



CprE 281: Digital Logic

Instructor: Alexander Stoytchev

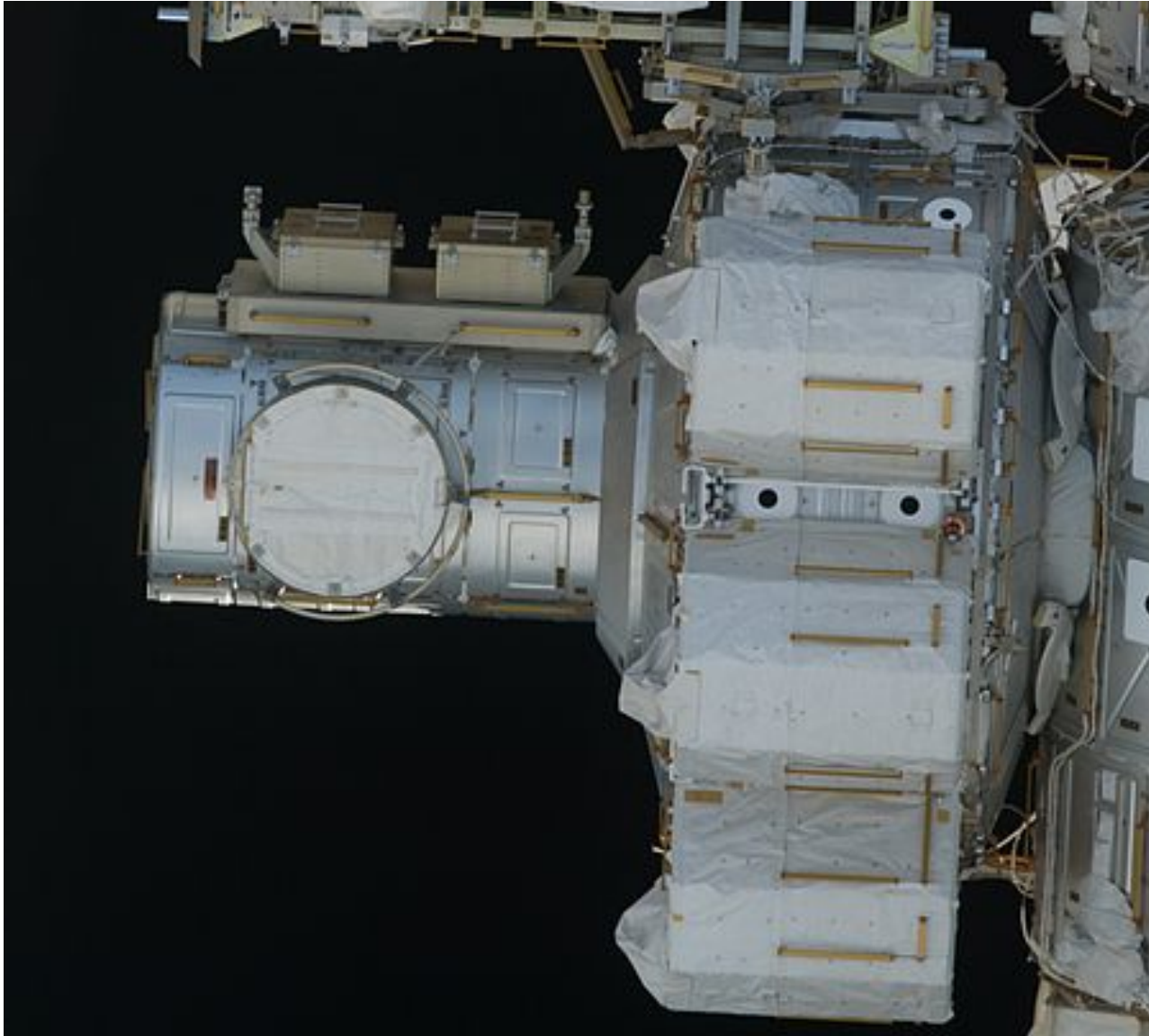
<http://www.ece.iastate.edu/~alexs/classes/>

Registers and Counters

CprE 281: Digital Logic
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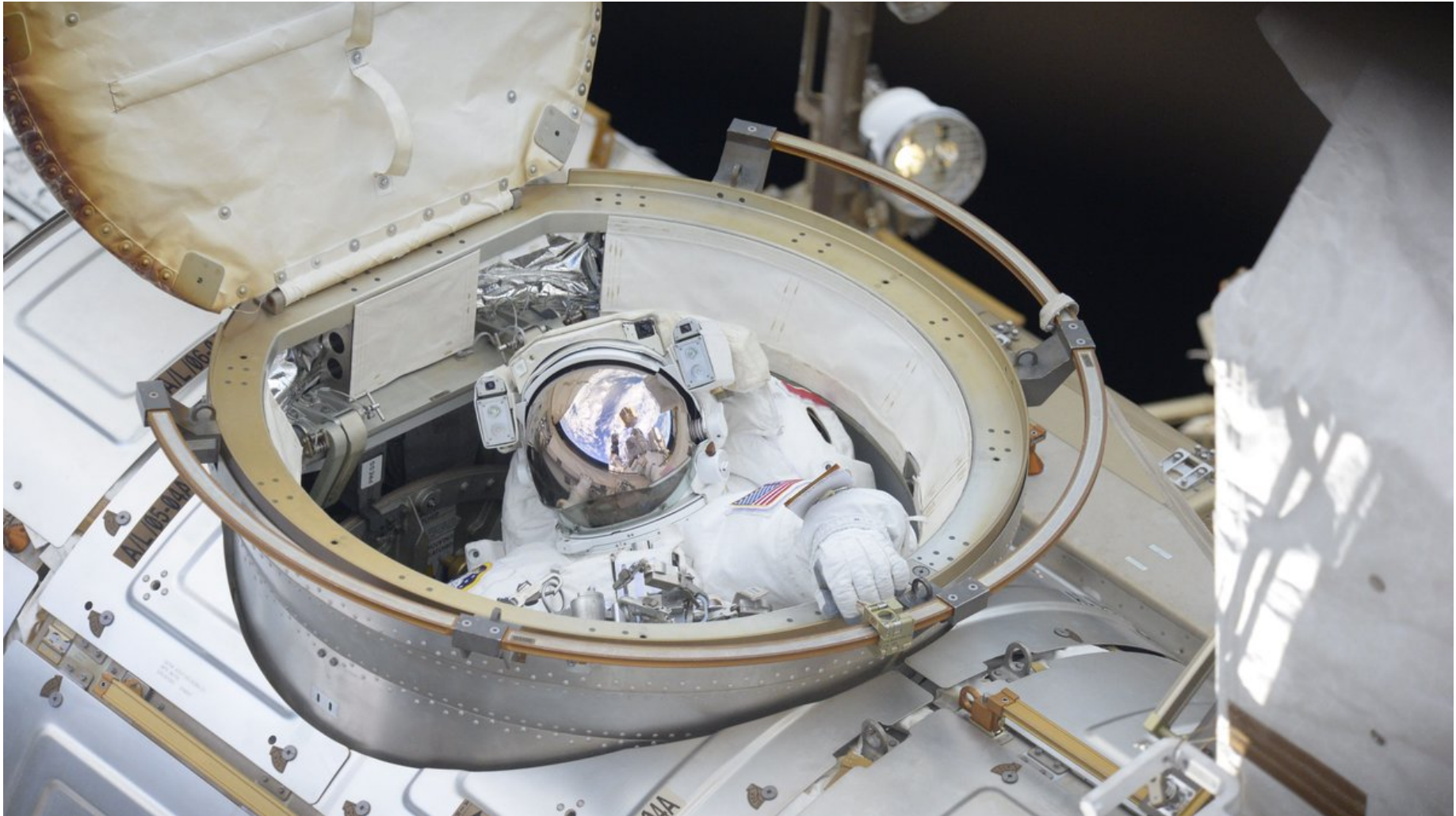
Flip-Flop Analogy

(Airlock)



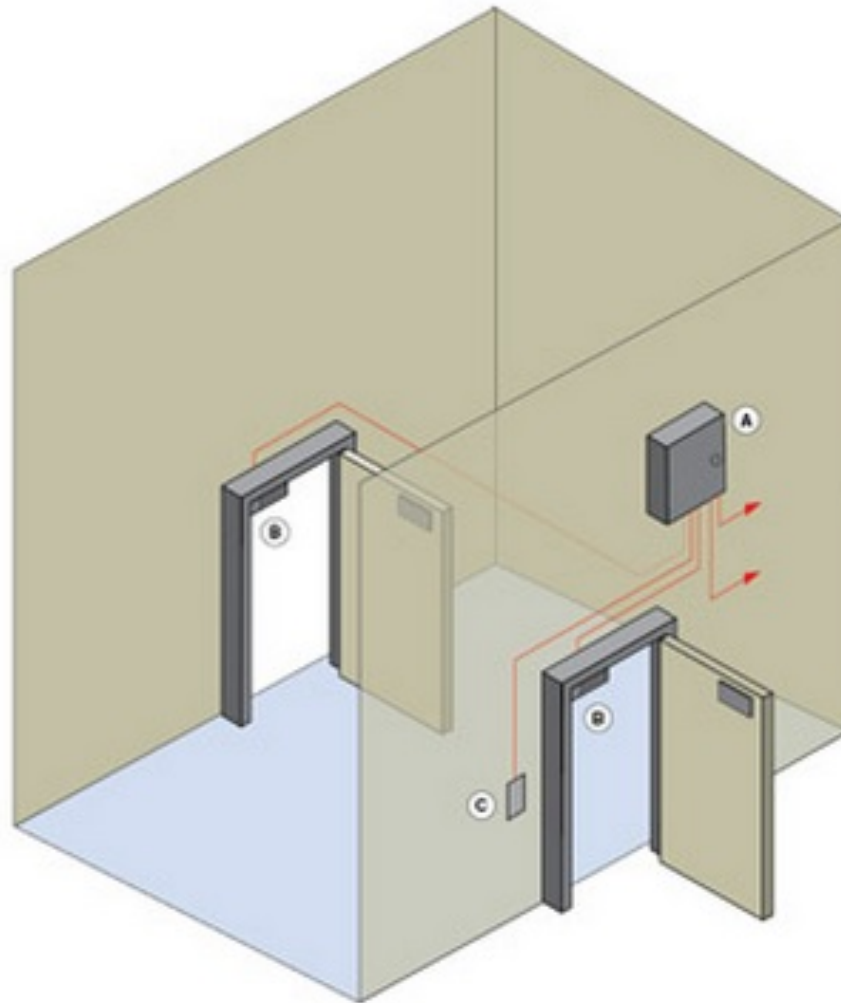








Airlock on Earth

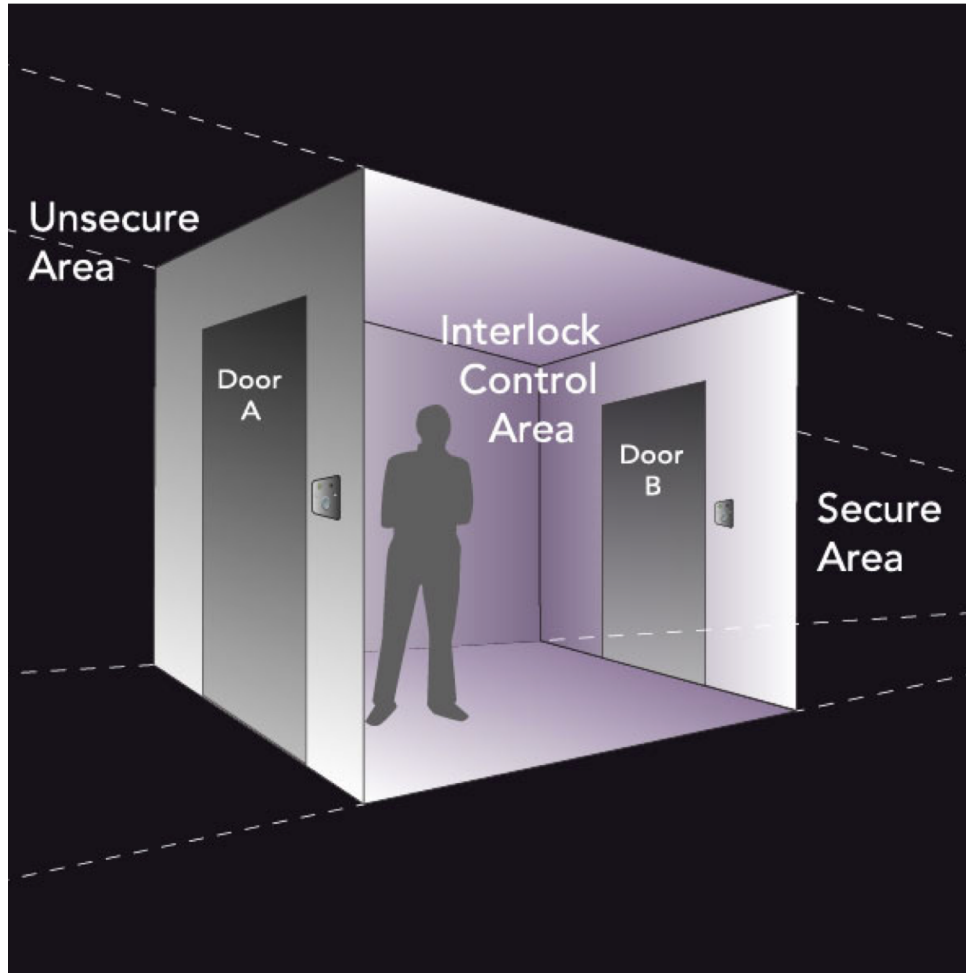


D Flip-Flop Analogy



Outer Door
Will Not Unlock When
Inner Door is Open

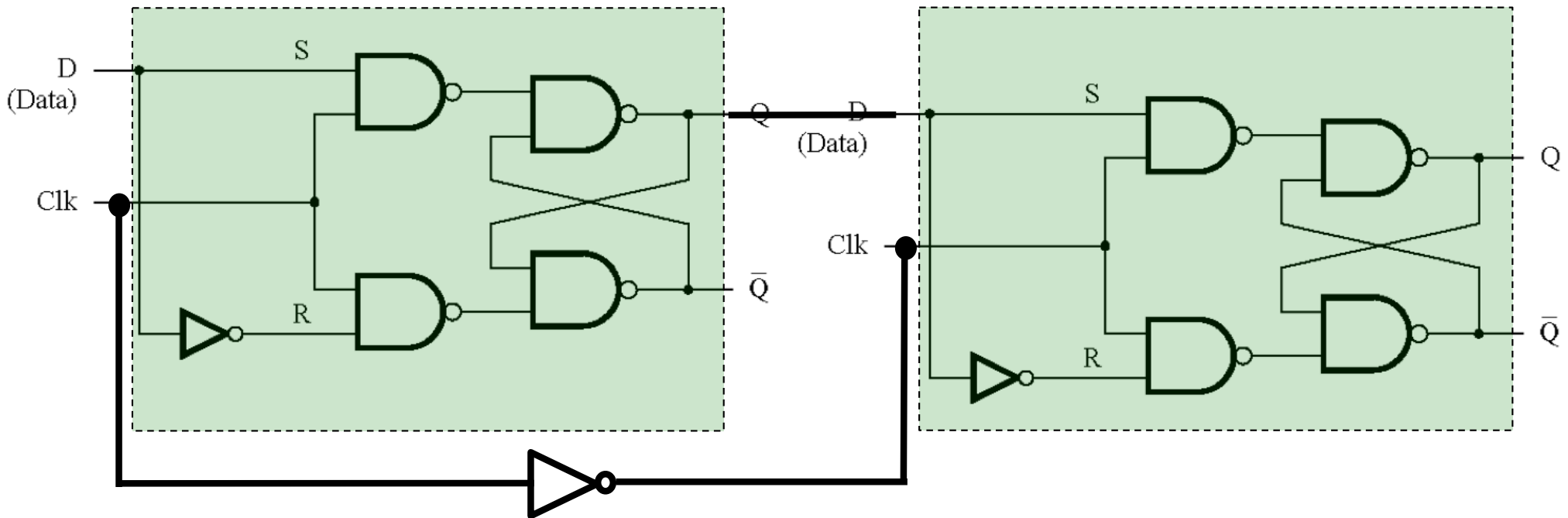
Inner Door
Will Not Unlock When
Outer Door is Open



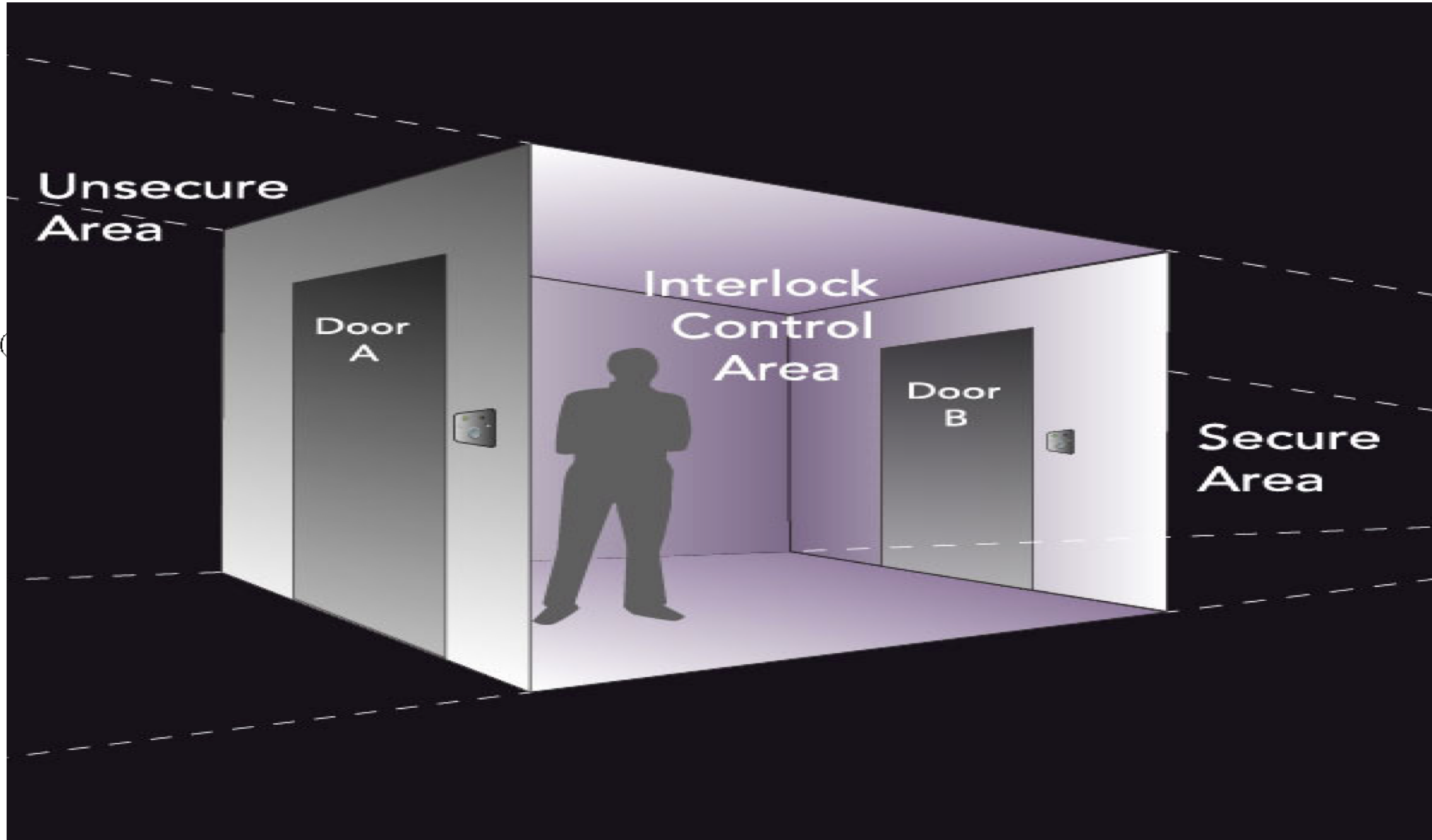
D Flip-Flop Analogy

Master

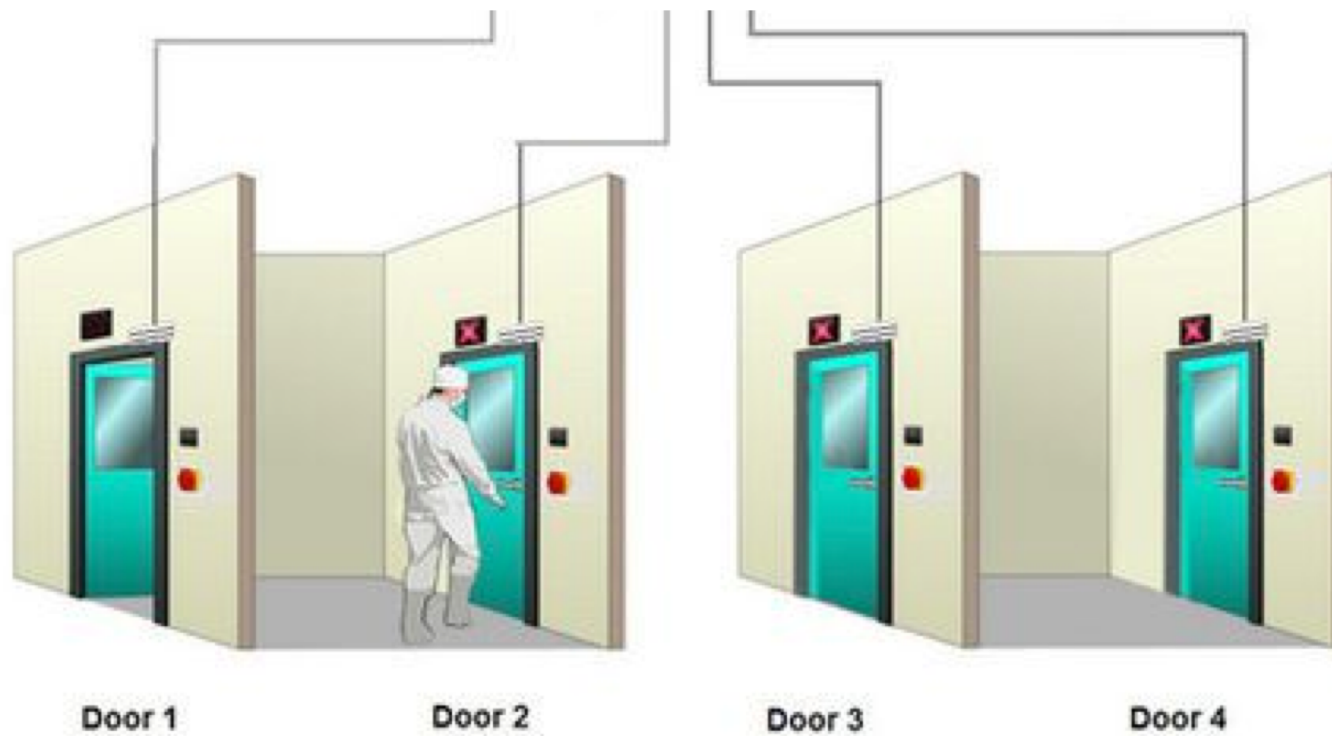
Slave



D Flip-Flop Analogy



Shift-Register Analogy



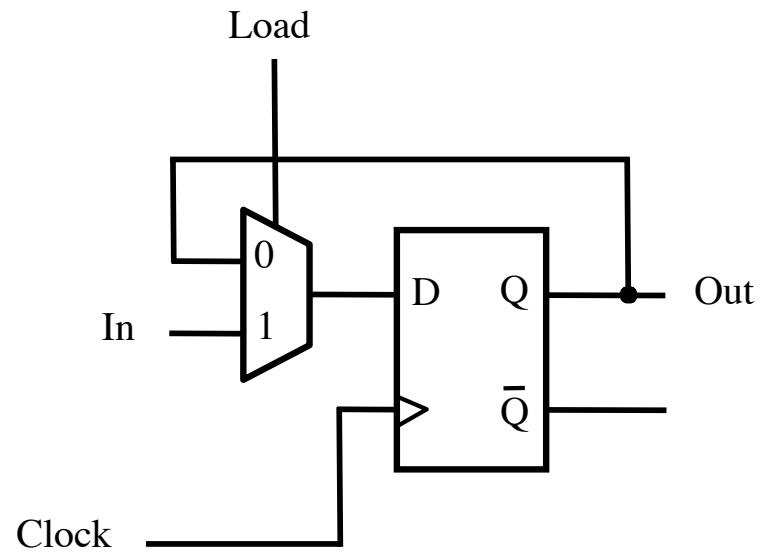
Registers

Register (Definition)

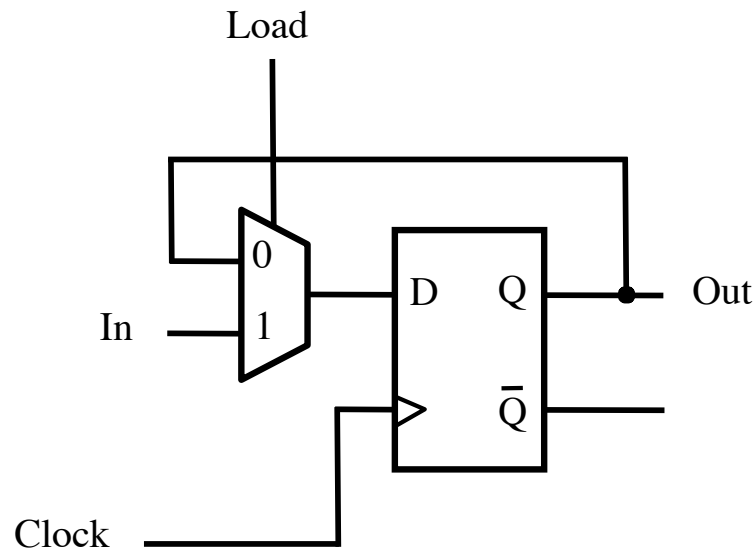
An n-bit structure consisting of flip-flops

Parallel-Access Register

1-Bit Parallel-Access Register



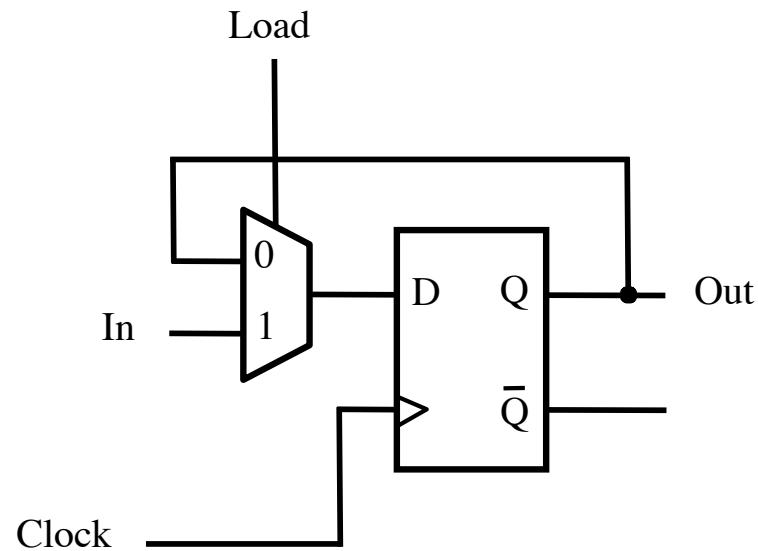
1-Bit Parallel-Access Register



The 2-to-1 multiplexer is used to select whether to load a new value into the D flip-flop or to retain the old value.

The output of this circuit is the Q output of the flip-flop.

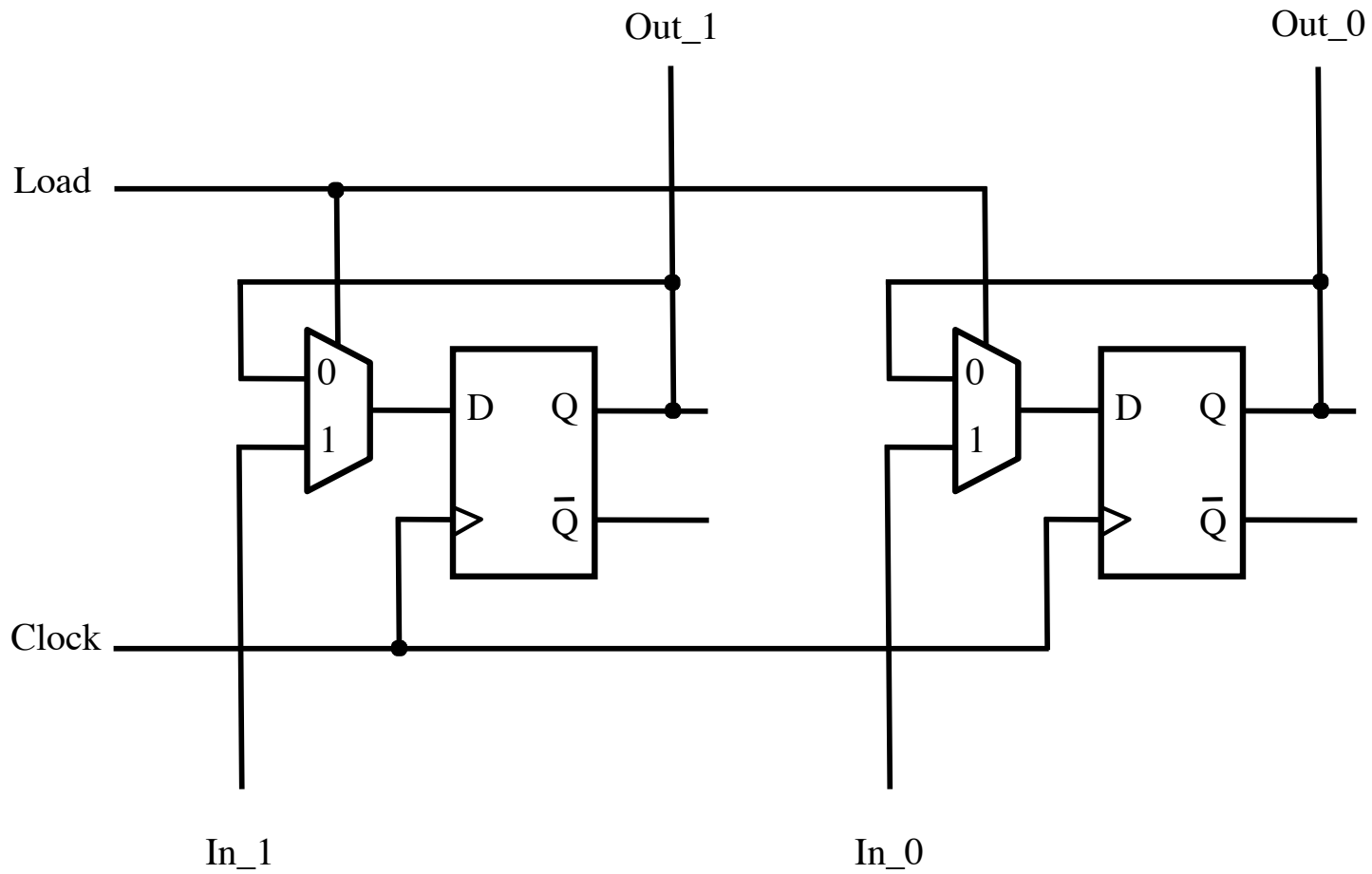
1-Bit Parallel-Access Register



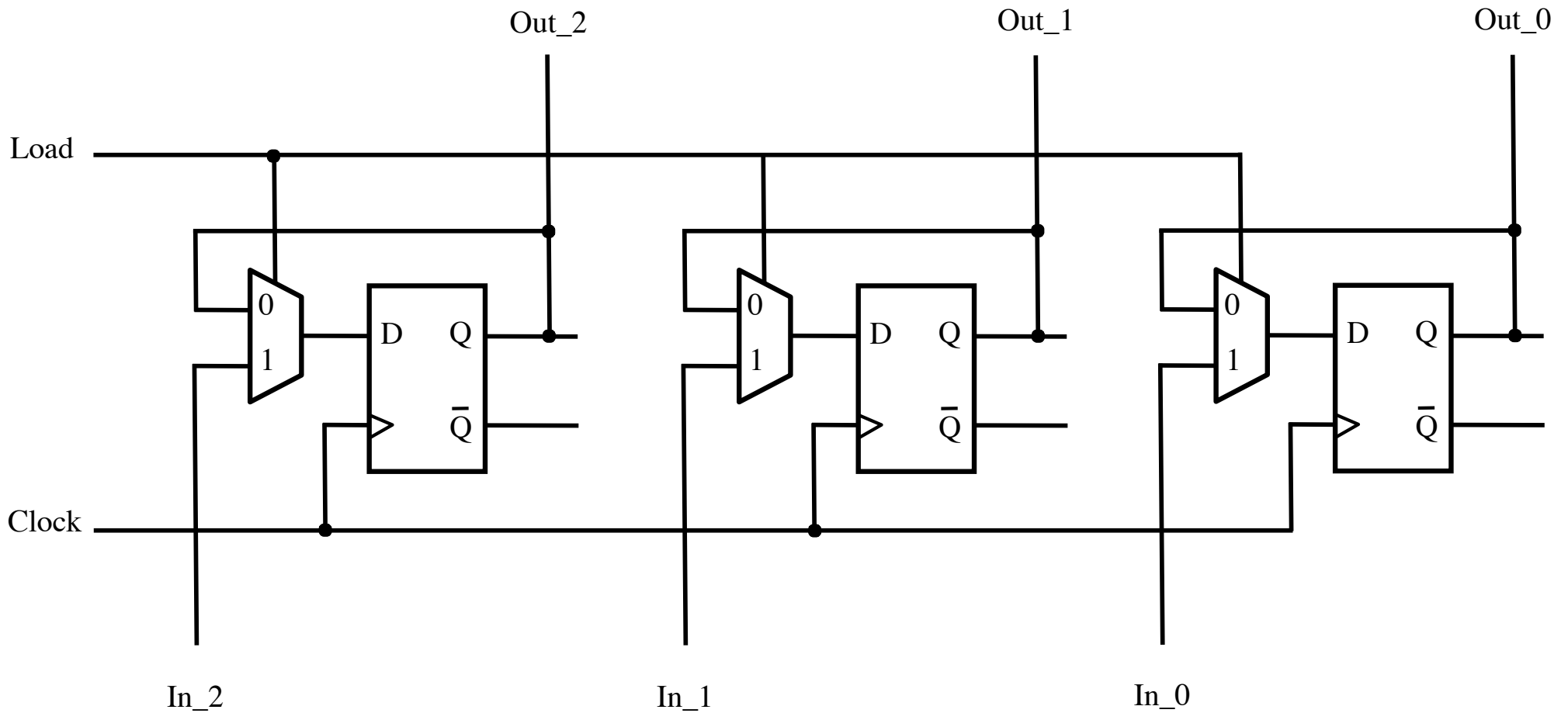
If Load = 0, then retain the old value.

If Load = 1, then load the new value from In.

2-Bit Parallel-Access Register

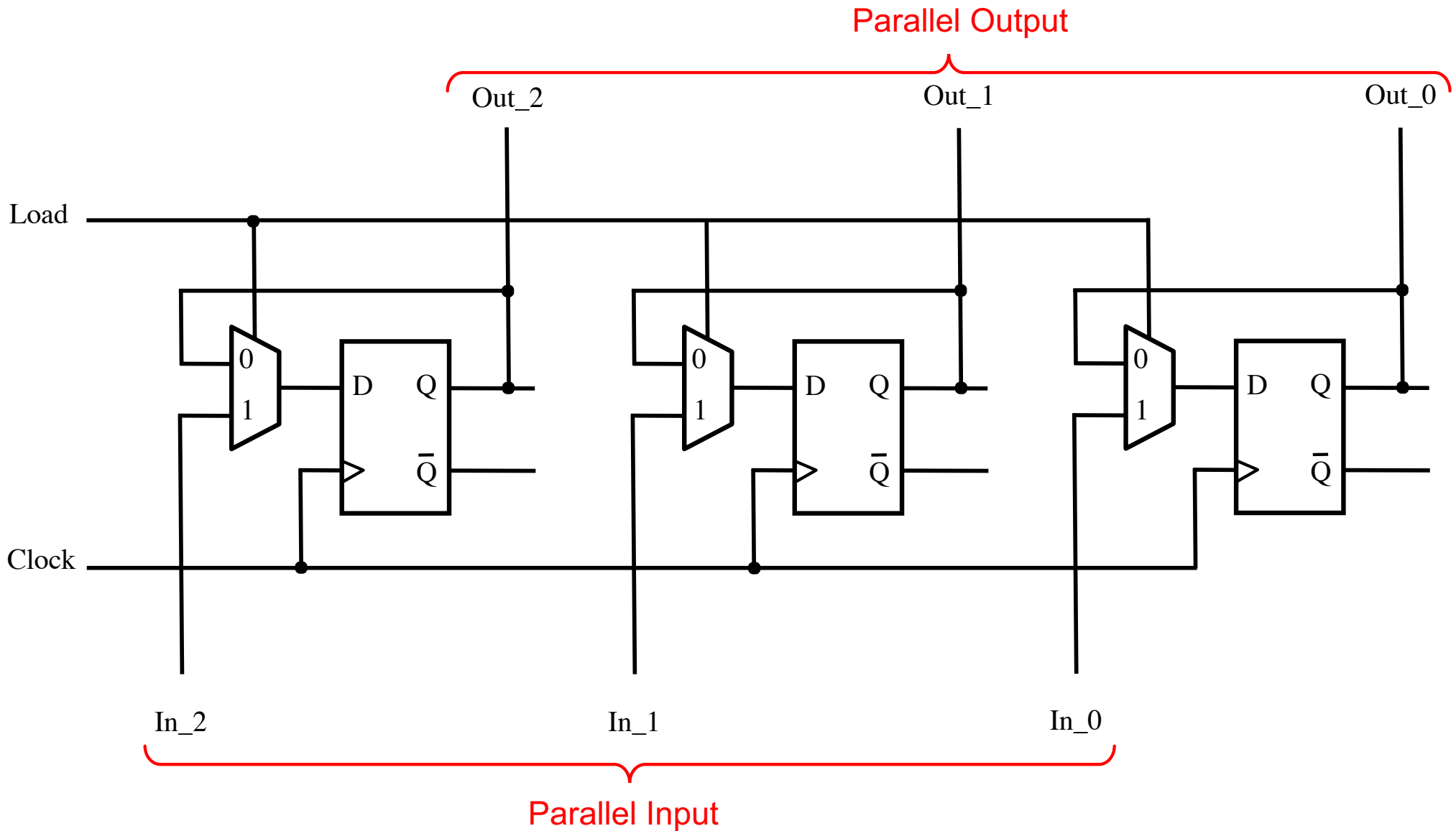


3-Bit Parallel-Access Register

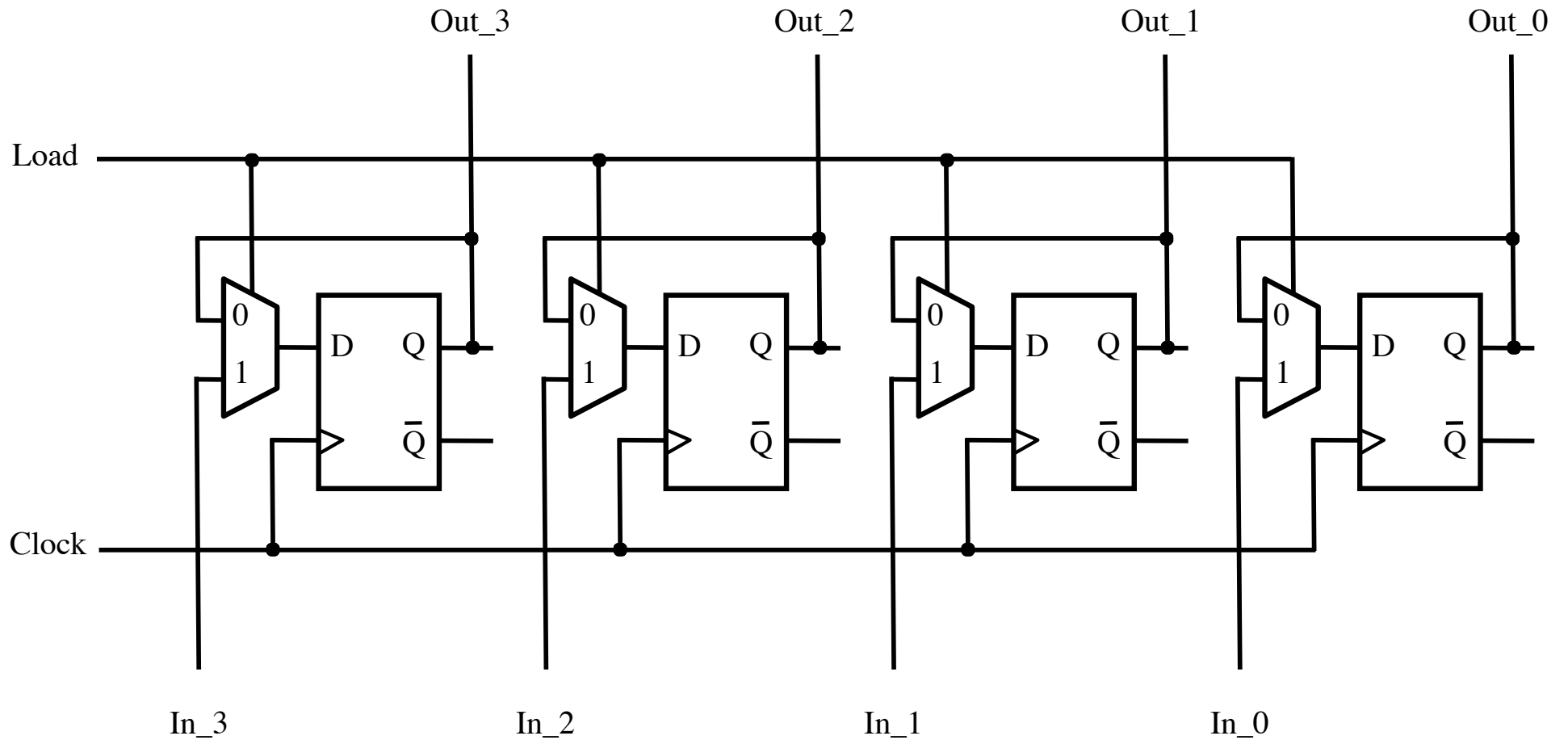


Notice that all flip-flops are on the same clock cycle.

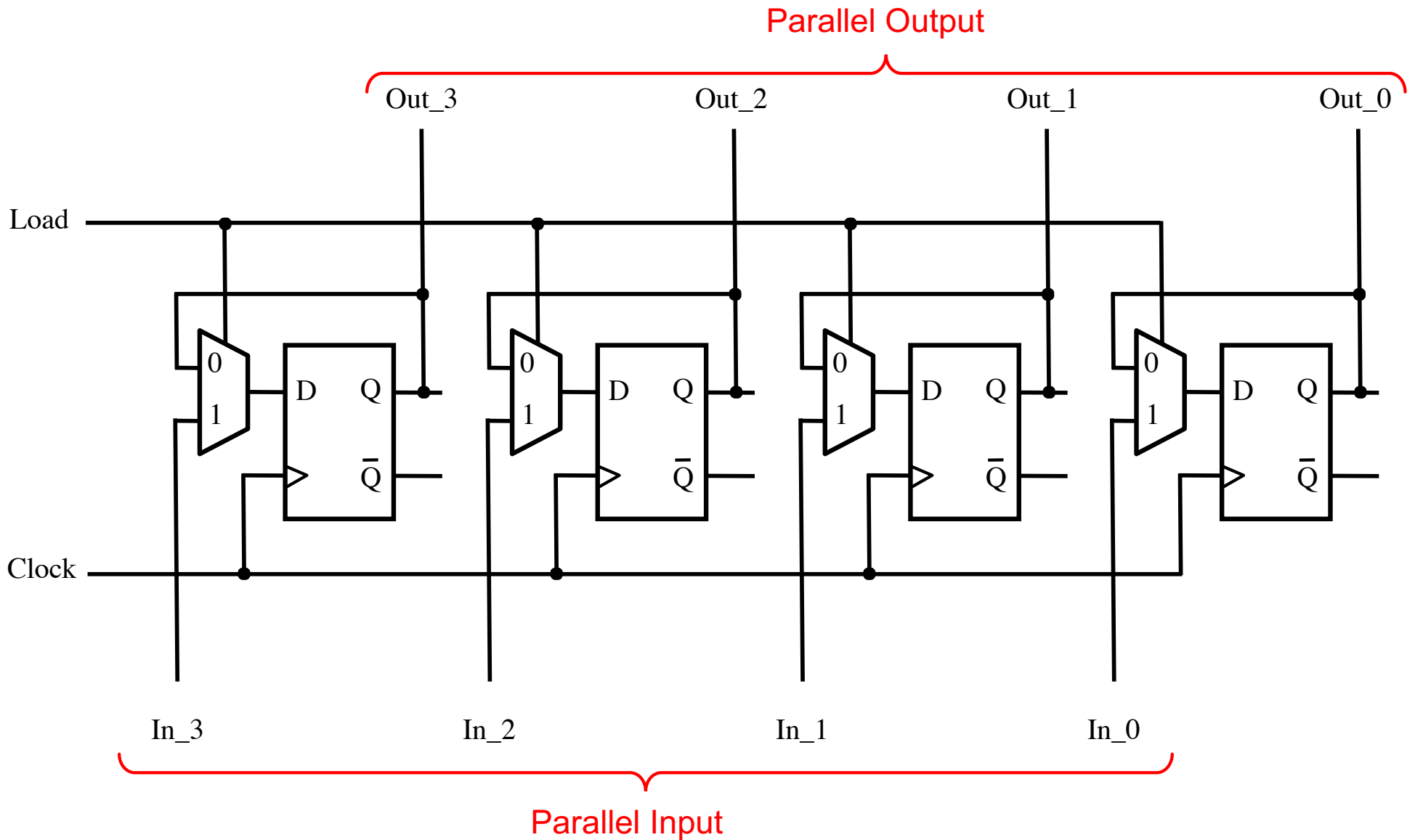
3-Bit Parallel-Access Register



4-Bit Parallel-Access Register

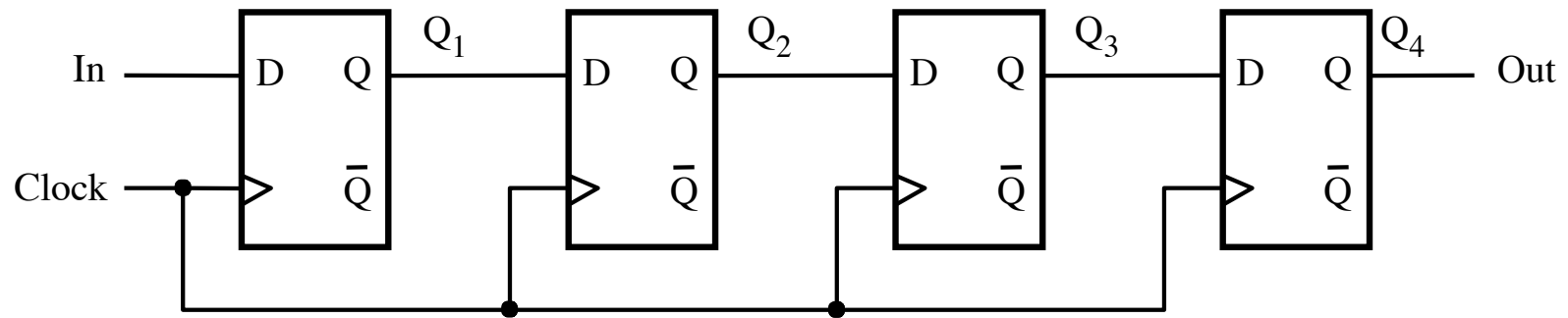


4-Bit Parallel-Access Register

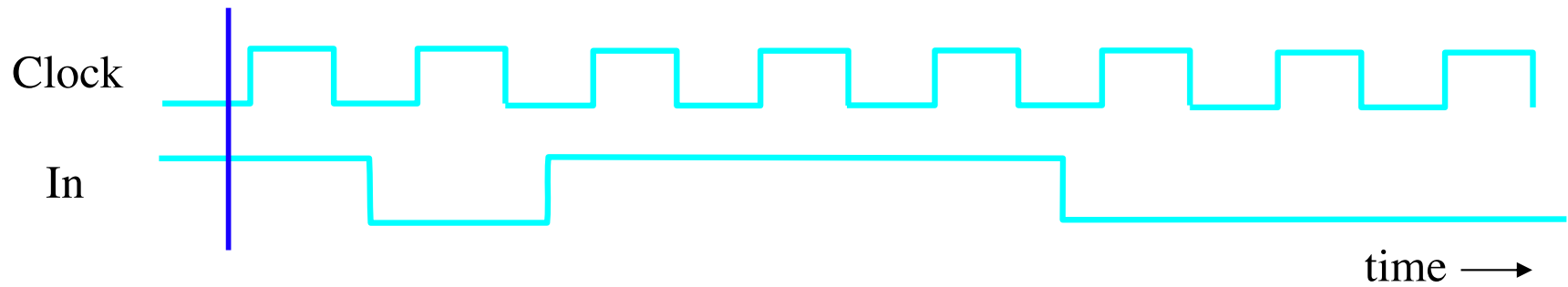
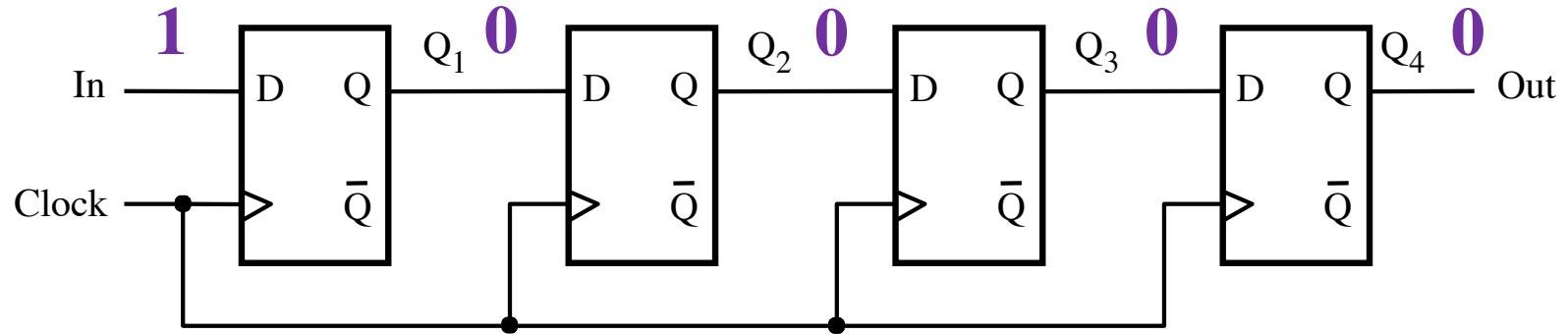


Shift Register

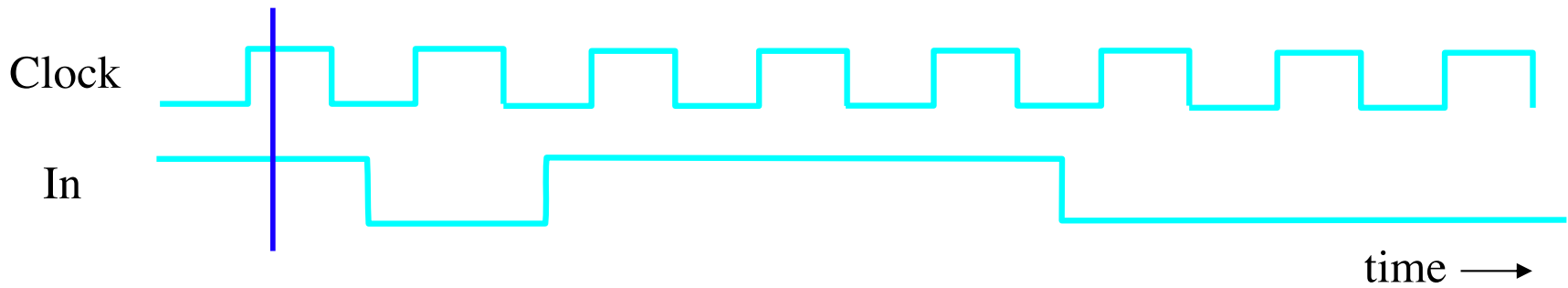
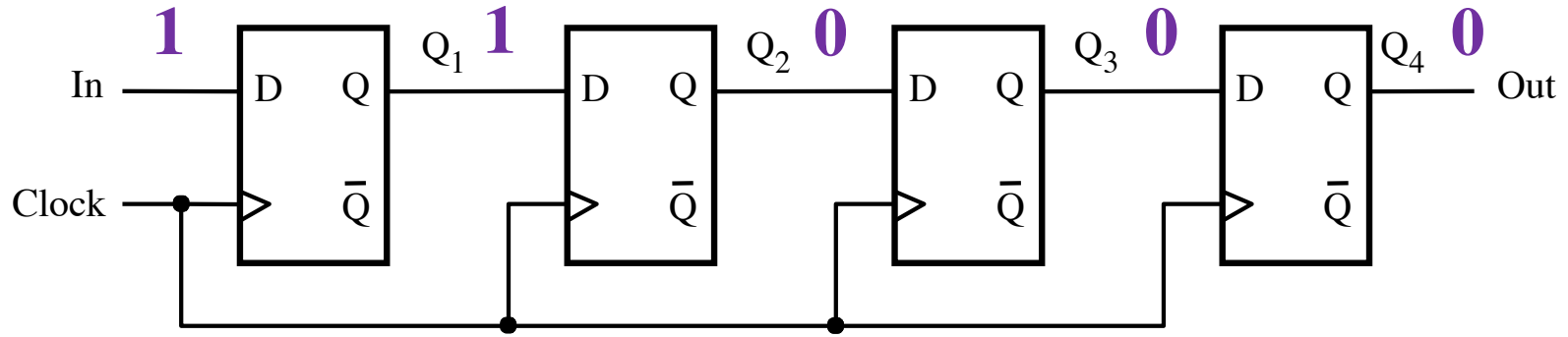
A simple shift register



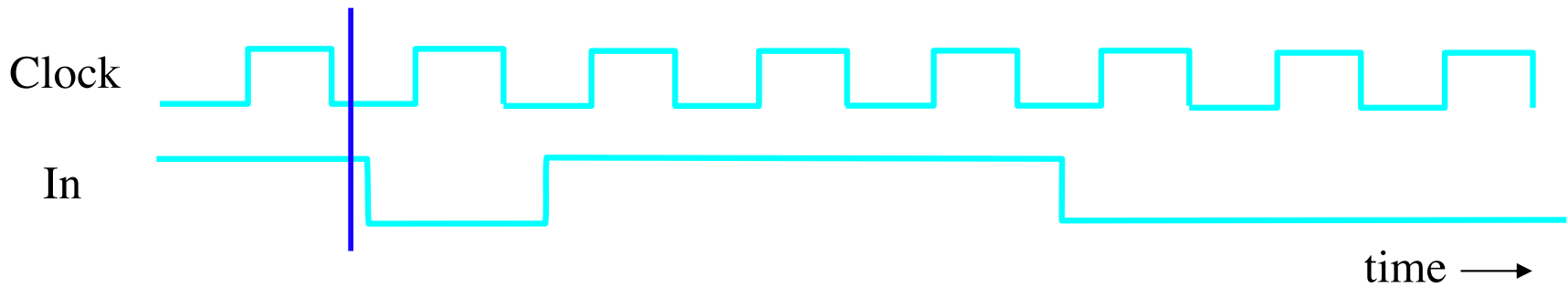
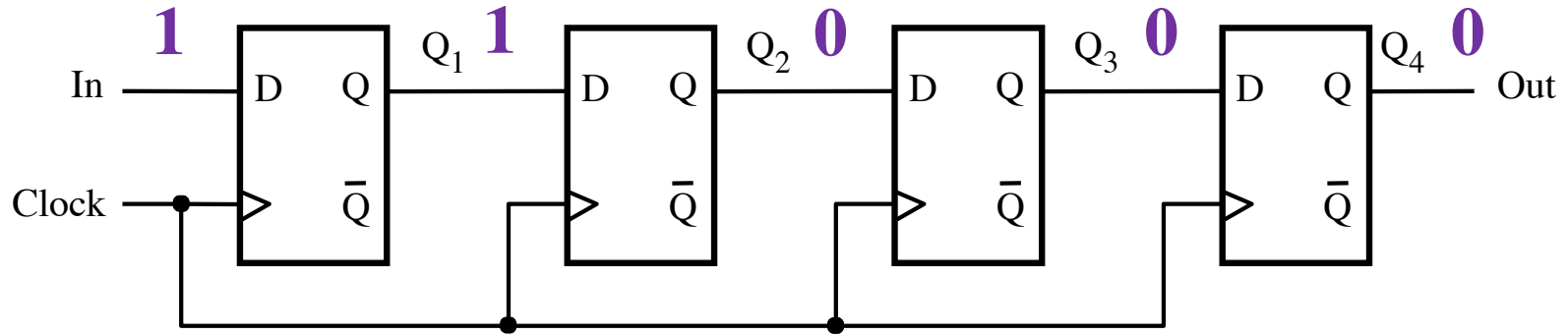
Shift Register Simulation



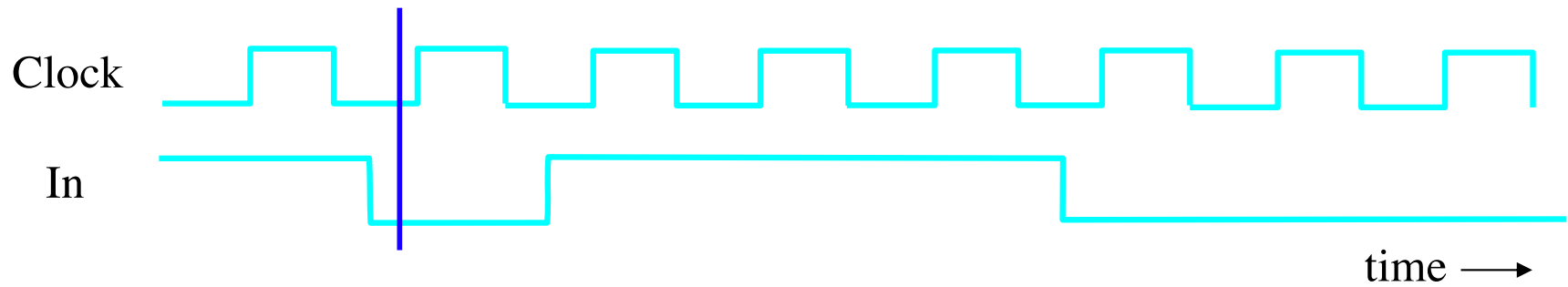
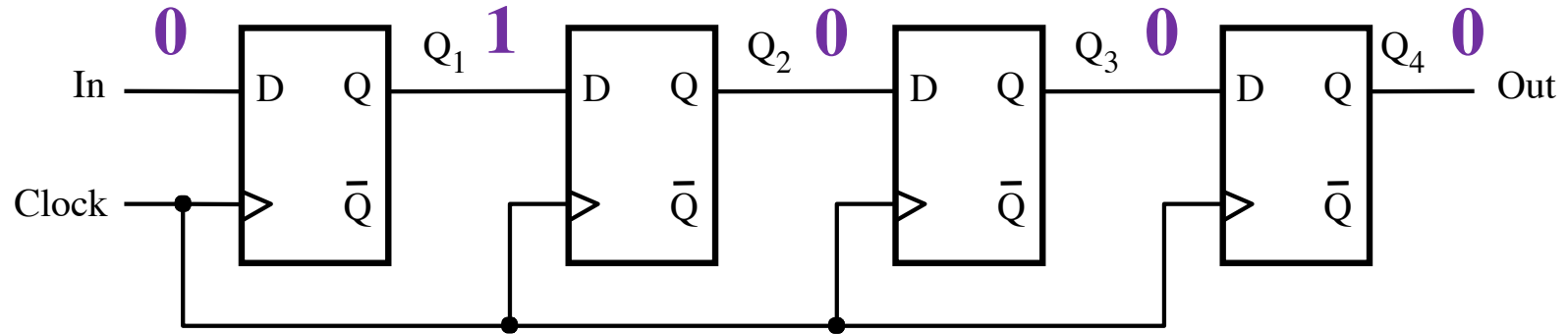
Shift Register Simulation



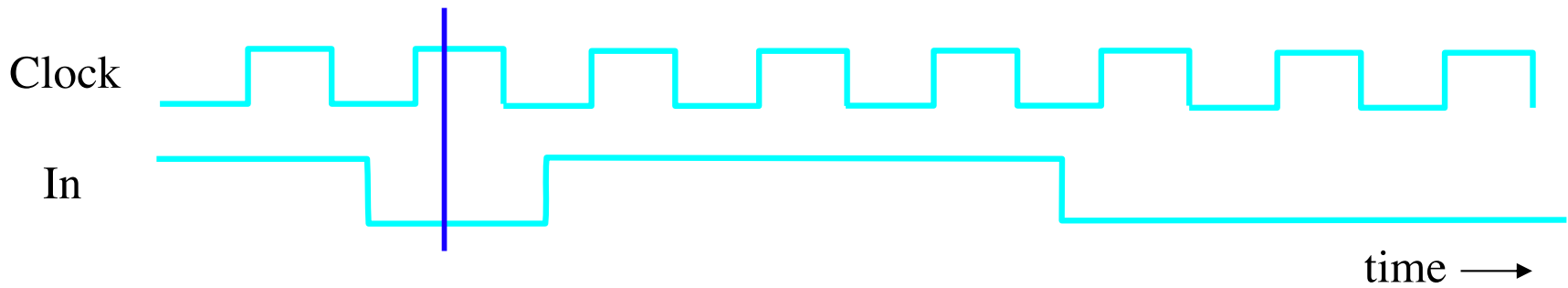
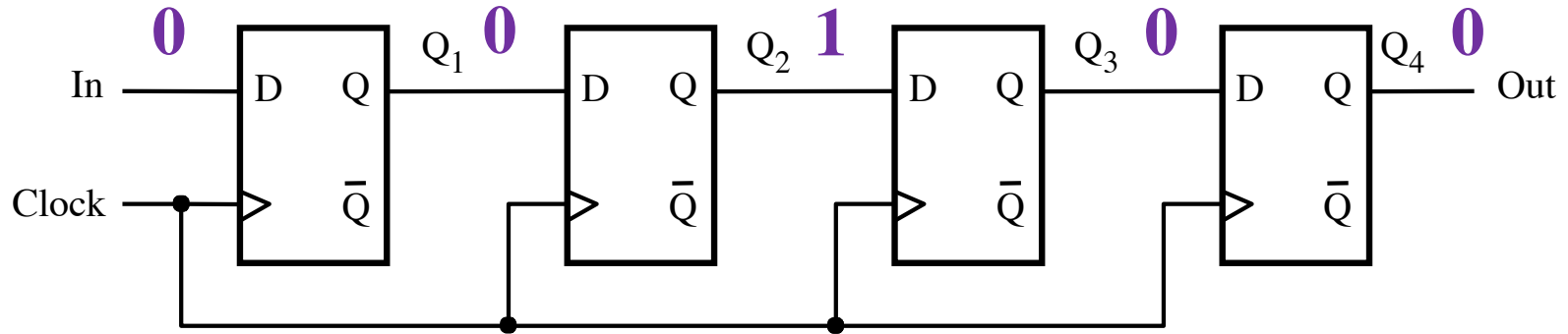
Shift Register Simulation



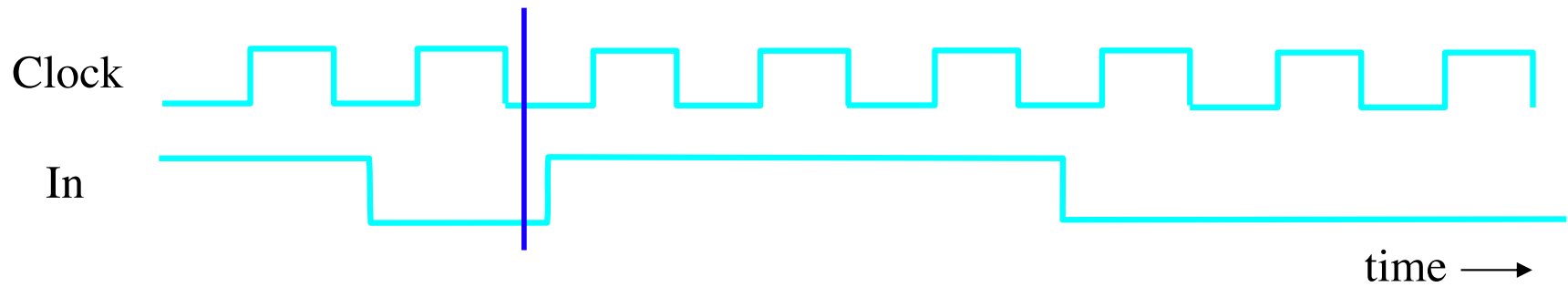
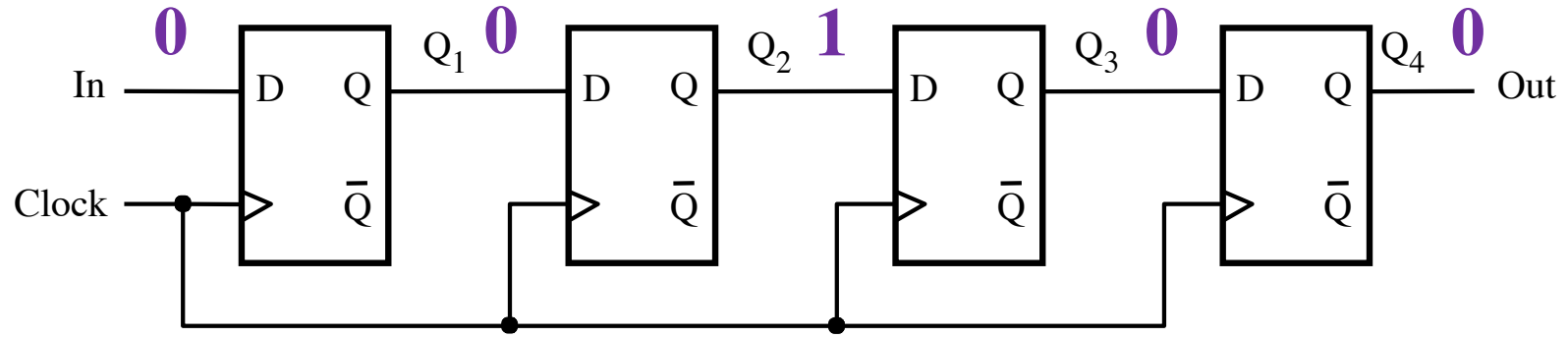
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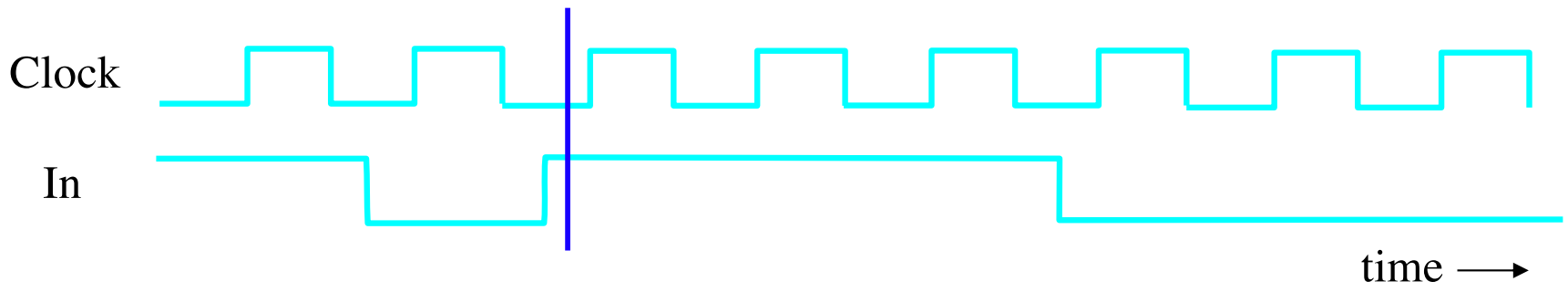
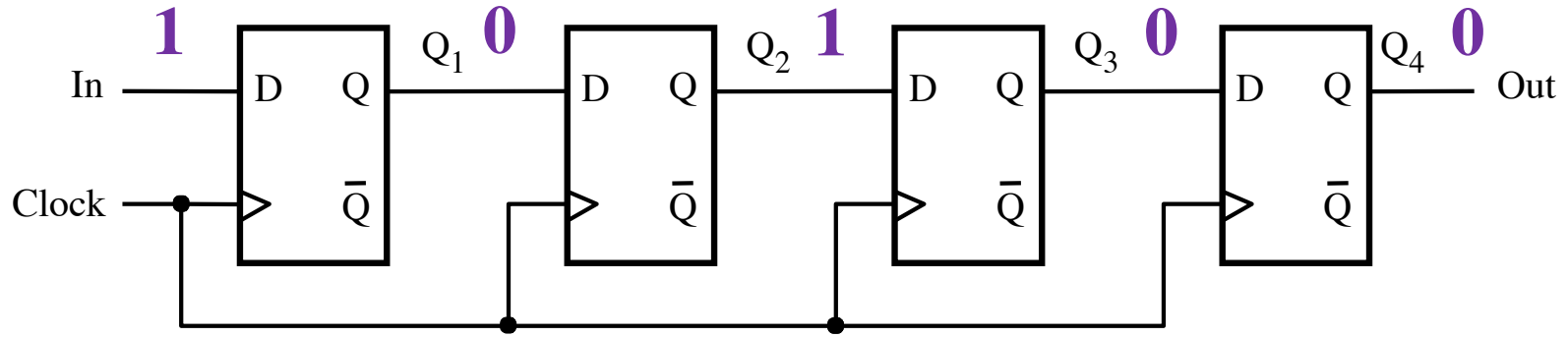
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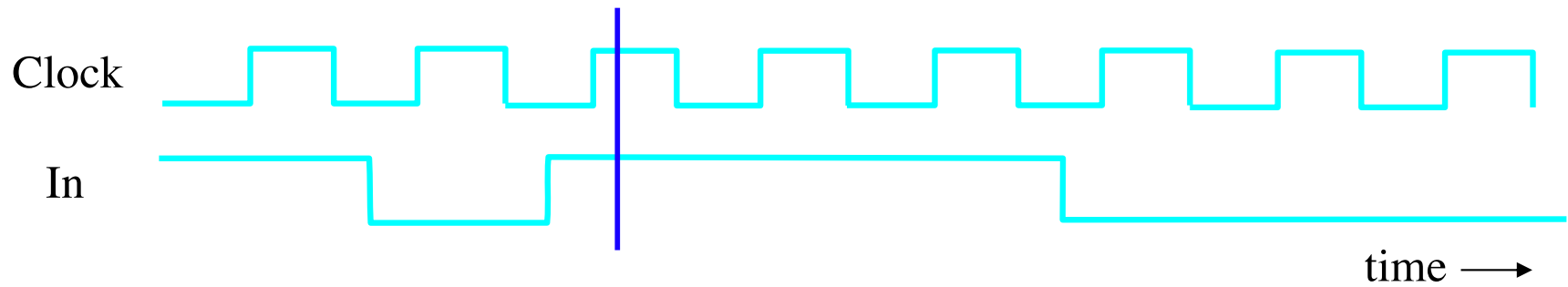
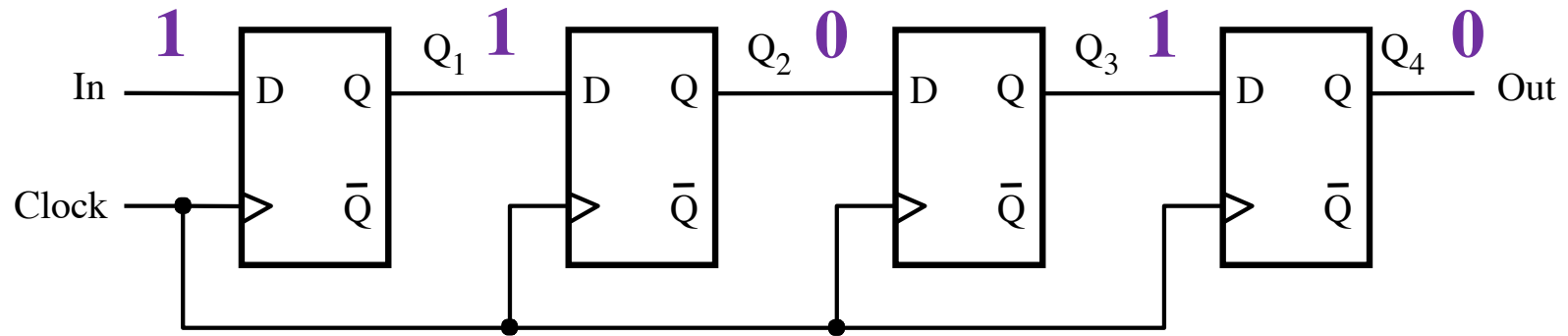
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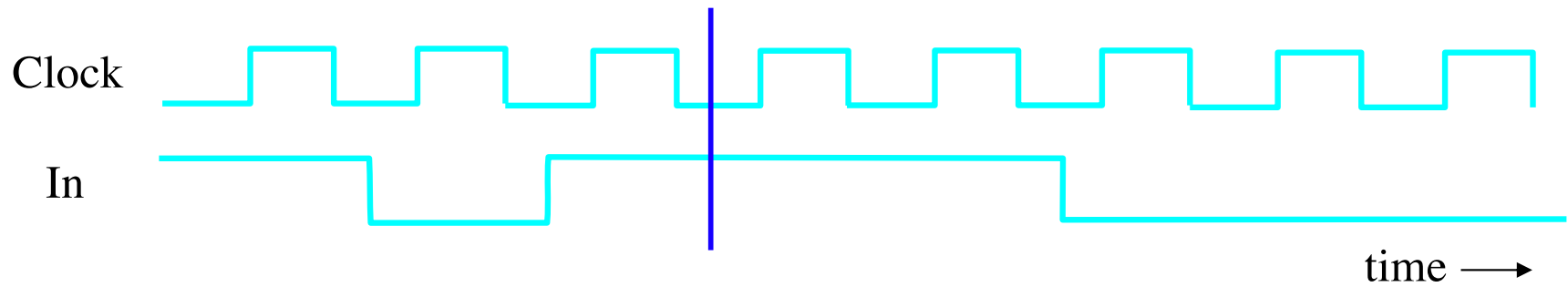
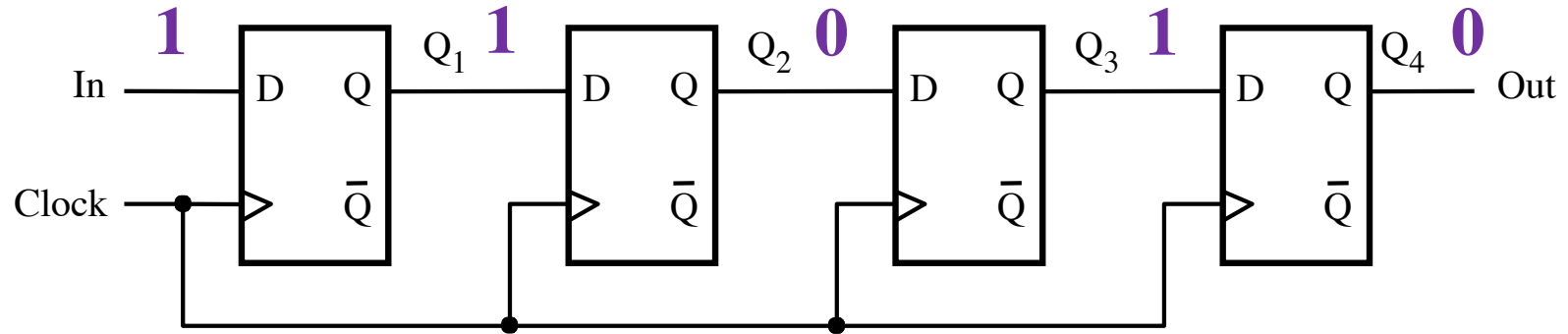
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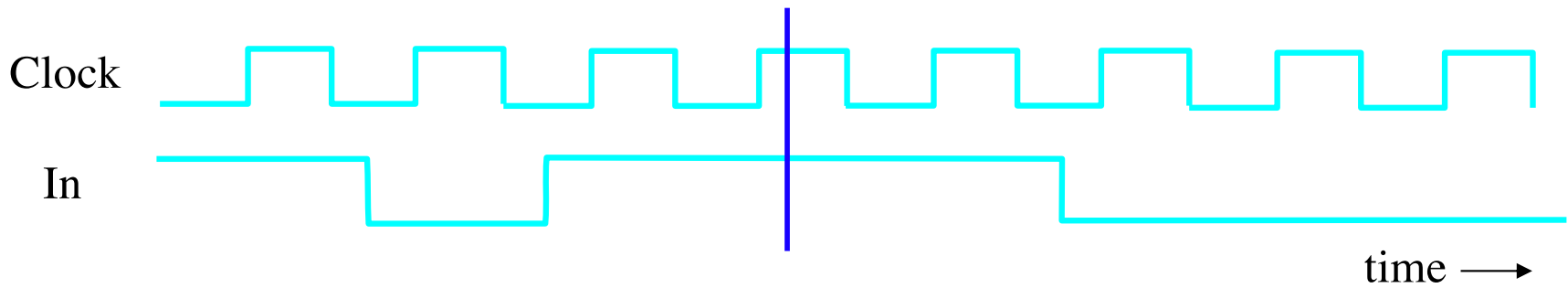
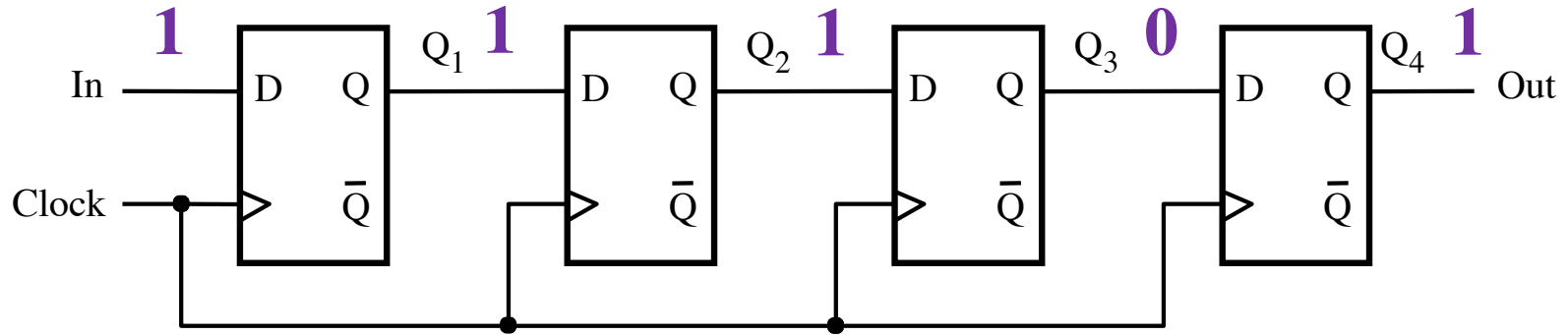
Shift Register Simulation



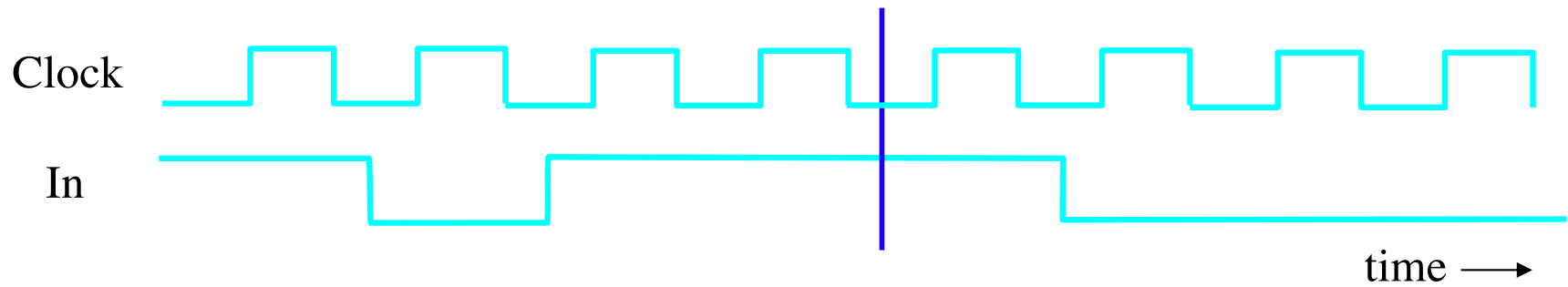
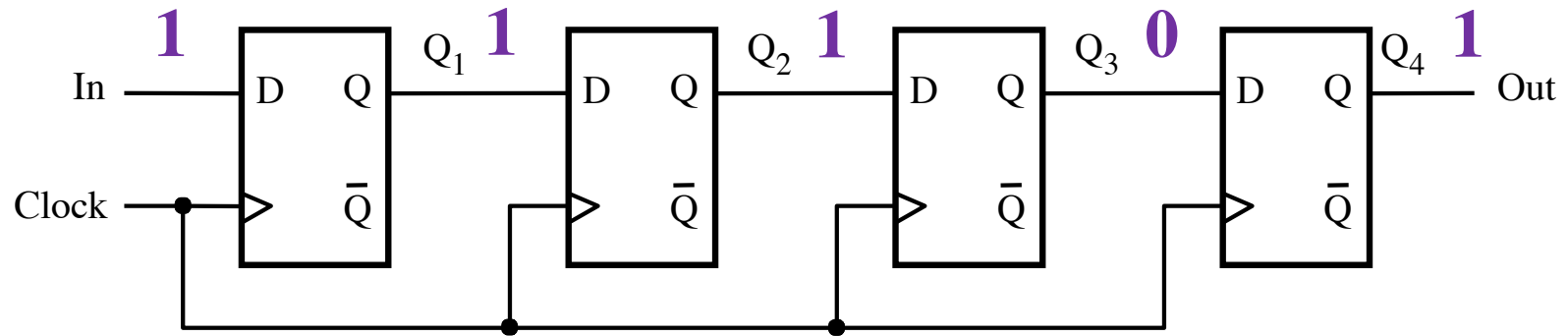
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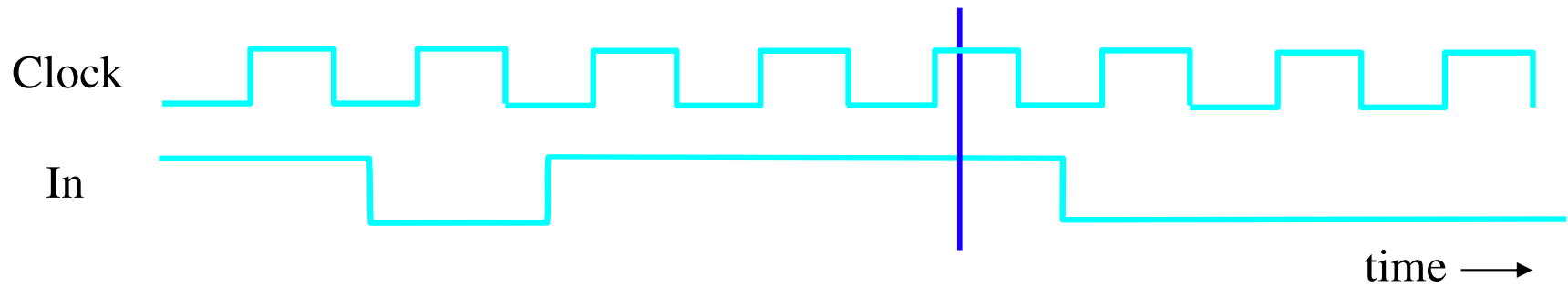
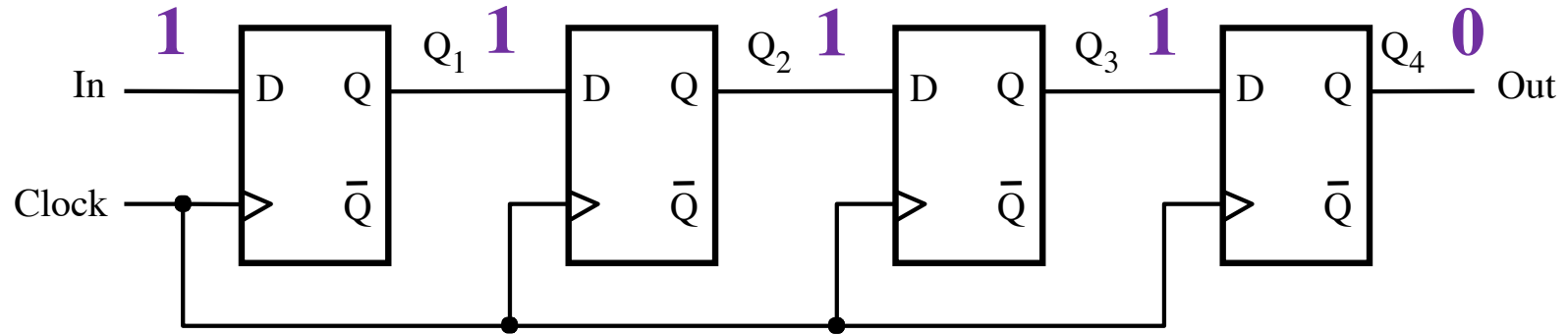
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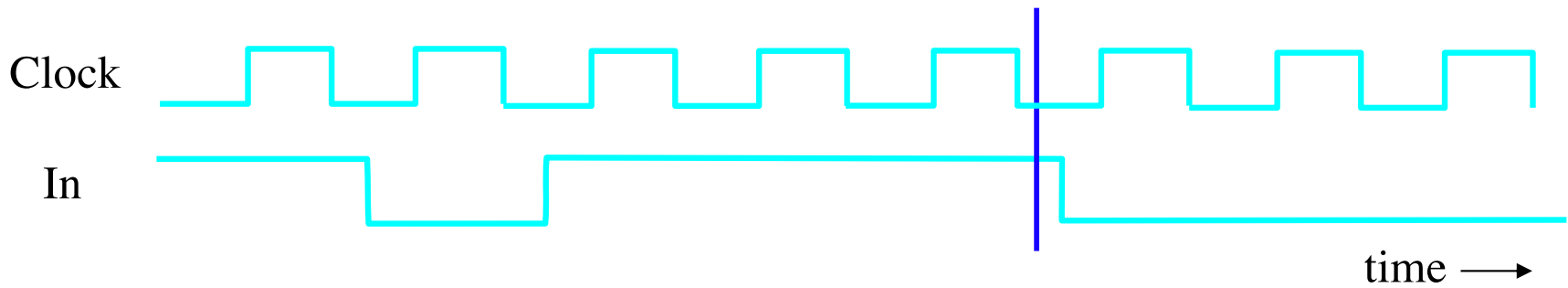
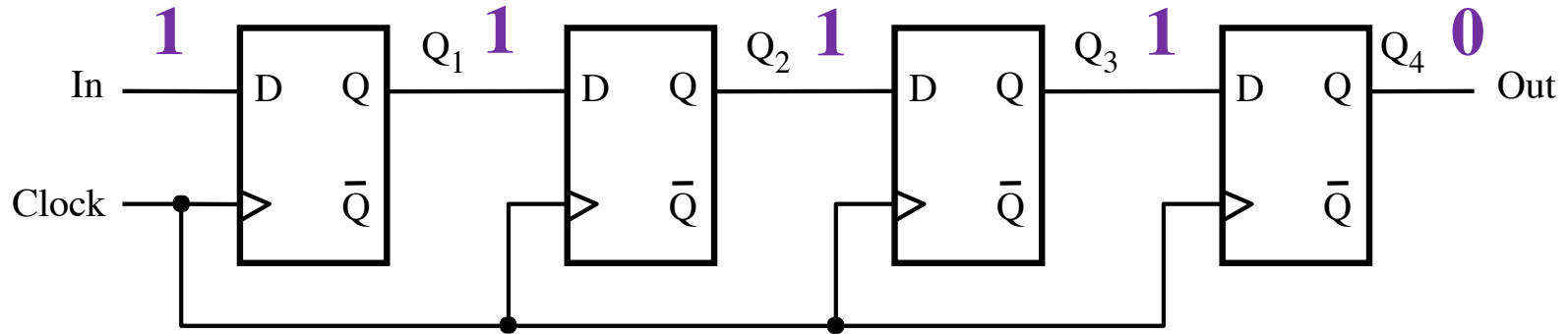
Shift Register Simulation



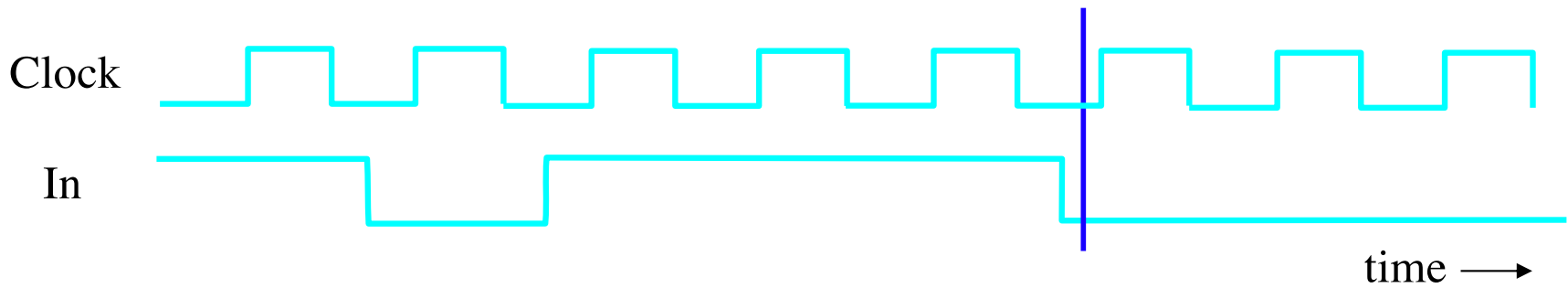
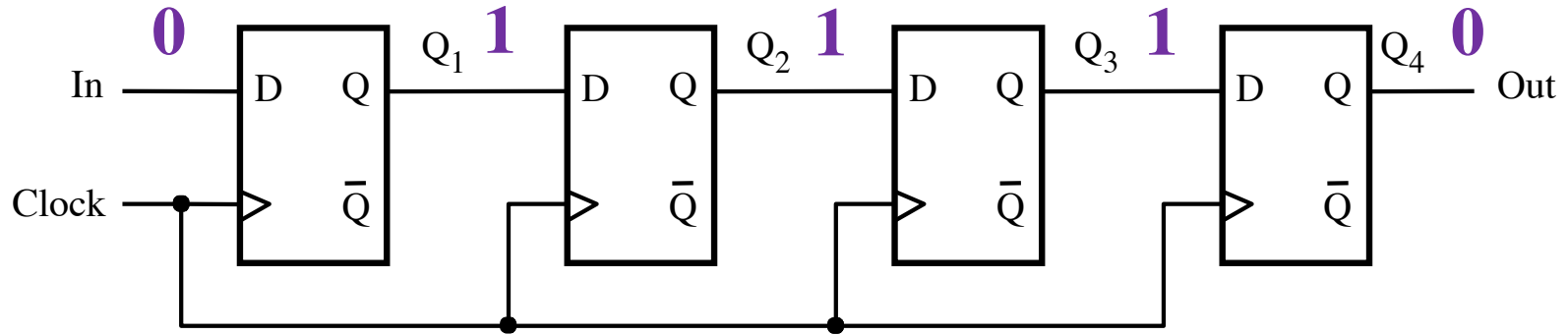
Shift Register Simulation



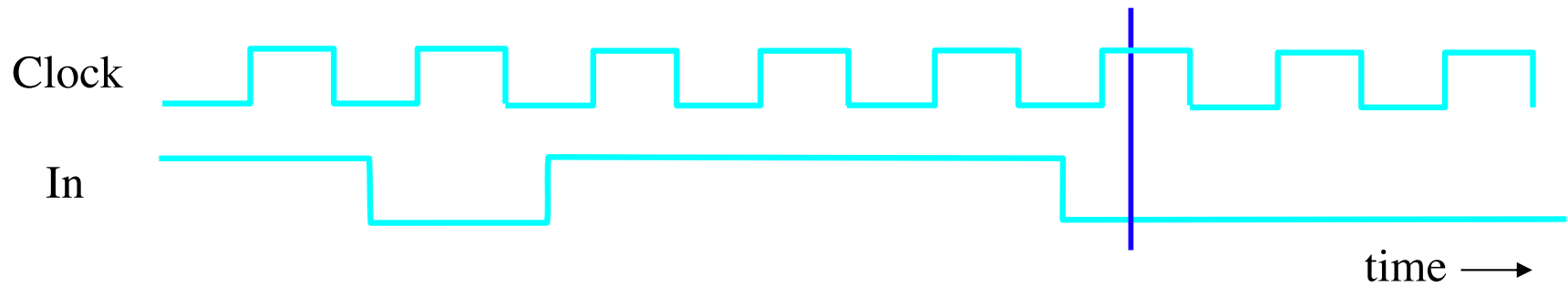
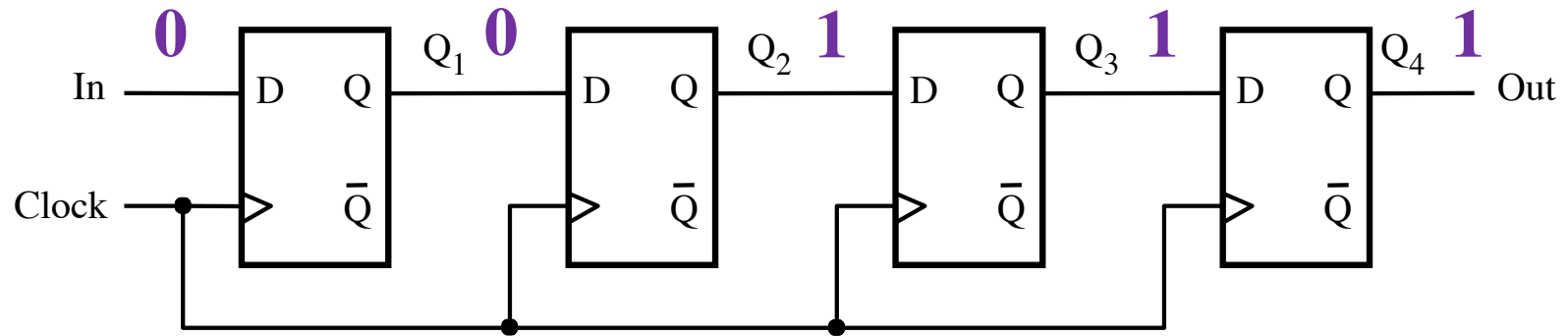
Shift Register Simulation



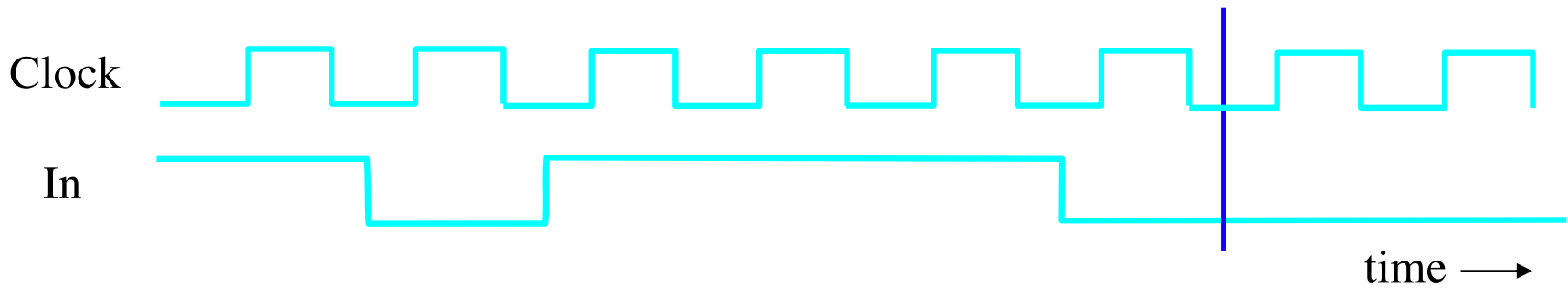
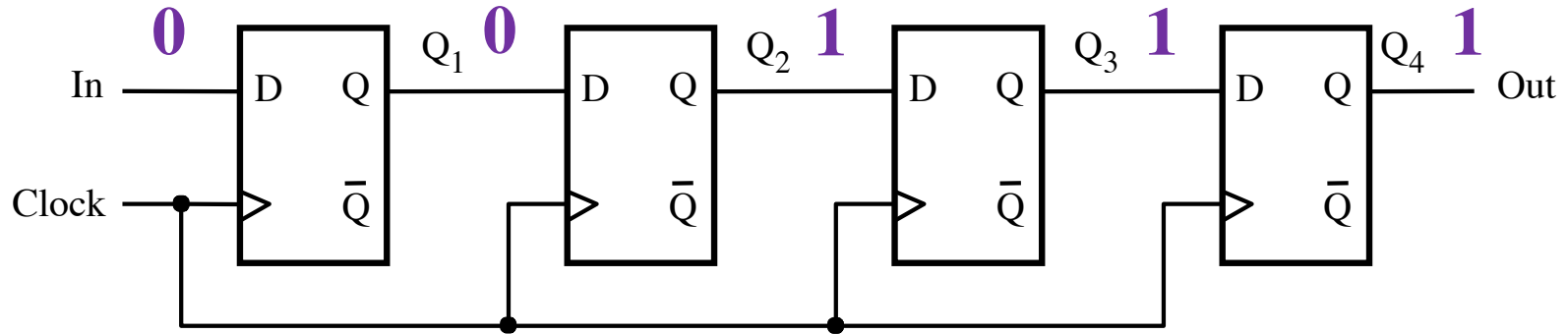
Shift Register Simulation



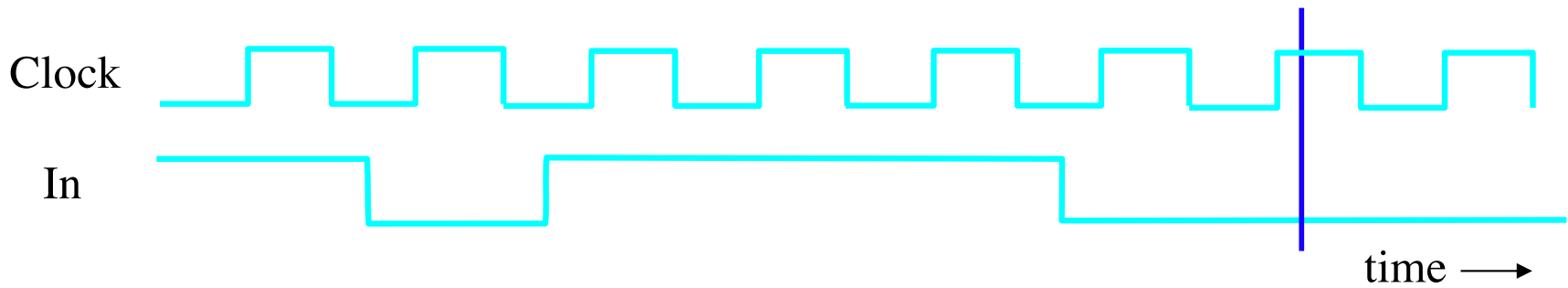
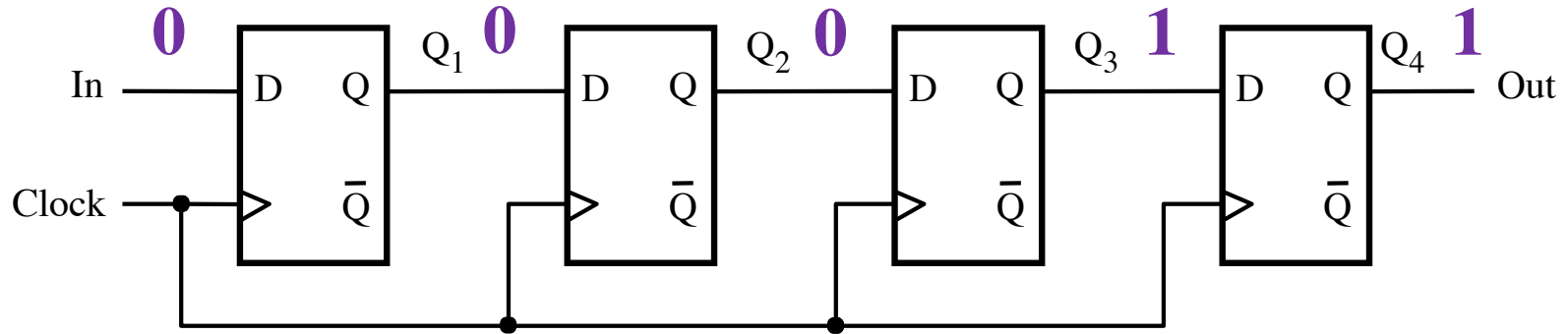
Shift Register Simulation



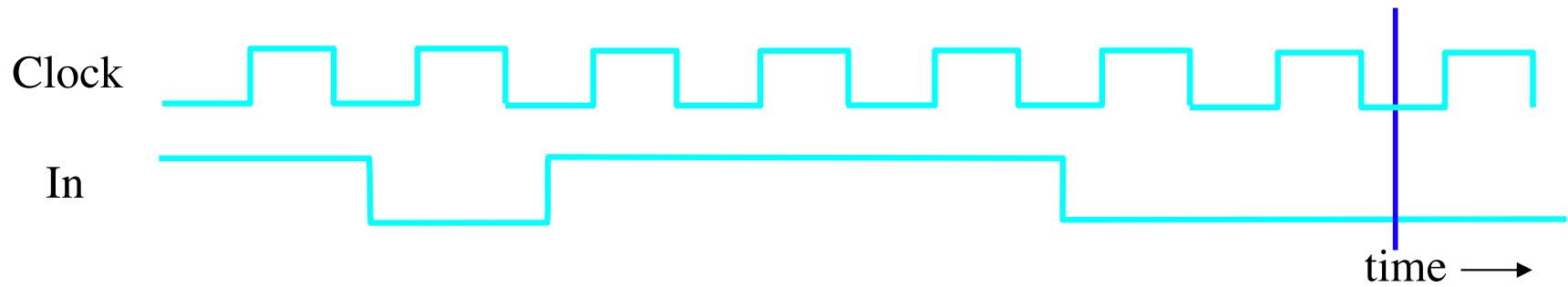
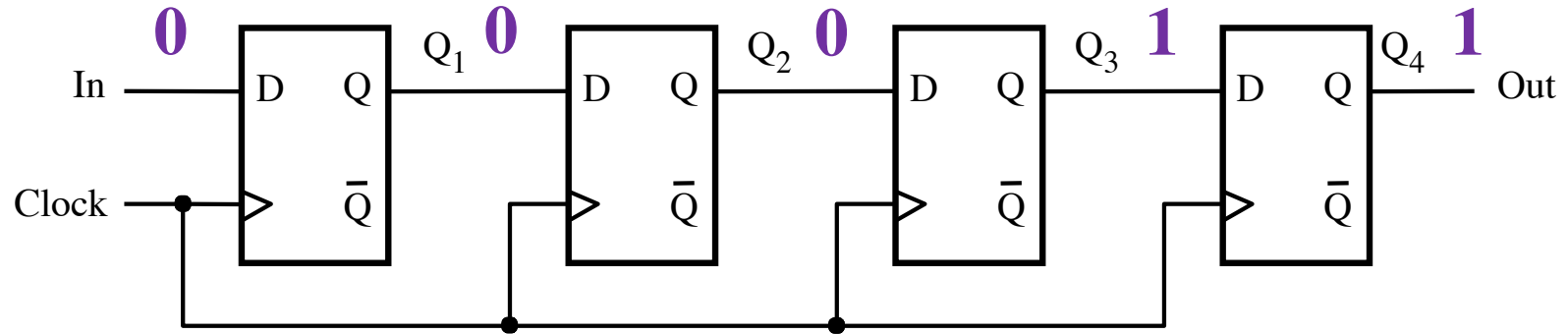
Shift Register Simulation



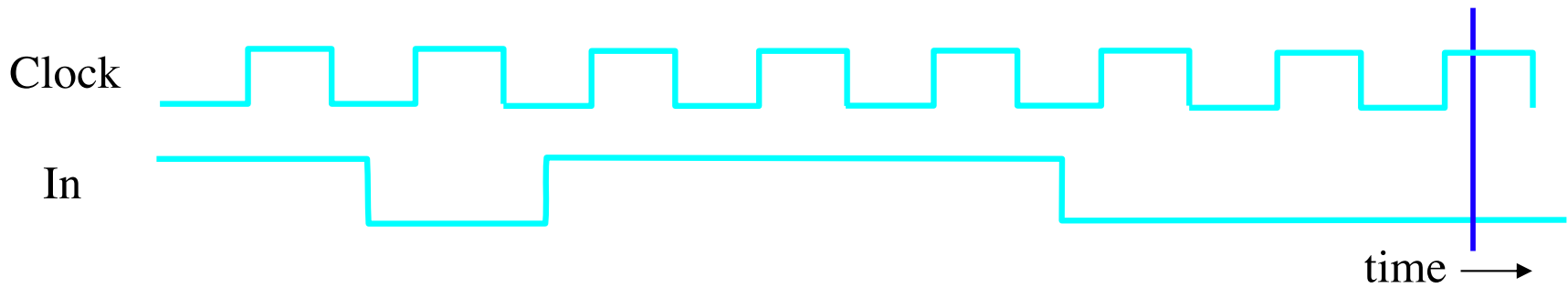
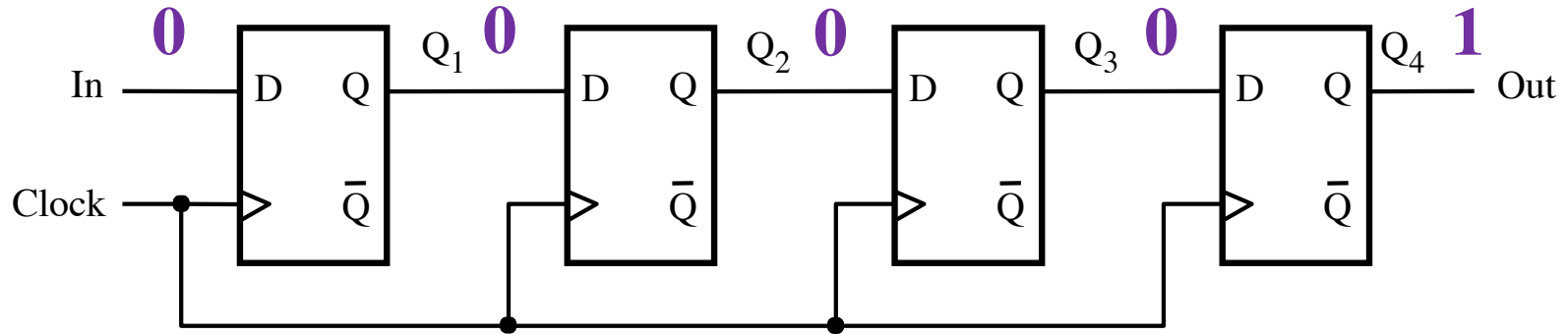
Shift Register Simulation



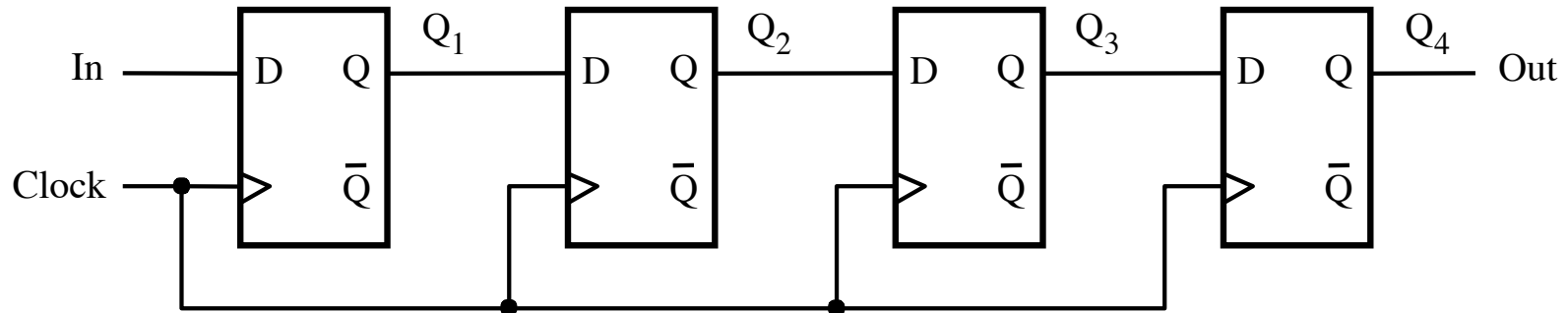
Shift Register Simulation



Shift Register Simulation



A simple shift register

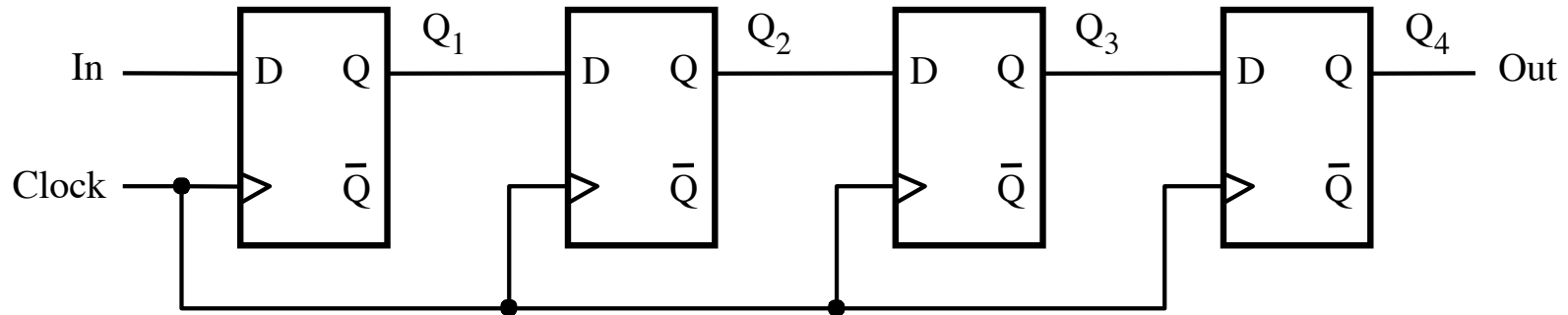


(a) Circuit

time	In	Q_1	Q_2	Q_3	$Q_4 = \text{Out}$
t_0	1	0	0	0	0
t_1	0	1	0	0	0
t_2	1	0	1	0	0
t_3	1	1	0	1	0
t_4	1	1	1	0	1
t_5	0	1	1	1	0
t_6	0	0	1	1	1
t_7	0	0	0	1	1

(b) A sample sequence

A simple shift register

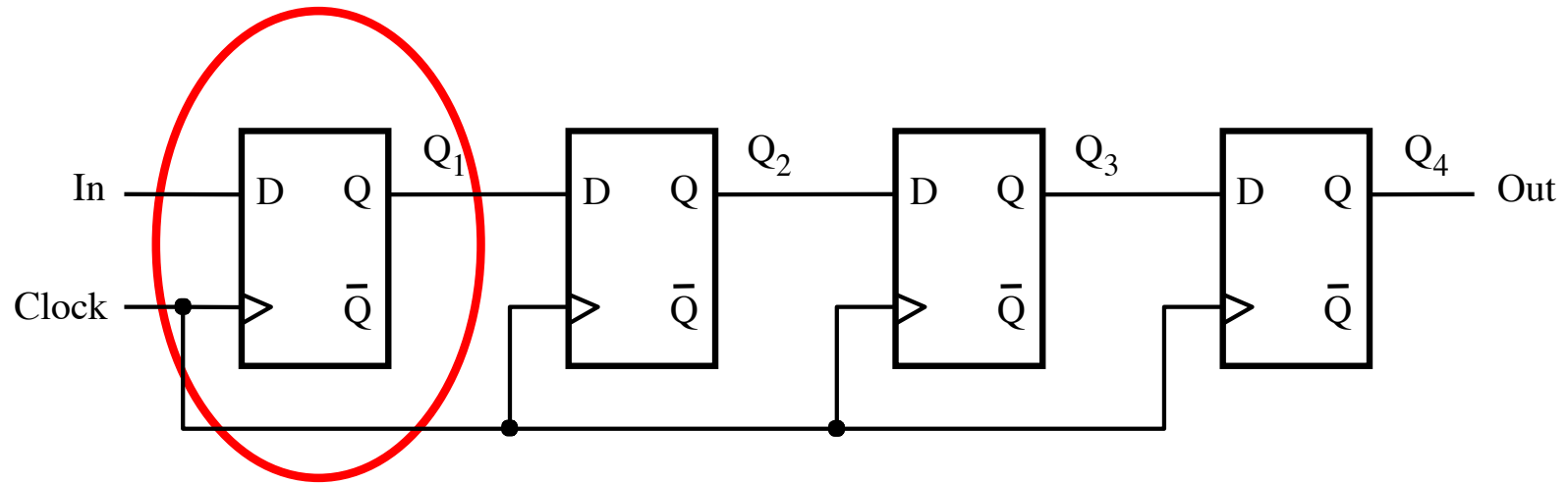


(a) Circuit

time	In	Q ₁	Q ₂	Q ₃	Q ₄ = Out
t_0	1	0	0	0	0
t_1	0	1	0	0	0
t_2	1	0	1	0	0
t_3	1	1	0	1	0
t_4	1	1	1	0	1
t_5	0	1	1	1	0
t_6	0	0	1	1	1
t_7	0	0	0	1	1
	0	0	0	0	1

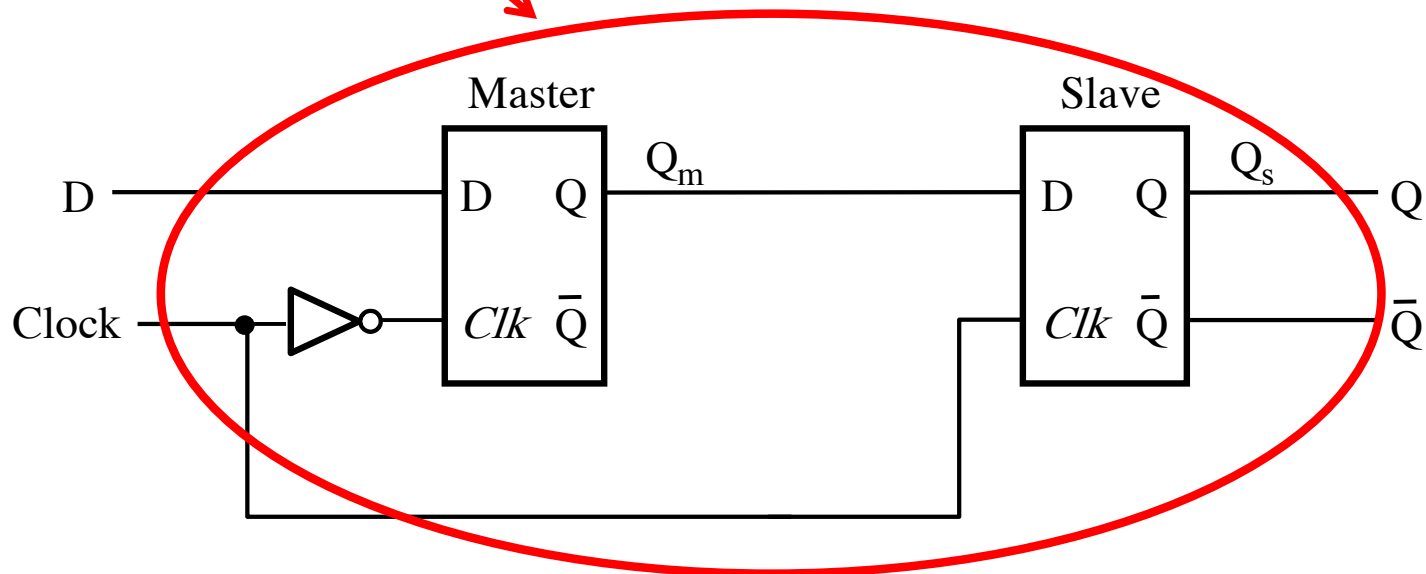
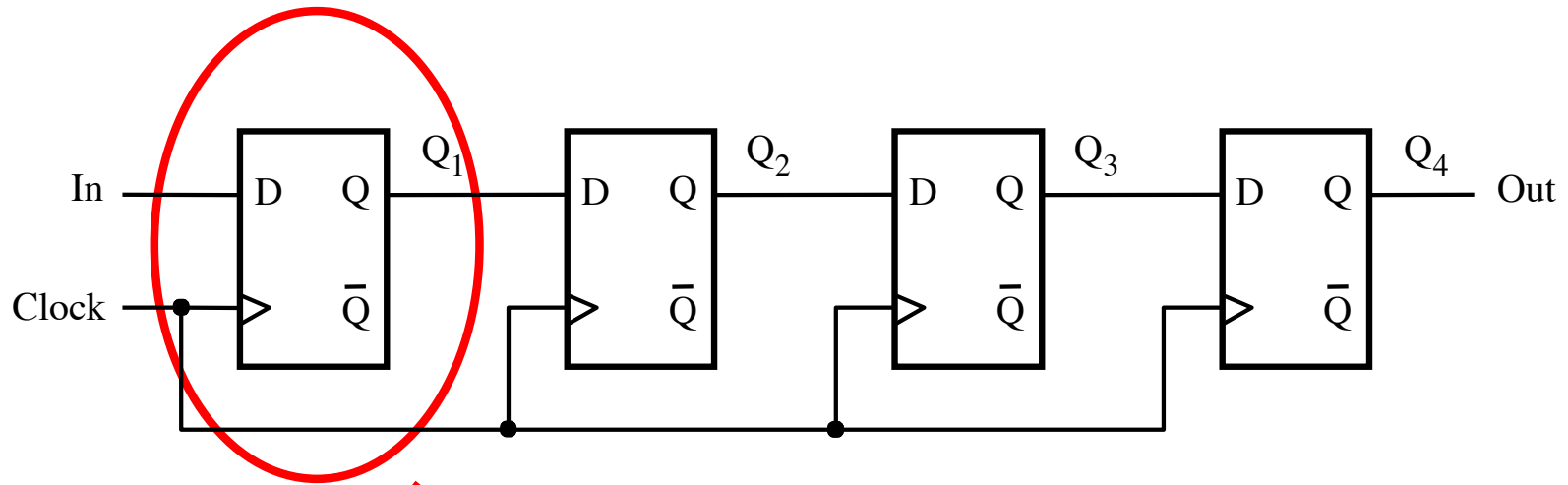
The simulation goes
one step further

A simple shift register

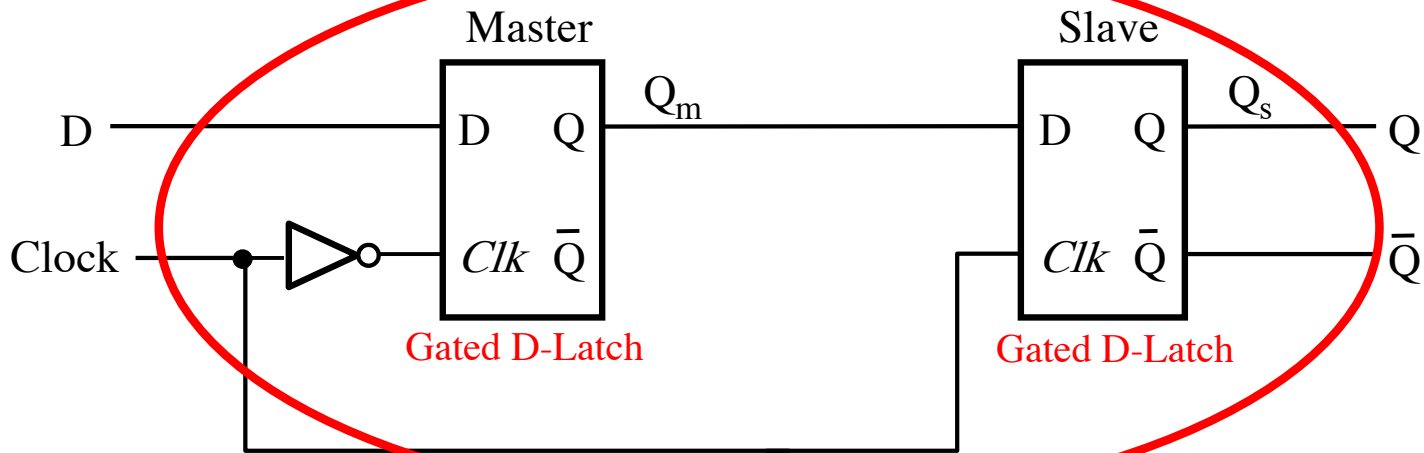
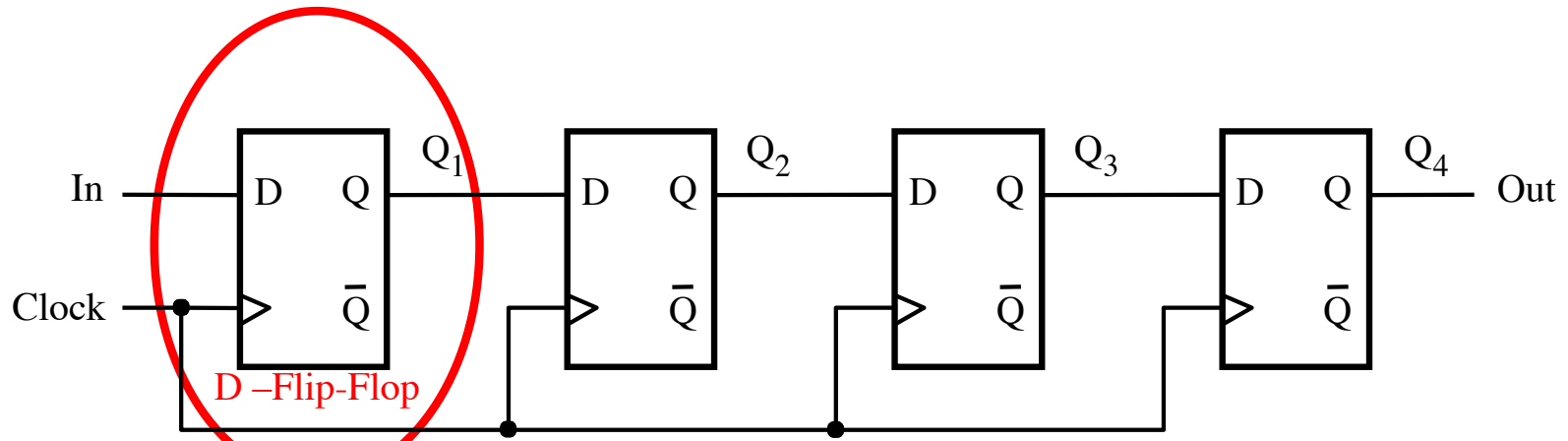


Positive-edge-triggered
D Flip-Flop

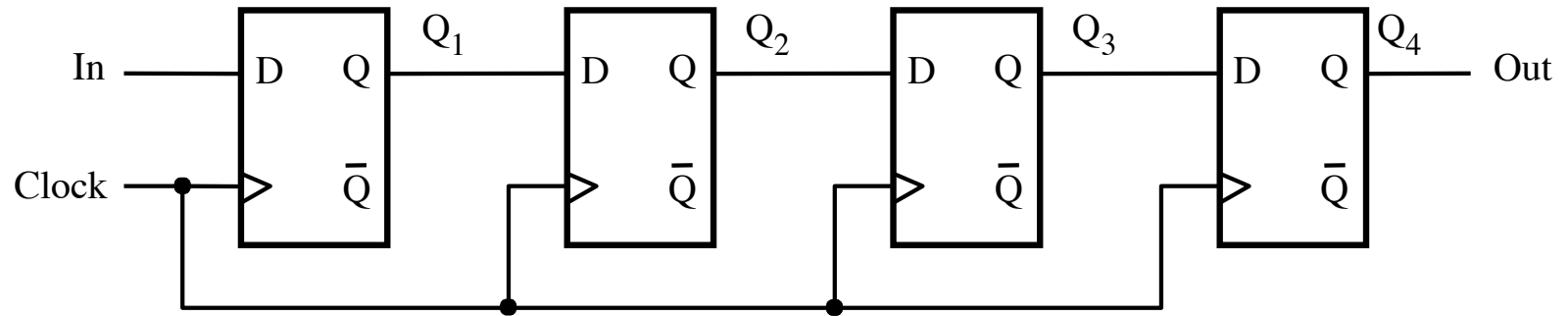
A simple shift register



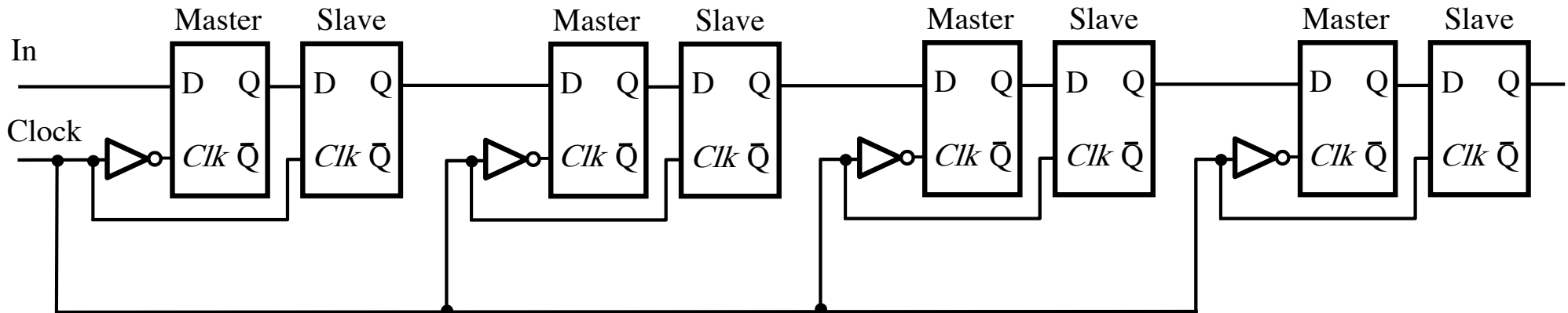
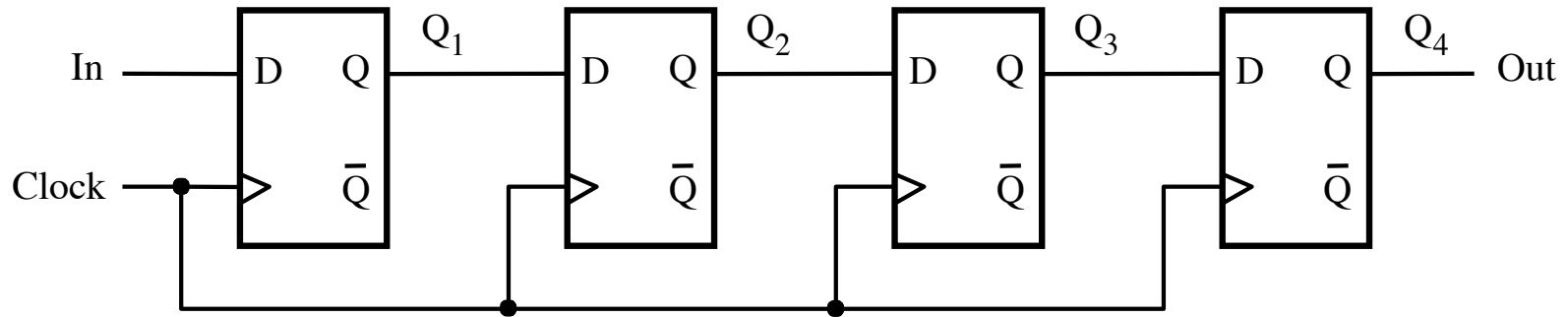
A simple shift register



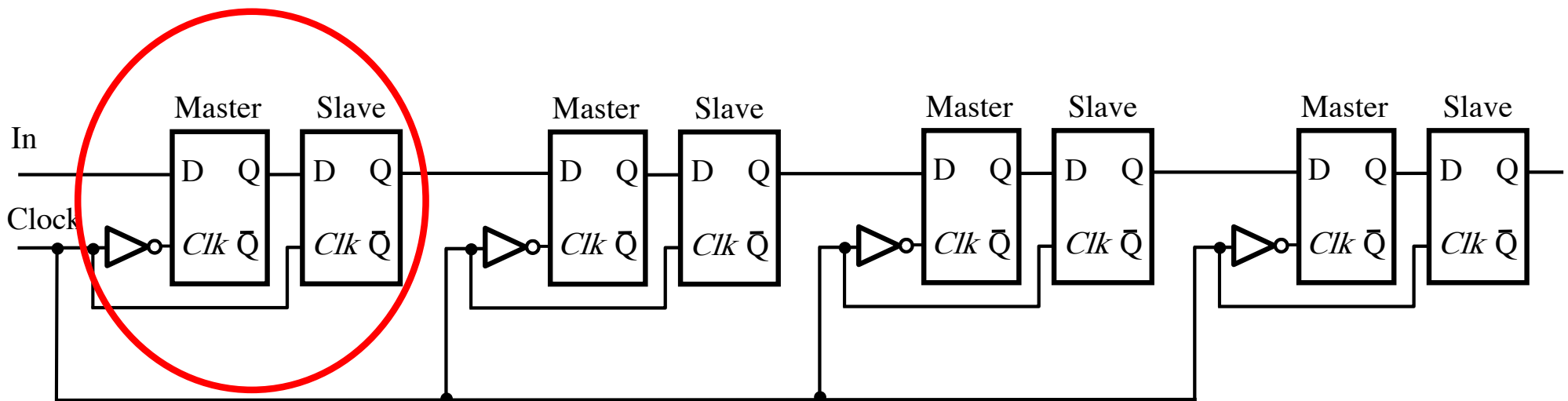
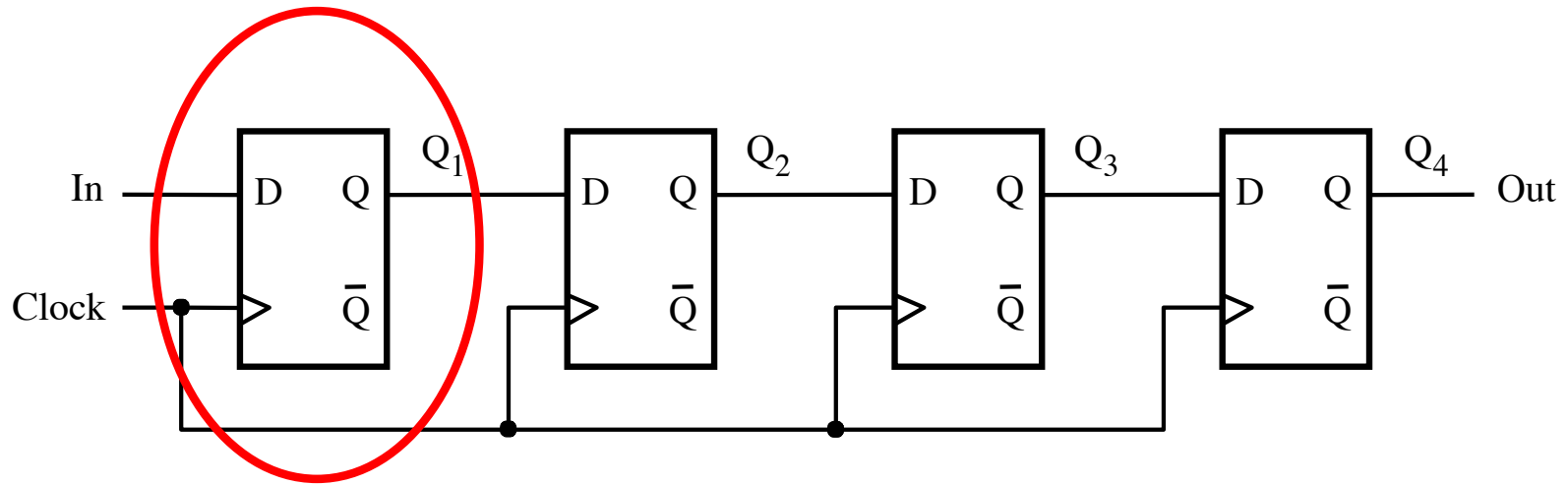
A simple shift register



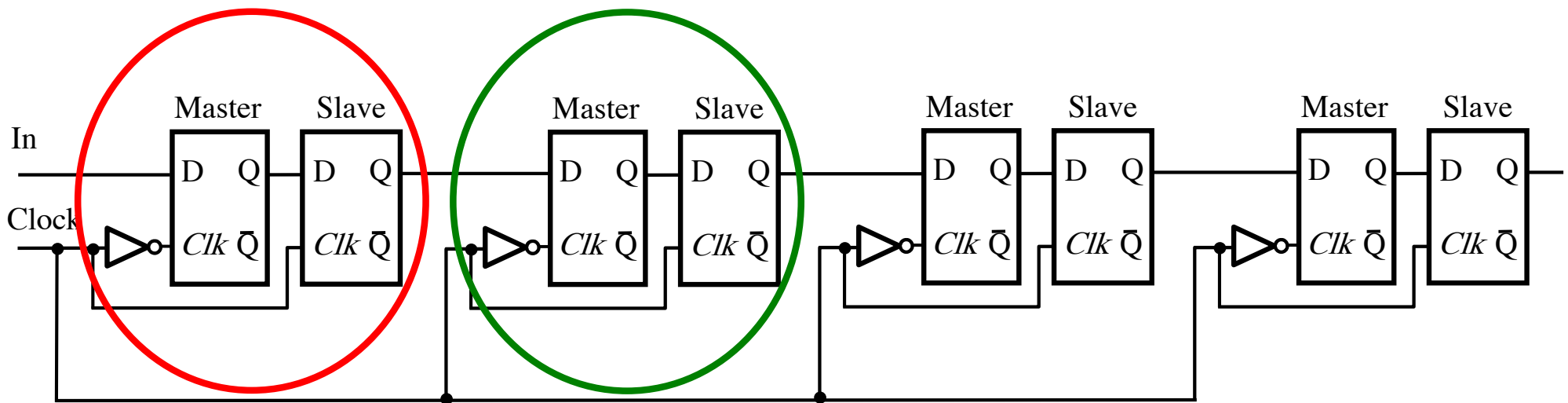
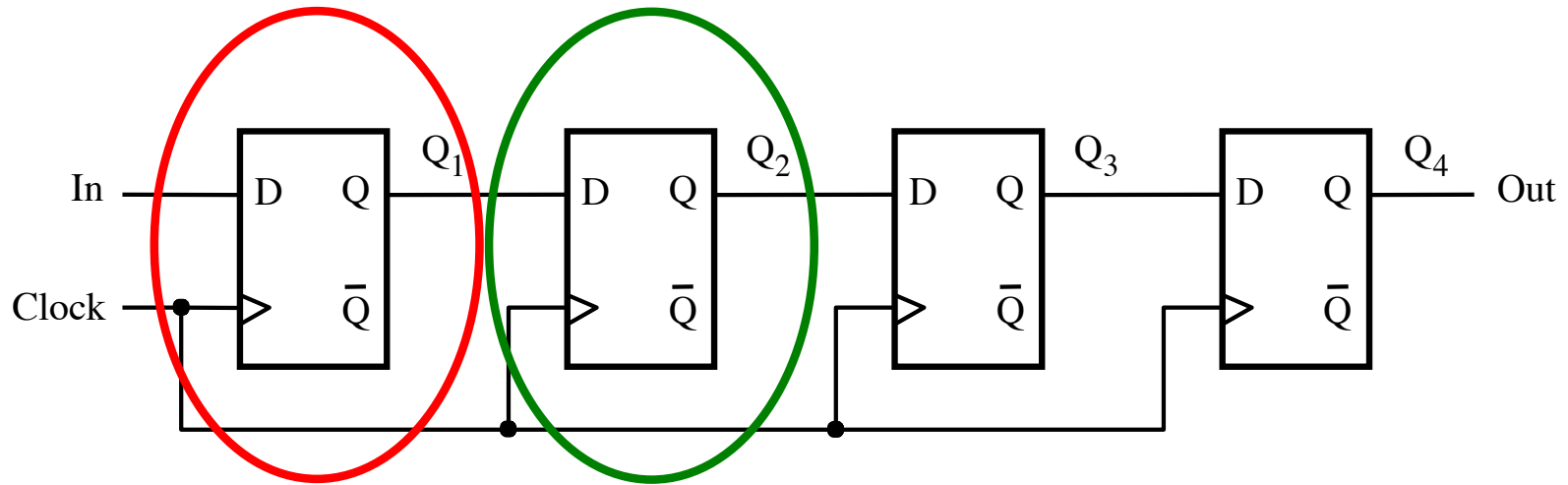
A simple shift register



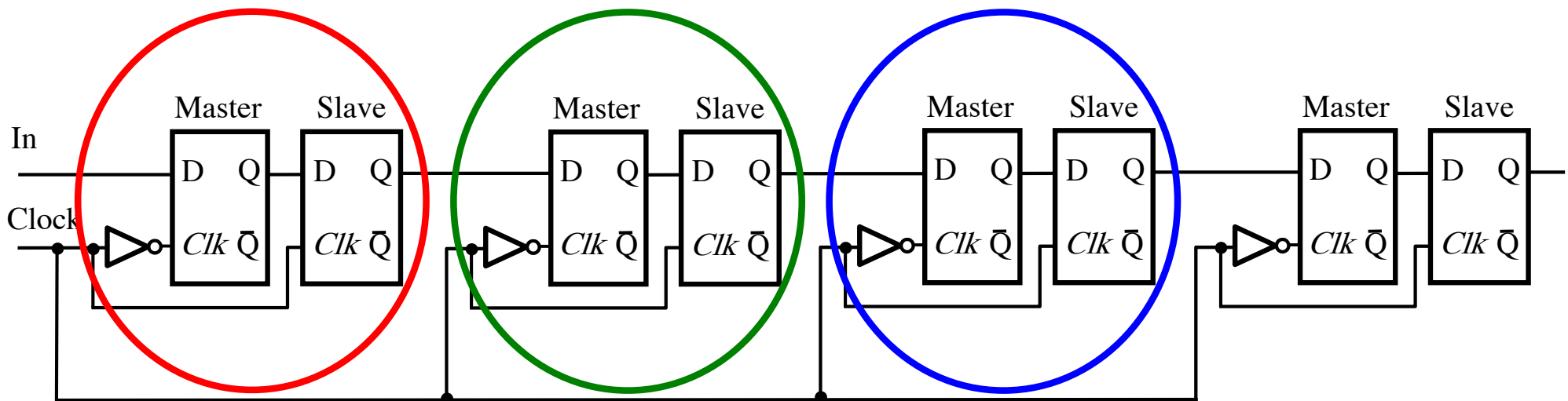
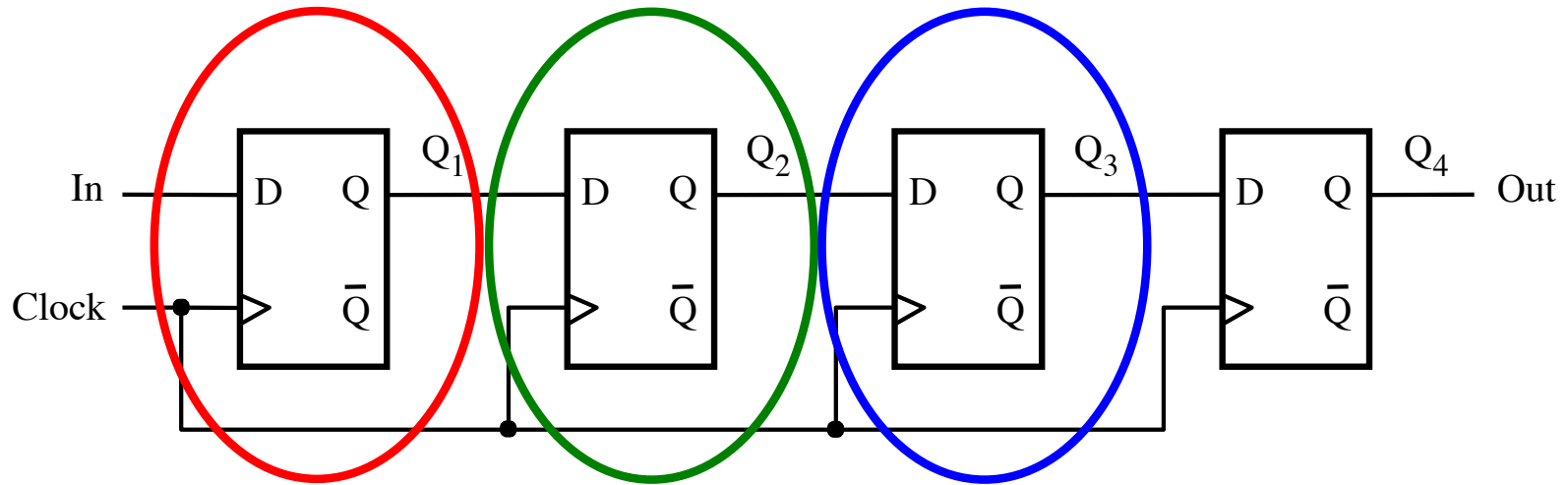
A simple shift register



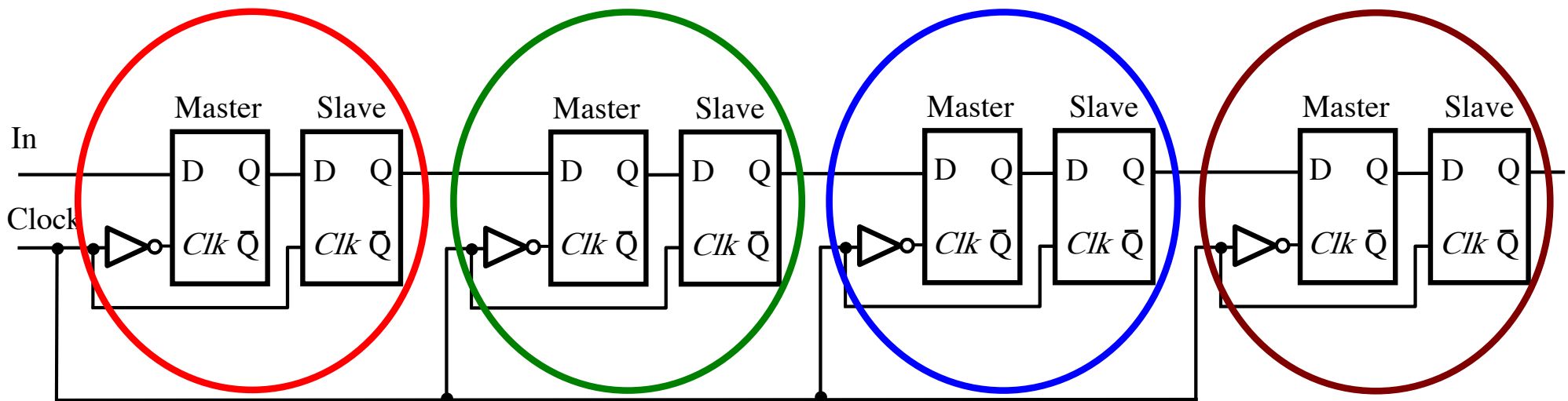
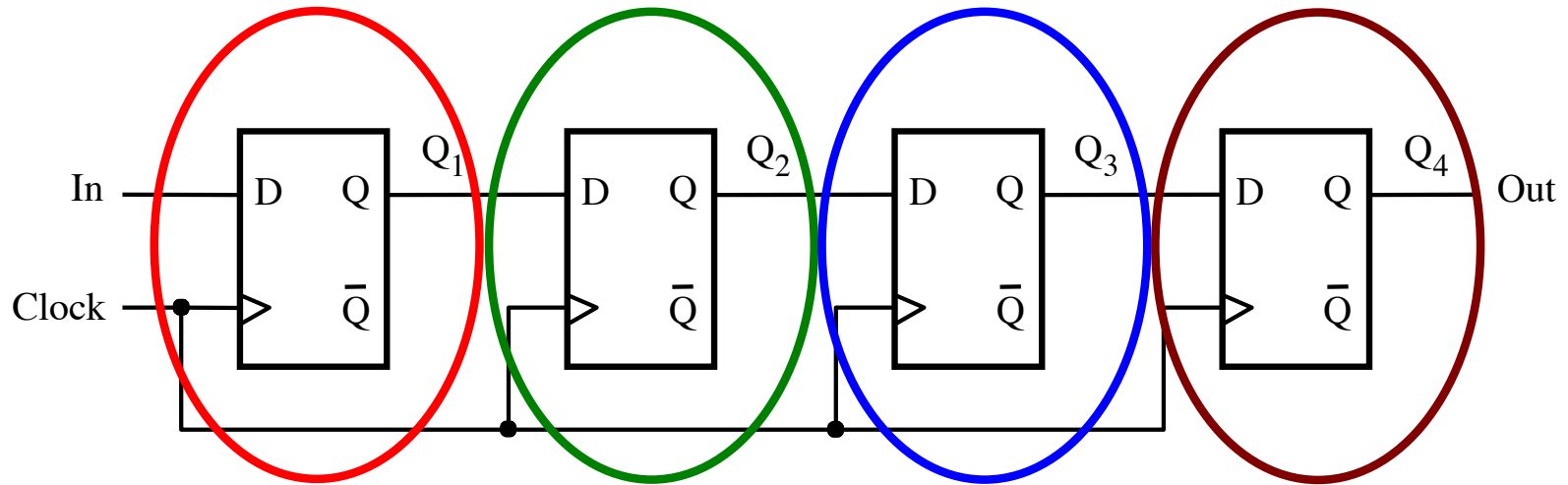
A simple shift register



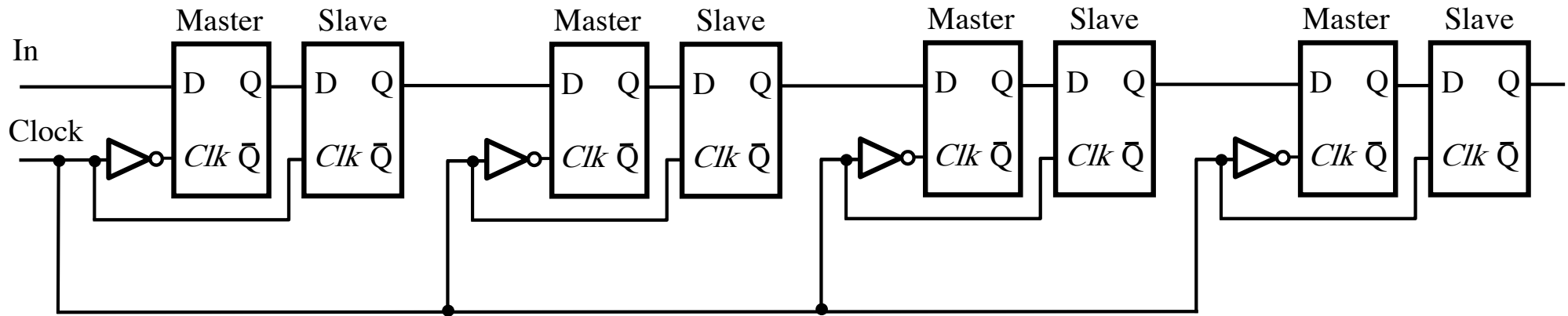
A simple shift register



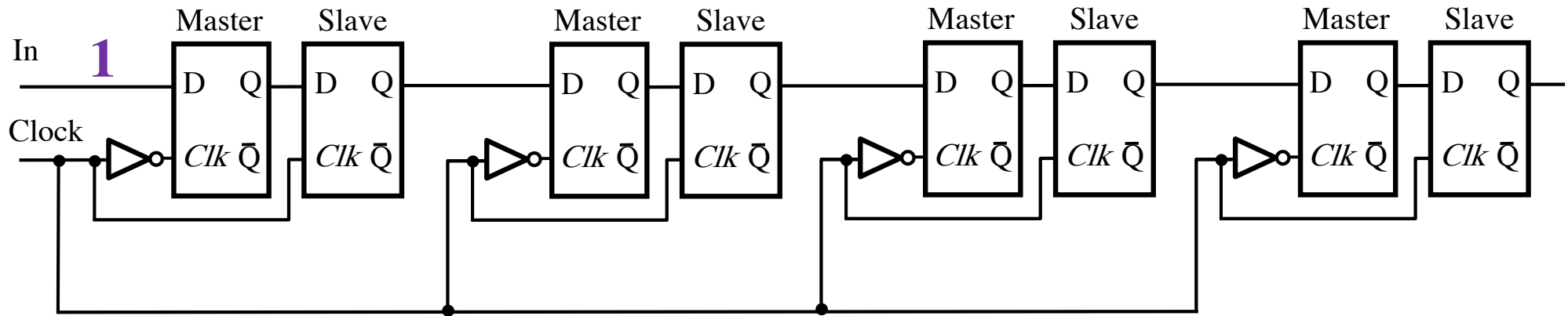
A simple shift register



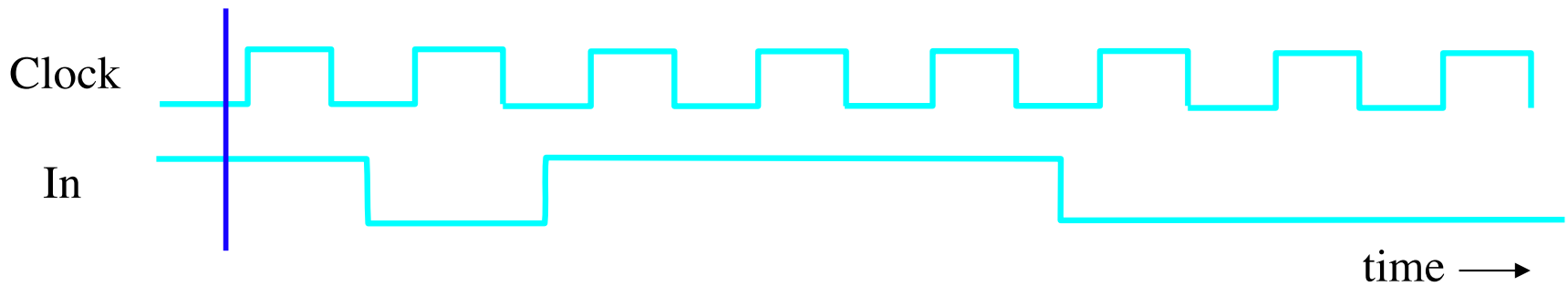
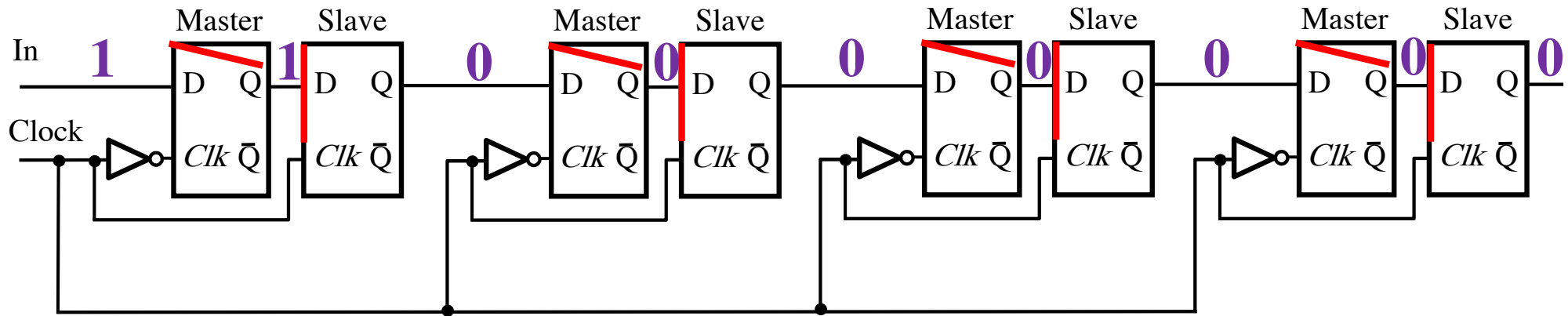
Simulating a shift register



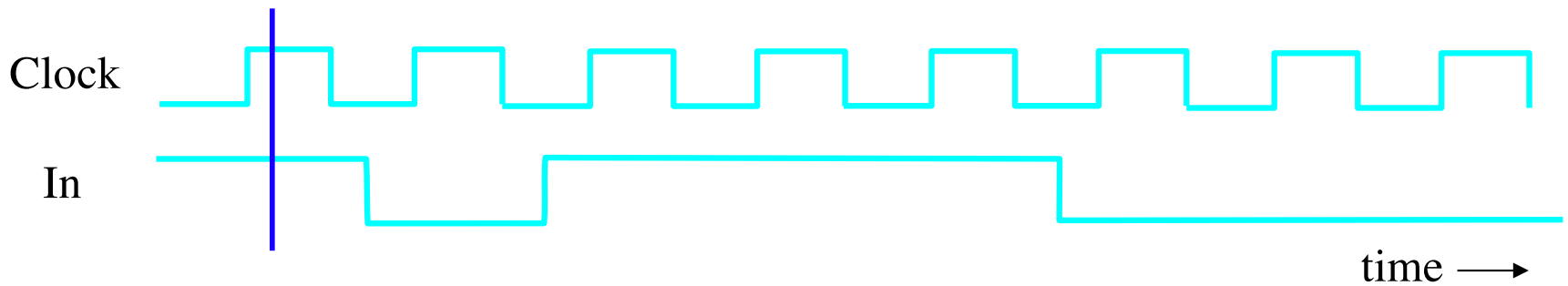
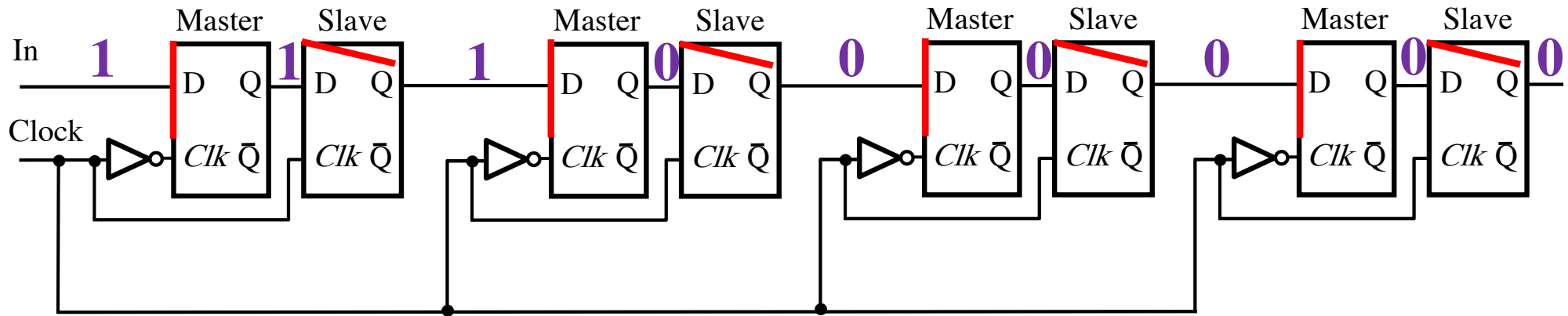
Simulating a shift register



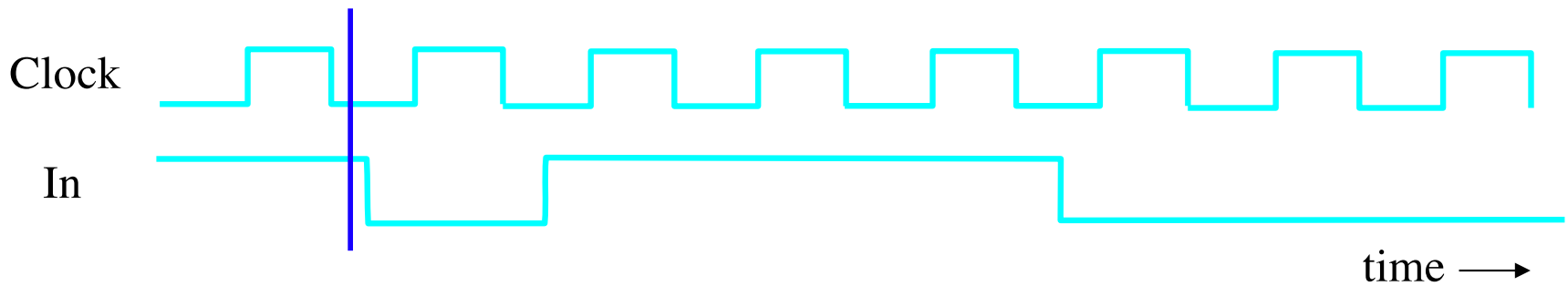
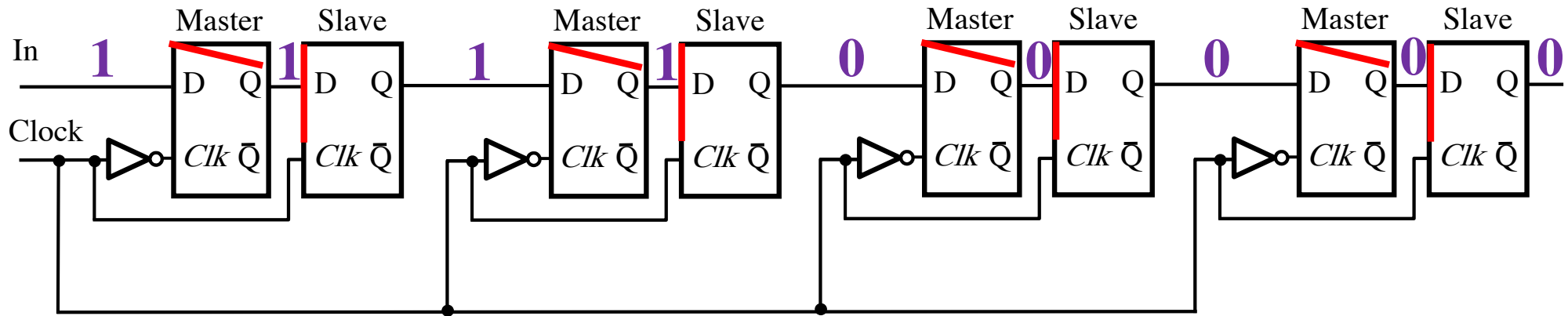
Simulating a shift register



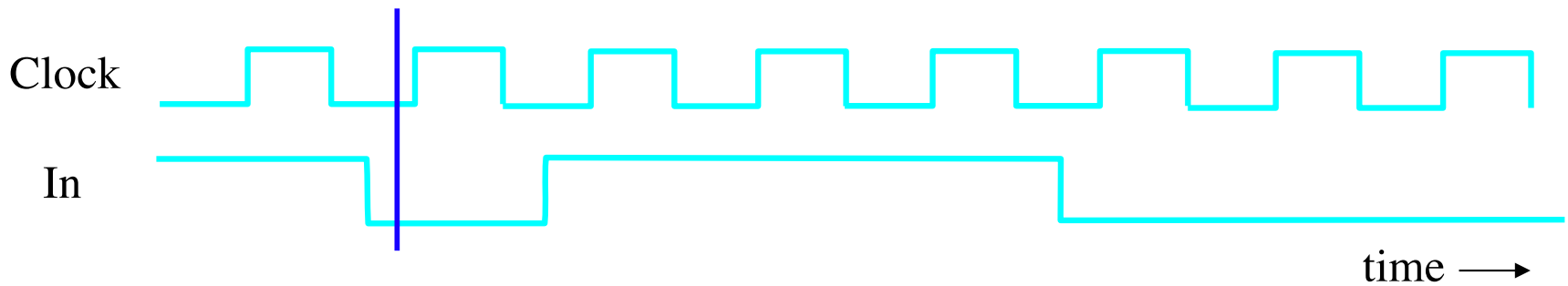
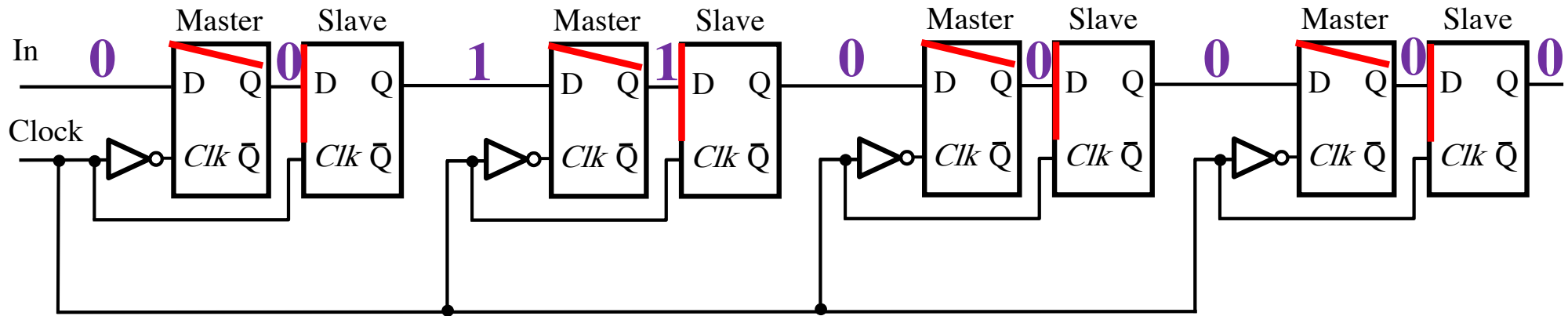
Simulating a shift register



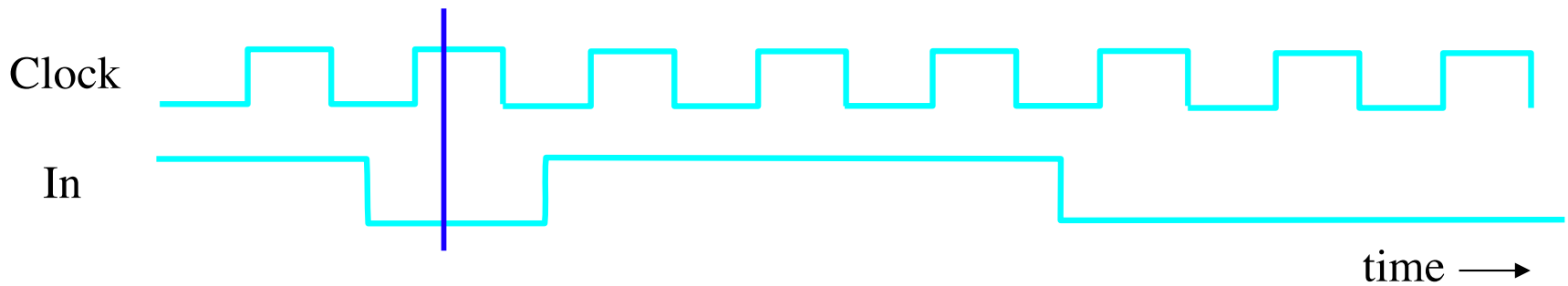
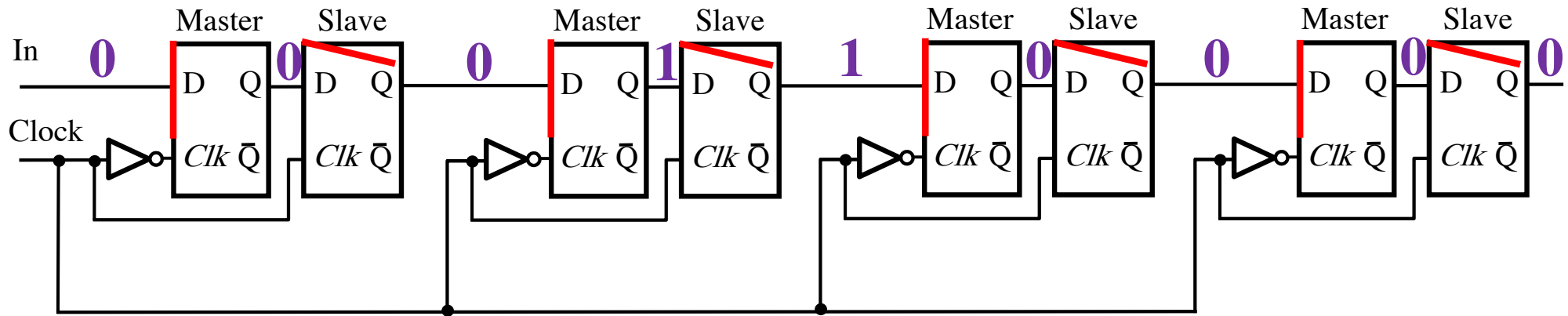
Simulating a shift register



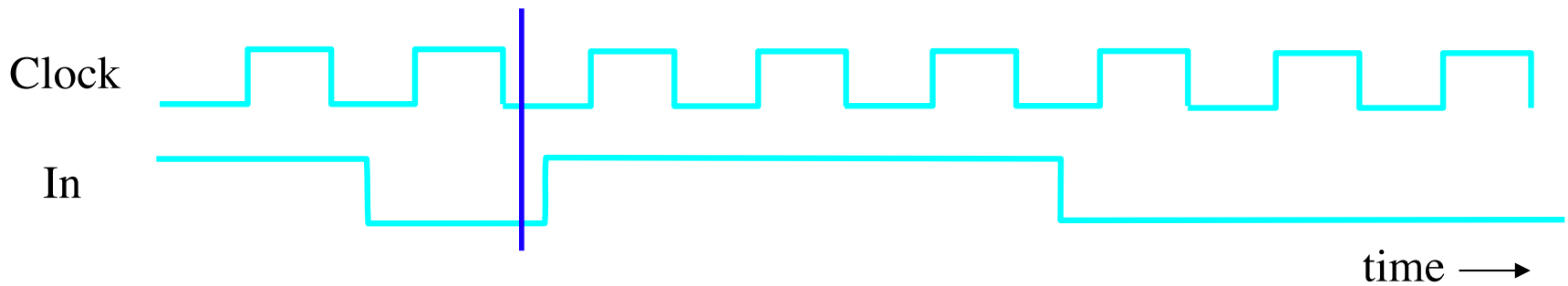
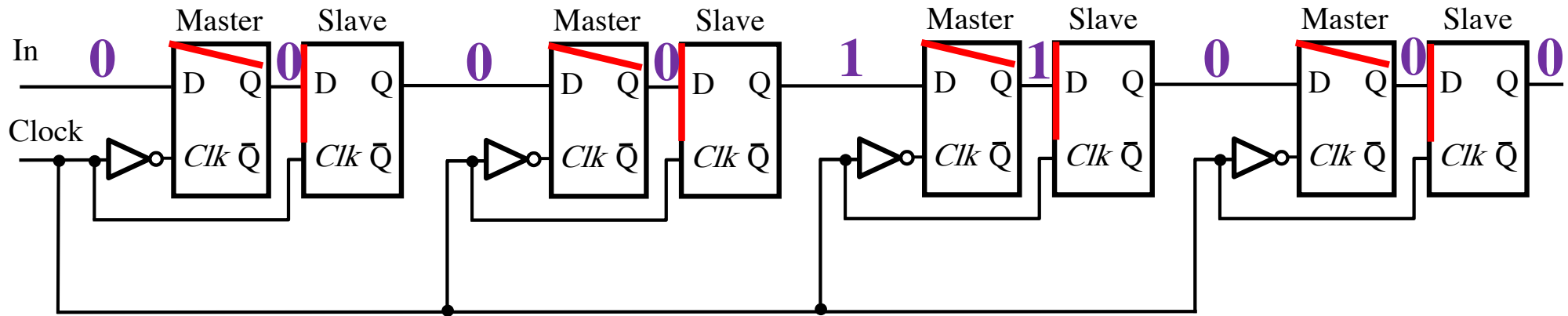
Simulating a shift register



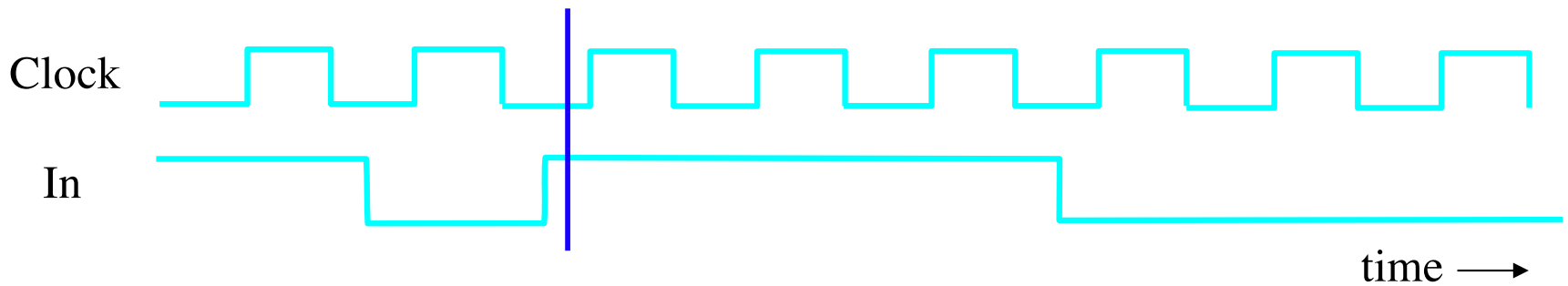
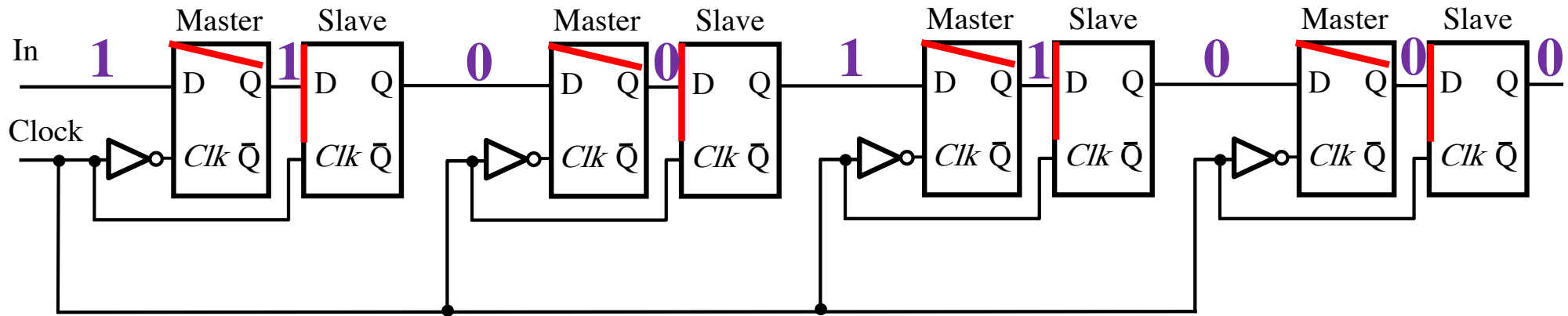
Simulating a shift register



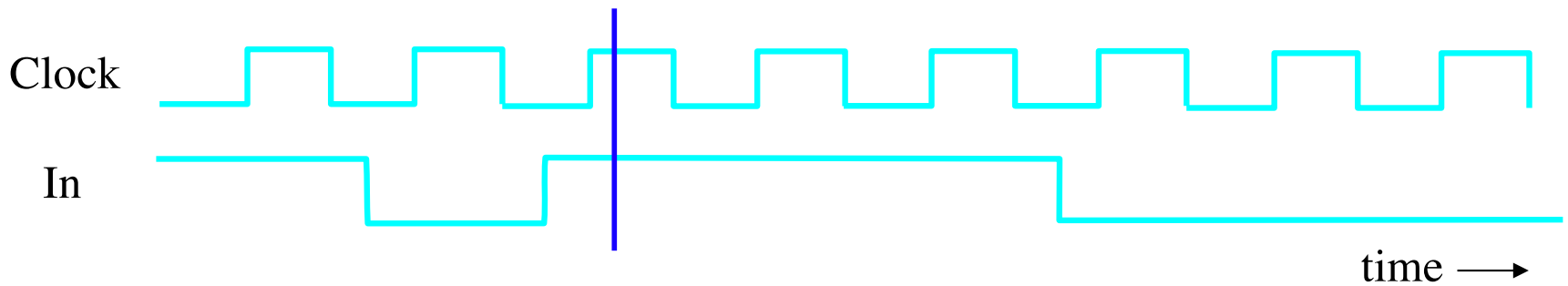
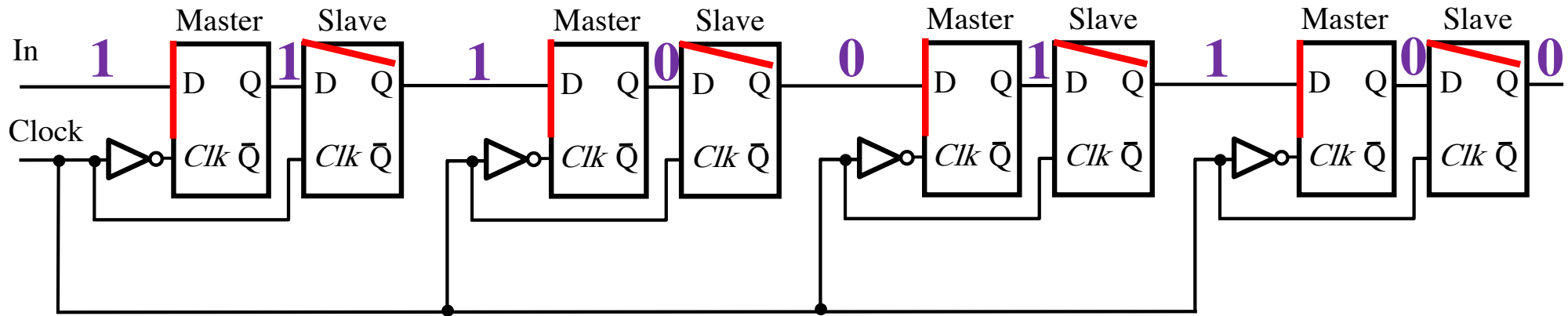
Simulating a shift register



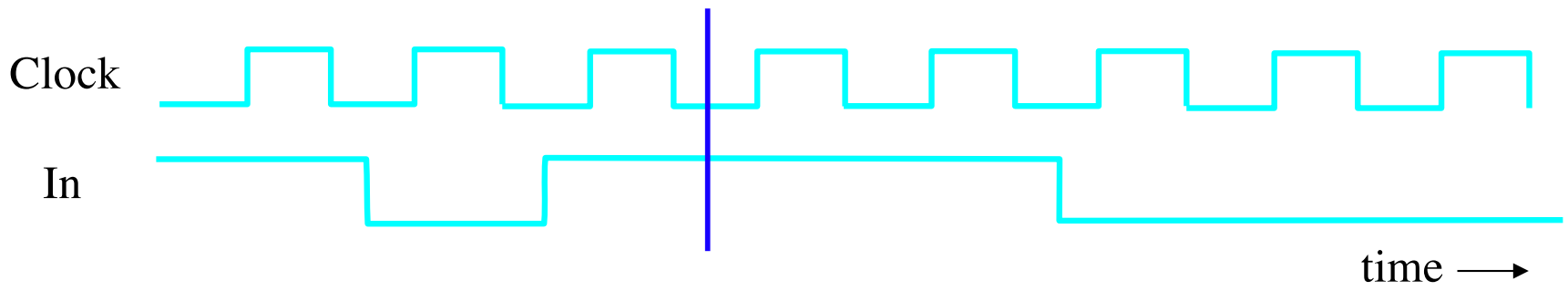
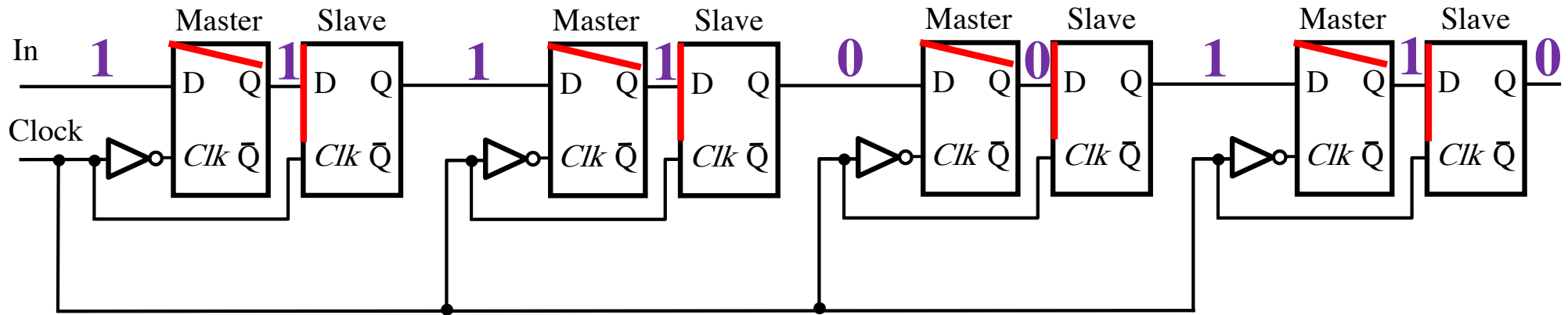
Simulating a shift register



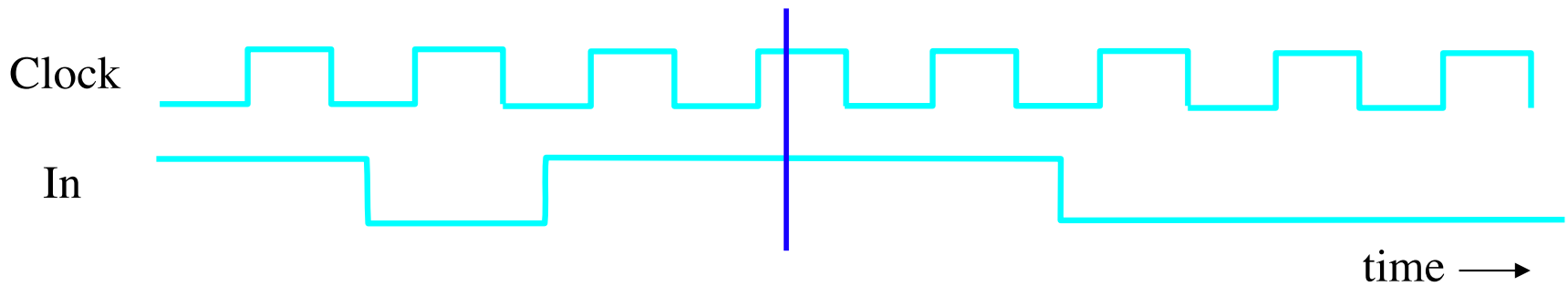
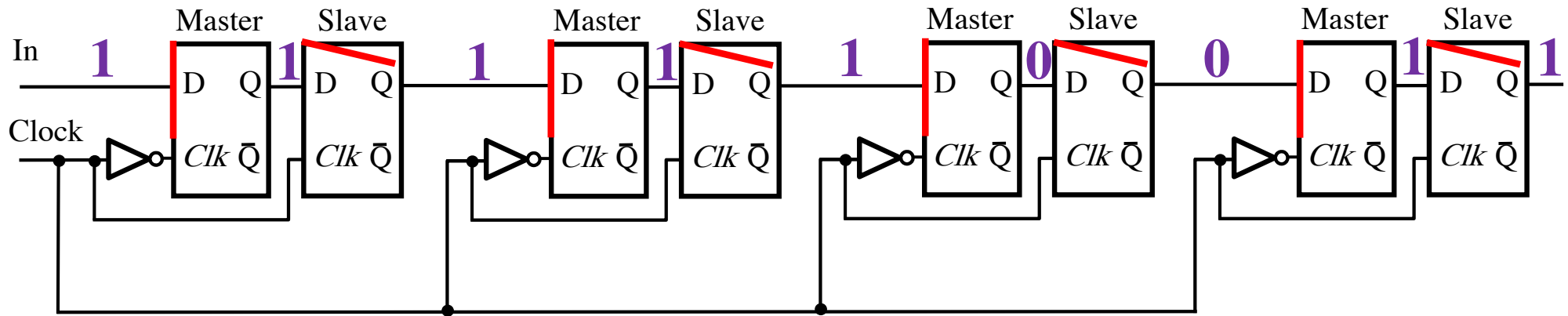
Simulating a shift register



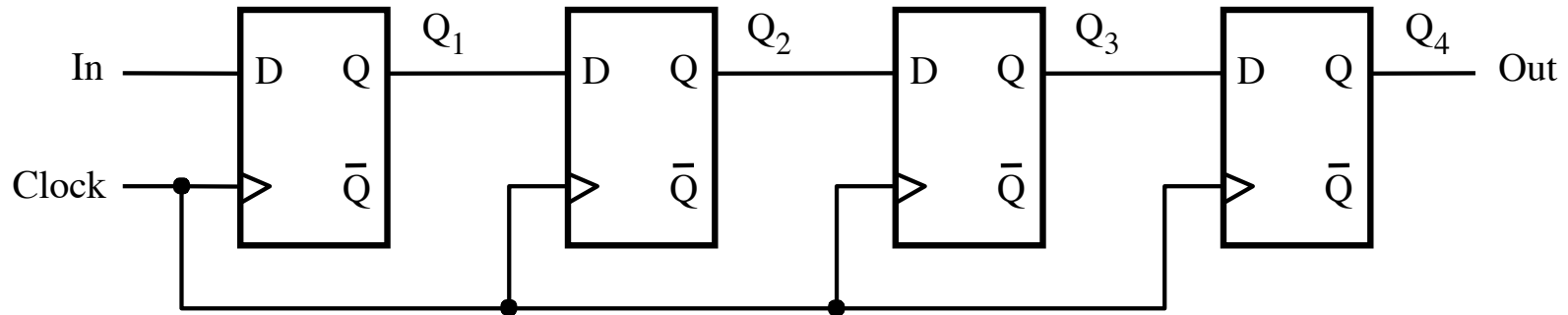
Simulating a shift register



Simulating a shift register



A simple shift register



(a) Circuit

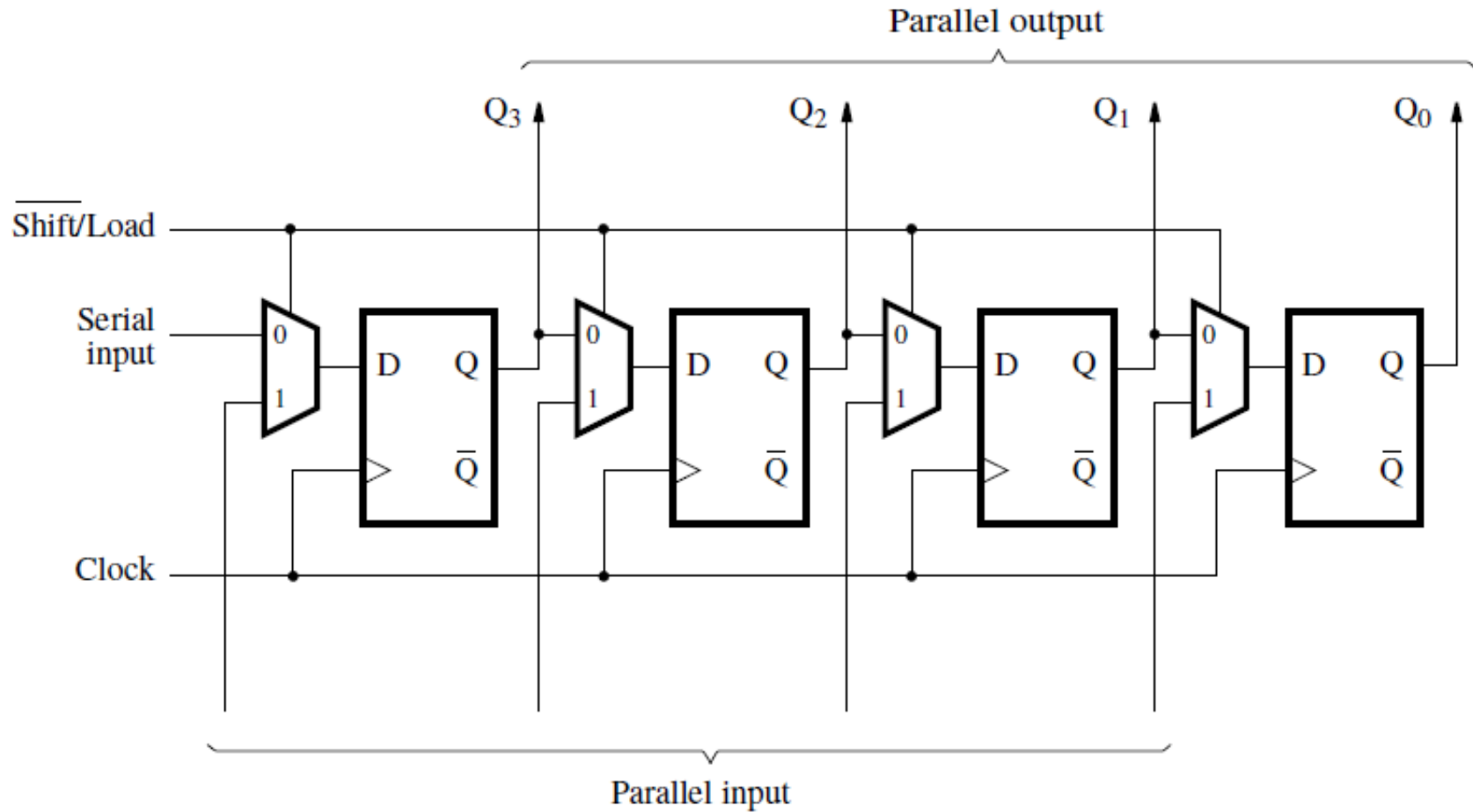
	In	Q ₁	Q ₂	Q ₃	Q ₄ = Out
t_0	1	0	0	0	0
t_1	0	1	0	0	0
t_2	1	0	1	0	0
t_3	1	1	0	1	0
t_4	1	1	1	0	1
t_5	0	1	1	1	0
t_6	0	0	1	1	1
t_7	0	0	0	1	1

This simulation goes only up to here

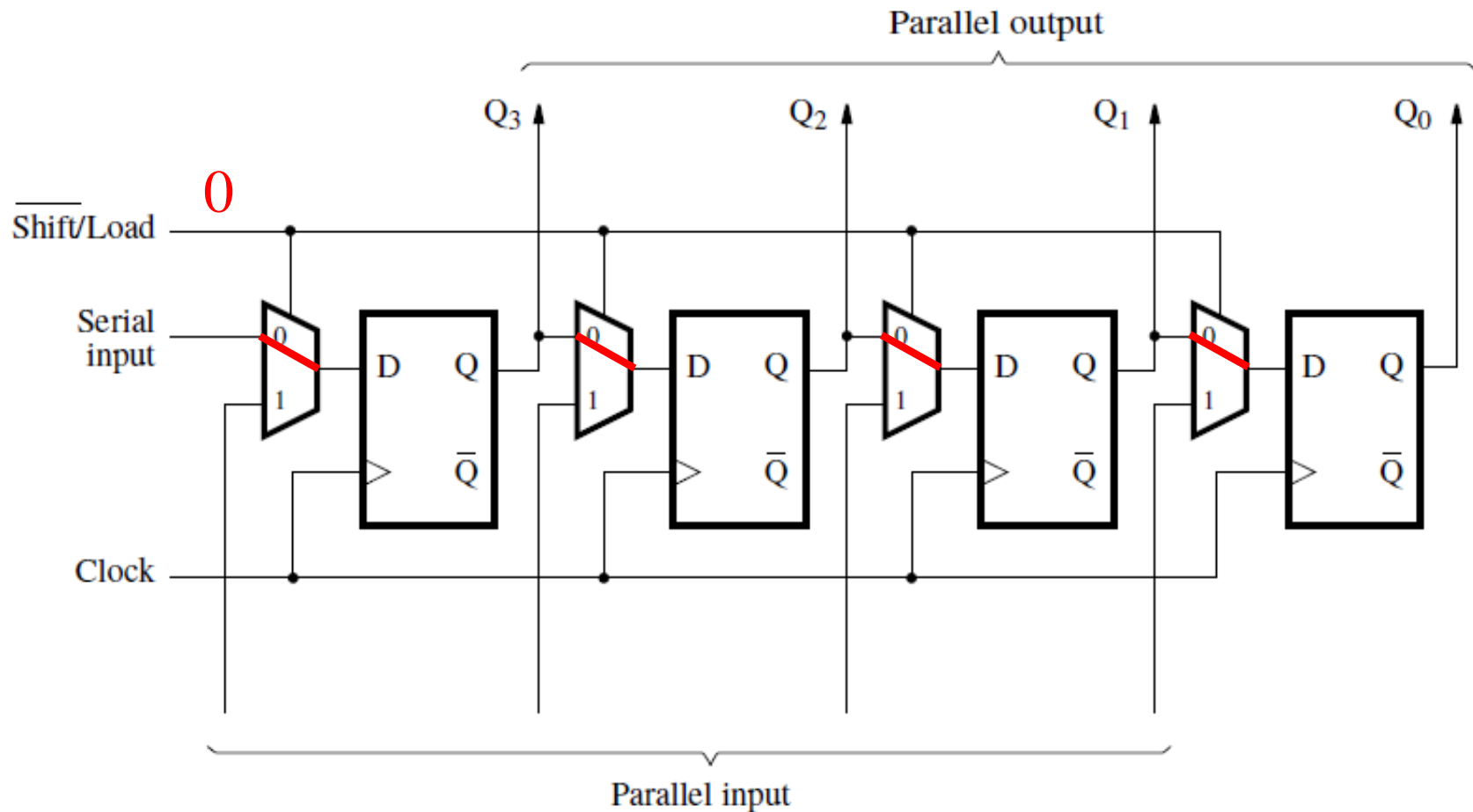
(b) A sample sequence

Parallel-Access Shift Register

Parallel-access shift register

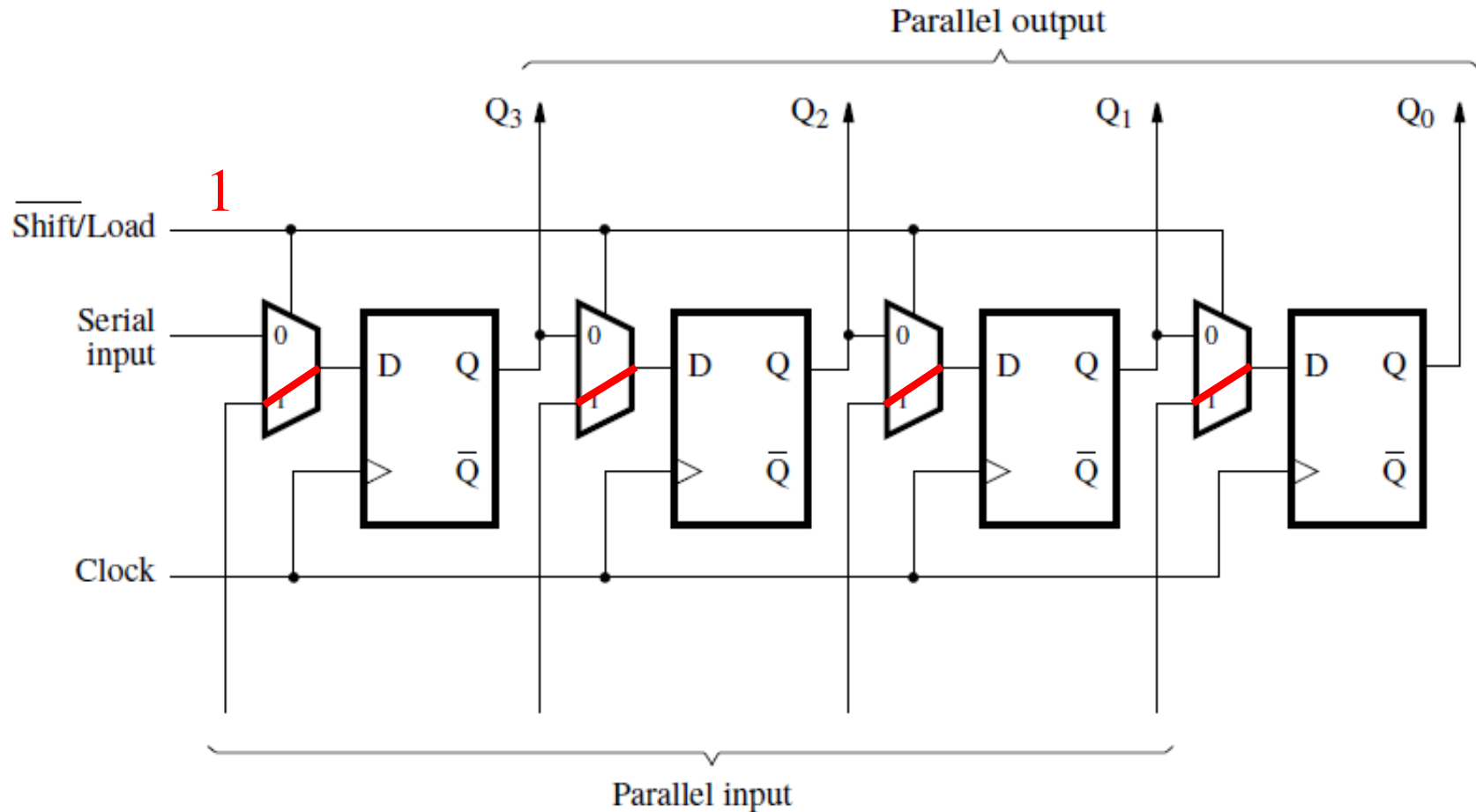


Parallel-access shift register



When Load=0, this behaves like a shift register.

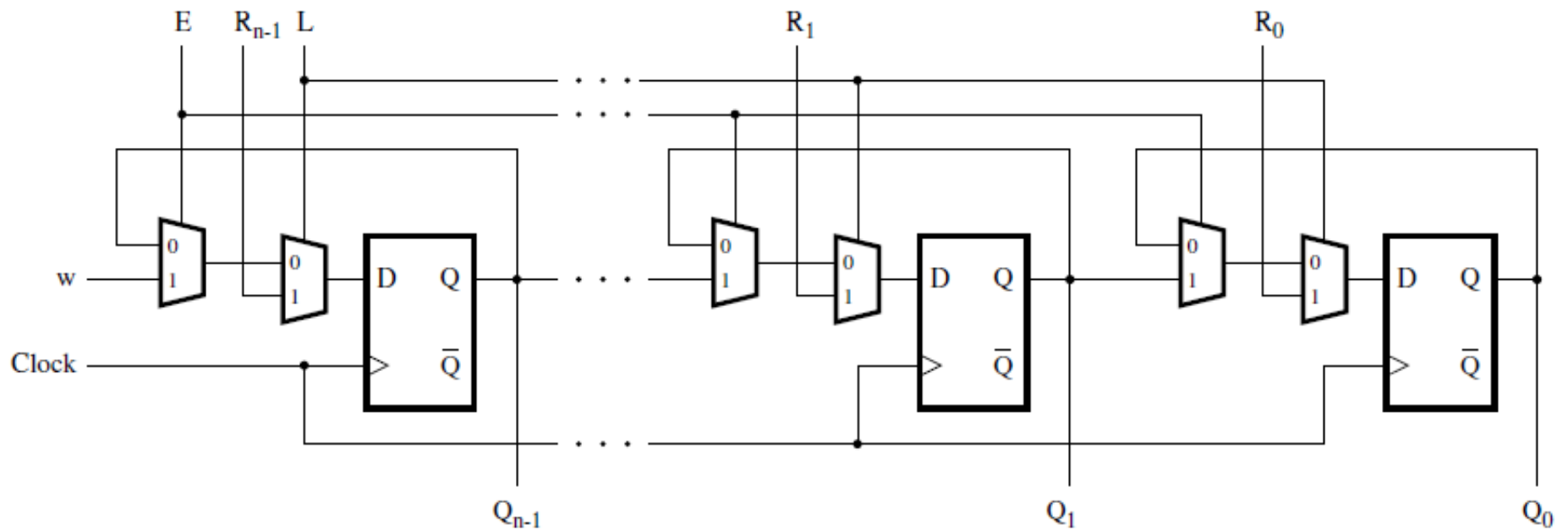
Parallel-access shift register



When Load=1, this behaves like a parallel-access register.

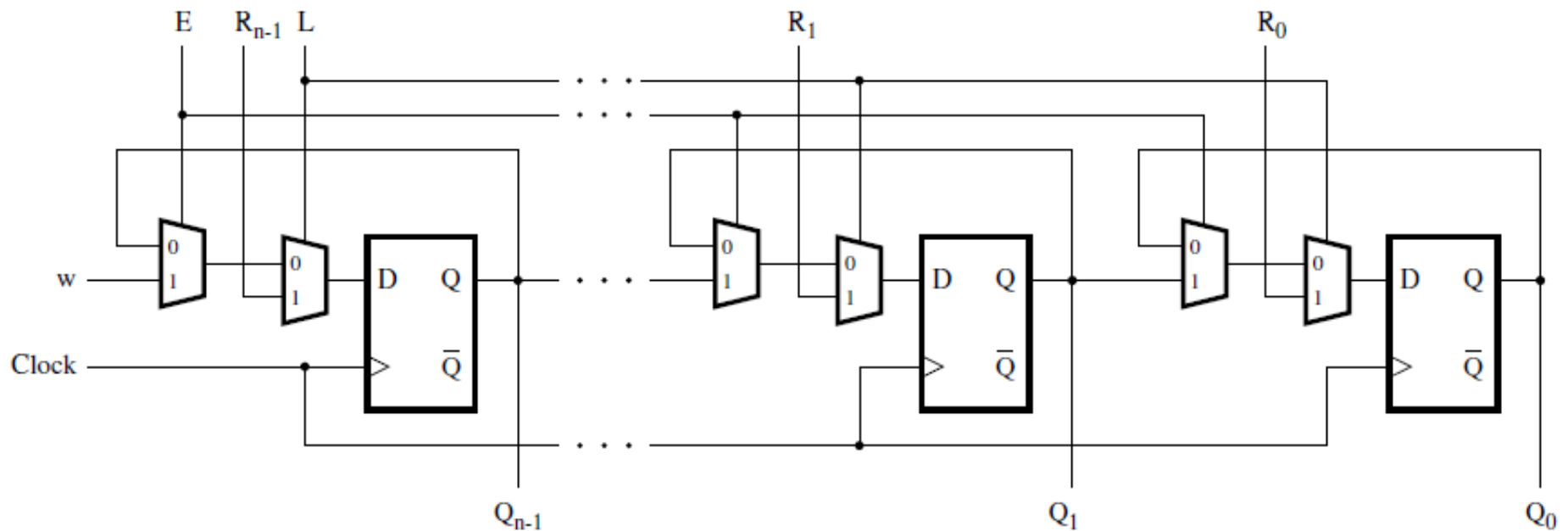
Shift Register With Parallel Load and Enable

A shift register with parallel load and enable control inputs



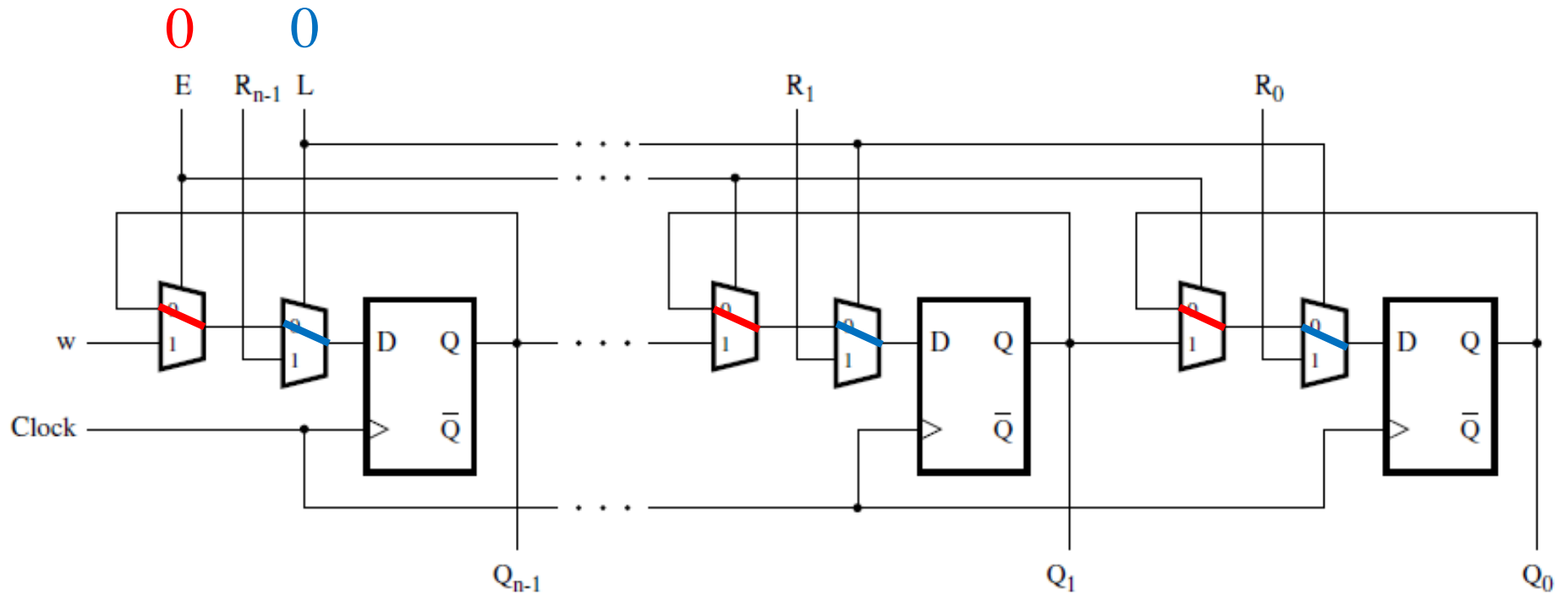
[Figure 5.59 from the textbook]

A shift register with parallel load and enable control inputs



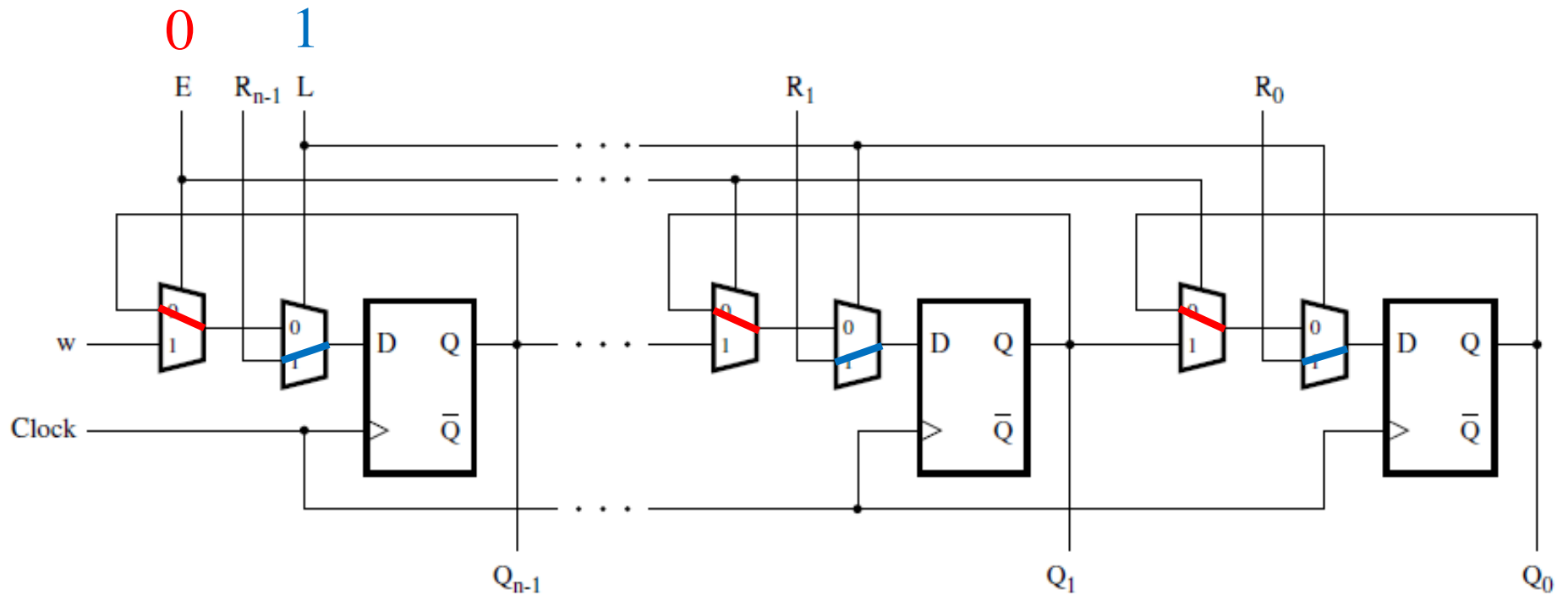
The directions of the input and output lines are switched relative to the previous slides.

A shift register with parallel load and enable control inputs



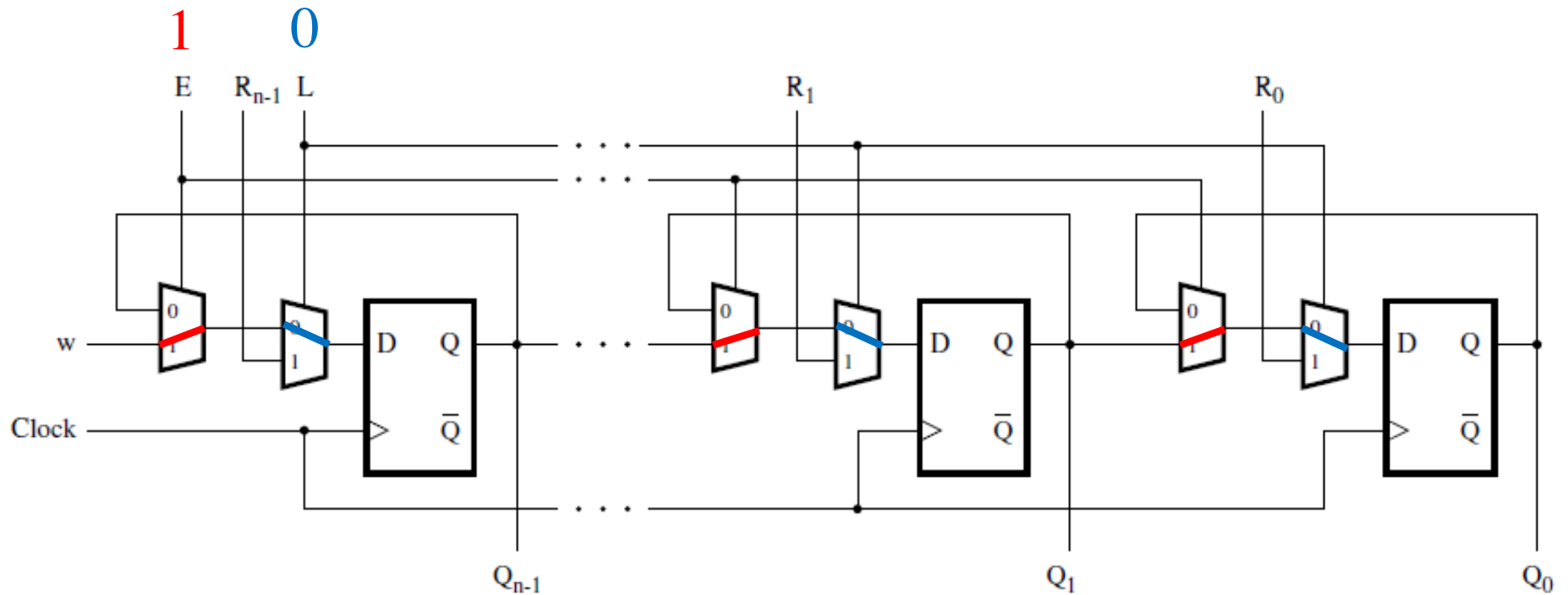
[Figure 5.59 from the textbook]

A shift register with parallel load and enable control inputs



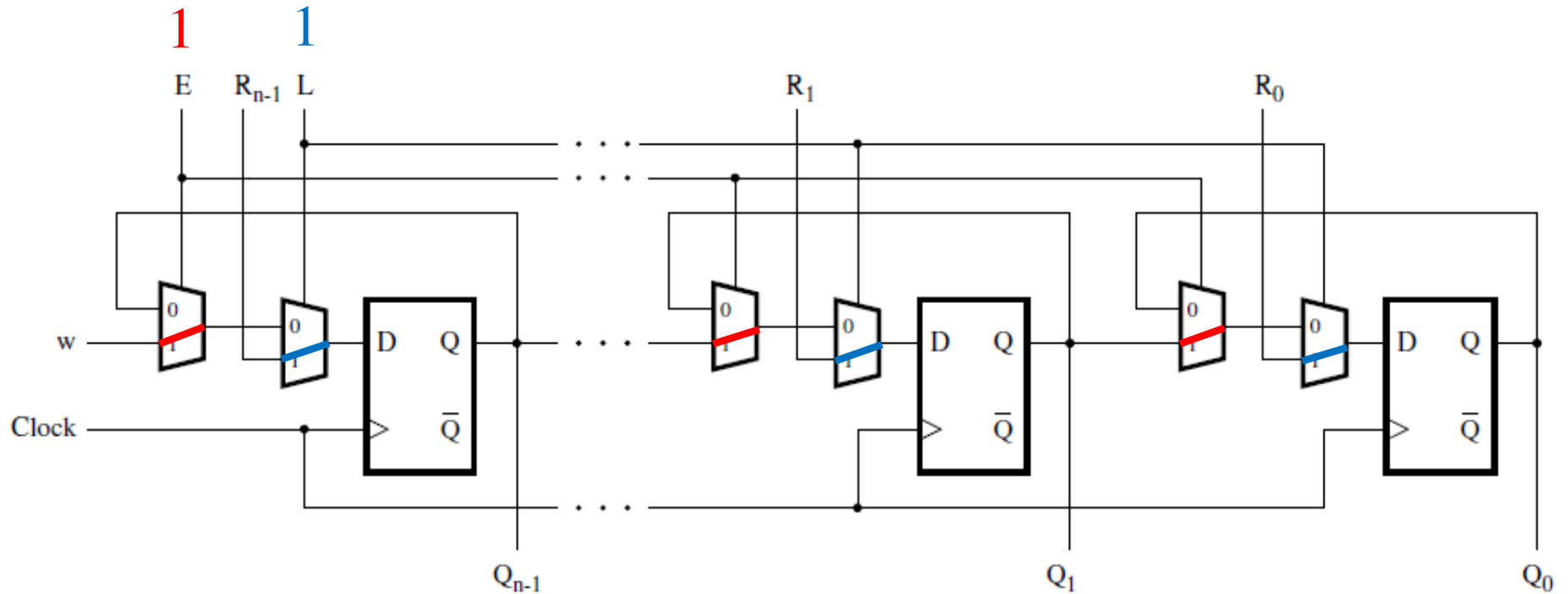
[Figure 5.59 from the textbook]

A shift register with parallel load and enable control inputs



[Figure 5.59 from the textbook]

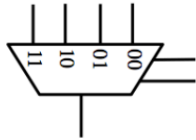
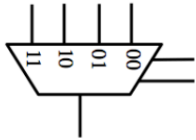
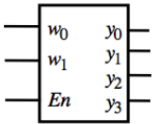
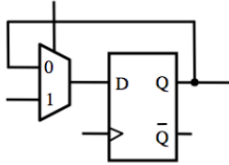
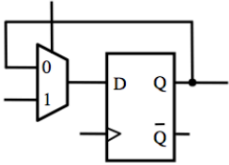
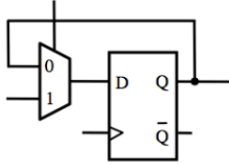
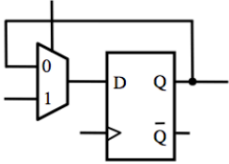
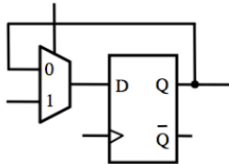
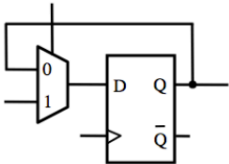
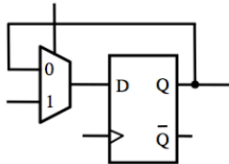
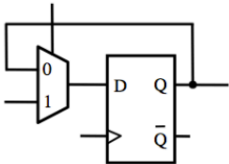
A shift register with parallel load and enable control inputs

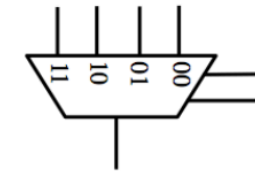
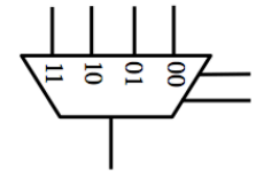
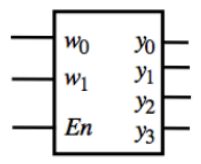
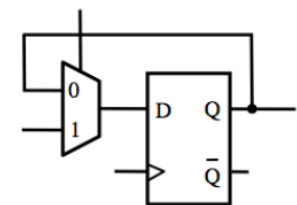
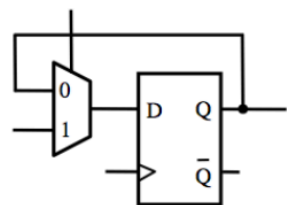
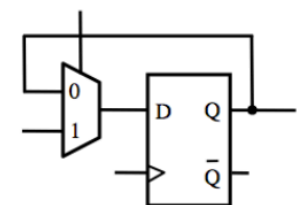
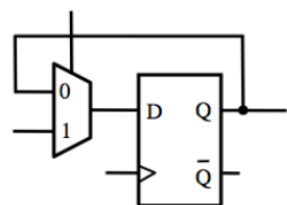
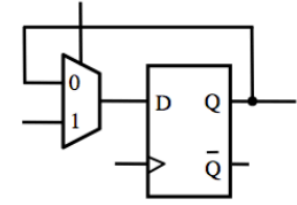
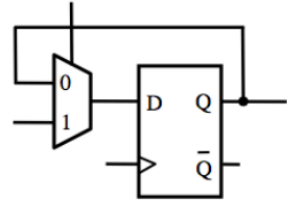
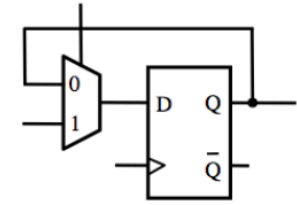
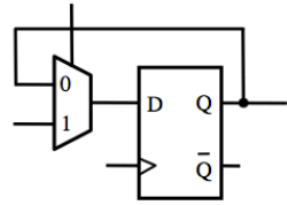


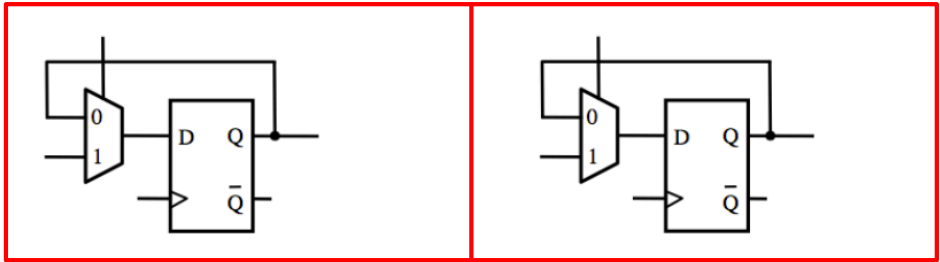
[Figure 5.59 from the textbook]

Register File

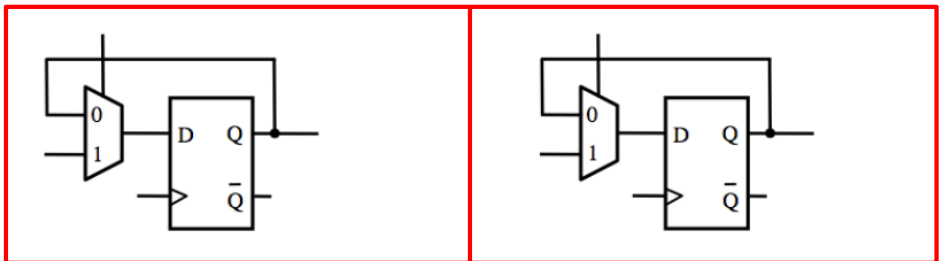
Complete the following circuit diagram to implement a register file with four 2-bit registers, one write port, one read port, and one write enable line.



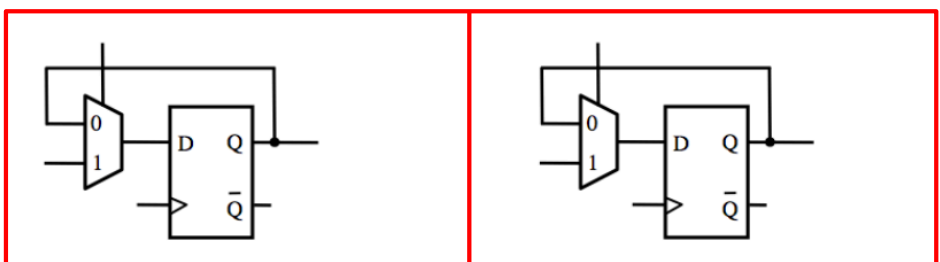




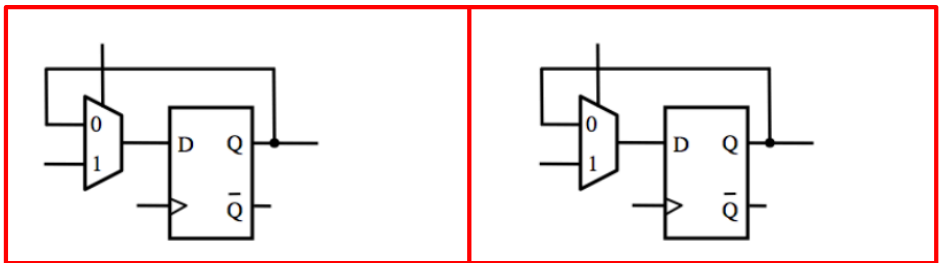
Register 0



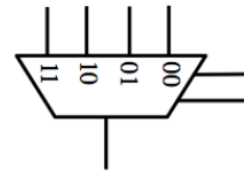
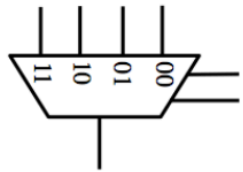
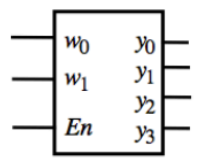
Register 1

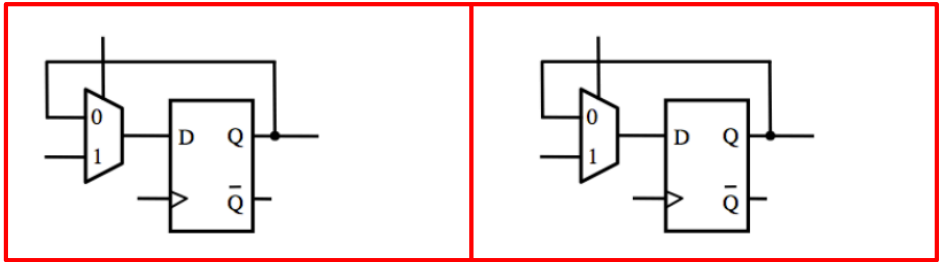


Register 2

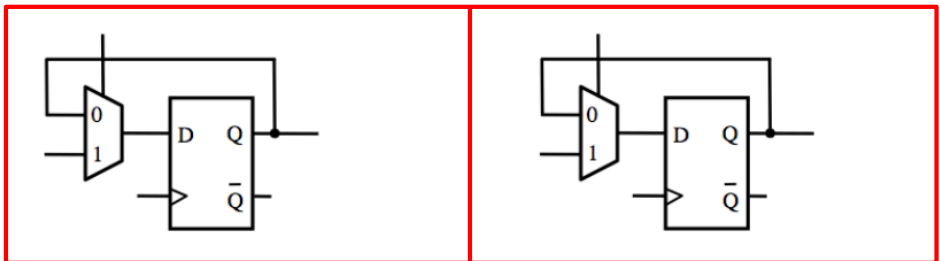


Register 3

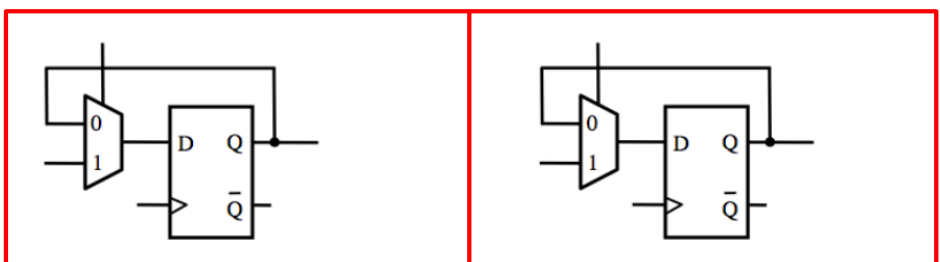




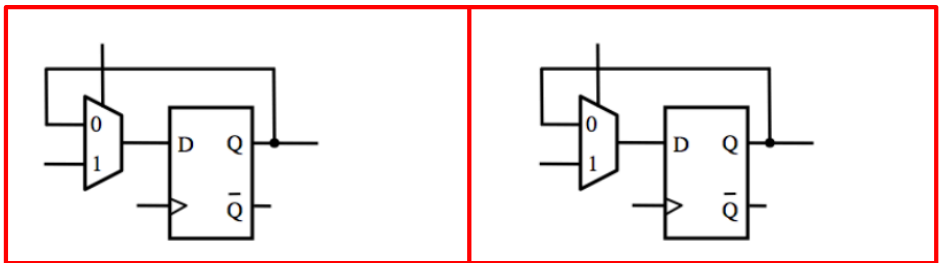
Register A



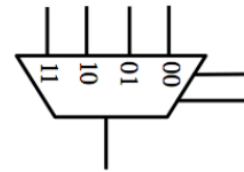
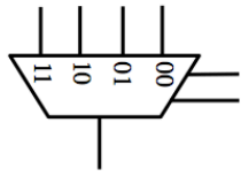
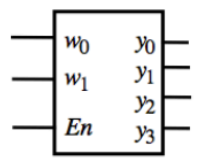
Register B

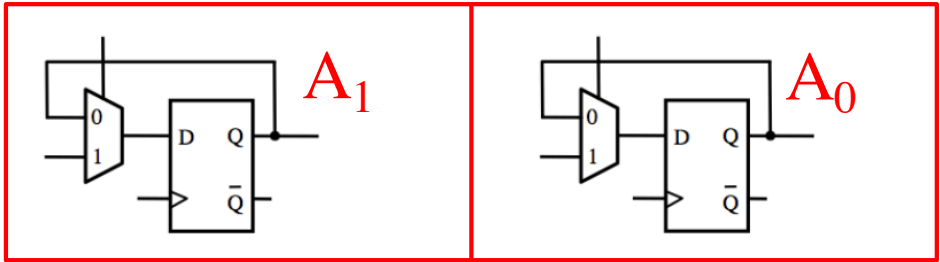


Register C

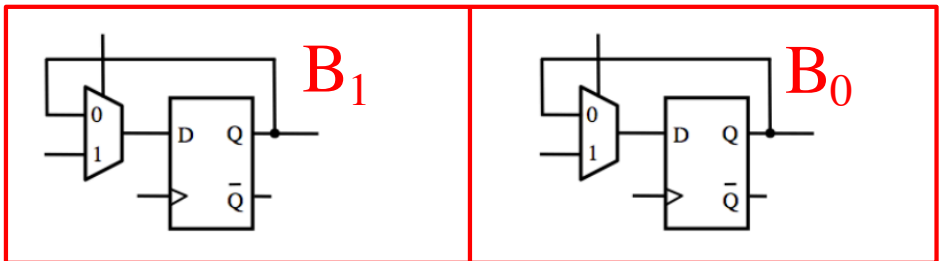


Register D

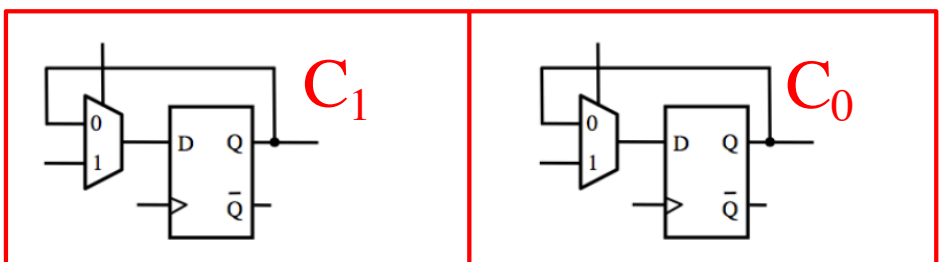




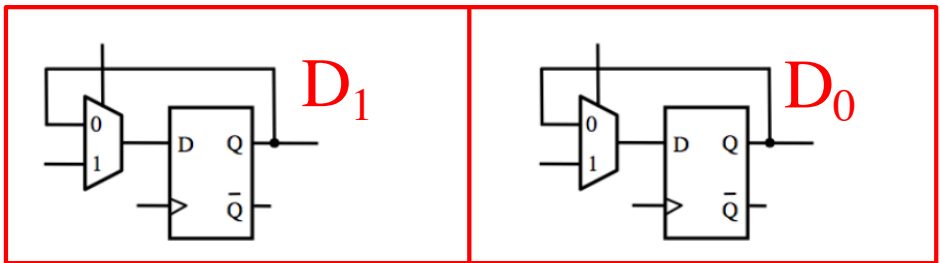
Register A



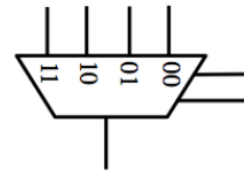
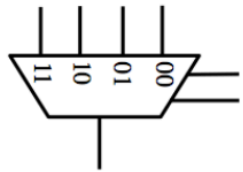
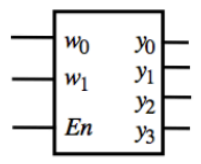
Register B

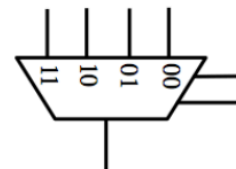
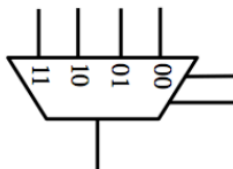
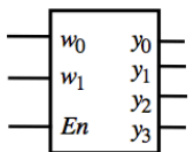
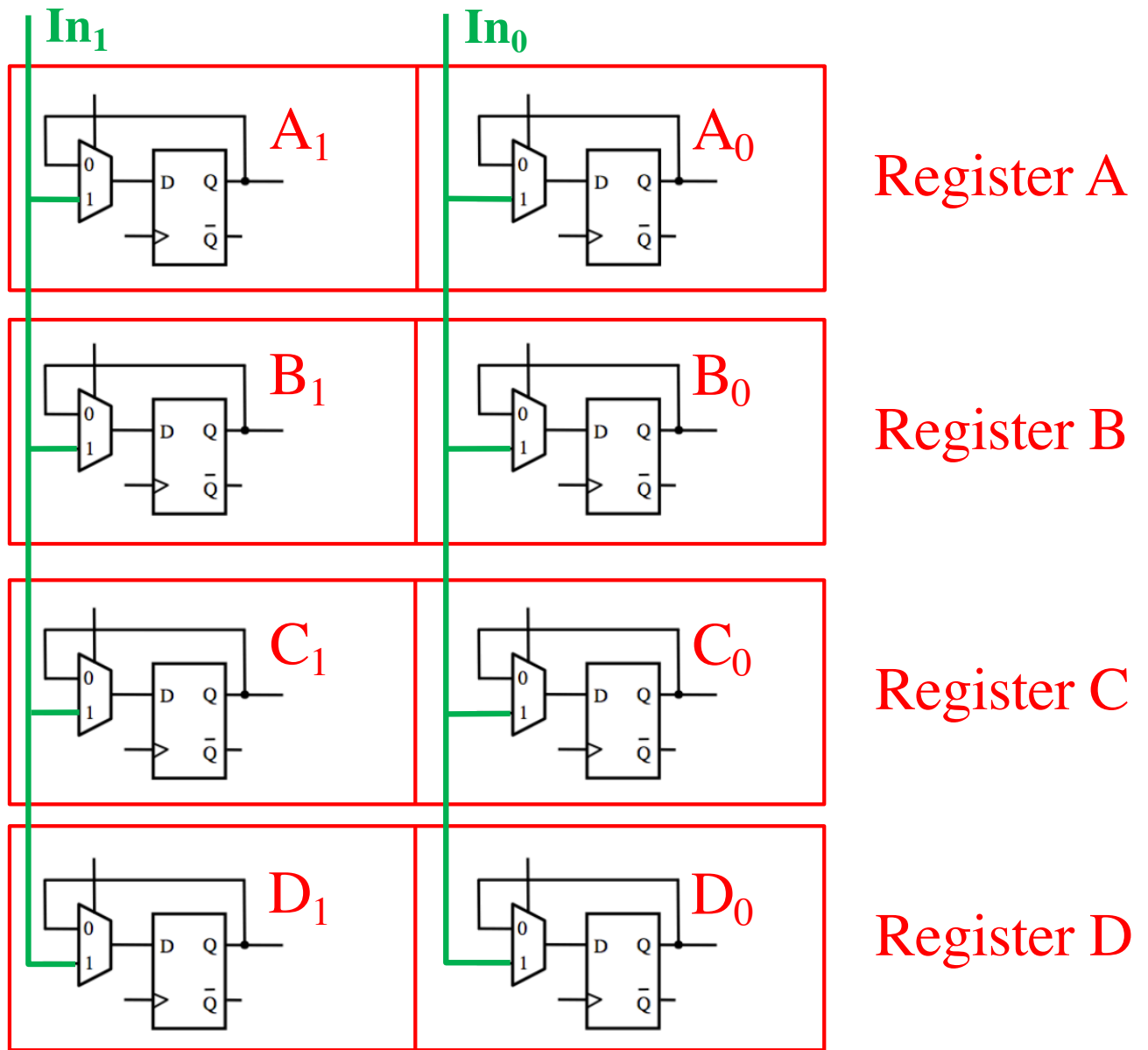


Register C



Register D





In_1

In_0

A_1

A_0

Register A

B_1

B_0

Register B

C_1

C_0

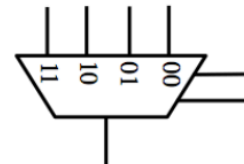
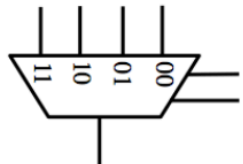
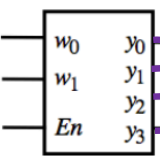
Register C

D_1

D_0

Register D

Write_address_0
Write_address_1
Write_enable



In₁

In₀

A₁

A₀

Register A

B₁

B₀

Register B

C₁

C₀

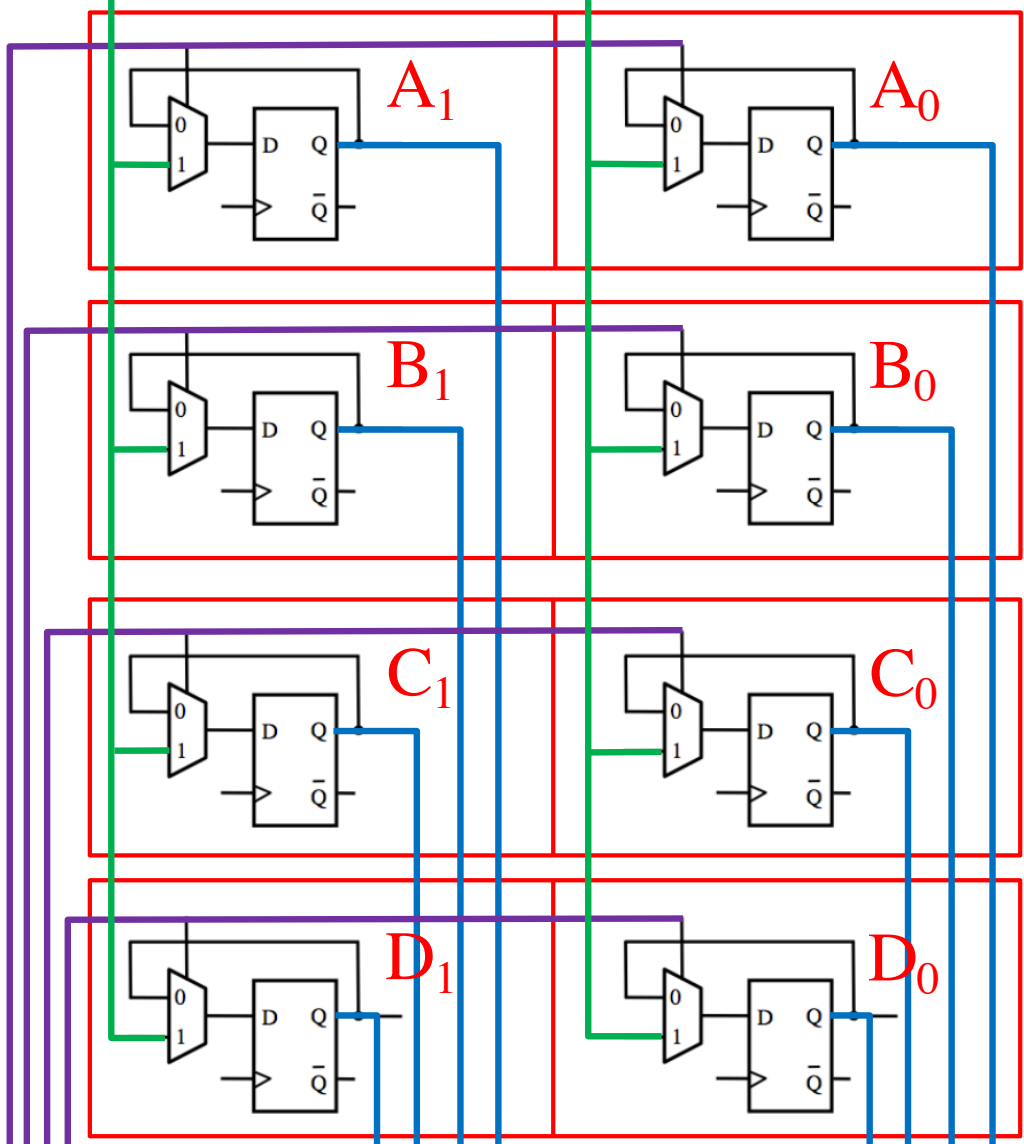
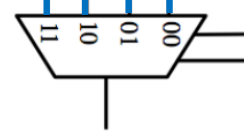
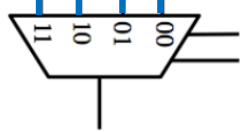
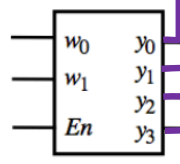
Register C

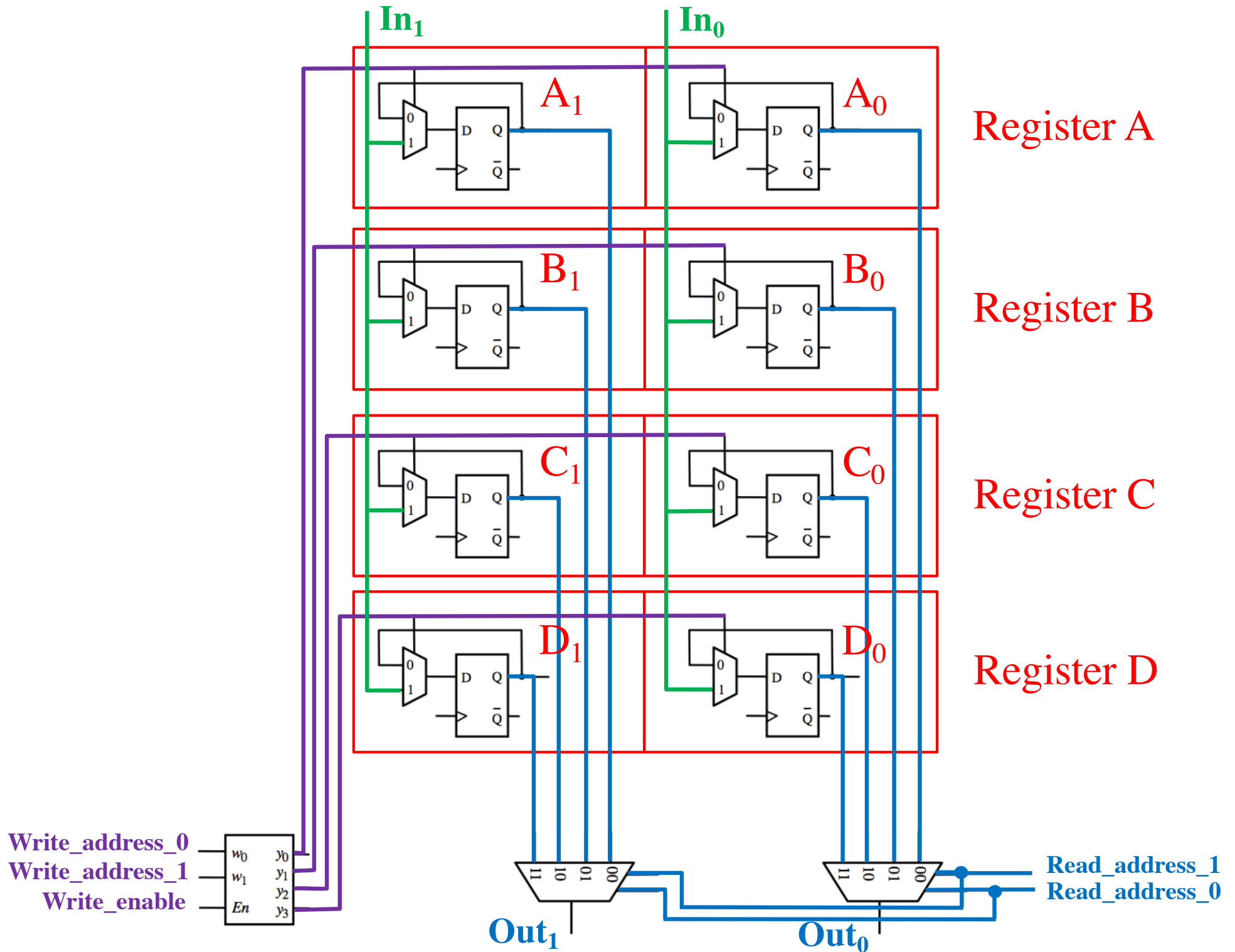
D₁

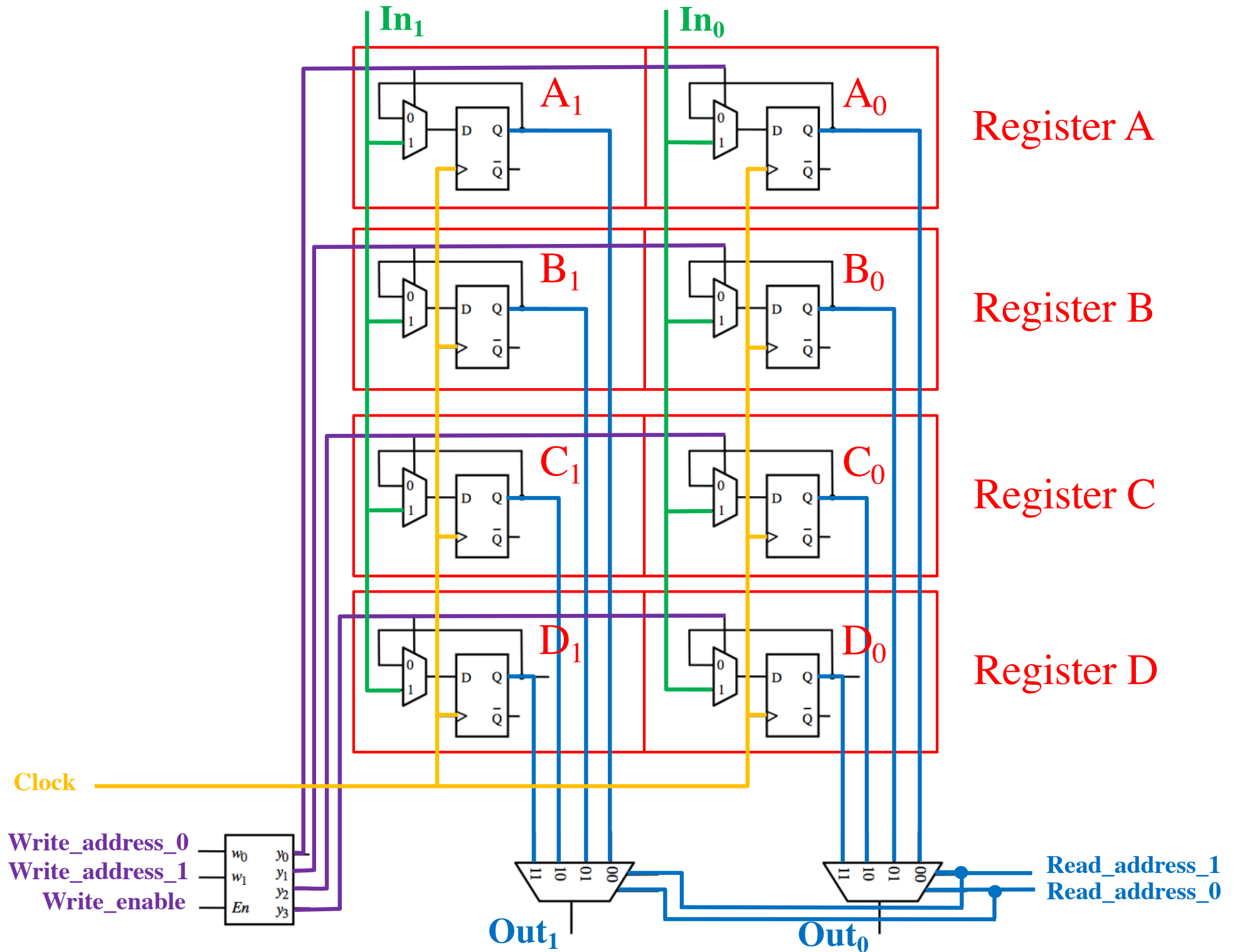
D₀

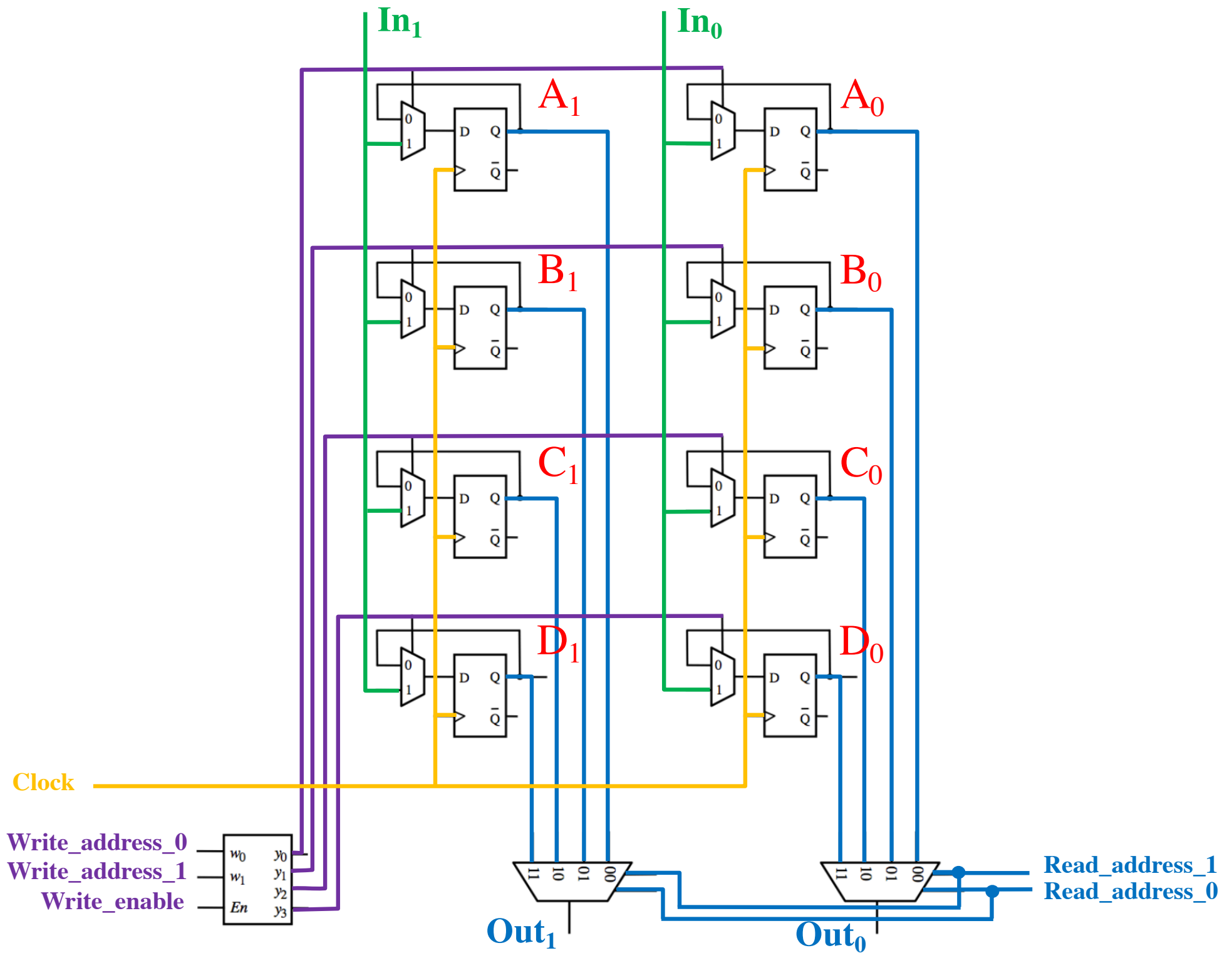
Register D

Write_address_0
Write_address_1
Write_enable









Components of the Register File

2-to-4 Decoder

In_1

In_0

A_1

A_0

B_1

B_0

C_1

C_0

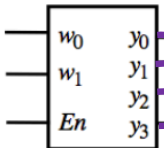
D_1

D_0

2-to-4 decoder
with enable

Clock

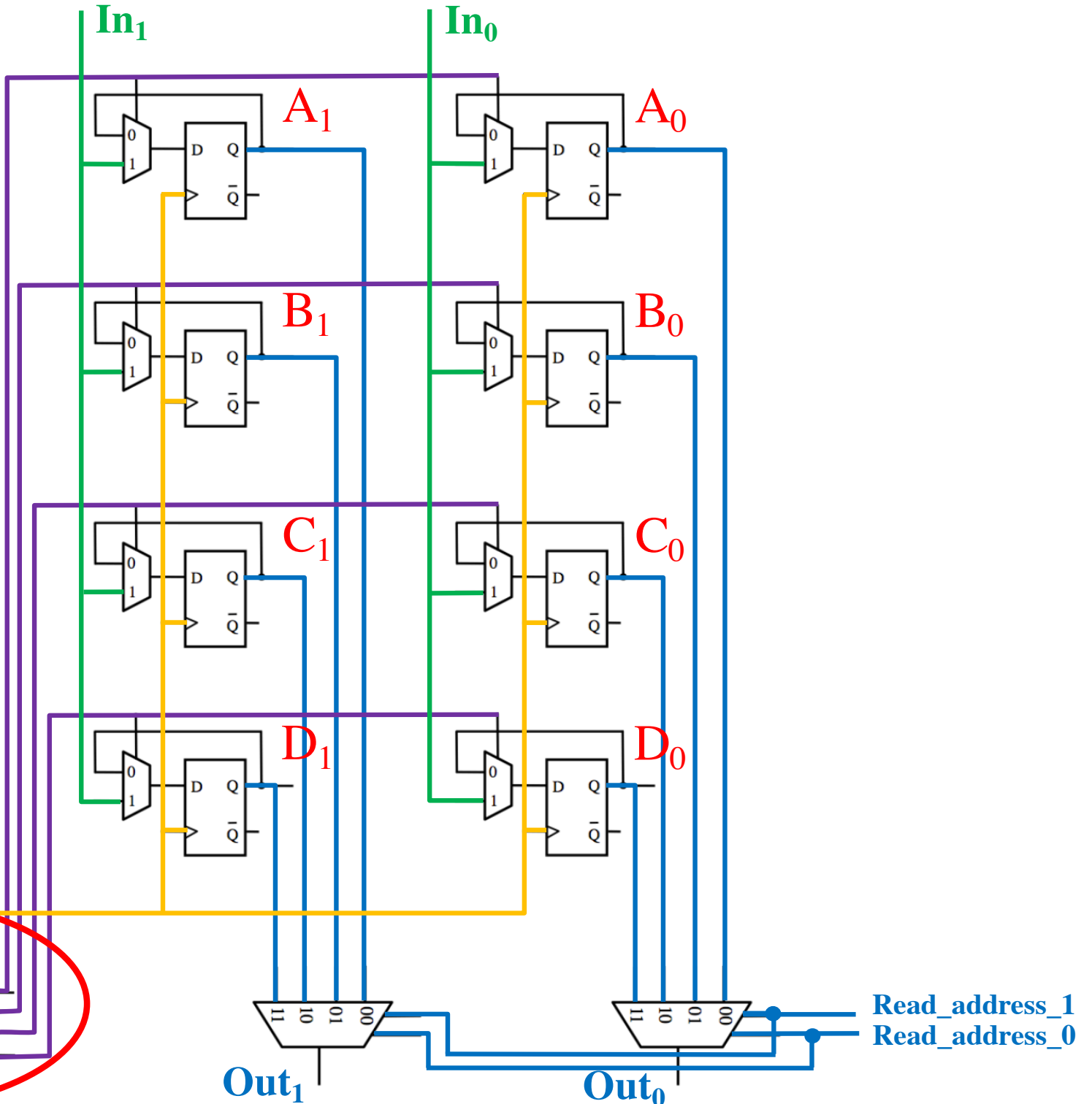
Write_address_0
Write_address_1
Write_enable



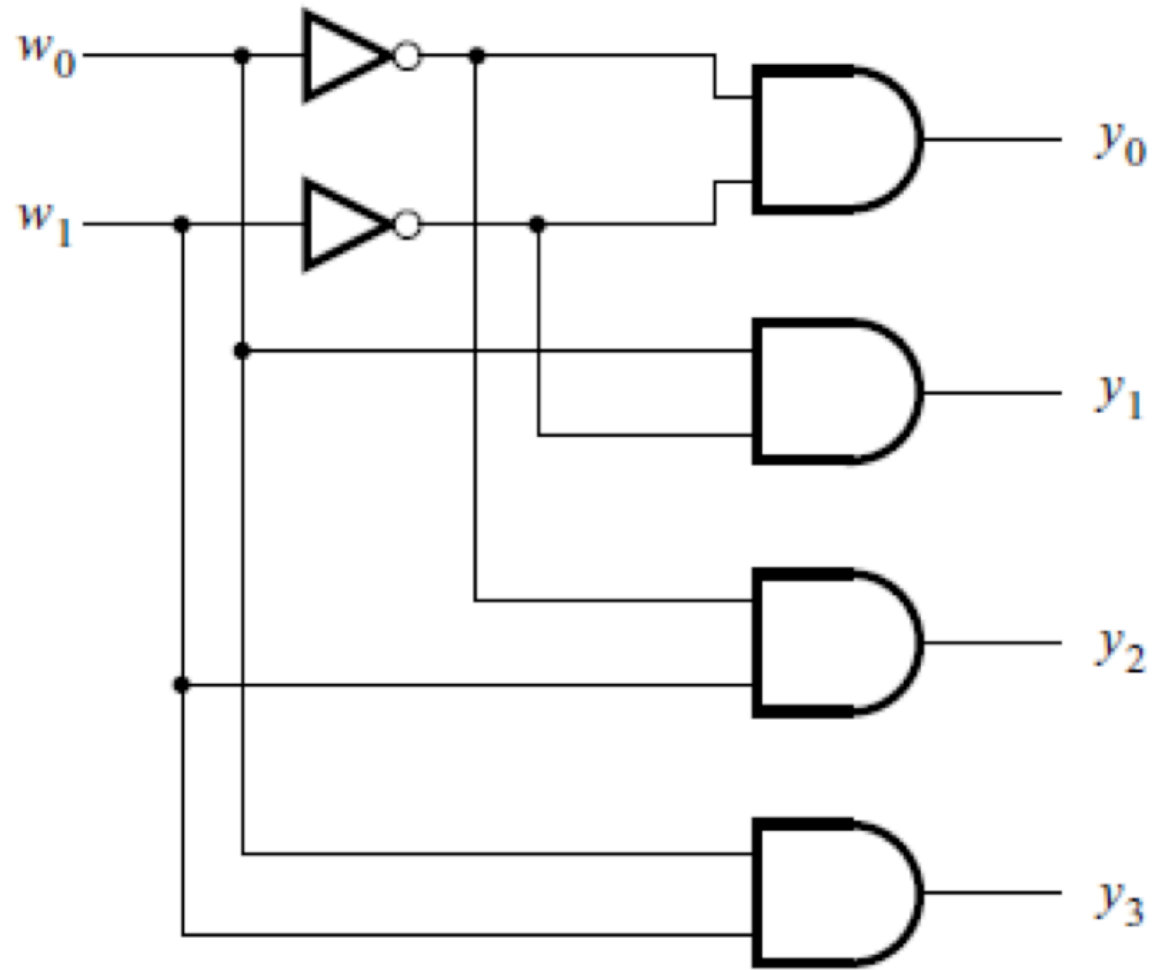
Out_1

Out_0

Read_address_1
Read_address_0

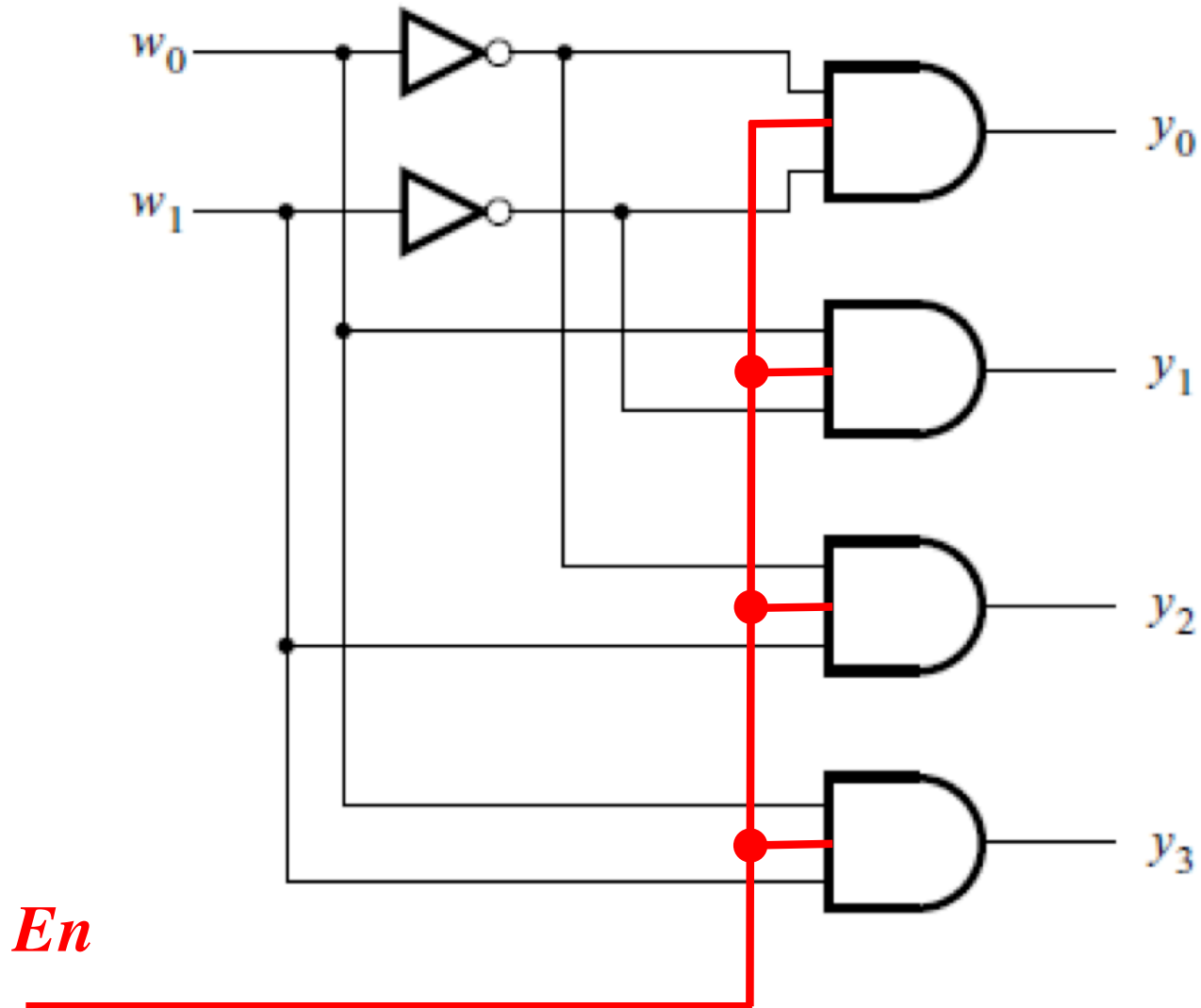


2-to-4 Decoder

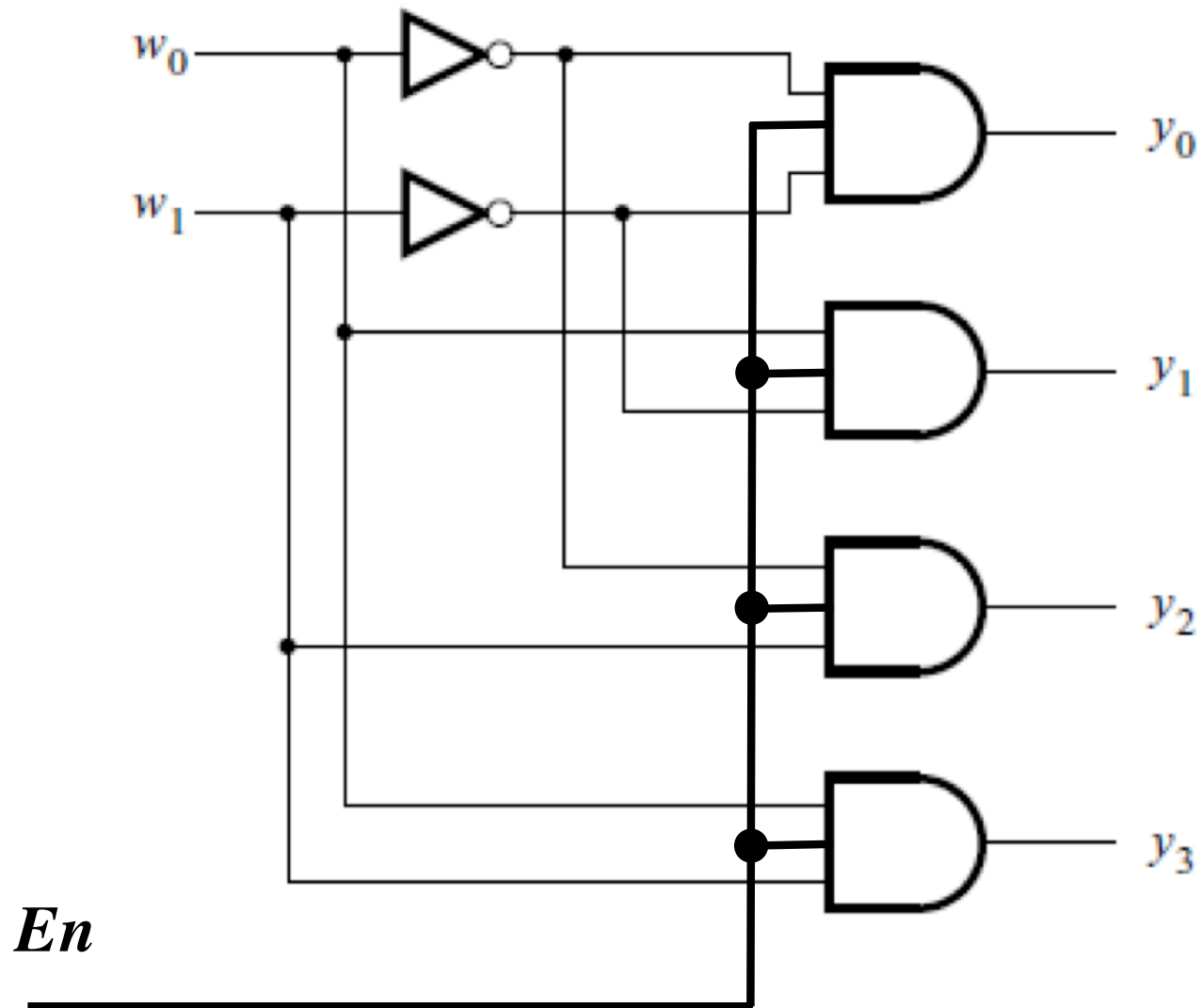


[Figure 4.14c from the textbook]

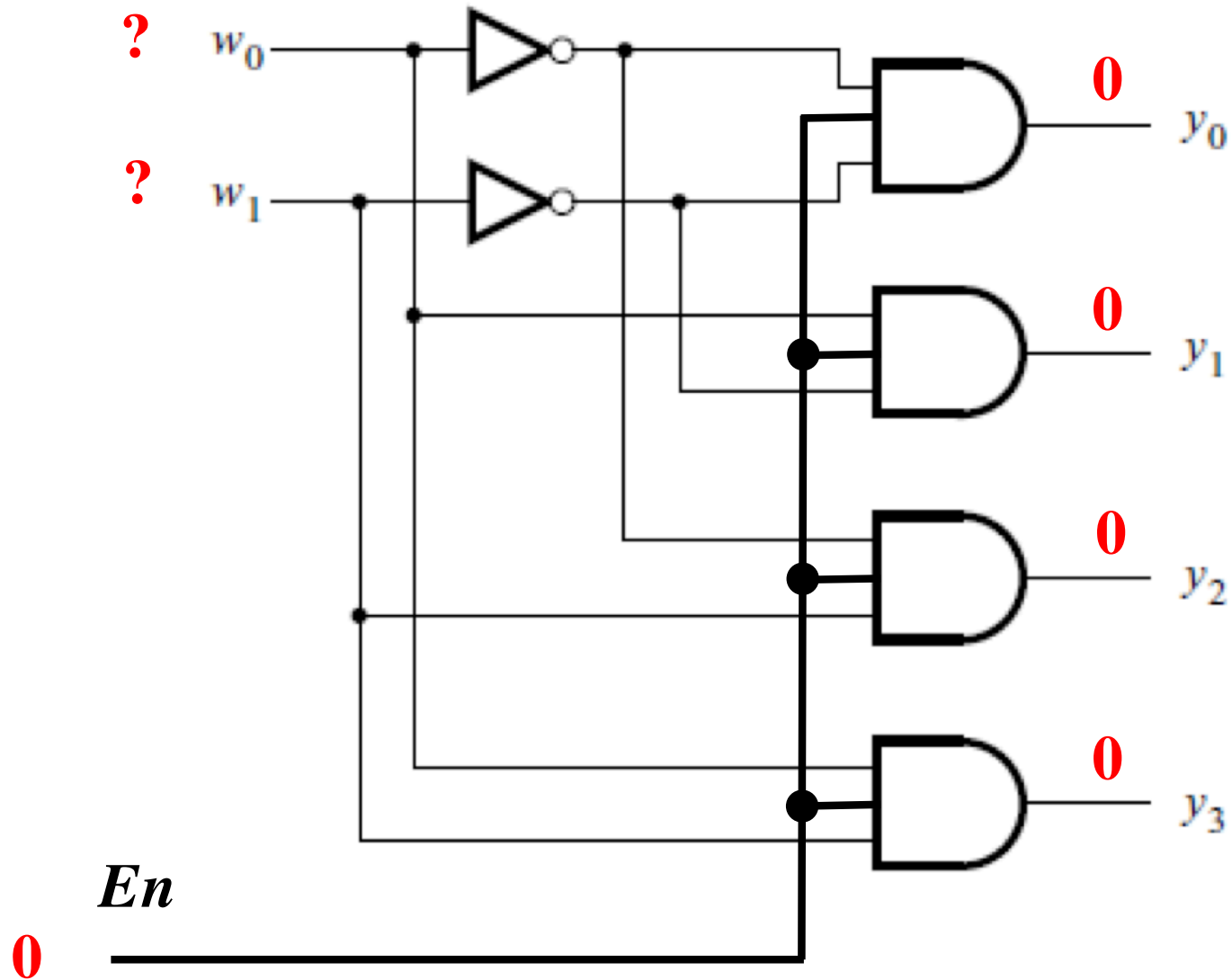
2-to-4 Decoder with Enable Input



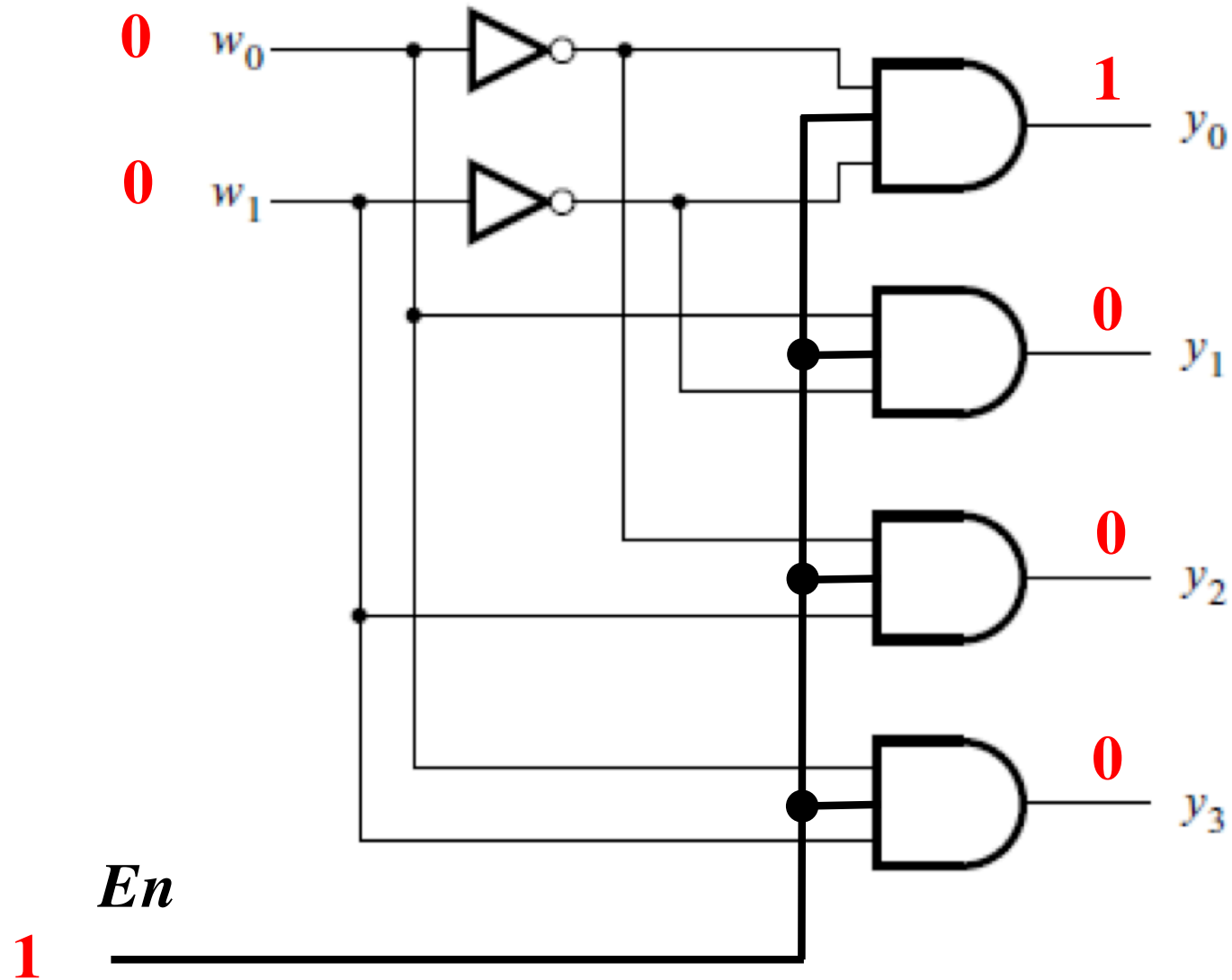
2-to-4 Decoder with Enable Input



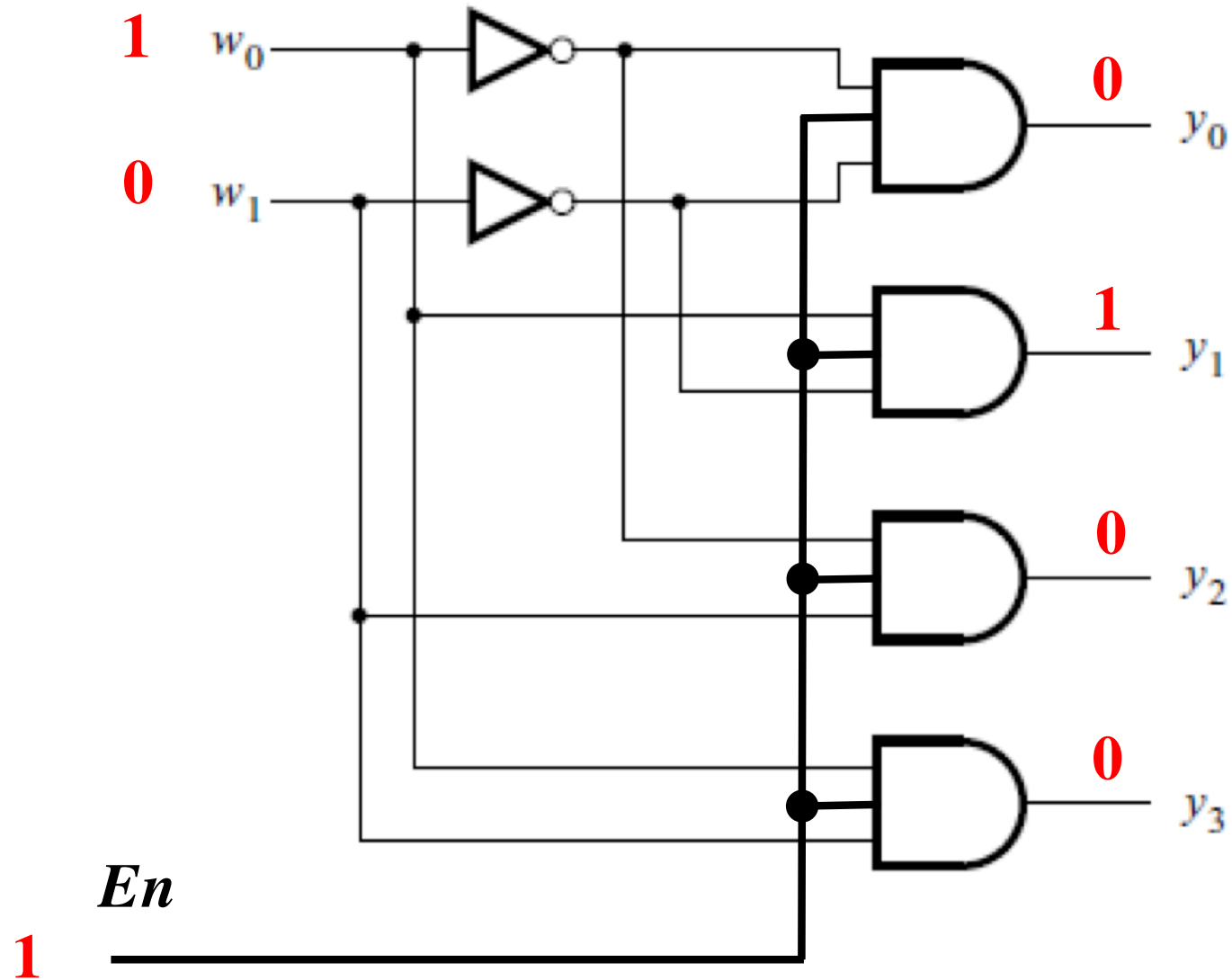
2-to-4 Decoder with Enable Input



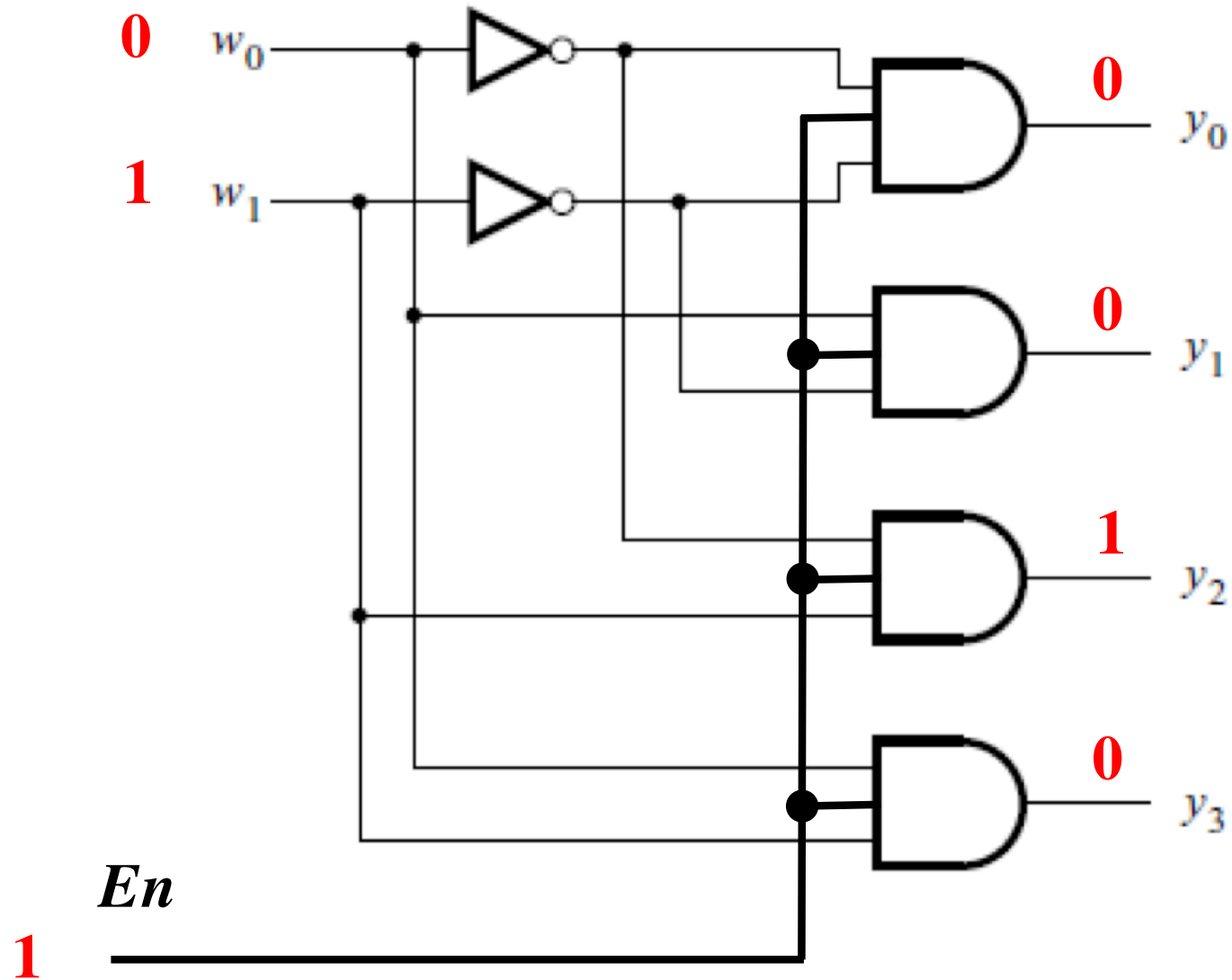
2-to-4 Decoder with Enable Input



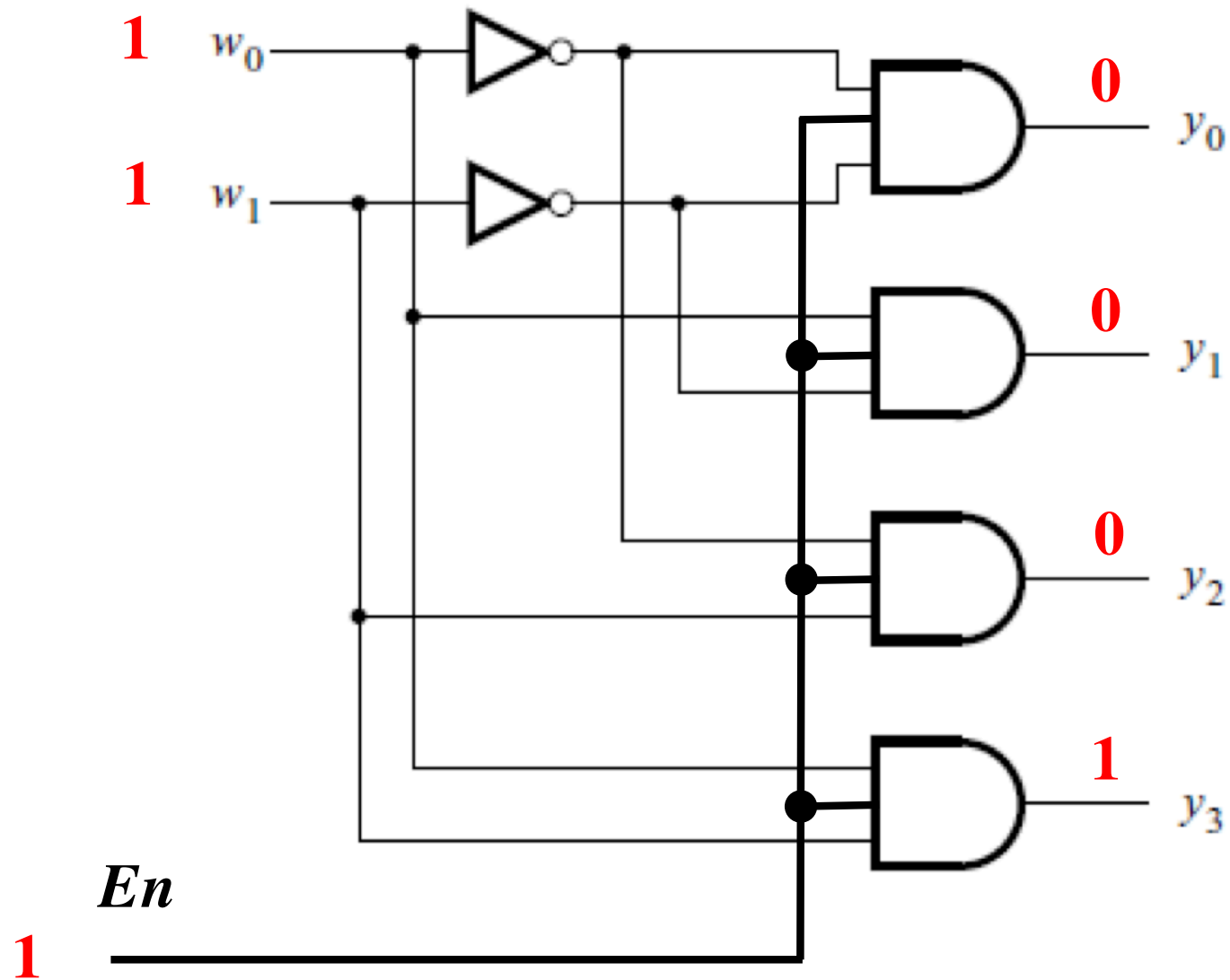
2-to-4 Decoder with Enable Input



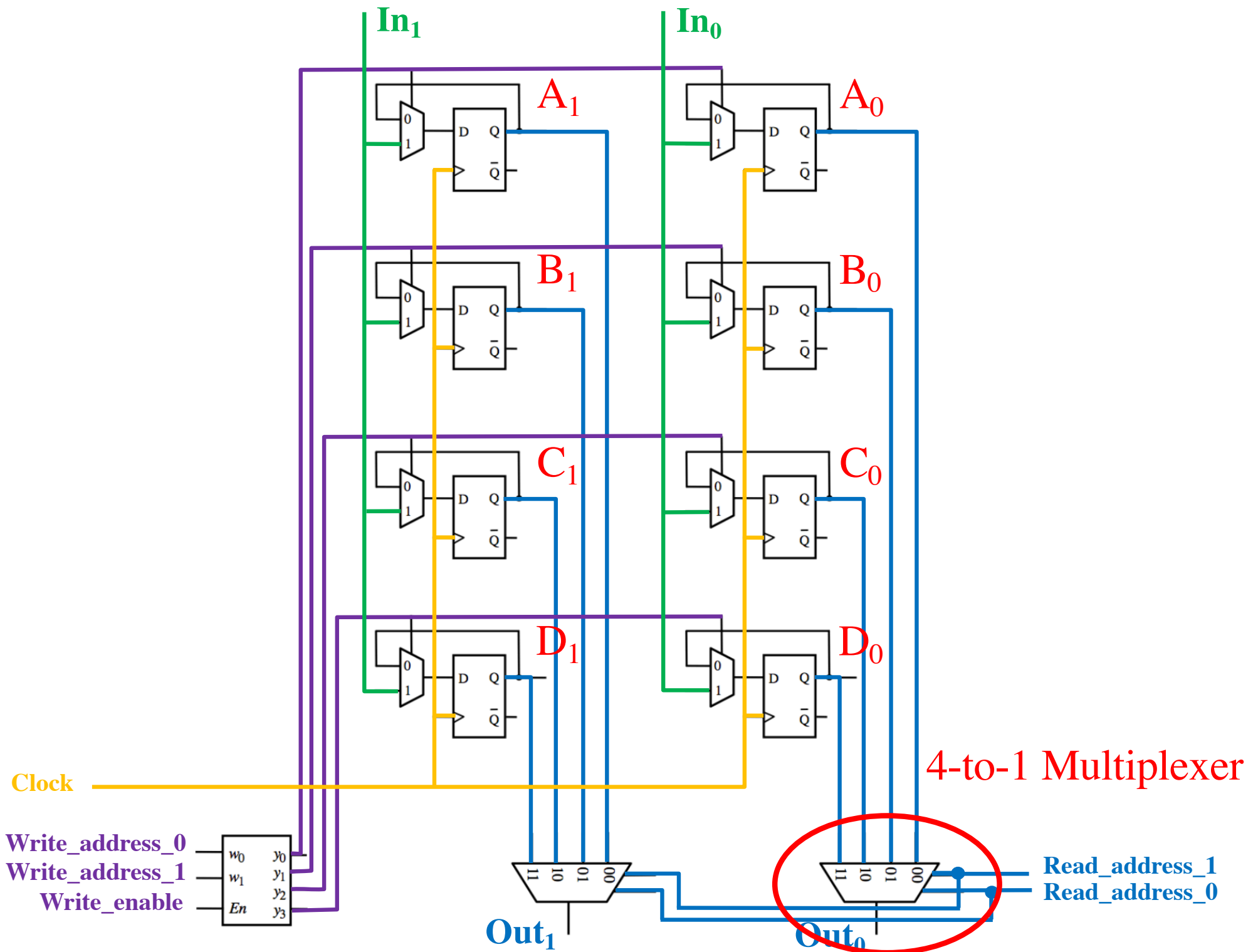
2-to-4 Decoder with Enable Input

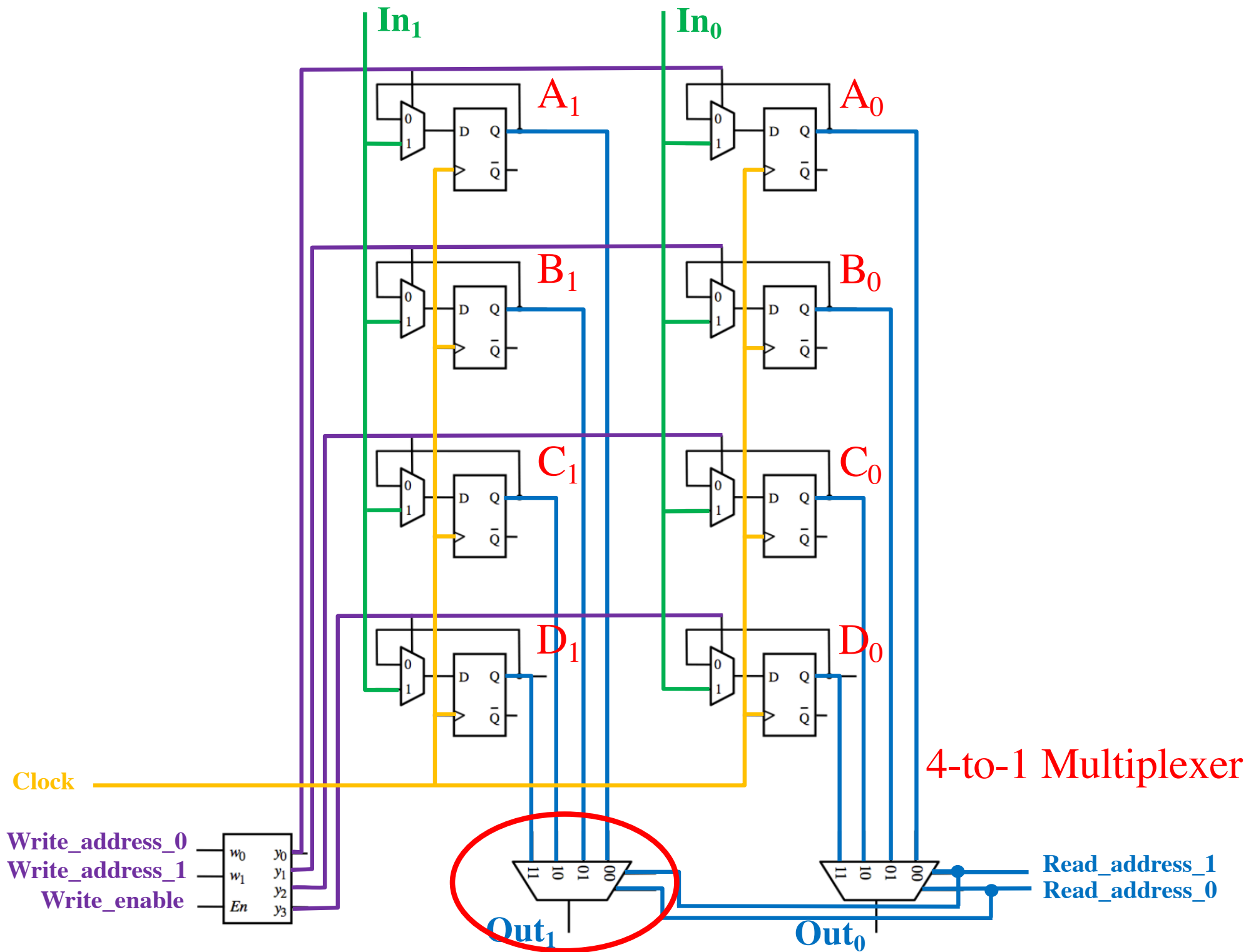


2-to-4 Decoder with Enable Input

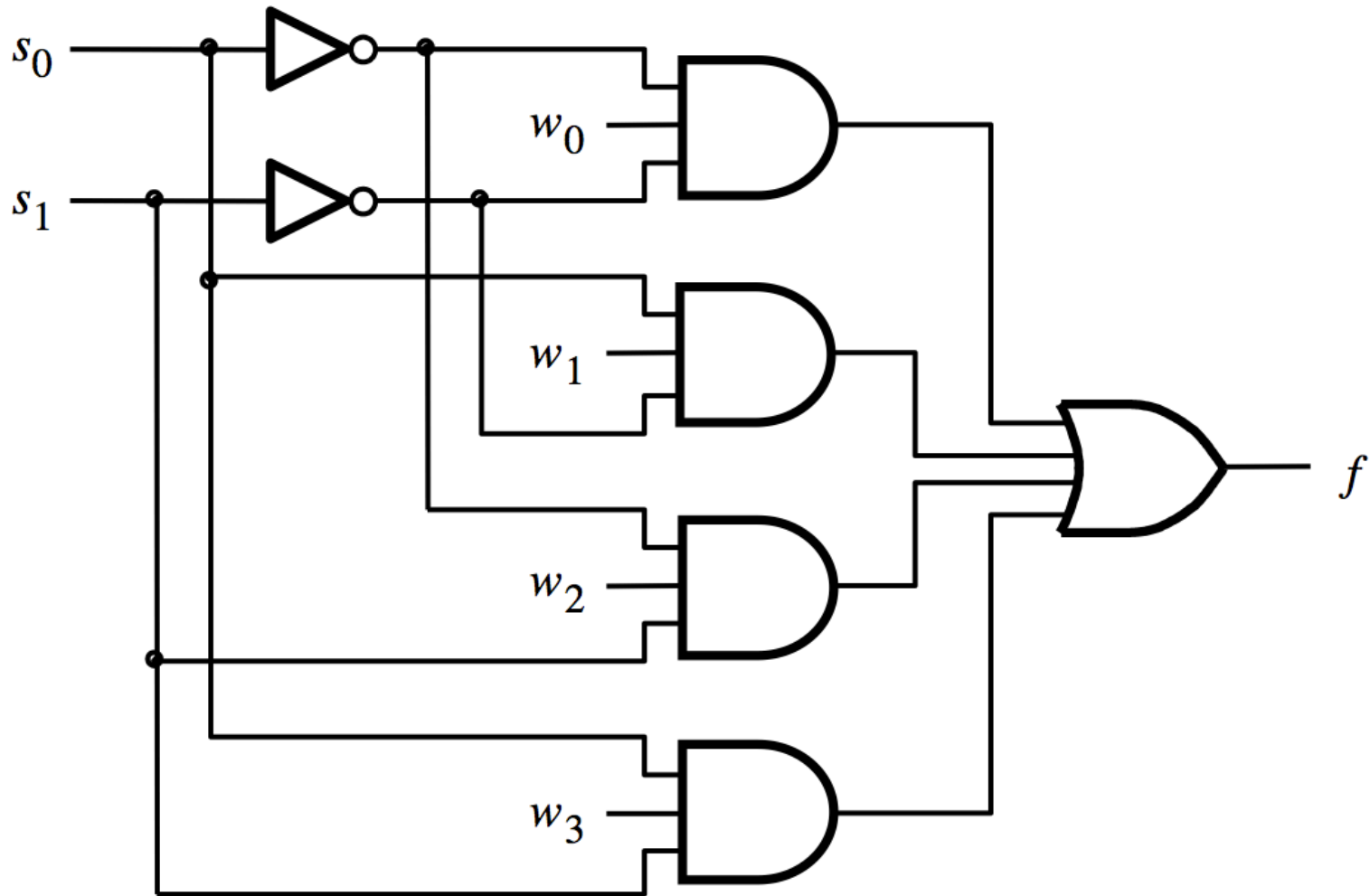


4-to-1 Multiplexer



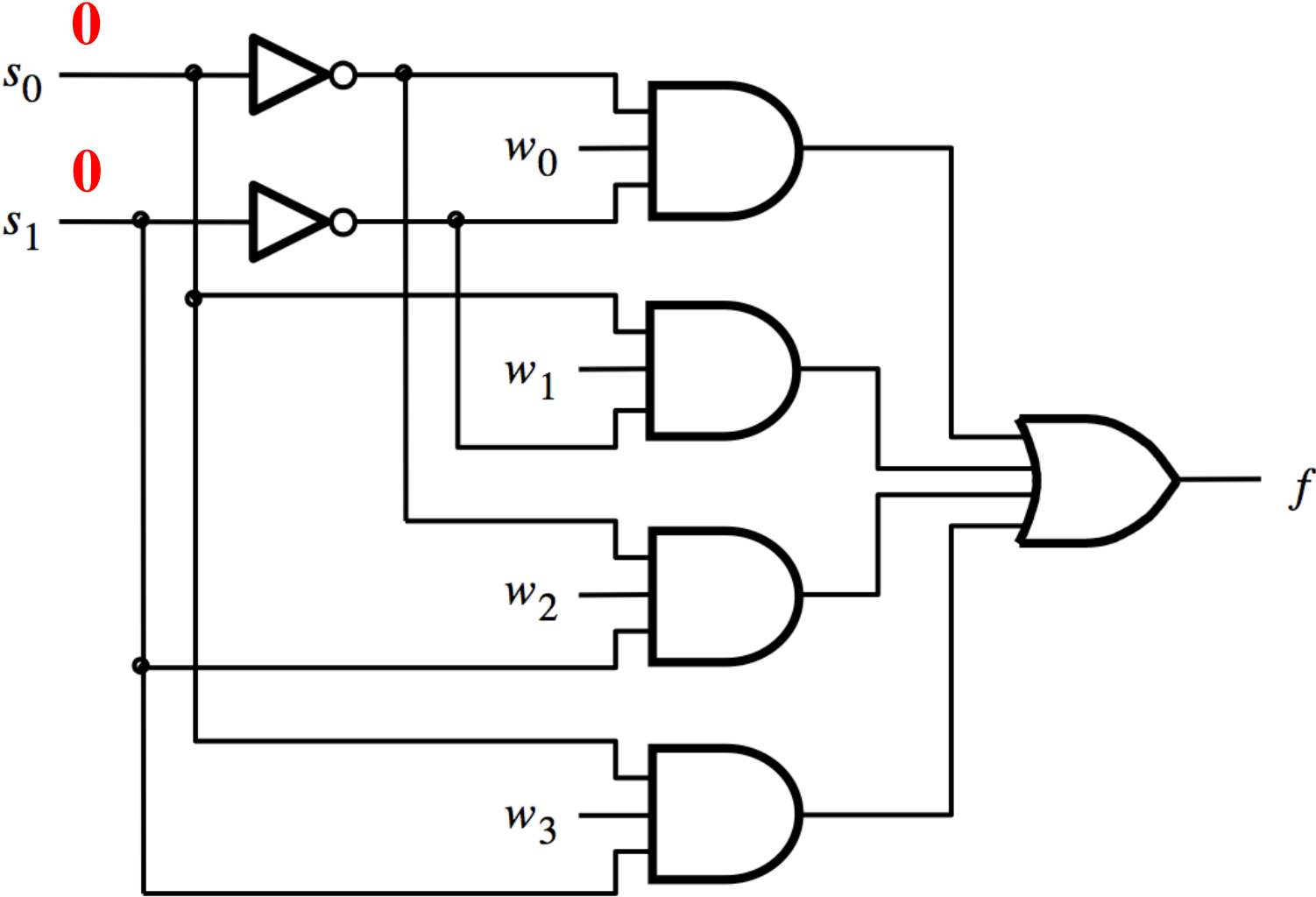


4-to-1 Multiplexer

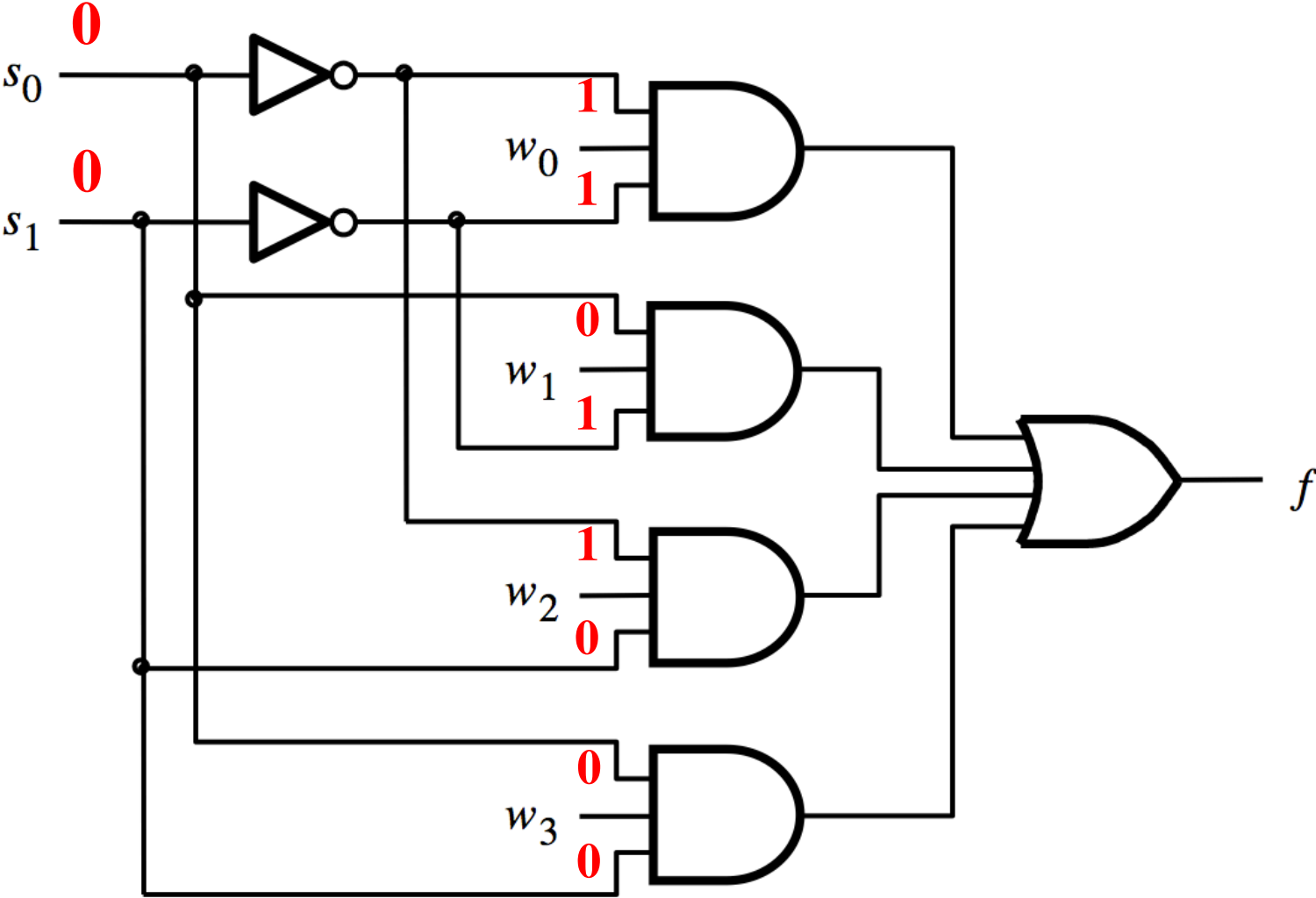


$$f = \overline{s_1} \overline{s_0} w_0 + \overline{s_1} s_0 w_1 + s_1 \overline{s_0} w_2 + s_1 s_0 w_3$$

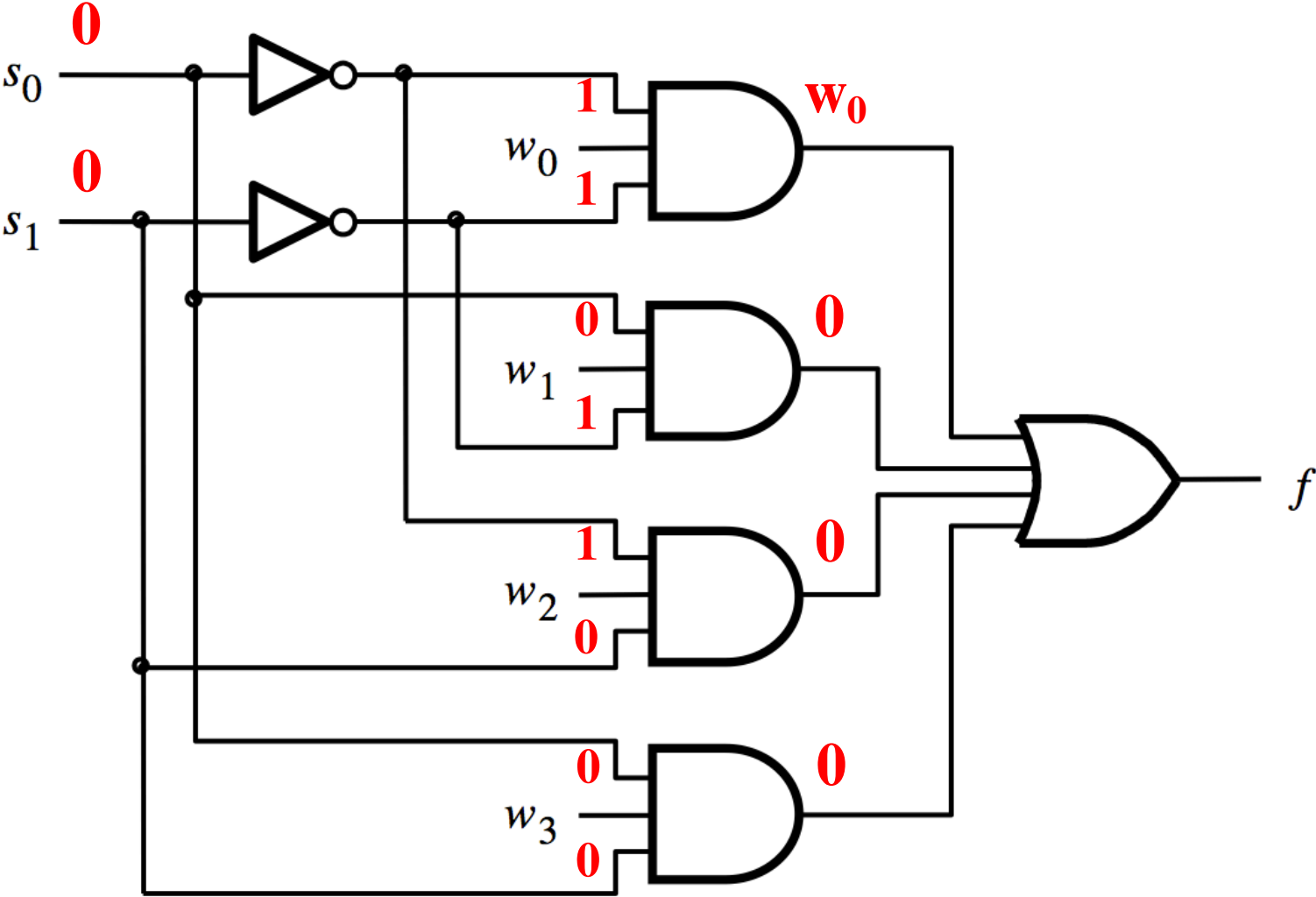
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=0$)



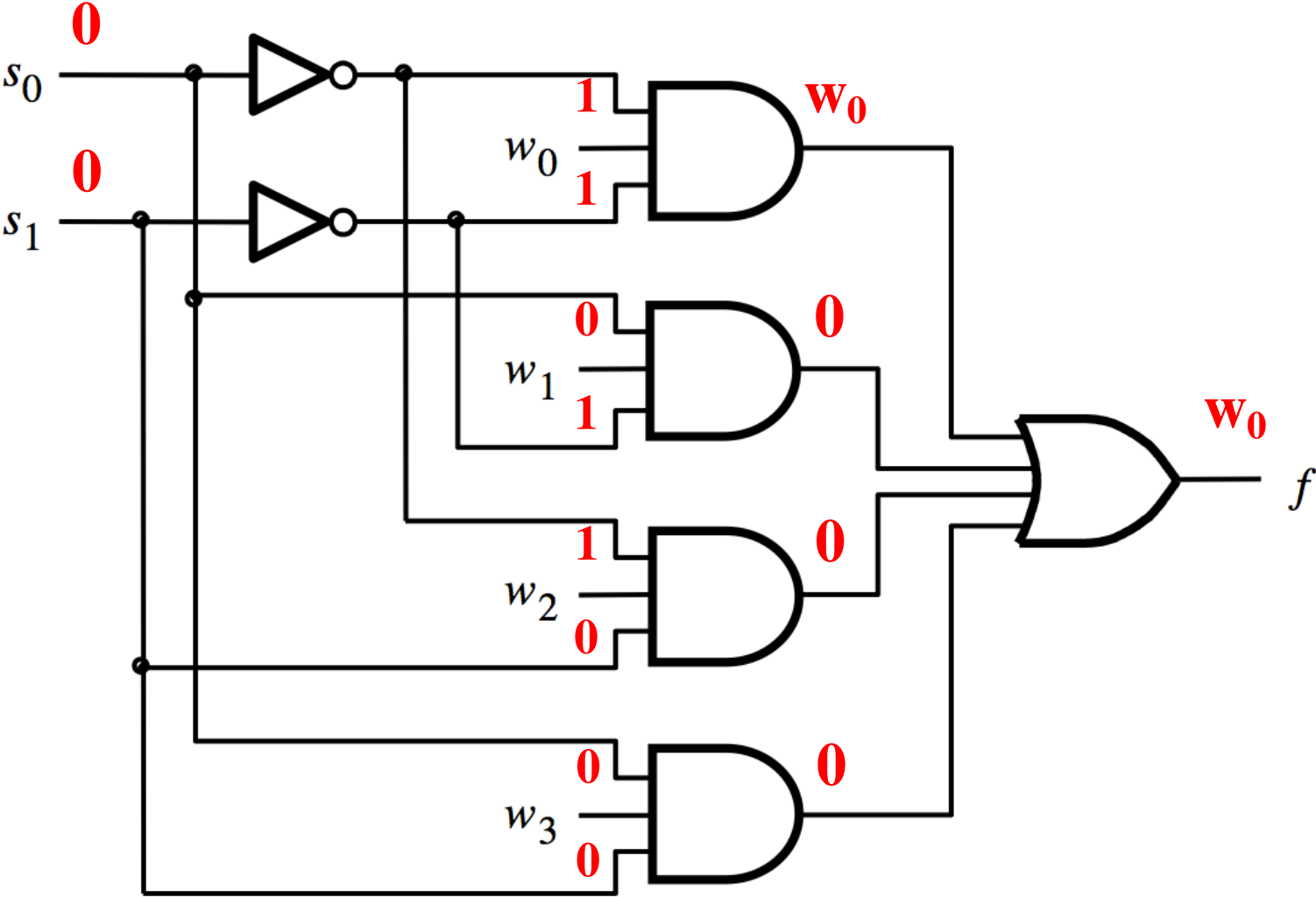
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=0$)



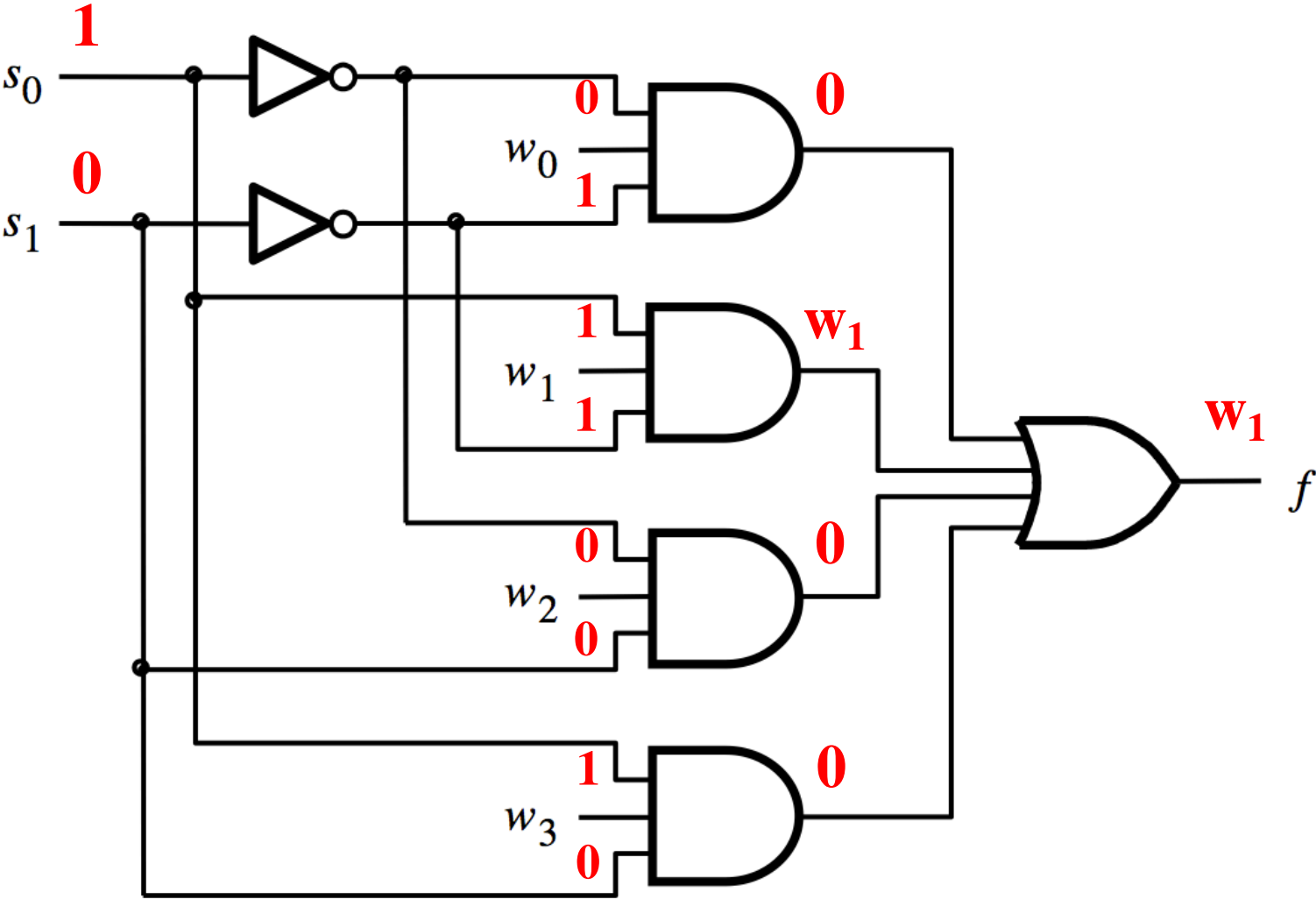
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=0$)



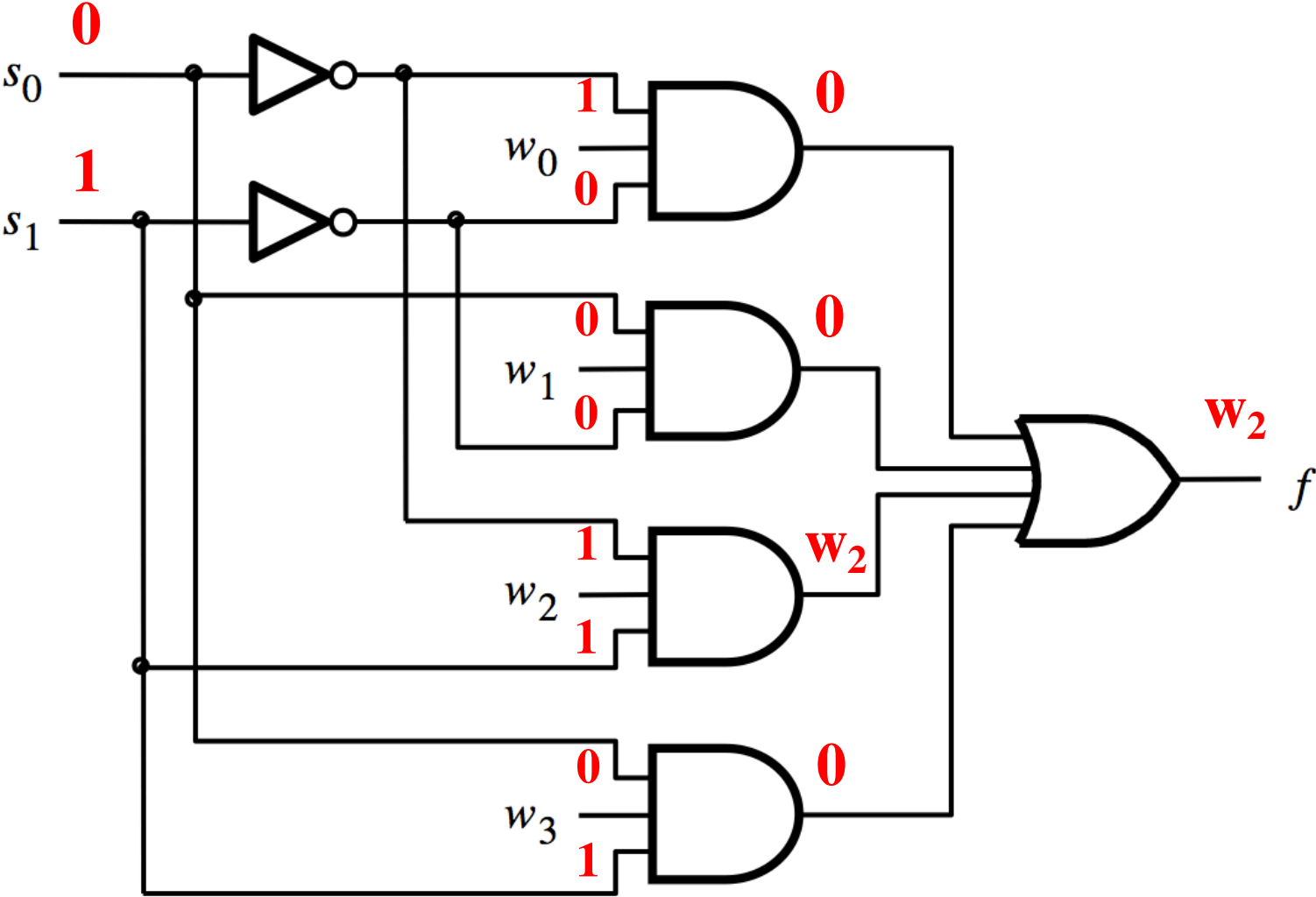
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=0$)



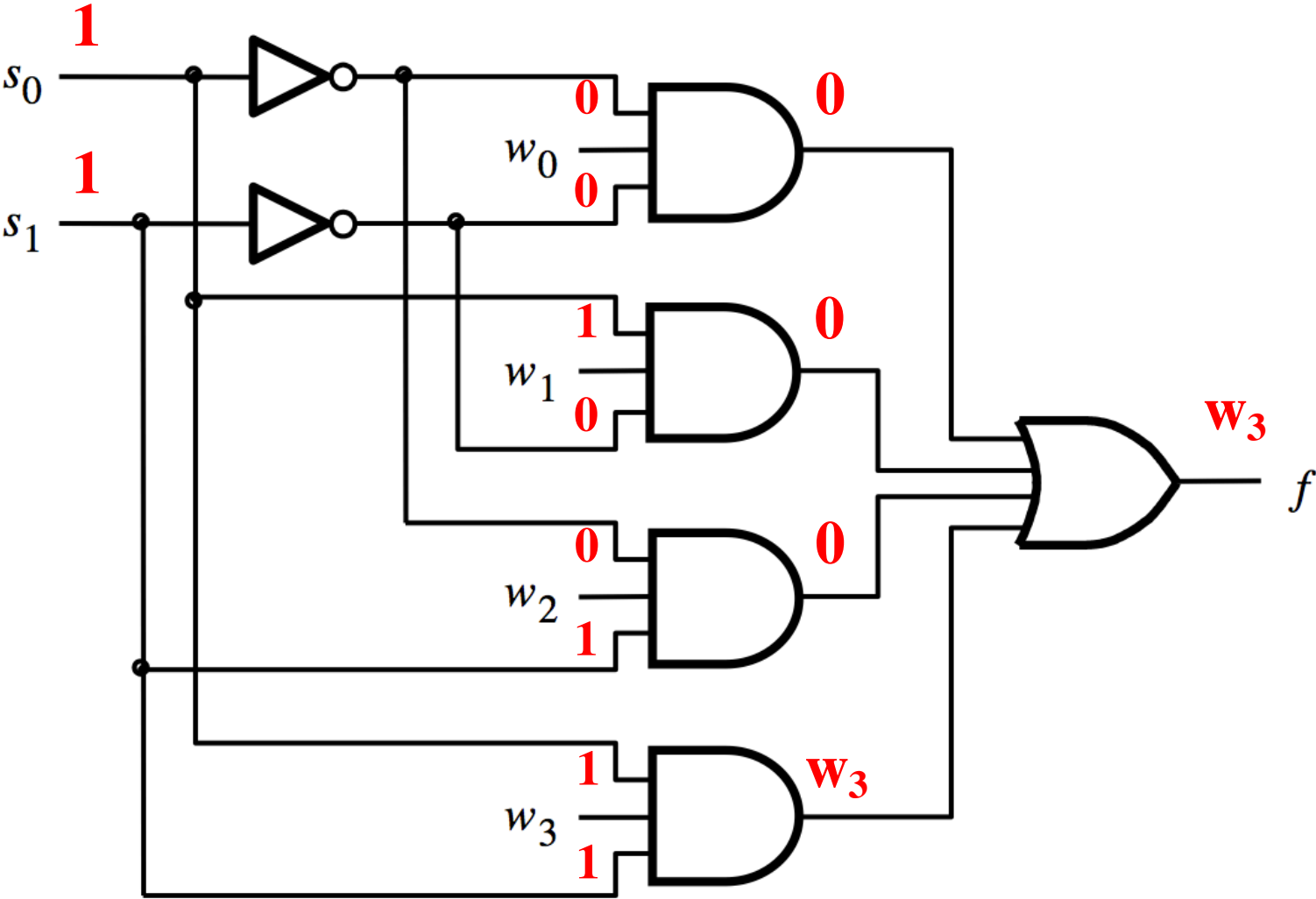
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=1$)



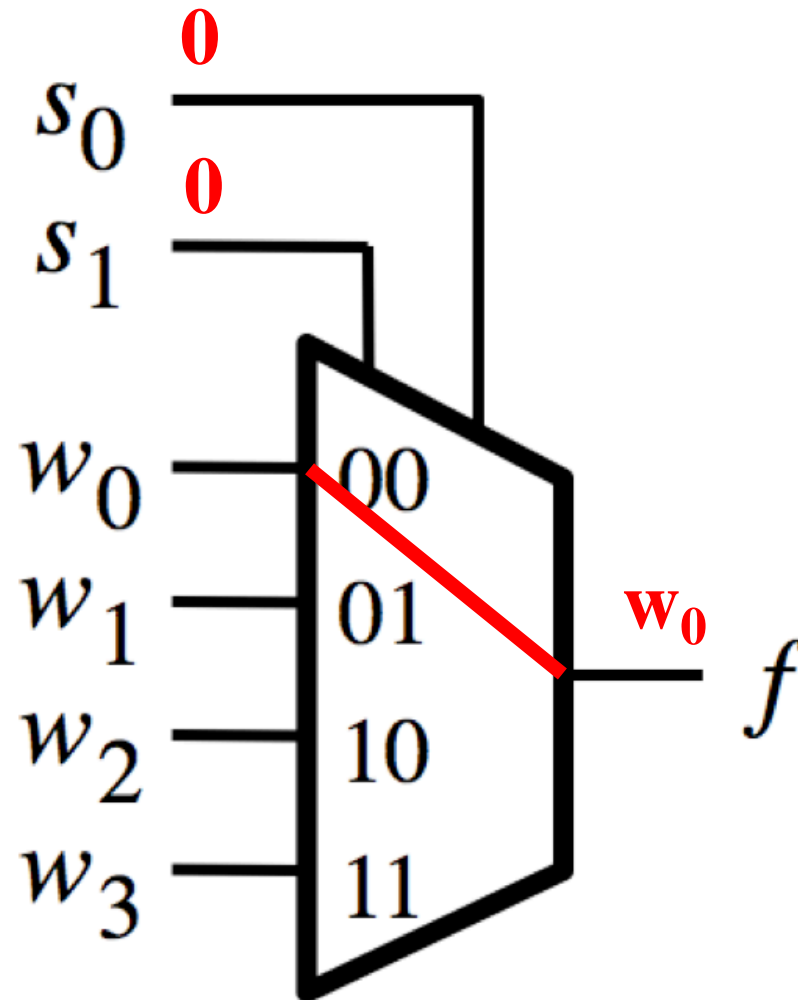
Analysis of the 4-to-1 Multiplexer ($s_1=1$ and $s_0=0$)



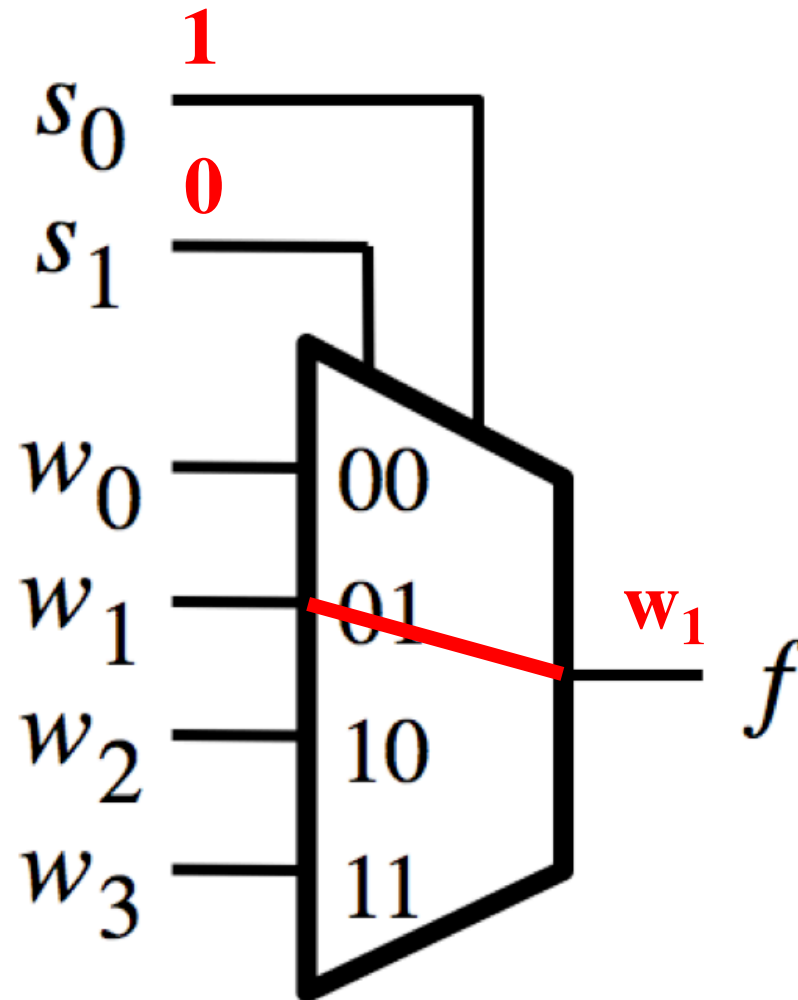
Analysis of the 4-to-1 Multiplexer ($s_1=1$ and $s_0=1$)



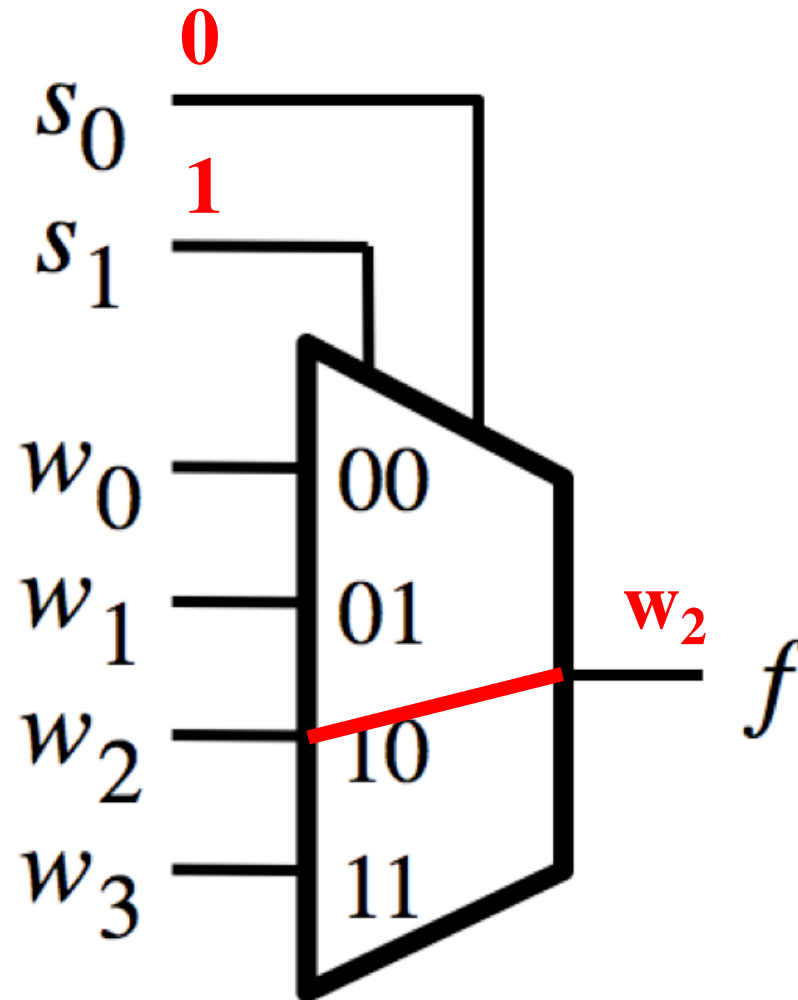
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=0$)



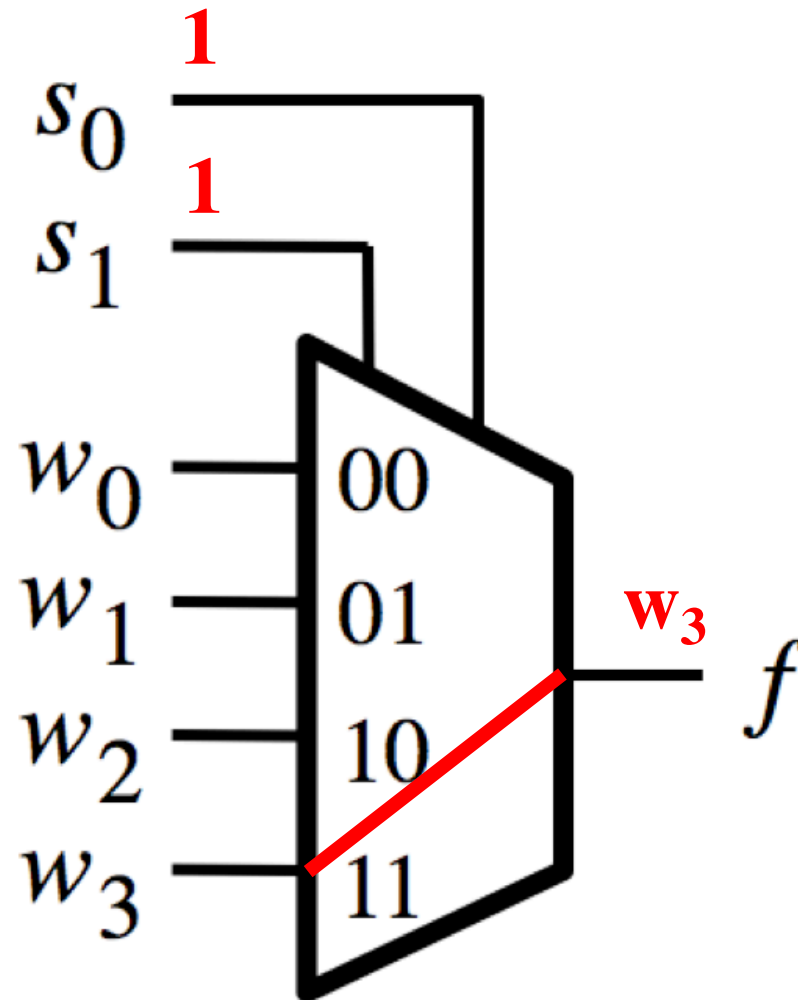
Analysis of the 4-to-1 Multiplexer ($s_1=0$ and $s_0=1$)



Analysis of the 4-to-1 Multiplexer ($s_1=1$ and $s_0=0$)



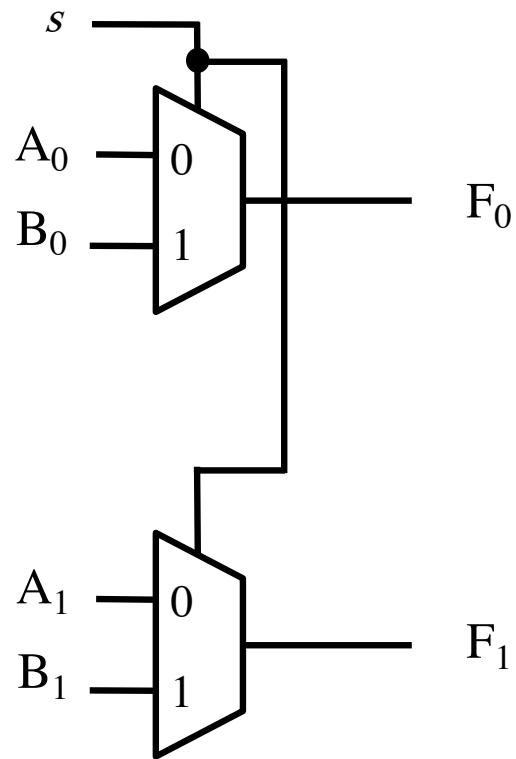
Analysis of the 4-to-1 Multiplexer ($s_1=1$ and $s_0=1$)



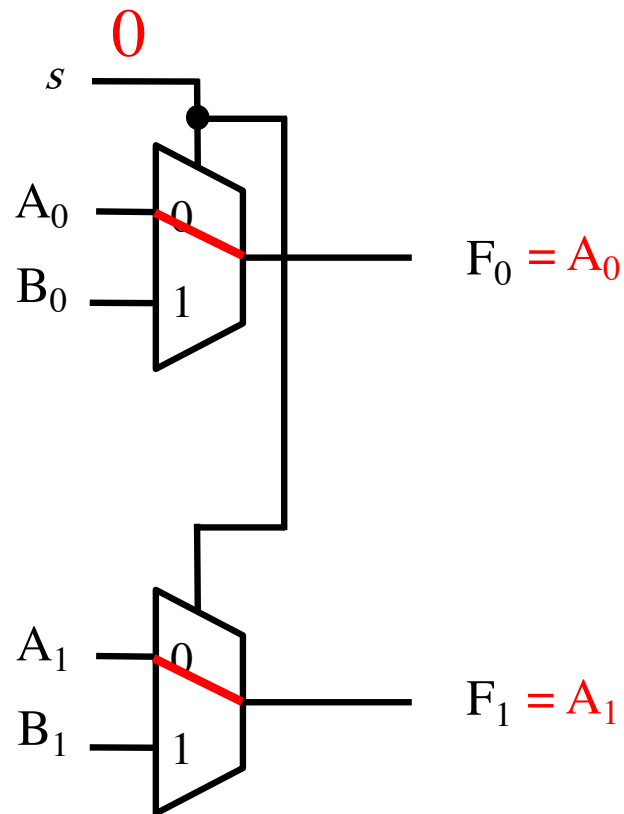
Multiplexer Tricks

(select one of two 2-bit numbers)

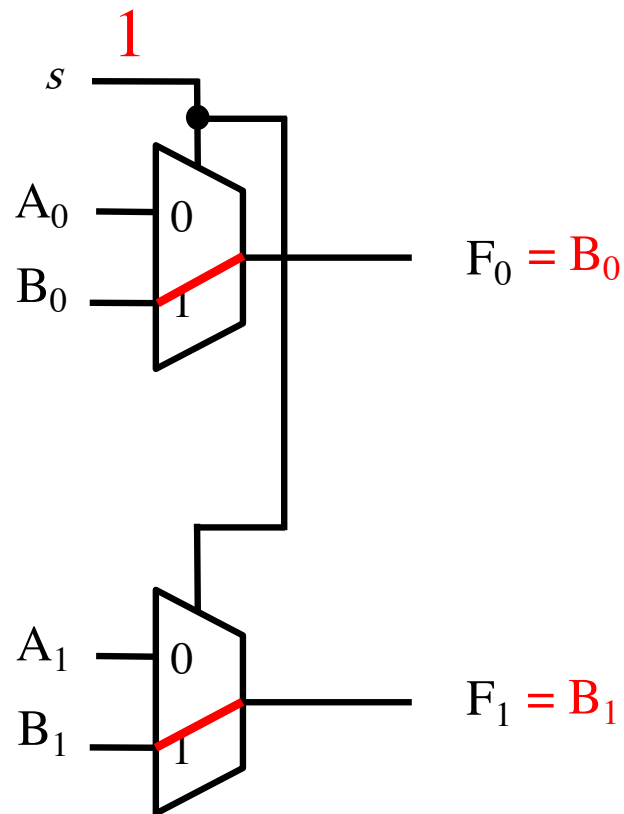
Select Either $A=A_1A_0$ or $B=B_1B_0$



Select Either $A=A_1A_0$ or $B=B_1B_0$



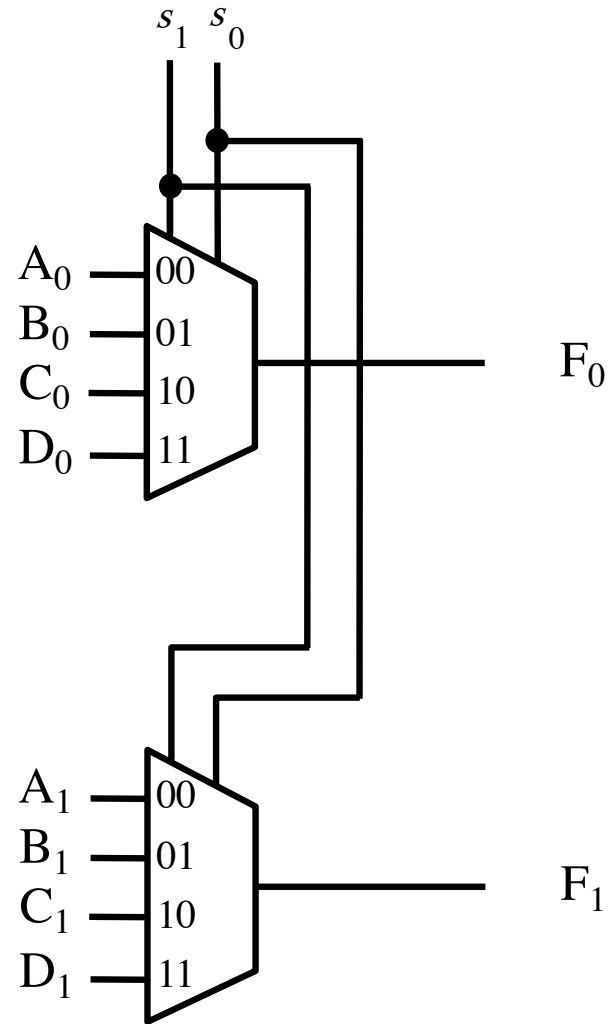
Select Either $A=A_1A_0$ or $B=B_1B_0$



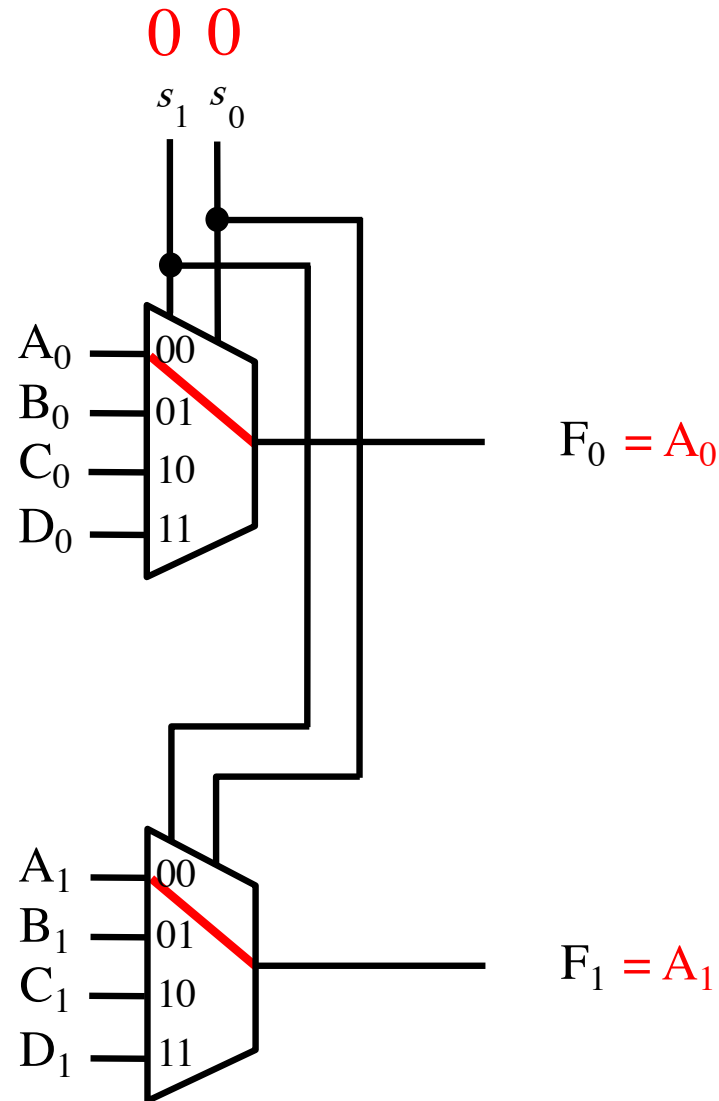
Multiplexer Tricks

(select one of four 2-bit numbers)

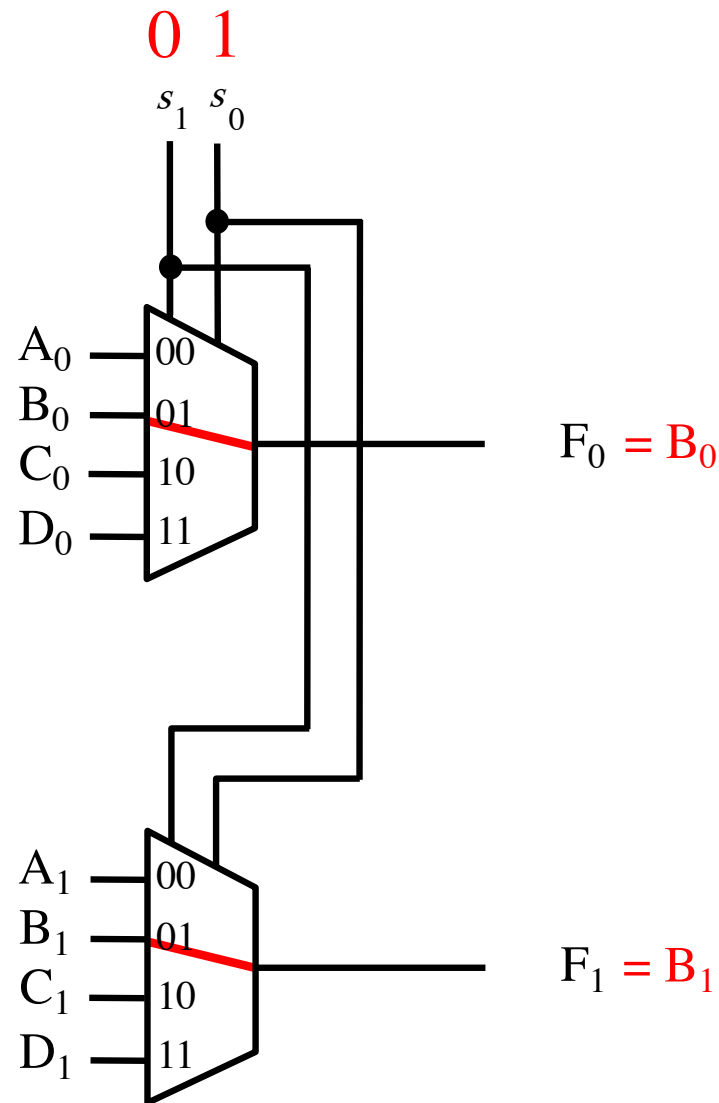
Select $A=A_1A_0$ or $B=B_1B_0$ or $C=C_1C_0$ or $D=D_1D_0$



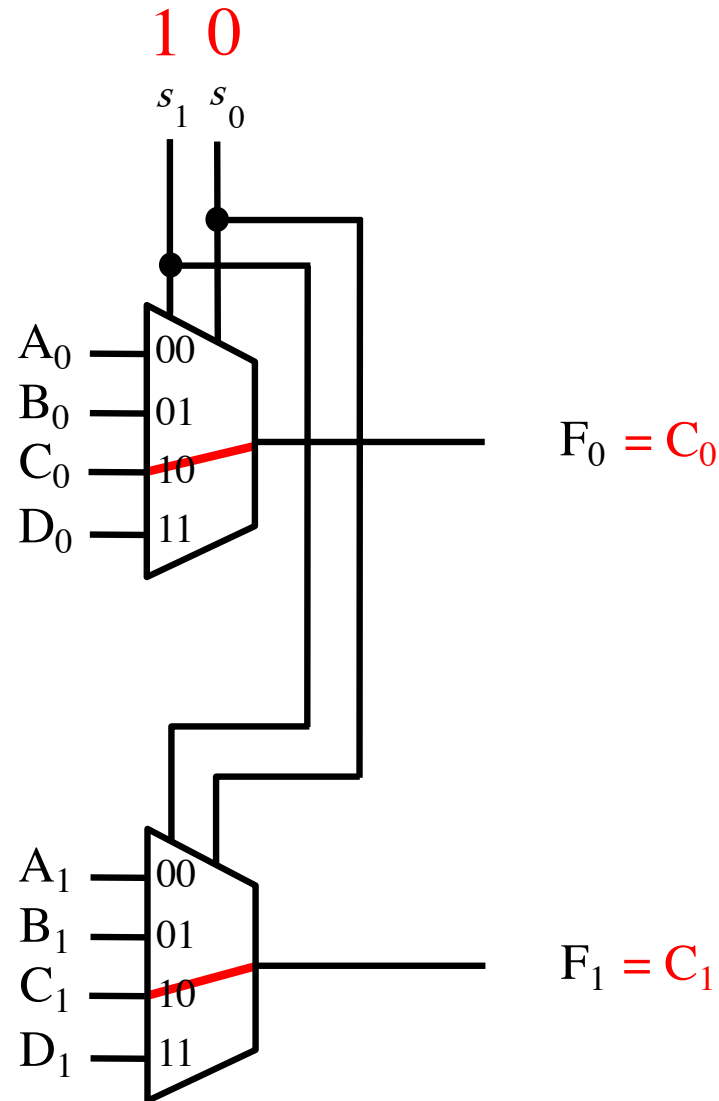
Select $A=A_1A_0$ or $B=B_1B_0$ or $C=C_1C_0$ or $D=D_1D_0$



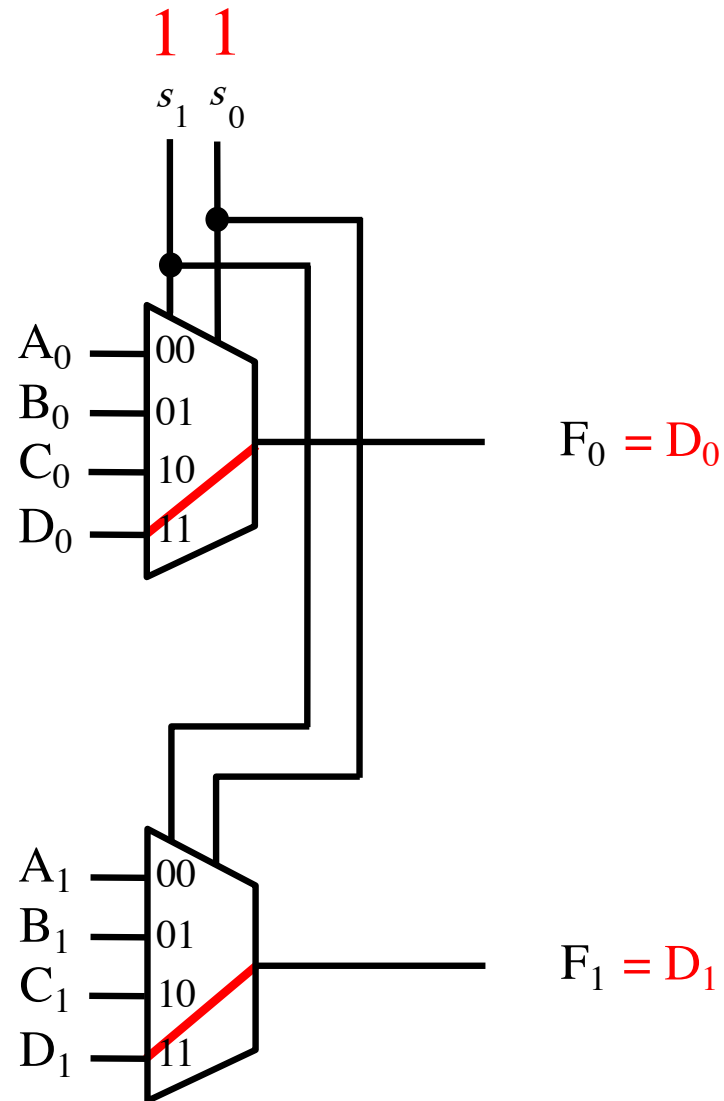
Select $A=A_1A_0$ or $B=B_1B_0$ or $C=C_1C_0$ or $D=D_1D_0$



Select $A=A_1A_0$ or $B=B_1B_0$ or $C=C_1C_0$ or $D=D_1D_0$



Select $A=A_1A_0$ or $B=B_1B_0$ or $C=C_1C_0$ or $D=D_1D_0$

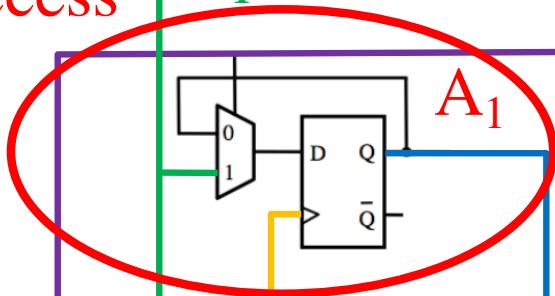


1-bit Parallel-Access Register

1-bit Parallel-Access Register

In₁

In₀



A₁

A₀

B₁

B₀

C₁

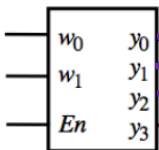
C₀

D₁

D₀

Clock

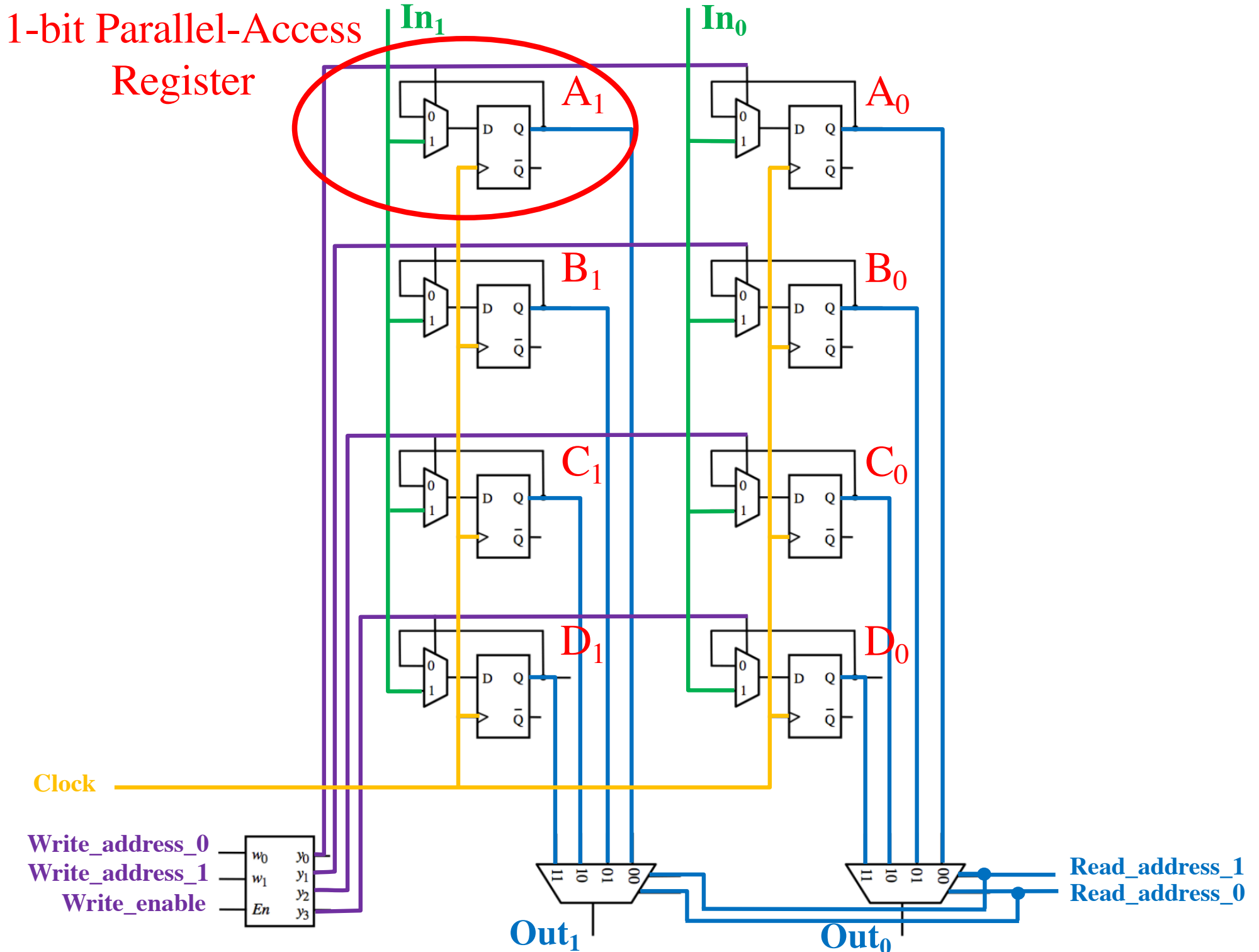
Write_address_0
Write_address_1
Write_enable



Out₁

Out₀

Read_address_1
Read_address_0



1-bit Parallel-Access Register

x 8

In₁

In₀

A₁

A₀

B₁

B₀

C₁

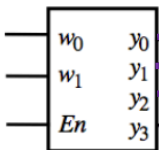
C₀

D₁

D₀

Clock

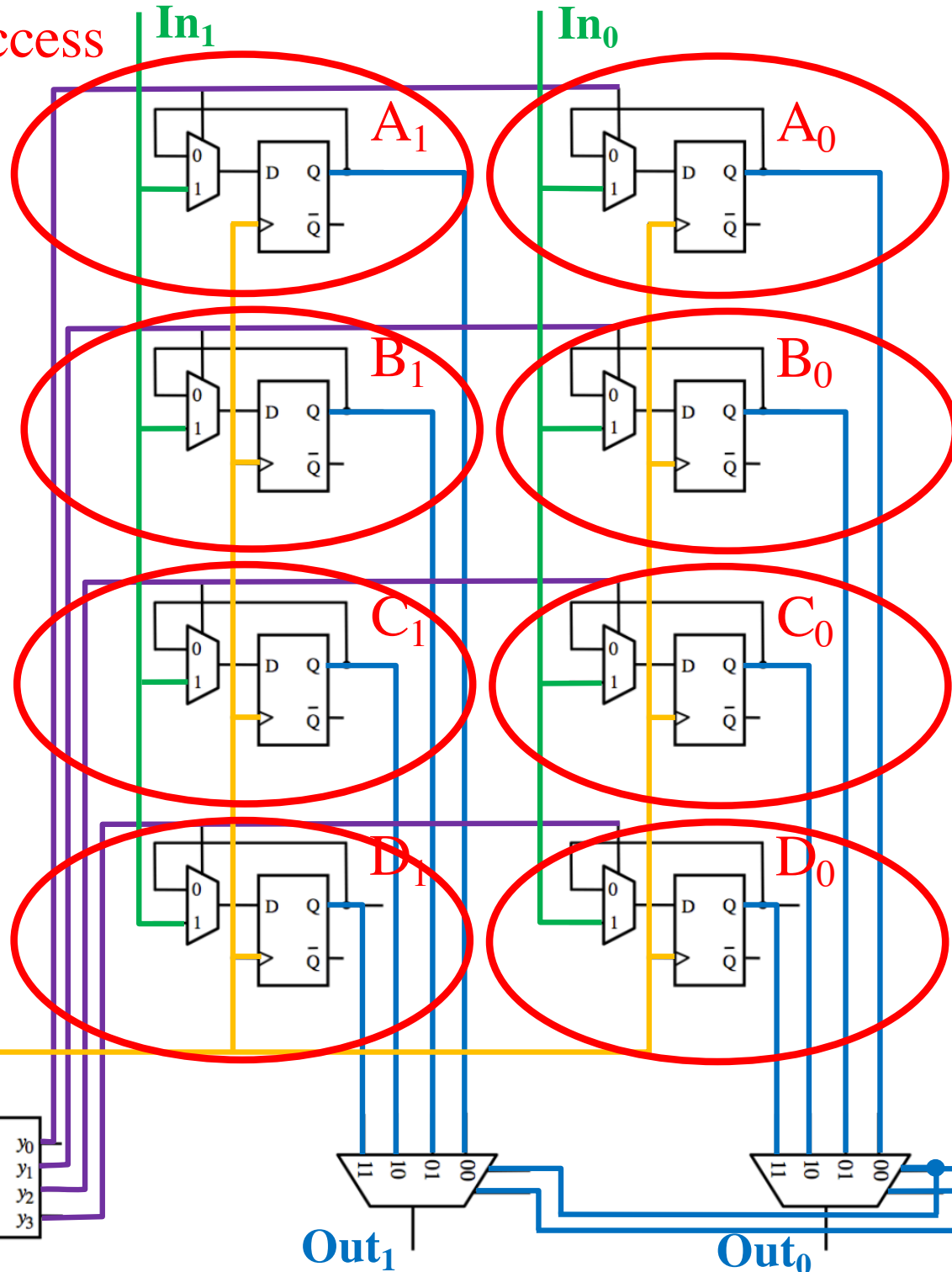
Write_address_0
Write_address_1
Write_enable



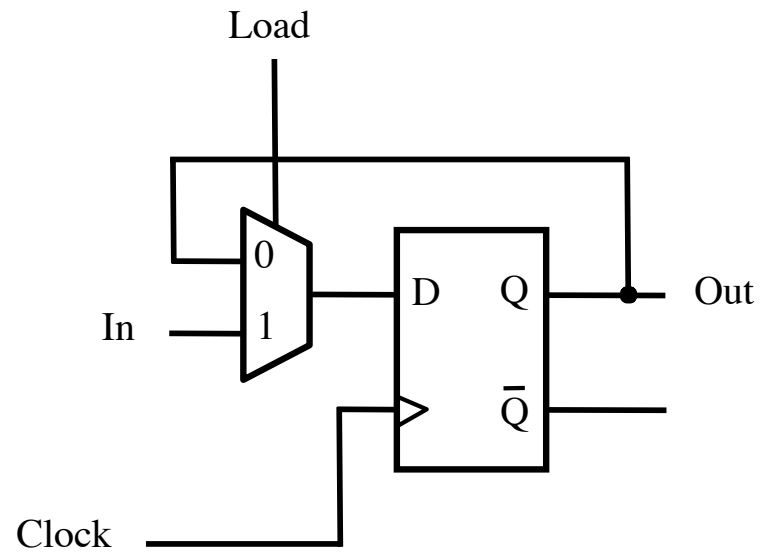
Out₁

Out₀

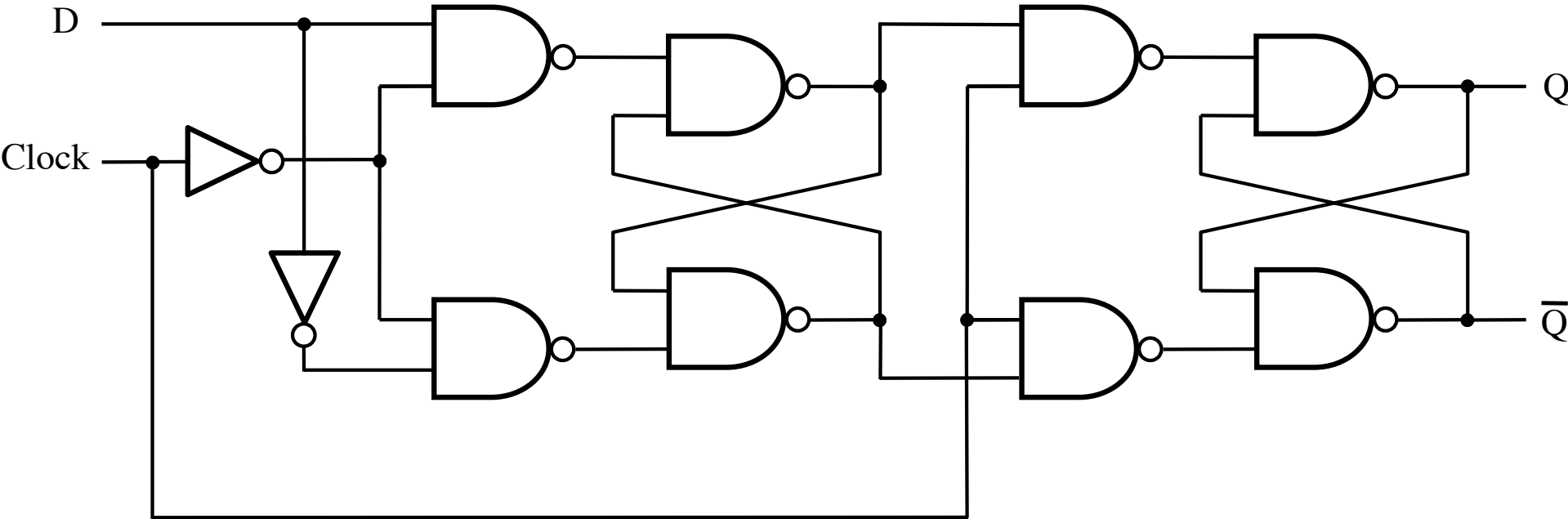
Read_address_1
Read_address_0



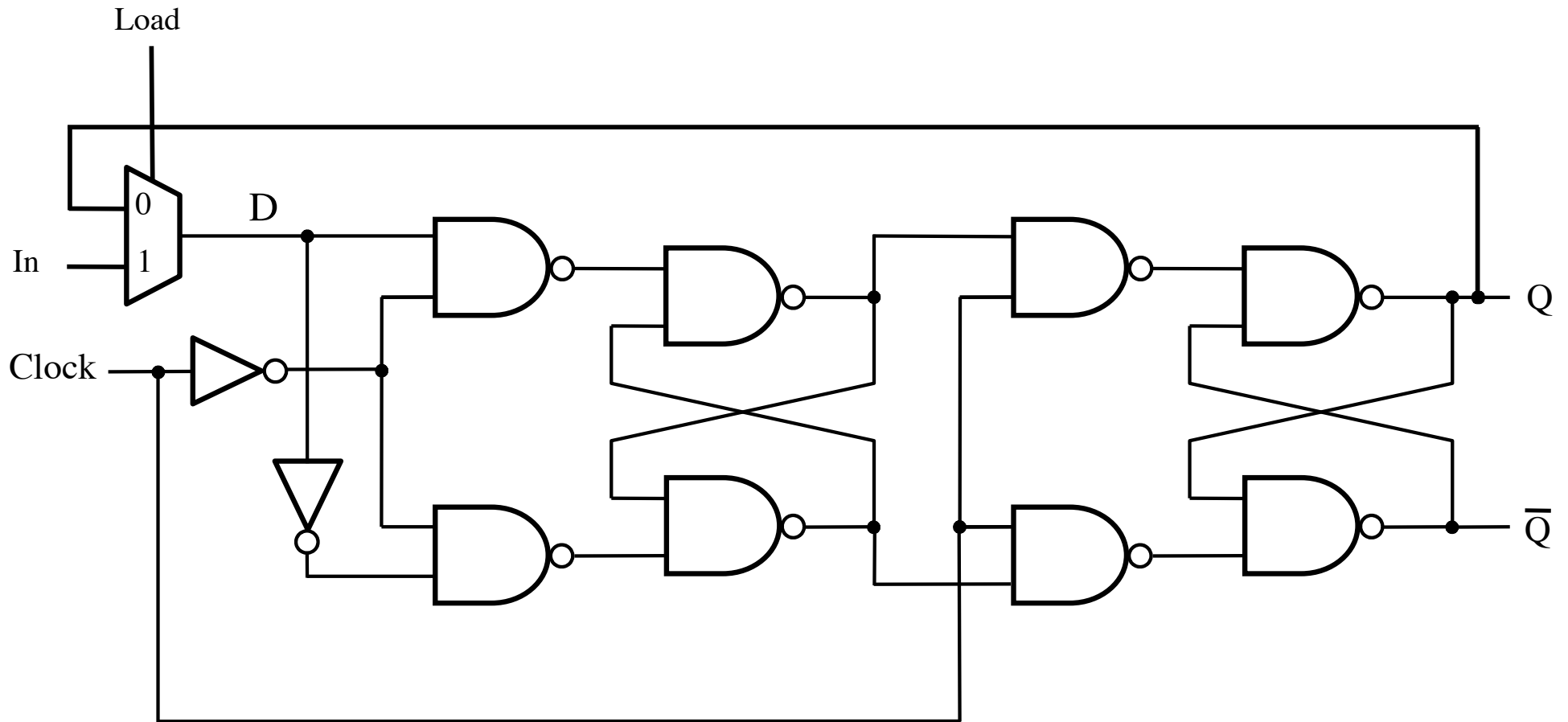
1-Bit Parallel-Access Register



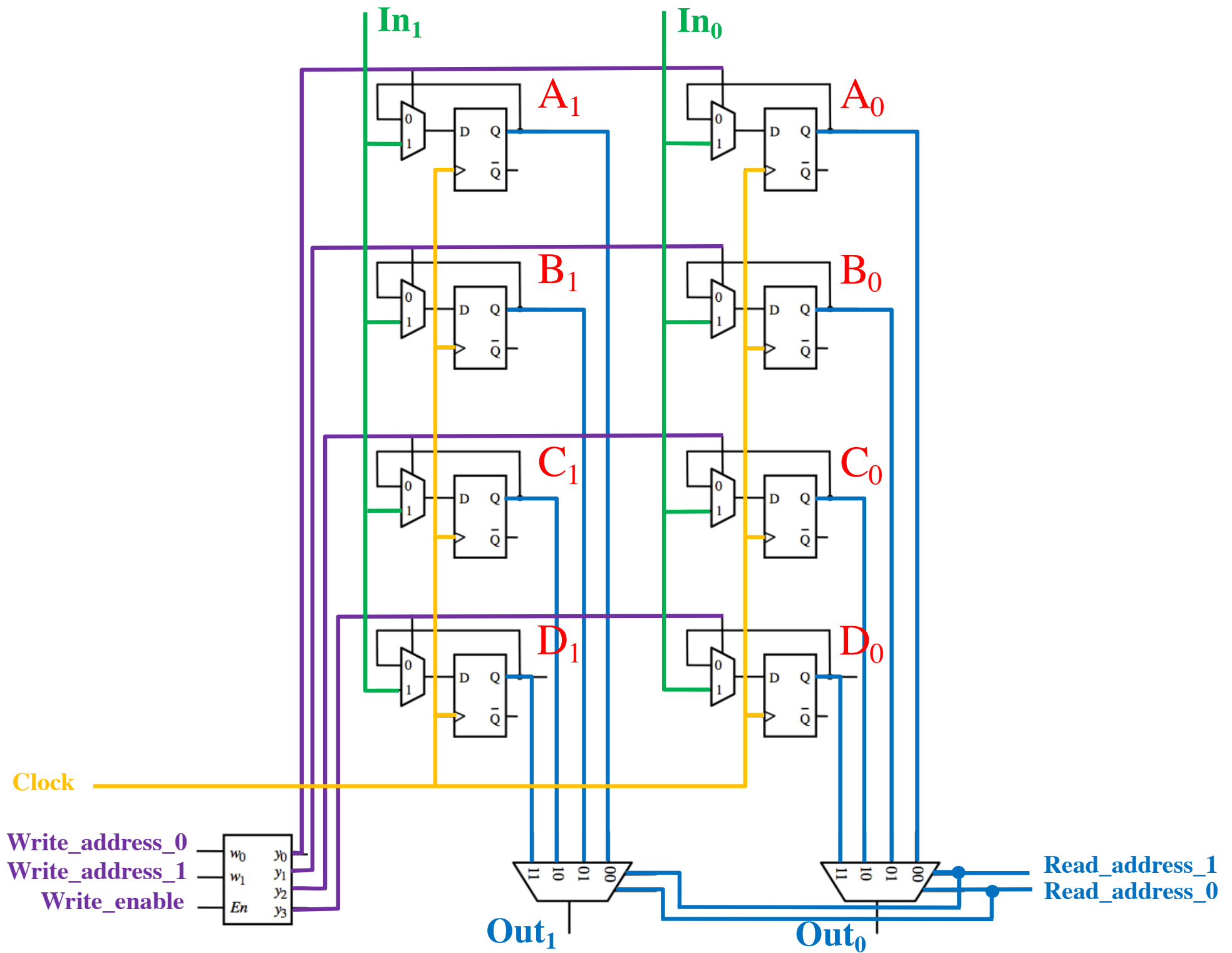
Positive-Edge-Triggered D Flip-Flop



1-bit Parallel-Access Register



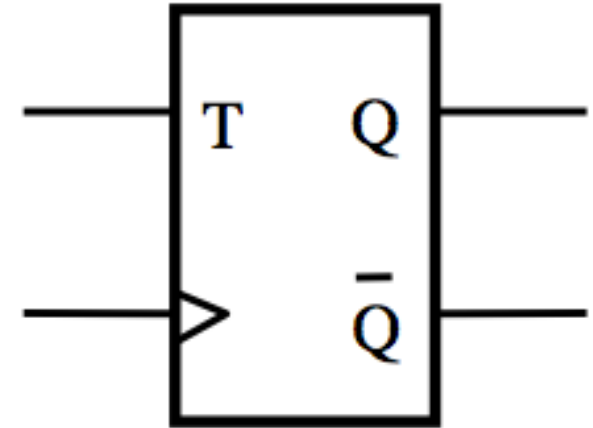
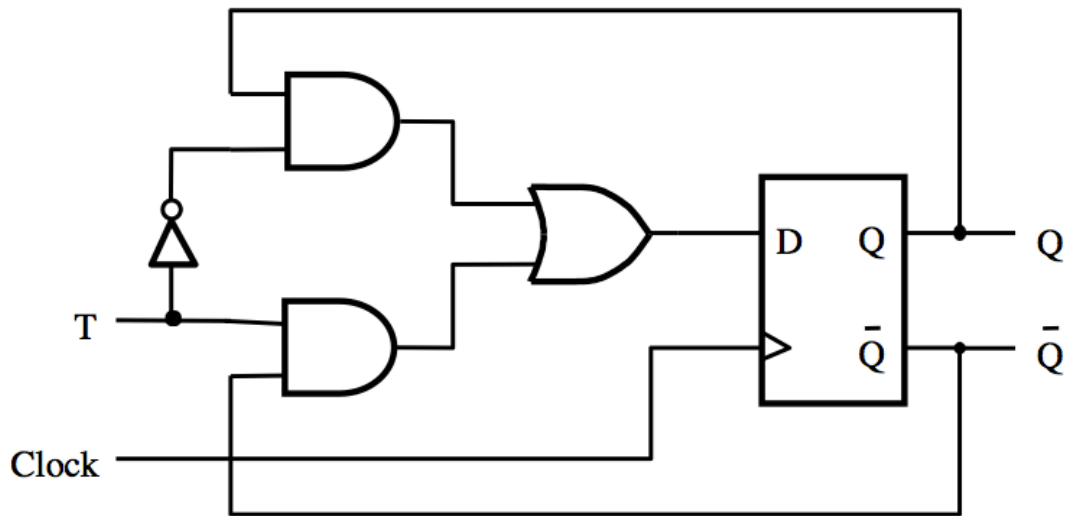
Putting it all Together



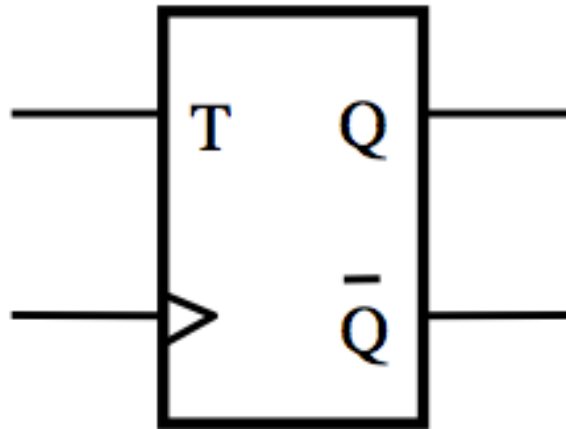
Counters

T Flip-Flop

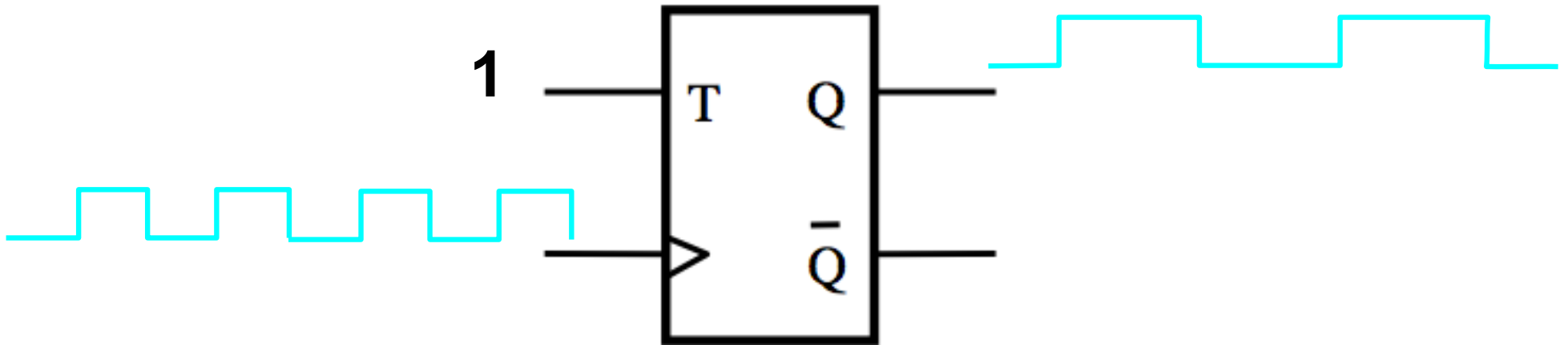
(circuit and graphical symbol)



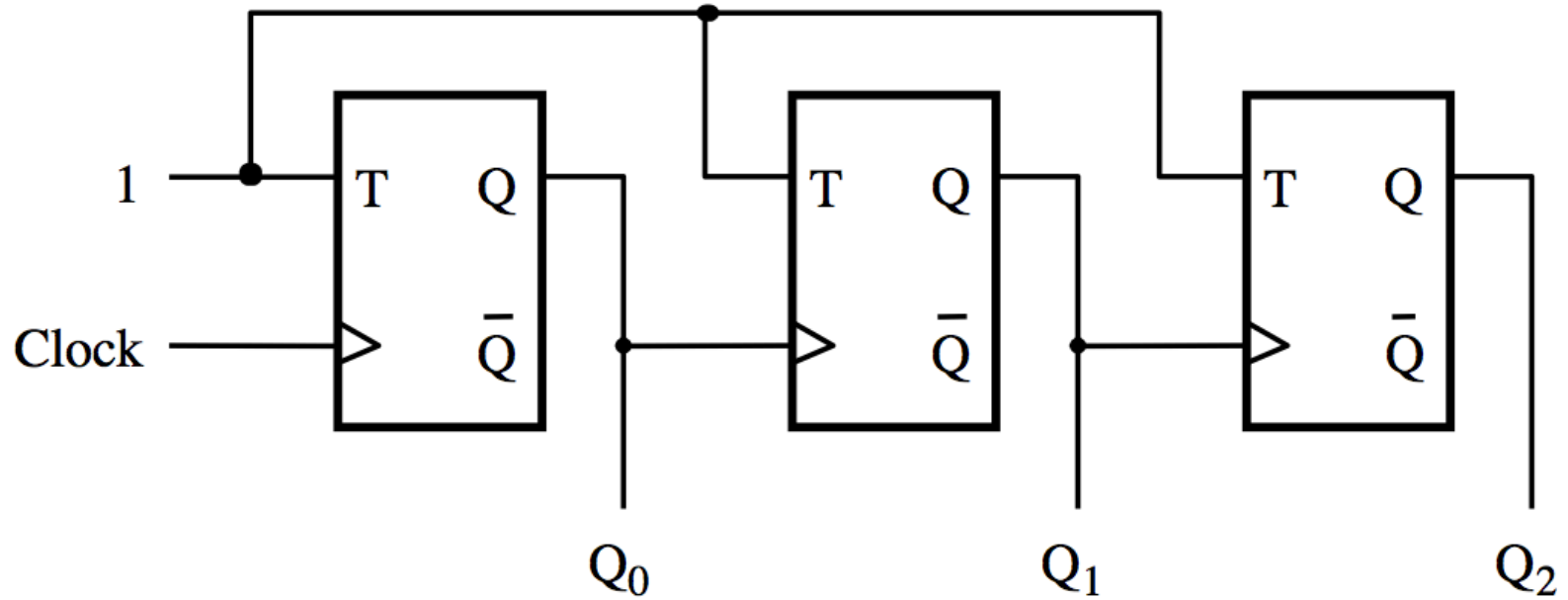
**The output of the T Flip-Flop
divides the frequency of the clock by 2**



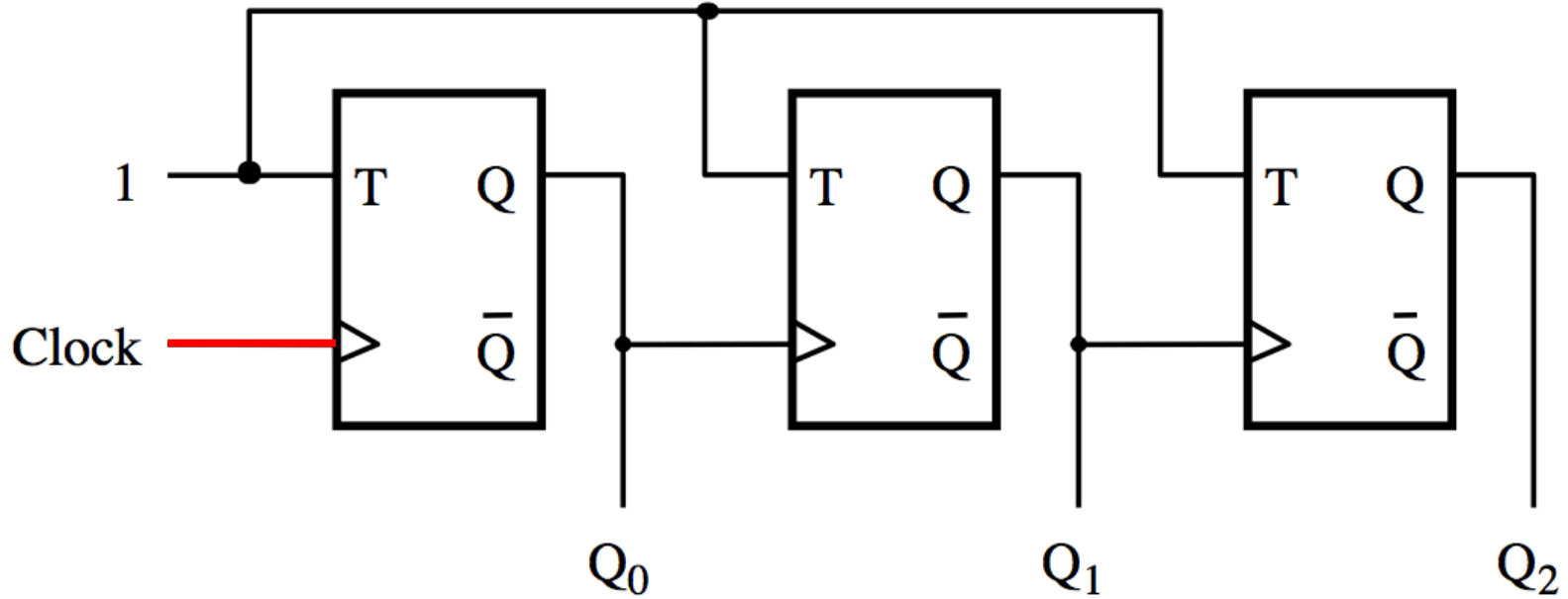
**The output of the T Flip-Flop
divides the frequency of the clock by 2**



A three-bit down-counter

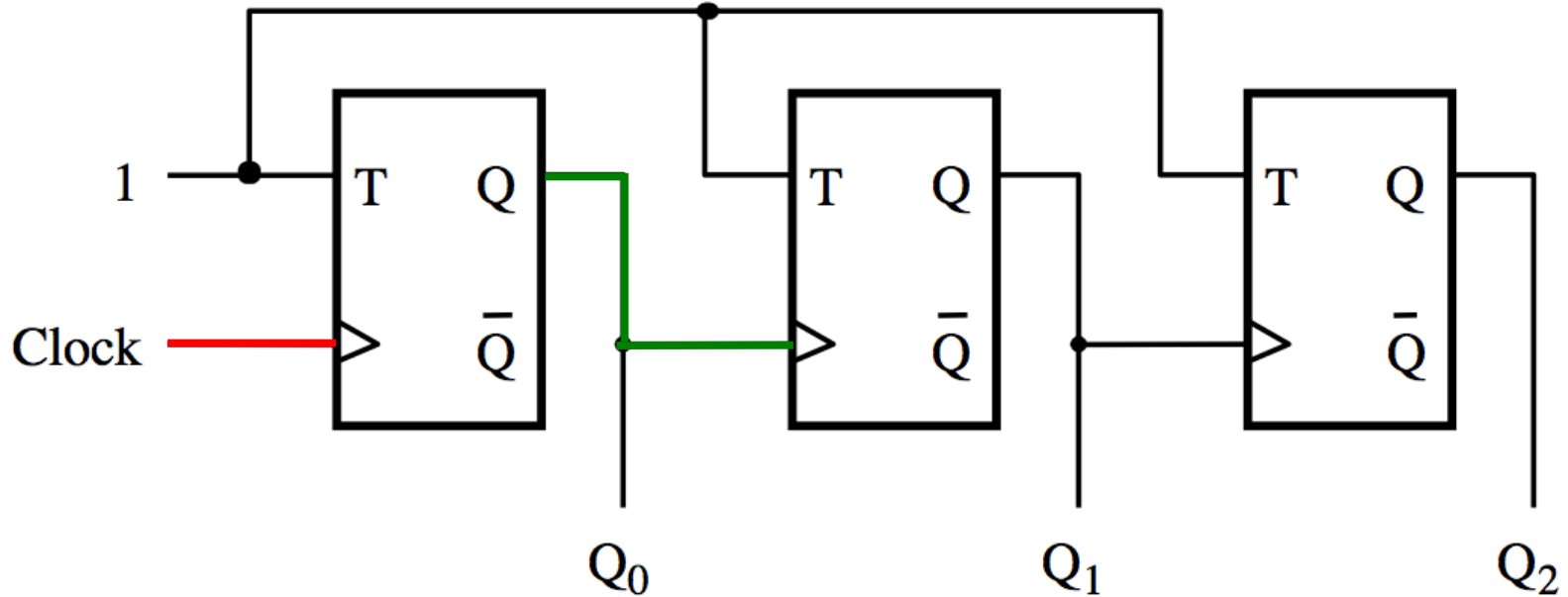


A three-bit down-counter



The first flip-flop changes
on the positive edge of the clock

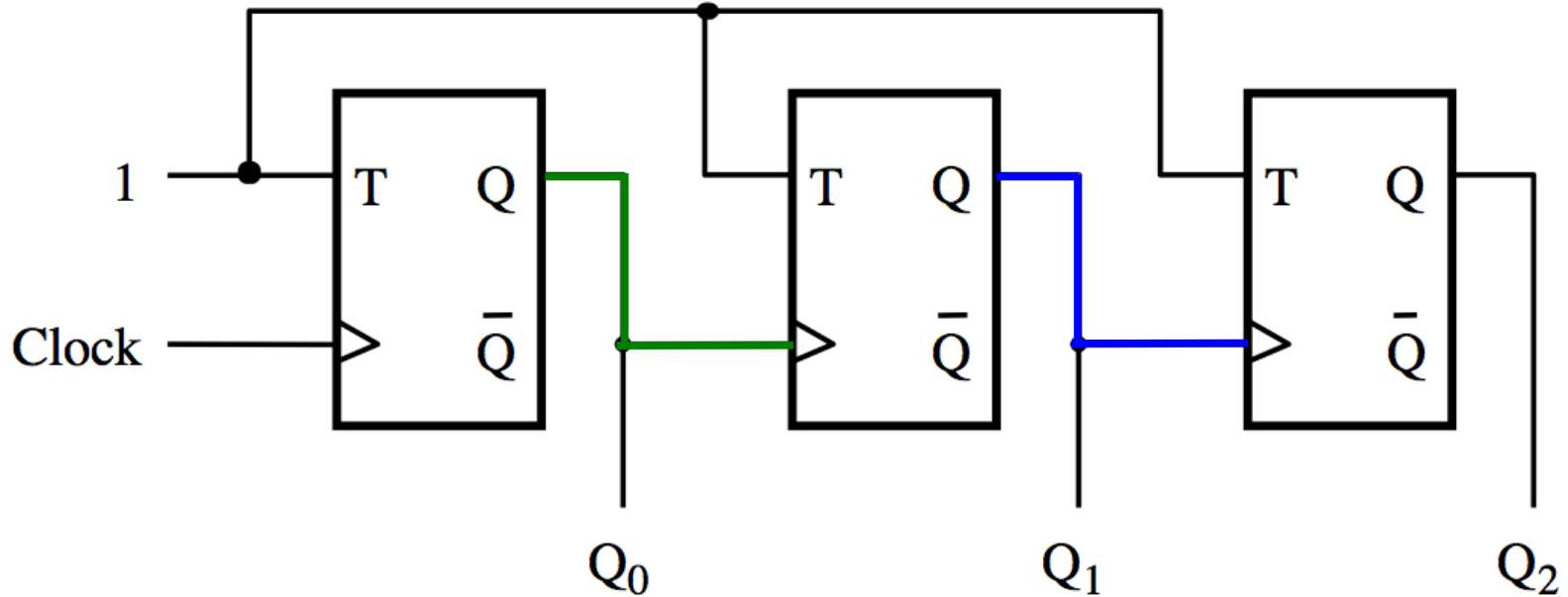
A three-bit down-counter



The first flip-flop changes
on the positive edge of the clock

The second flip-flop changes
on the positive edge of Q_0

A three-bit down-counter

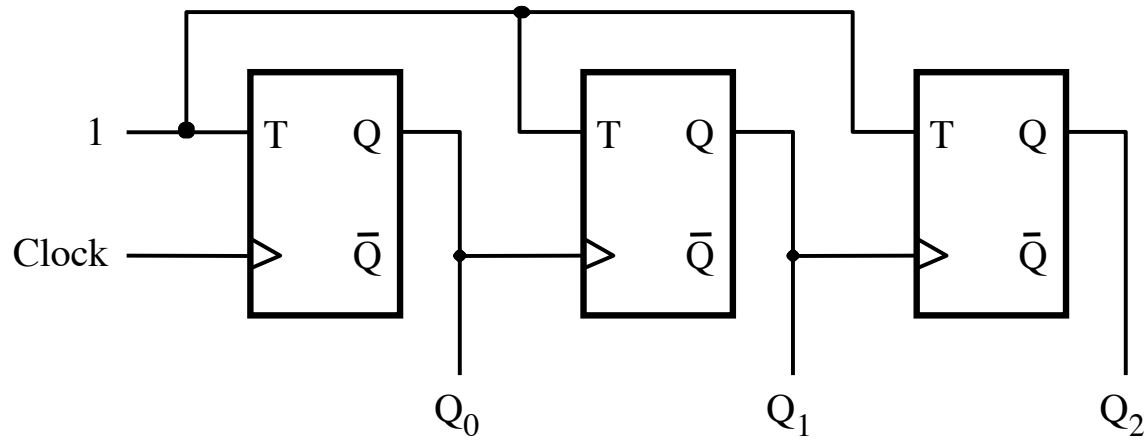


The first flip-flop changes
on the positive edge of the clock

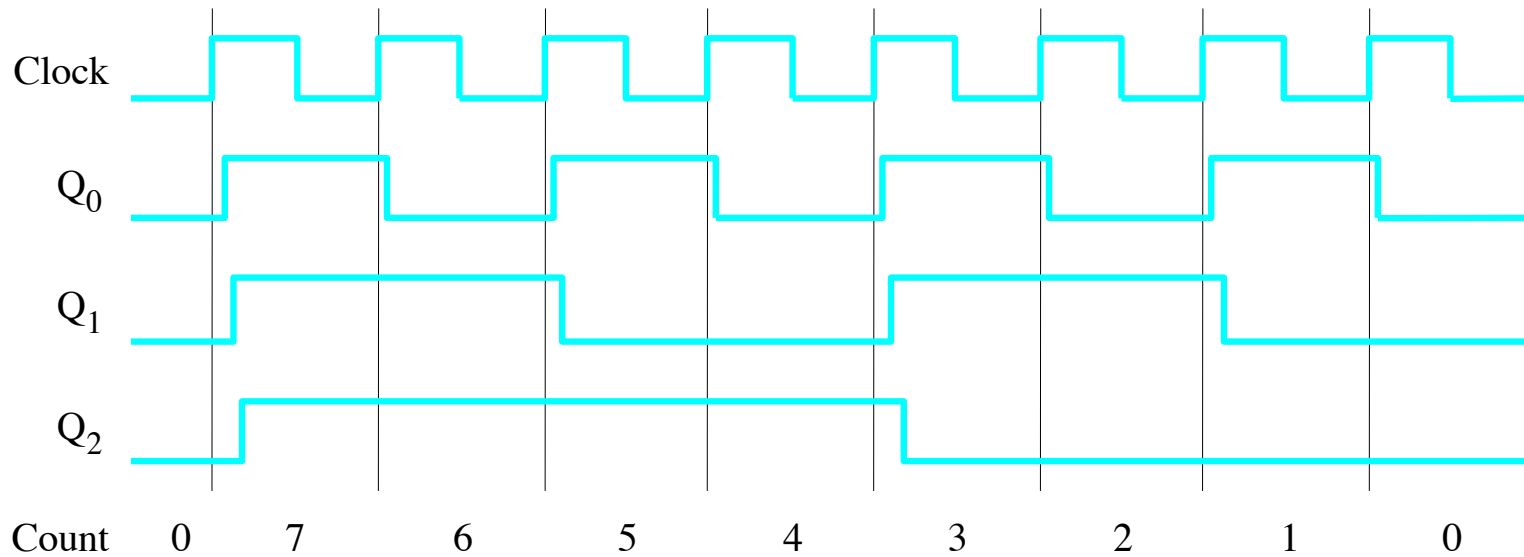
The second flip-flop changes
on the positive edge of Q_0

The third flip-flop changes
on the positive edge of Q_1

A three-bit down-counter



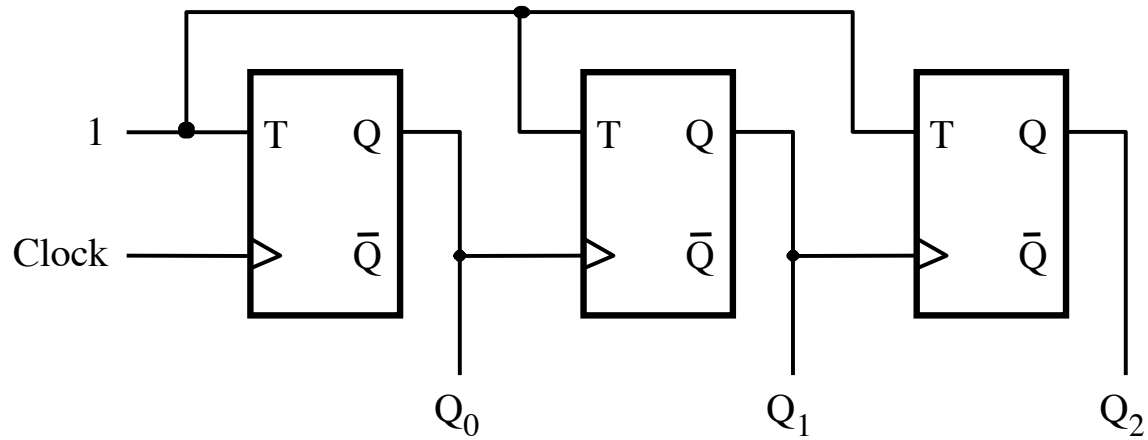
(a) Circuit



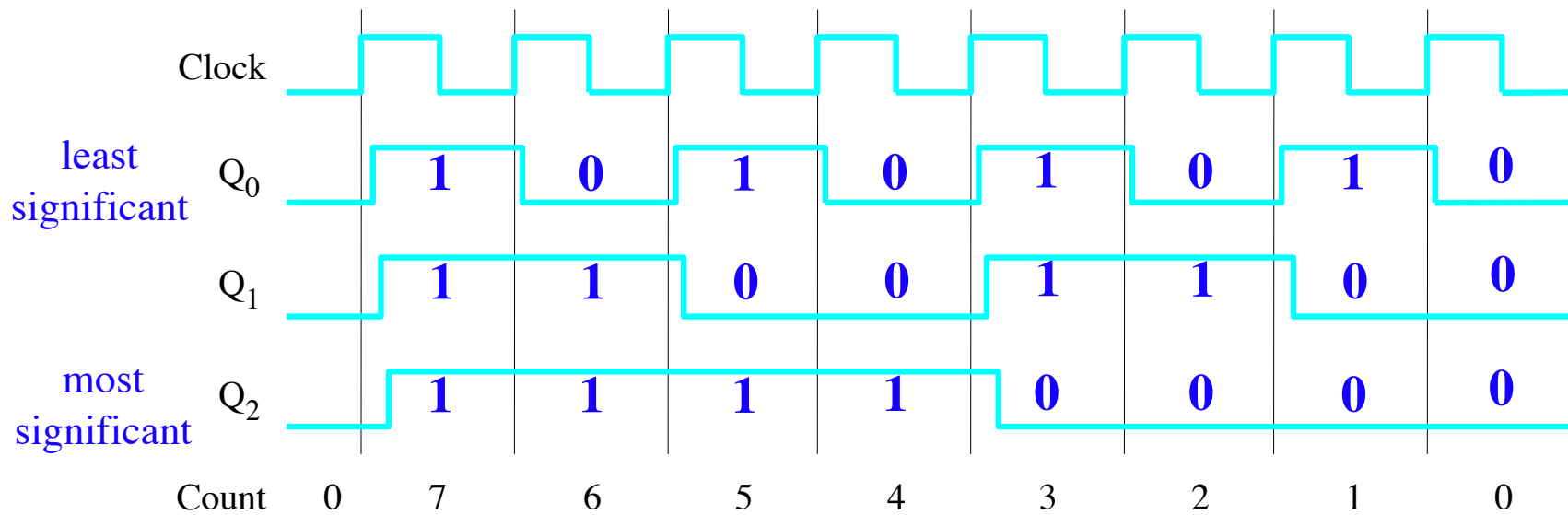
(b) Timing diagram

[Figure 5.20 from the textbook]

A three-bit down-counter

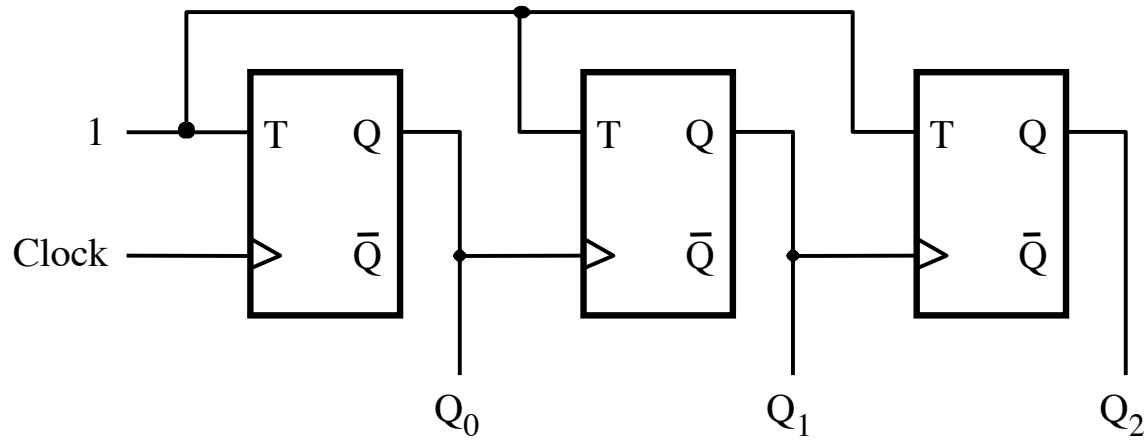


(a) Circuit

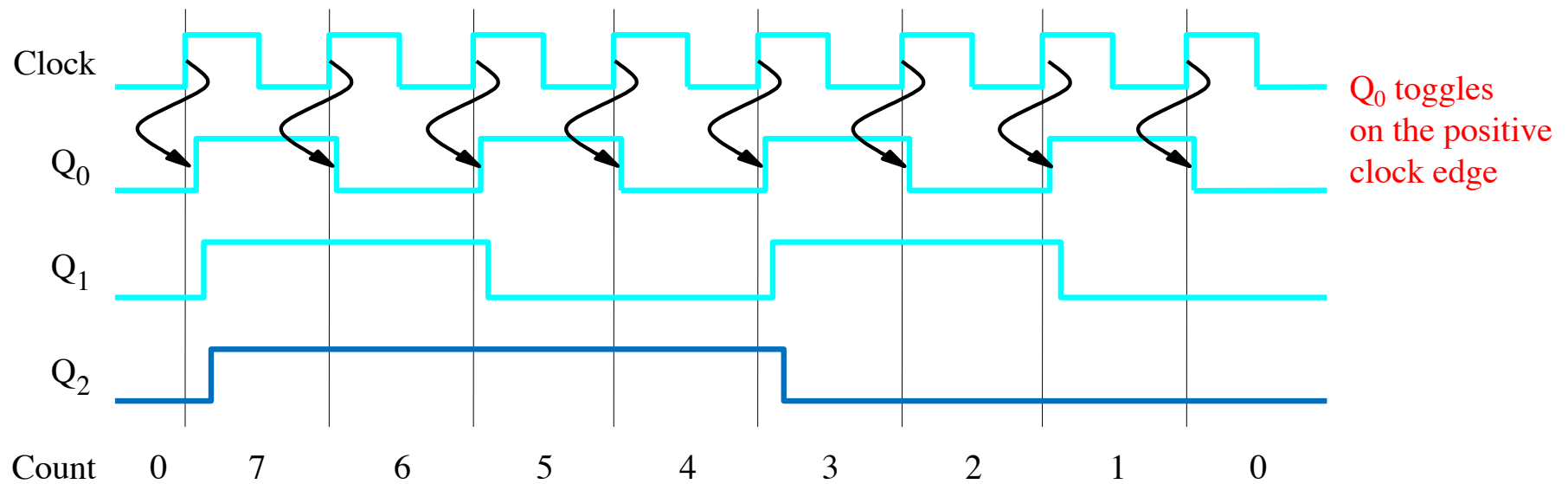


(b) Timing diagram

A three-bit down-counter

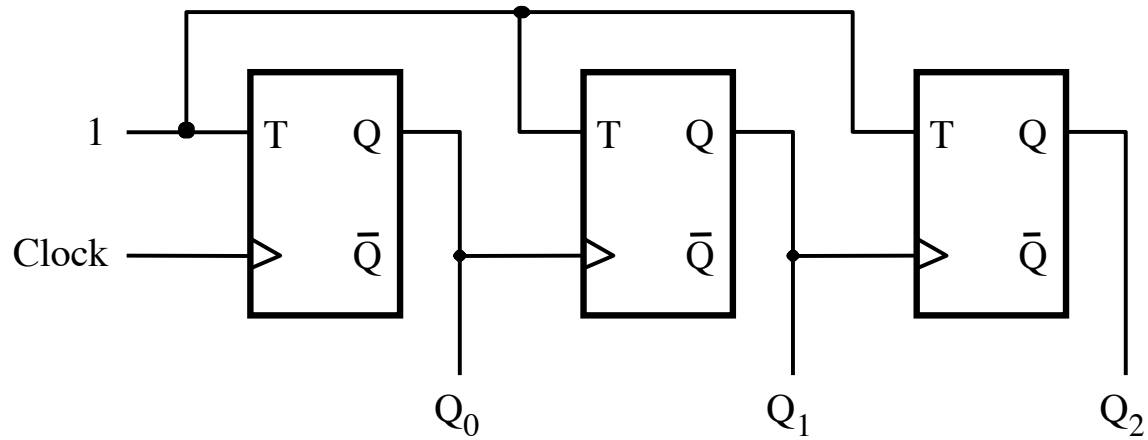


(a) Circuit

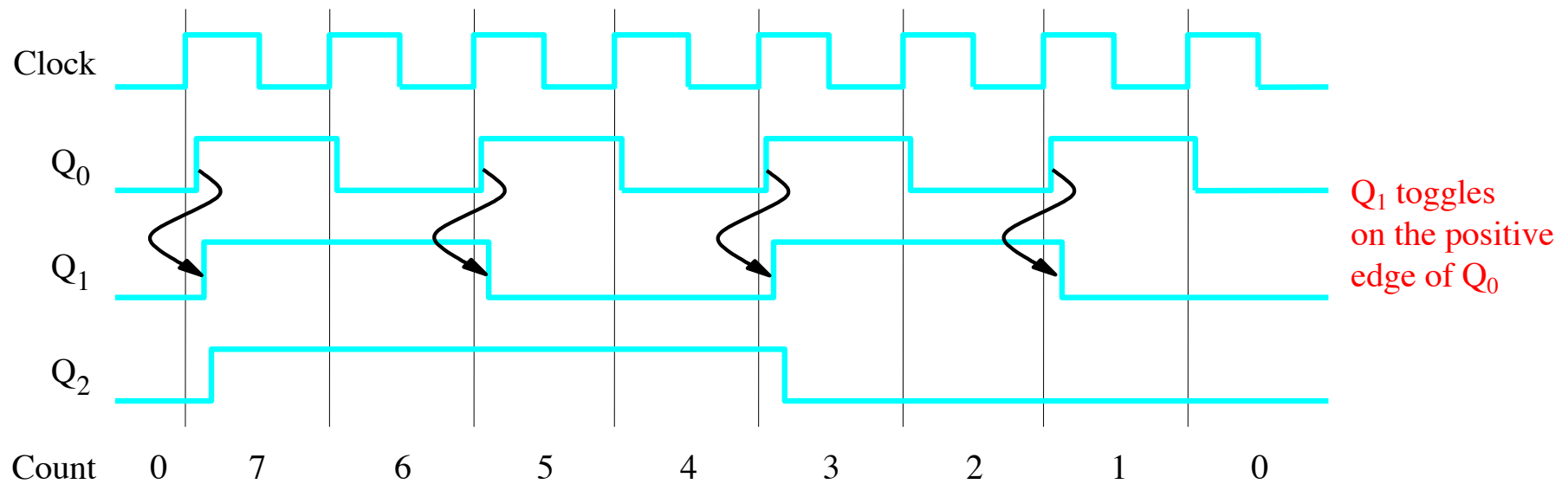


(b) Timing diagram

A three-bit down-counter

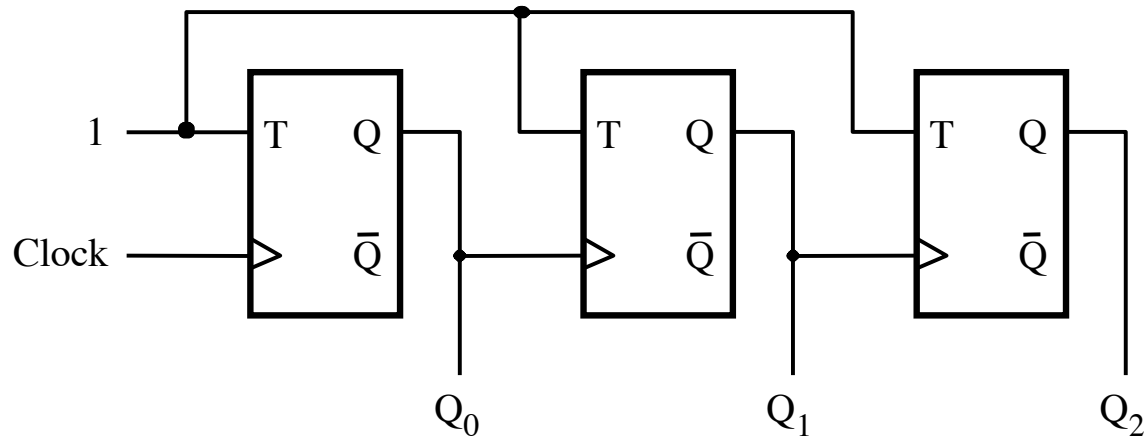


(a) Circuit

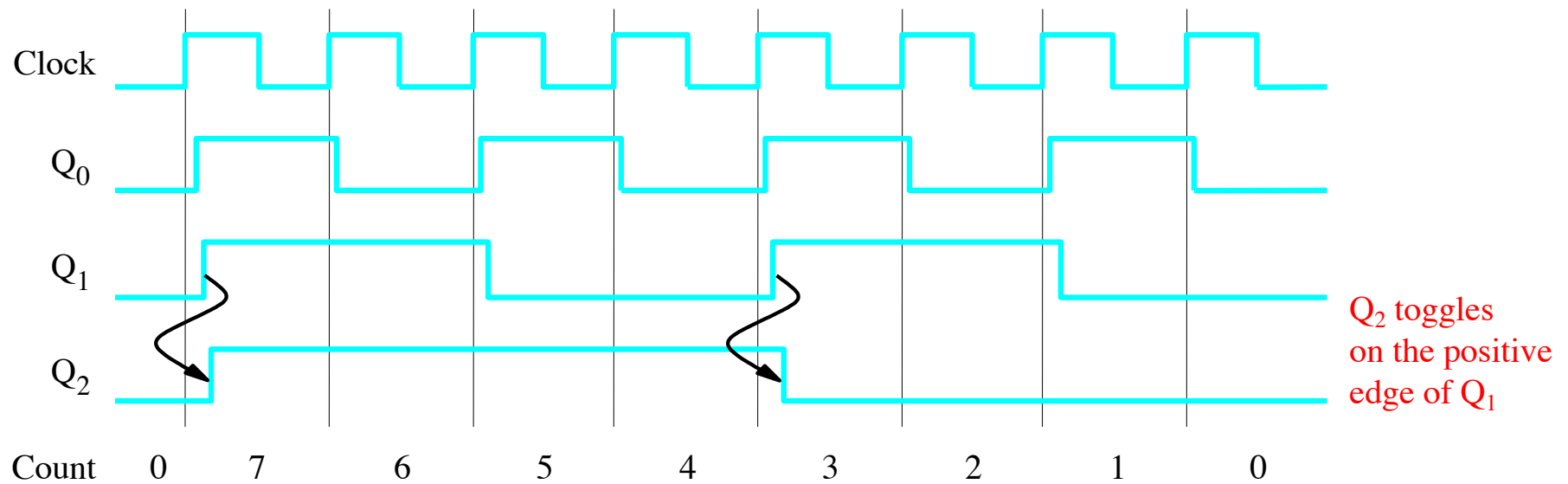


(b) Timing diagram

A three-bit down-counter

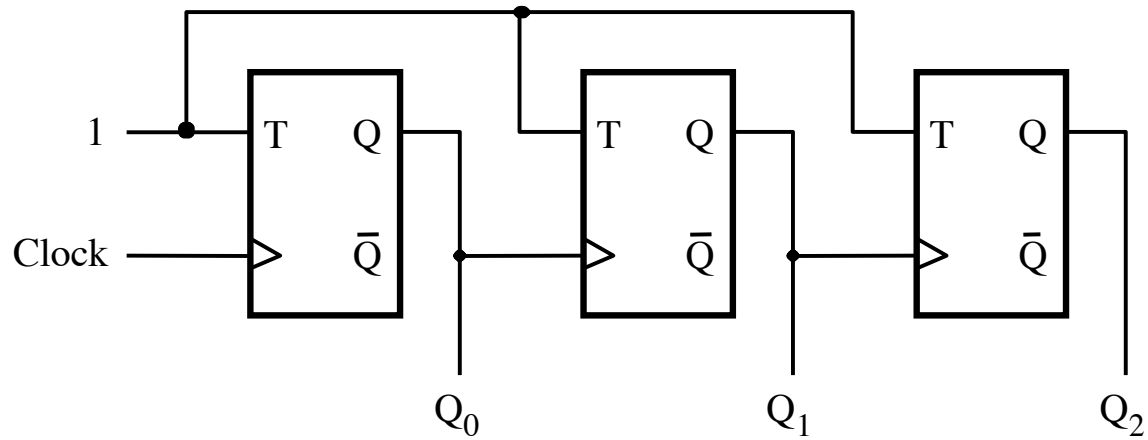


(a) Circuit



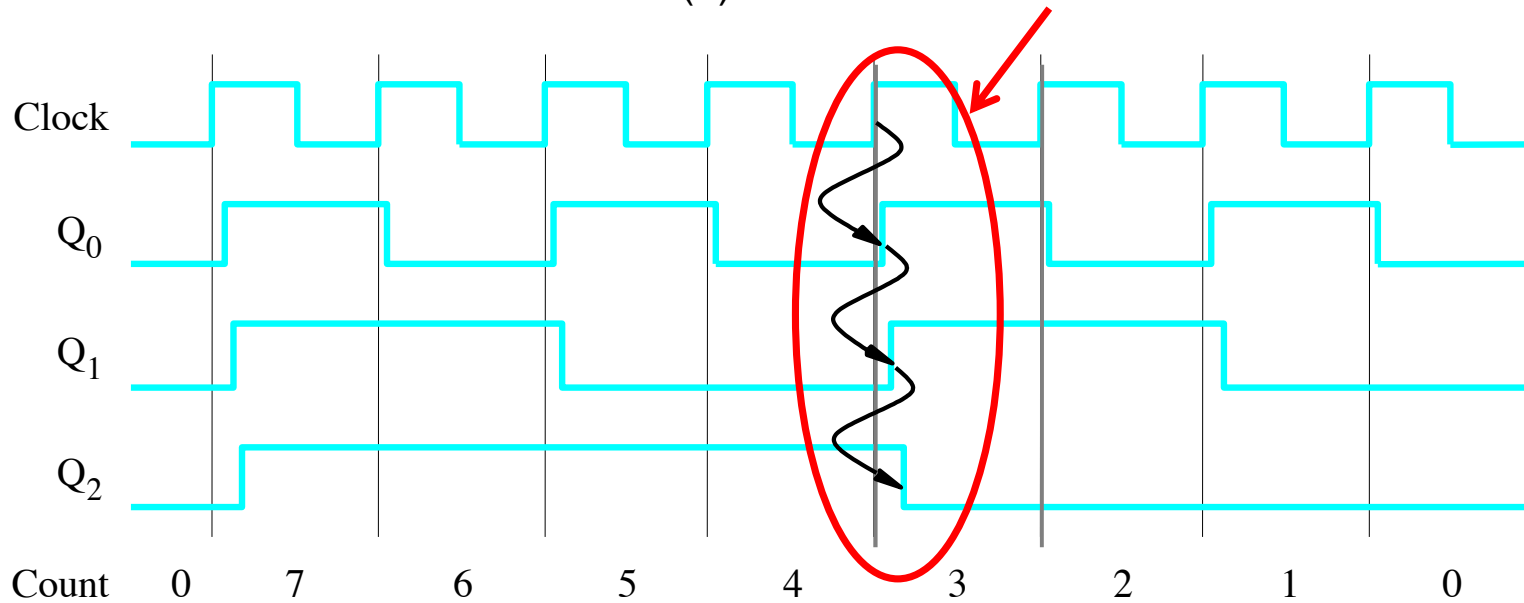
(b) Timing diagram

A three-bit down-counter



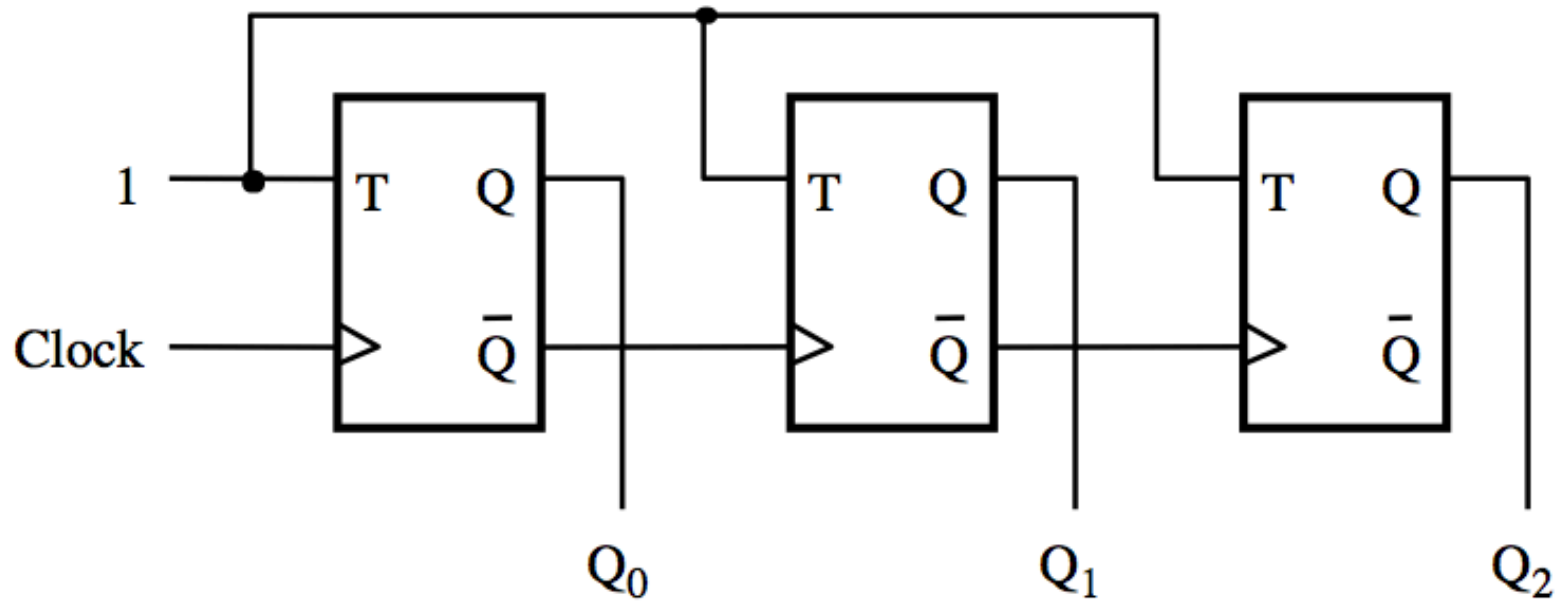
(a) Circuit

The propagation delays get longer

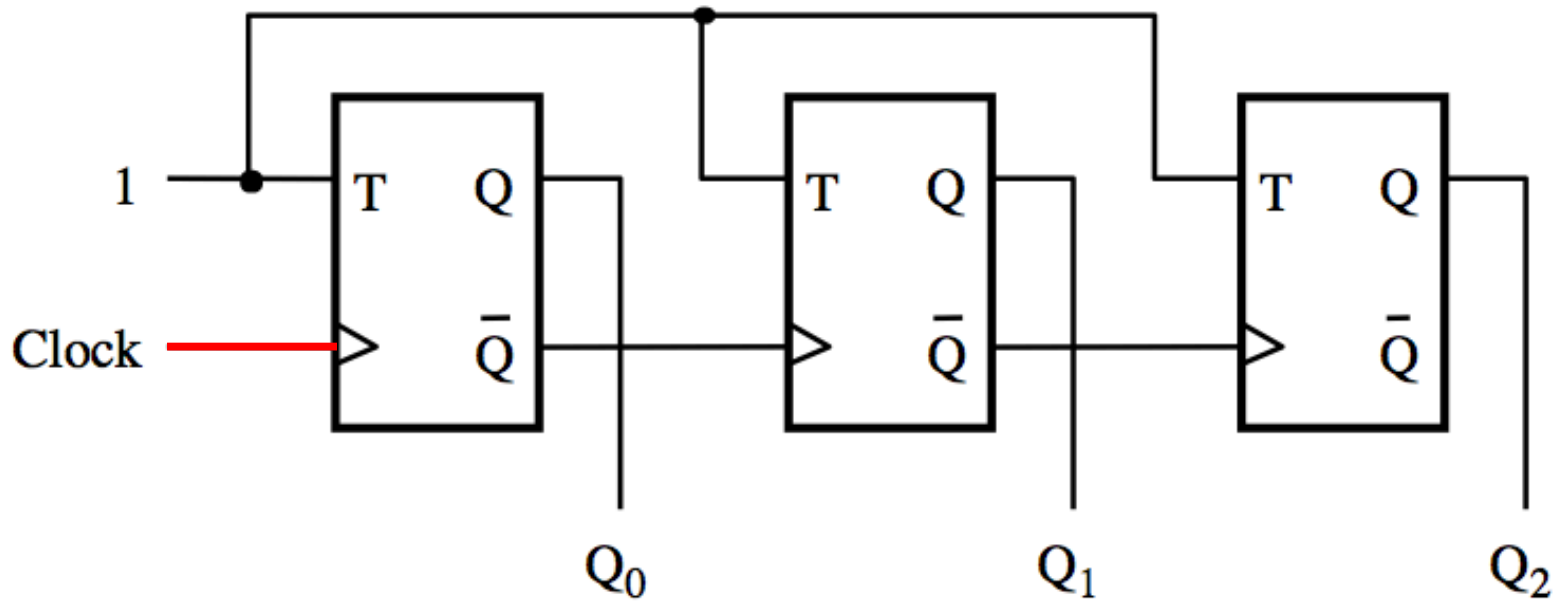


(b) Timing diagram

A three-bit up-counter

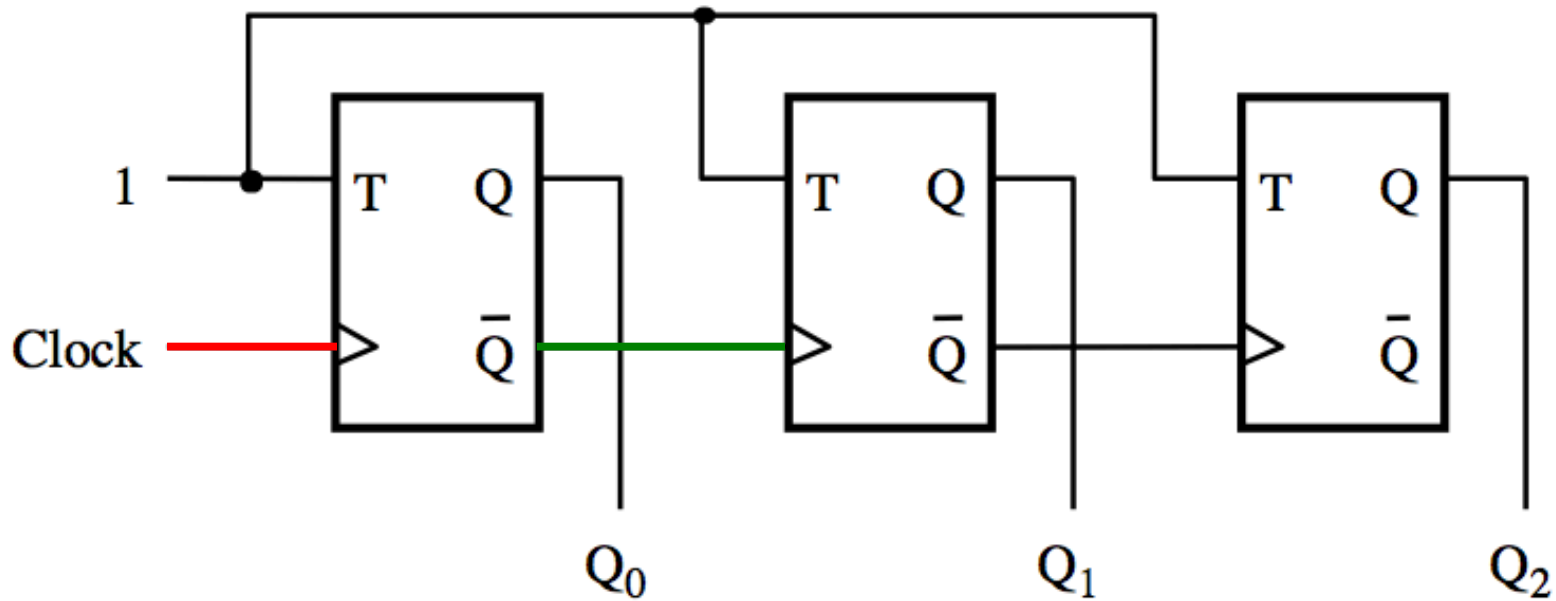


A three-bit up-counter



The first flip-flop changes
on the positive edge of the clock

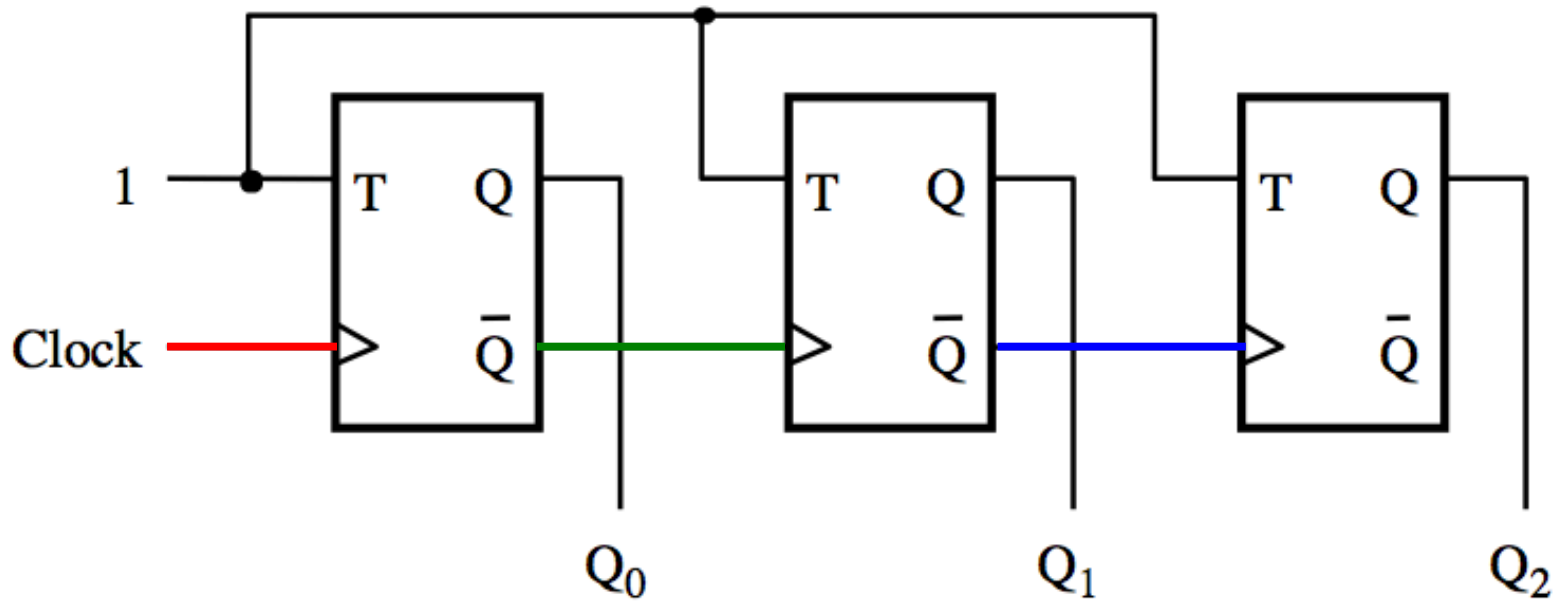
A three-bit up-counter



The first flip-flop changes
on the positive edge of the clock

The second flip-flop changes
on the positive edge of \bar{Q}_0

A three-bit up-counter

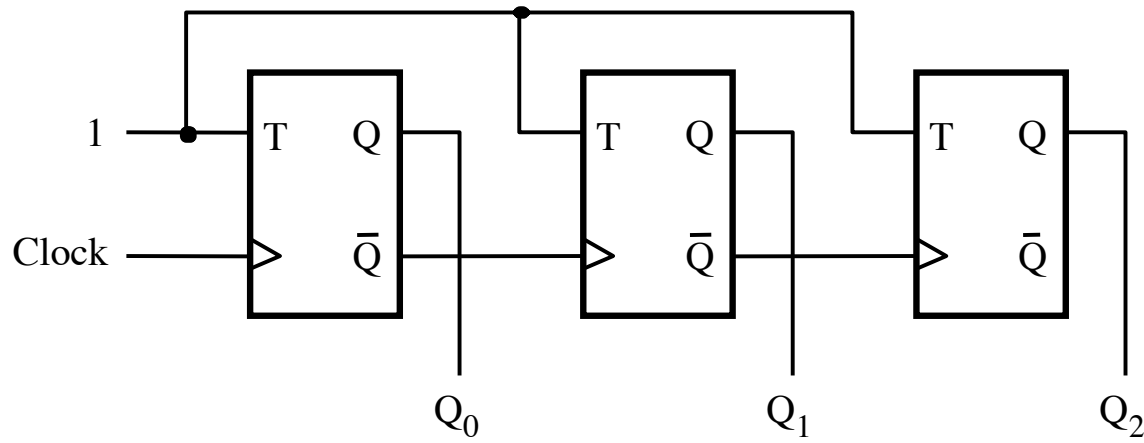


The first flip-flop changes on the positive edge of the clock

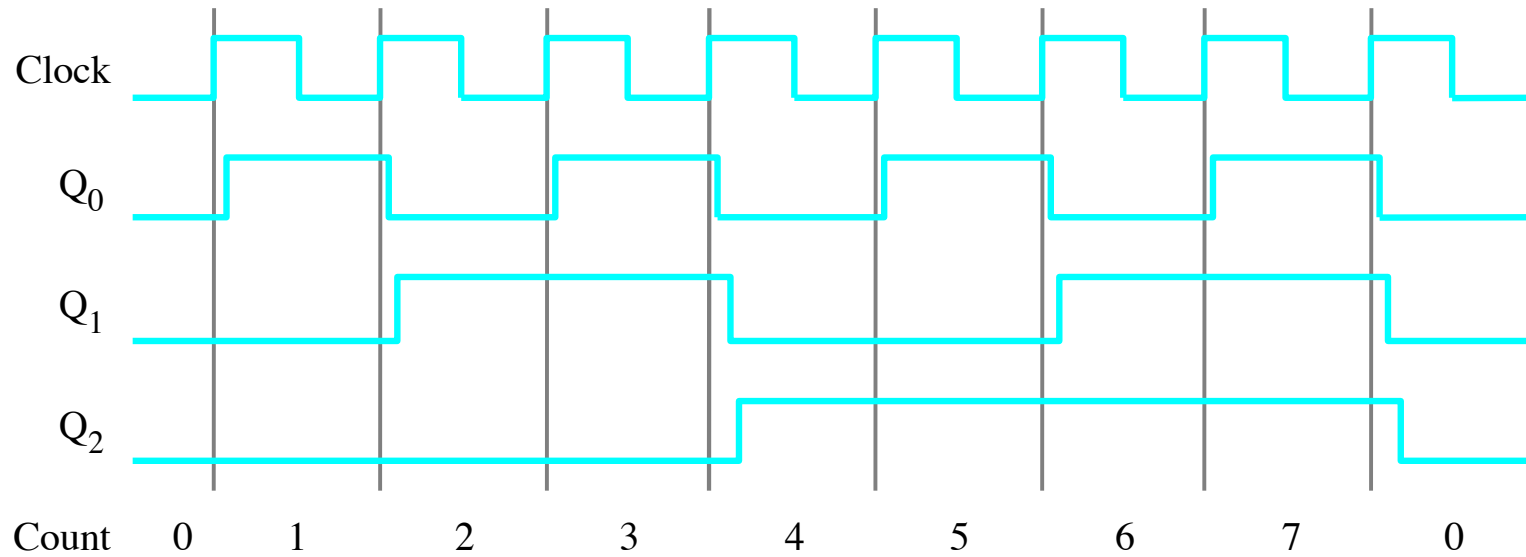
The second flip-flop changes on the positive edge of \bar{Q}_0

The third flip-flop changes on the positive edge of \bar{Q}_1

A three-bit up-counter

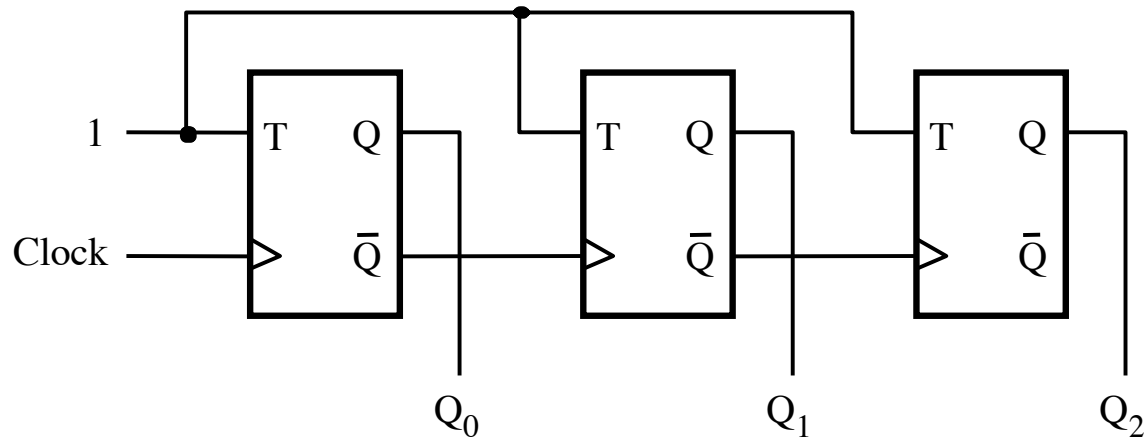


(a) Circuit

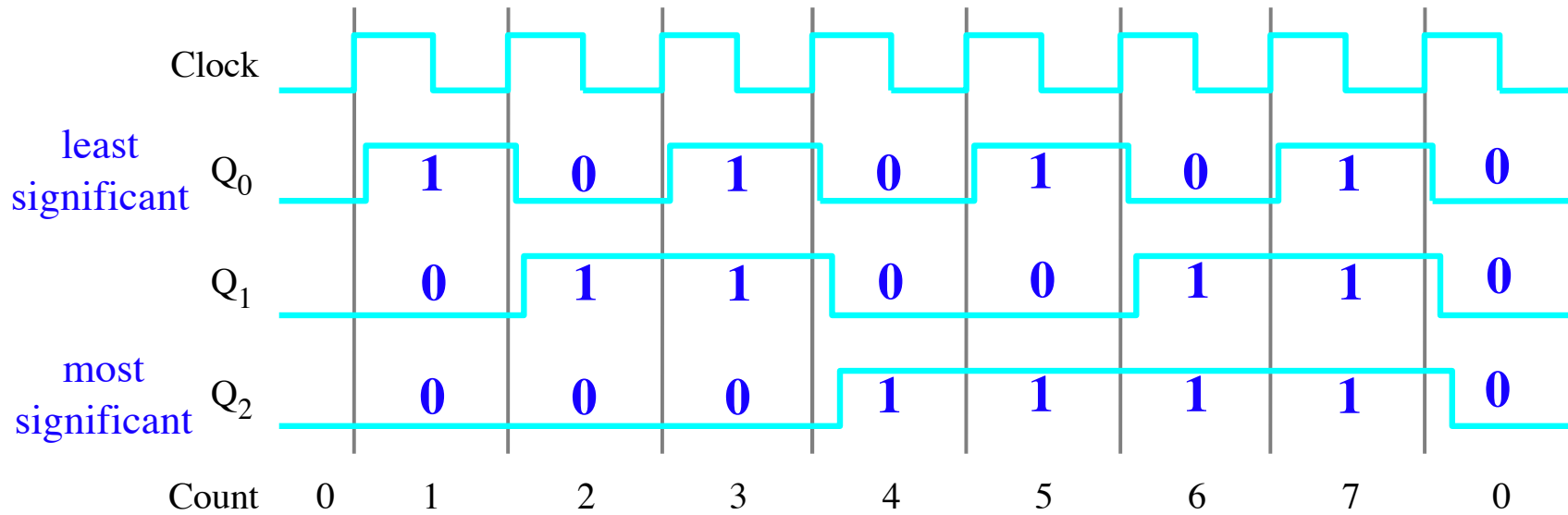


(b) Timing diagram

A three-bit up-counter

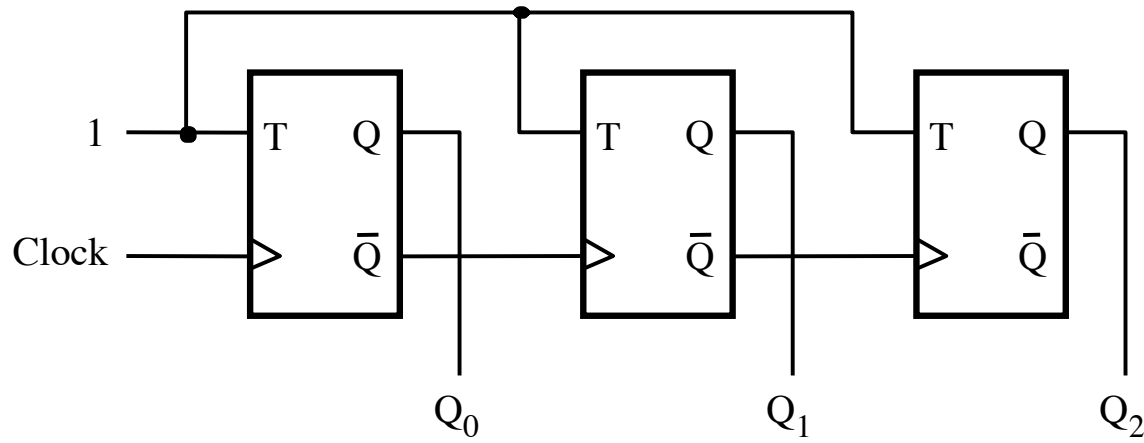


(a) Circuit

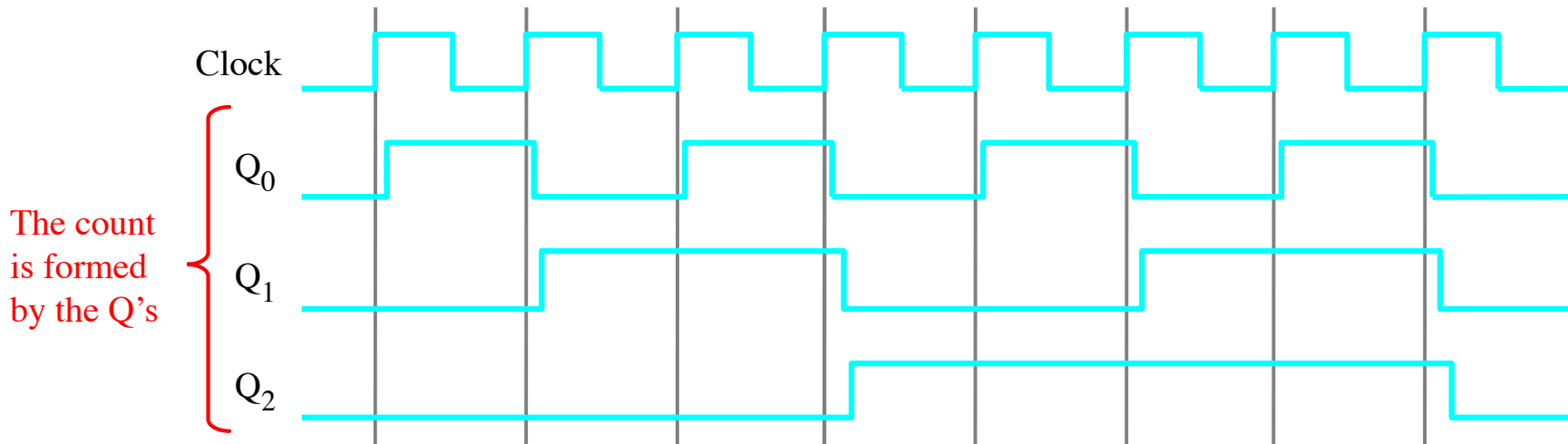


(b) Timing diagram

A three-bit up-counter

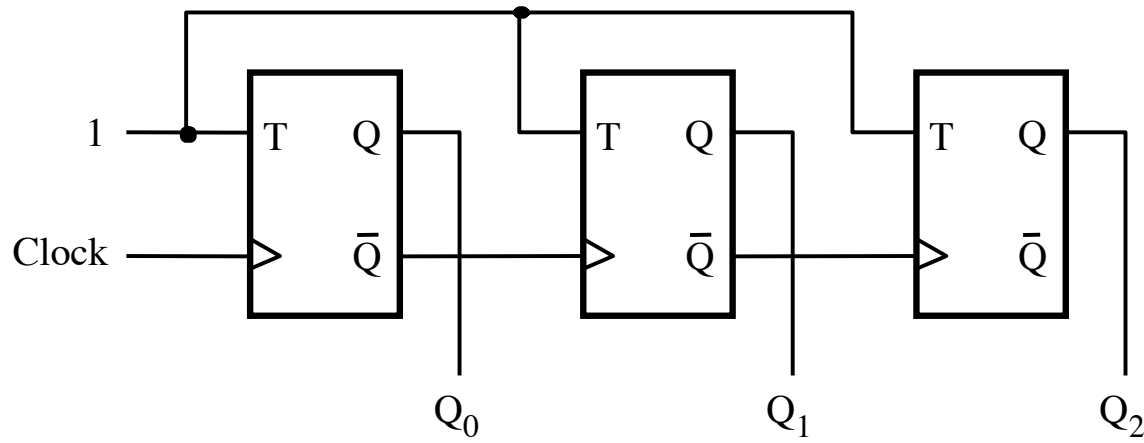


(a) Circuit

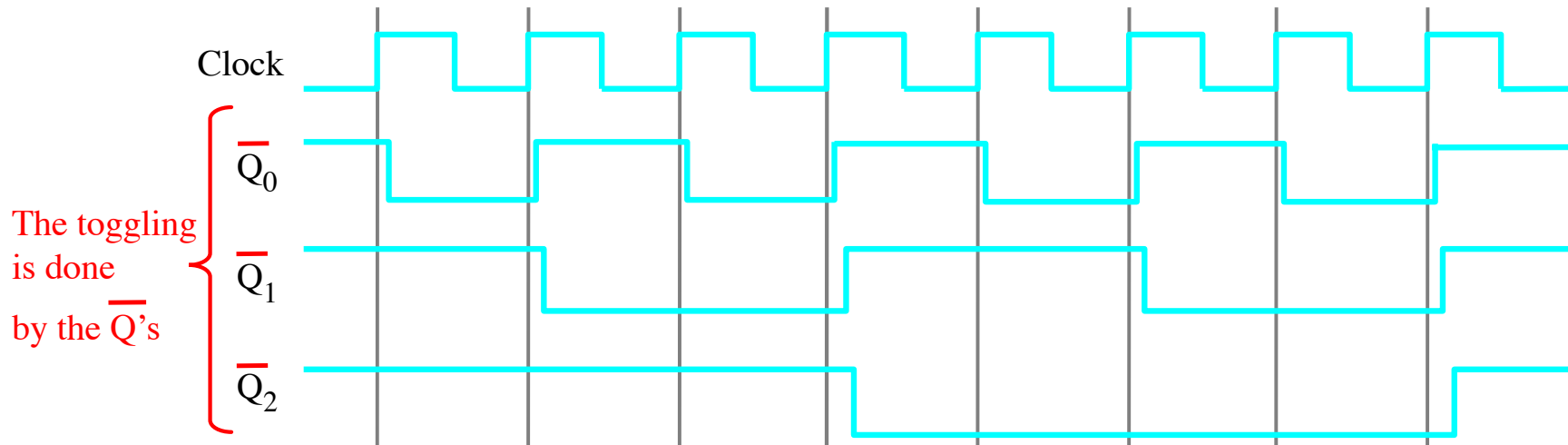


(b) Timing diagram

A three-bit up-counter

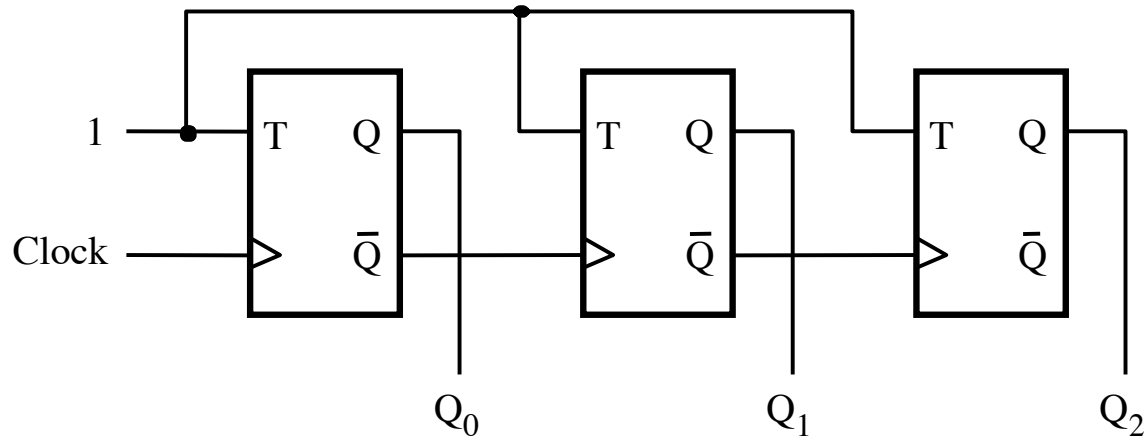


(a) Circuit

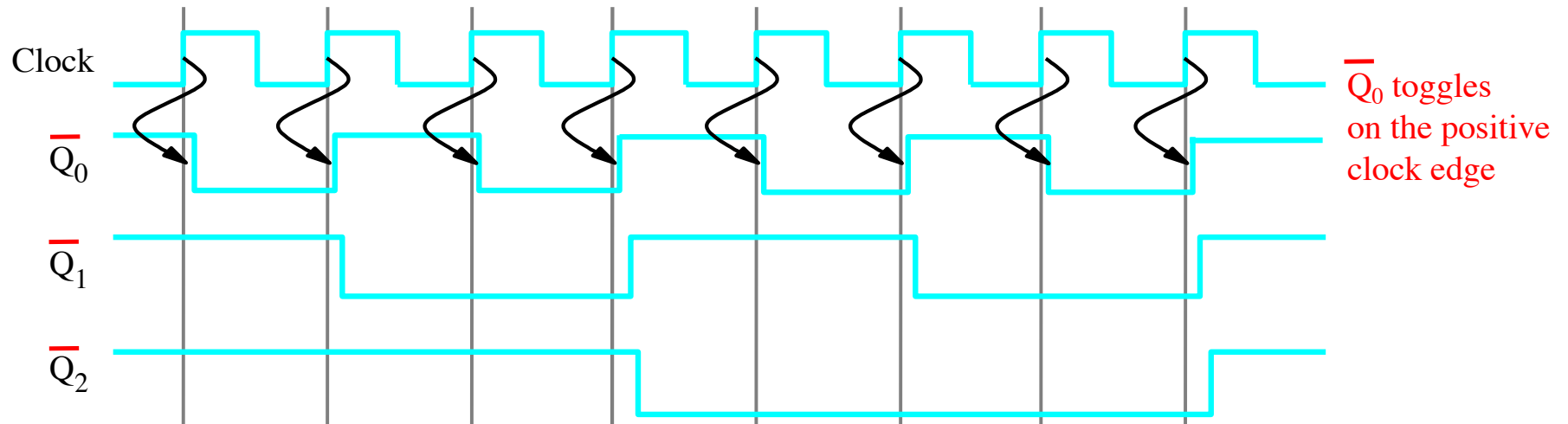


(b) Timing diagram

A three-bit up-counter

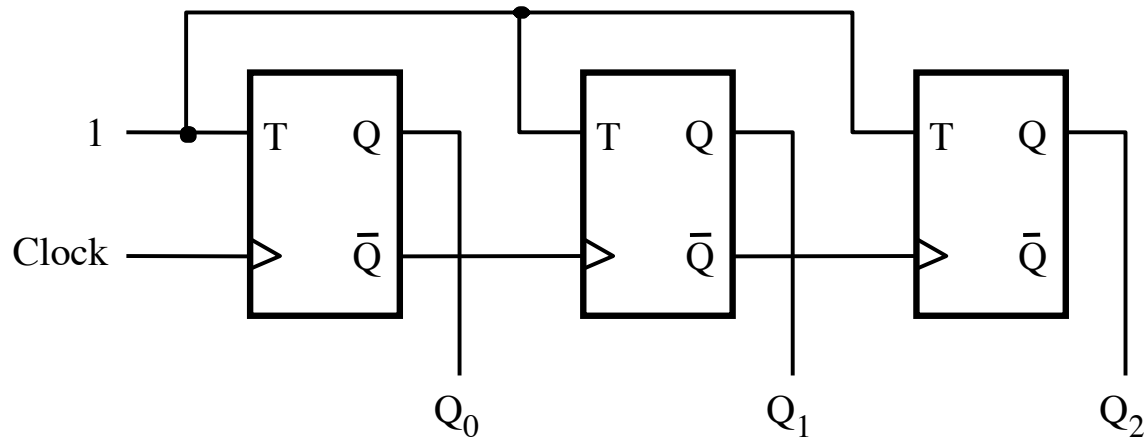


(a) Circuit

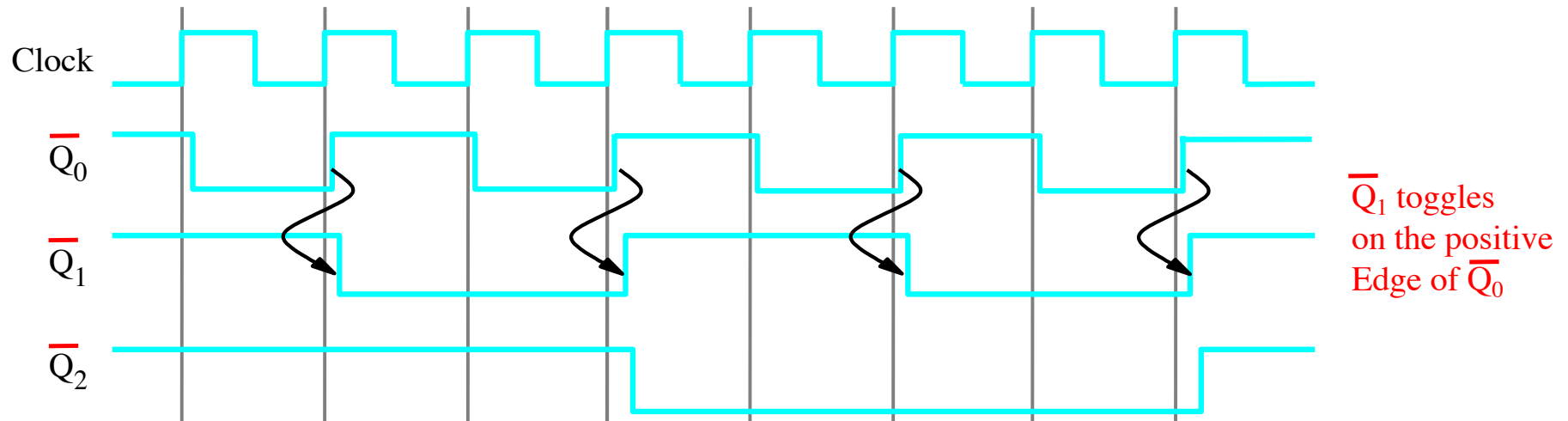


(b) Timing diagram

A three-bit up-counter

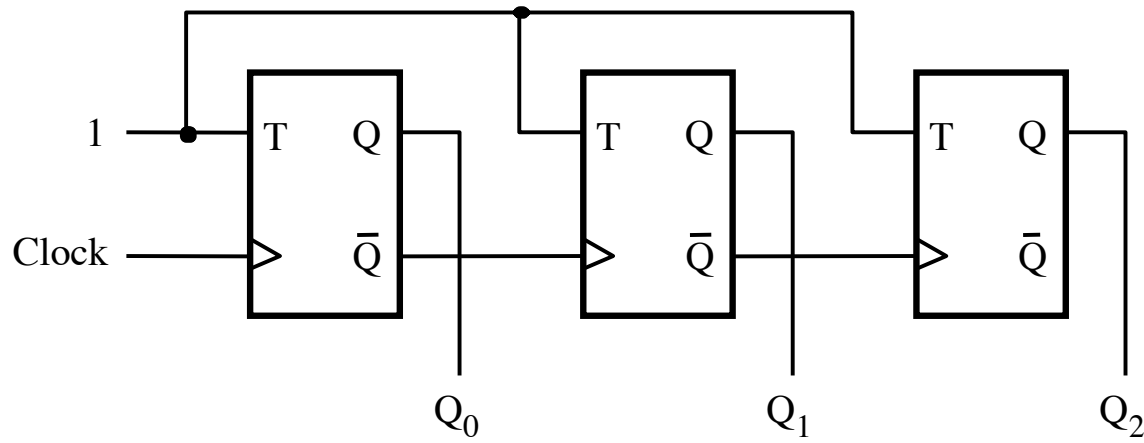


(a) Circuit

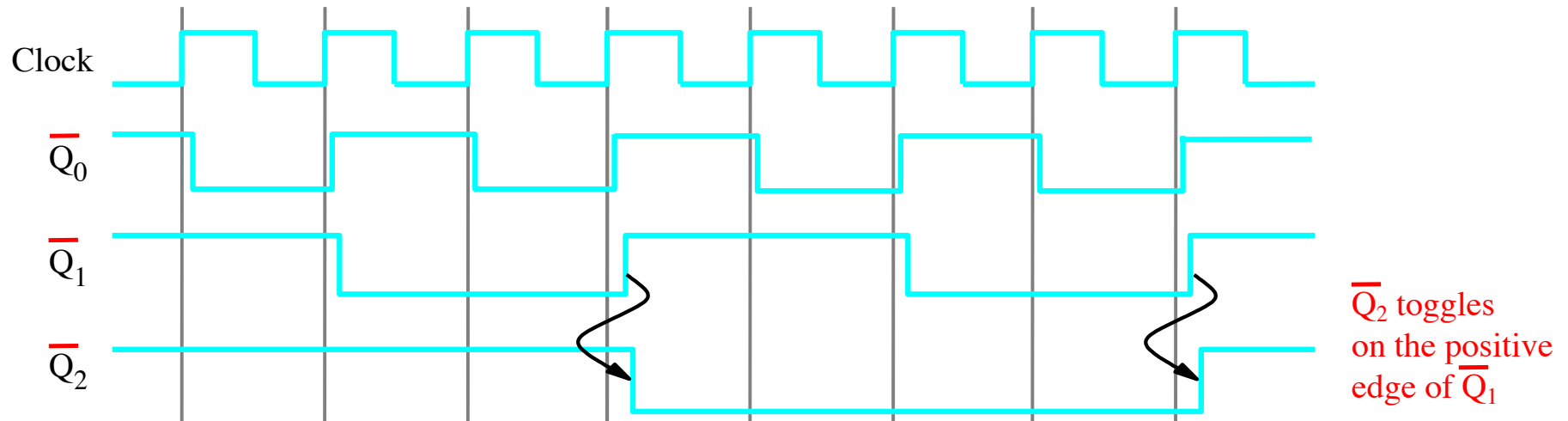


(b) Timing diagram

A three-bit up-counter

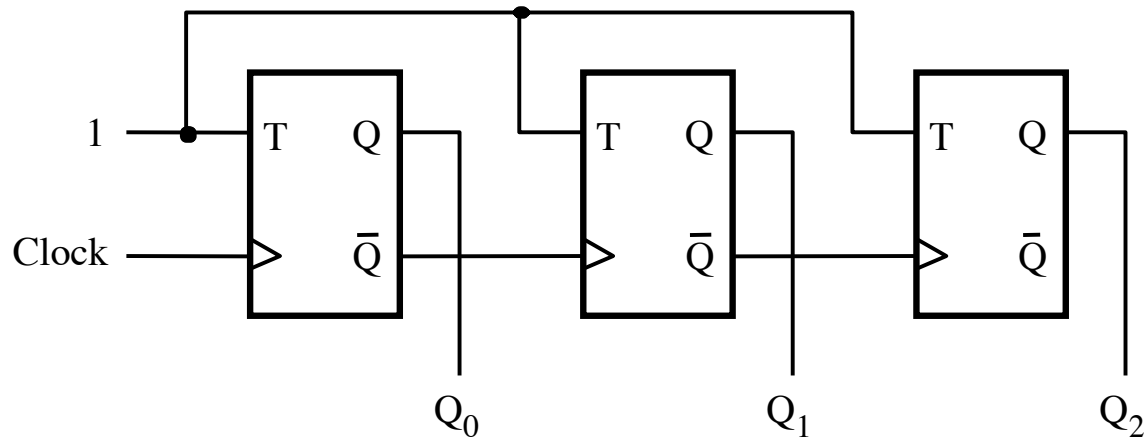


(a) Circuit

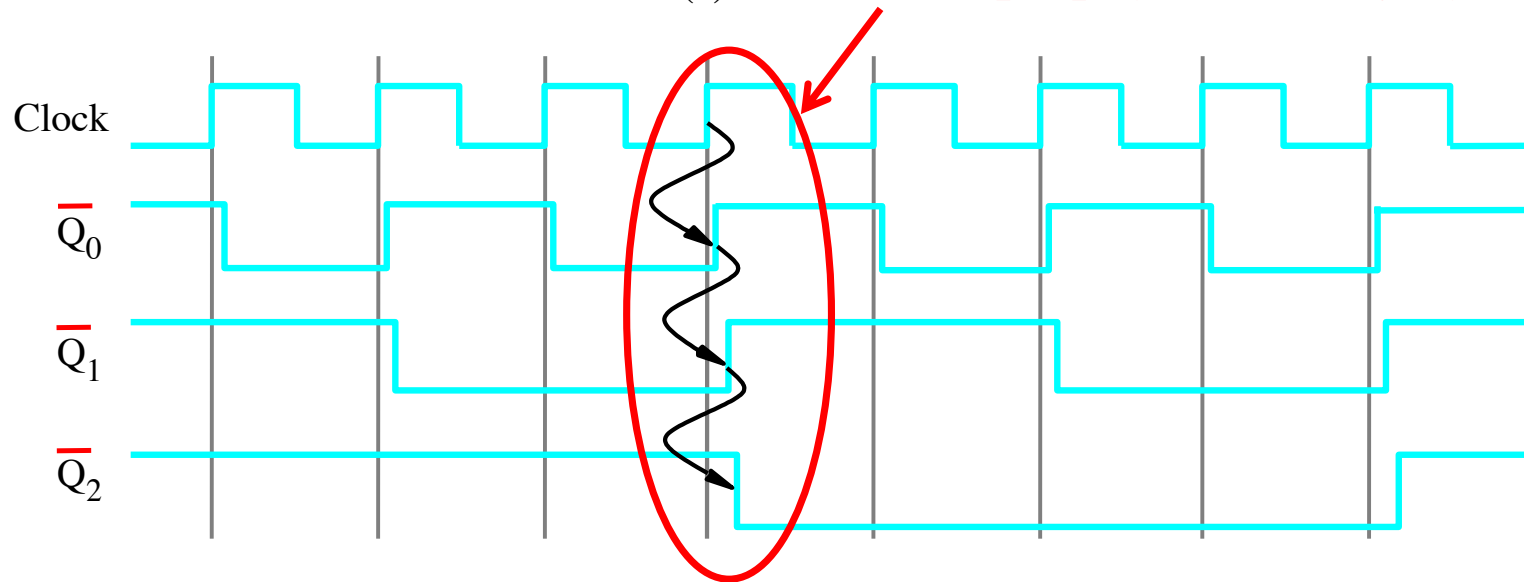


(b) Timing diagram

A three-bit up-counter

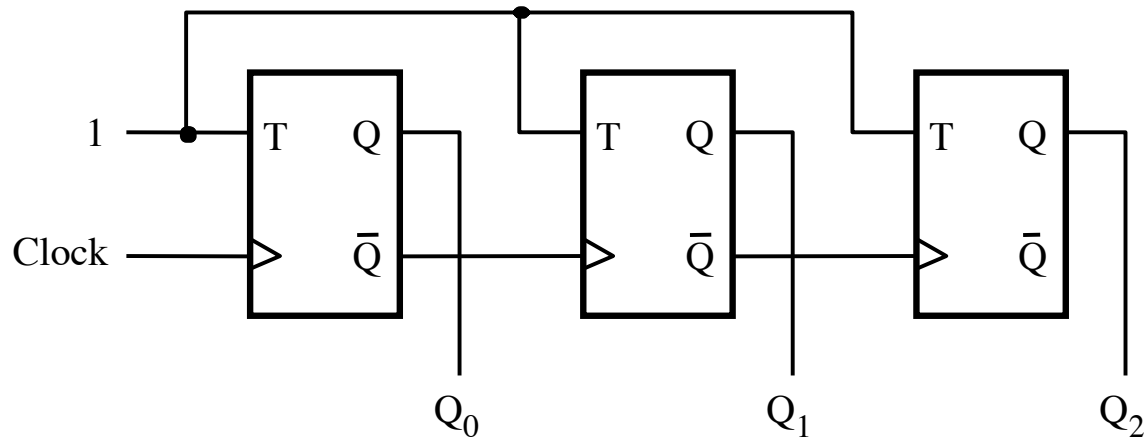


(a) Circuit **The propagation delays get longer**

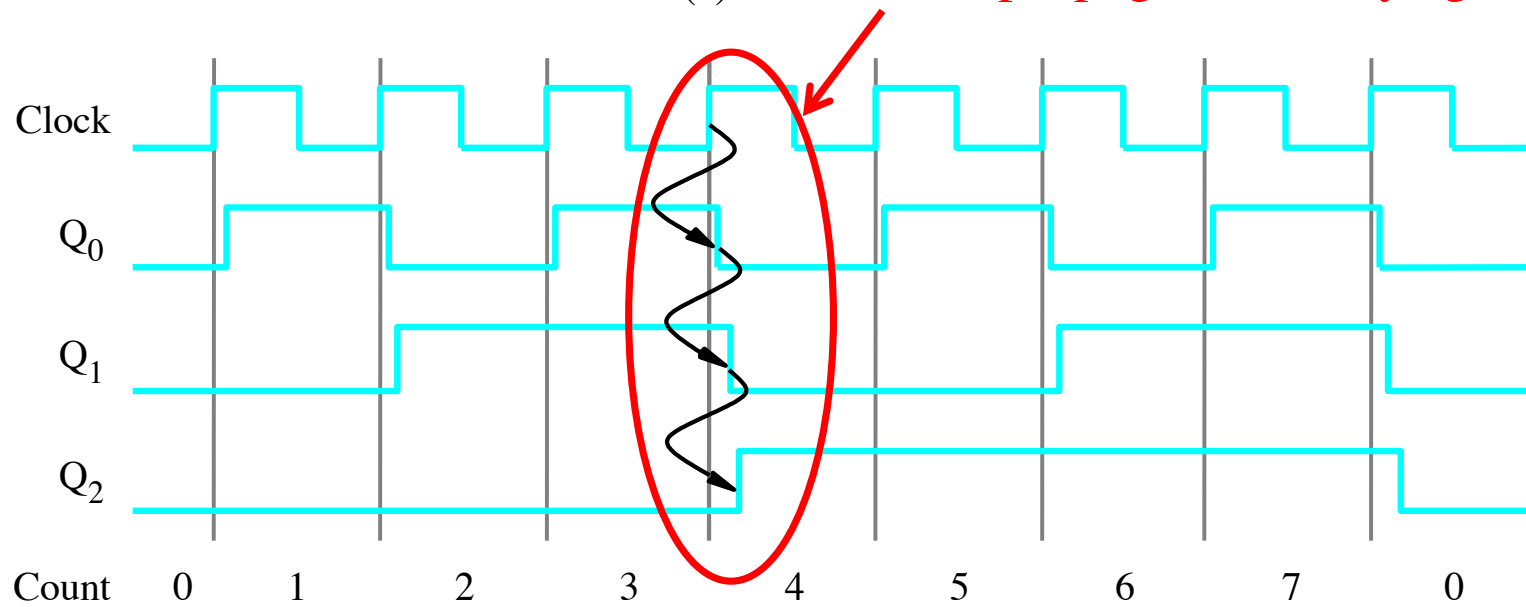


(b) Timing diagram

A three-bit up-counter



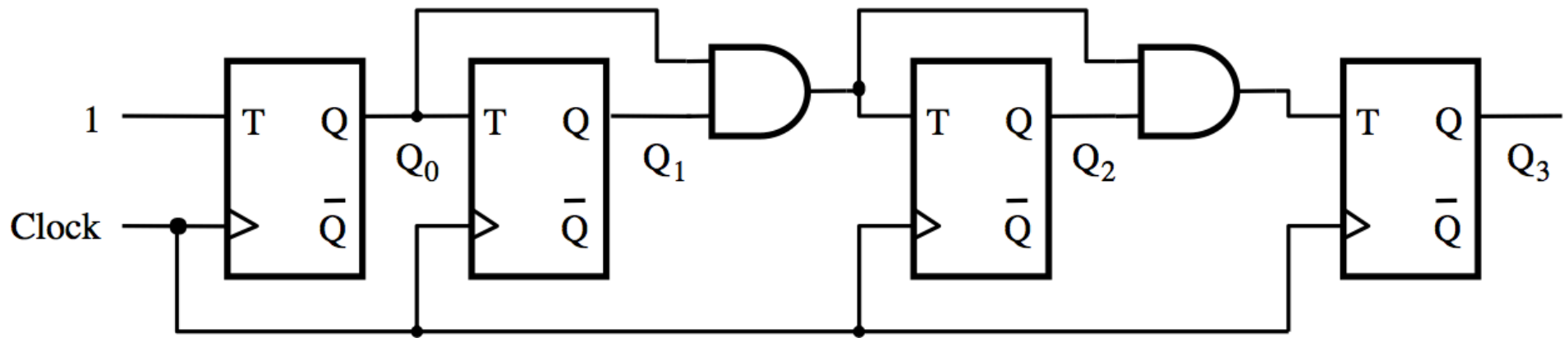
(a) Circuit **The propagation delays get longer**



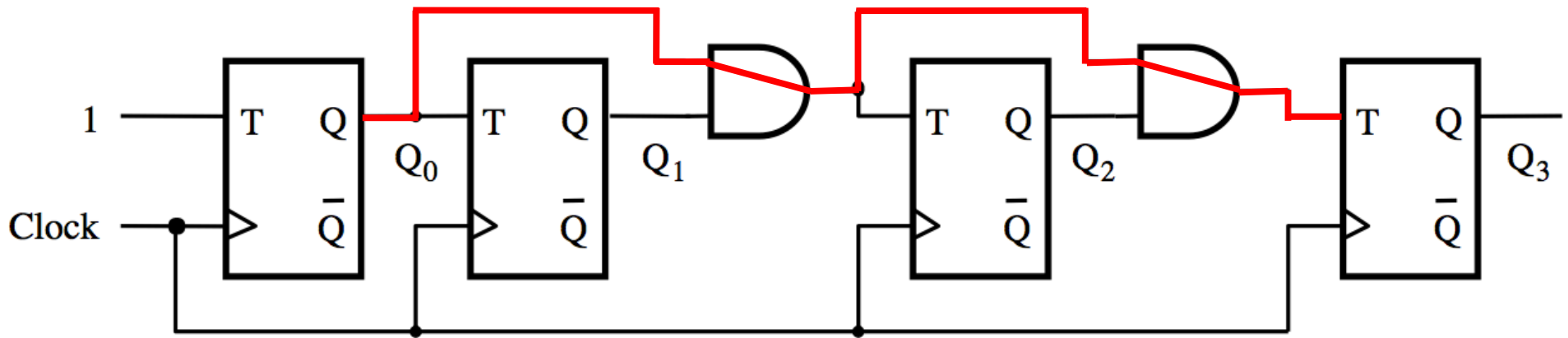
(b) Timing diagram

Synchronous Counters

A four-bit synchronous up-counter

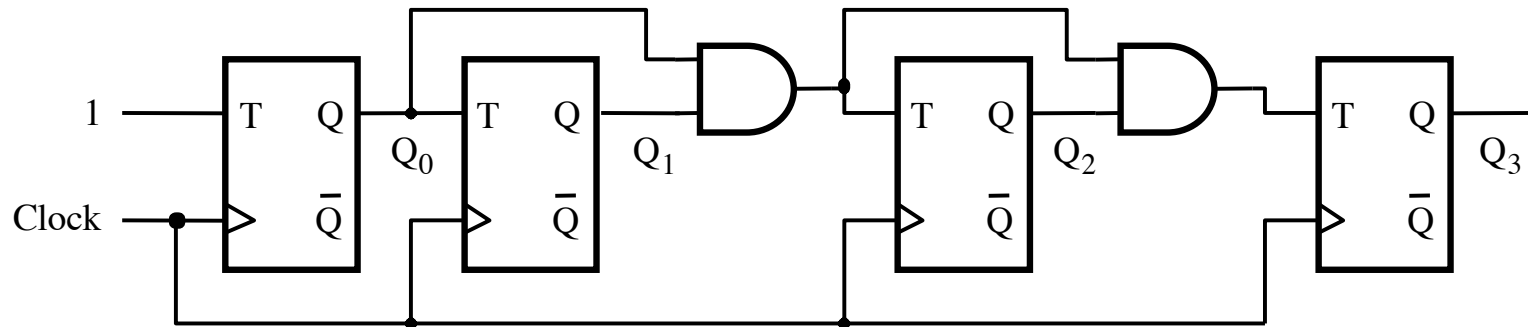


A four-bit synchronous up-counter

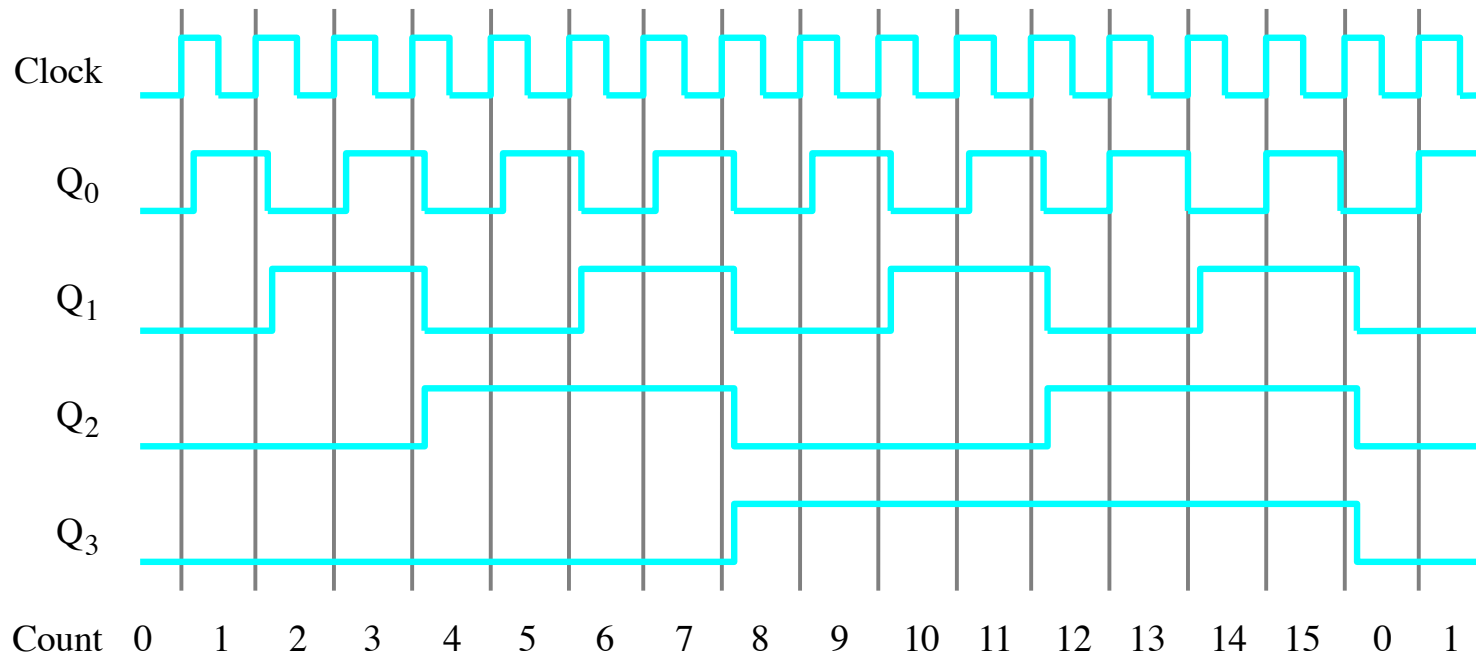


The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

A four-bit synchronous up-counter



(a) Circuit



(b) Timing diagram

Derivation of the synchronous up-counter

Clock cycle	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0

The diagram illustrates the timing of the counter outputs. The signals Q₀, Q₁, and Q₂ are shown as horizontal lines. Q₀ is the least significant bit and changes every clock cycle. Q₁ changes at the falling edge of Q₀. Q₂ changes at the falling edge of Q₁. The labels "Q₁ changes" and "Q₂ changes" point to the falling edges of Q₀ and Q₁ respectively.

Derivation of the synchronous up-counter

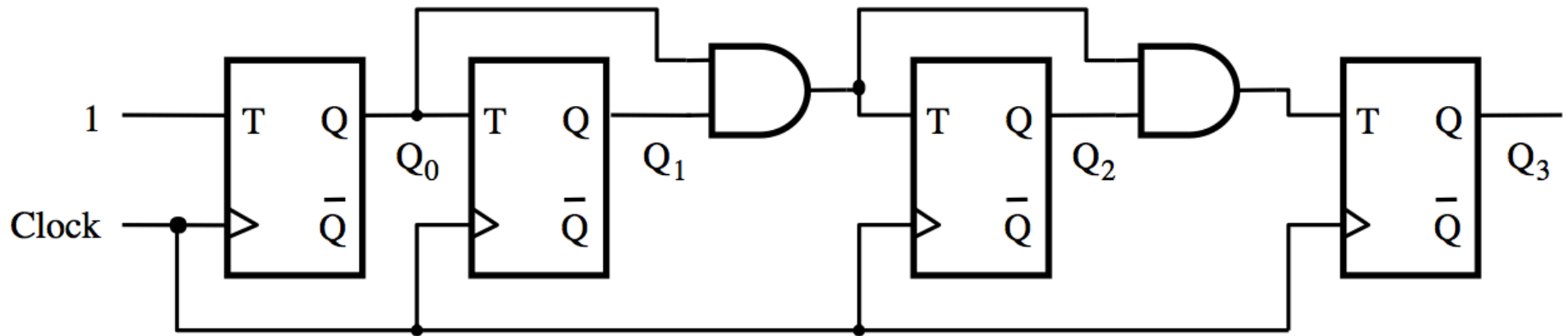
Clock cycle	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

A four-bit synchronous up-counter



$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

In general we have

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

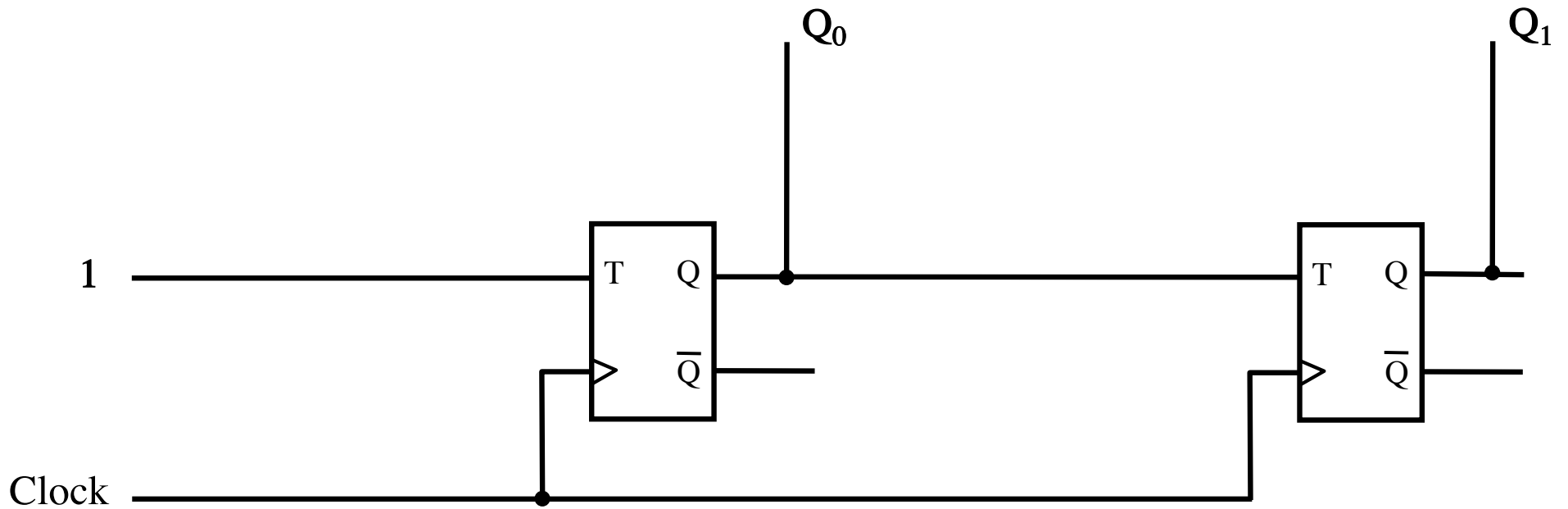
$$T_3 = Q_0 Q_1 Q_2$$

...

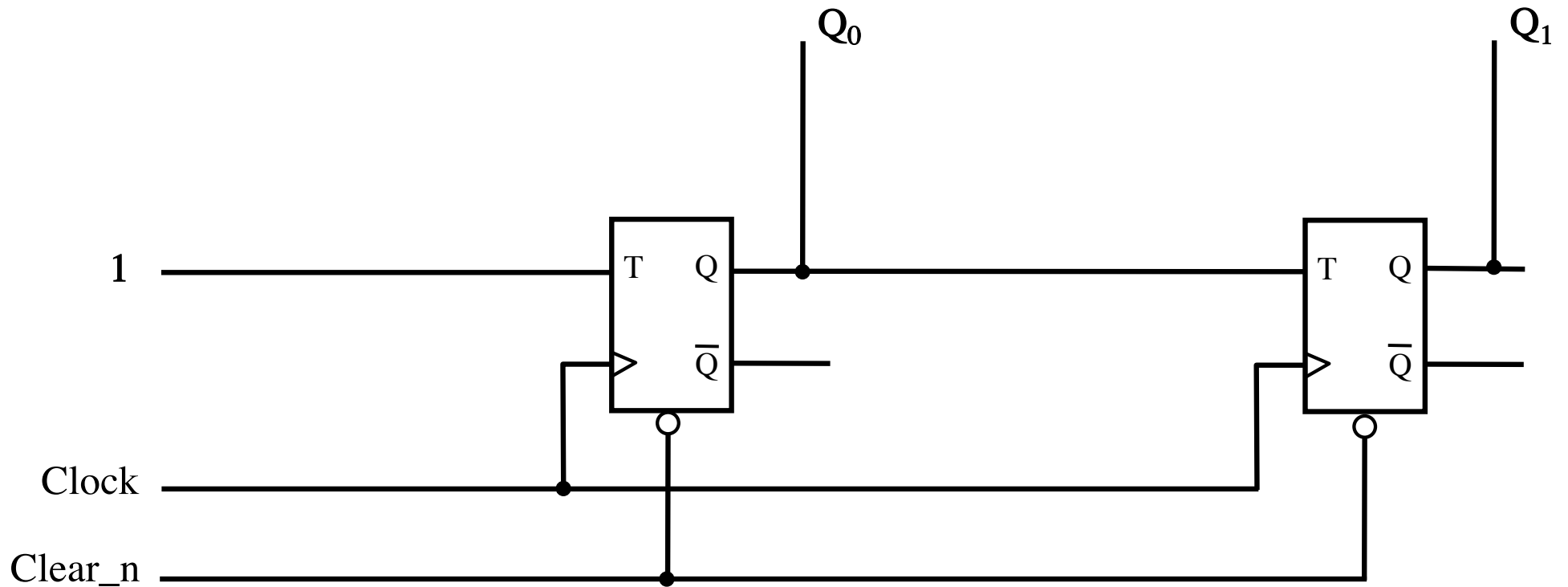
$$T_n = Q_0 Q_1 Q_2 \cdots Q_{n-1}$$

Synchronous v.s. Asynchronous Clear

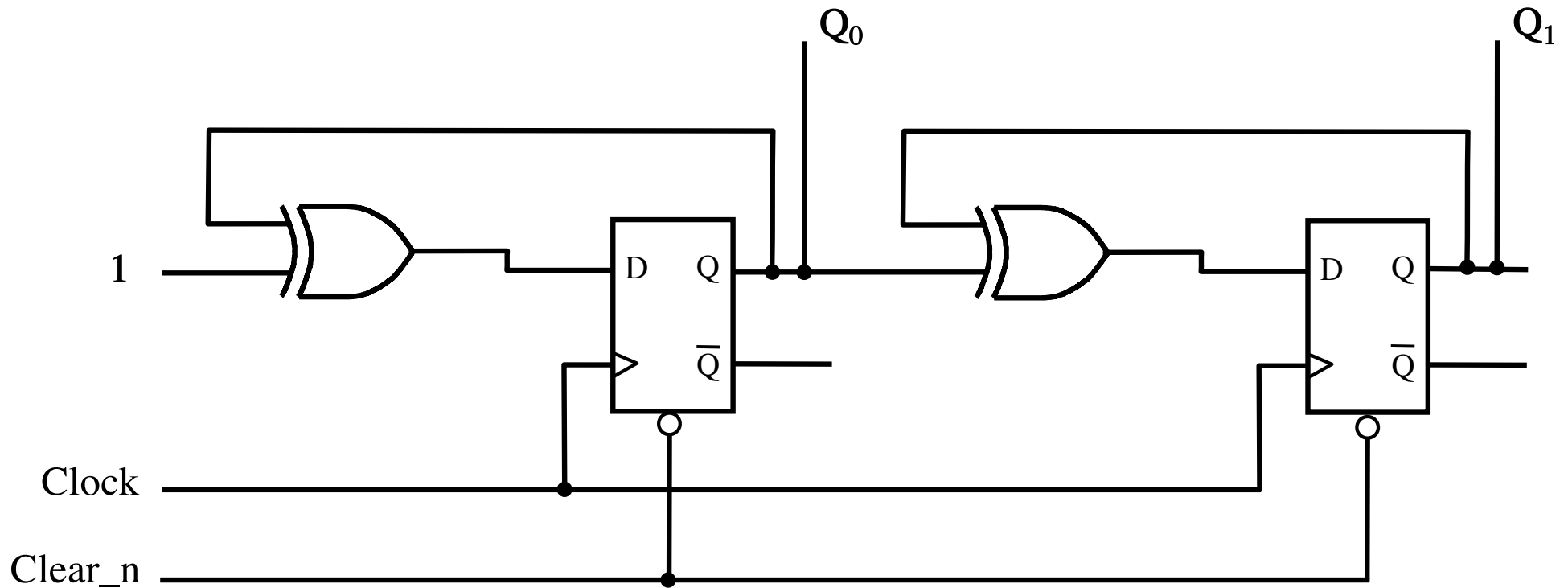
2-Bit Synchronous Up-Counter (without clear capability)



2-Bit Synchronous Up-Counter (with asynchronous clear)

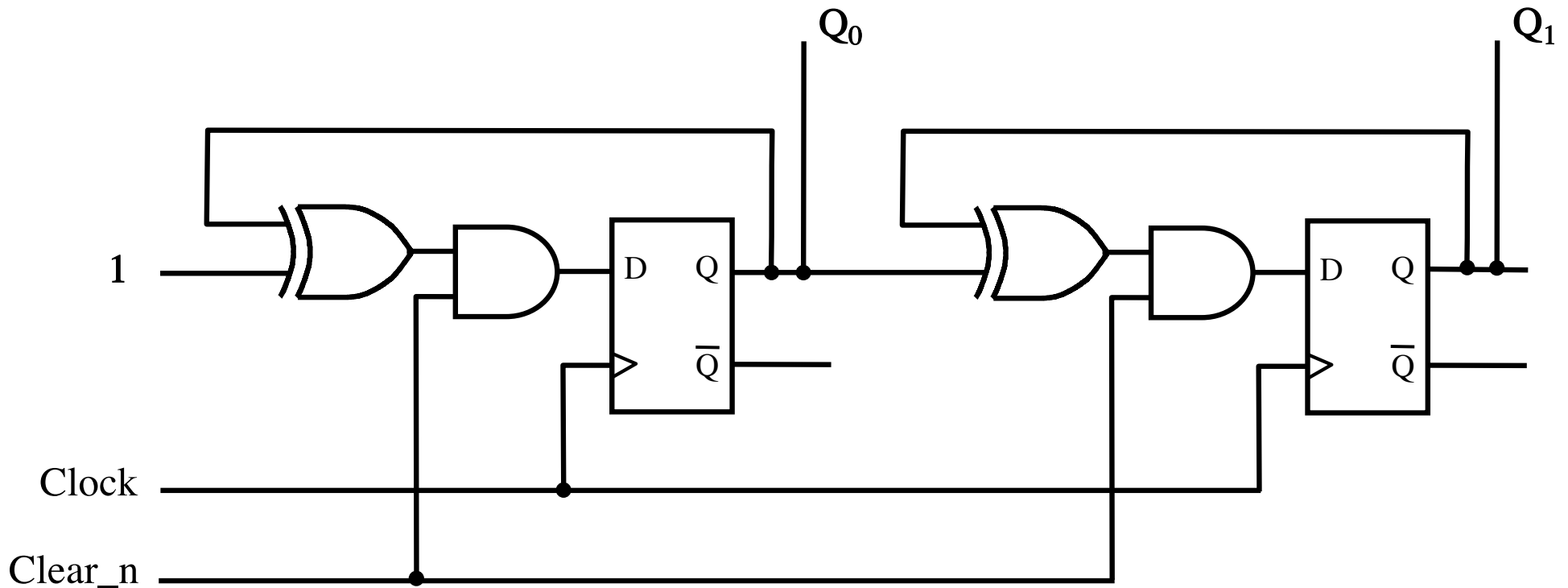


2-Bit Synchronous Up-Counter (with asynchronous clear)



This is the same circuit but uses D Flip-Flops.

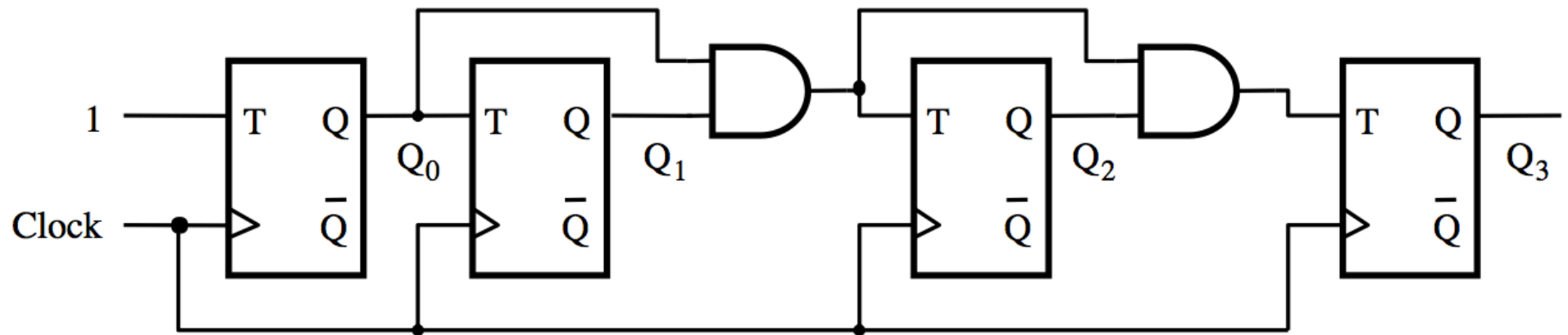
2-Bit Synchronous Up-Counter (with synchronous clear)



This counter can be cleared only on the positive clock edge.

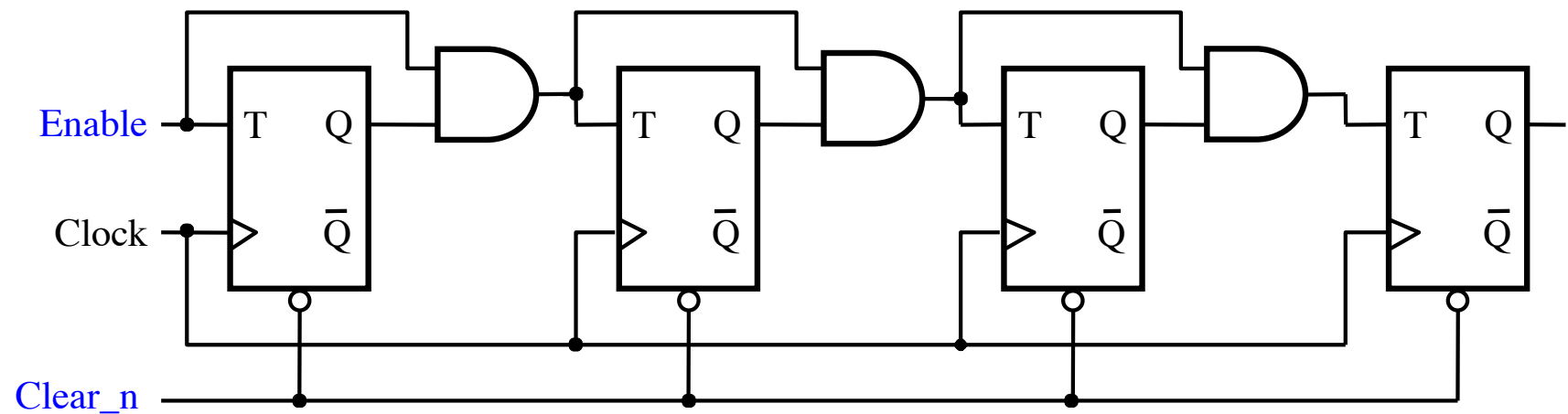
Adding Enable Capability

A four-bit synchronous up-counter



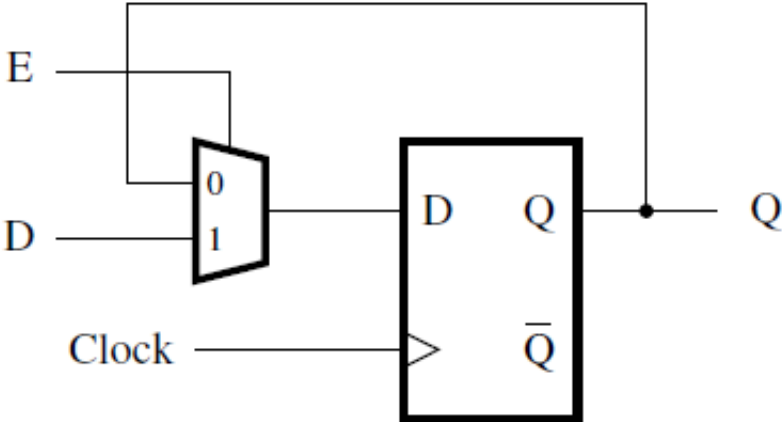
[Figure 5.21 from the textbook]

Inclusion of Enable and Clear Capability

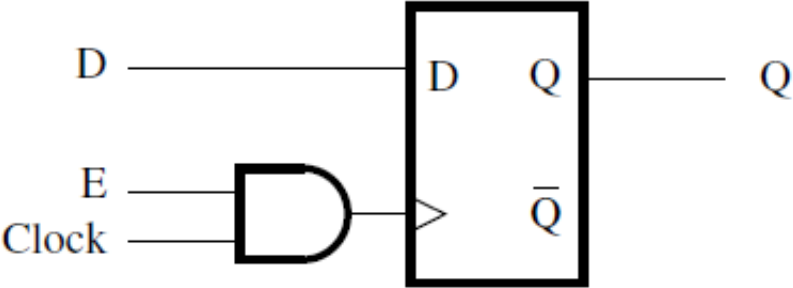


[Figure 5.22 from the textbook]

Providing an enable input for a D flip-flop



(a) Using a multiplexer

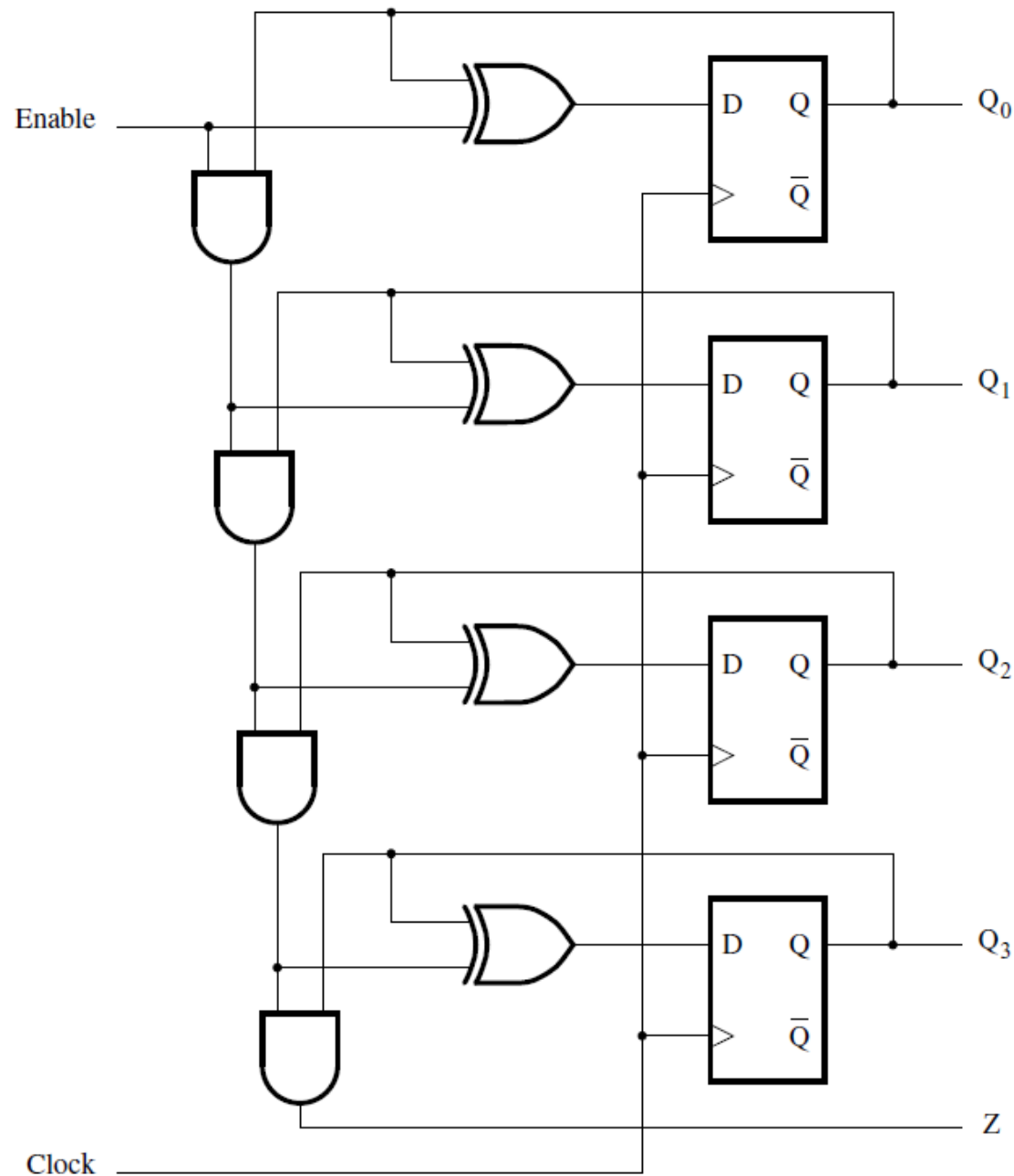


(b) Clock gating

[Figure 5.56 from the textbook]

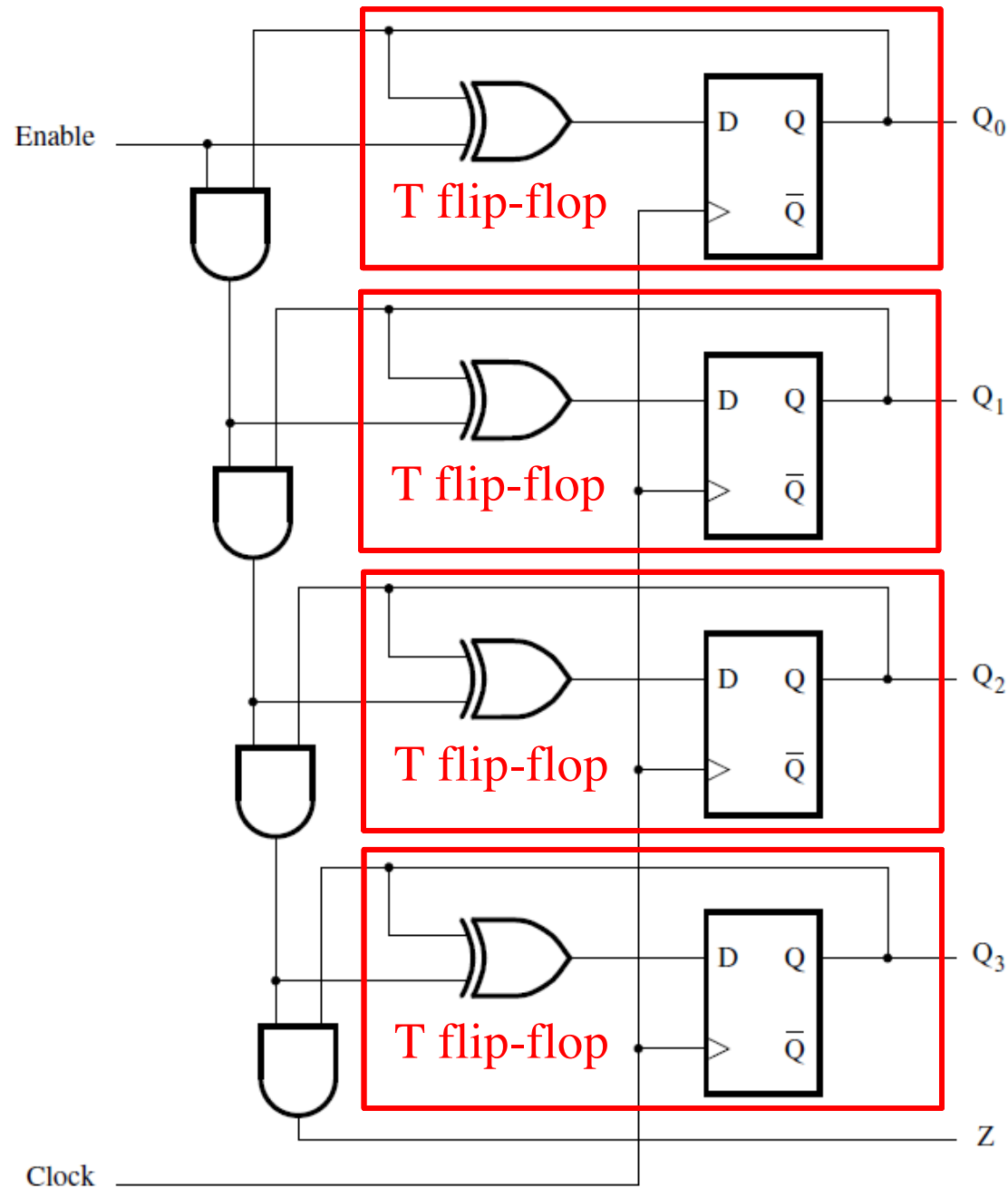
Synchronous Counter (with D Flip-Flops)

A 4-bit up-counter with D flip-flops



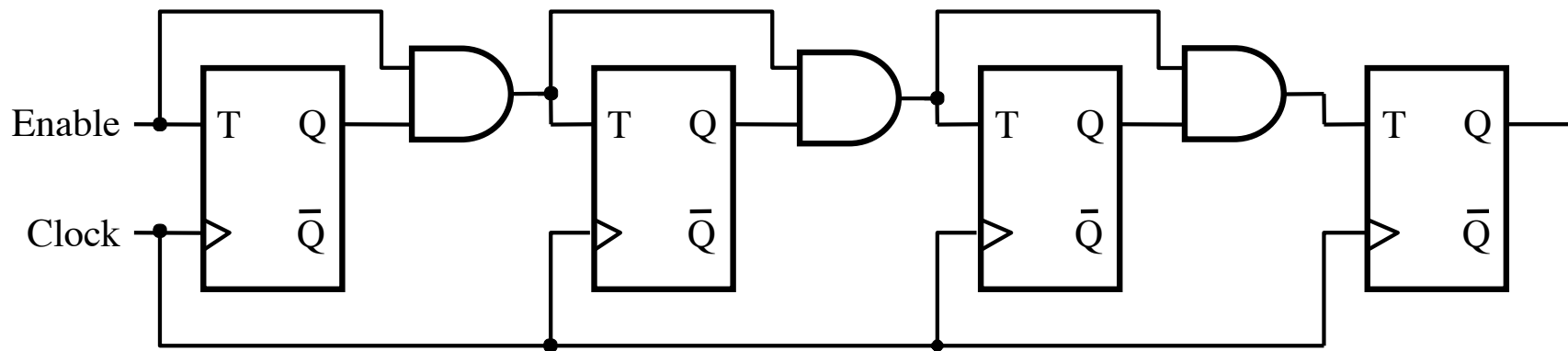
[Figure 5.23 from the textbook]

A 4-bit up-counter with D flip-flops

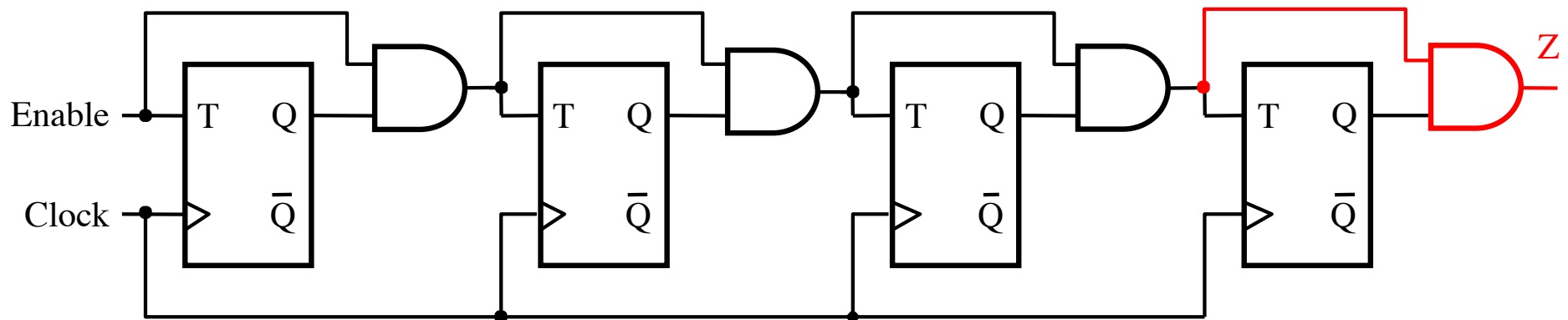


[Figure 5.23 from the textbook]

Equivalent to this circuit with T flip-flops



Equivalent to this circuit with T flip-flops

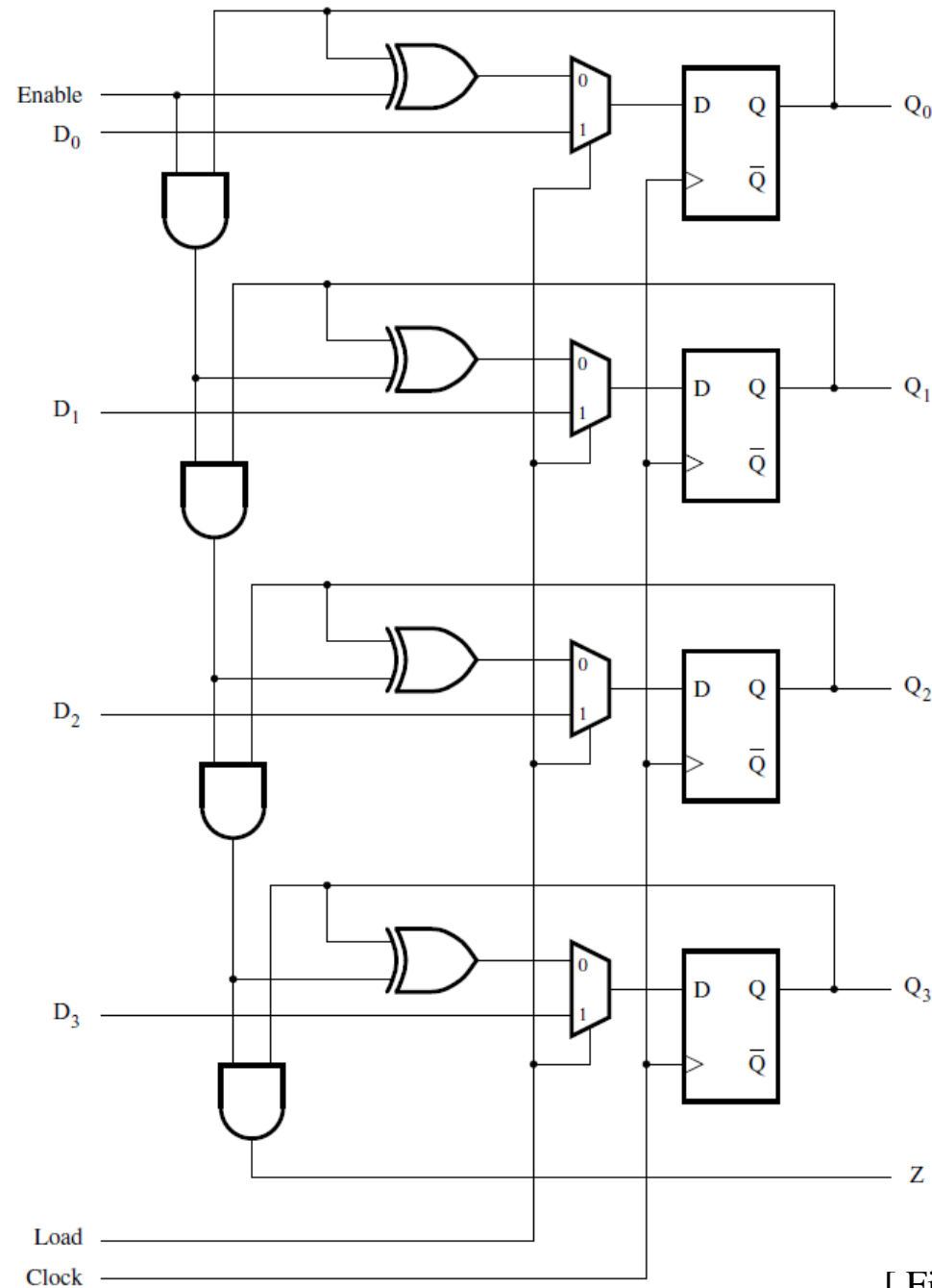


But has one extra output called Z, which can be used to connect two 4-bit counters to make an 8-bit counter.

When $Z=1$ the counter will go to 0000 on the next clock edge, i.e., the outputs of all flip-flops are currently 1 (maximum count value).

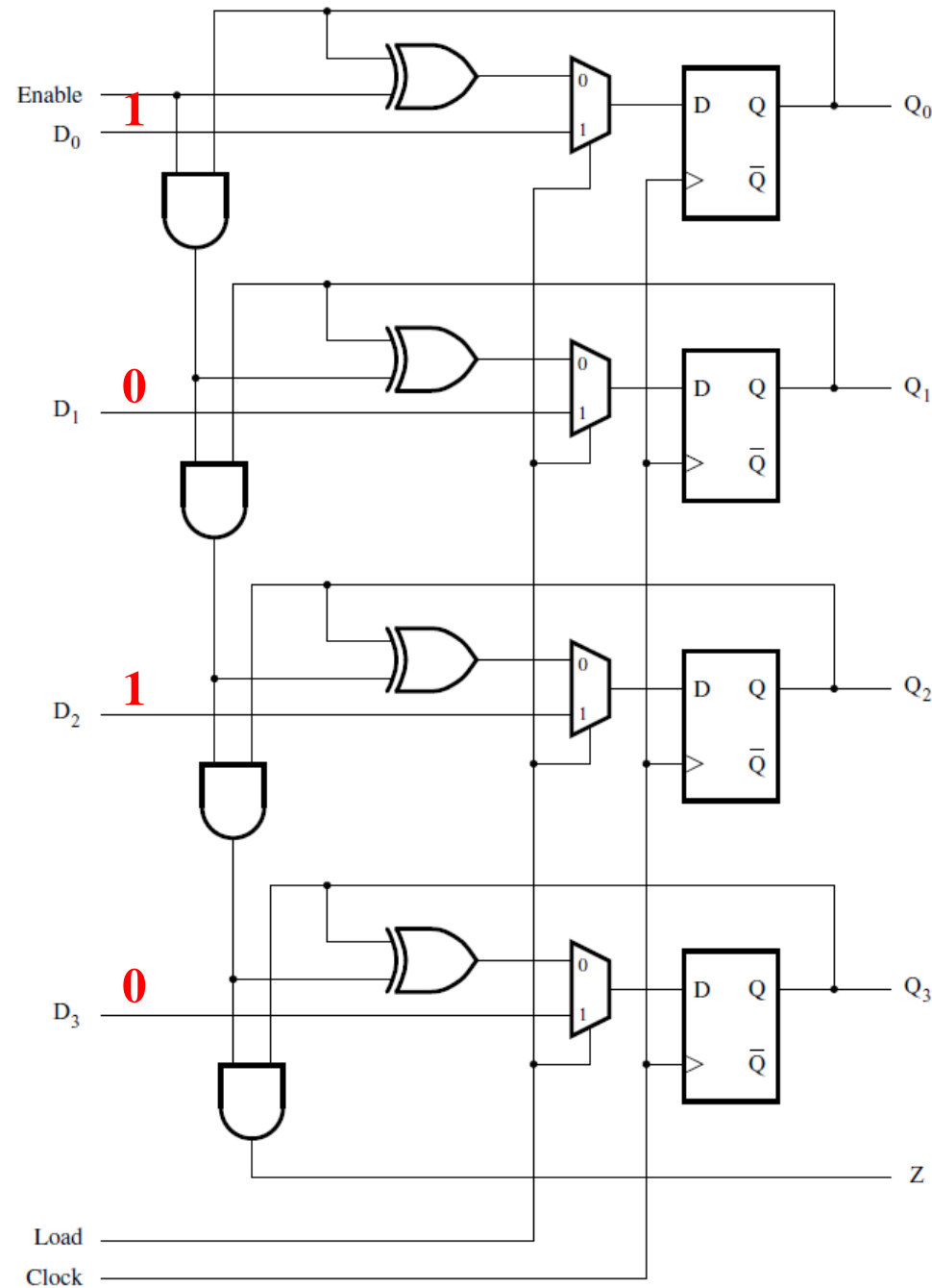
Counters with Parallel Load

A counter with parallel-load capability



[Figure 5.24 from the textbook]

How to load the initial count value



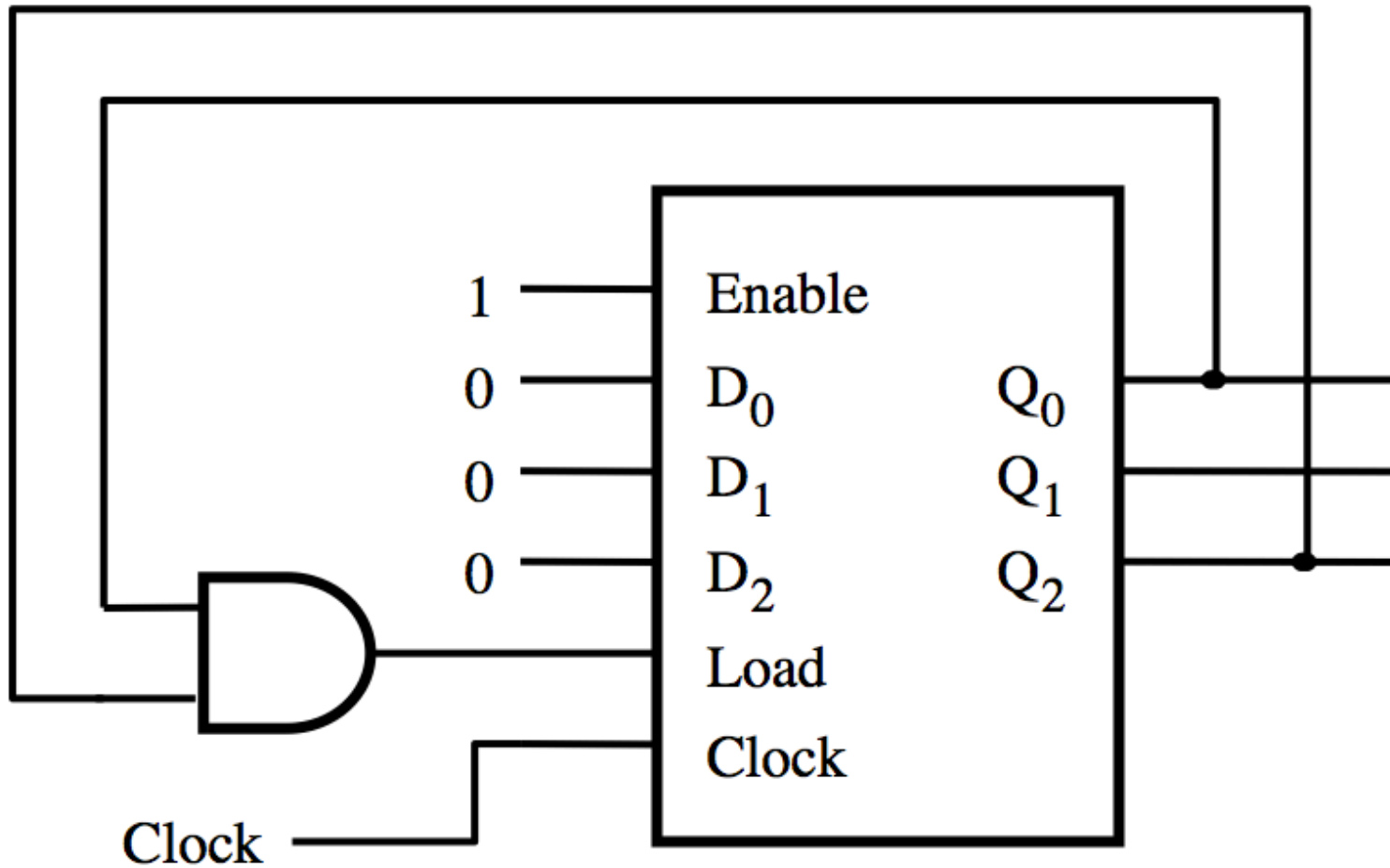
Set the initial count on the parallel load lines (in this case 5).

Reset Synchronization

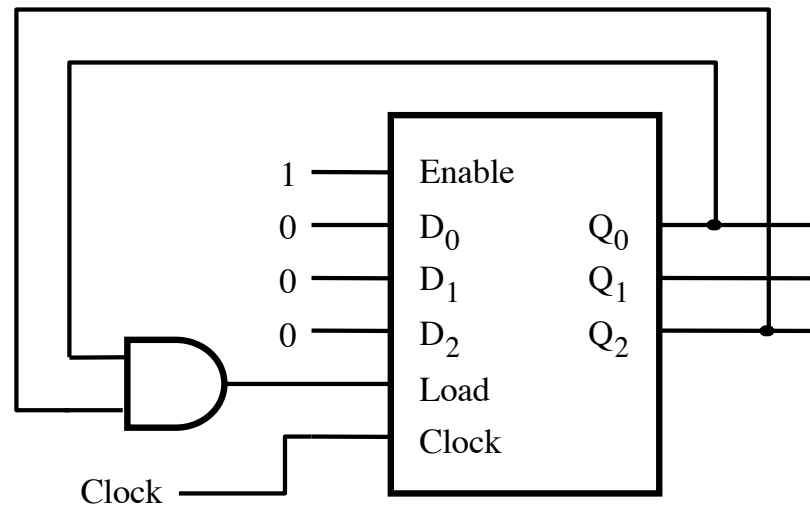
Motivation

- **An n-bit counter counts from 0, 1, ..., 2^n-1**
- **For example a 3-bit counter counts up as follow**
 - **0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, ...**
- **What if we want it to count like this**
 - **0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0, 1, ...**
- **In other words, what is the cycle is not a power of 2?**

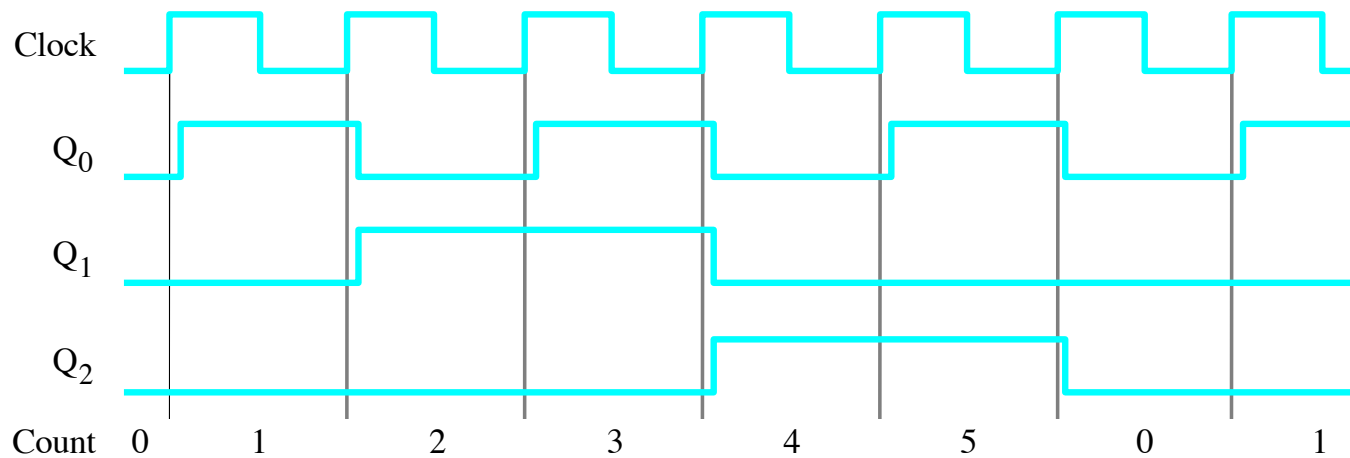
What does this circuit do?



A modulo-6 counter with synchronous reset

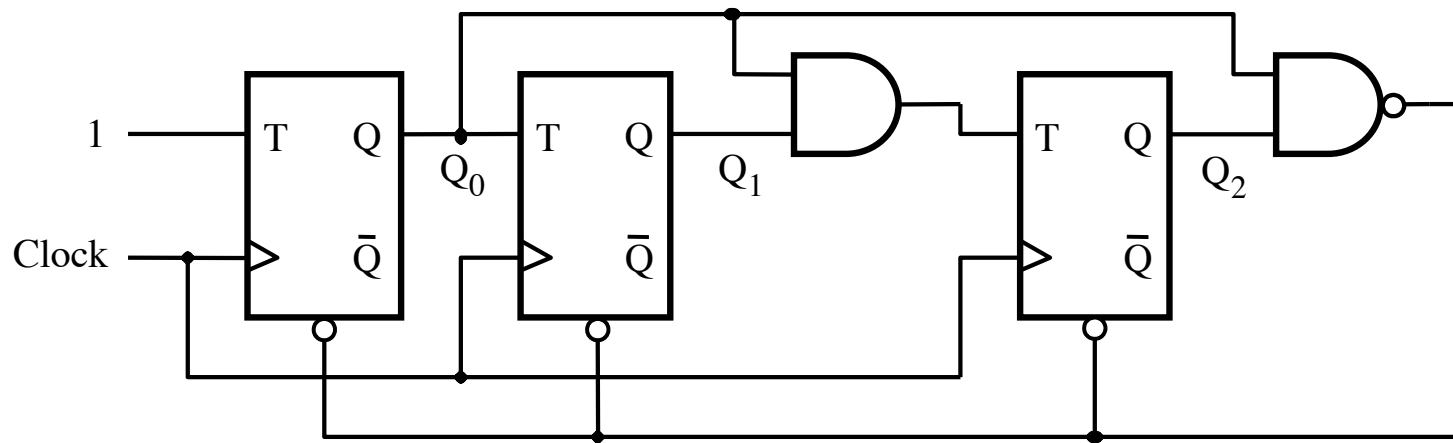


(a) Circuit

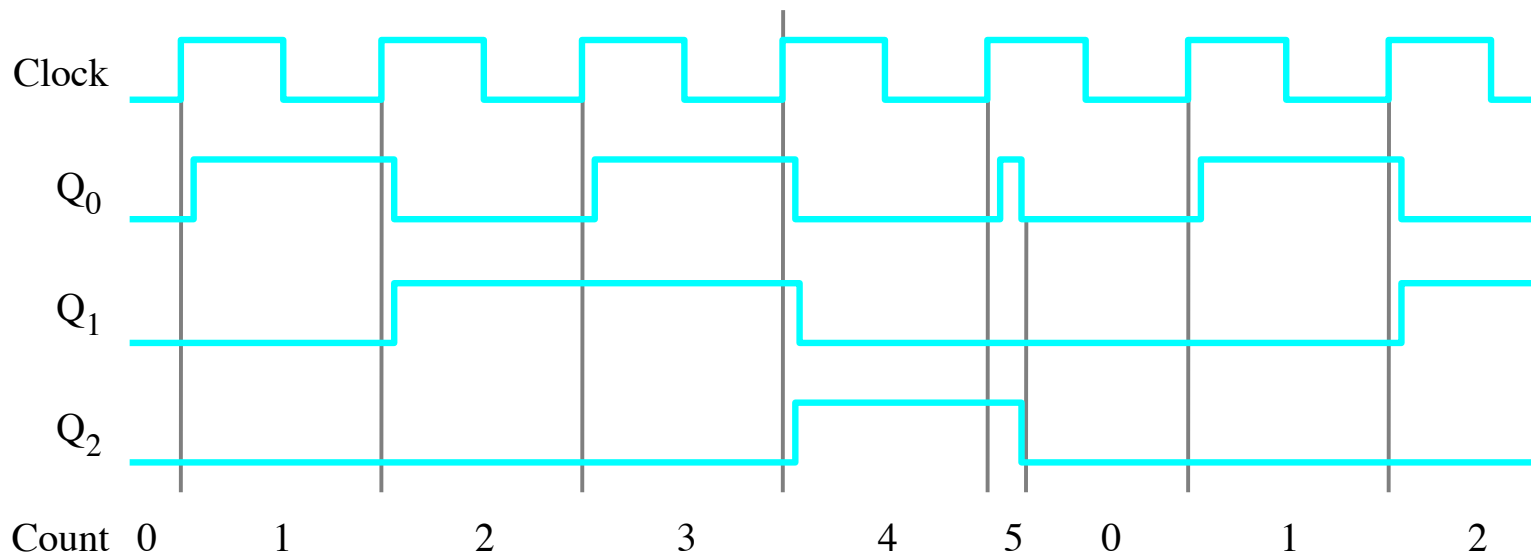


(b) Timing diagram

A modulo-6 counter with asynchronous reset

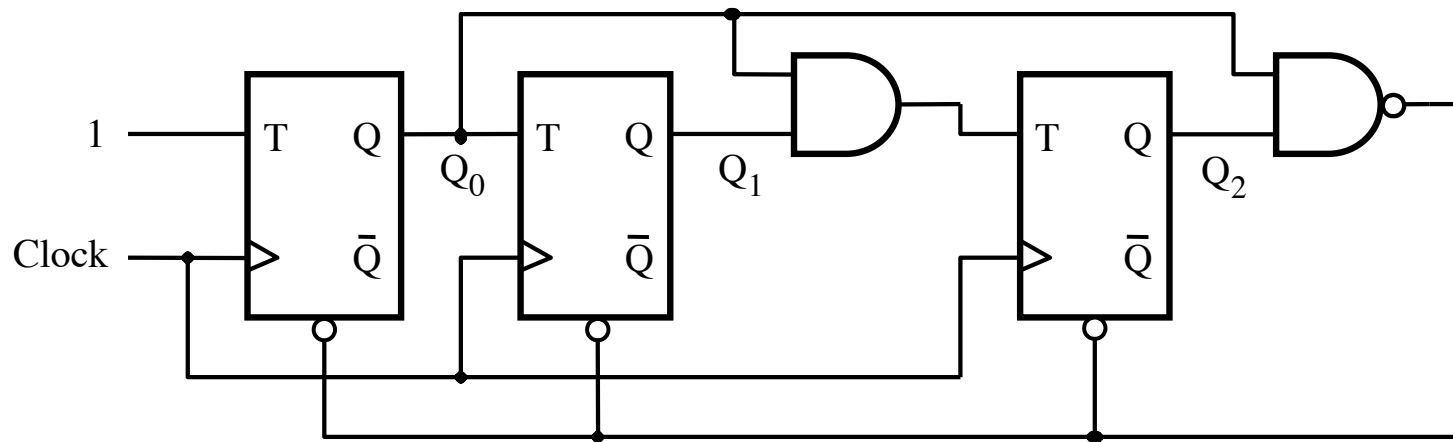


(a) Circuit



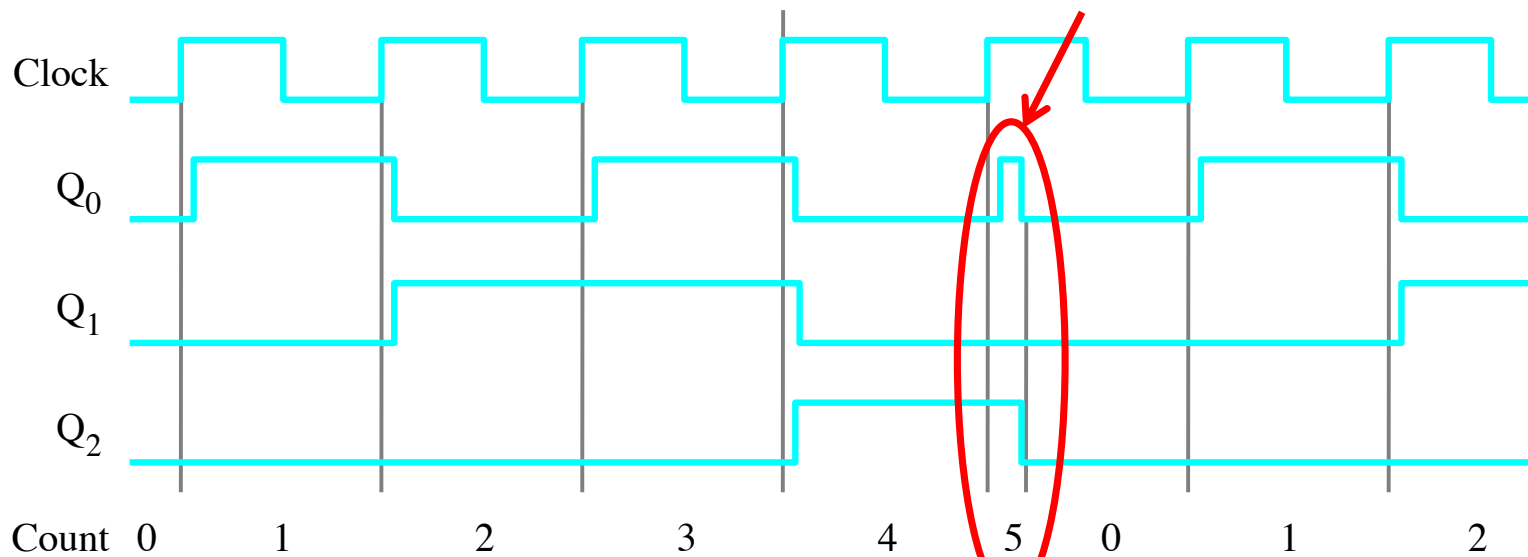
(b) Timing diagram

A modulo-6 counter with asynchronous reset



(a) Circuit

The number 5 is displayed for a very short amount of time



(b) Timing diagram

Questions?

THE END