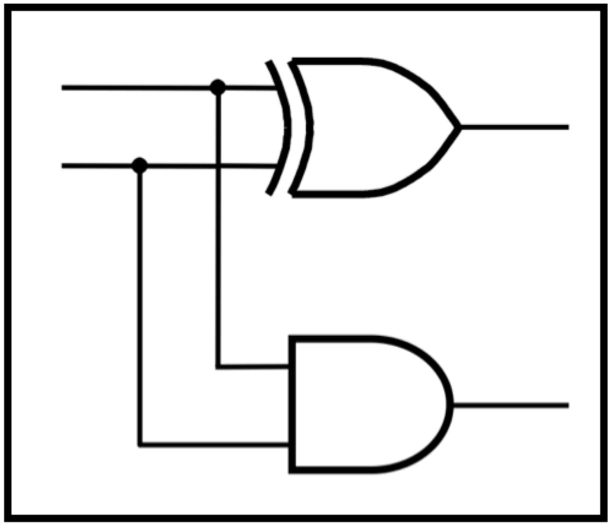


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Multiplexers

CprE 281: Digital Logic
Iowa State University, Ames, IA

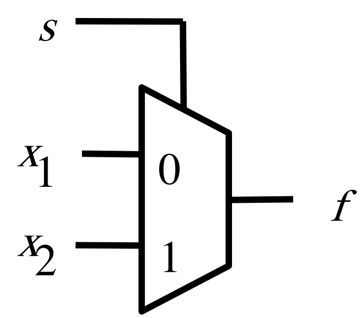
Administrative Stuff

- **HW 6 is due on Monday Oct 5**

2-to-1 Multiplexer (Definition)

- Has two inputs: x_1 and x_2
- Also has another input line s
- If s=0, then the output is equal to x_1
- If s=1, then the output is equal to x_2

Graphical Symbol for a 2-to-1 Multiplexer



[Figure 2.33c from the textbook]

Truth Table for a 2-to-1 Multiplexer

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

[Figure 2.33a from the textbook]

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Where should we put the negation signs?

$s\ x_1\ x_2$

$s\ x_1\ x_2$

$s\ x_1\ x_2$

$s\ x_1\ x_2$

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\overline{s}\ x_1\ \overline{x}_2$
0 1 1	1	$\overline{s}\ x_1\ x_2$
1 0 0	0	
1 0 1	1	$s\ \overline{x}_1\ x_2$
1 1 0	0	
1 1 1	1	$s\ x_1\ x_2$

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	1	$\overline{s}\, x_1\, \overline{x_2}$
0 1 1	1	$\overline{s}\, x_1\, x_2$
1 0 0	0	
1 0 1	1	$s\, \overline{x_1}\, x_2$
1 1 0	0	
1 1 1	1	$s\, x_1\, x_2$

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

$$f(s, x_1, x_2) = \overline{s}\, x_1\, (\overline{x_2} + x_2) + s\, (\overline{x_1} + x_1)\, x_2$$

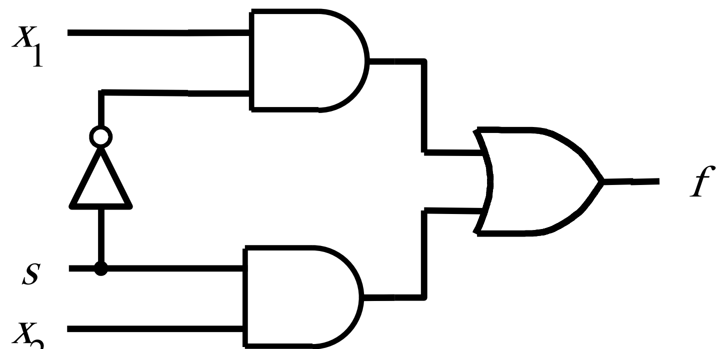
Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

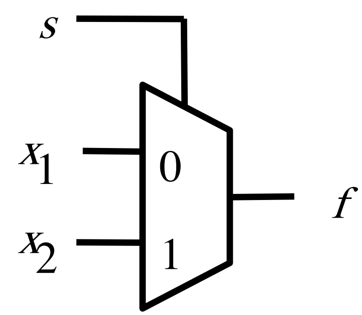
$$f(s, x_1, x_2) = \overline{s}\, x_1\, (\overline{x_2} + x_2) + s\, (\overline{x_1} + x_1)\, x_2$$

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

Circuit for 2-1 Multiplexer



(b) Circuit

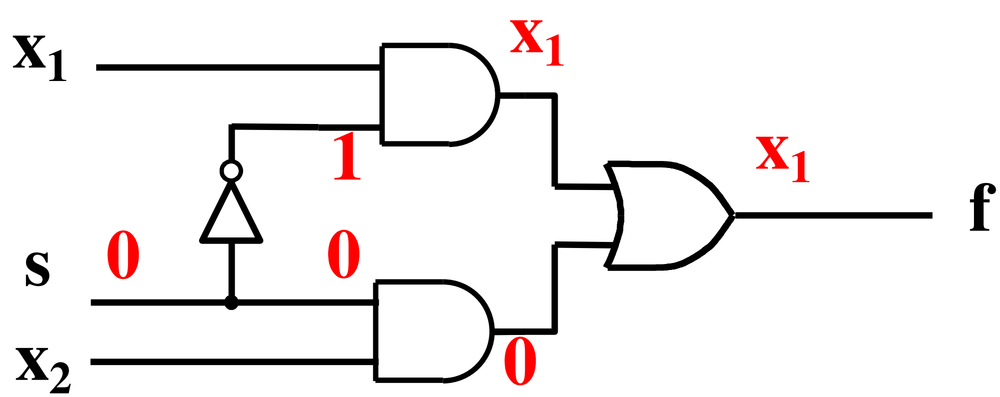


(c) Graphical symbol

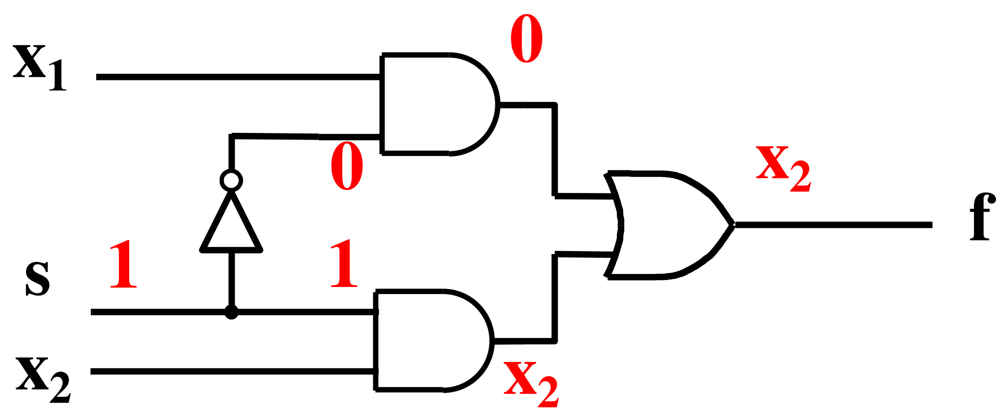
$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

[Figure 2.33b-c from the textbook]

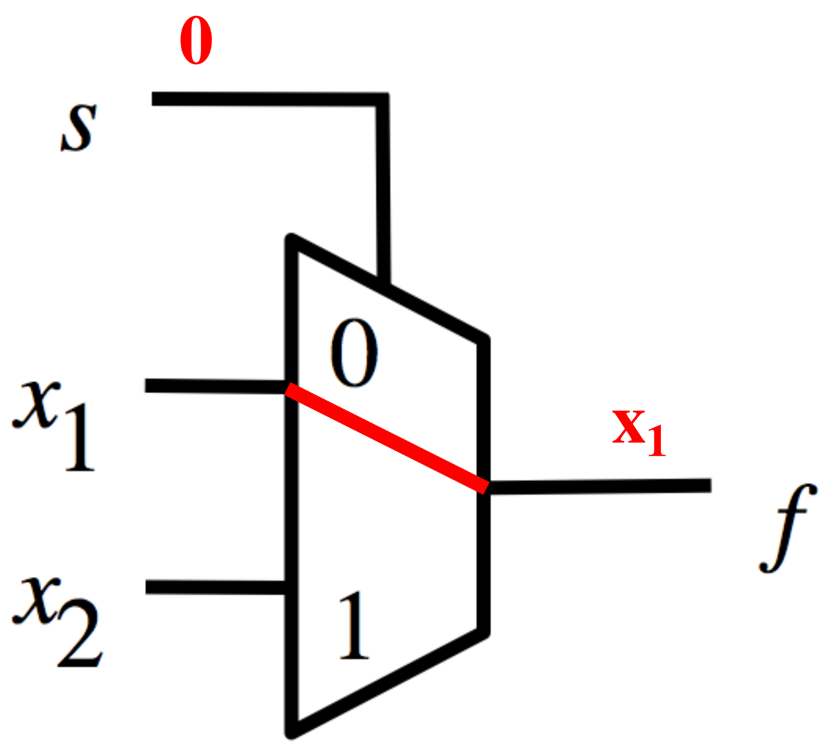
Analysis of the 2-to-1 Multiplexer (when the input s=0)



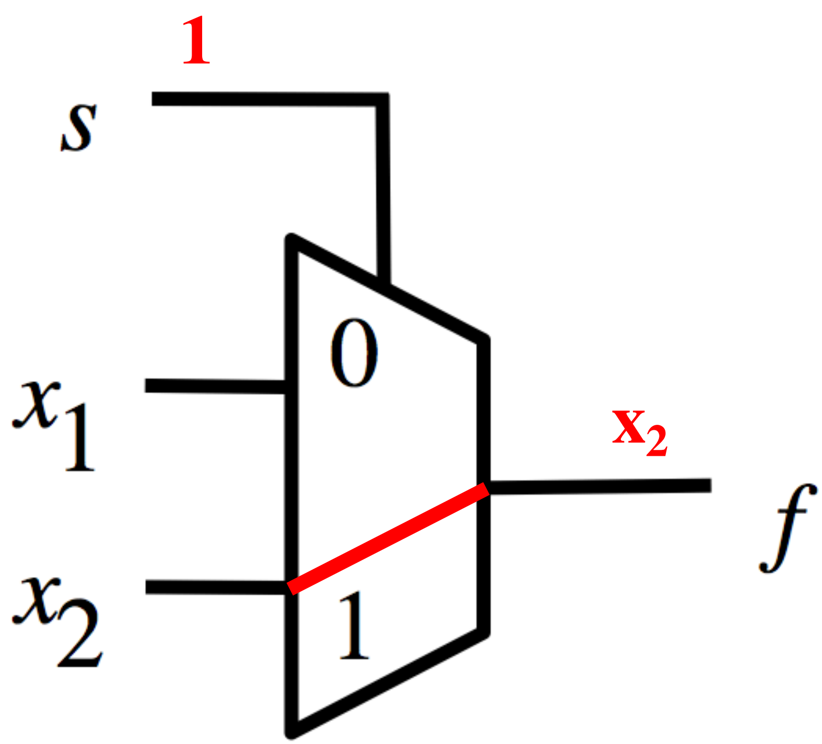
Analysis of the 2-to-1 Multiplexer (when the input s=1)



Analysis of the 2-to-1 Multiplexer (when the input s=0)



Analysis of the 2-to-1 Multiplexer (when the input s=1)



More Compact Truth-Table Representation

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

(a)Truth table

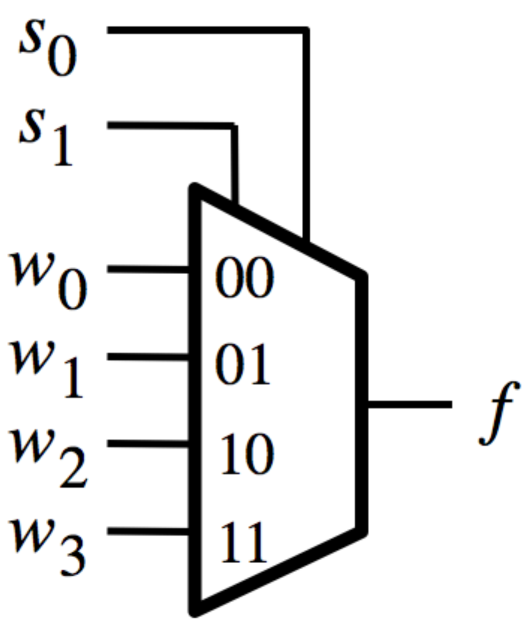
s	$f(s, x_1, x_2)$
0	x_1
1	x_2

[Figure 2.33 from the textbook]

4-to-1 Multiplexer (Definition)

- **Has four inputs: w_0 , w_1, w_2, w_3**
- **Also has two select lines: s_1 and s_0**
- **If s_1=0 and s_0=0, then the output f is equal to w_0**
- **If s_1=0 and s_0=1, then the output f is equal to w_1**
- **If s_1=1 and s_0=0, then the output f is equal to w_2**
- **If s_1=1 and s_0=1, then the output f is equal to w_3**

Graphical Symbol and Truth Table



(a) Graphic symbol

s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3

(b) Truth table

[Figure 4.2a-b from the textbook]

The long-form truth table

The long-form truth table

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	0	0
		0	0	0	1	1
		0	0	1	0	0
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	1
		0	1	1	0	0
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	1
		1	0	1	0	0
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	1
		1	1	1	0	0
		1	1	1	1	1

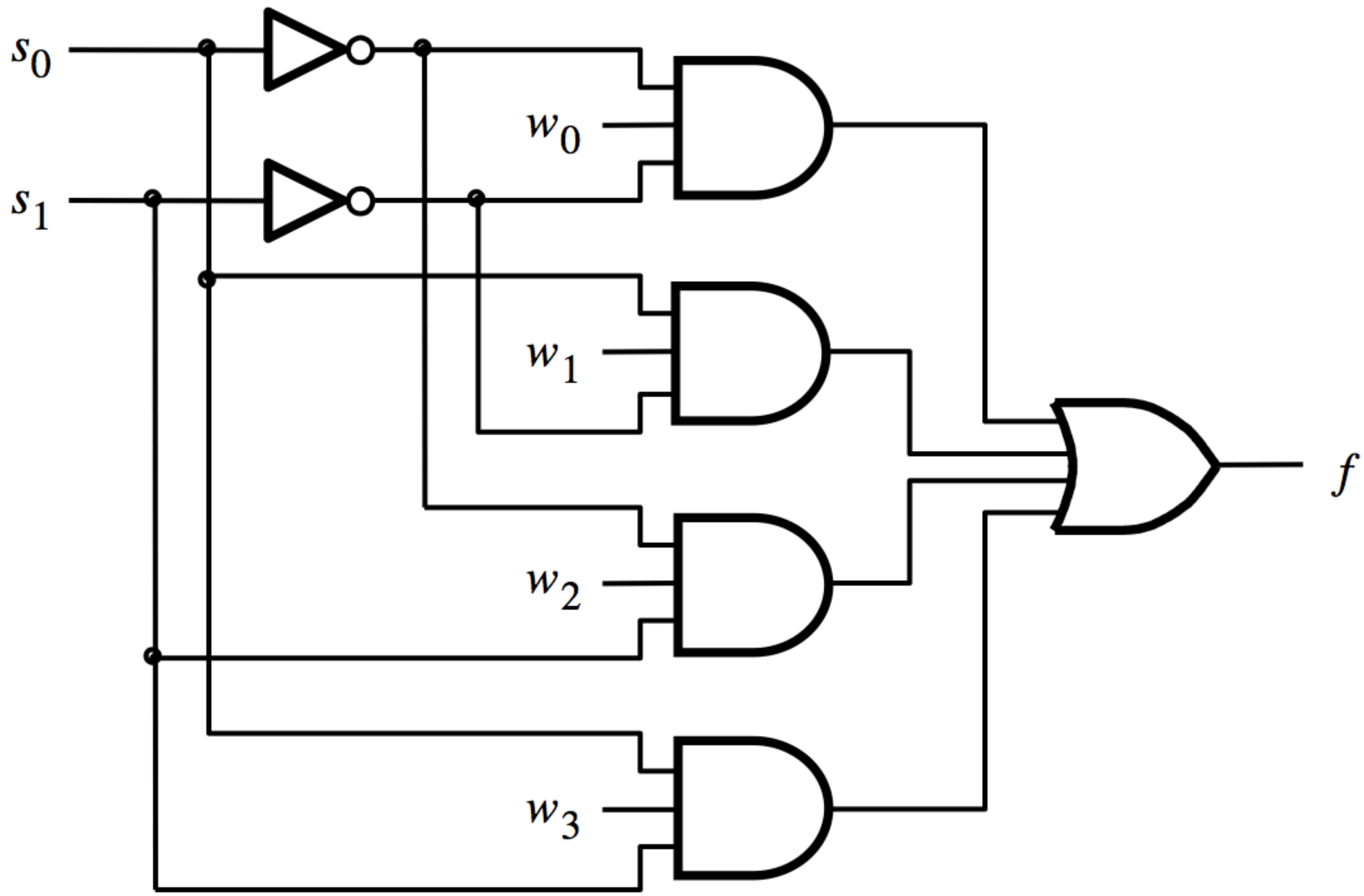
S_1	S_0	I_3	I_2	I_1	I_0	F
0	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	1
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	0
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	0	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	1
		0	1	0	1	1
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	0
		1	0	1	1	0
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	0
		0	1	1	1	0
		1	0	0	0	1
		1	0	0	1	1
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

[http://www.absoluteastronomy.com/topics/Multiplexer]

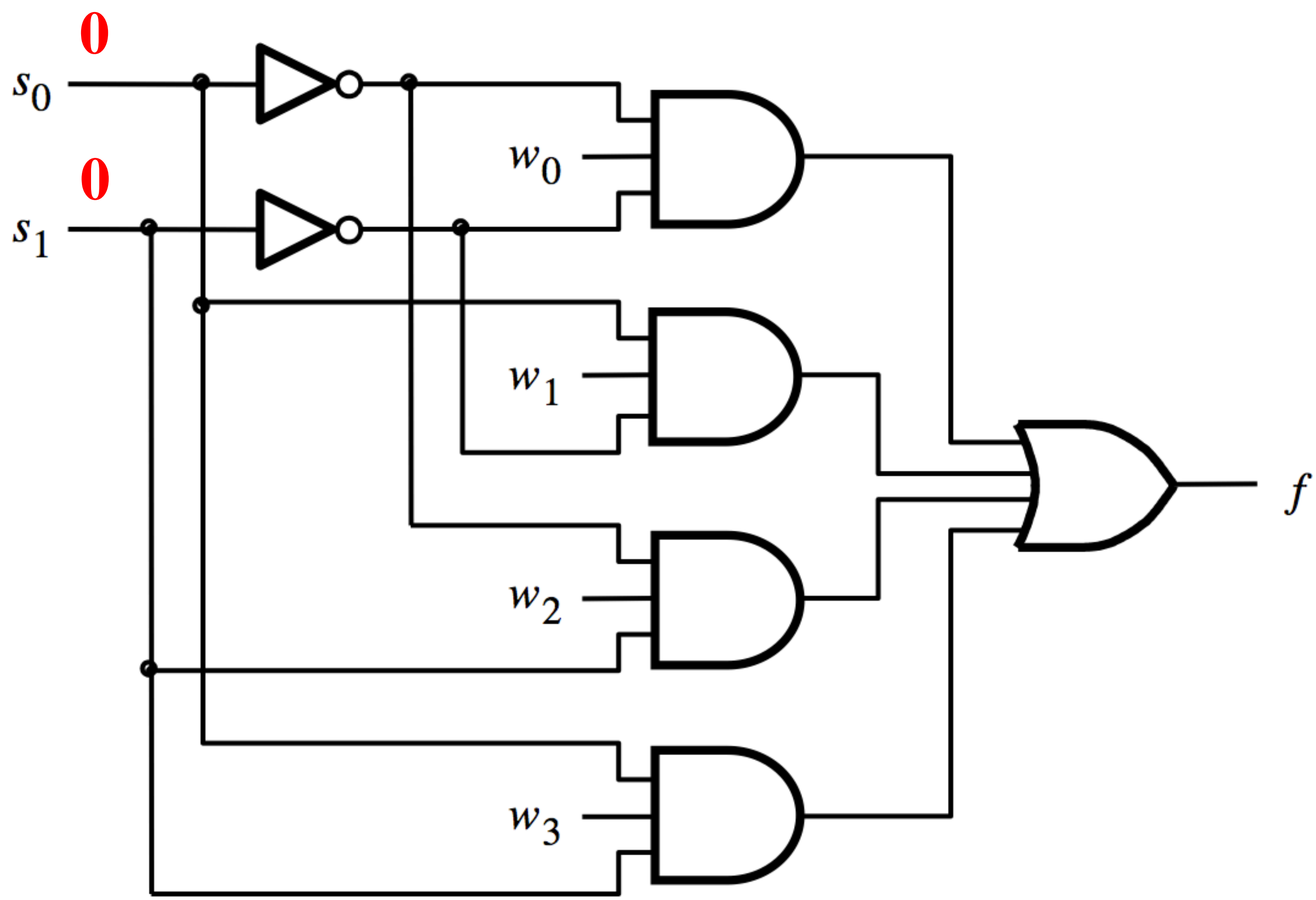
4-to-1 Multiplexer (SOP circuit)



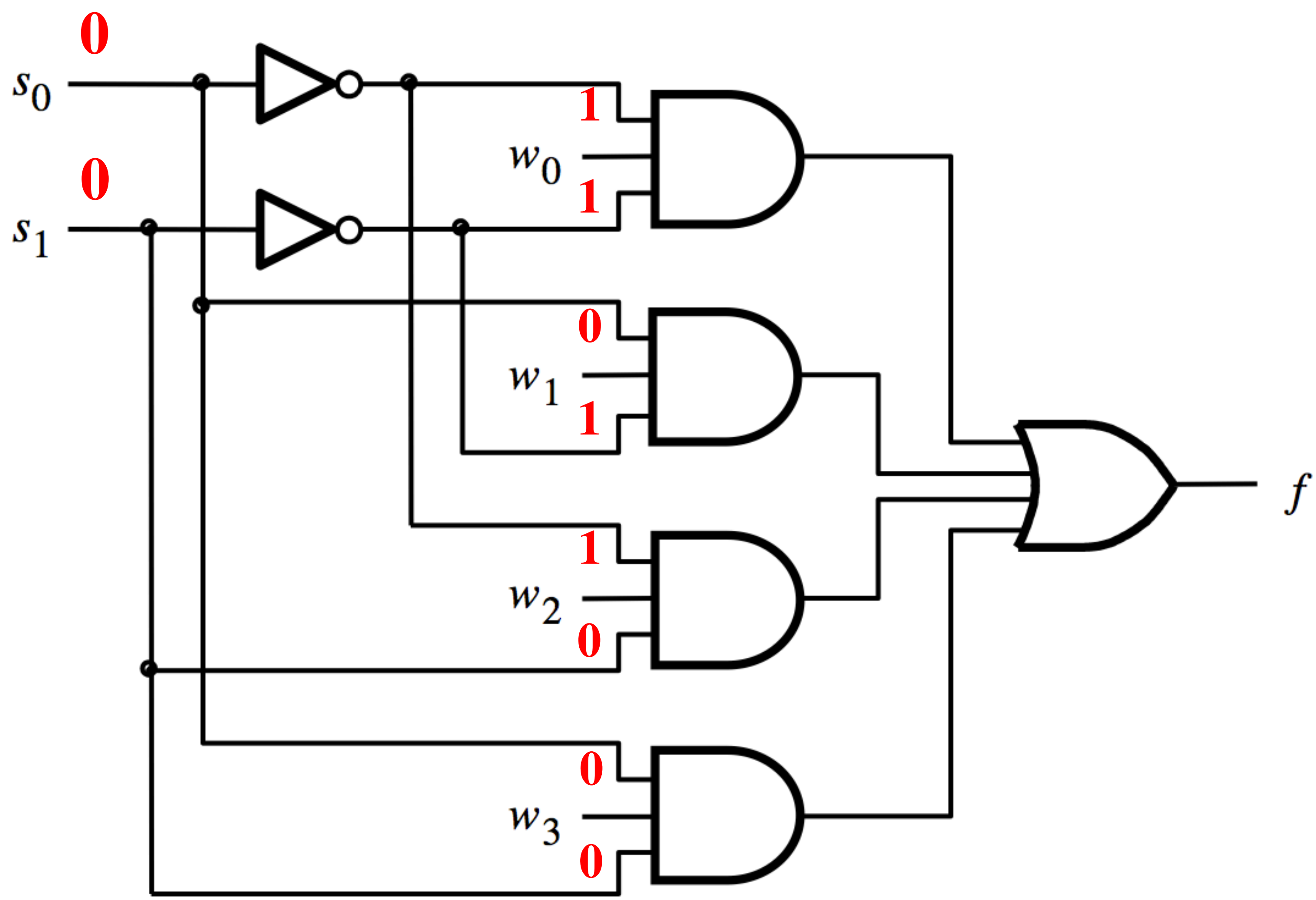
$$f = \overline{s_1}\,\overline{s_0}\,w_0 + \overline{s_1}\,s_0\,w_1 + s_1\,\overline{s_0}\,w_2 + s_1\,s_0\,w_3$$

[Figure 4.2c from the textbook]

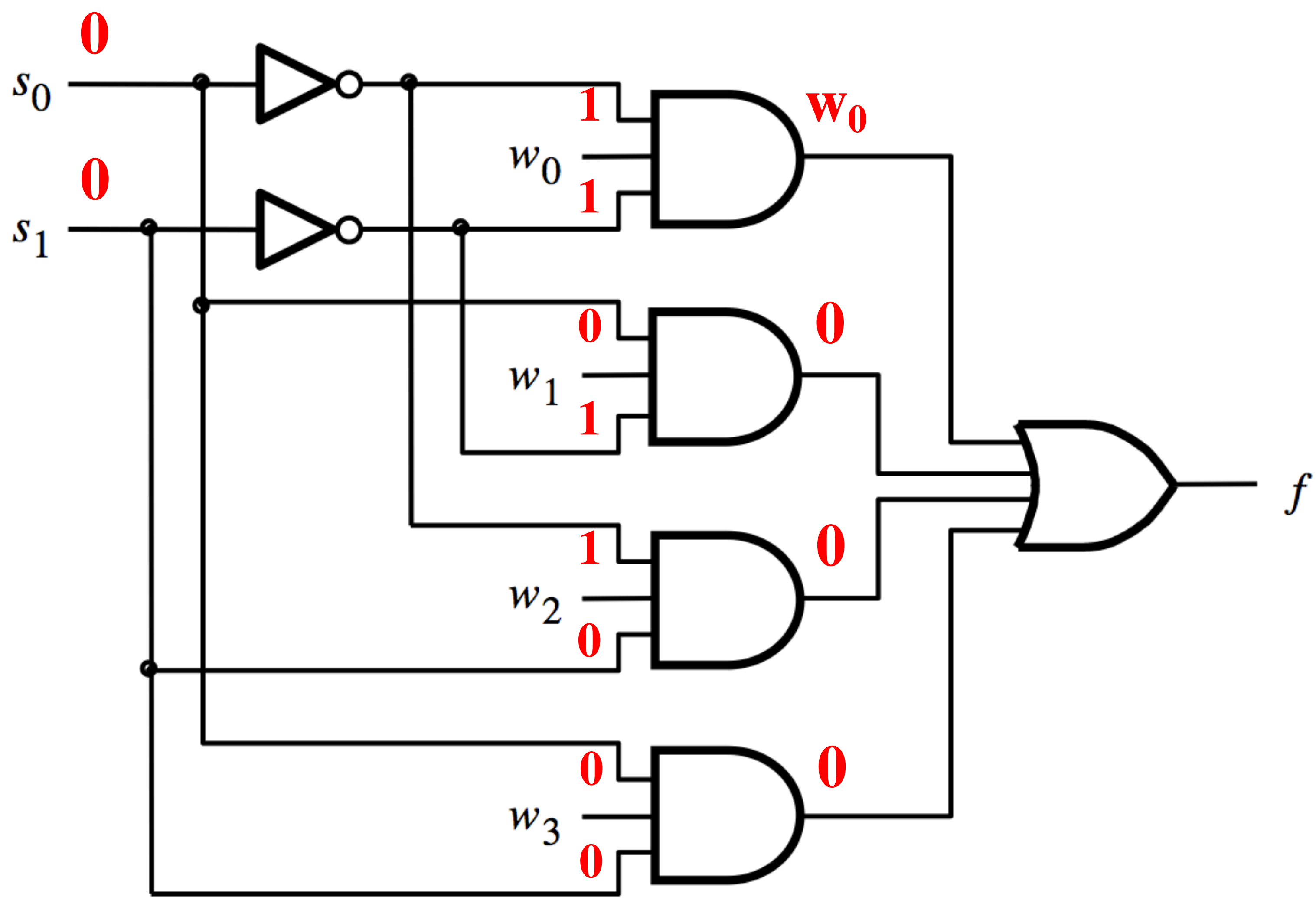
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=0)



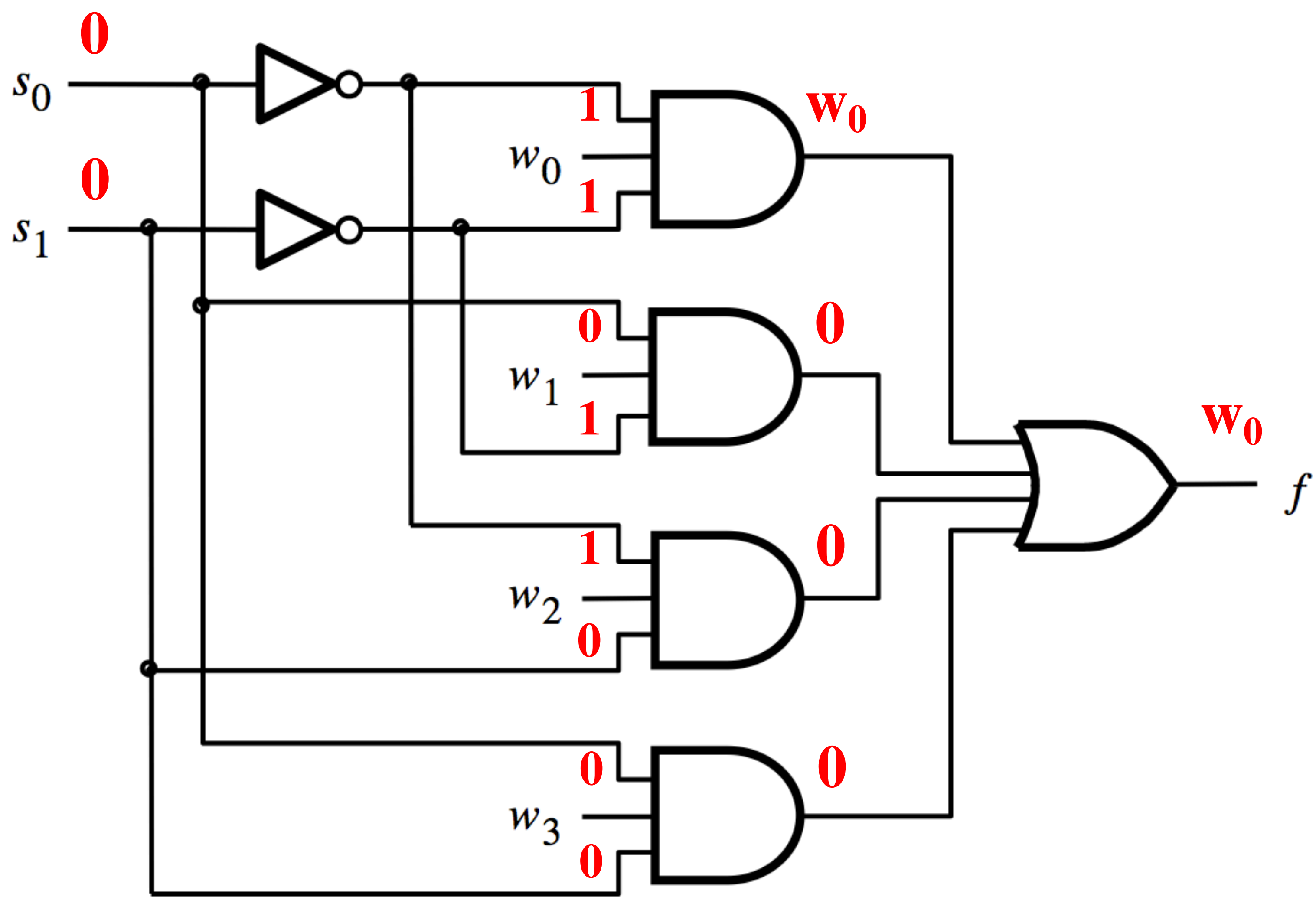
Analysis of the 4-to-1 Multiplexer (s_1=0 and s_0=0)



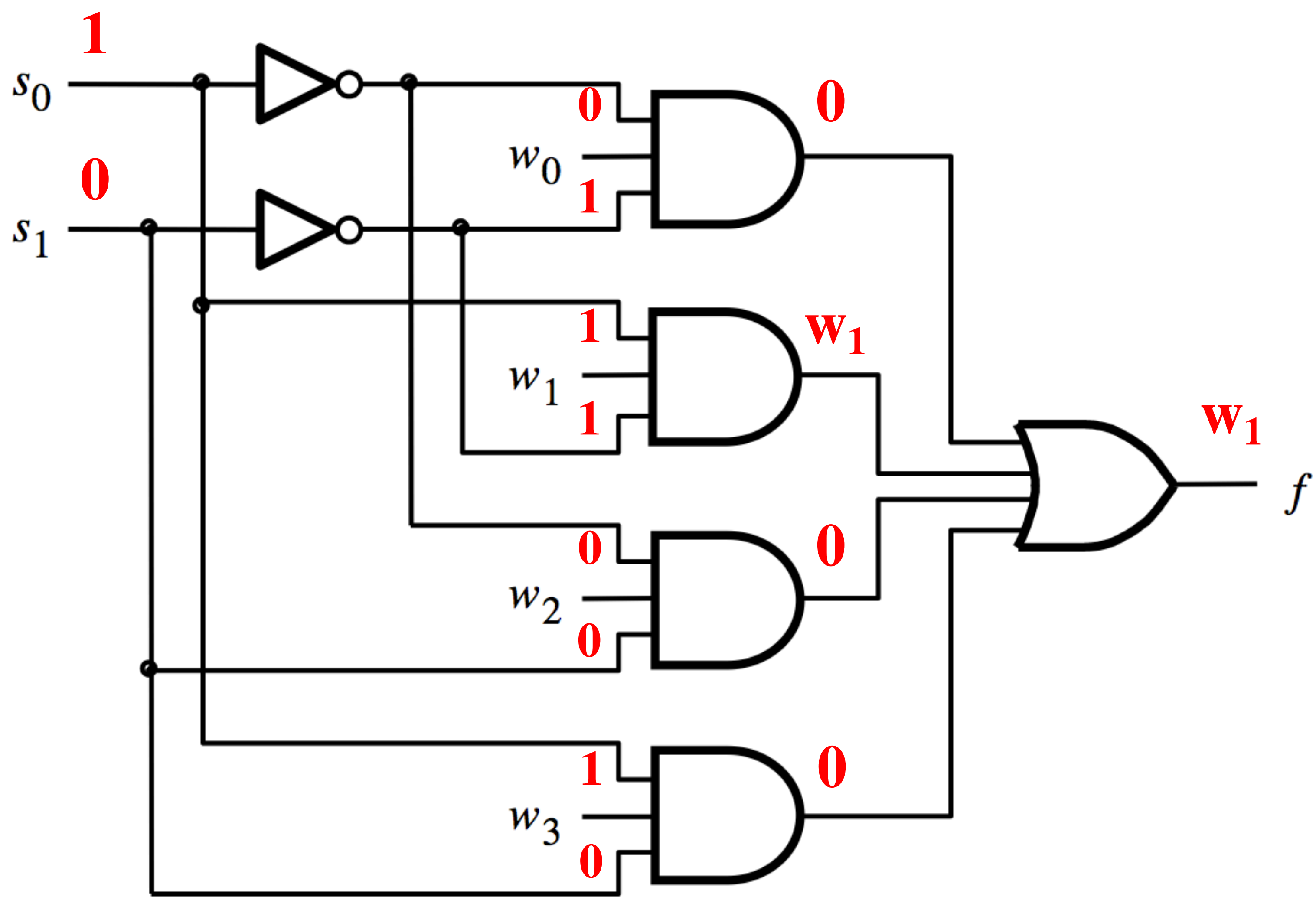
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=0)



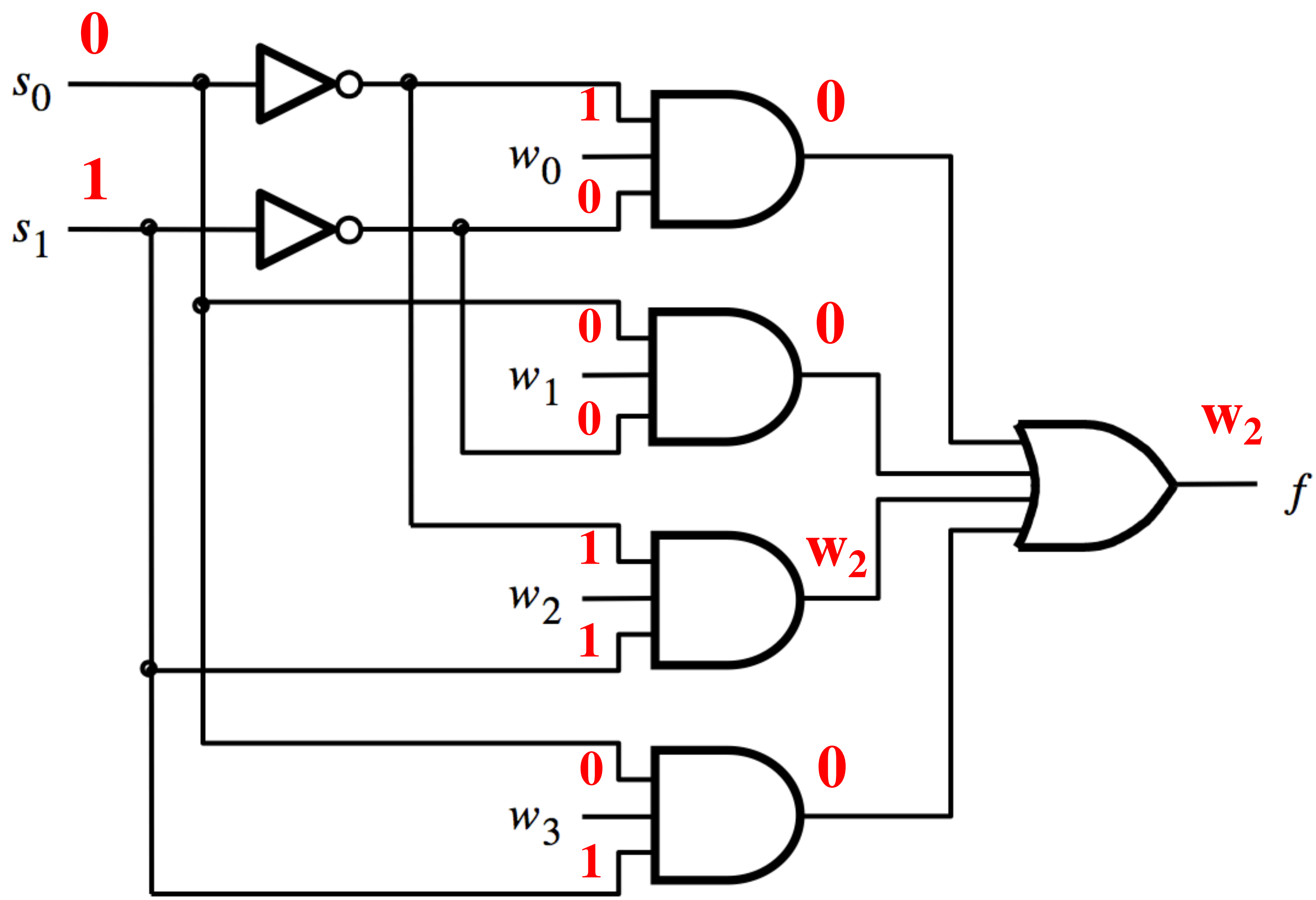
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=0)



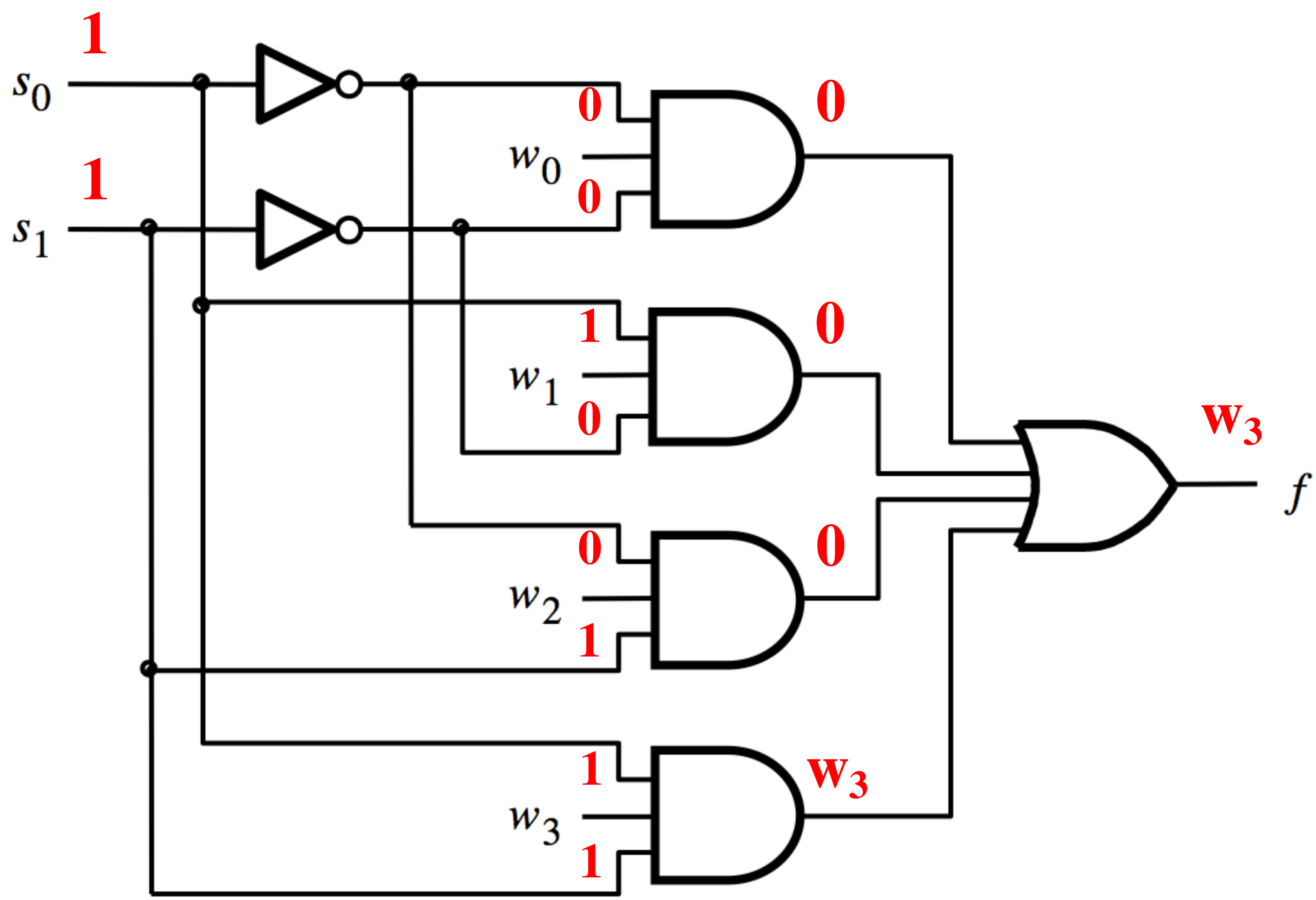
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=1)



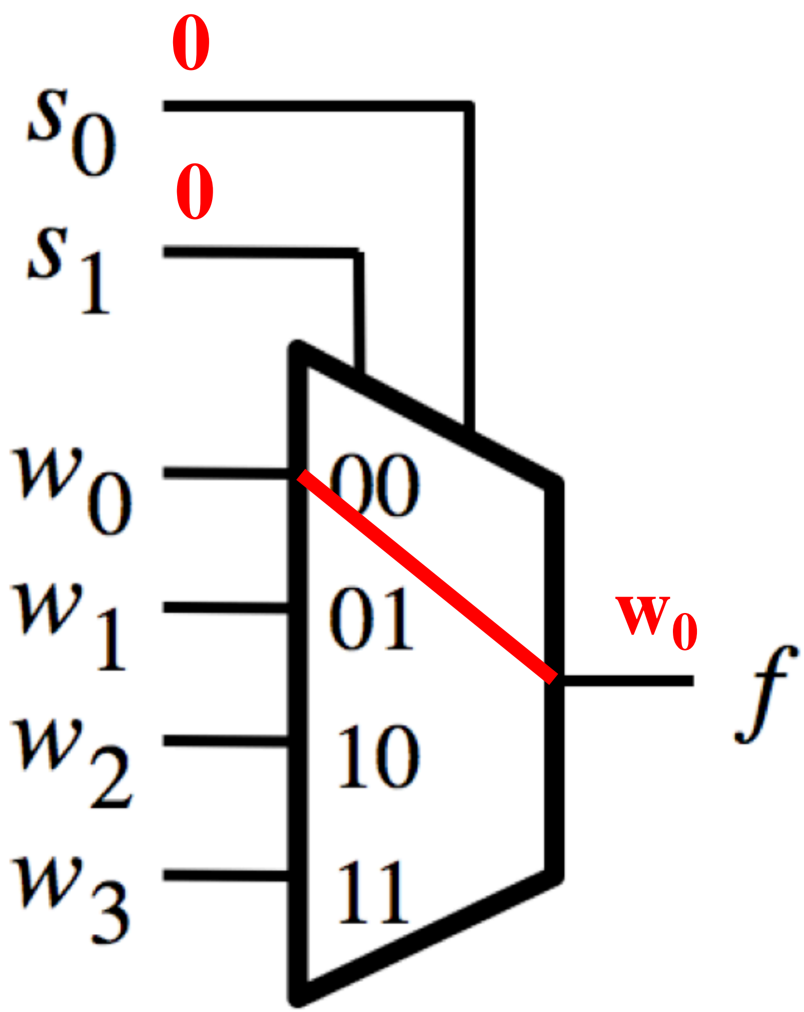
Analysis of the 4-to-1 Multiplexer
(s_1=1 and s_0=0)



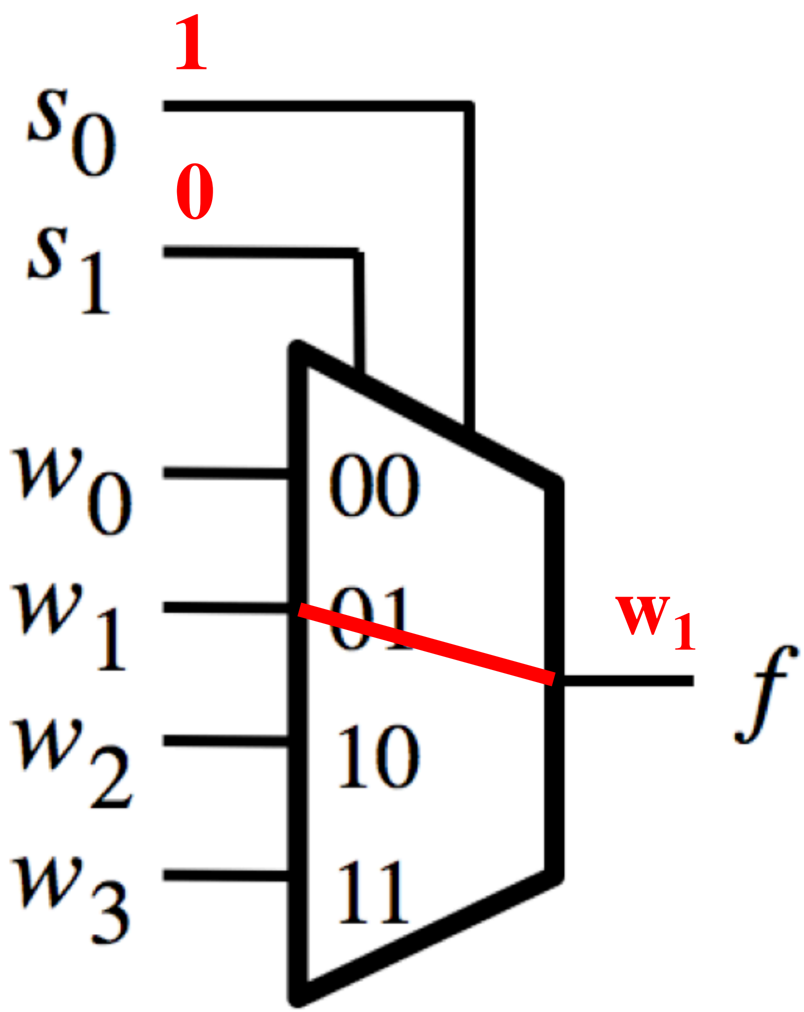
Analysis of the 4-to-1 Multiplexer
(s_1=1 and s_0=1)



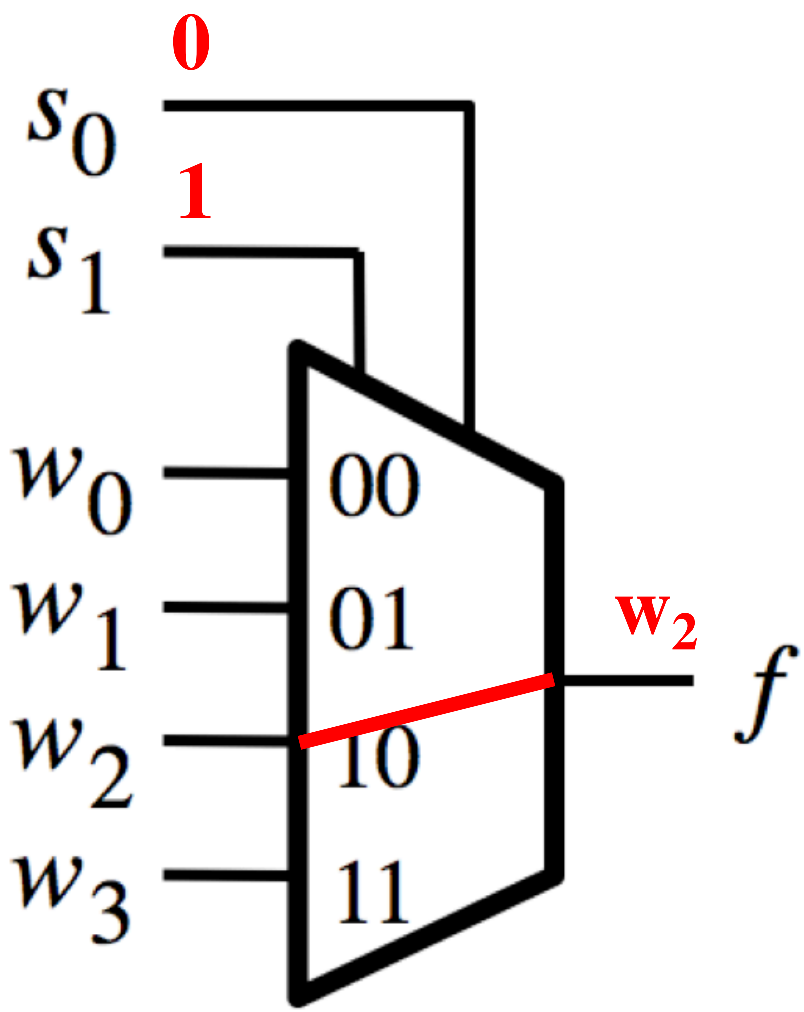
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=0)



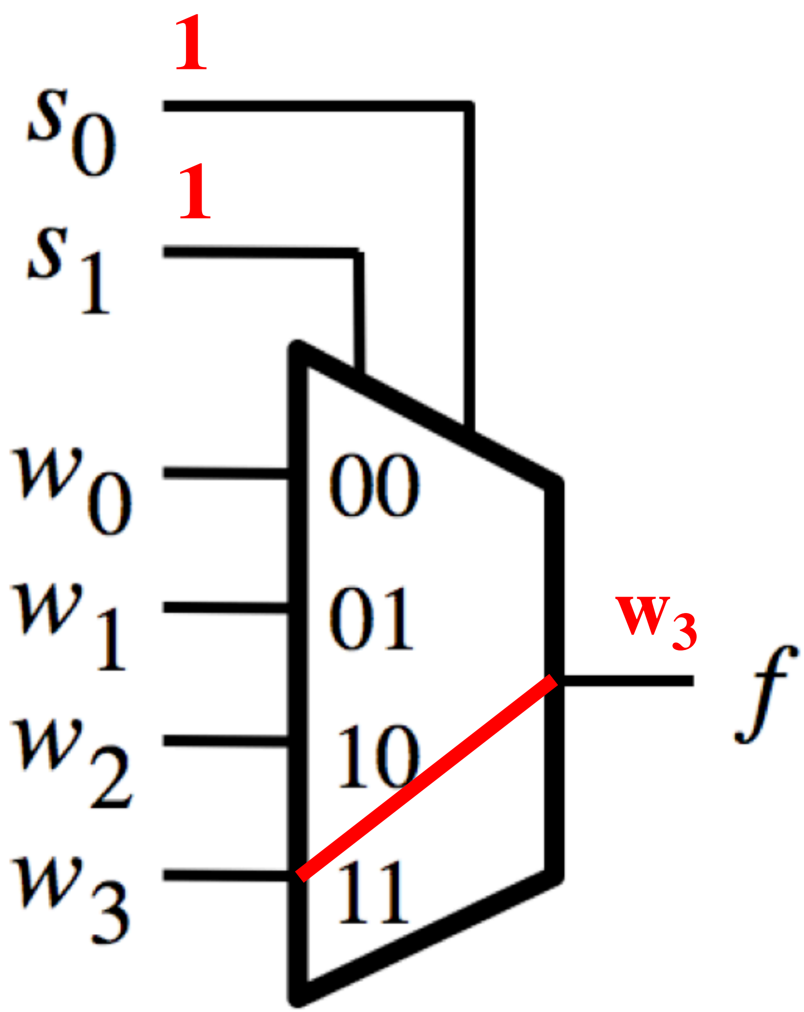
Analysis of the 4-to-1 Multiplexer
(s_1=0 and s_0=1)



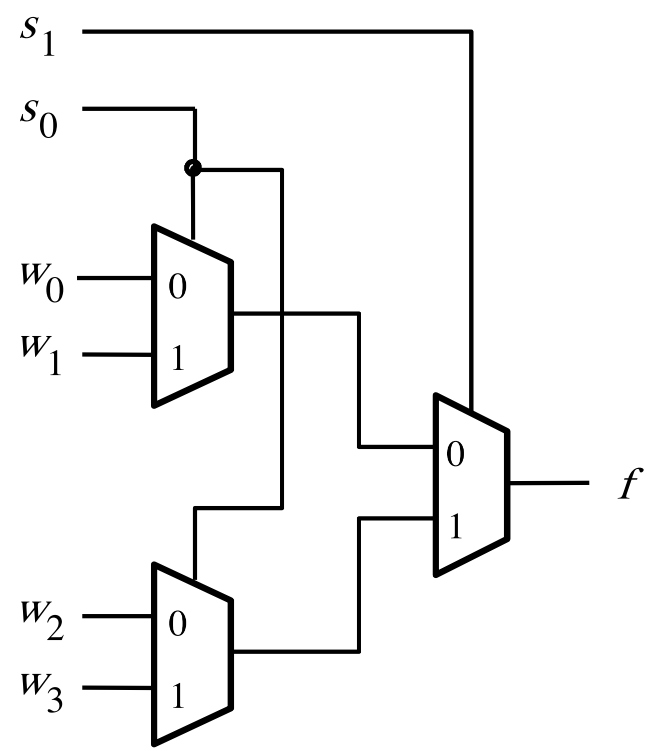
Analysis of the 4-to-1 Multiplexer
(s_1=1 and s_0=0)



Analysis of the 4-to-1 Multiplexer
(s_1=1 and s_0=1)

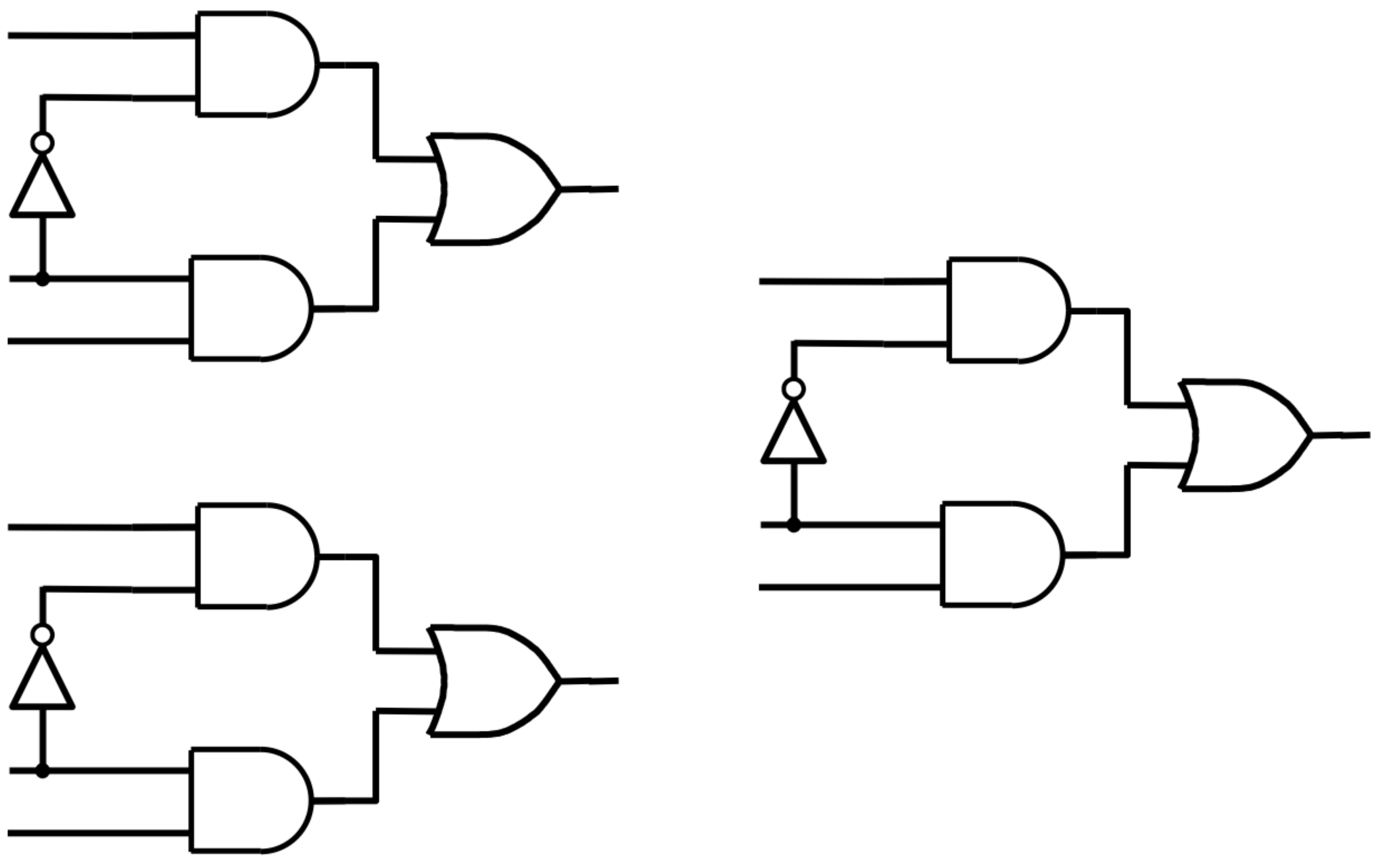


Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer

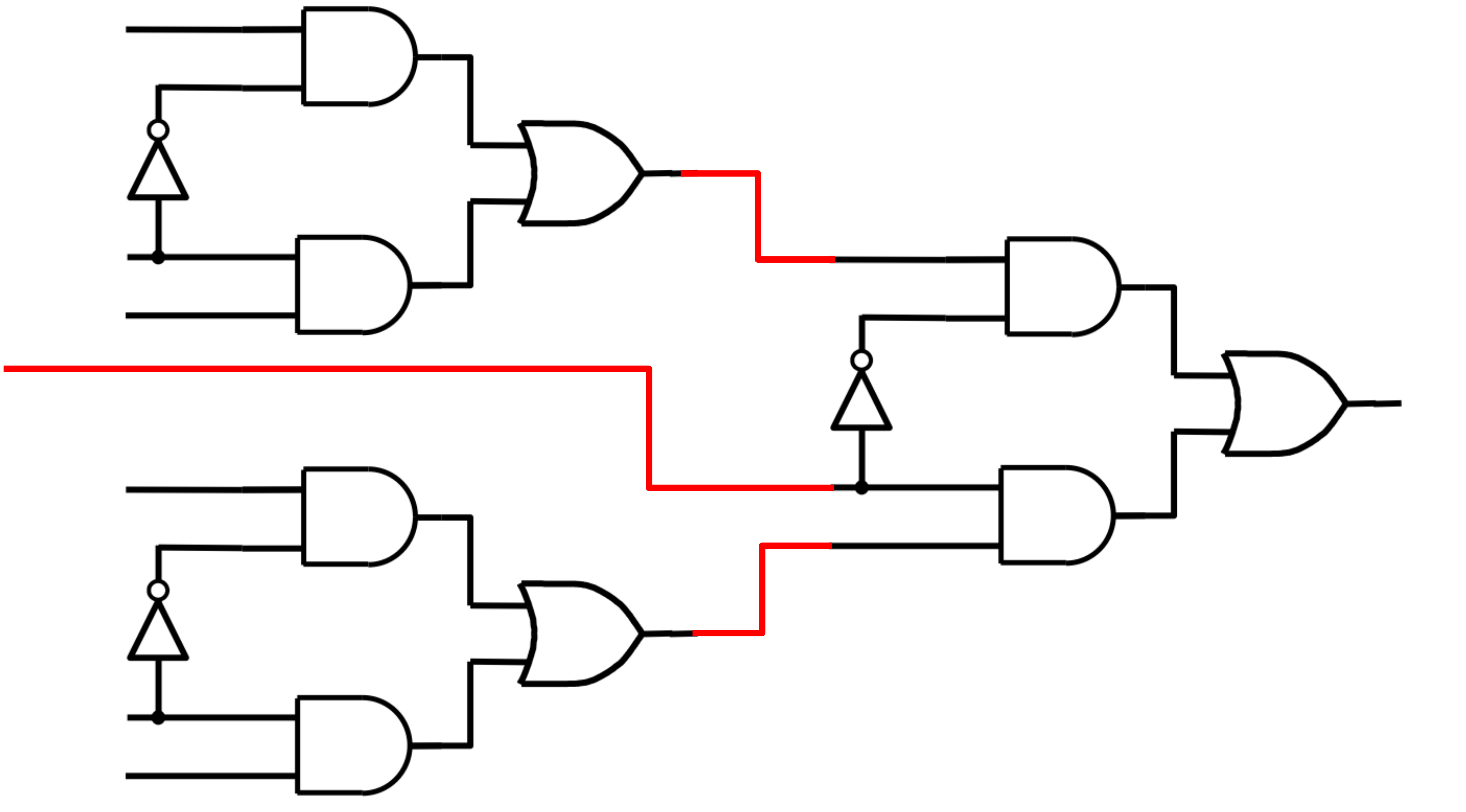


[Figure 4.3 from the textbook]

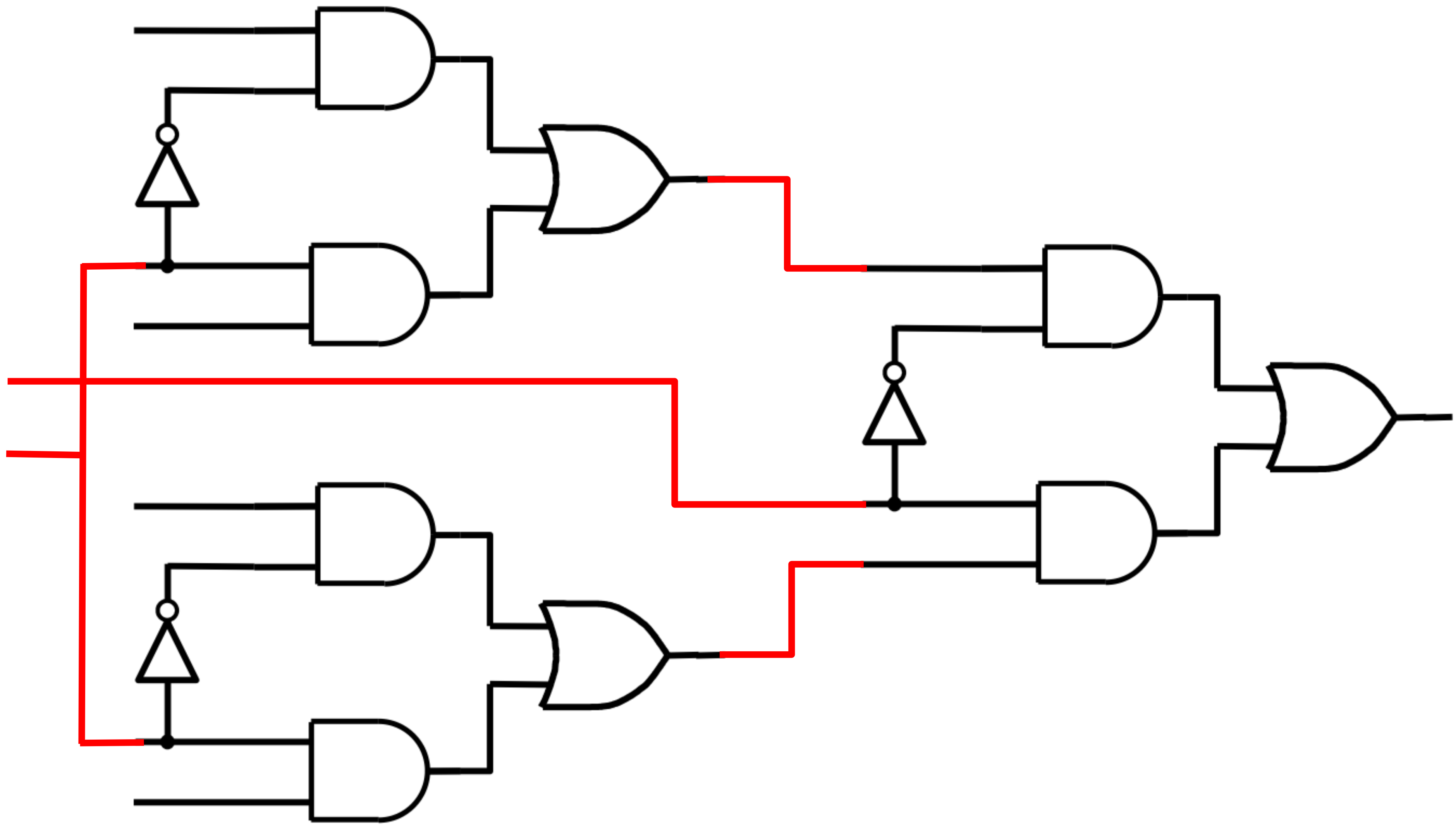
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



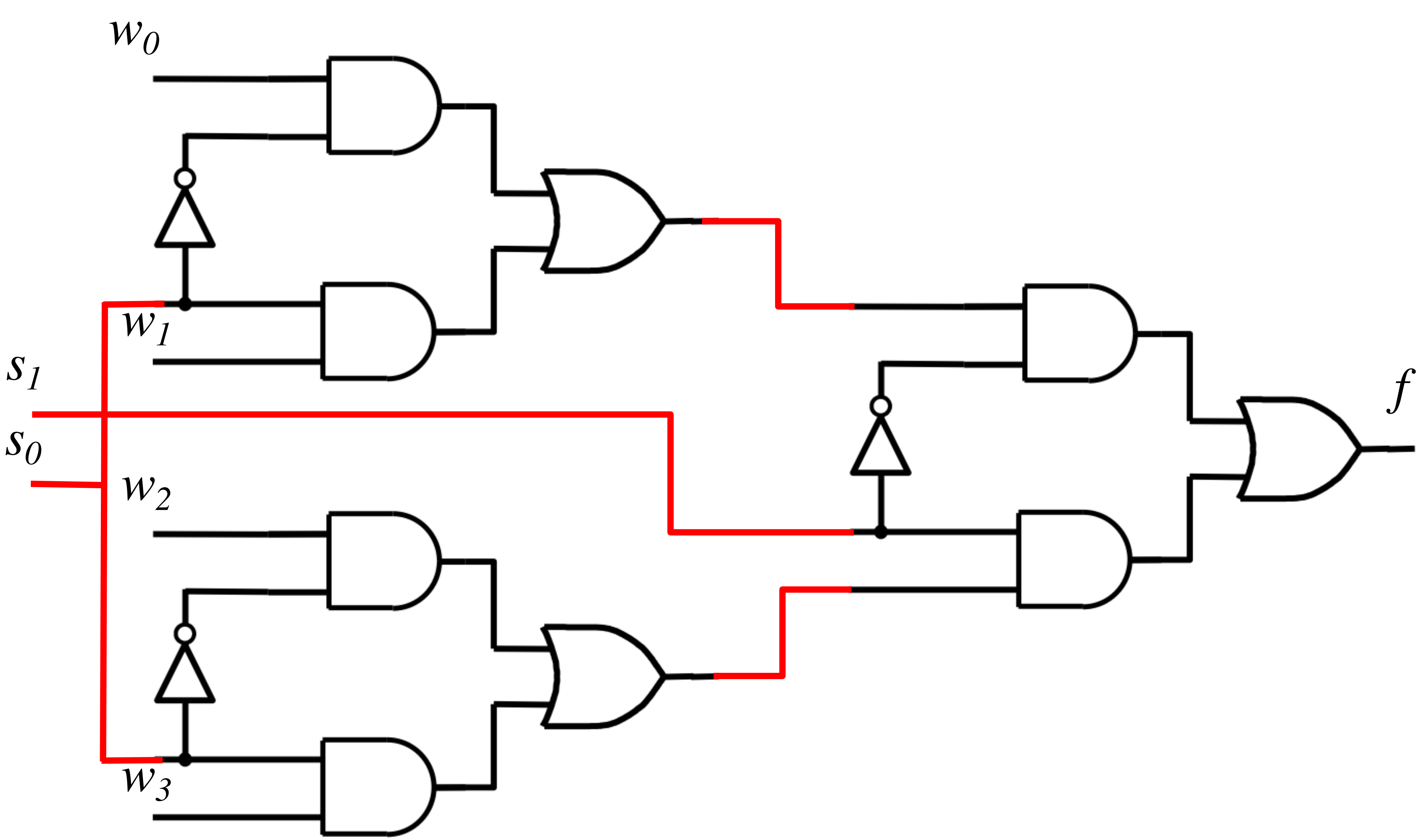
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



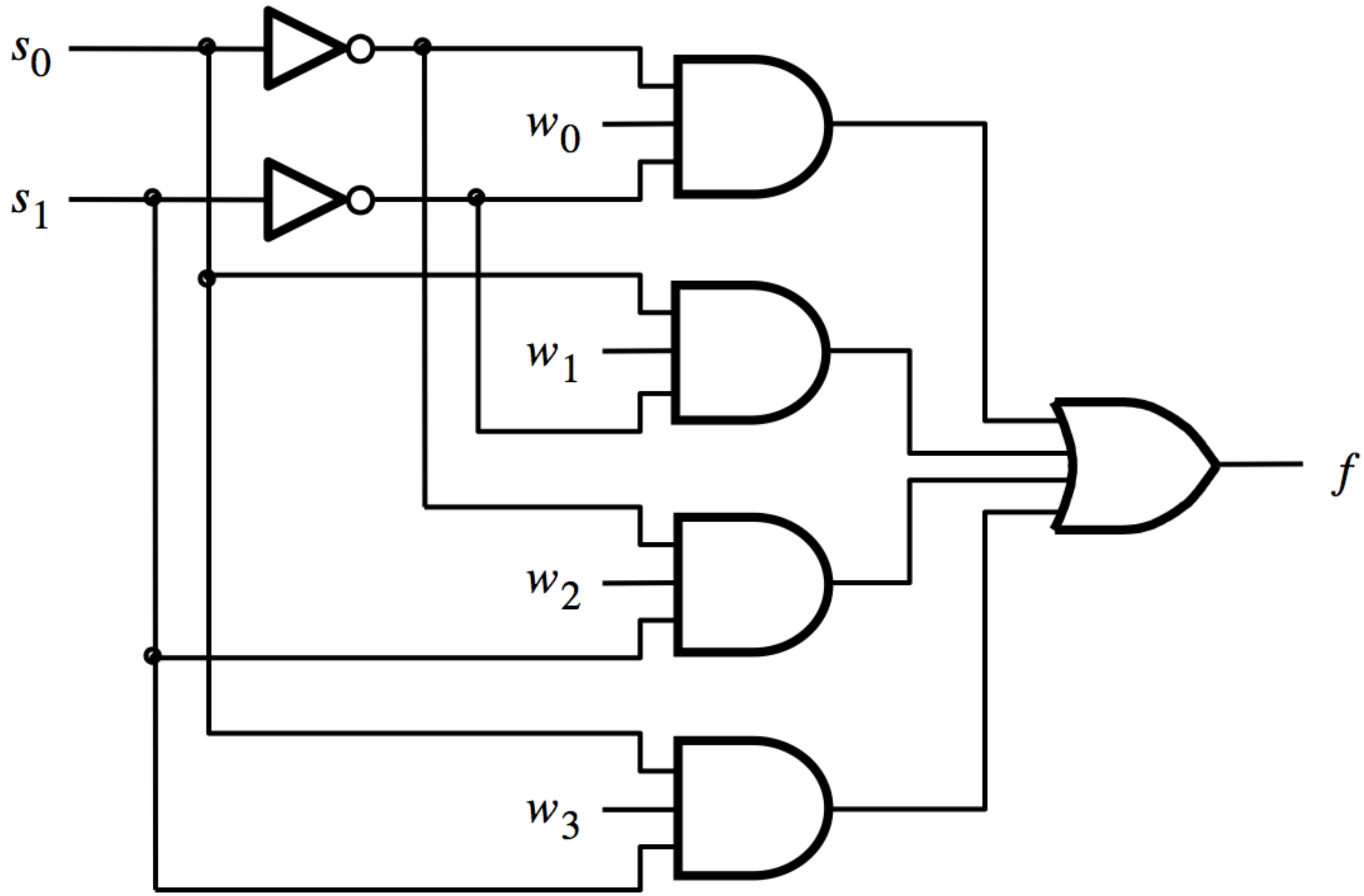
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



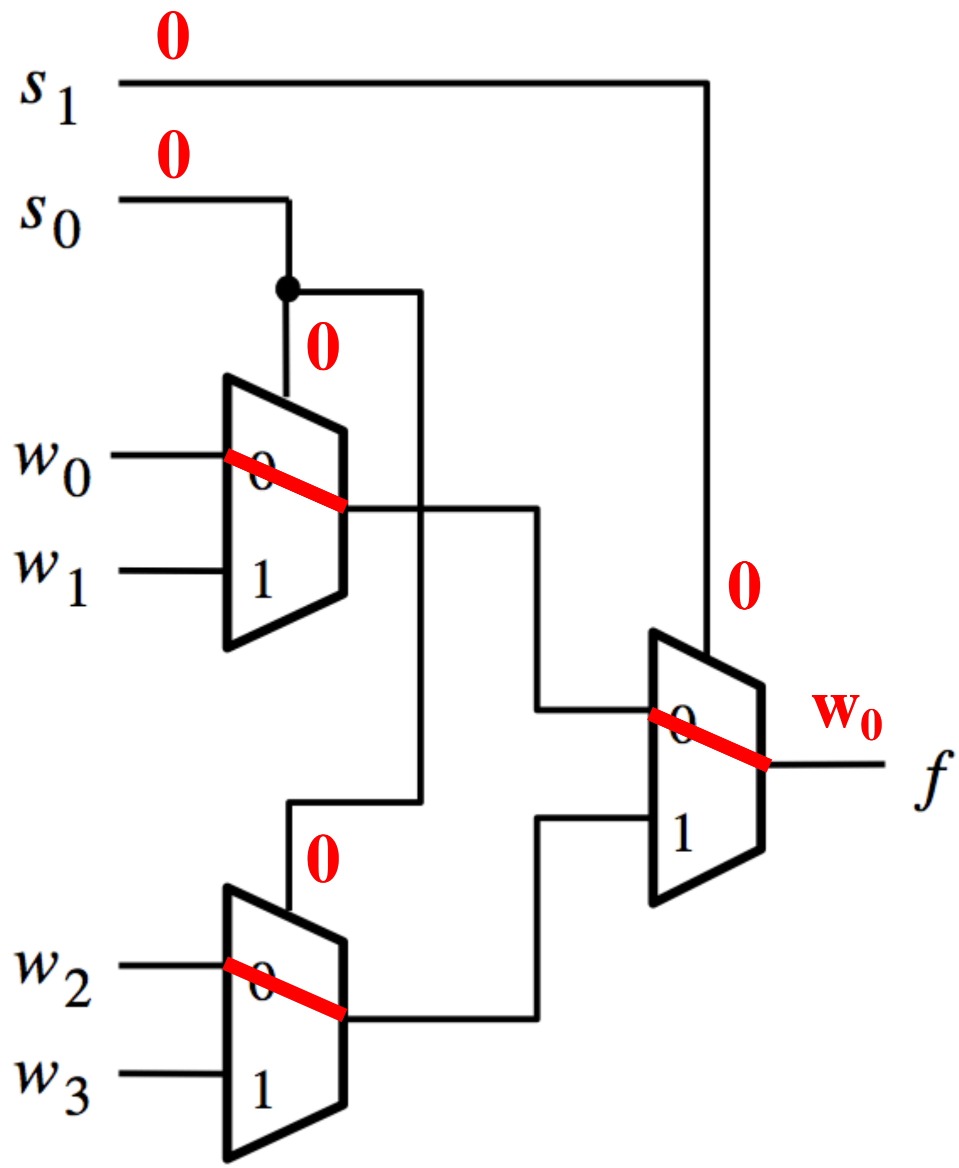
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



That is different from the SOP form of the 4-to-1 multiplexer shown below

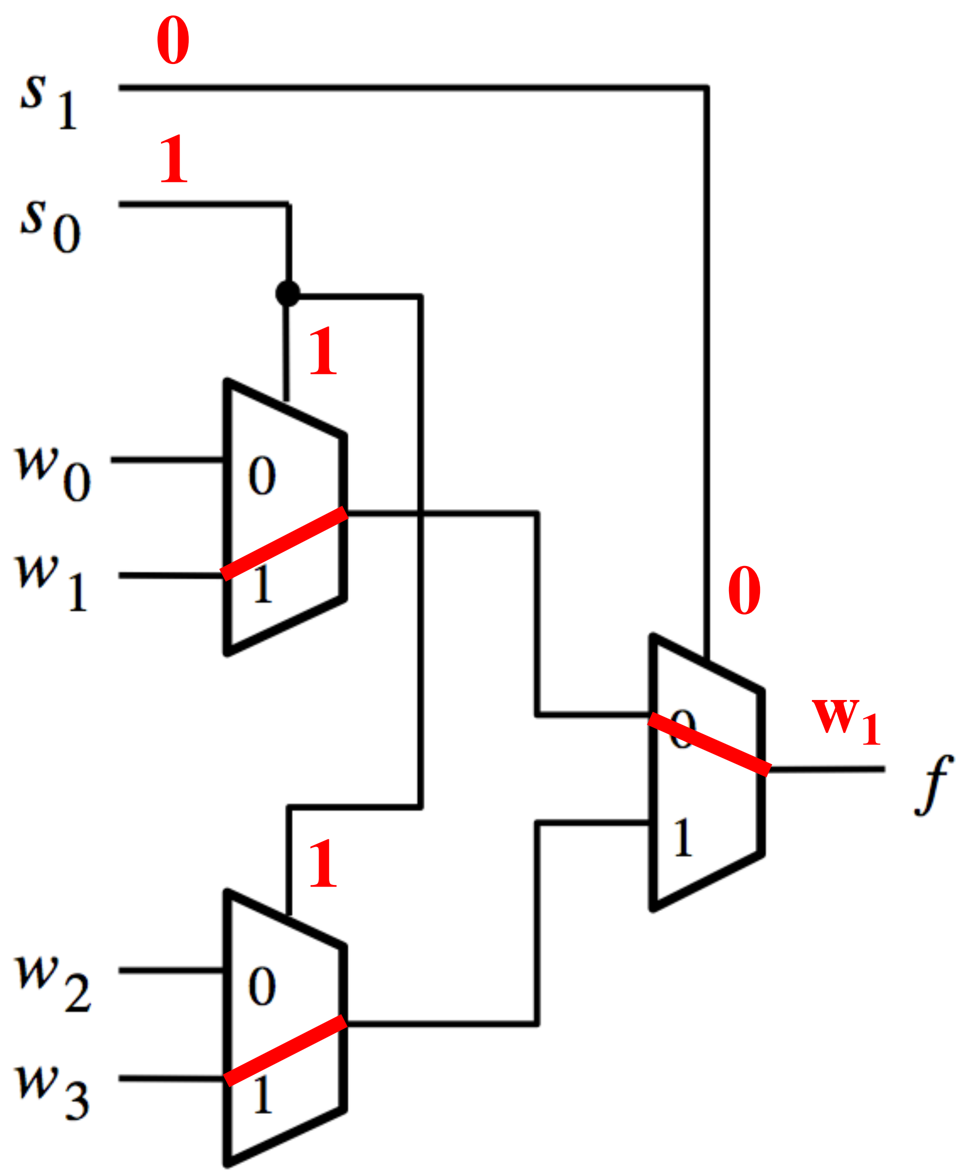


Analysis of the Hierarchical Implementation
(s_1=0 and s_0=0)



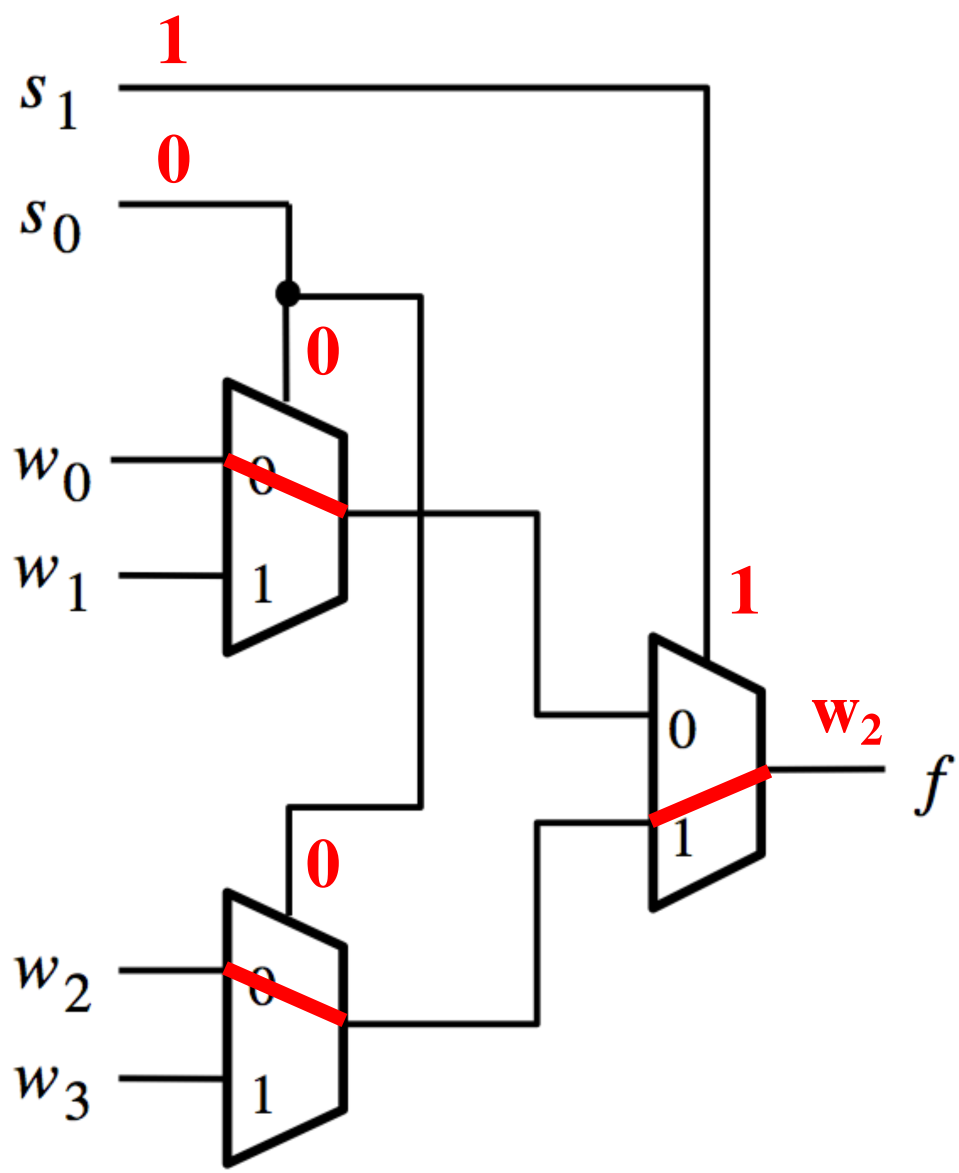
[Figure 4.3 from the textbook]

Analysis of the Hierarchical Implementation
(s_1=0 and s_0=1)



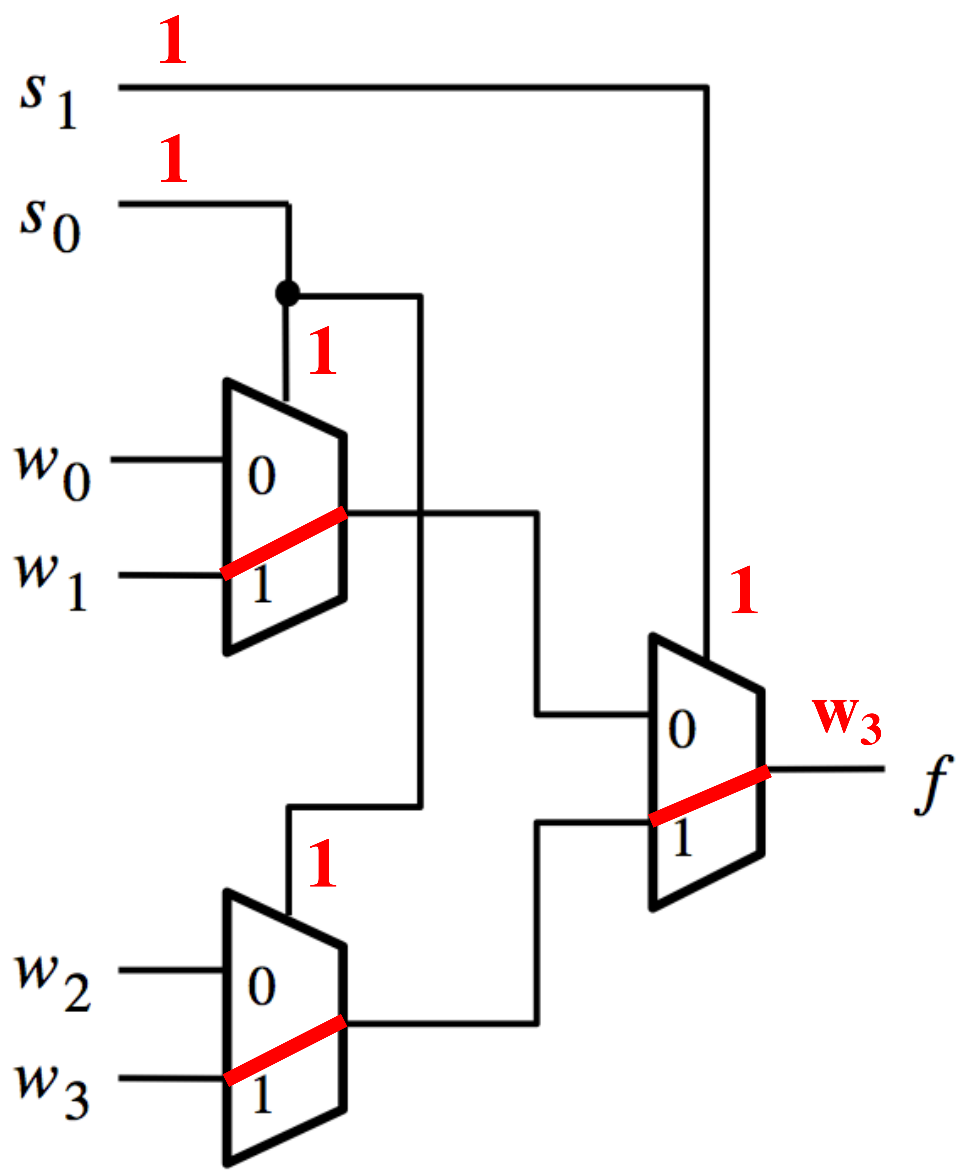
[Figure 4.3 from the textbook]

Analysis of the Hierarchical Implementation (s_1=1 and s_0=0)



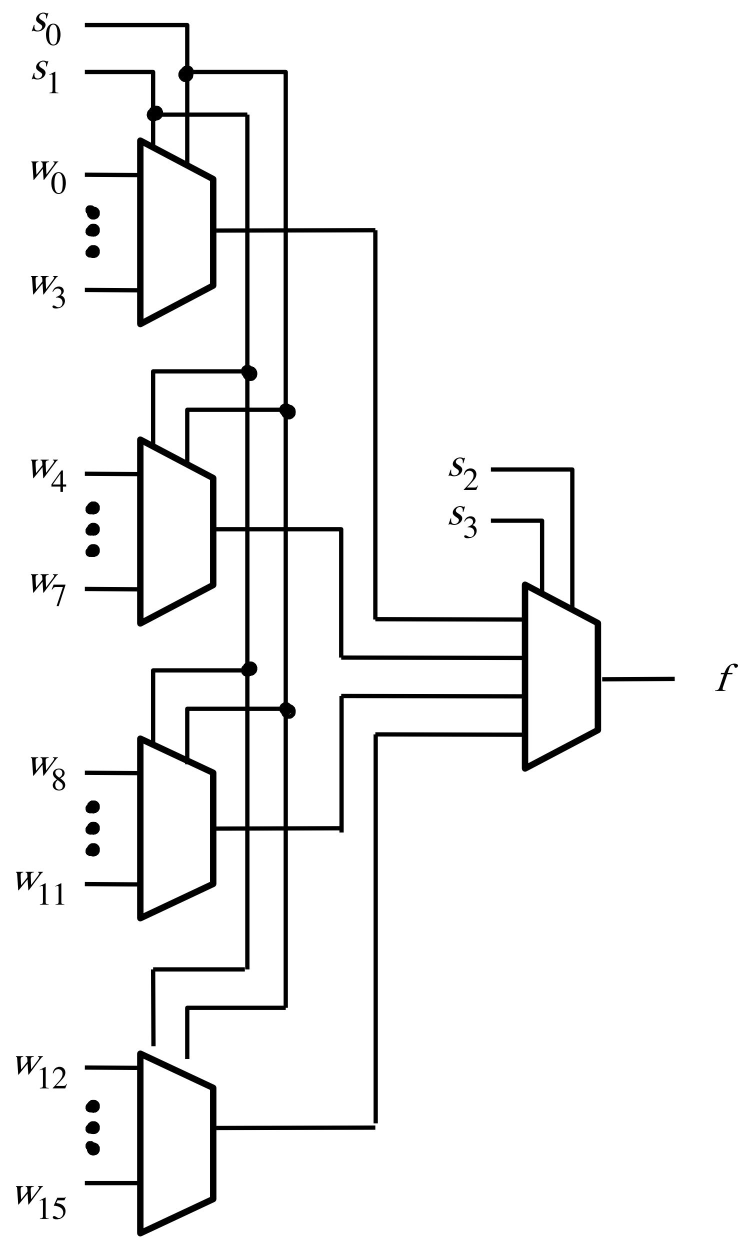
[Figure 4.3 from the textbook]

Analysis of the Hierarchical Implementation (s_1=1 and s_0=1)



[Figure 4.3 from the textbook]

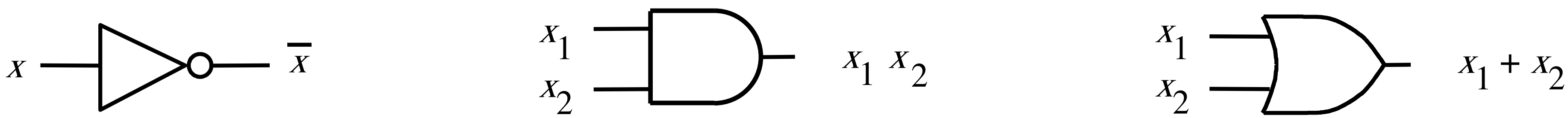
16-1 Multiplexer



[Figure 4.4 from the textbook]

Multiplexers Are Special

The Three Basic Logic Gates



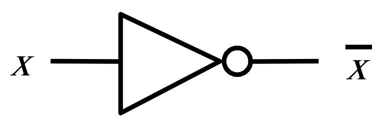
NOT gate

AND gate

OR gate

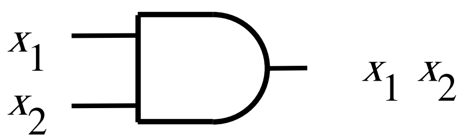
[Figure 2.8 from the textbook]

Truth Table for NOT



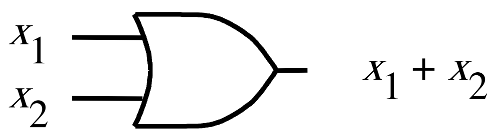
x	$\overline{x}$
0	1
1	0

Truth Table for AND



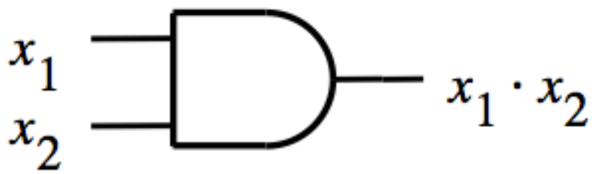
x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Truth Table for OR

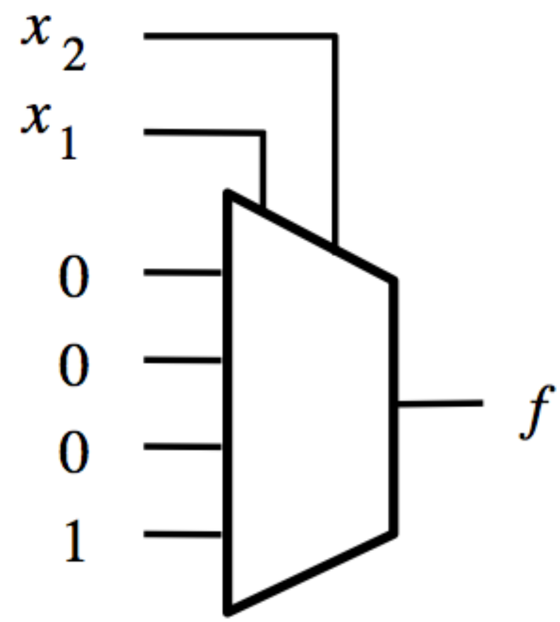


x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

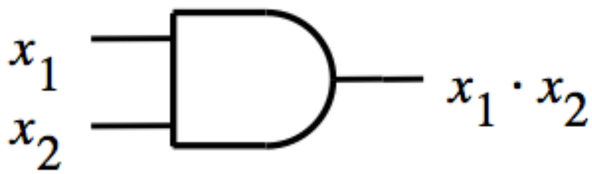
Building an AND Gate with 4-to-1 Mux



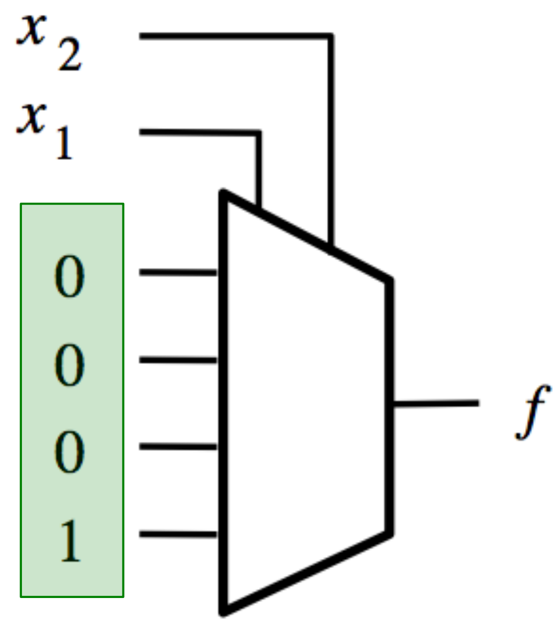
x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1



Building an AND Gate with 4-to-1 Mux

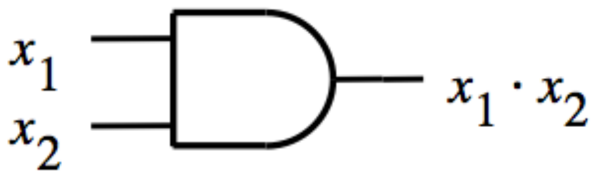


x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

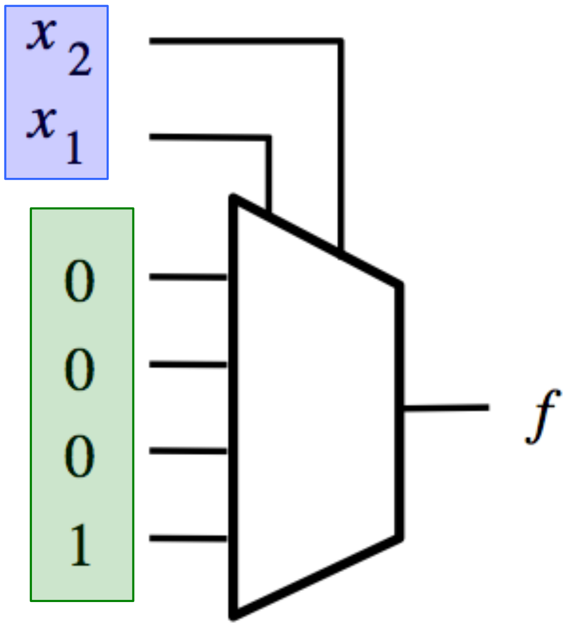


These two are the same.

Building an AND Gate with 4-to-1 Mux



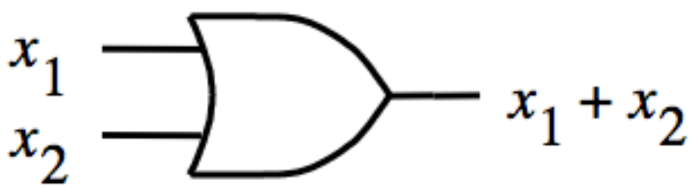
x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1



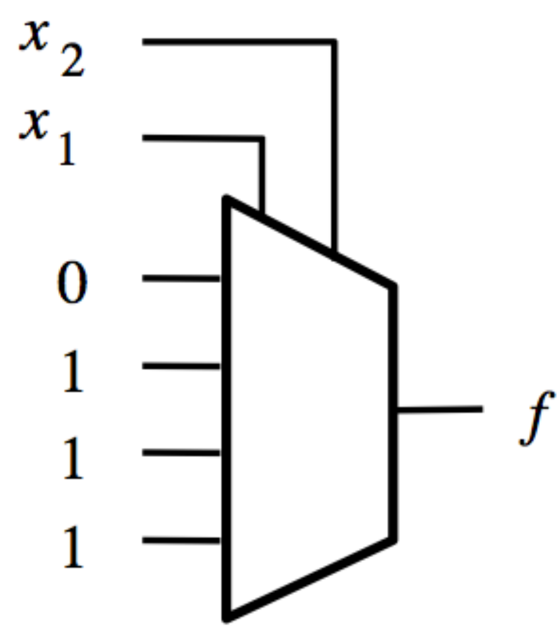
These two are the same.

And so are these two.

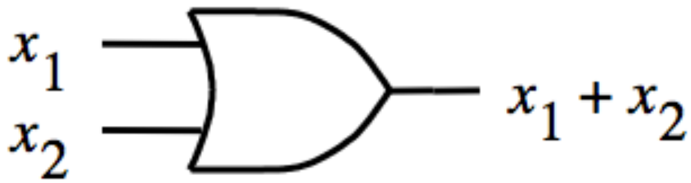
Building an OR Gate with 4-to-1 Mux



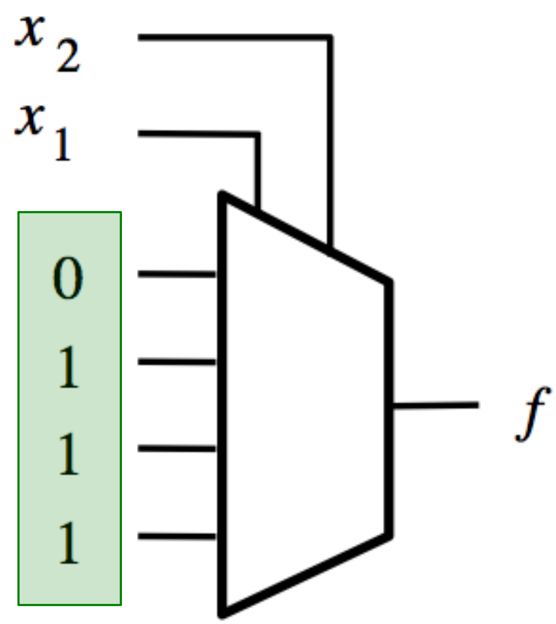
x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1



Building an OR Gate with 4-to-1 Mux

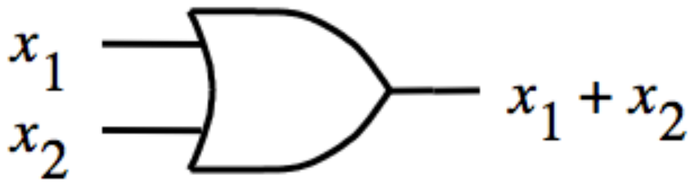


x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1

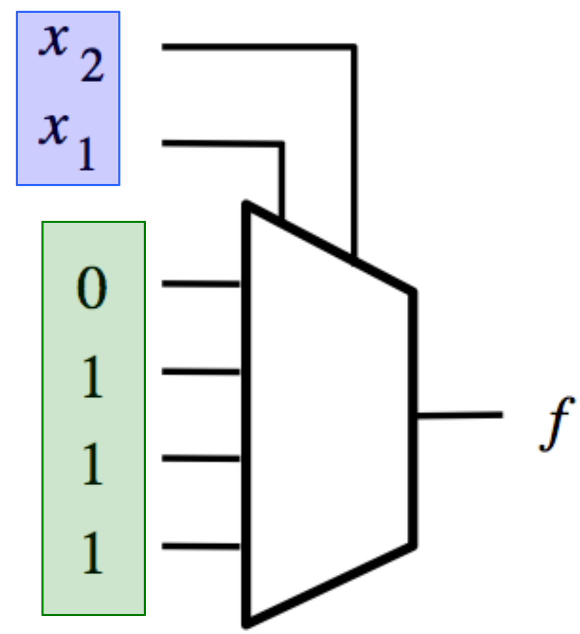


These two are the same.

Building an OR Gate with 4-to-1 Mux



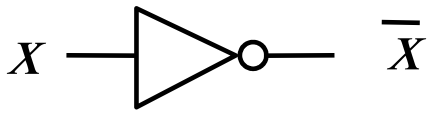
x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1



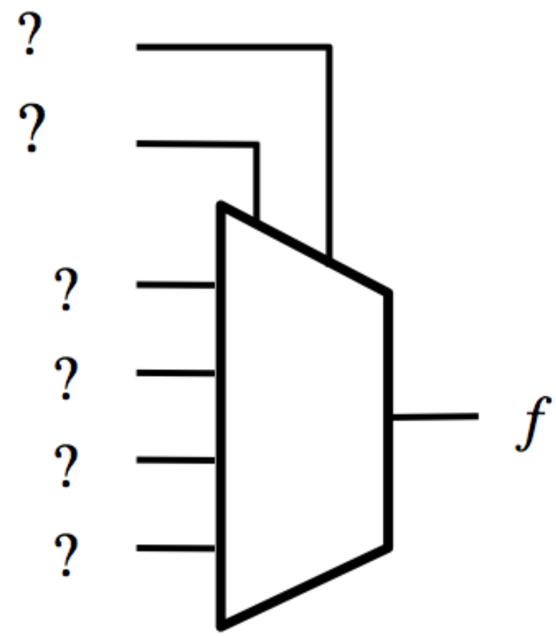
These two are the same.

And so are these two.

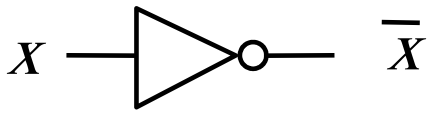
Building a NOT Gate with 4-to-1 Mux



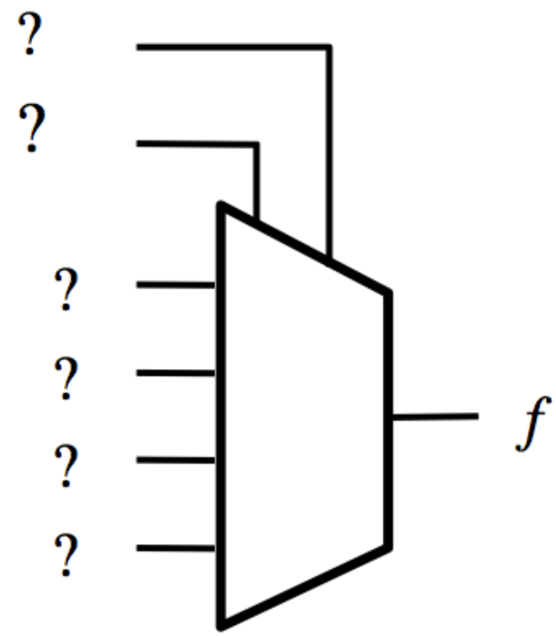
x	$\overline{x}$
0	1
1	0



Building a NOT Gate with 4-to-1 Mux

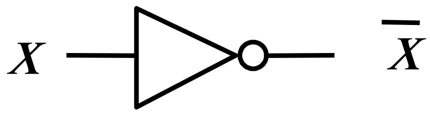


x	y	f
0	0	1
0	1	1
1	0	0
1	1	0

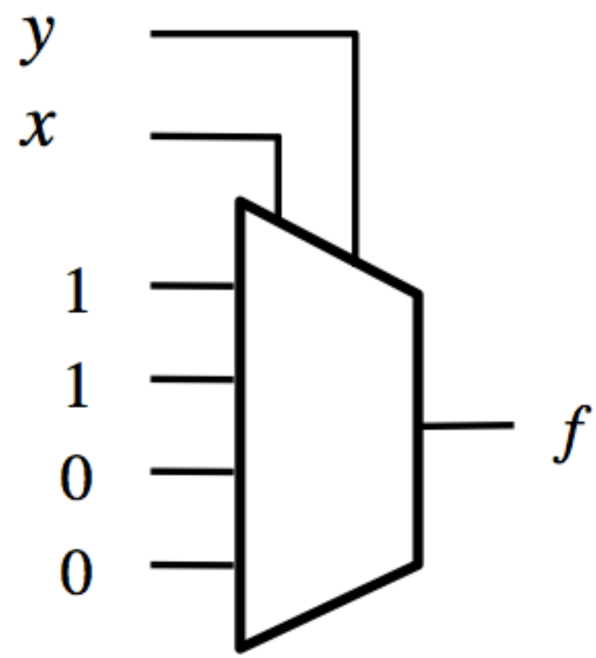


Introduce a dummy variable y.

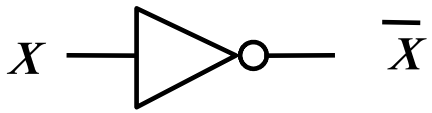
Building a NOT Gate with 4-to-1 Mux



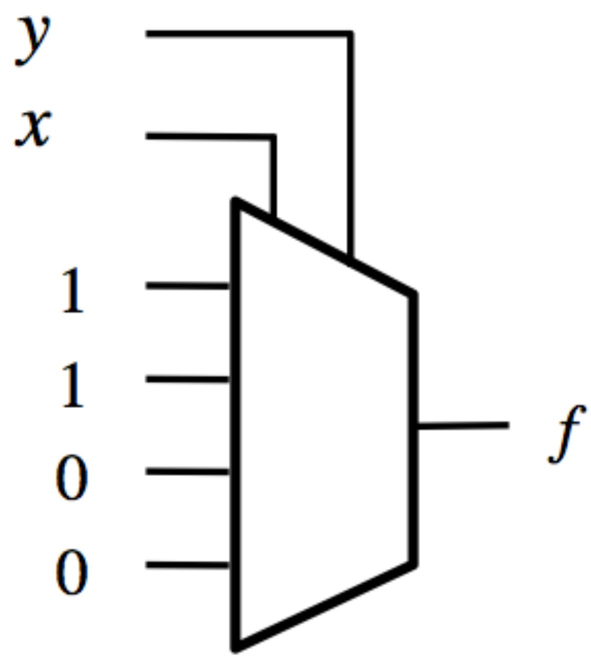
x	y	f
0	0	1
0	1	1
1	0	0
1	1	0



Building a NOT Gate with 4-to-1 Mux

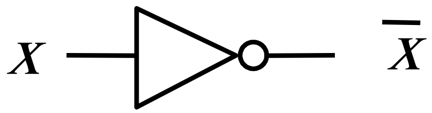


x	y	f
0	0	1
0	1	1
1	0	0
1	1	0

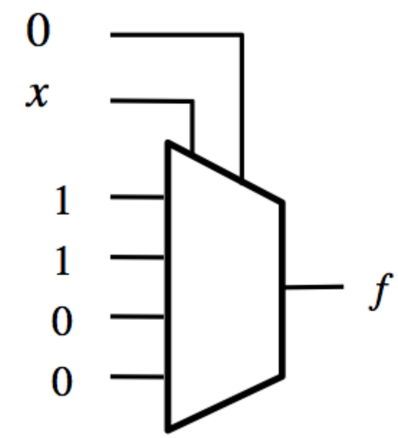


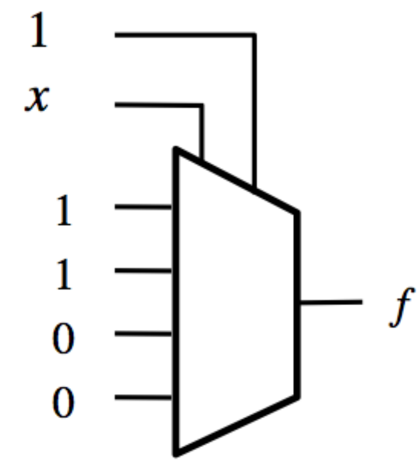
Now set y to either 0 or 1 (both will work). Why?

Building a NOT Gate with 4-to-1 Mux



x	$\overline{x}$
0	1
1	0



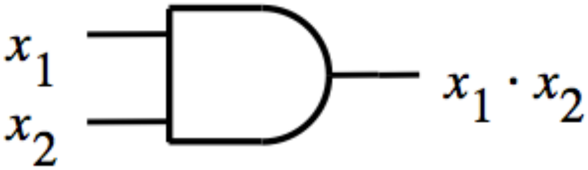


Two alternative solutions.

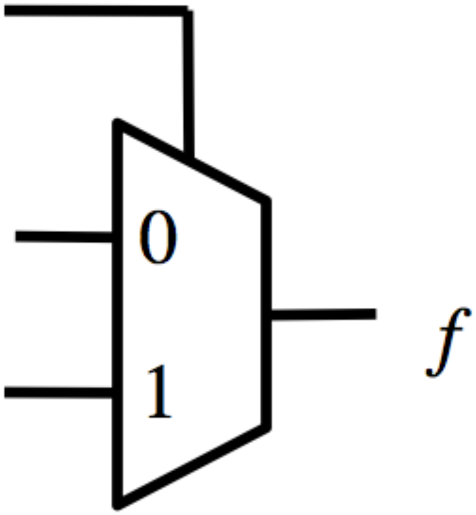
Implications

Any Boolean function can be implemented using only 4-to-1 multiplexers!

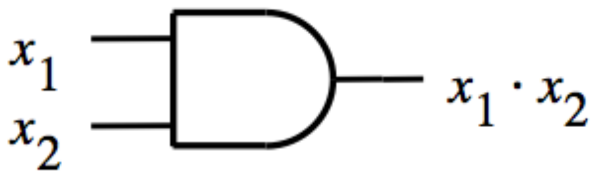
Building an AND Gate with 2-to-1 Mux



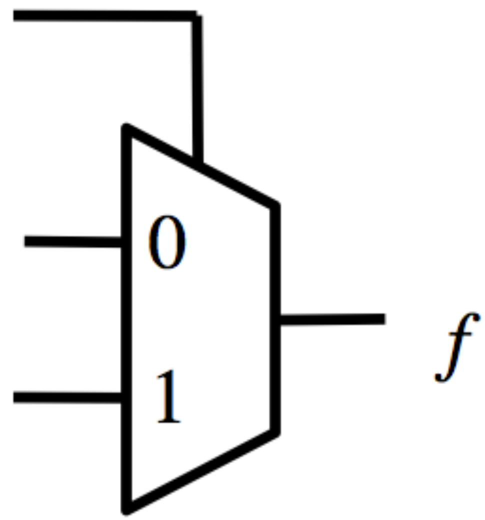
x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1



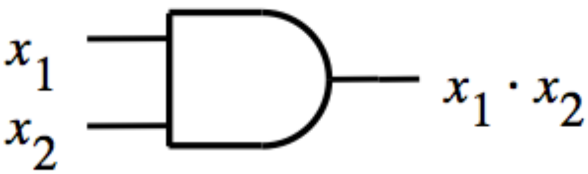
Building an AND Gate with 2-to-1 Mux



x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1



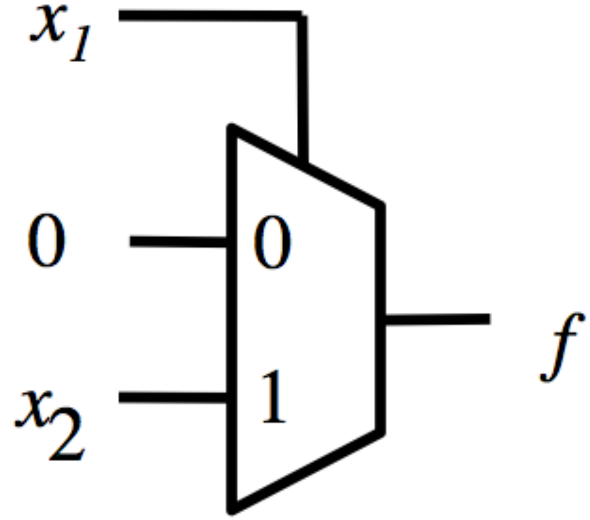
Building an AND Gate with 2-to-1 Mux



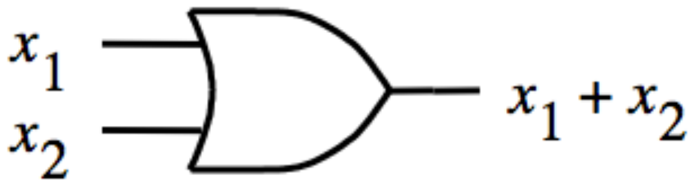
x_1	x_2	$x_1 \cdot x_2$
0	0	0
0	1	0
1	0	0
1	1	1

Rows with $x_1 = 0$: 0

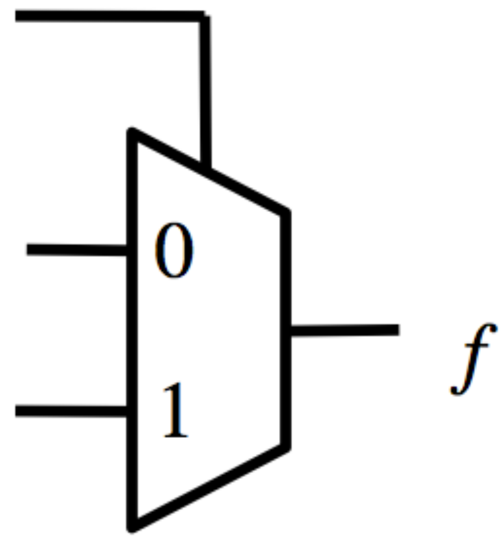
Rows with $x_1 = 1$: x_2



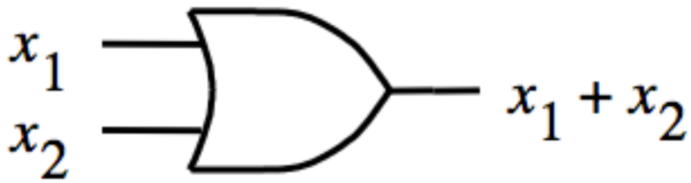
Building an OR Gate with 2-to-1 Mux



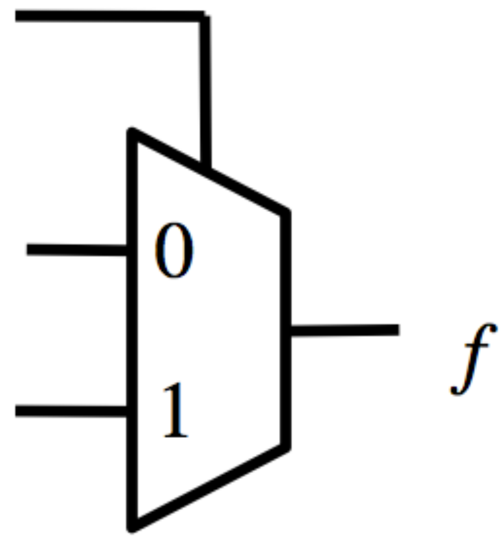
x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1



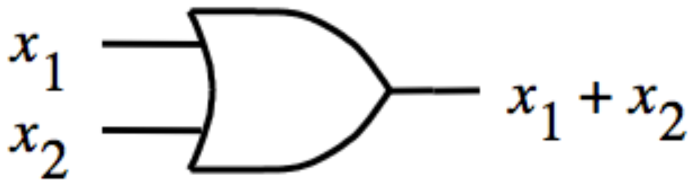
Building an OR Gate with 2-to-1 Mux



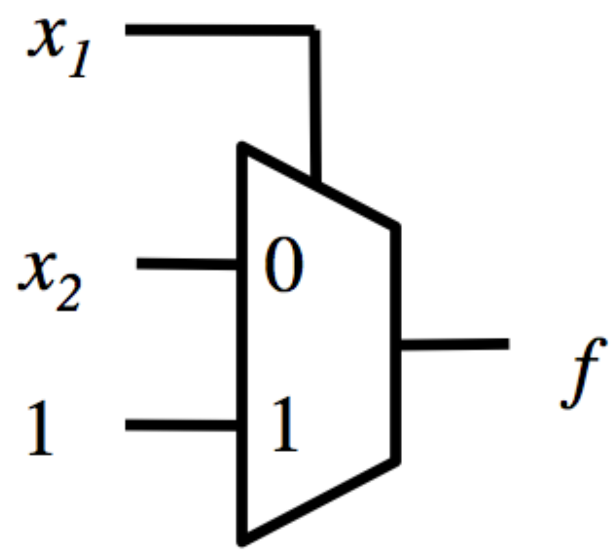
x_1	x_2	$x_1 + x_2$
0	0	0
0	1	1
1	0	1
1	1	1



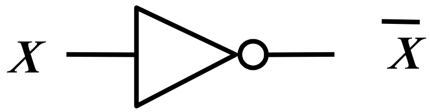
Building an OR Gate with 2-to-1 Mux



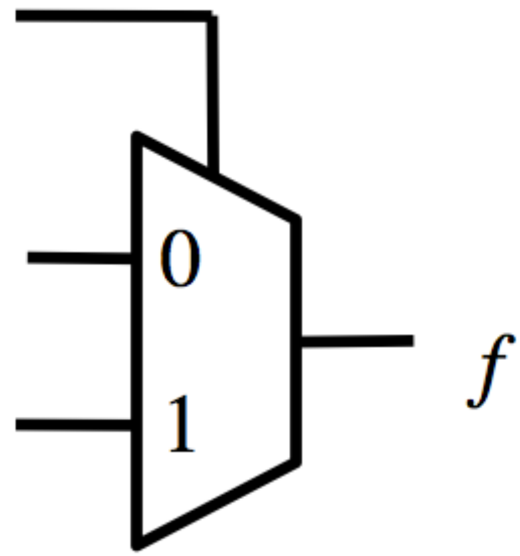
x_1	x_2	$x_1 + x_2$	
0	0	0	x_2
0	1	1	
1	0	1	1
1	1	1	



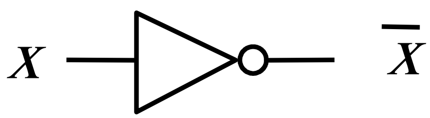
Building a NOT Gate with 2-to-1 Mux



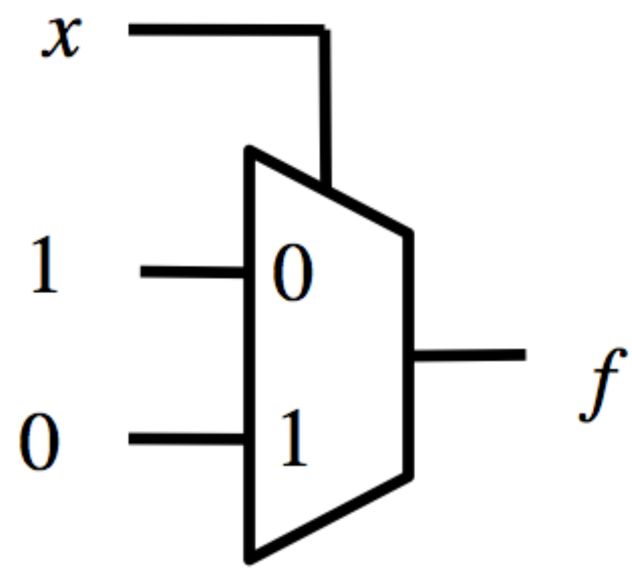
x	$\overline{x}$
0	1
1	0



Building a NOT Gate with 2-to-1 Mux



x	$\overline{x}$
0	1
1	0

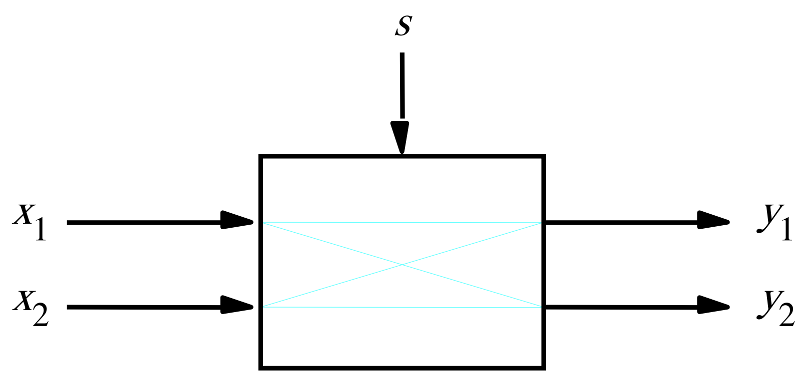


Implications

Any Boolean function can be implemented using only 2-to-1 multiplexers!

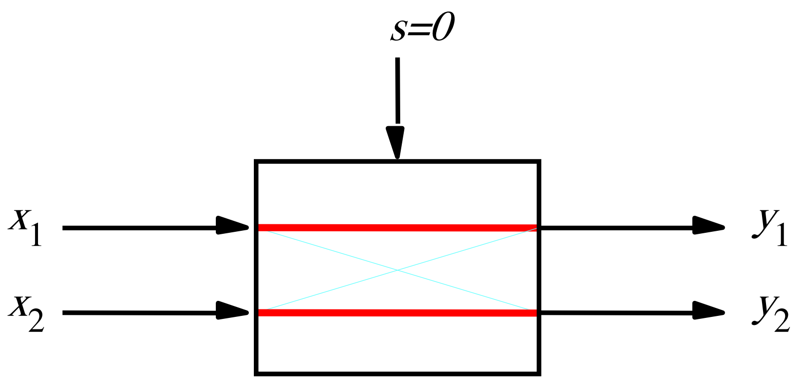
Synthesis of Logic Circuits Using Multiplexers

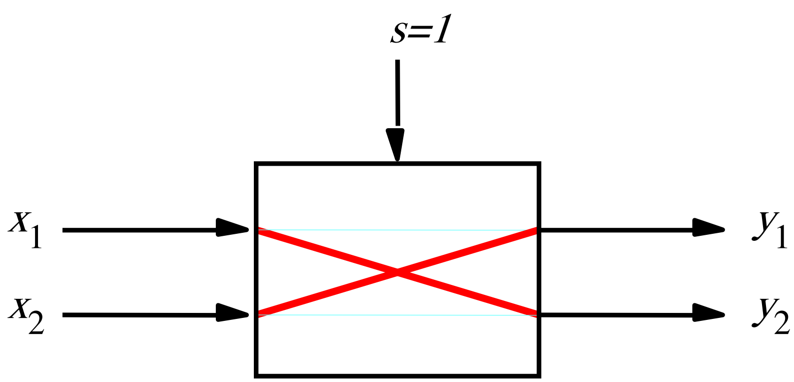
2 x 2 Crossbar switch



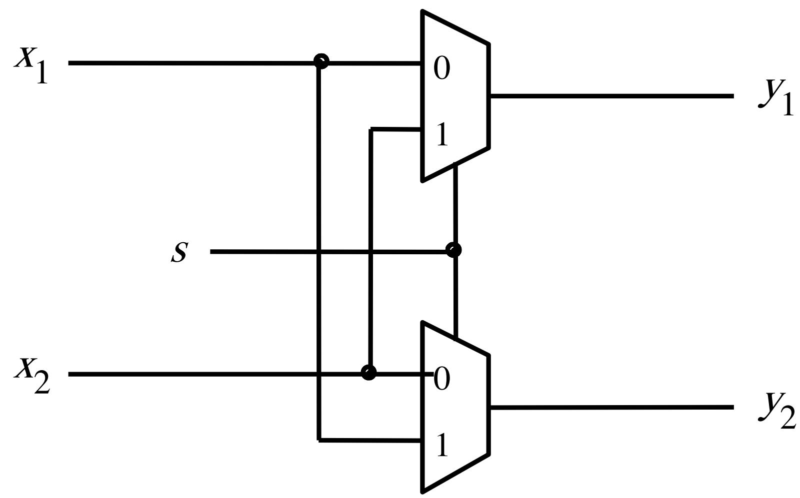
[Figure 4.5a from the textbook]

2 x 2 Crossbar switch



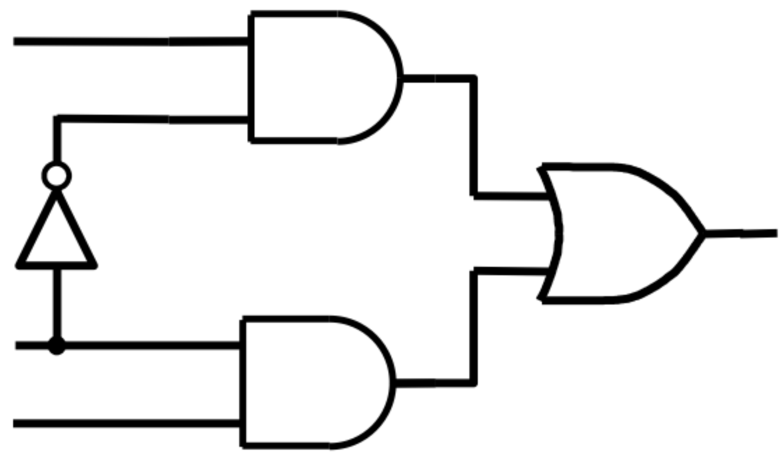


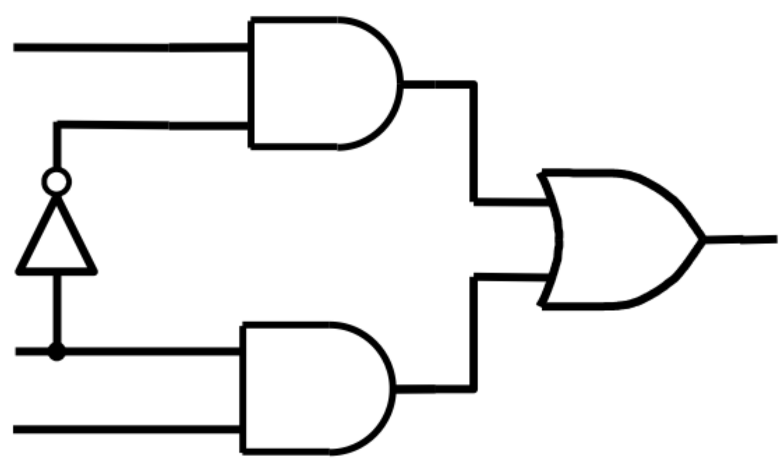
Implementation of a 2 x 2 crossbar switch with multiplexers



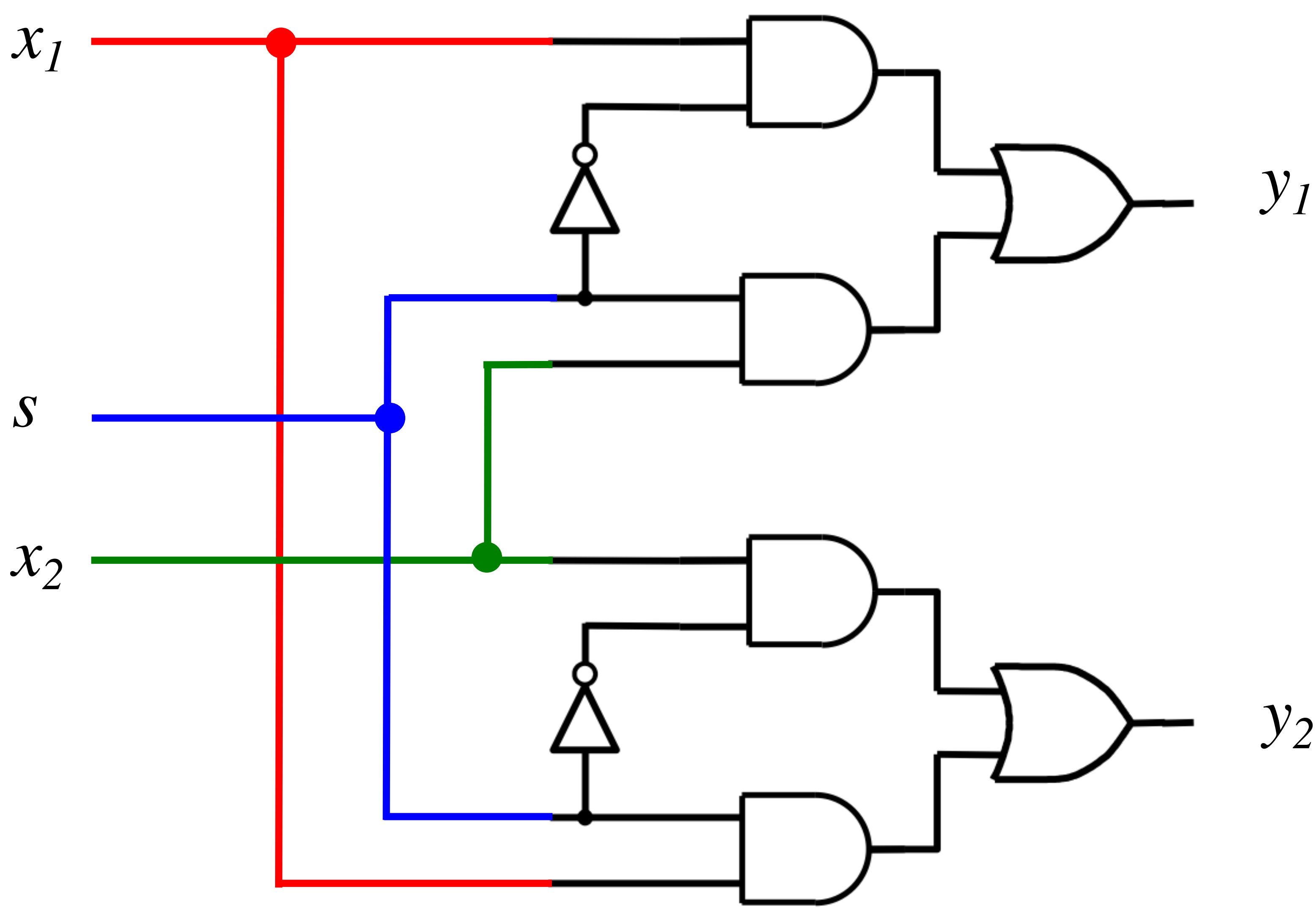
[Figure 4.5b from the textbook]

Implementation of a 2 x 2 crossbar switch with multiplexers



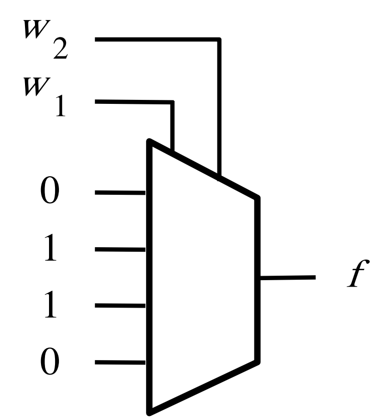


Implementation of a 2 x 2 crossbar switch with multiplexers



Implementation of a logic function with a 4-to-1 multiplexer

w_1	w_2	f
0	0	0
0	1	1
1	0	1
1	1	0



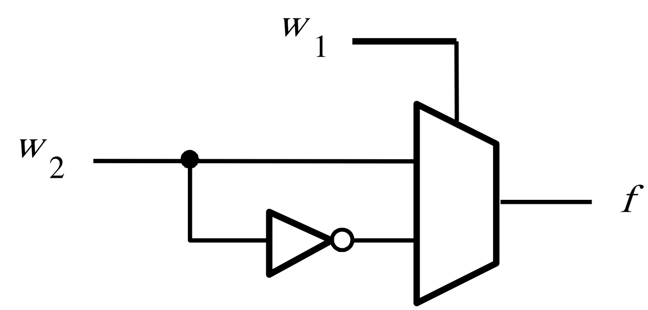
[Figure 4.6a from the textbook]

Implementation of the same logic function with a 2-to-1 multiplexer

w_1	w_2	f
0	0	0
0	1	1
1	0	1
1	1	0

w_1	f
0	w_2
1	$\overline{w}_2$

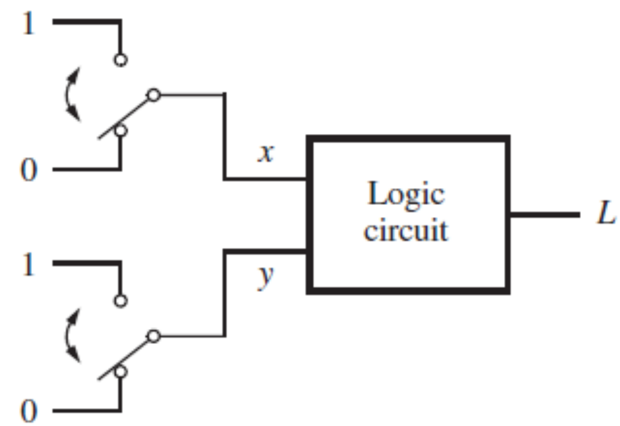
(b) Modified truth table



(c) Circuit

[Figure 4.6b-c from the textbook]

The XOR Logic Gate



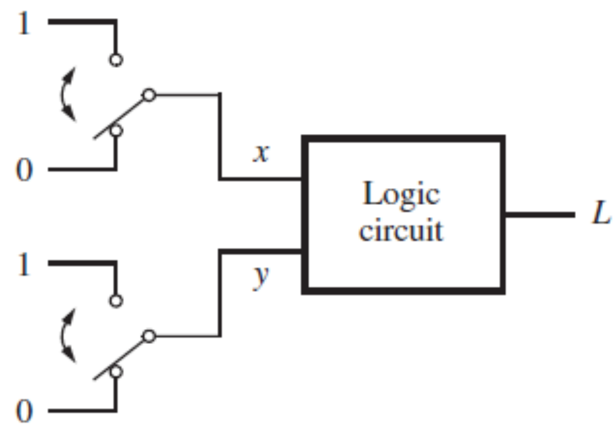
(a) Two switches that control a light

x	y	L
0	0	0
0	1	1
1	0	1
1	1	0

(b) Truth table

[Figure 2.11 from the textbook]

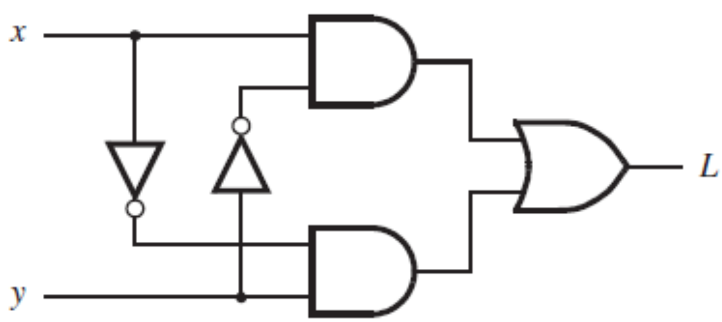
The XOR Logic Gate



(a) Two switches that control a light

x	y	L
0	0	0
0	1	1
1	0	1
1	1	0

(b) Truth table

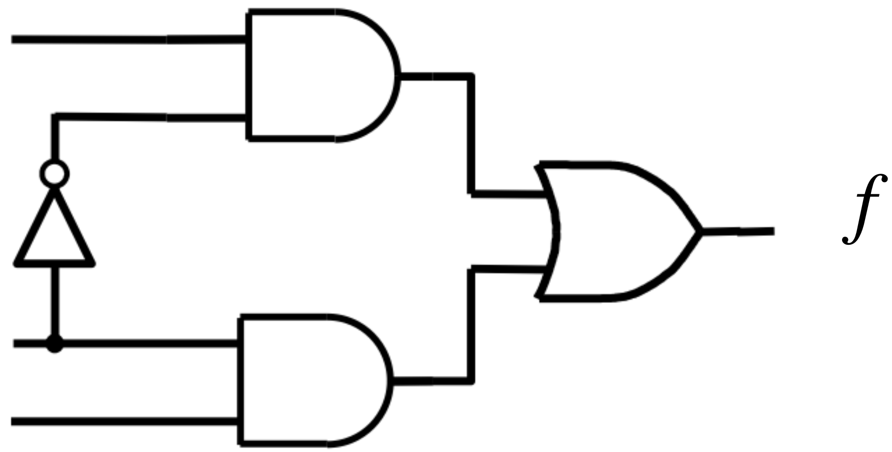


(c) Logic network

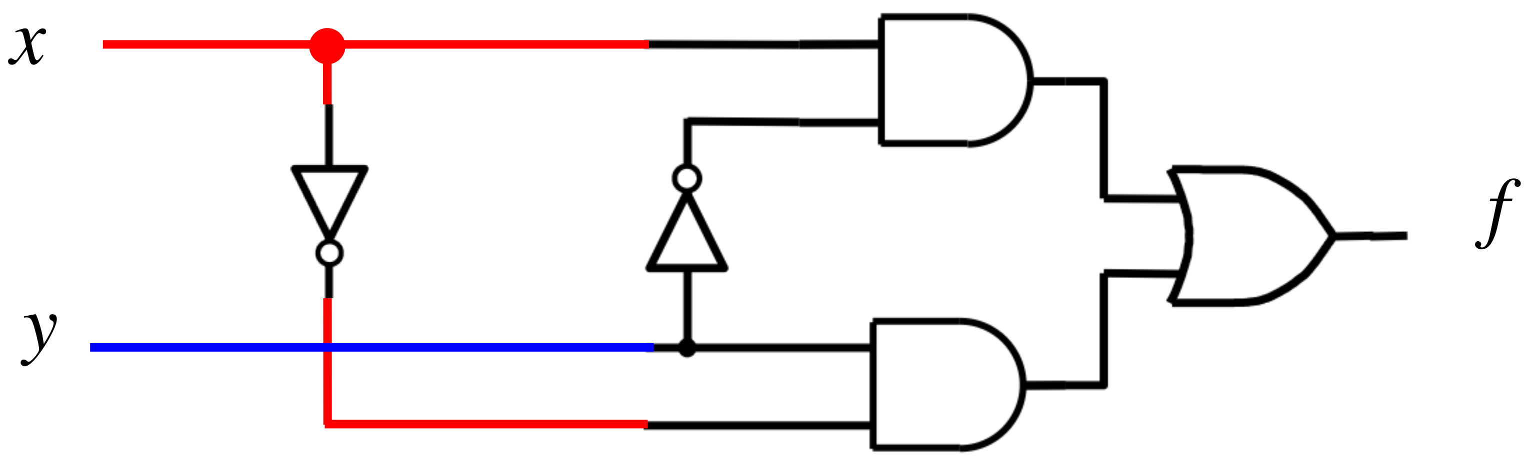
(d) XOR gate symbol

[Figure 2.11 from the textbook]

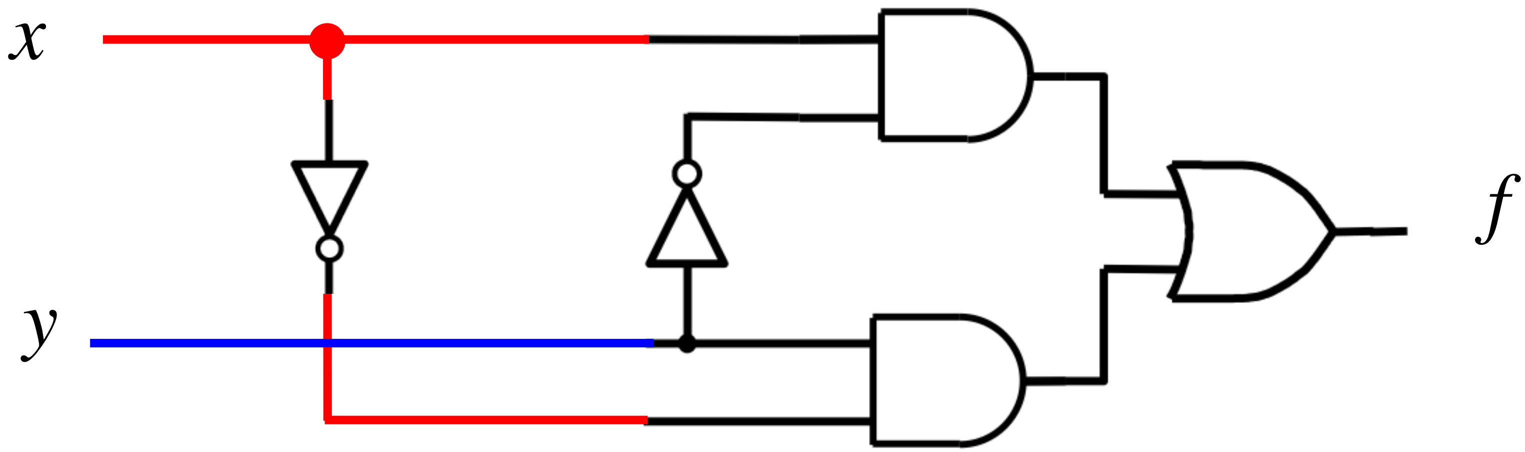
Implementation of the XOR Logic Gate with a 2-to-1 multiplexer and one NOT

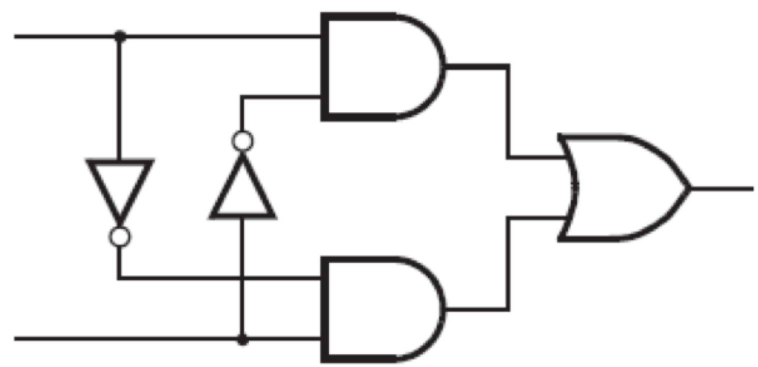


Implementation of the XOR Logic Gate with a 2-to-1 multiplexer and one NOT



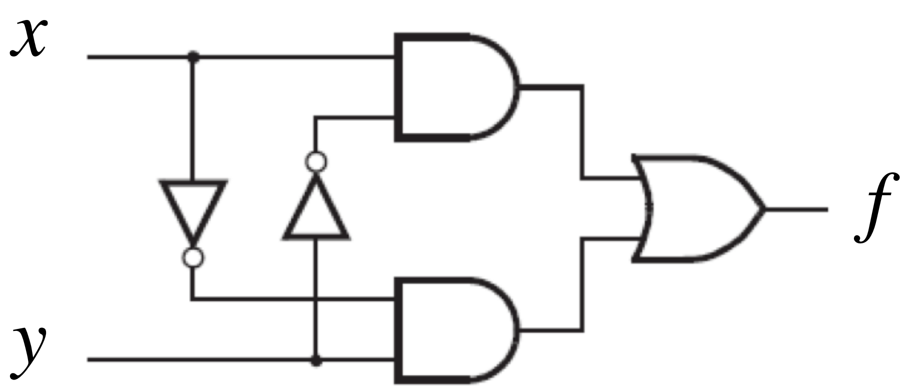
Implementation of the XOR Logic Gate with a 2-to-1 multiplexer and one NOT

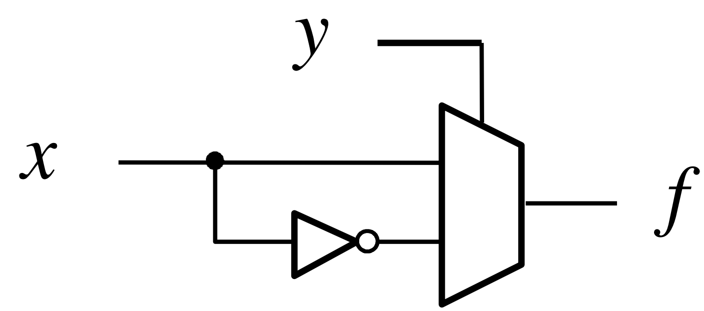


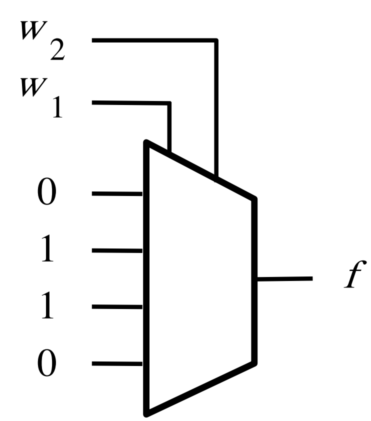


These two circuits are equivalent
(the wires of the bottom AND gate are flipped)

In other words,
all four of these are equivalent!







Implementation of another logic function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 4.7 from the textbook]

Implementation of another logic function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 4.7 from the textbook]

Implementation of another logic function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

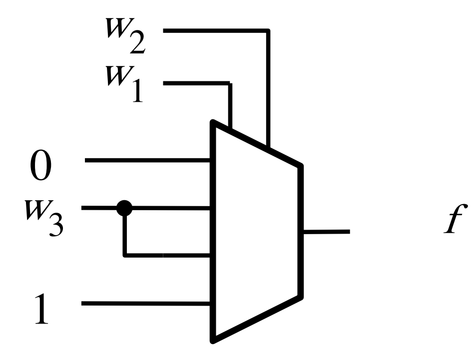
w_1	w_2	f
0	0	0
0	1	w_3
1	0	w_3
1	1	1

[Figure 4.7 from the textbook]

Implementation of another logic function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

w_1	w_2	f
0	0	0
0	1	w_3
1	0	$\overline{w_3}$
1	1	1



[Figure 4.7 from the textbook]

Another Example
(3-input XOR)

Implementation of 3-input XOR with 2-to-1 Multiplexers

w_1	w_2	w_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[Figure 4.8a from the textbook]

Implementation of 3-input XOR with 2-to-1 Multiplexers

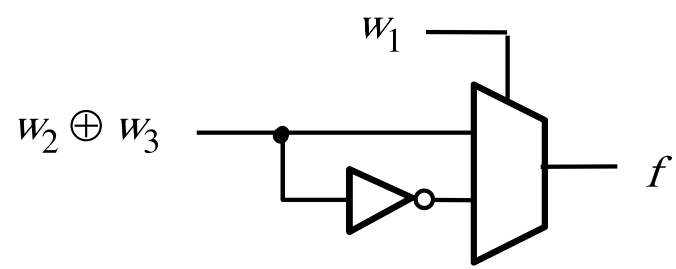
w_1	w_2	w_3	f	
0	0	0	0	$w_2 \oplus w_3$
0	0	1	1	
0	1	0	1	
0	1	1	0	
1	0	0	1	$\overline{w_2 \oplus w_3}$
1	0	1	0	
1	1	0	0	
1	1	1	1	

[Figure 4.8a from the textbook]

Implementation of 3-input XOR with 2-to-1 Multiplexers

w_1	w_2	w_3	f	
0	0	0	0	$w_2 \oplus w_3$
0	0	1	1	
0	1	0	1	
0	1	1	0	
1	0	0	1	$\overline{w_2 \oplus w_3}$
1	0	1	0	
1	1	0	0	
1	1	1	1	

(a) Truth table



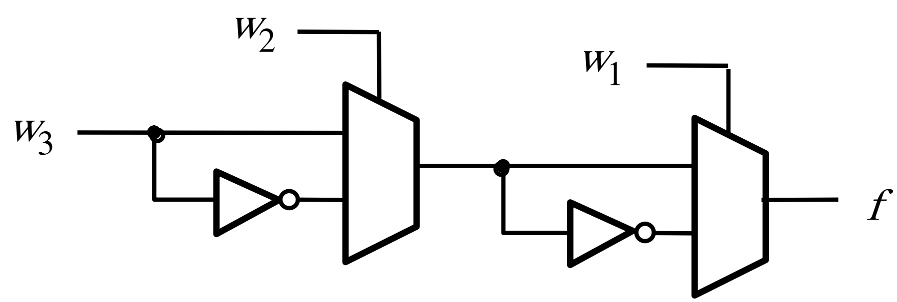
(b) Circuit

[Figure 4.8 from the textbook]

Implementation of 3-input XOR with 2-to-1 Multiplexers

w_1	w_2	w_3	f	
0	0	0	0	w_3
0	0	1	1	
0	1	0	1	$\overline{w_3}$
0	1	1	0	
1	0	0	1	
1	0	1	0	
1	1	0	0	
1	1	1	1	

(a) Truth table



(b) Circuit

[Figure 4.8 from the textbook]

Implementation of 3-input XOR with a 4-to-1 Multiplexer

w_1	w_2	w_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[Figure 4.9a from the textbook]

Implementation of 3-input XOR with a 4-to-1 Multiplexer

w_1	w_2	w_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[Figure 4.9a from the textbook]

Implementation of 3-input XOR with a 4-to-1 Multiplexer

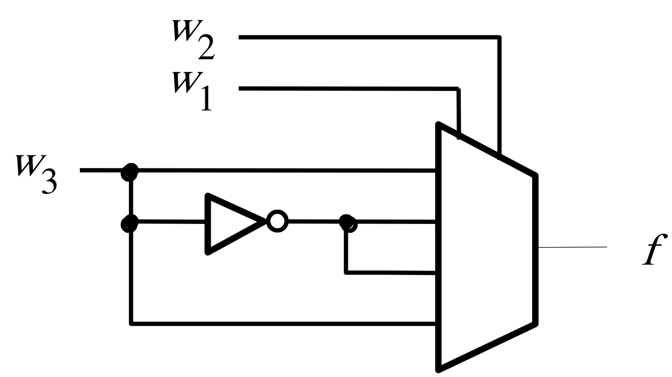
w_1	w_2	w_3	f	
0	0	0	0	w_3
0	0	1	1	
0	1	0	1	$\overline{w_3}$
0	1	1	0	
1	0	0	1	$\overline{w_3}$
1	0	1	0	
1	1	0	0	w_3
1	1	1	1	

[Figure 4.9a from the textbook]

Implementation of 3-input XOR with a 4-to-1 Multiplexer

w_1	w_2	w_3	f	
0	0	0	0	w_3
0	0	1	1	
0	1	0	1	$\overline{w_3}$
0	1	1	0	
1	0	0	1	$\overline{w_3}$
1	0	1	0	
1	1	0	0	w_3
1	1	1	1	

(a) Truth table



(b) Circuit

[Figure 4.9 from the textbook]

Multiplexor Synthesis
Using Shannon's Expansion

Three-input majority function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

[Figure 4.10a from the textbook]

Three-input majority function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

w_1	f
0	
1	

[Figure 4.10a from the textbook]

Three-input majority function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

w_1	f
0	$w_2 w_3$
1	$w_2 + w_3$

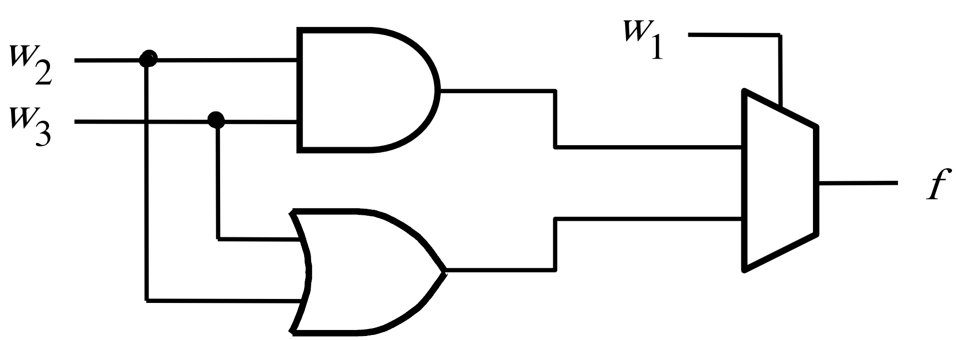
[Figure 4.10a from the textbook]

Three-input majority function

w_1	w_2	w_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

w_1	f
0	$w_2 w_3$
1	$w_2 + w_3$

(b) Truth table



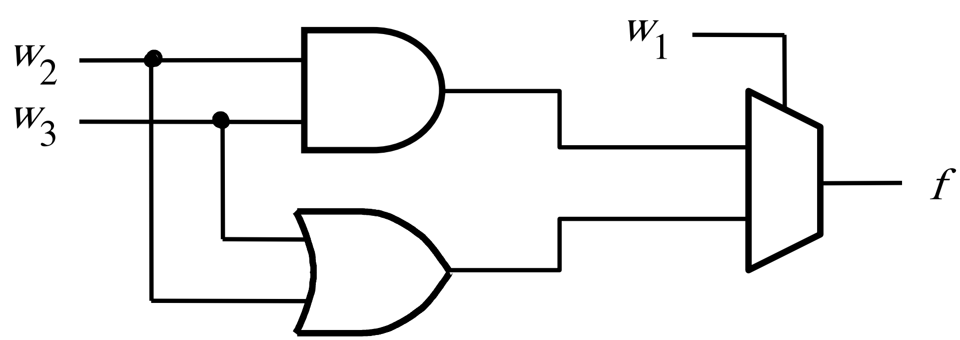
(b) Circuit

[Figure 4.10a from the textbook]

Three-input majority function

$$f = \overline{w}_1 w_2 w_3 + w_1 \overline{w}_2 w_3 + w_1 w_2 \overline{w}_3 + w_1 w_2 w_3$$

$$f = \overline{w}_1 (w_2 w_3) + w_1 (\overline{w}_2 w_3 + w_2 \overline{w}_3 + w_2 w_3)$$
$$= \overline{w}_1 (w_2 w_3) + w_1 (w_2 + w_3)$$



Shannon's Expansion Theorem

Any Boolean function $f(w_1, \ldots, w_n)$ can be rewritten in the form:

$$f(w_1, w_2, \ldots, w_n) = \overline{w}_1 \cdot f(0, w_2, \ldots, w_n) + w_1 \cdot f(1, w_2, \ldots, w_n)$$

Shannon's Expansion Theorem

Any Boolean function $f(w_1, \ldots, w_n)$ can be rewritten in the form:

$$f(w_1, w_2, \ldots, w_n) = \overline{w}_1 \cdot f(0, w_2, \ldots, w_n) + w_1 \cdot f(1, w_2, \ldots, w_n)$$

$$f = \overline{w}_1 f_{\overline{w}_1} + w_1 f_{w_1}$$

Shannon's Expansion Theorem

Any Boolean function $f(w_1, \ldots, w_n)$ can be rewritten in the form:

$$f(w_1, w_2, \ldots, w_n) = \overline{w}_1 \cdot f(0, w_2, \ldots, w_n) + w_1 \cdot f(1, w_2, \ldots, w_n)$$

$$f = \overline{w}_1 f_{\overline{w}_1} + w_1 f_{w_1}$$

cofactor cofactor

Shannon's Expansion Theorem (Example)

$$f(w_1, w_2, w_3) = w_1 w_2 + w_1 w_3 + w_2 w_3$$

Shannon's Expansion Theorem (Example)

$$f(w_1, w_2, w_3) = w_1w_2 + w_1w_3 + w_2w_3$$

$$f(w_1, w_2, w_3) = w_1w_2 + w_1w_3 + w_2w_3\,(\overline{w_1} + w_1)$$

Shannon's Expansion Theorem (Example)

$$f(w_1, w_2, w_3) = w_1w_2 + w_1w_3 + w_2w_3$$

$$f(w_1, w_2, w_3) = w_1w_2 + w_1w_3 + w_2w_3 \, (\overline{w_1} + w_1)$$

$$\begin{aligned} f &= \overline{w}_1(0 \cdot w_2 + 0 \cdot w_3 + w_2w_3) + w_1(1 \cdot w_2 + 1 \cdot w_3 + w_2w_3) \\ &= \overline{w}_1(w_2w_3) + w_1(w_2 + w_3) \end{aligned}$$

Shannon's Expansion Theorem
(In terms of more than one variable)

$$\begin{aligned} f(w_1, \ldots, w_n) = {} & \overline{w}_1\overline{w}_2 \cdot f(0, 0, w_3, \ldots, w_n) + \overline{w}_1 w_2 \cdot f(0, 1, w_3, \ldots, w_n) \\ & + w_1\overline{w}_2 \cdot f(1, 0, w_3, \ldots, w_n) + w_1 w_2 \cdot f(1, 1, w_3, \ldots, w_n) \end{aligned}$$

This form is suitable for implementation with a 4x1 multiplexer.

Another Example

Factor and implement the following function with a 2-to-1 multiplexer

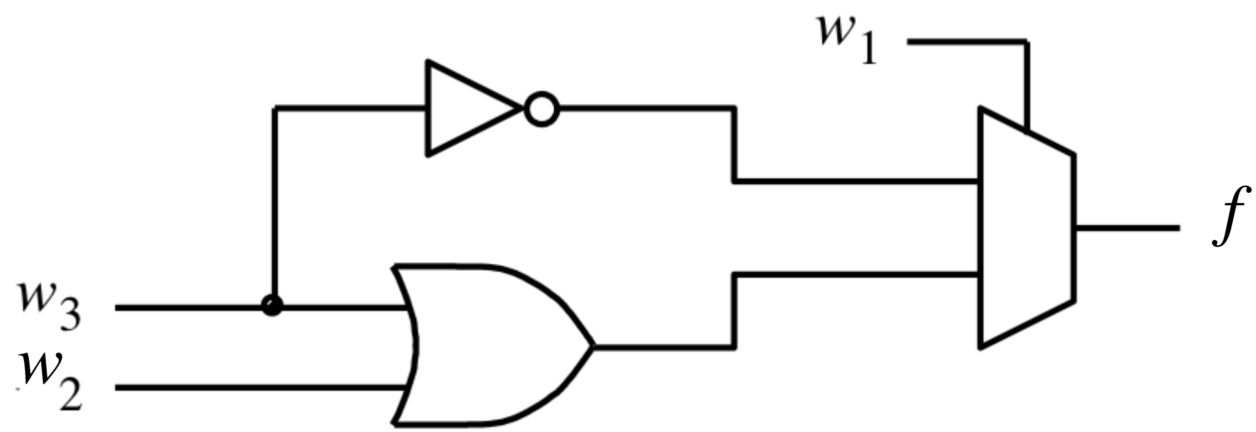
$$f = \overline{w}_1\overline{w}_3 + w_1w_2 + w_1w_3$$

Factor and implement the following function with a 2-to-1 multiplexer

$$f = \overline{w}_1\overline{w}_3 + w_1w_2 + w_1w_3$$

$$\begin{aligned} f &= \overline{w}_1 f_{\overline{w}_1} + w_1 f_{w_1} \\ &= \overline{w}_1(\overline{w}_3) + w_1(w_2 + w_3) \end{aligned}$$

Factor and implement the following function with a 2-to-1 multiplexer



$$f = \overline{w}_1 f_{\overline{w}_1} + w_1 f_{w_1}$$
$$= \overline{w}_1(\overline{w}_3) + w_1(w_2 + w_3)$$

[Figure 4.11a from the textbook]

Factor and implement the following function with a 4-to-1 multiplexer

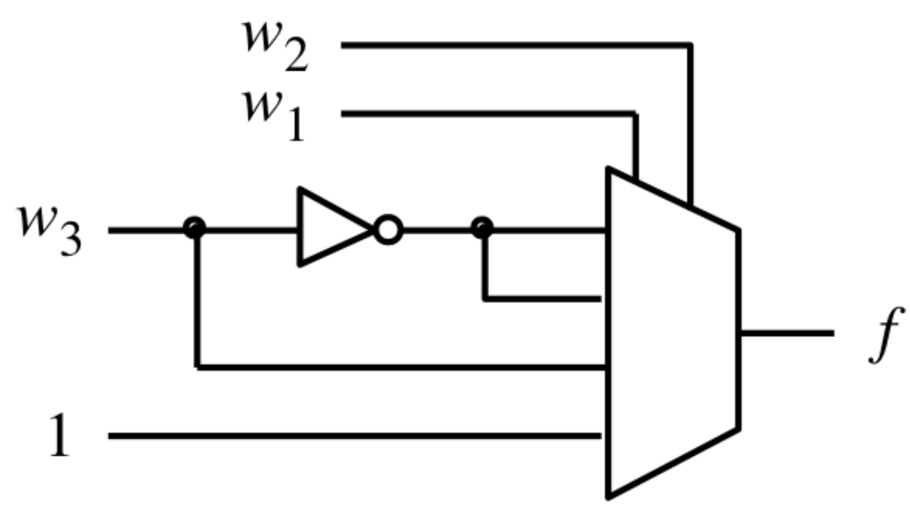
$$f = \overline{w}_1 \overline{w}_3 + w_1 w_2 + w_1 w_3$$

Factor and implement the following function with a 4-to-1 multiplexer

$$f = \overline{w}_1\overline{w}_3 + w_1w_2 + w_1w_3$$

$$\begin{aligned} f &= \overline{w}_1\overline{w}_2 f_{\overline{w}_1\overline{w}_2} + \overline{w}_1 w_2 f_{\overline{w}_1 w_2} + w_1\overline{w}_2 f_{w_1\overline{w}_2} + w_1 w_2 f_{w_1 w_2} \\ &= \overline{w}_1\overline{w}_2(\overline{w}_3) + \overline{w}_1 w_2(\overline{w}_3) + w_1\overline{w}_2(w_3) + w_1 w_2(1) \end{aligned}$$

Factor and implement the following function with a 4-to-1 multiplexer



$$f = \overline{w}_1\overline{w}_2 f_{\overline{w}_1\overline{w}_2} + \overline{w}_1 w_2 f_{\overline{w}_1 w_2} + w_1\overline{w}_2 f_{w_1\overline{w}_2} + w_1 w_2 f_{w_1 w_2}$$

$$= \overline{w}_1\overline{w}_2(\overline{w}_3) + \overline{w}_1 w_2(\overline{w}_3) + w_1\overline{w}_2(w_3) + w_1 w_2(1)$$

[Figure 4.11b from the textbook]

Yet Another Example

Factor and implement the following function using <u>only</u> 2-to-1 multiplexers

$$f = w_1 w_2 + w_1 w_3 + w_2 w_3$$

Factor and implement the following function using only 2-to-1 multiplexers

$$f = w_1 w_2 + w_1 w_3 + w_2 w_3$$

$$f = \overline{w}_1 (w_2 w_3) + w_1 (w_2 + w_3 + w_2 w_3)$$
$$= \overline{w}_1 (w_2 w_3) + w_1 (w_2 + w_3)$$

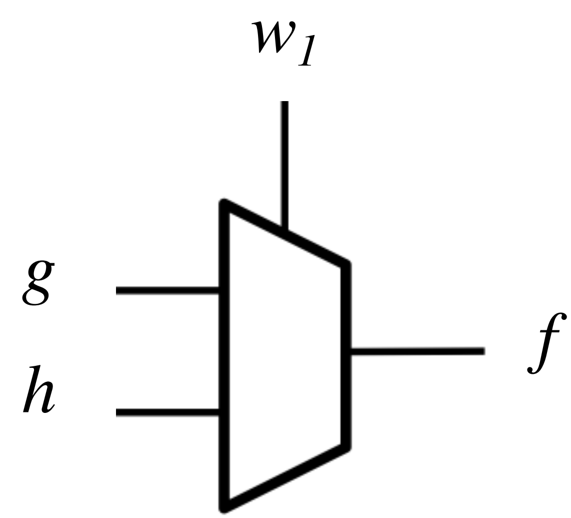
Factor and implement the following function using <u>only</u> 2-to-1 multiplexers

$$f = w_1 w_2 + w_1 w_3 + w_2 w_3$$

$$f = \overline{w}_1 (w_2 w_3) + w_1 (w_2 + w_3 + w_2 w_3)$$
$$= \overline{w}_1 \underbrace{(w_2 w_3)} + w_1 \underbrace{(w_2 + w_3)}$$

$$g = w_2 w_3 \qquad h = w_2 + w_3$$

Factor and implement the following function using only 2-to-1 multiplexers



$$f = \overline{w}_1(w_2w_3) + w_1(w_2 + w_3 + w_2w_3)$$

$$= \overline{w}_1(w_2w_3) + w_1(w_2 + w_3)$$

$$g = w_2w_3 \qquad h = w_2 + w_3$$

Factor and implement the following function using only 2-to-1 multiplexers

$g = w_2 w_3$

$h = w_2 + w_3$

Factor and implement the following function using <u>only</u> 2-to-1 multiplexers

$$g = w_2 w_3$$

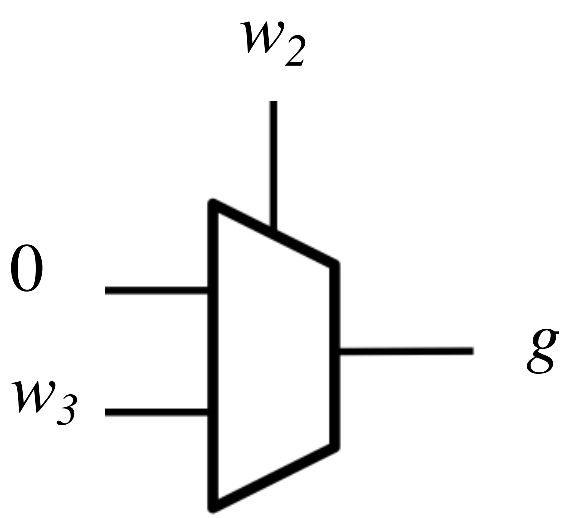
$$h = w_2 + w_3$$

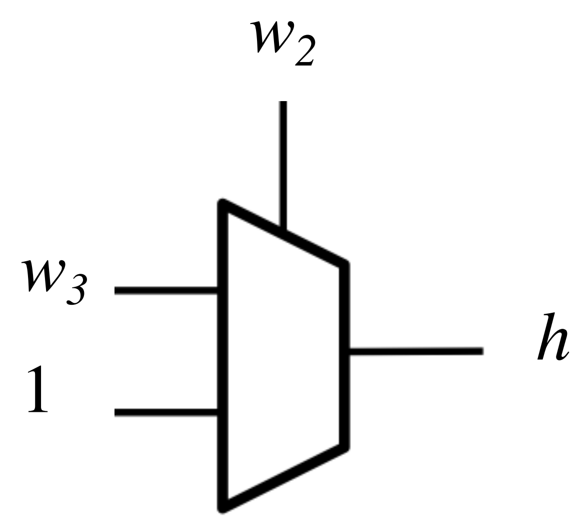
↓

$$g = \overline{w}_2(0) + w_2(w_3)$$

$$h = \overline{w}_2(w_3) + w_2(1)$$

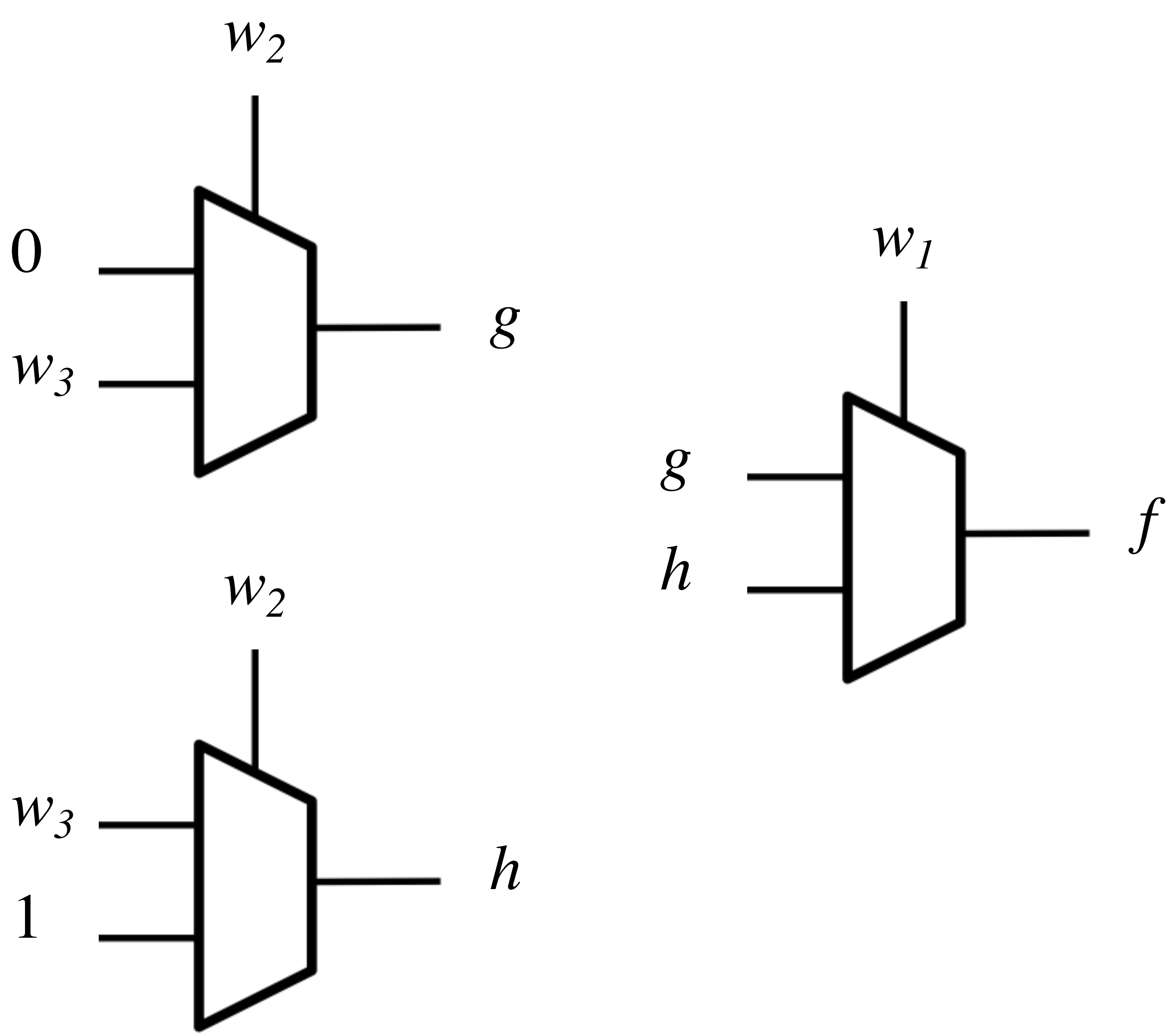
Factor and implement the following function using <u>only</u> 2-to-1 multiplexers



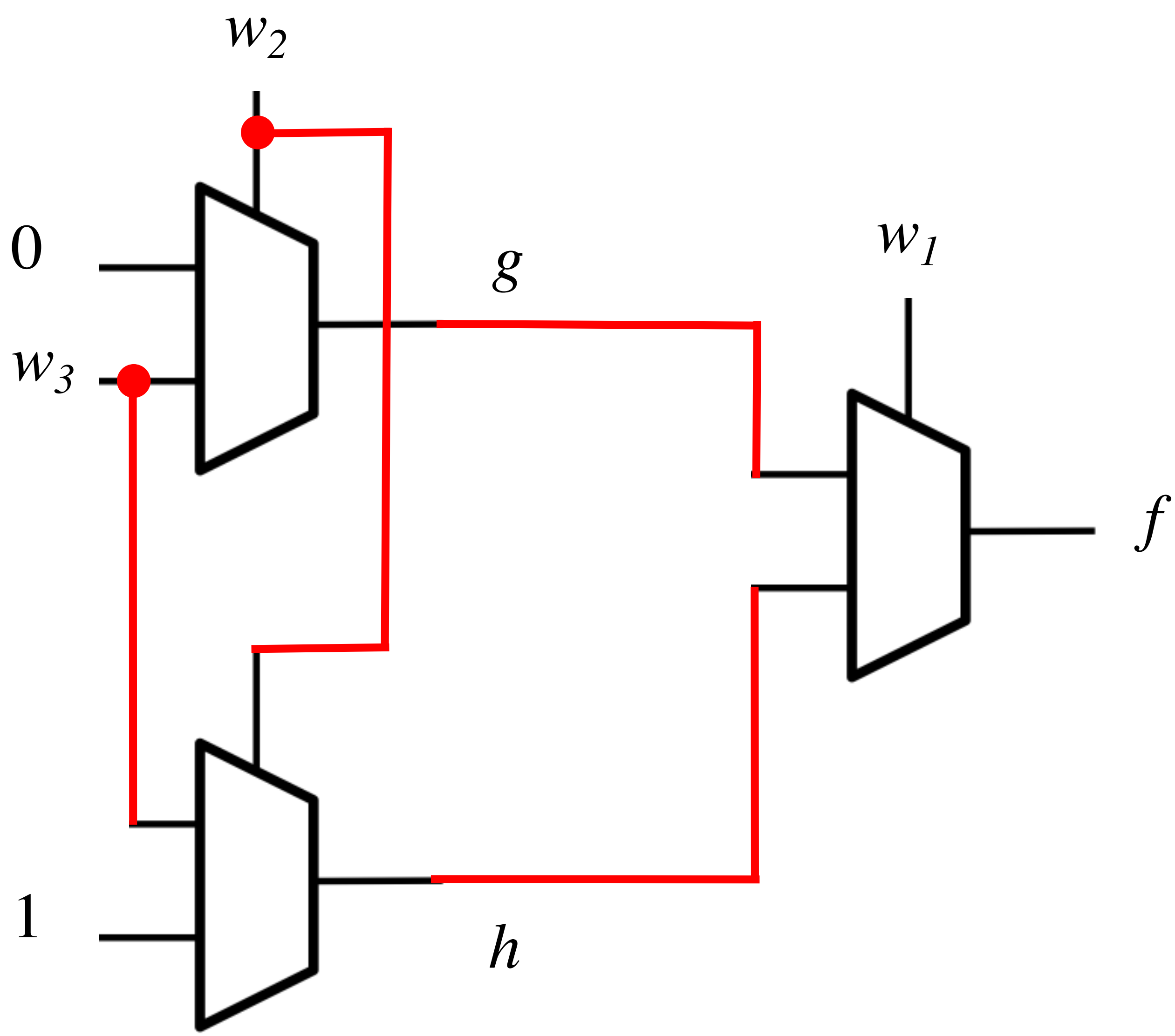


$$g = \overline{w}_2(0) + w_2(w_3) \qquad h = \overline{w}_2(w_3) + w_2(1)$$

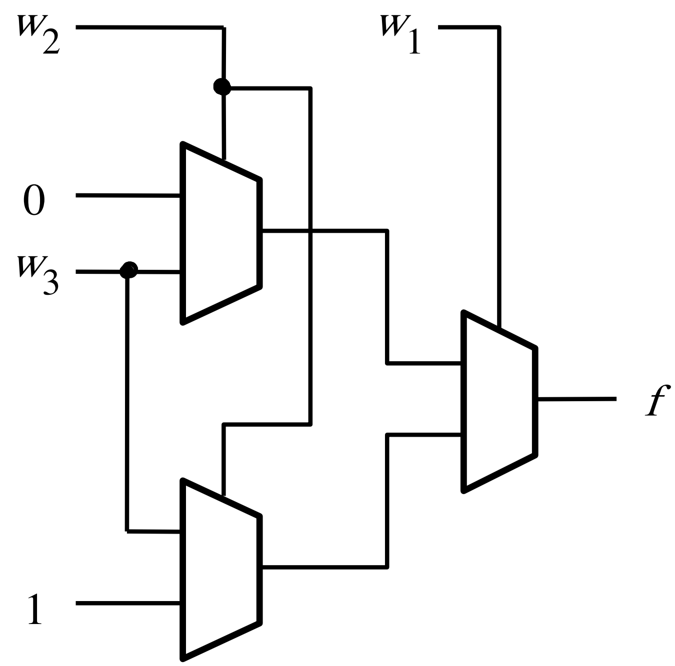
Finally, we are ready to draw the circuit



Finally, we are ready to draw the circuit



Finally, we are ready to draw the circuit



[Figure 4.12 from the textbook]

Questions?

THE END