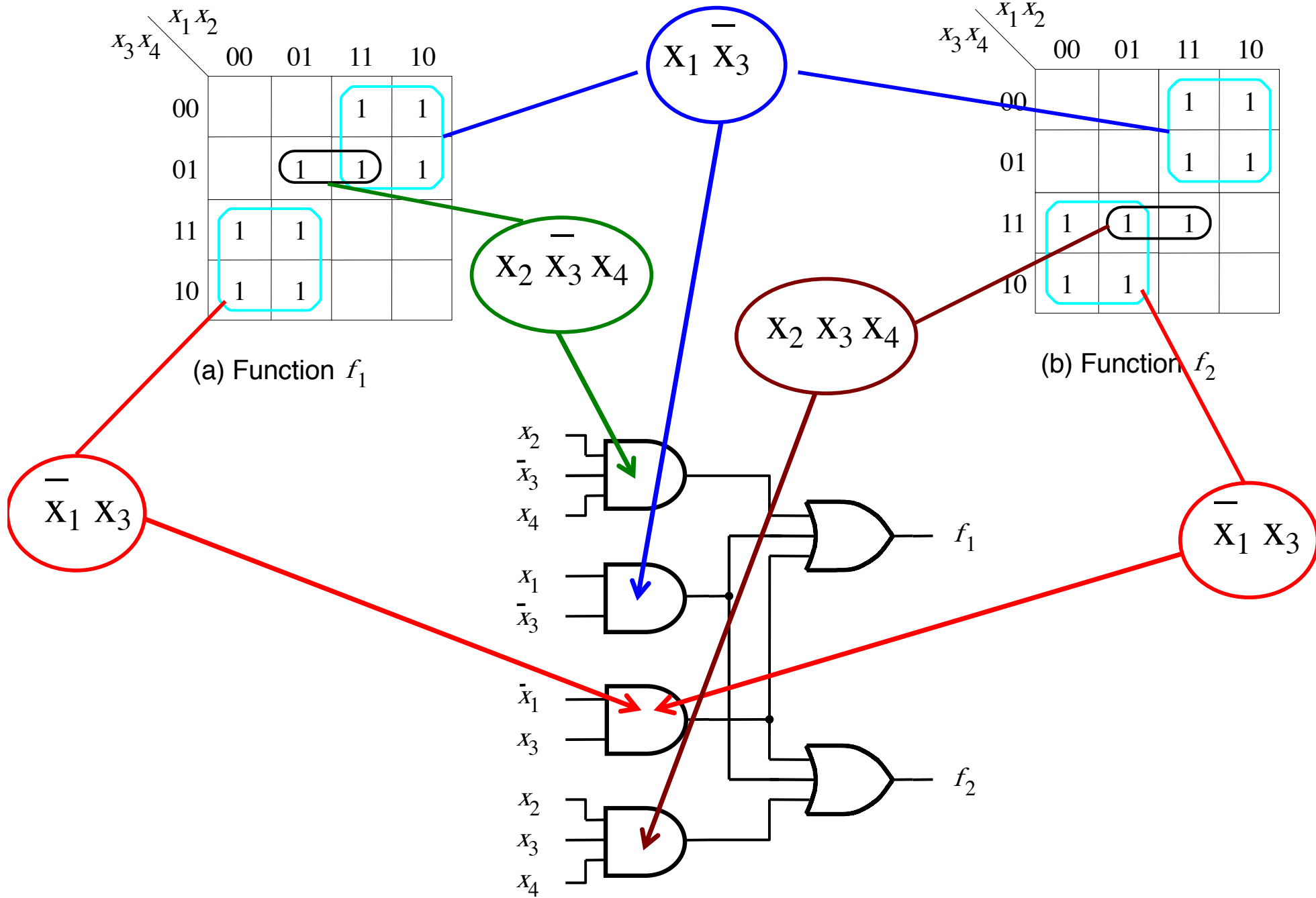
(c) Combined circuit for f_1 and f_2

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function f_2



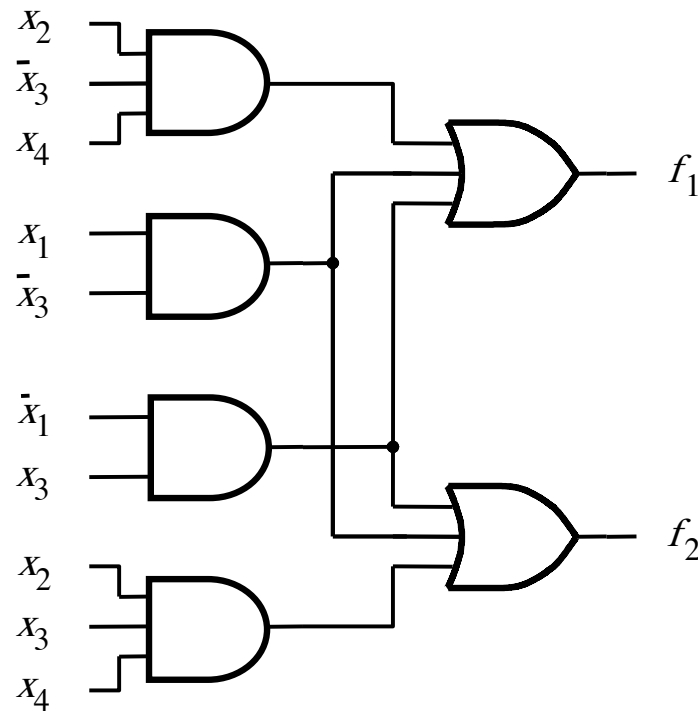
(c) Combined circuit for f_1 and f_2

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function f_1

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

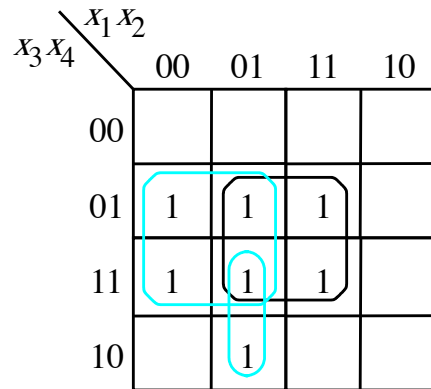
(b) Function f_2



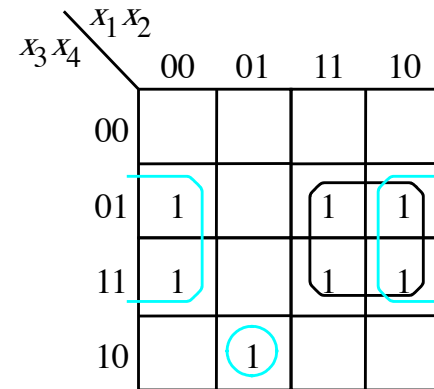
(c) Combined circuit for f_1 and f_2

Yet Another Example

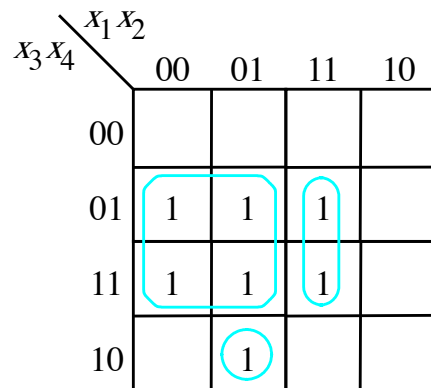
Individual vs Joint Optimization



(a) Optimal realization of f_3

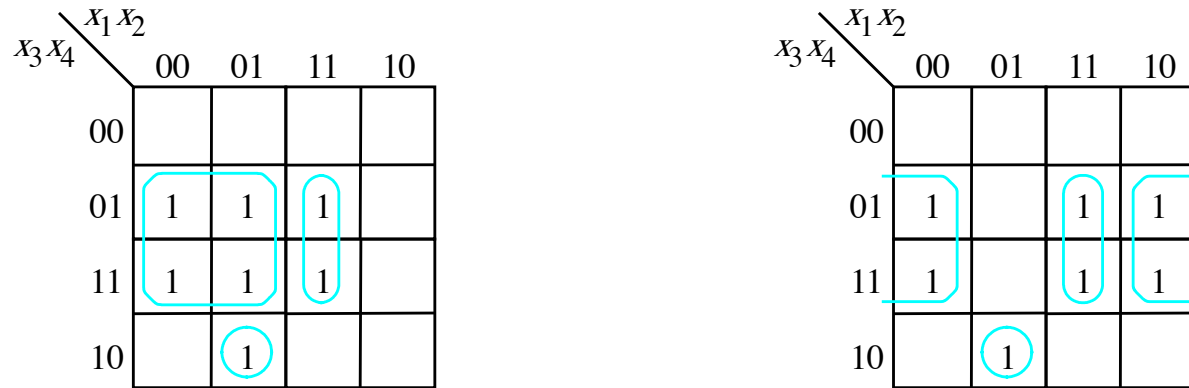


(b) Optimal realization of f_4

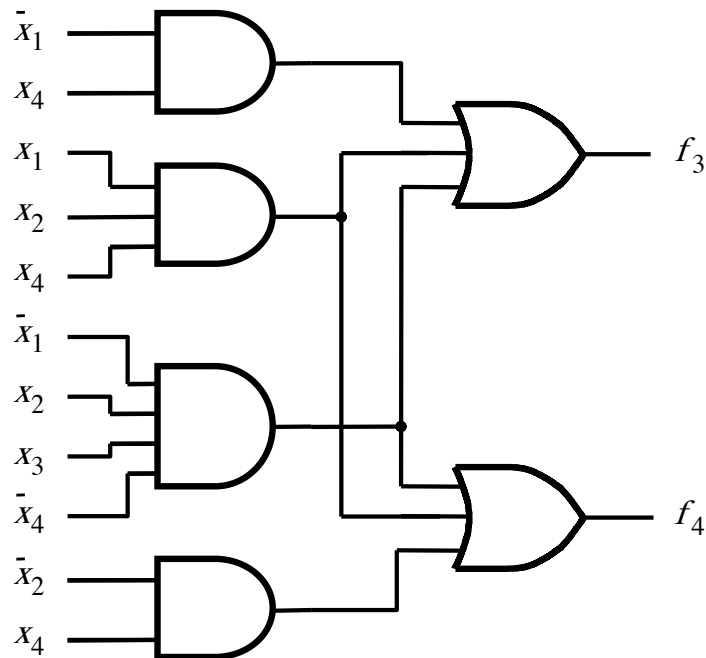


(c) Optimal realization of f_3 and f_4 together

Individual vs Joint Optimization

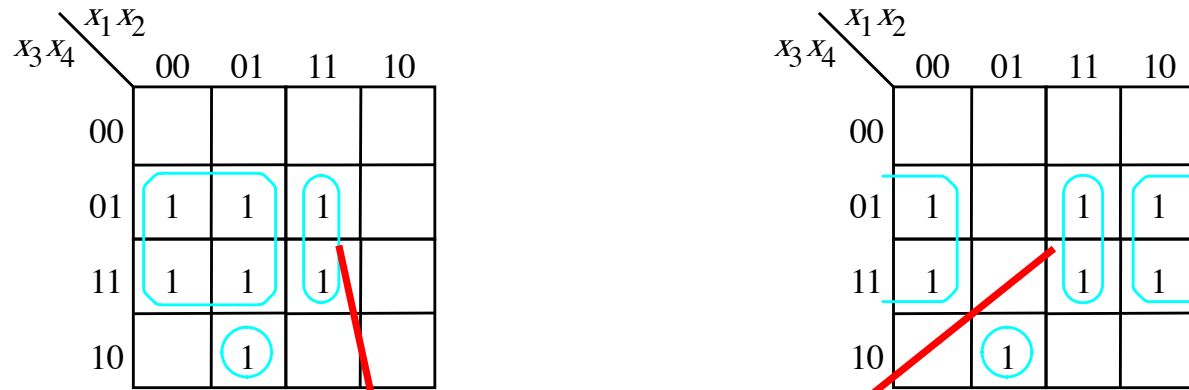


(c) Optimal realization of f_3 and f_4 together

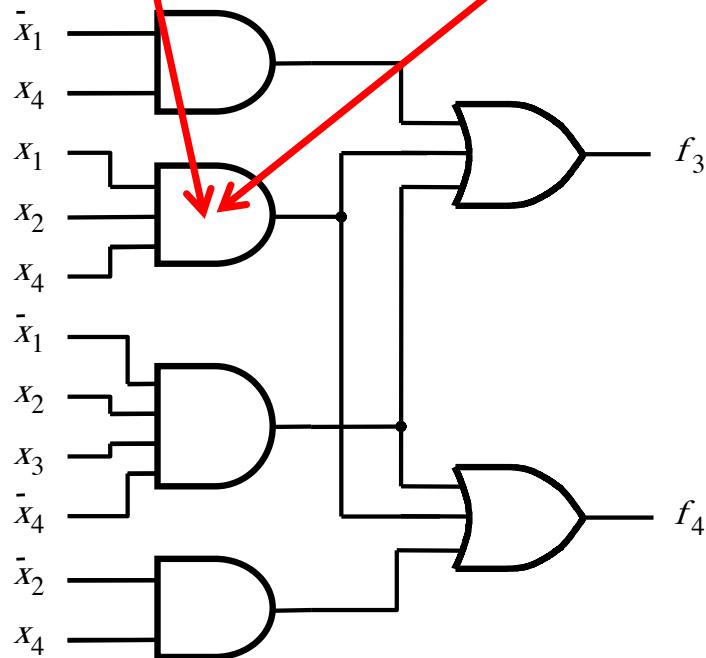


(d) Combined circuit for f_3 and f_4

Individual vs Joint Optimization

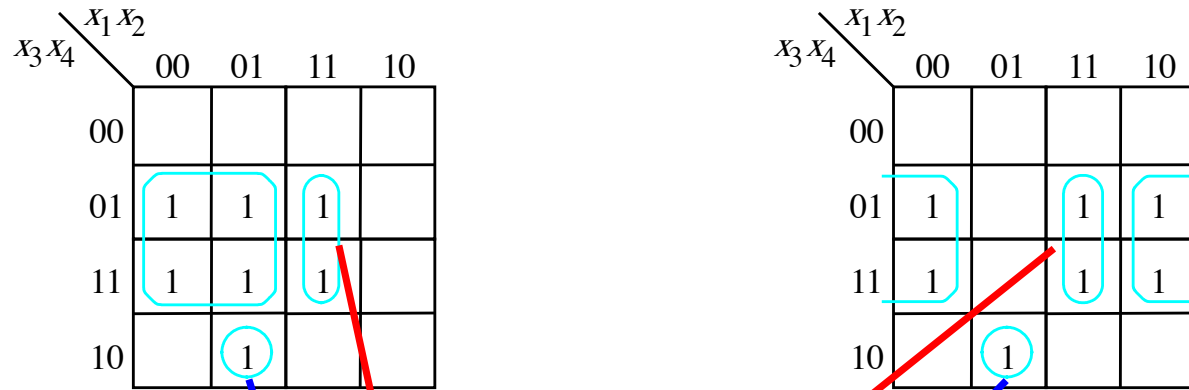


(c) Optimal realization of f_3 and f_4 together

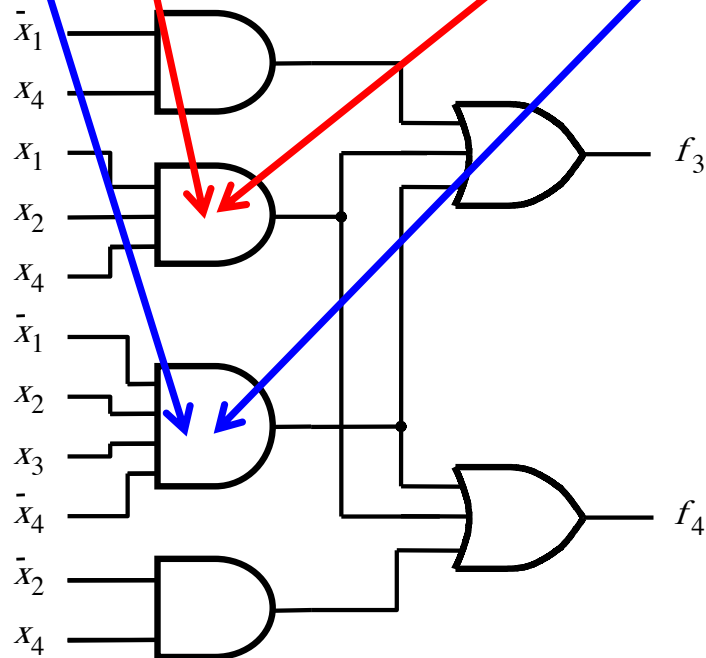


(d) Combined circuit for f_3 and f_4

Individual vs Joint Optimization



(c) Optimal realization of f_3 and f_4 together



(d) Combined circuit for f_3 and f_4

Questions?

THE END