

CprE 281: Digital Logic

Instructor: Alexander Stoytchev

<http://www.ece.iastate.edu/~alexs/classes/>

Counters & Solved Problems

CprE 281: Digital Logic
Iowa State University, Ames, IA
Copyright © 2013

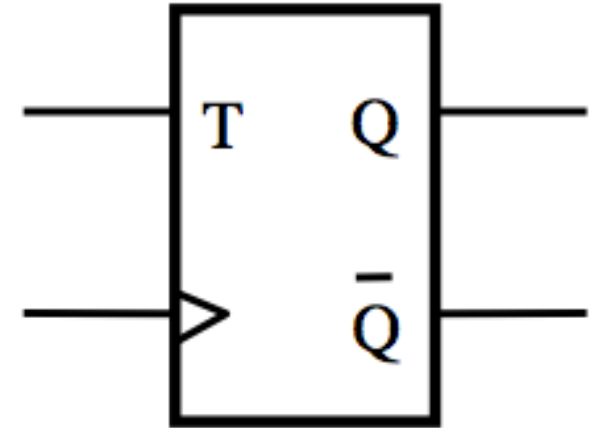
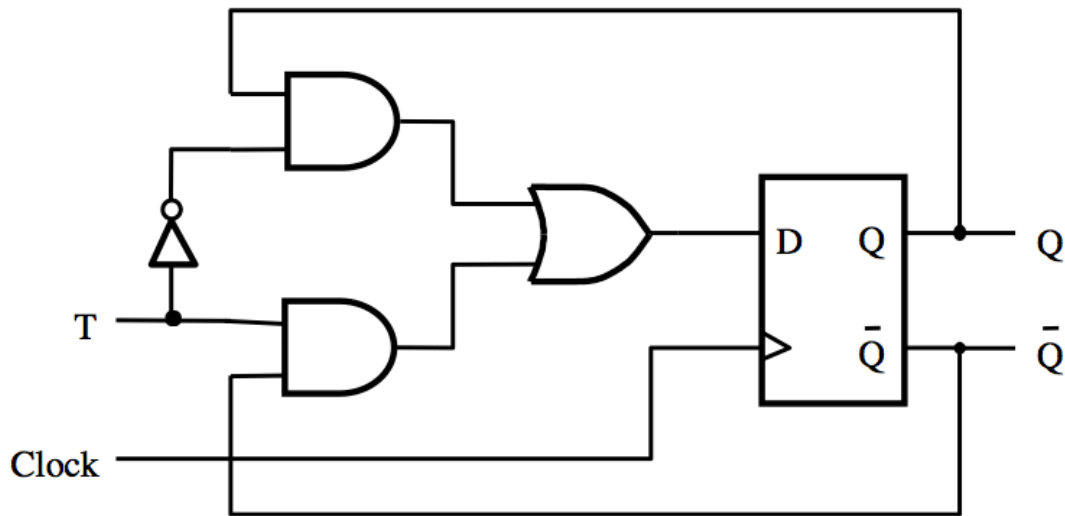
Administrative Stuff

- **Homework 8 is due on Monday @ 4pm**
- **Homework 9 will be due on Monday Oct 26 @ 4pm**

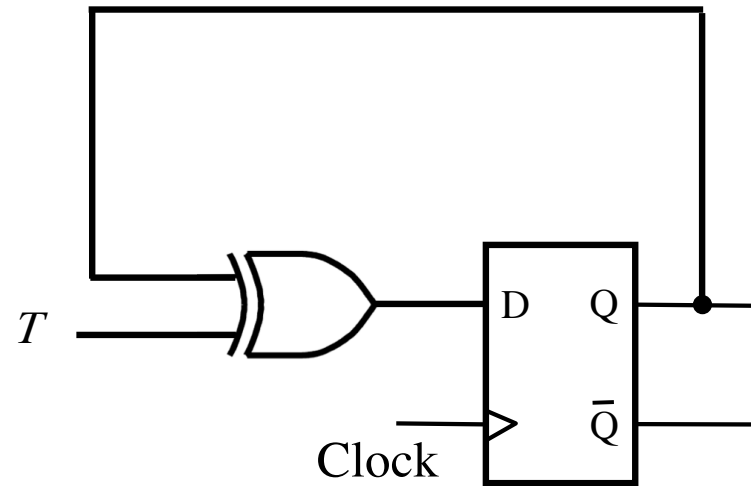
Counters

T Flip-Flop

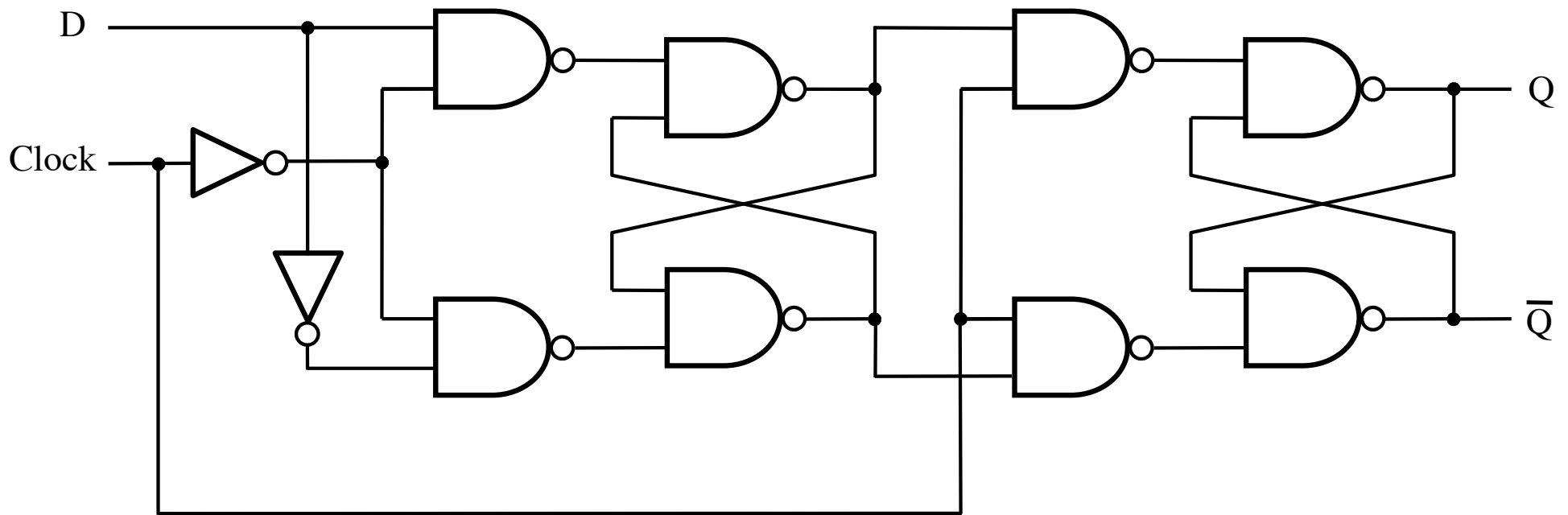
(circuit and graphical symbol)



Yet Another Way to Draw a T Flip-Flop

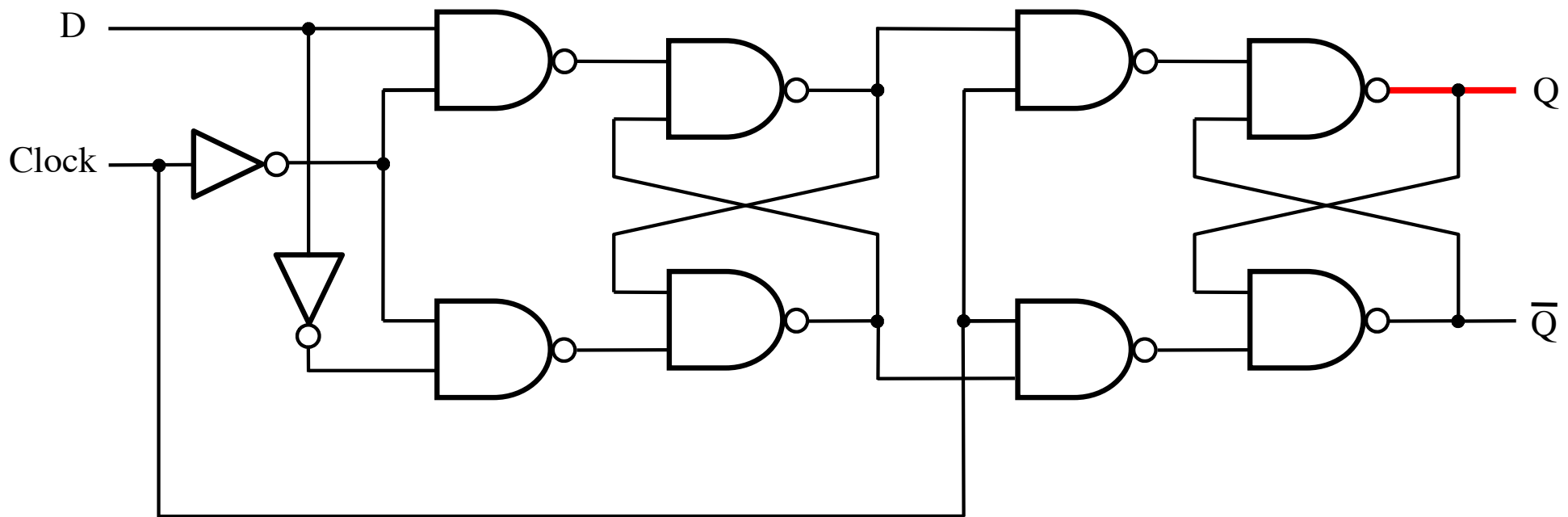


The Complete Wiring Diagram for a Positive-Edge-Triggered D Flip-Flop

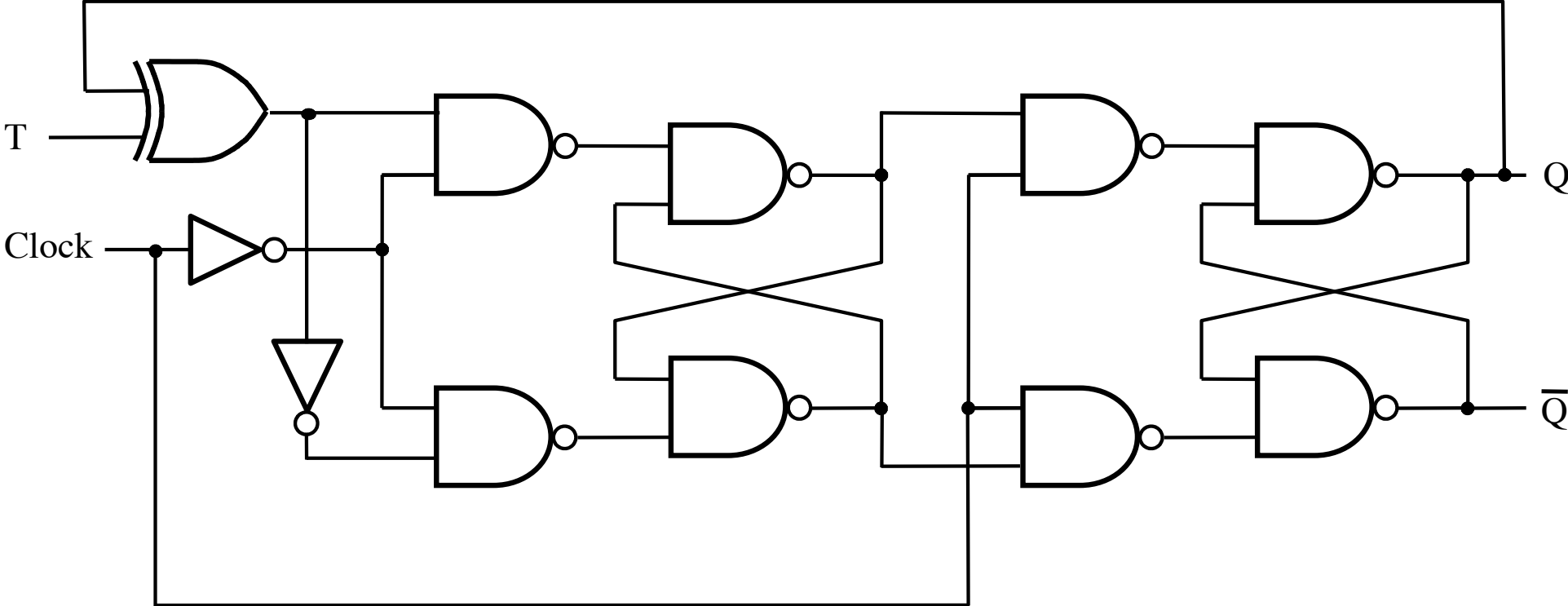


The Complete Wiring Diagram for a Positive-Edge-Triggered D Flip-Flop

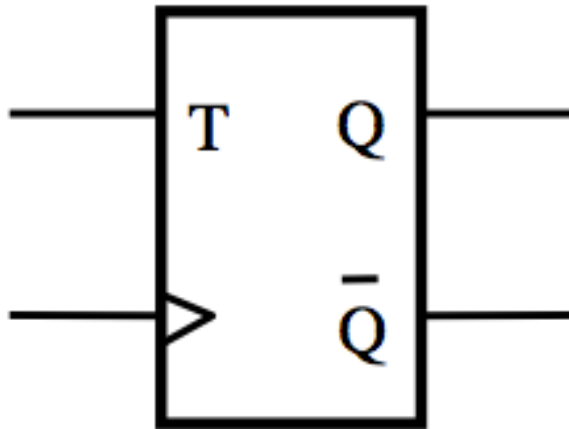
We need all of this just to store 1 bit!



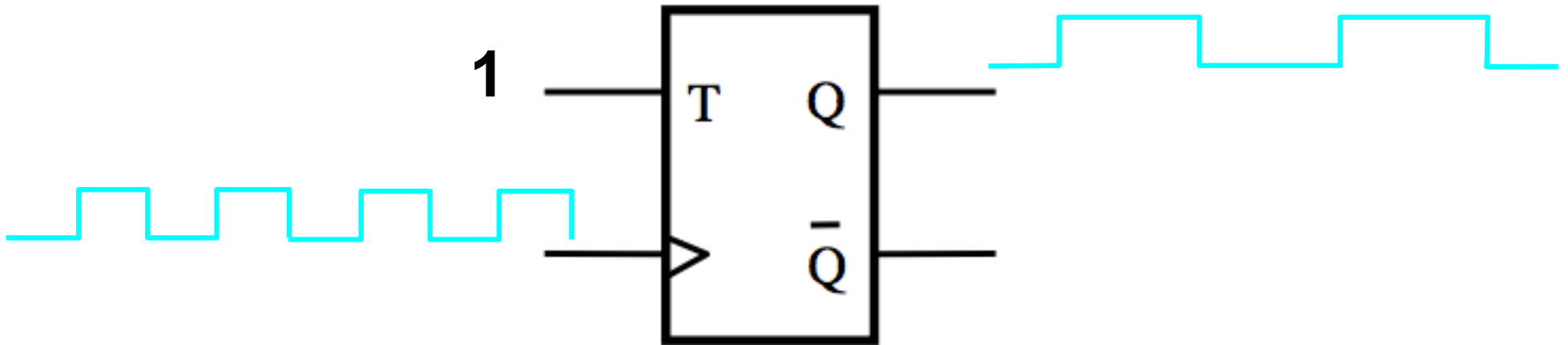
The Complete Wiring Diagram for a Positive-Edge-Triggered T Flip-Flop



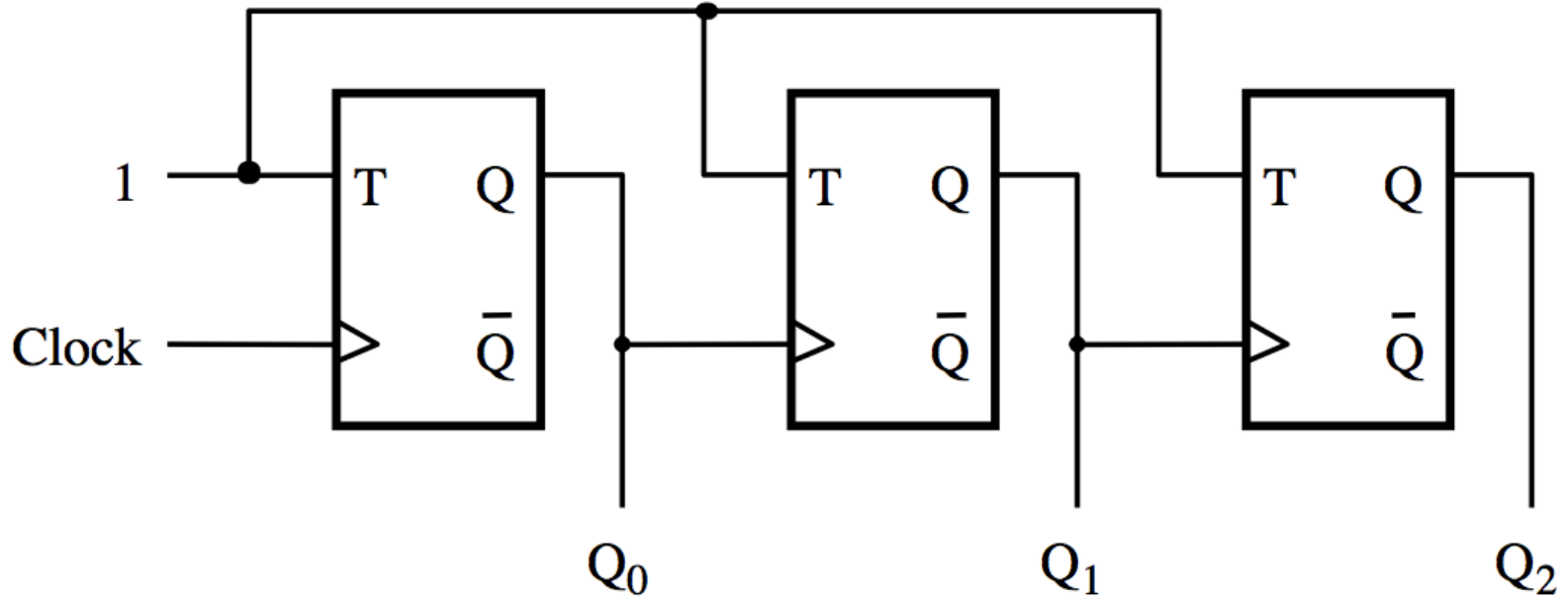
**The output of the T Flip-Flop
divides the frequency of the clock by 2**



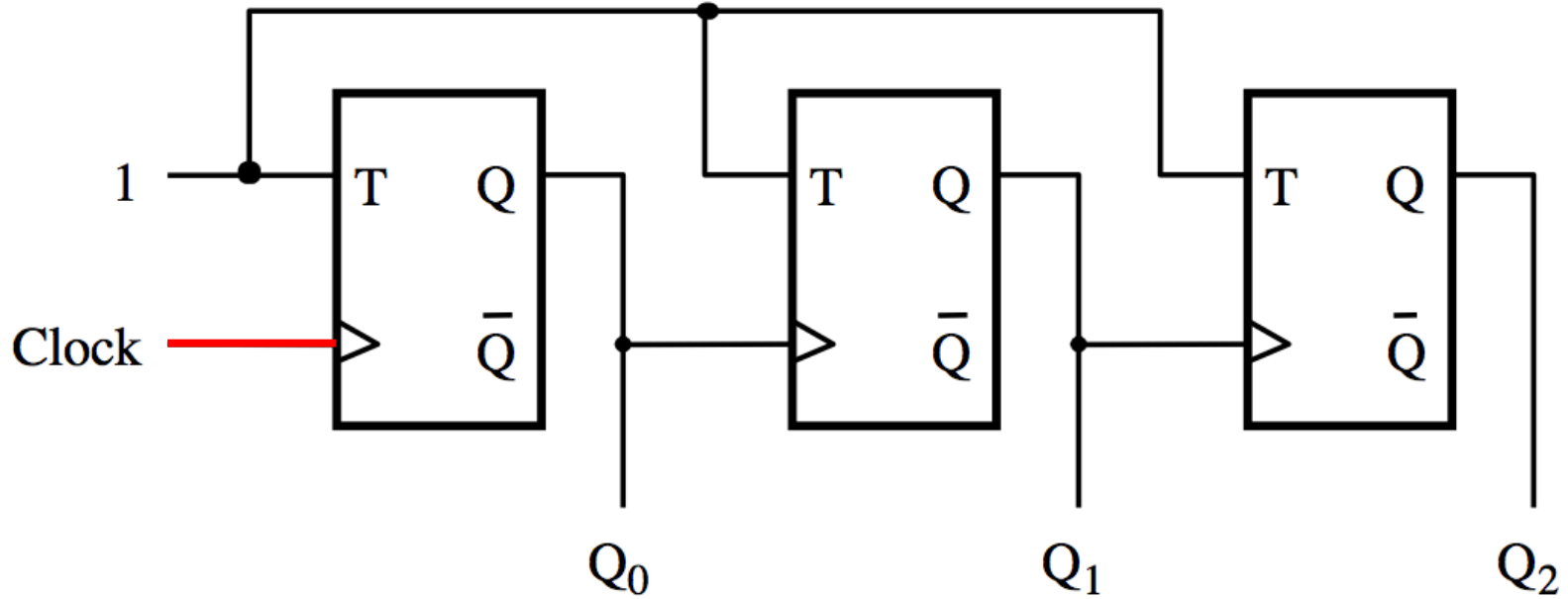
**The output of the T Flip-Flop
divides the frequency of the clock by 2**



A three-bit down-counter

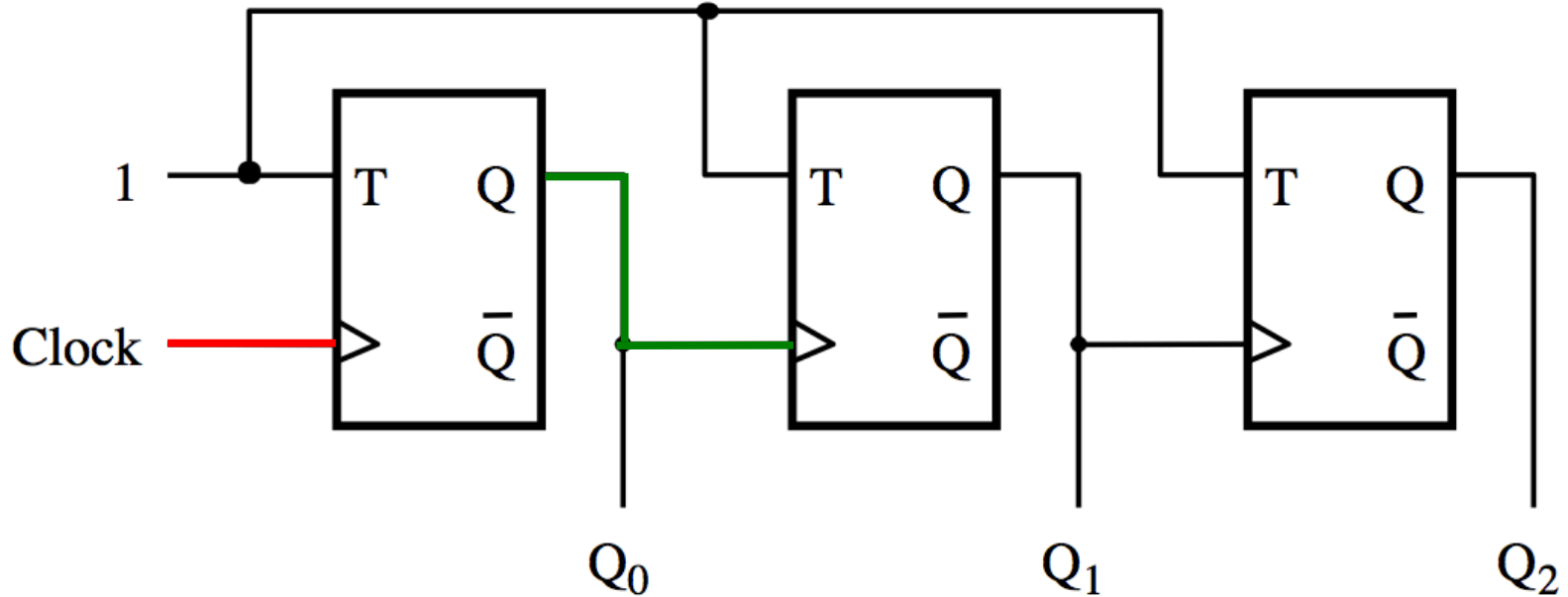


A three-bit down-counter



The first flip-flop changes
on the positive edge of the clock

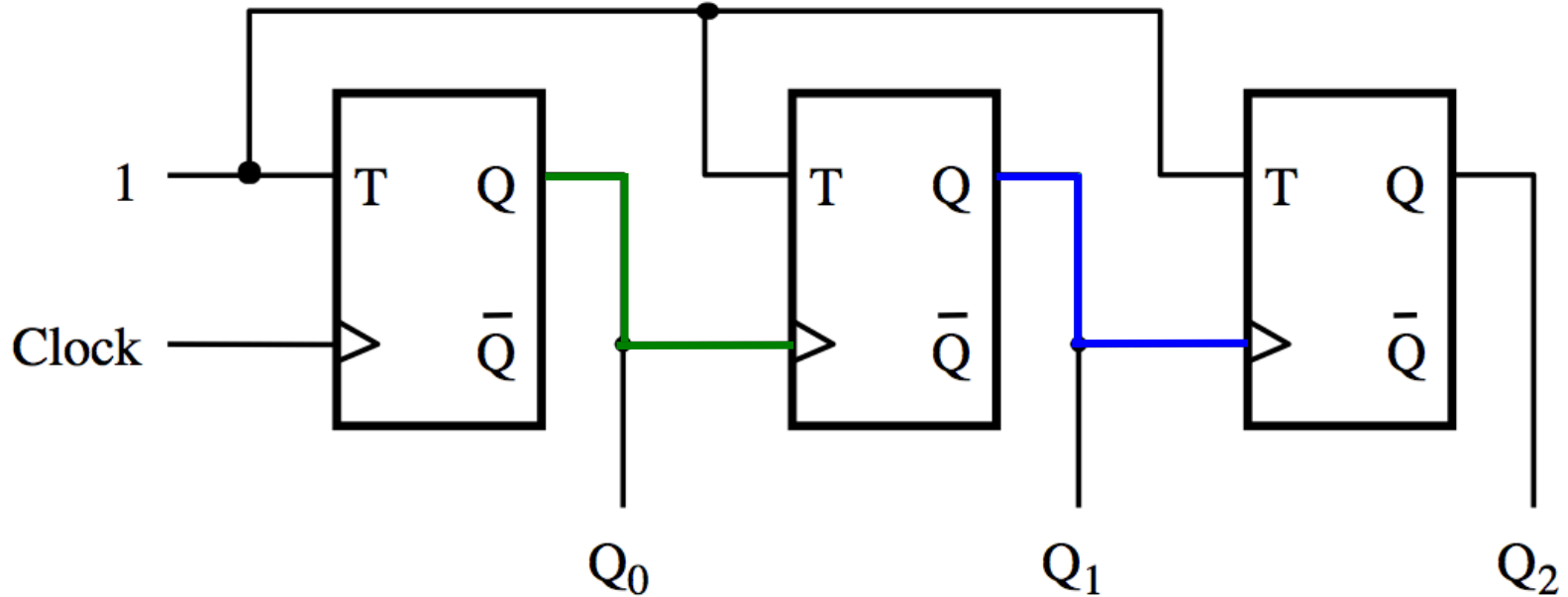
A three-bit down-counter



The first flip-flop changes
on the positive edge of the clock

The second flip-flop changes
on the positive edge of Q_0

A three-bit down-counter

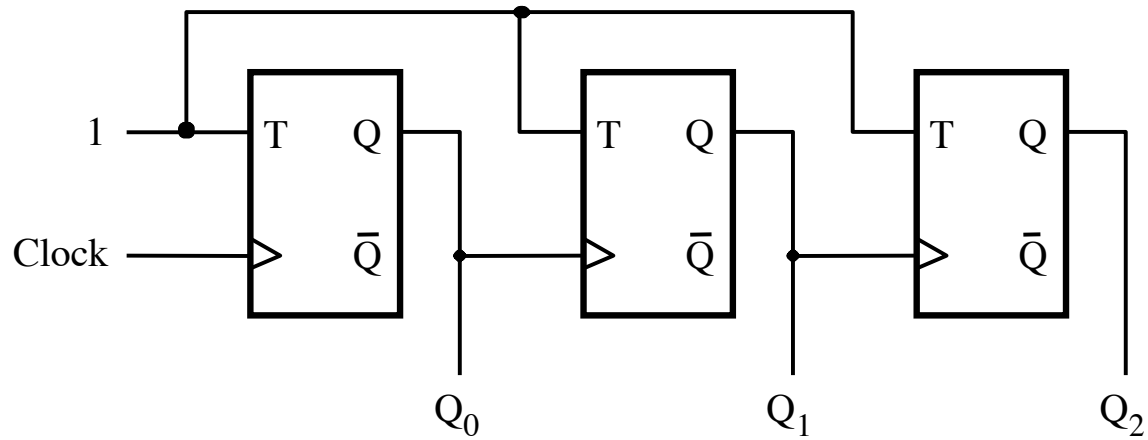


The first flip-flop changes
on the positive edge of the clock

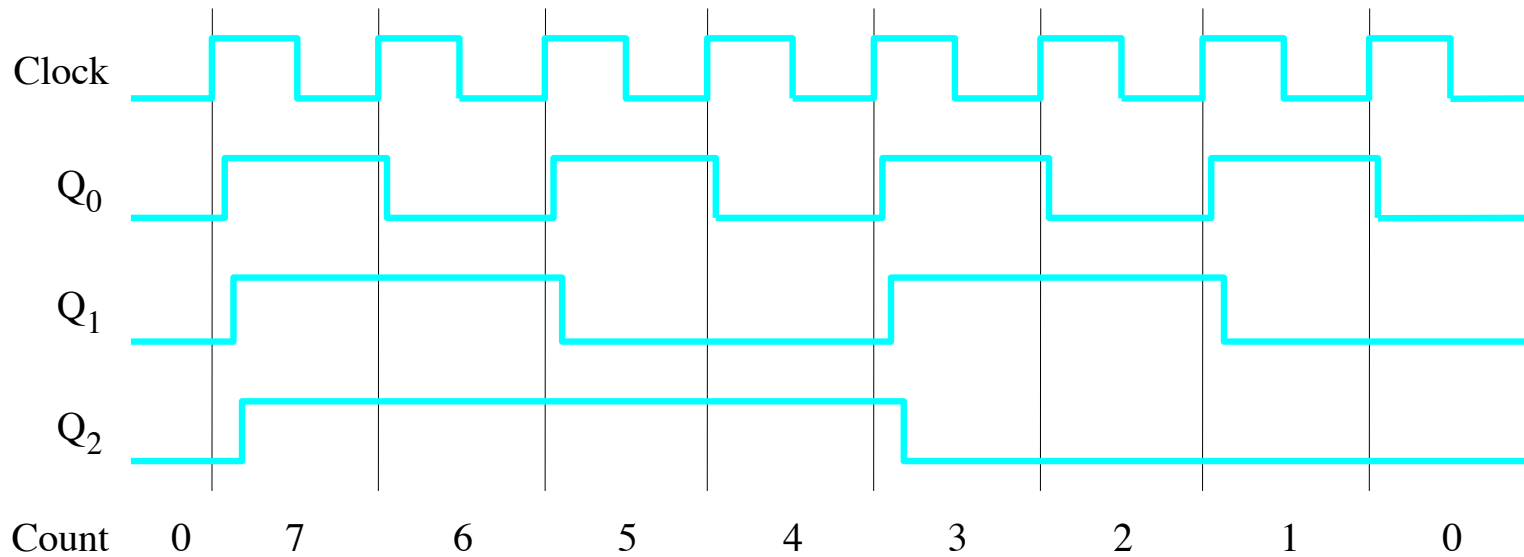
The second flip-flop changes
on the positive edge of Q_0

The third flip-flop changes
on the positive edge of Q_1

A three-bit down-counter



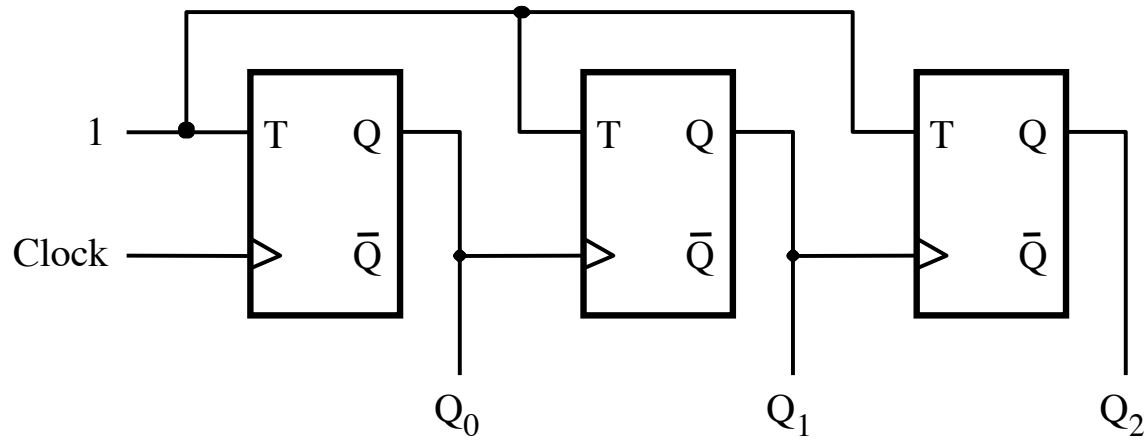
(a) Circuit



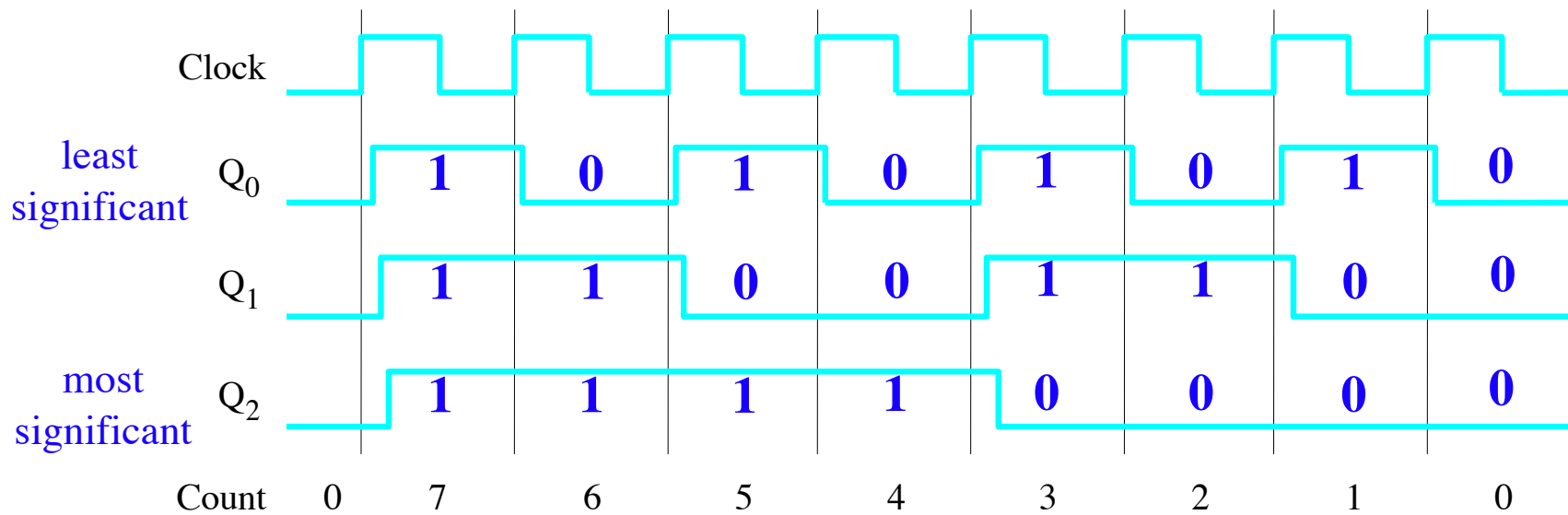
(b) Timing diagram

[Figure 5.20 from the textbook]

A three-bit down-counter

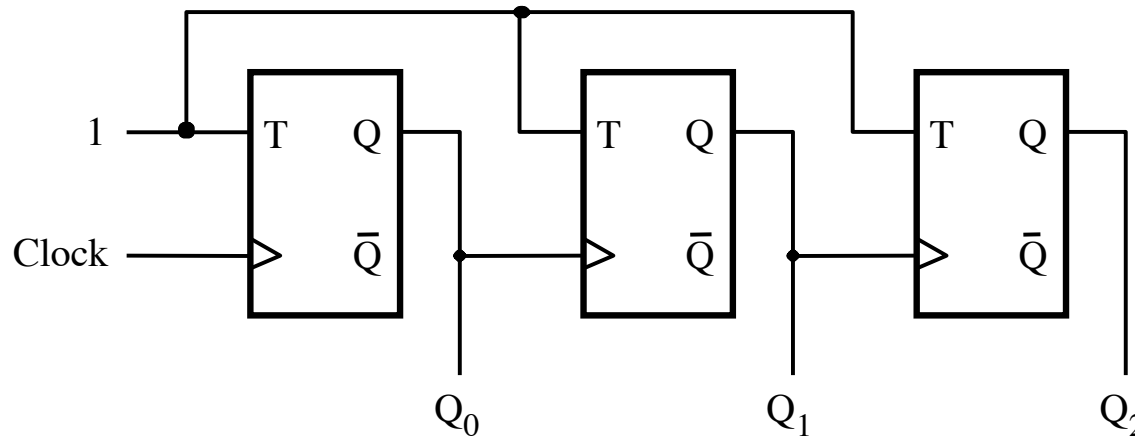


(a) Circuit

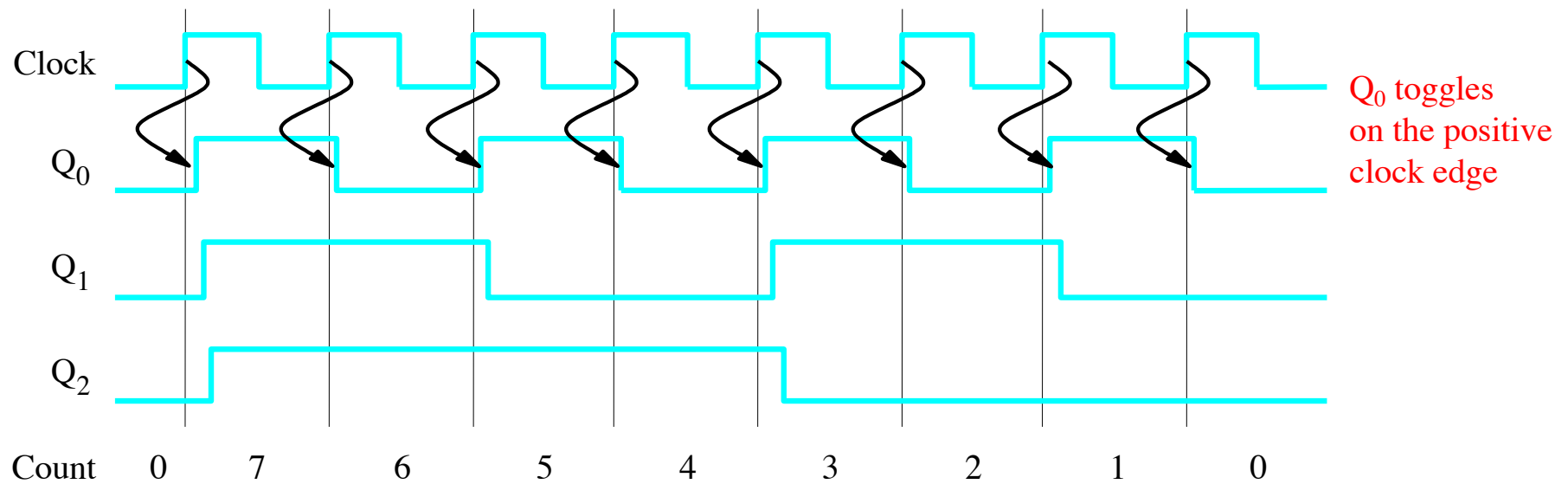


(b) Timing diagram

A three-bit down-counter

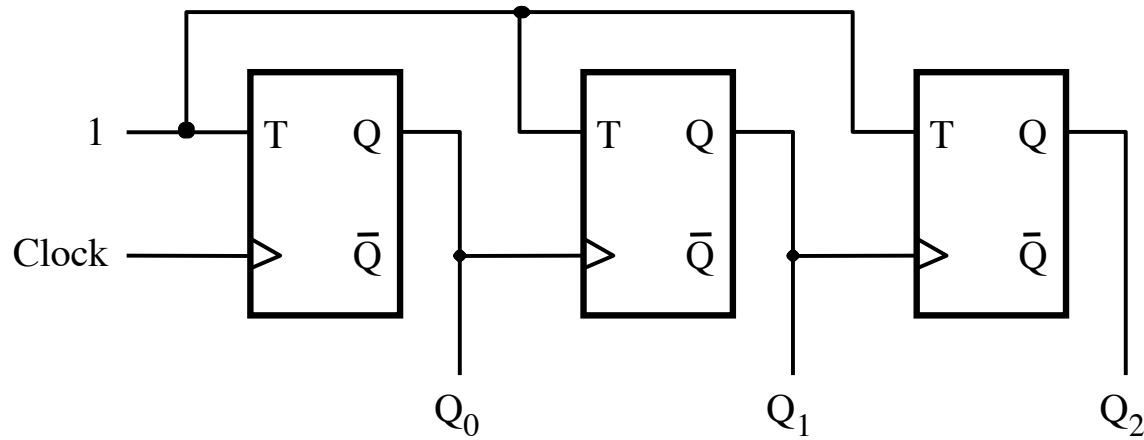


(a) Circuit

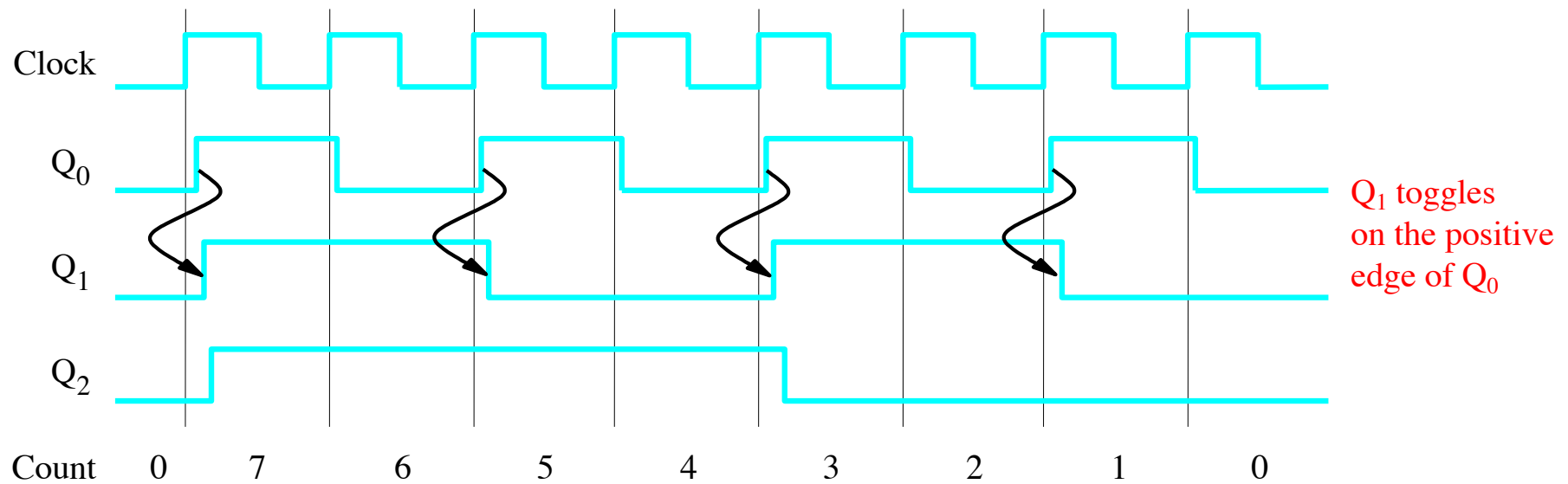


(b) Timing diagram

A three-bit down-counter

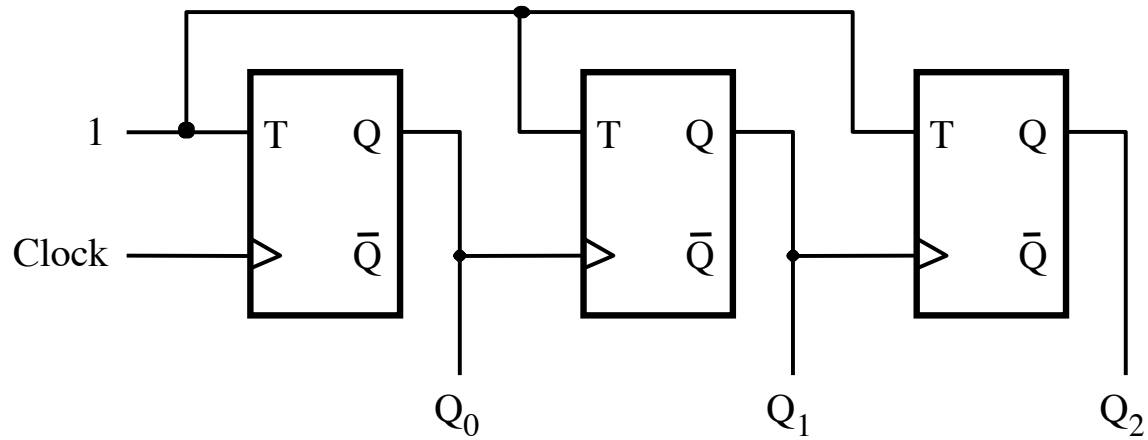


(a) Circuit

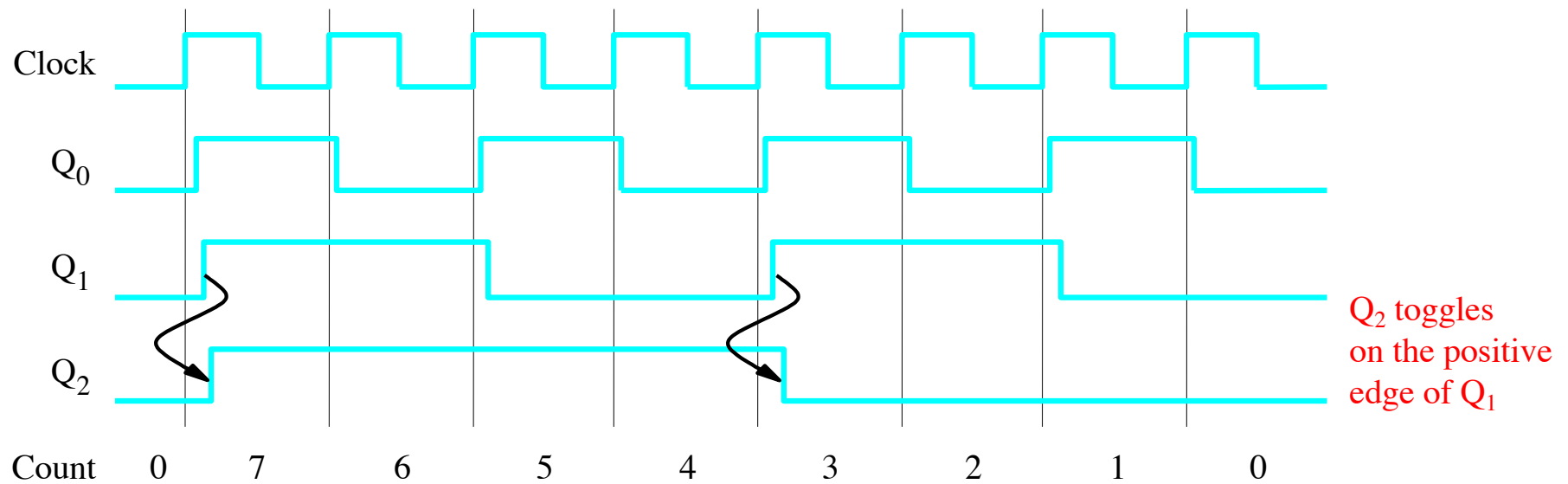


(b) Timing diagram

A three-bit down-counter

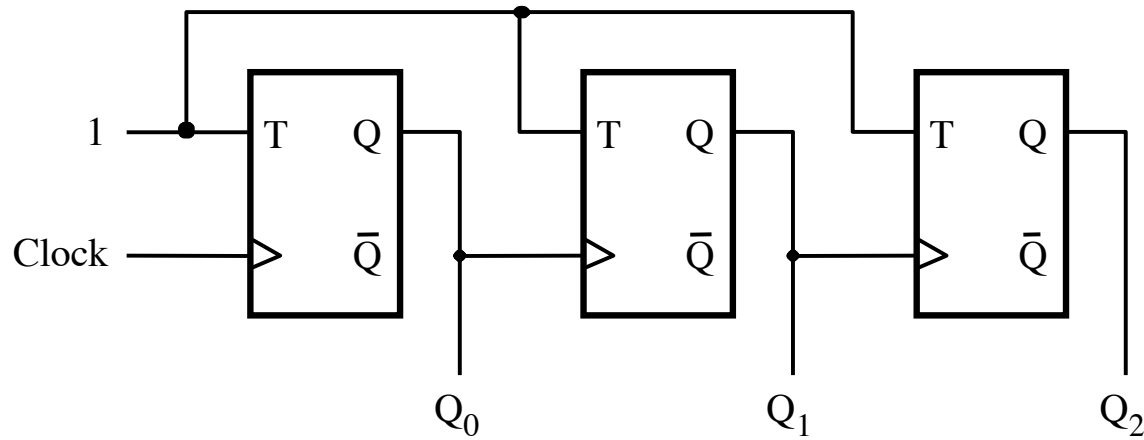


(a) Circuit



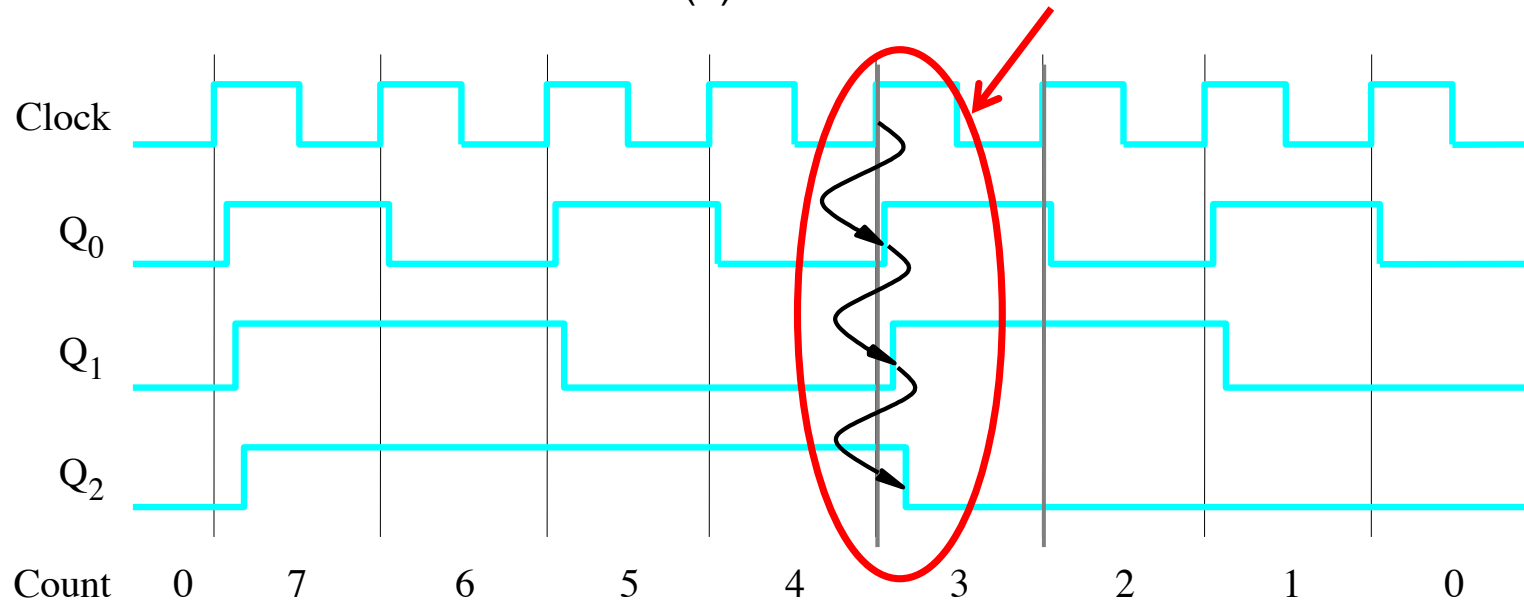
(b) Timing diagram

A three-bit down-counter



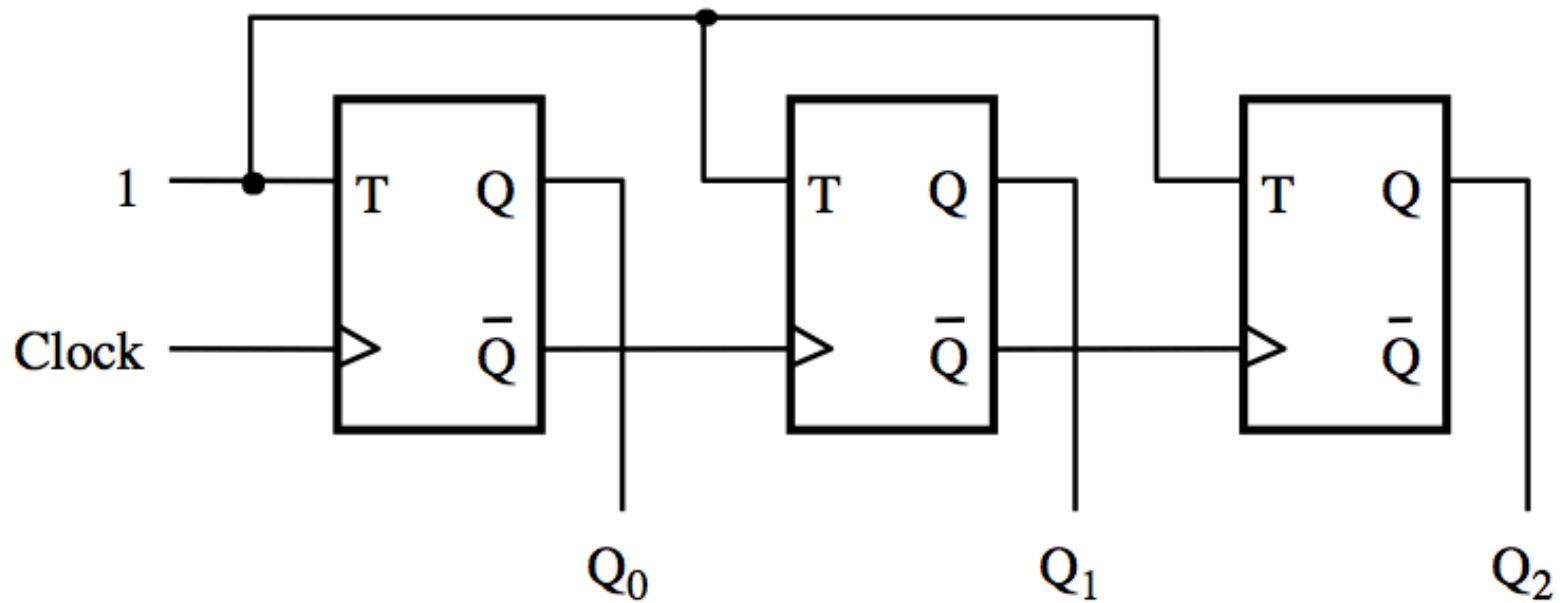
(a) Circuit

The propagation delays get longer

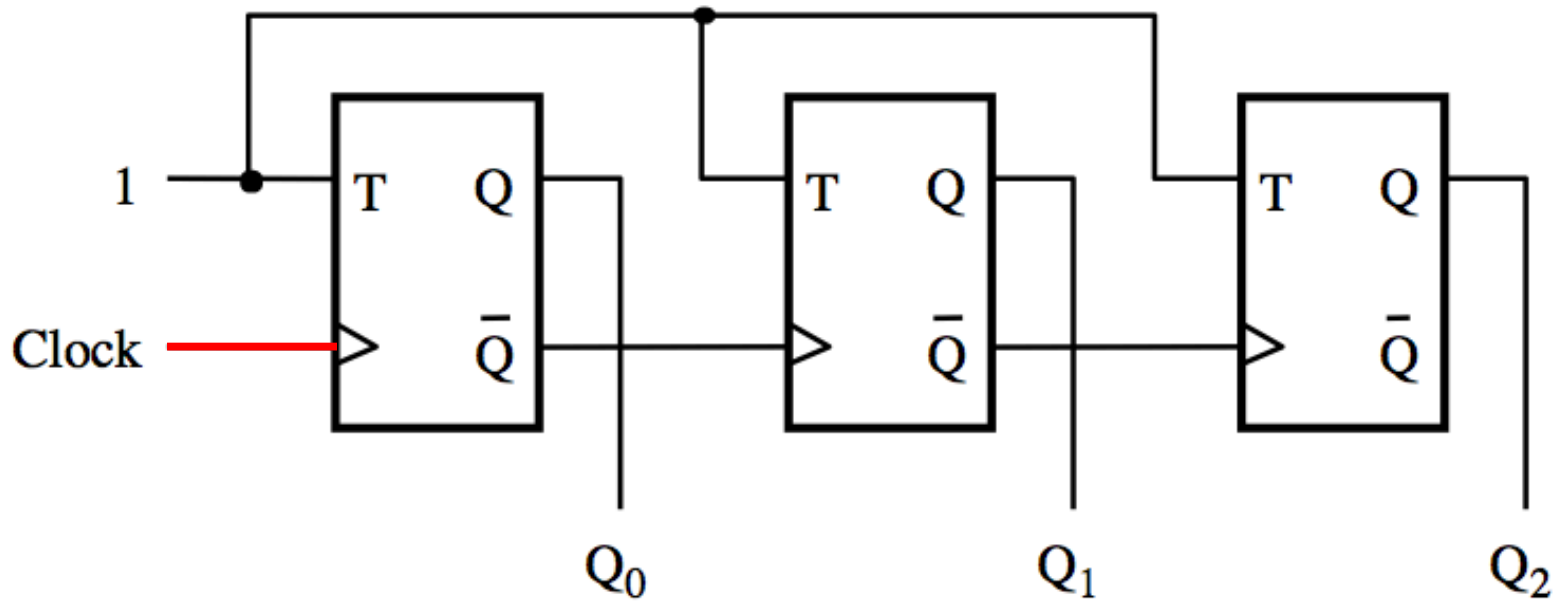


(b) Timing diagram

A three-bit up-counter

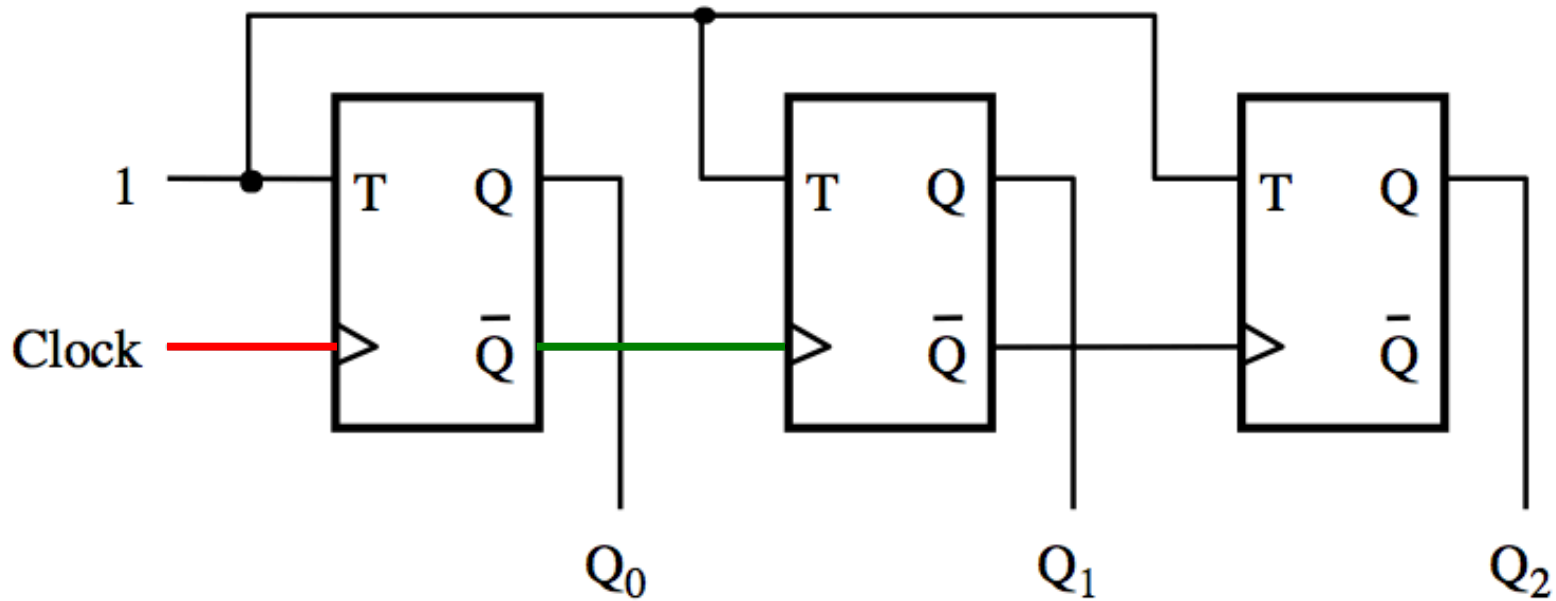


A three-bit up-counter



The first flip-flop changes
on the positive edge of the clock

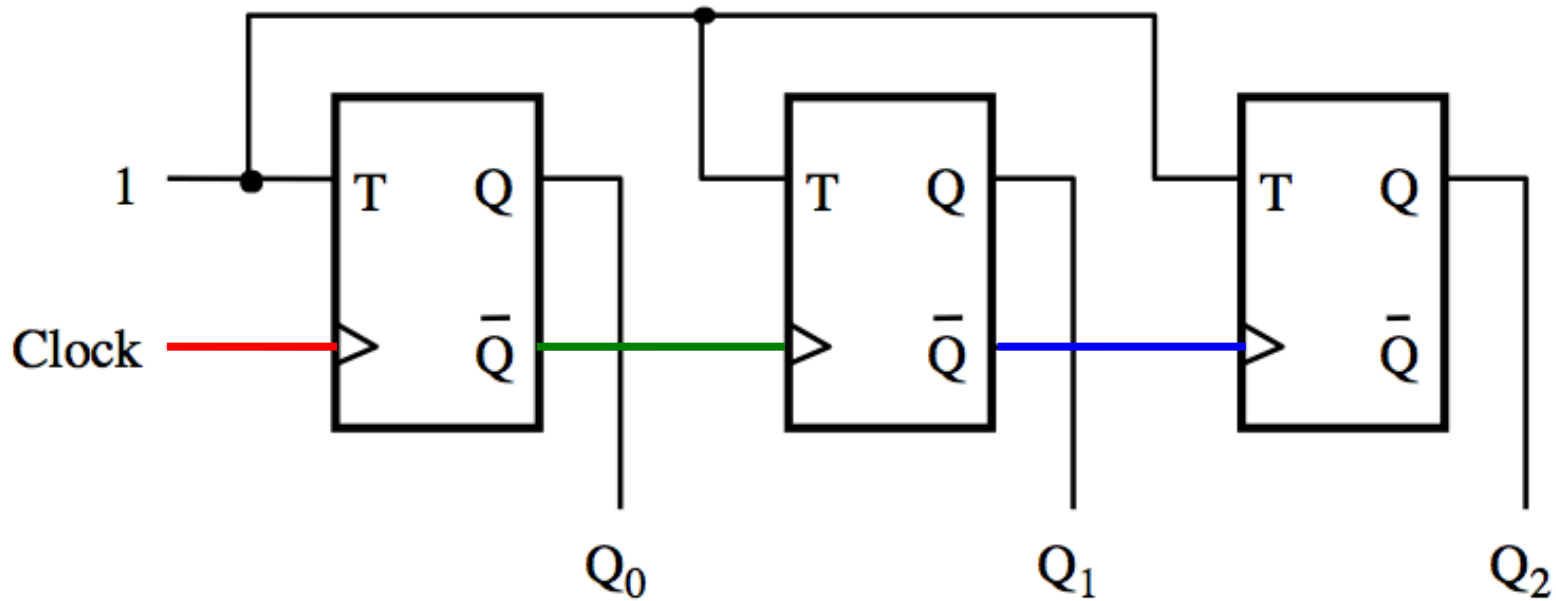
A three-bit up-counter



The first flip-flop changes
on the positive edge of the clock

The second flip-flop changes
on the positive edge of \bar{Q}_0

A three-bit up-counter

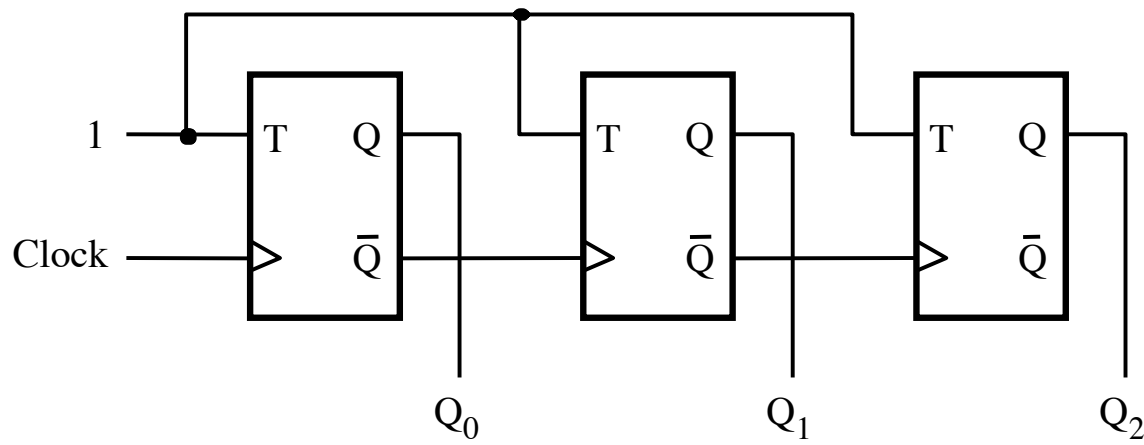


The first flip-flop changes on the positive edge of the clock

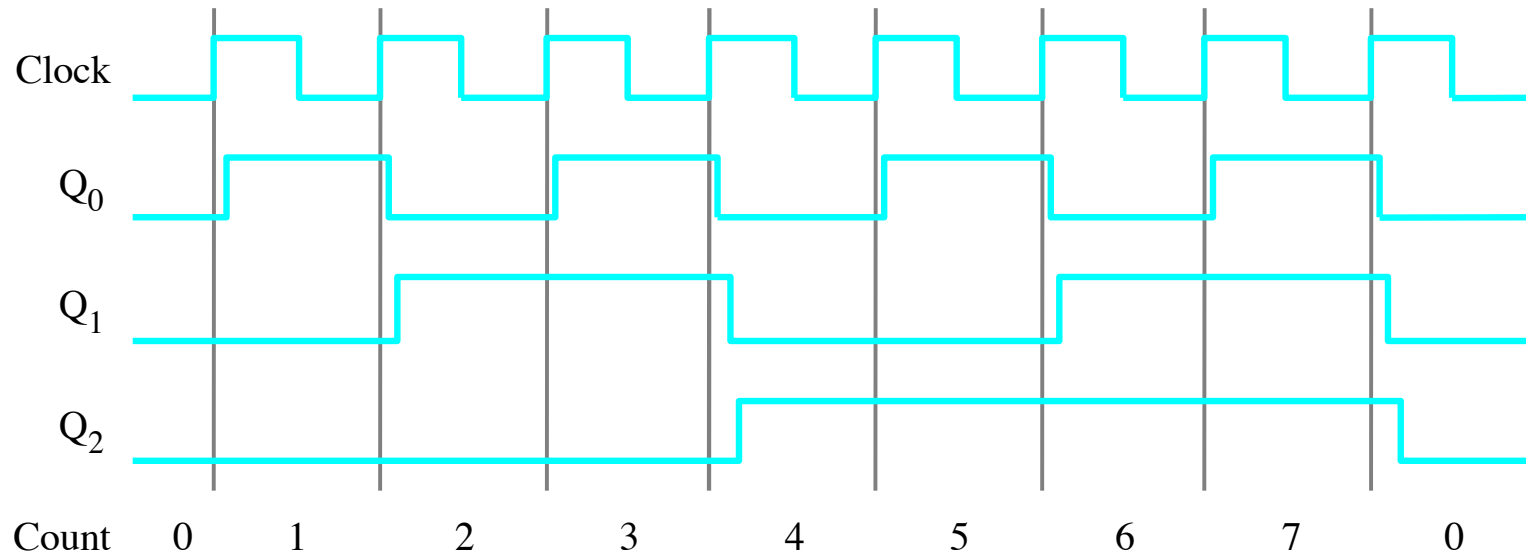
The second flip-flop changes on the positive edge of \bar{Q}_0

The third flip-flop changes on the positive edge of \bar{Q}_1

A three-bit up-counter

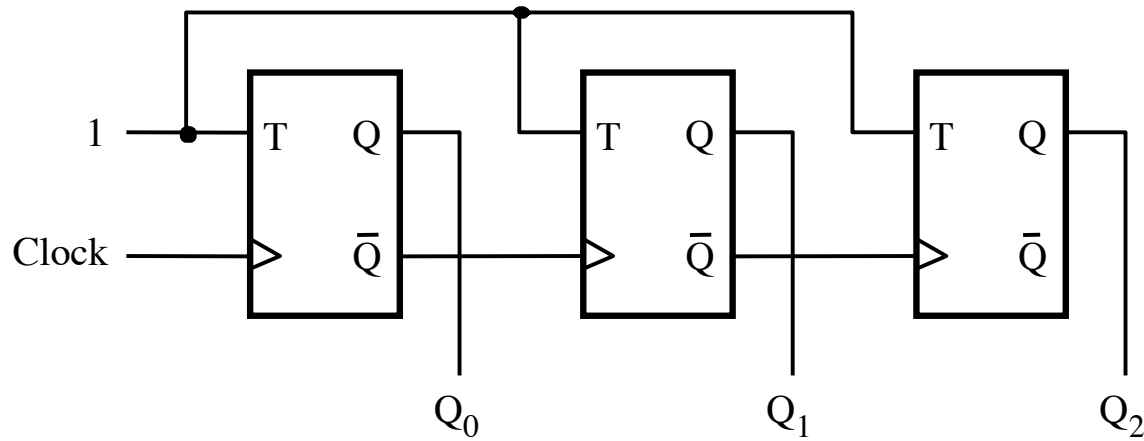


(a) Circuit

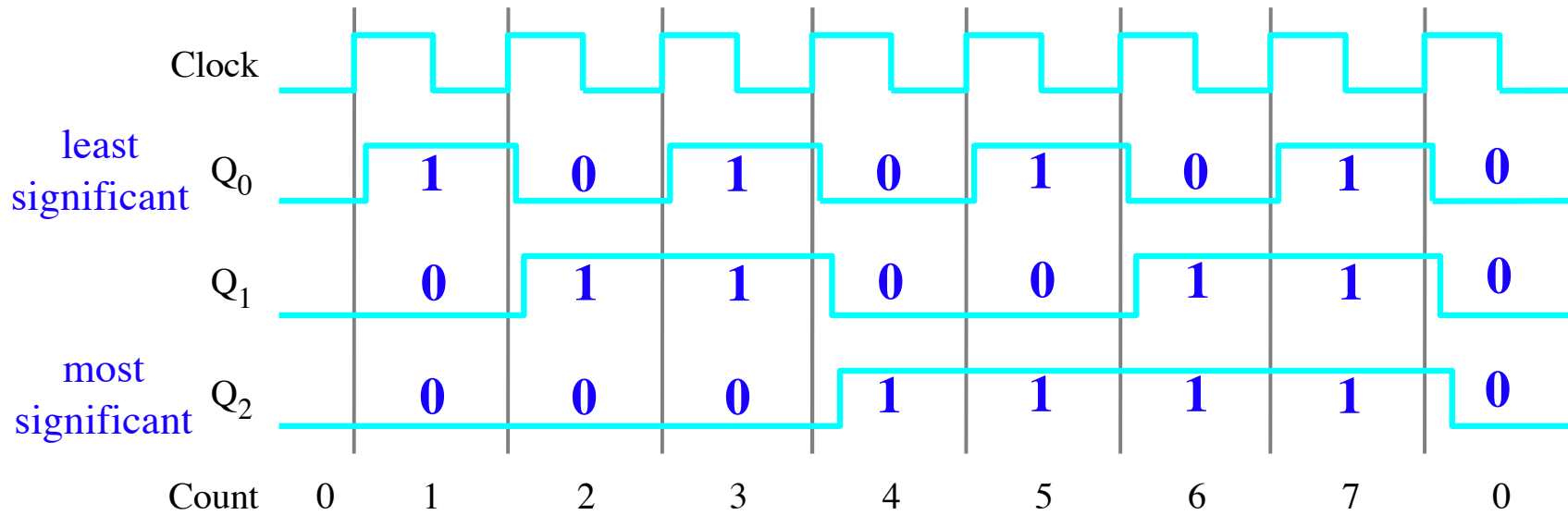


(b) Timing diagram

A three-bit up-counter

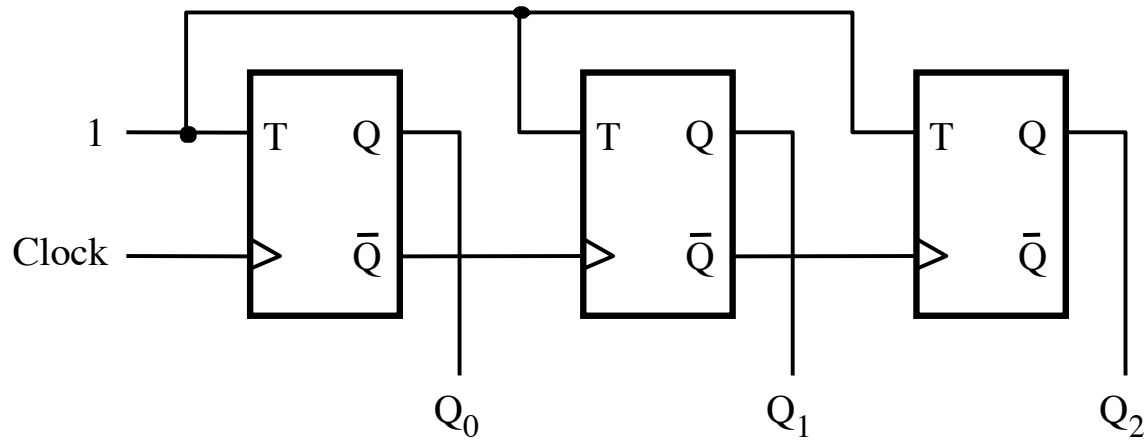


(a) Circuit

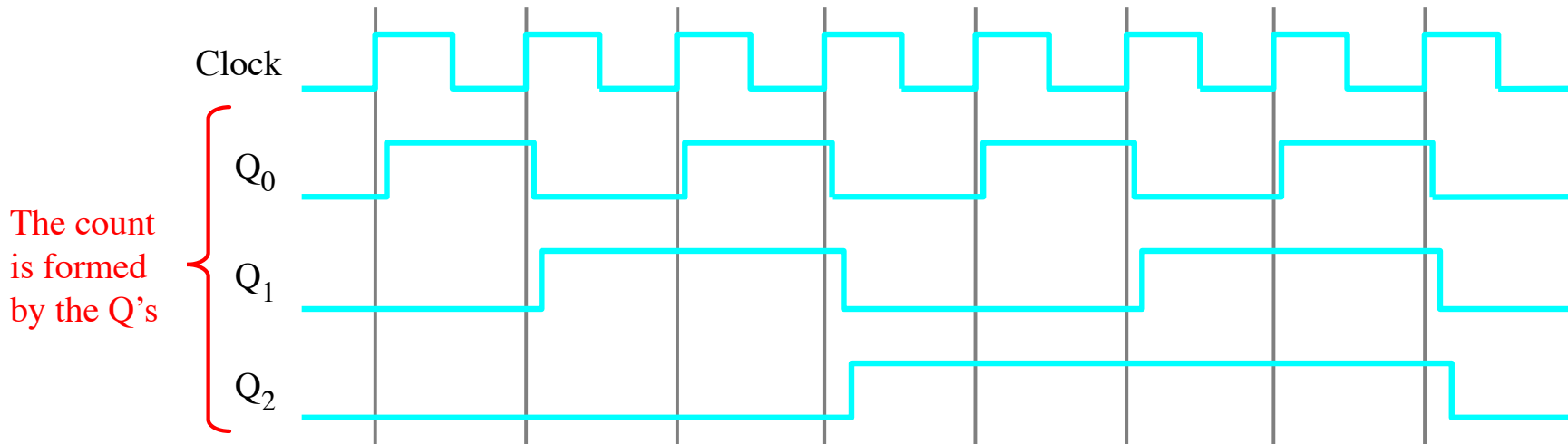


(b) Timing diagram

A three-bit up-counter

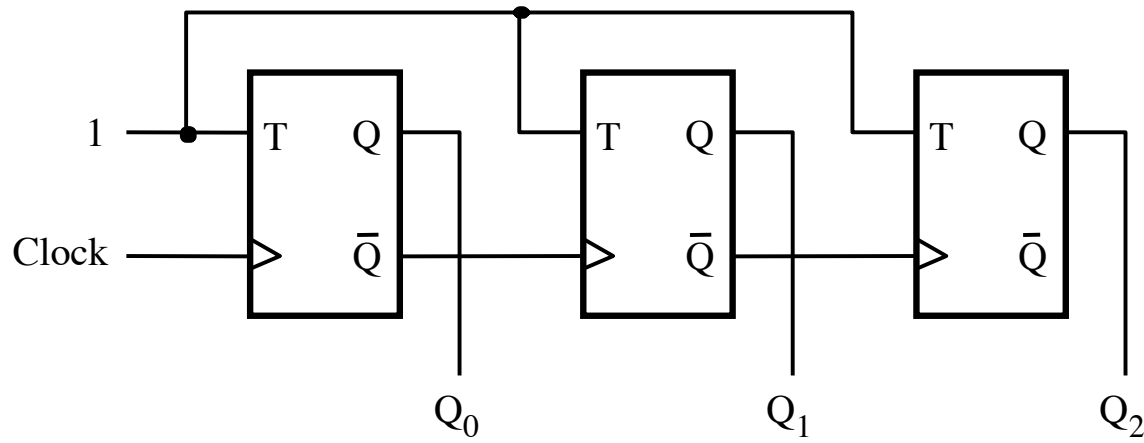


(a) Circuit

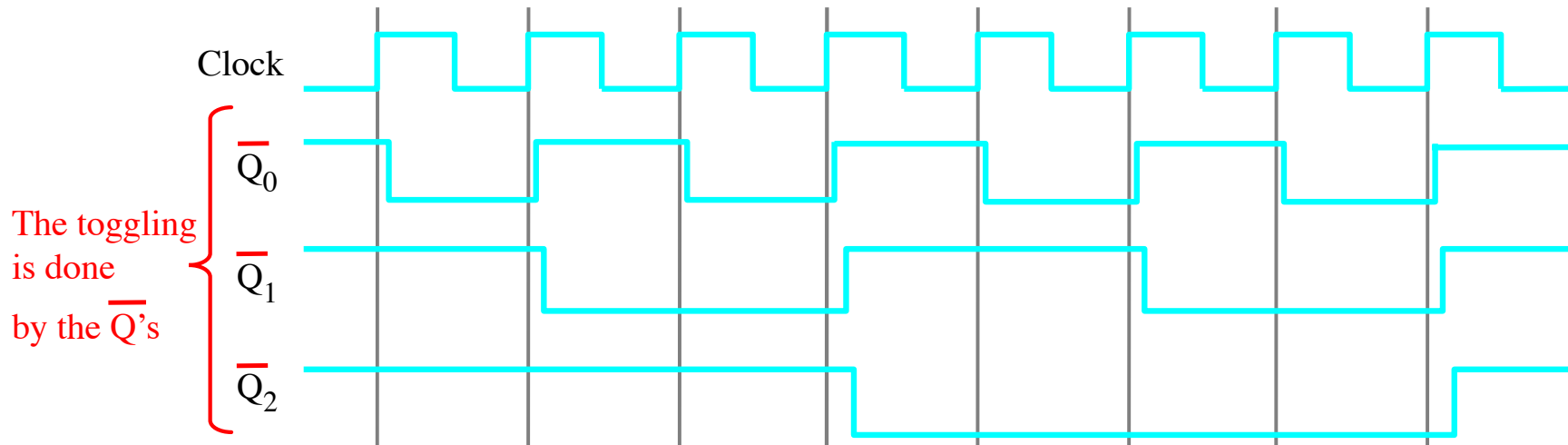


(b) Timing diagram

A three-bit up-counter

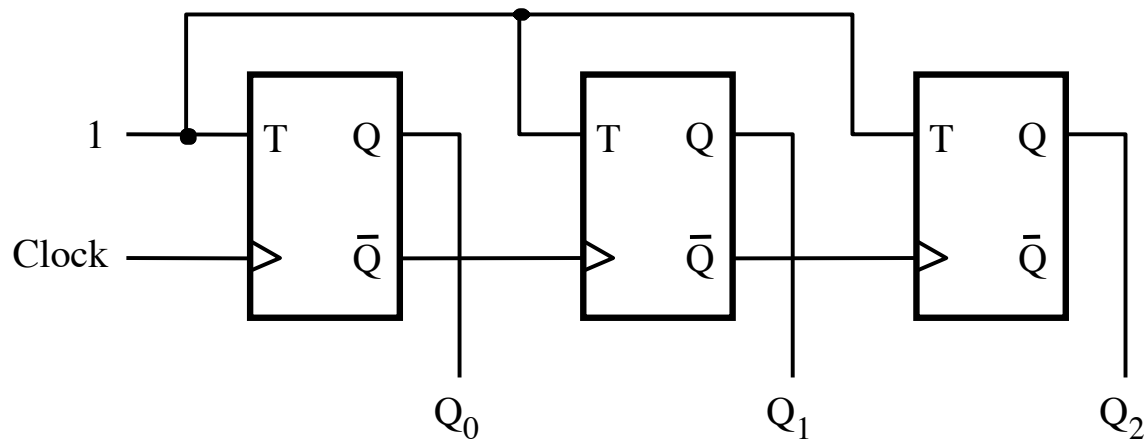


(a) Circuit

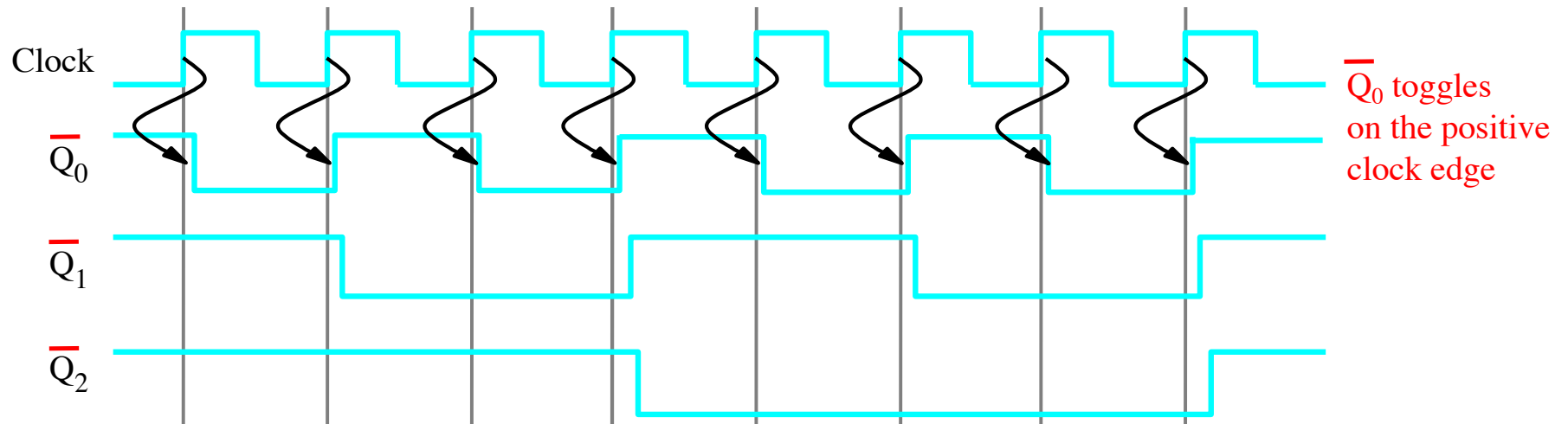


(b) Timing diagram

A three-bit up-counter

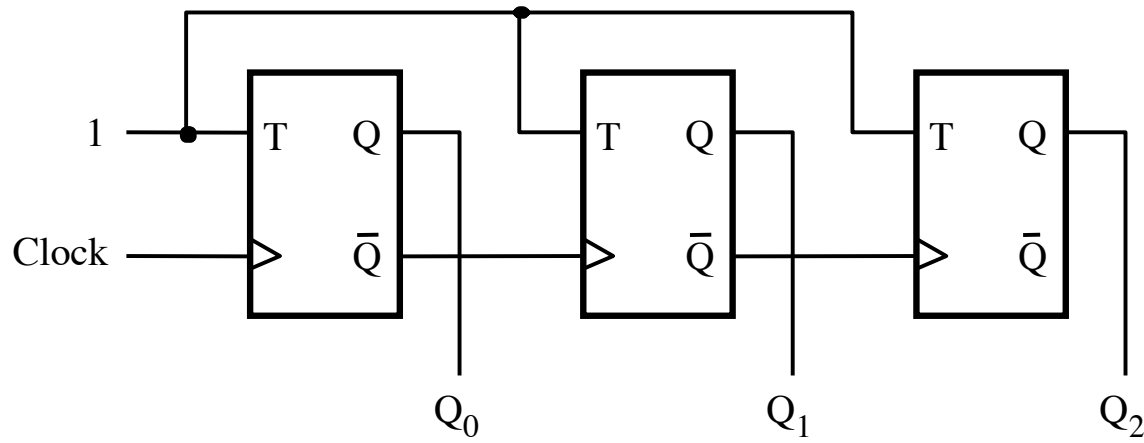


(a) Circuit

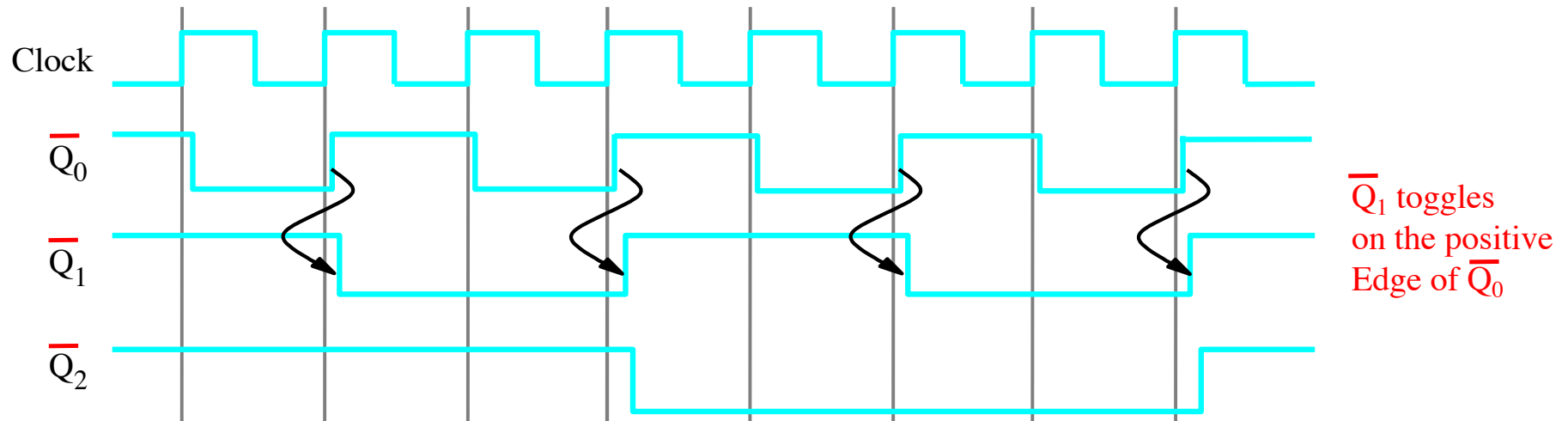


(b) Timing diagram

A three-bit up-counter

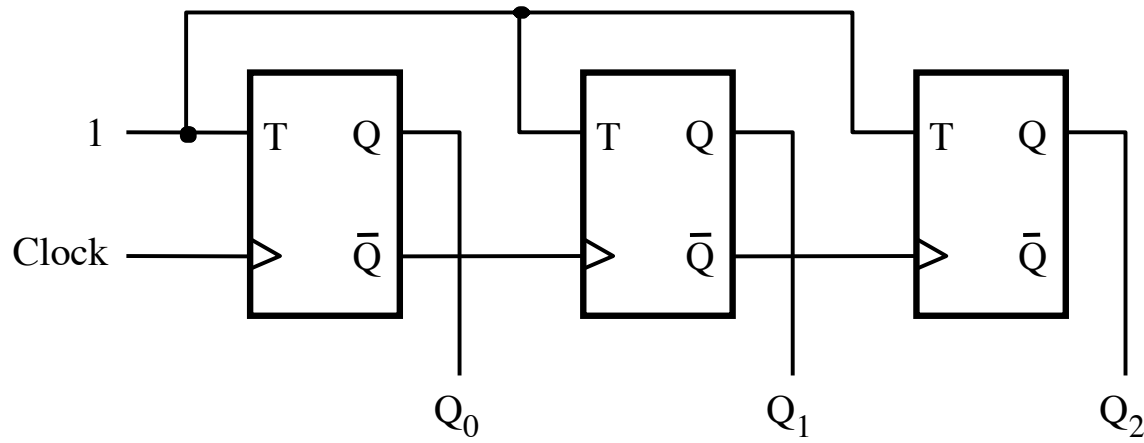


(a) Circuit

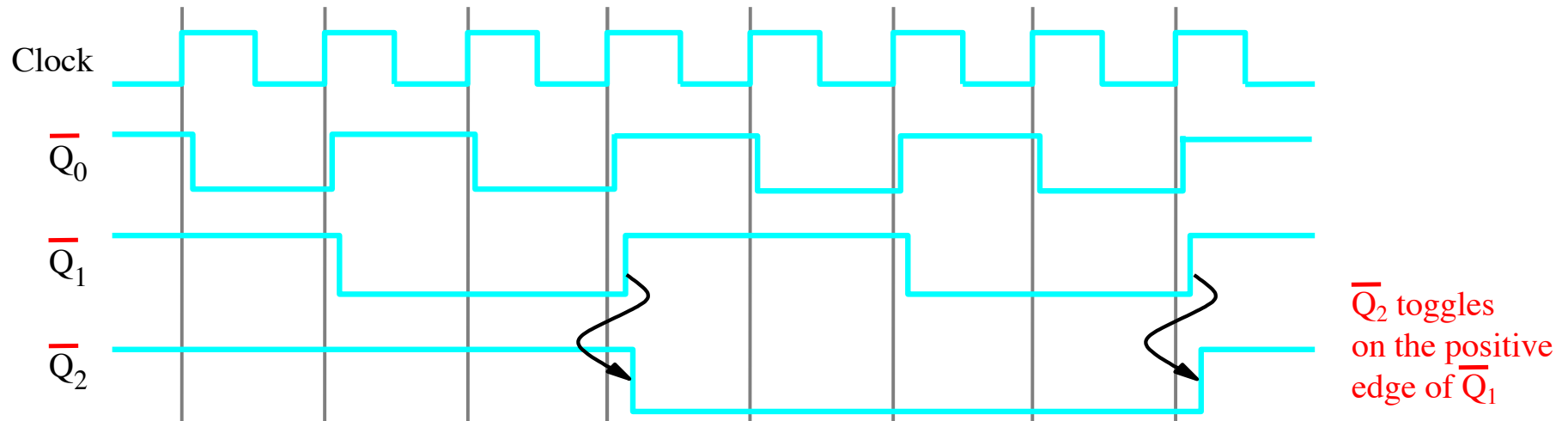


(b) Timing diagram

A three-bit up-counter

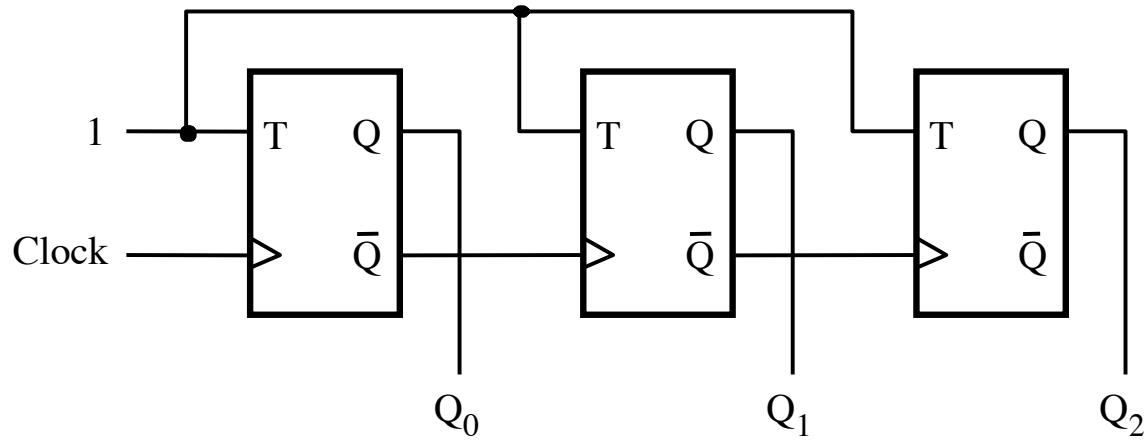


(a) Circuit

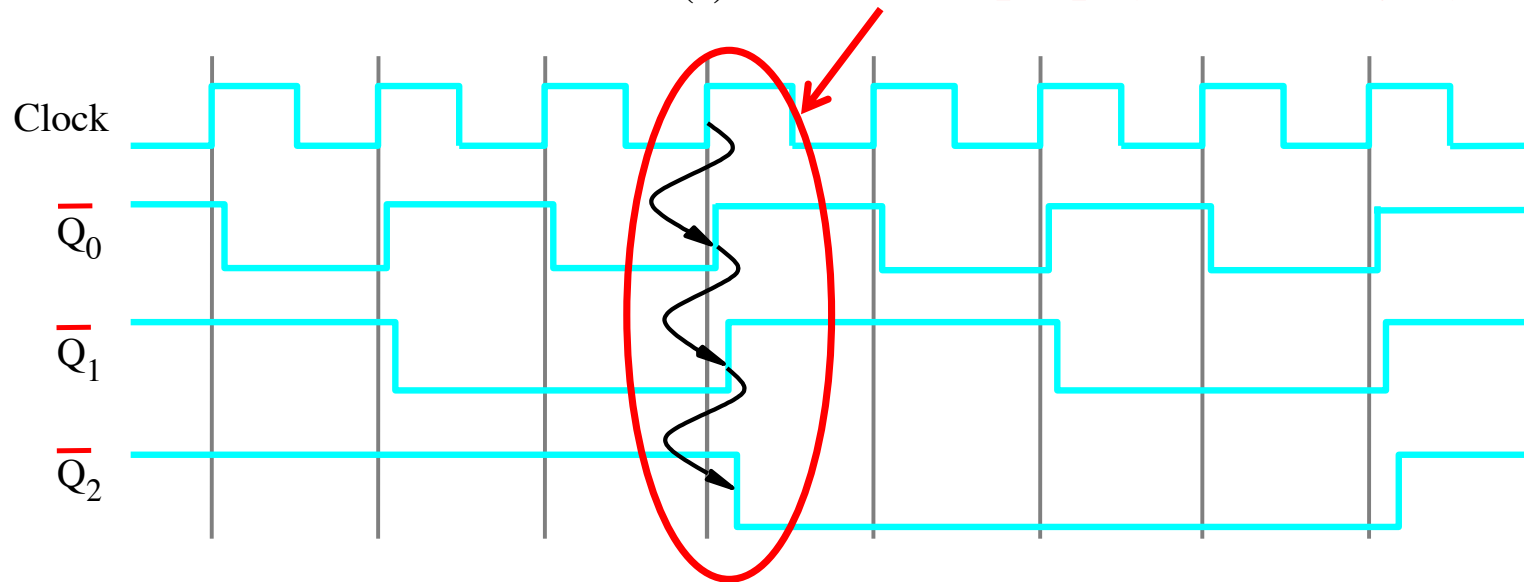


(b) Timing diagram

A three-bit up-counter

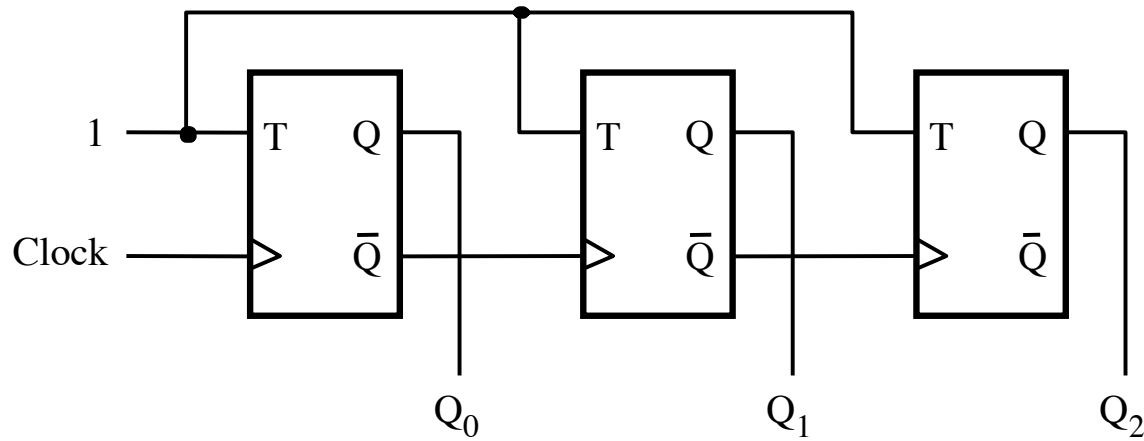


(a) Circuit **The propagation delays get longer**

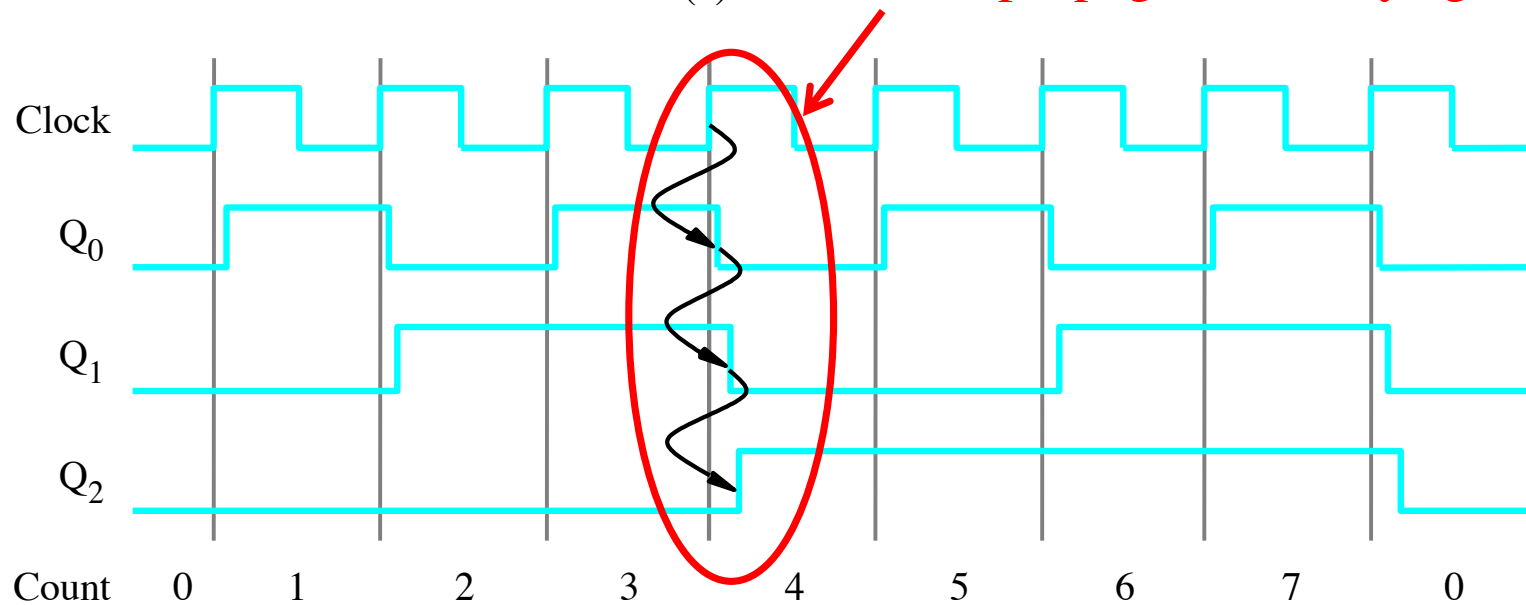


(b) Timing diagram

A three-bit up-counter



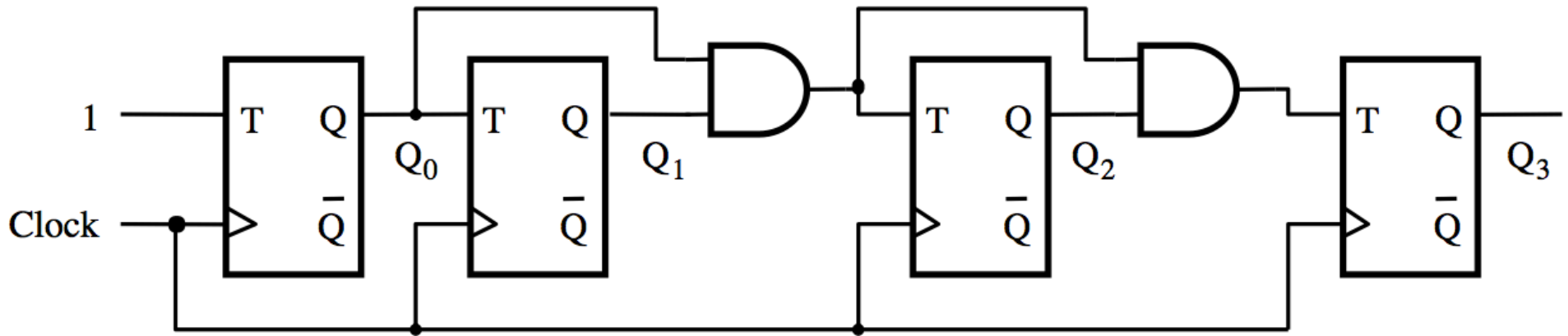
(a) Circuit **The propagation delays get longer**



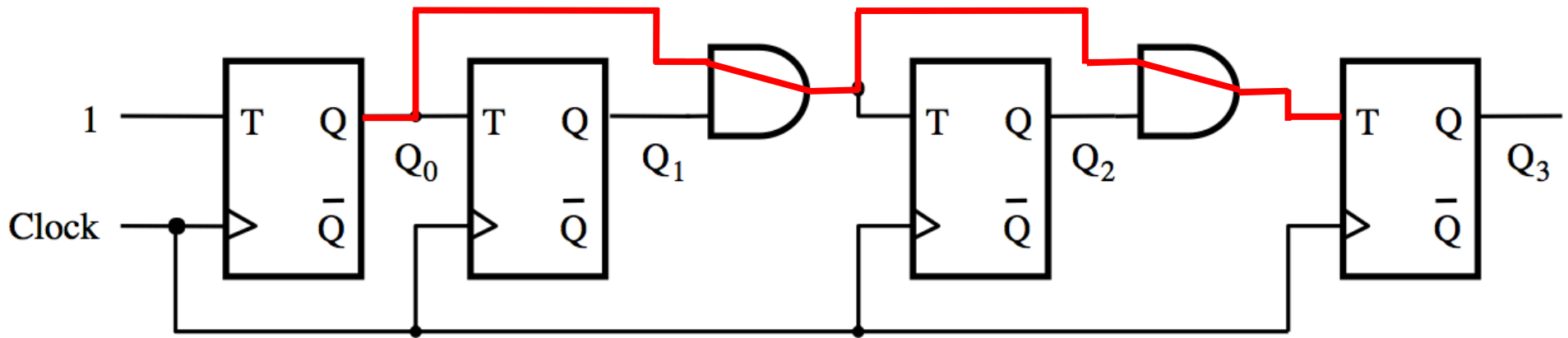
(b) Timing diagram

Synchronous Counters

A four-bit synchronous up-counter

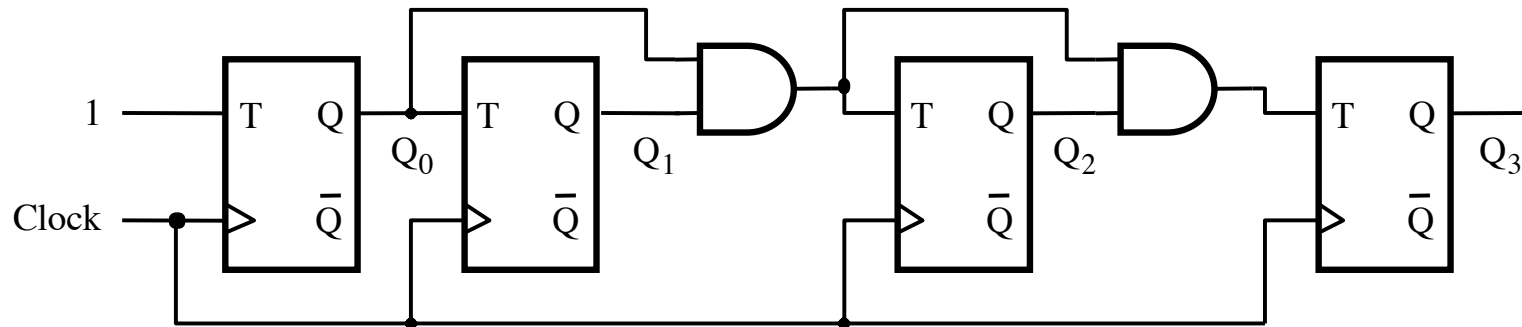


A four-bit synchronous up-counter

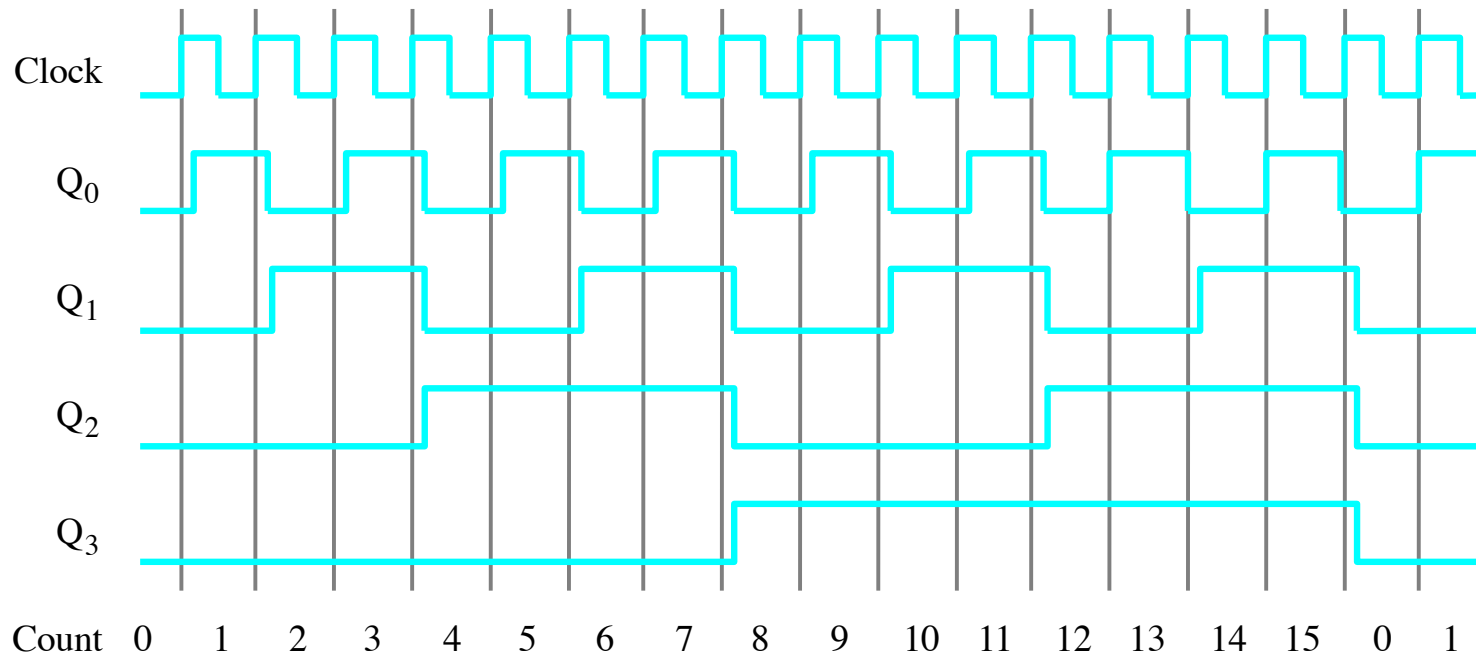


The propagation delay through all AND gates combined must not exceed the clock period minus the setup time for the flip-flops

A four-bit synchronous up-counter



(a) Circuit



(b) Timing diagram

Derivation of the synchronous up-counter

Clock cycle	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0

Q₁ changes

Q₂ changes

Derivation of the synchronous up-counter

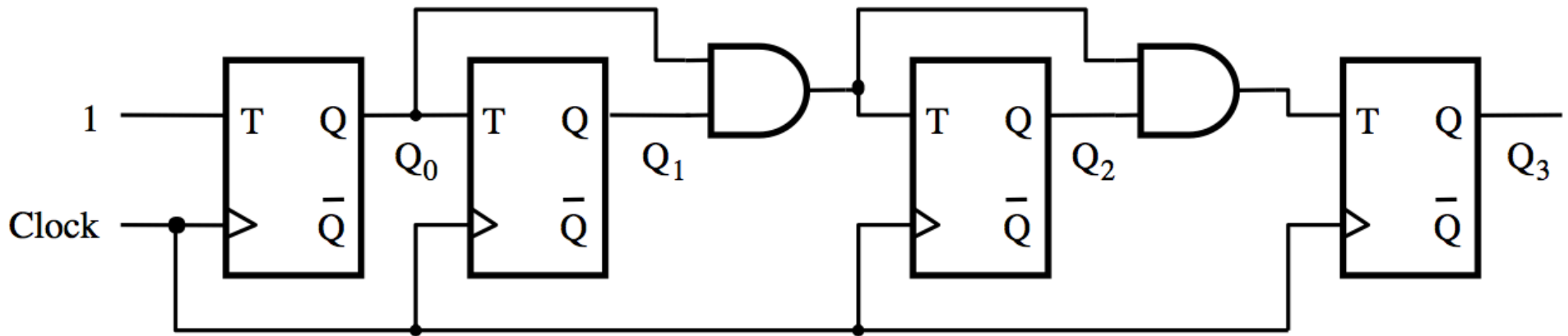
Clock cycle	Q ₂	Q ₁	Q ₀
0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1
8	0	0	0

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

A four-bit synchronous up-counter



$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

In general we have

$$T_0 = 1$$

$$T_1 = Q_0$$

$$T_2 = Q_0 Q_1$$

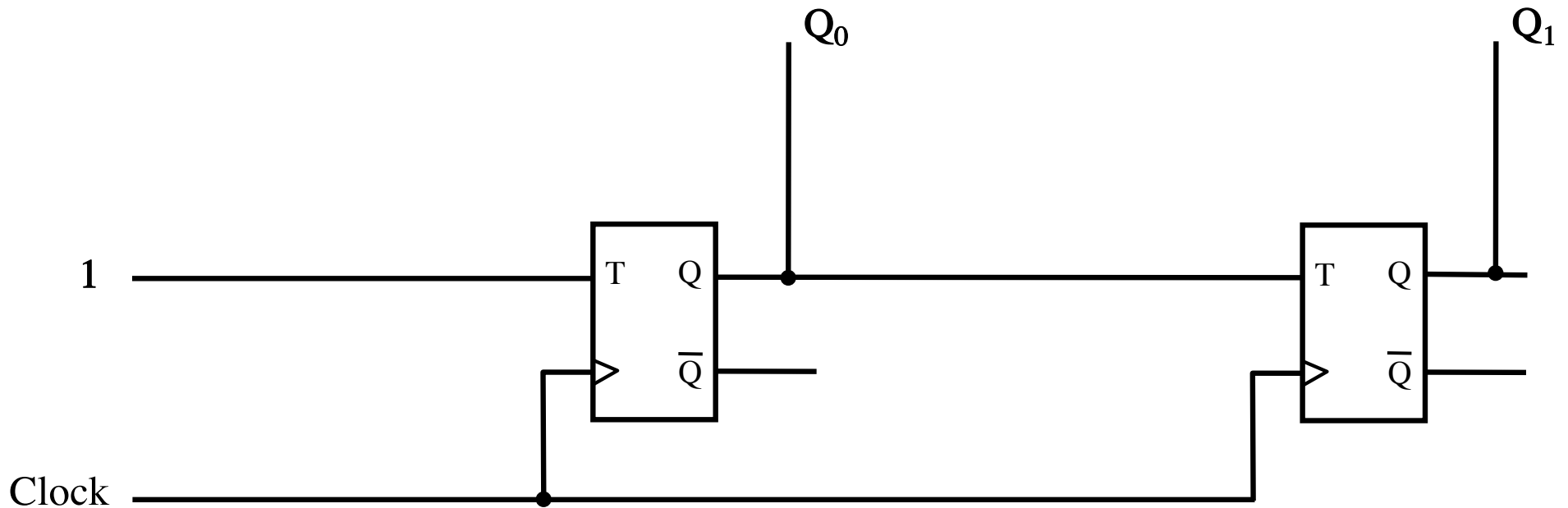
$$T_3 = Q_0 Q_1 Q_2$$

...

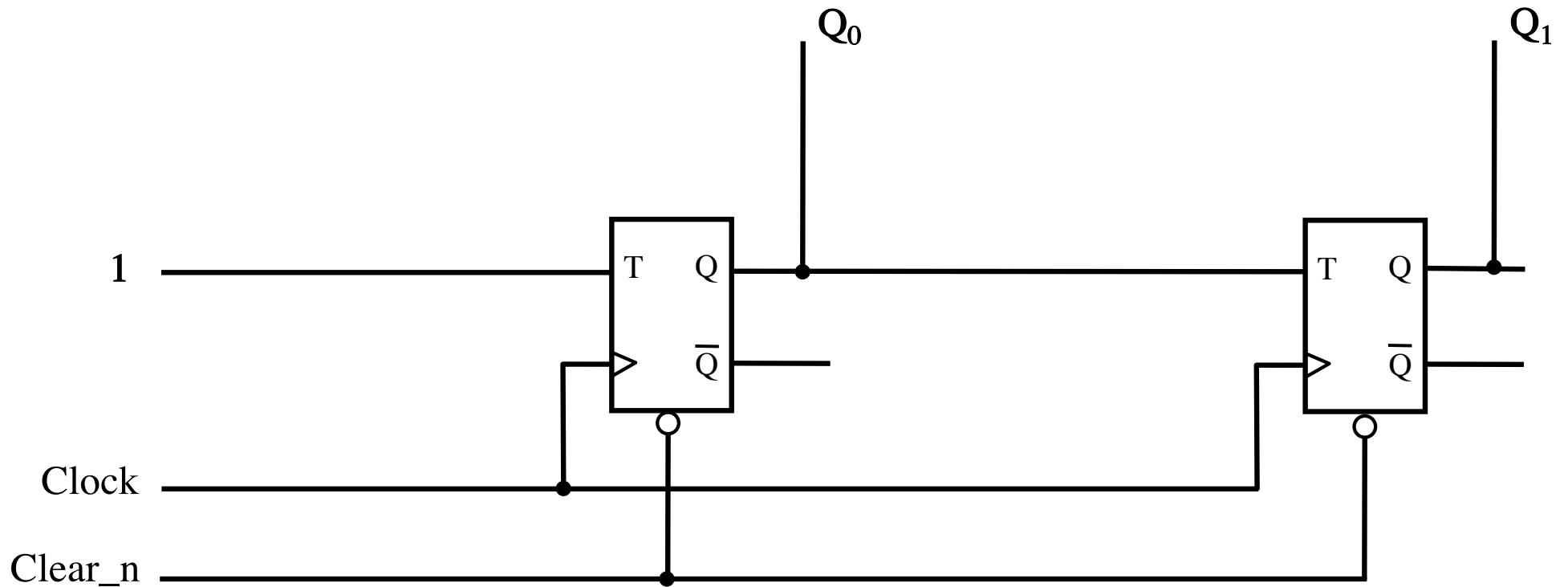
$$T_n = Q_0 Q_1 Q_2 \cdots Q_{n-1}$$

Synchronous v.s. Asynchronous Clear

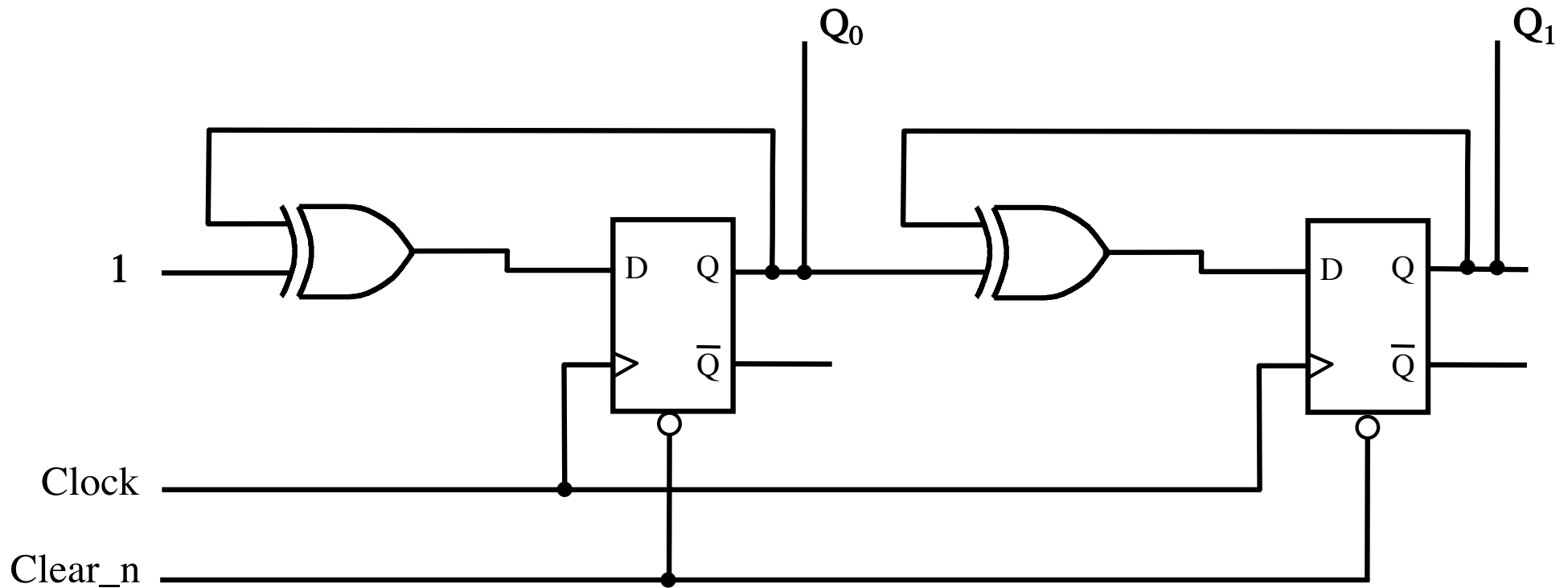
2-Bit Synchronous Up-Counter (without clear capability)



2-Bit Synchronous Up-Counter (with asynchronous clear)

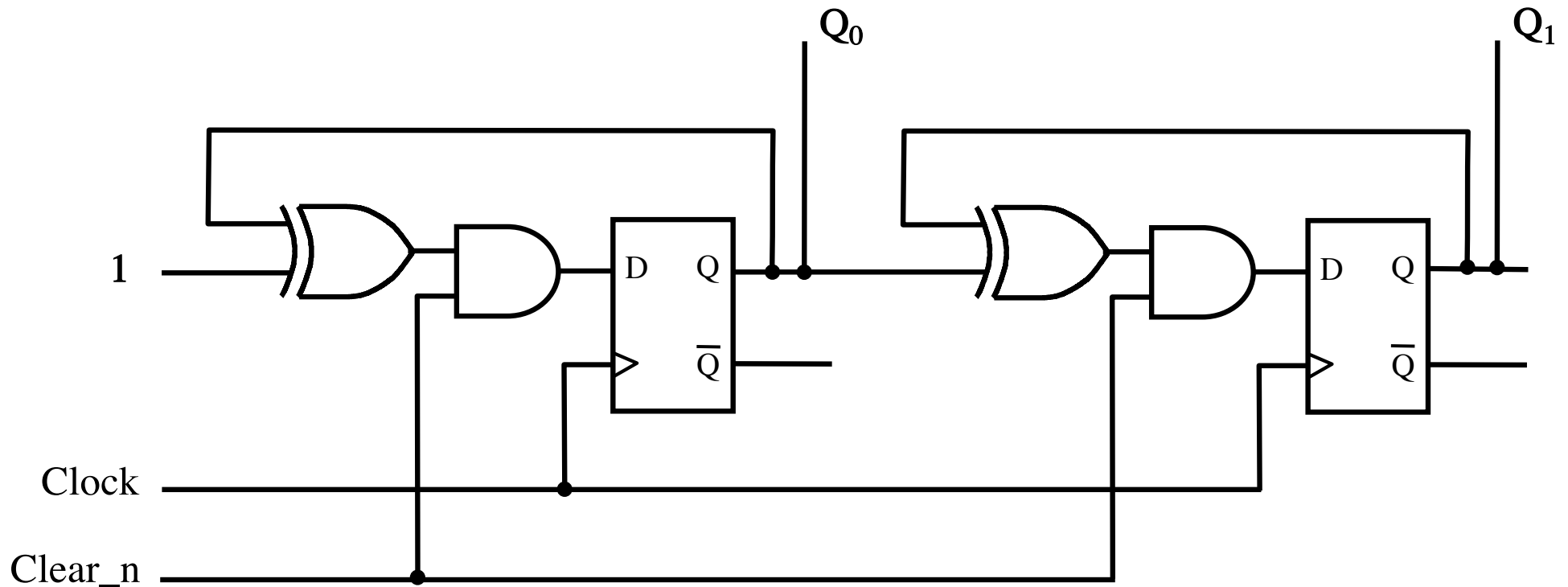


2-Bit Synchronous Up-Counter (with asynchronous clear)



This is the same circuit but uses D Flip-Flops.

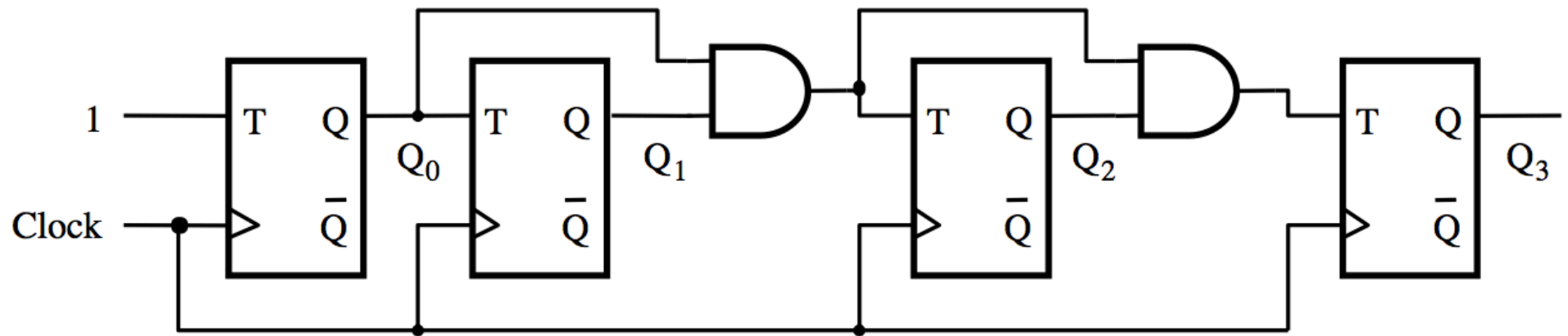
2-Bit Synchronous Up-Counter (with synchronous clear)



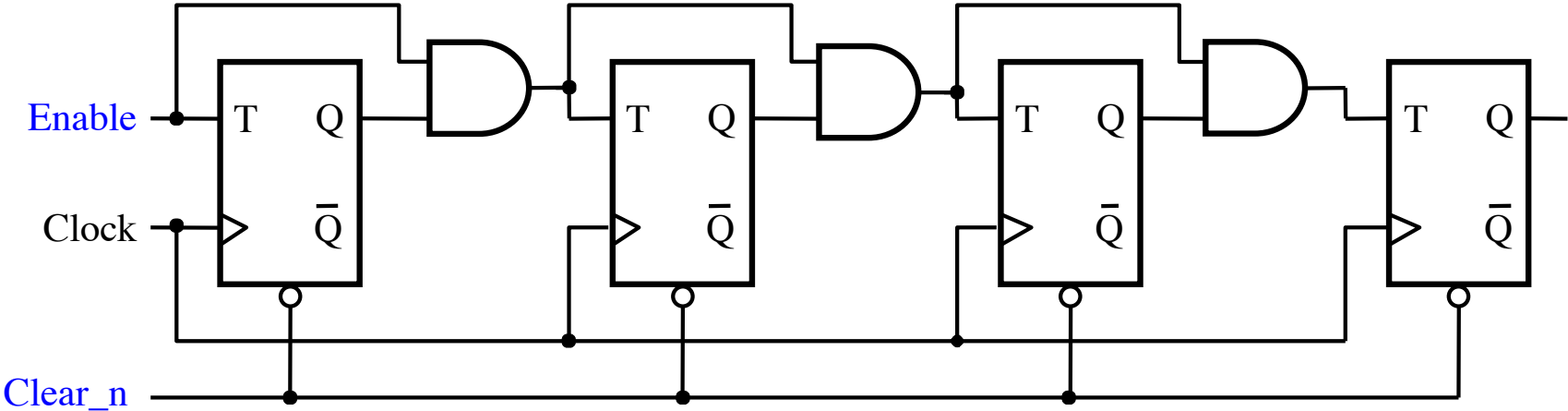
This counter can be cleared only on the positive clock edge.

Adding Enable Capability

A four-bit synchronous up-counter



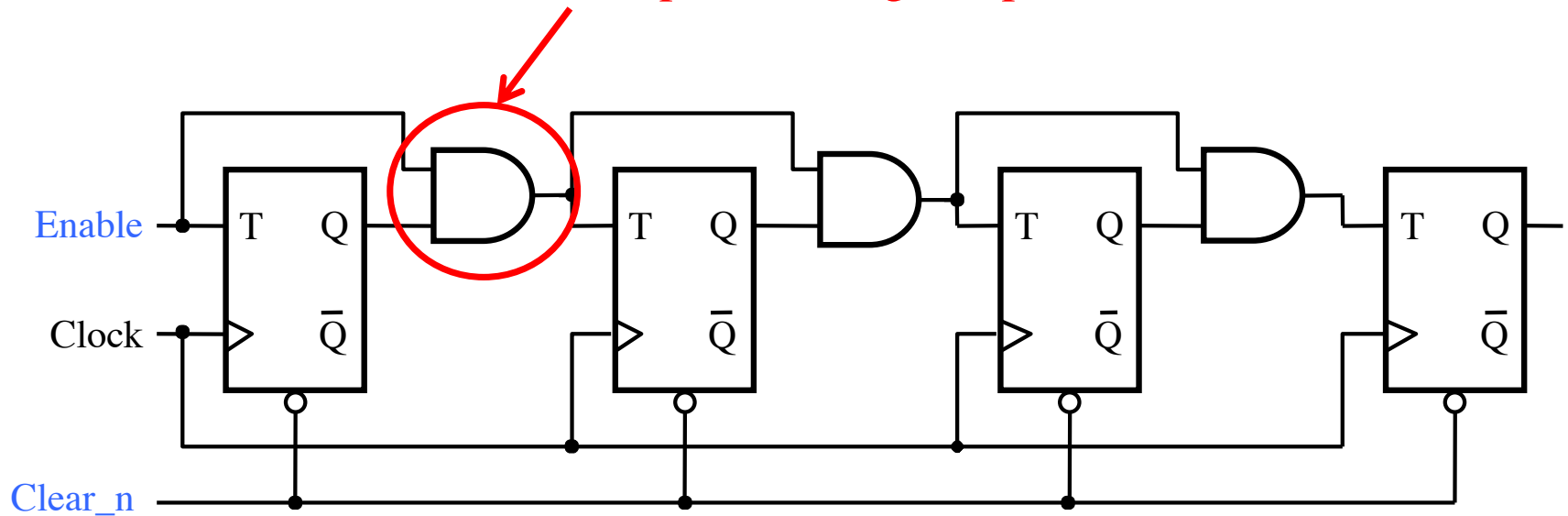
Inclusion of Enable and Clear Capability



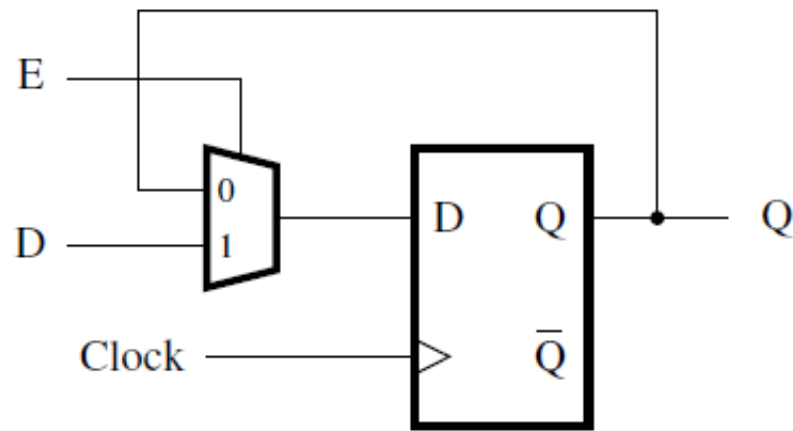
[Figure 5.22 from the textbook]

Inclusion of Enable and Clear Capability

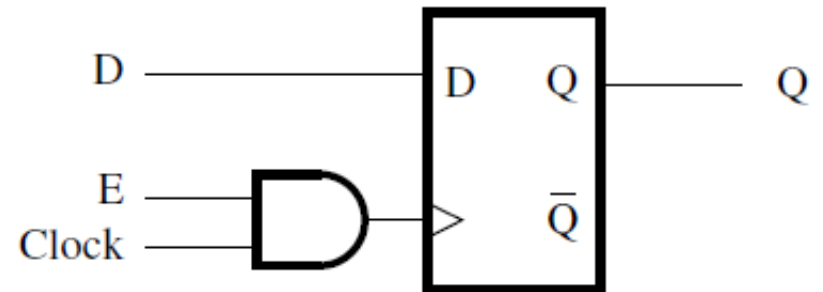
This is the new thing relative to the previous figure, plus the clear_n line



Providing an enable input for a D flip-flop



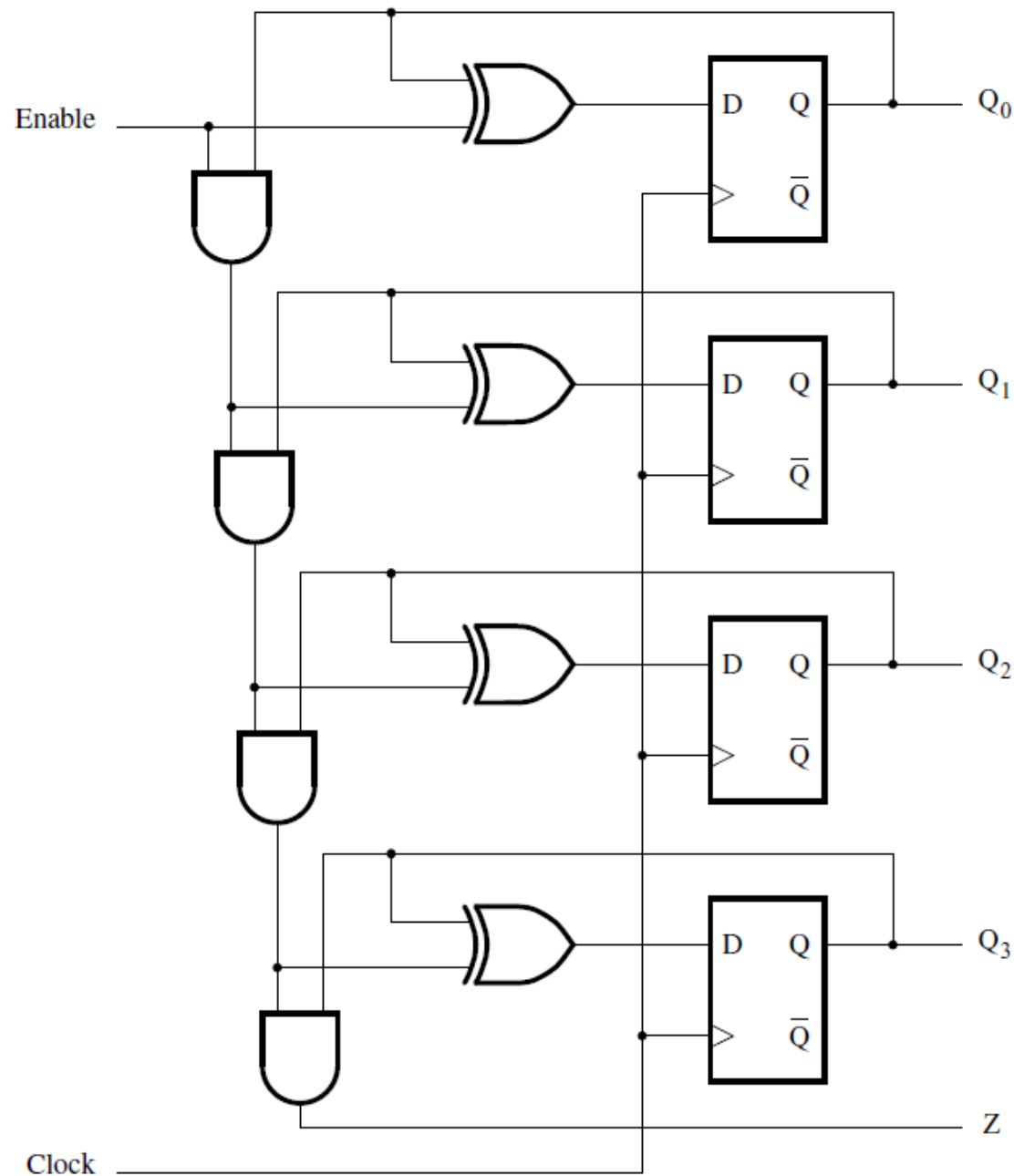
(a) Using a multiplexer



(b) Clock gating

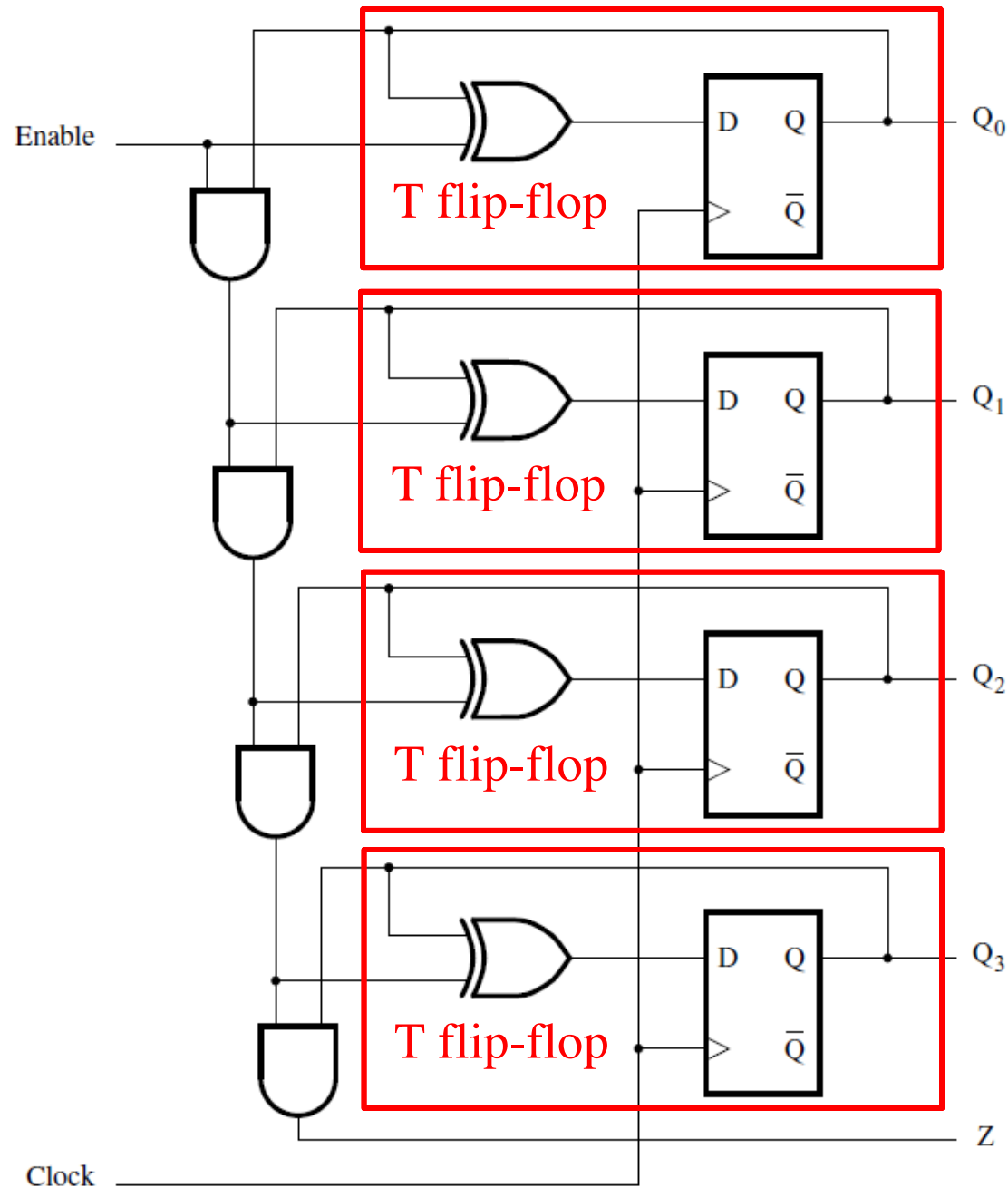
Synchronous Counter (with D Flip-Flops)

A 4-bit up-counter with D flip-flops



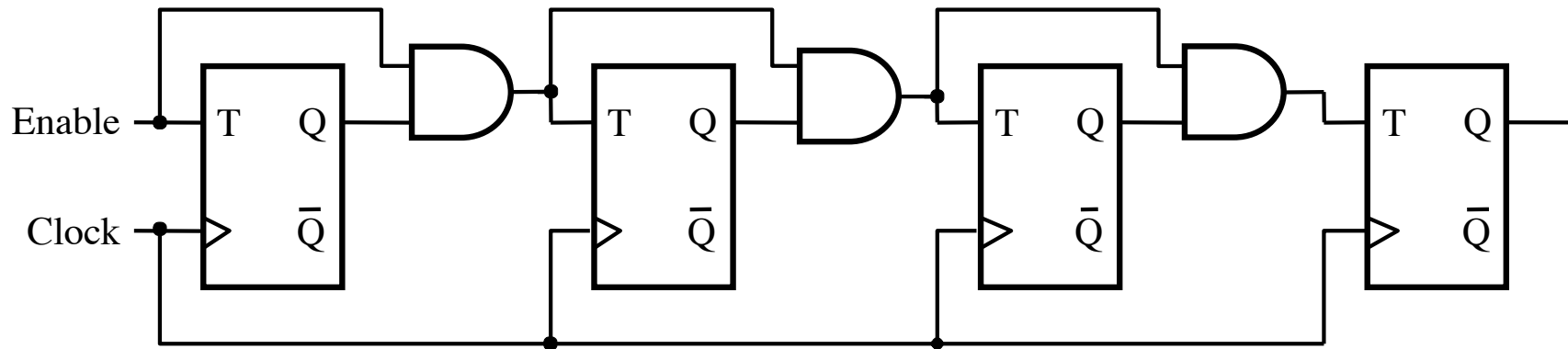
[Figure 5.23 from the textbook]

A 4-bit up-counter with D flip-flops

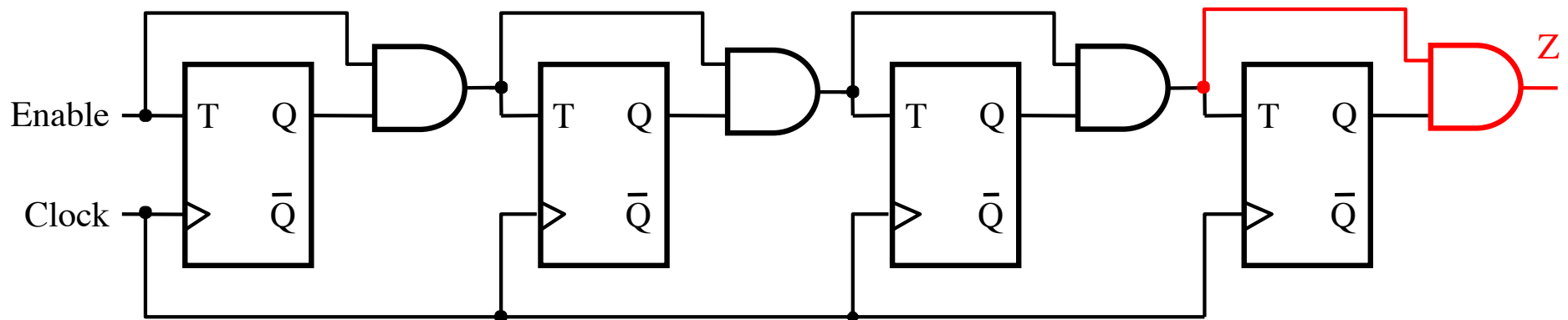


[Figure 5.23 from the textbook]

Equivalent to this circuit with T flip-flops



Equivalent to this circuit with T flip-flops



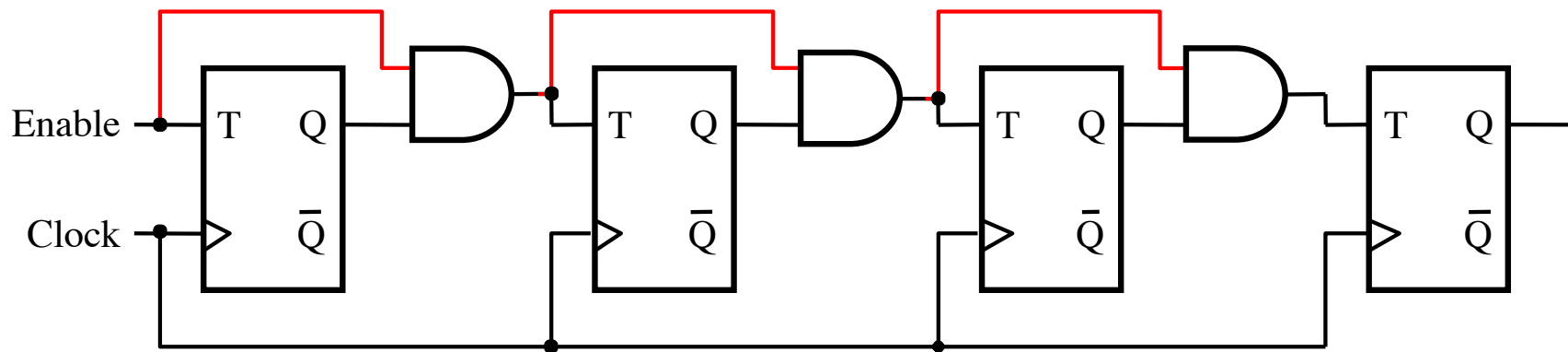
But has one extra output called Z, which can be used to connect two 4-bit counters to make an 8-bit counter.

When $Z=1$ the counter will go 0000 on the next clock edge, i.e., the outputs of all flip-flops are currently 1 (maximum count value).

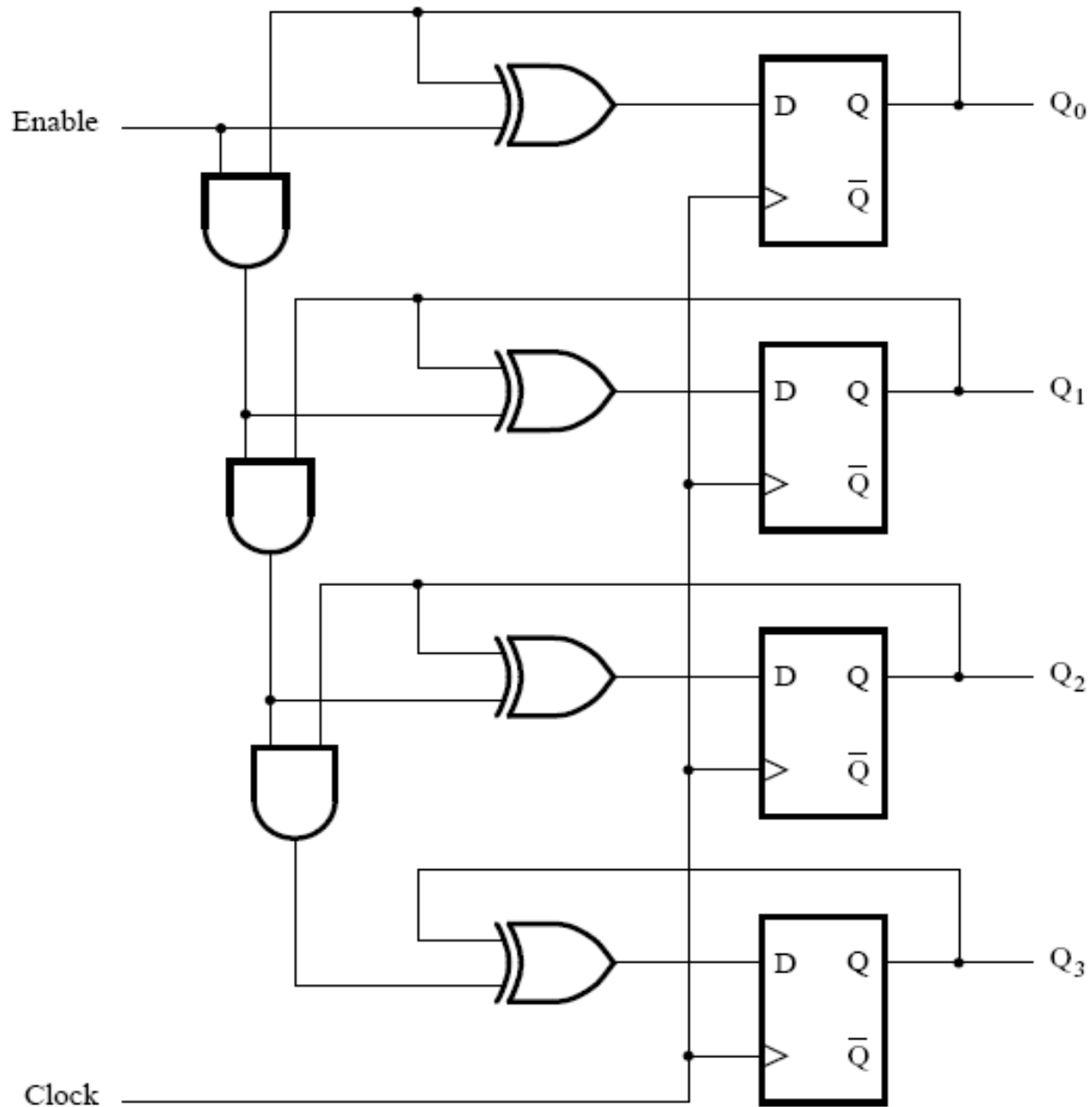
Faster 4-bit Counter

- **Want to increase the speed of the 4-bit counter?**
- **Use a similar method as the one used in 4-bit adder.**
- **Replace the series of 2-input AND gates with AND gates with more input lines.**

Is it possible to reduce this delay?

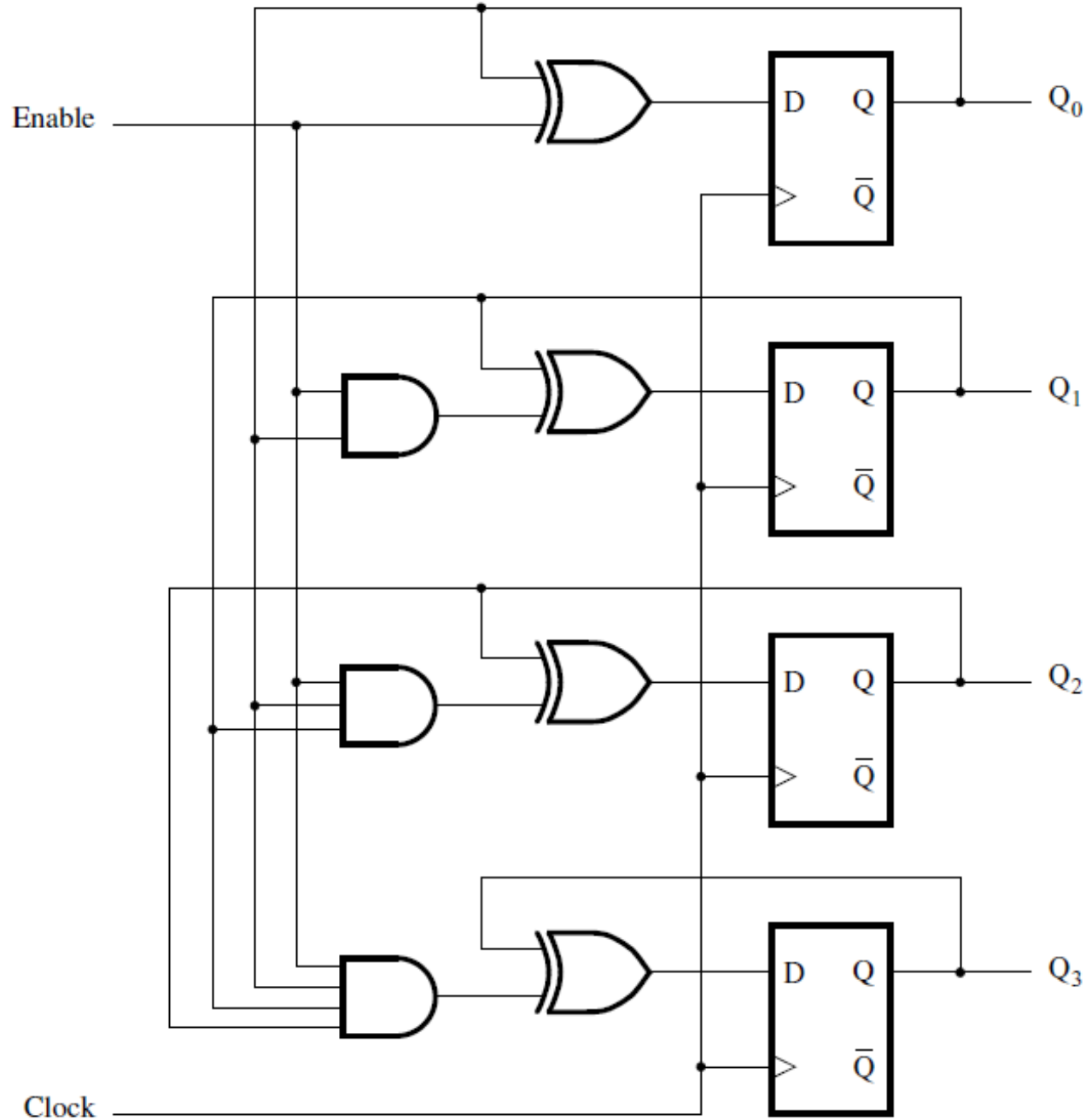


Equivalent 4-bit counter



[Figure 5.67 from the textbook]

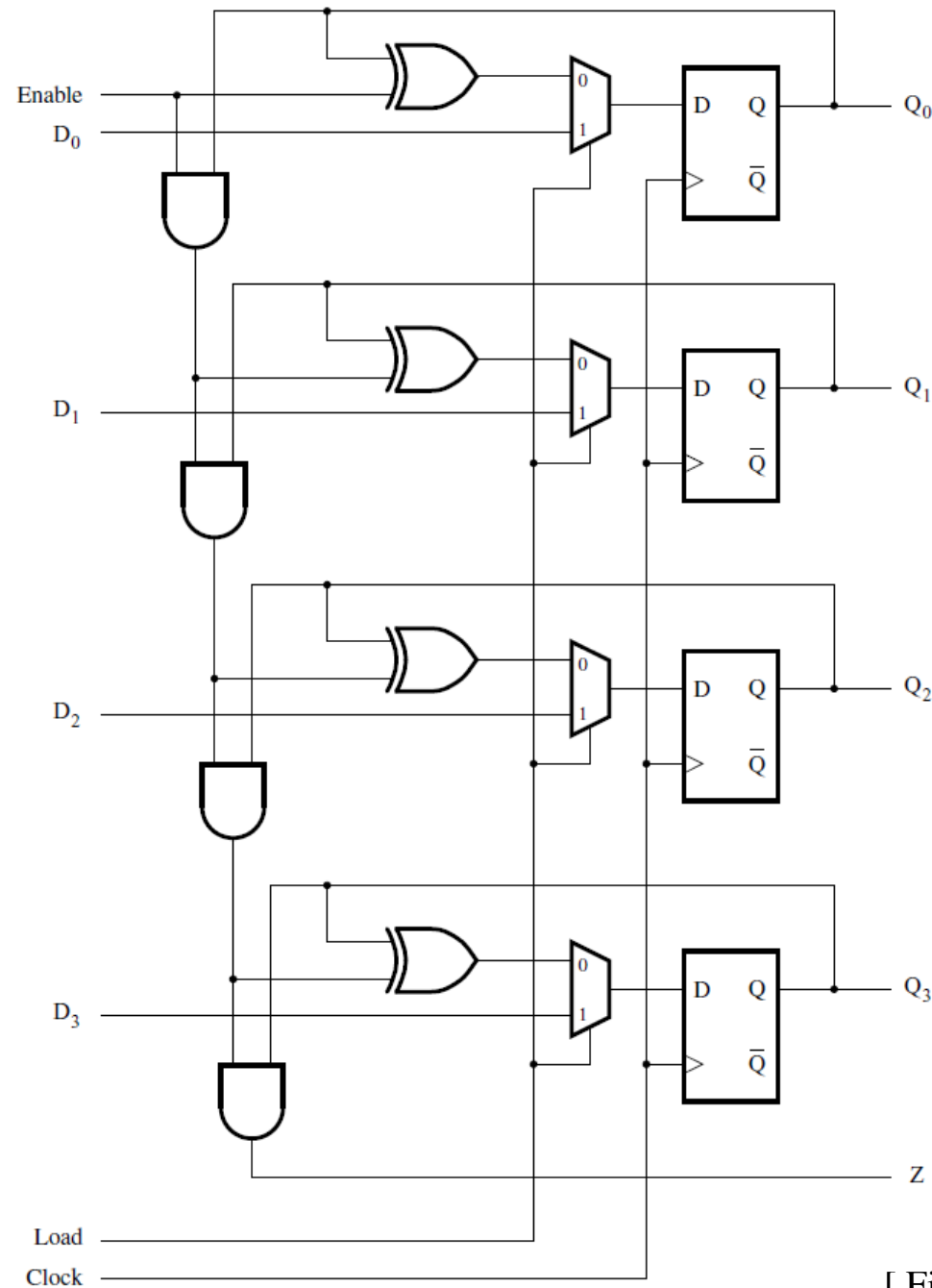
A faster 4-bit counter



[Figure 5.75 from the textbook]

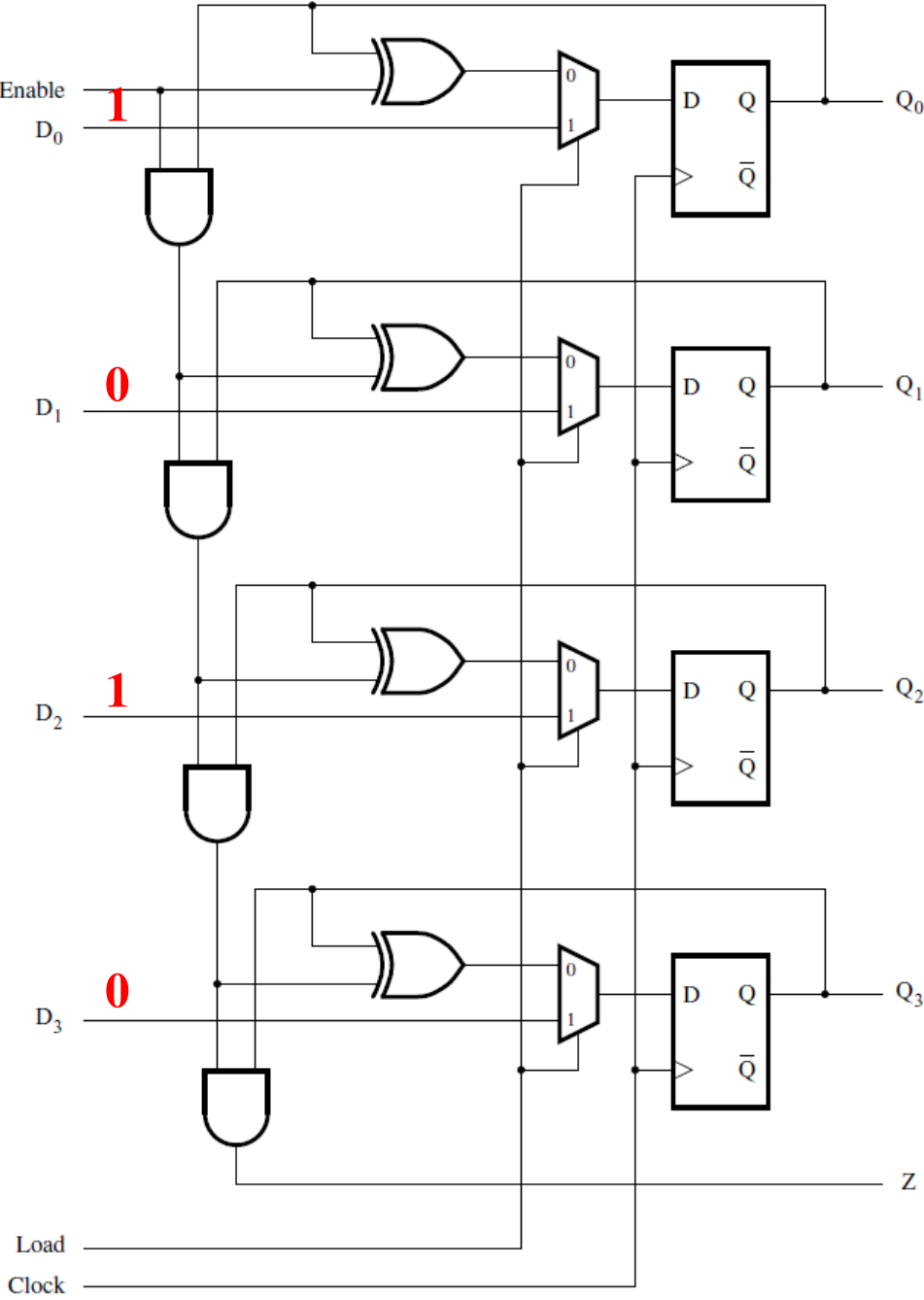
Counters with Parallel Load

A counter with parallel-load capability



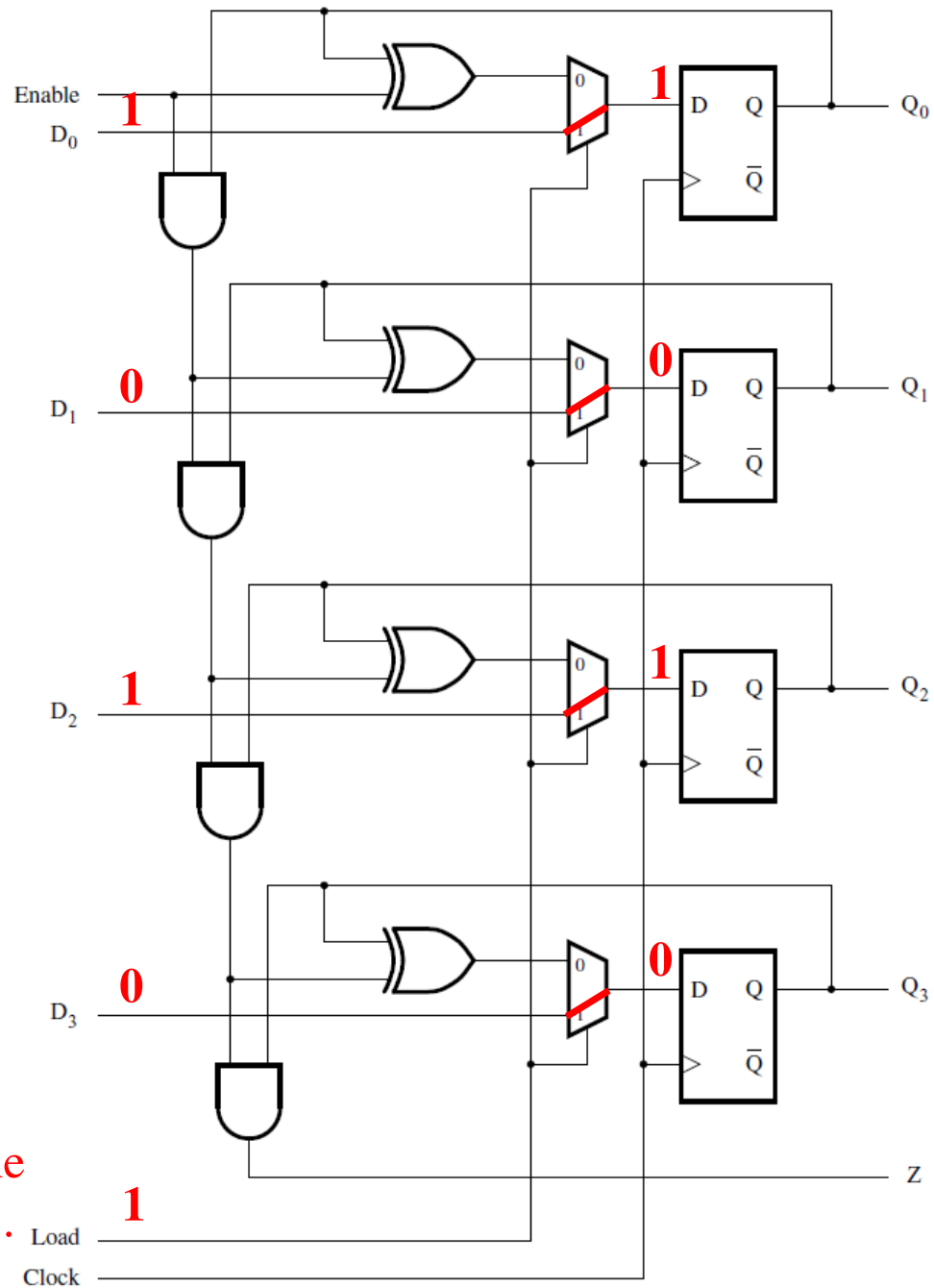
[Figure 5.24 from the textbook]

How to load the initial count value



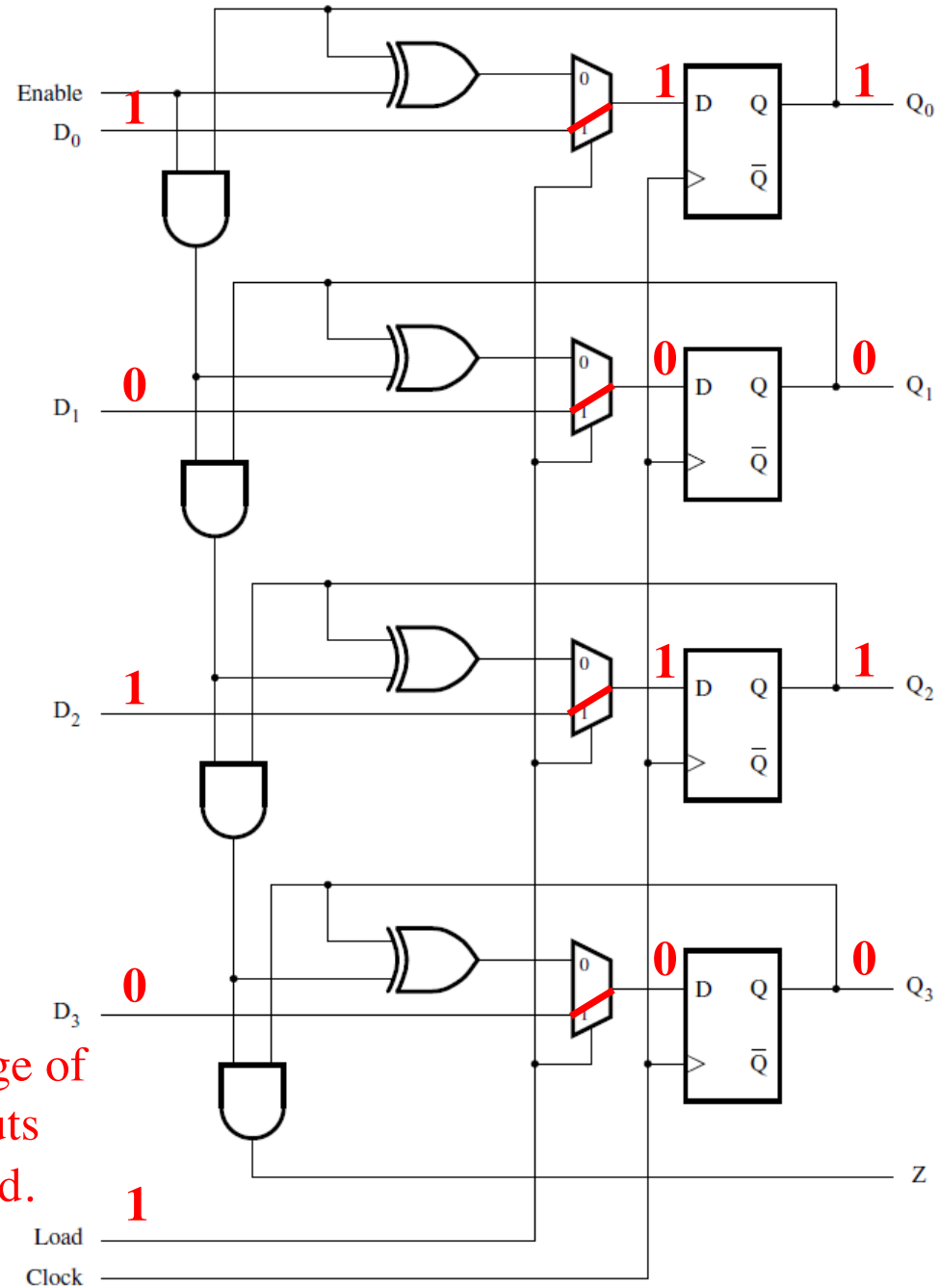
Set the initial count on the parallel load lines (in this case 5).

How to zero a counter



Set "Load" to 1, to open the "1" line of the multiplexers.

How to zero a counter



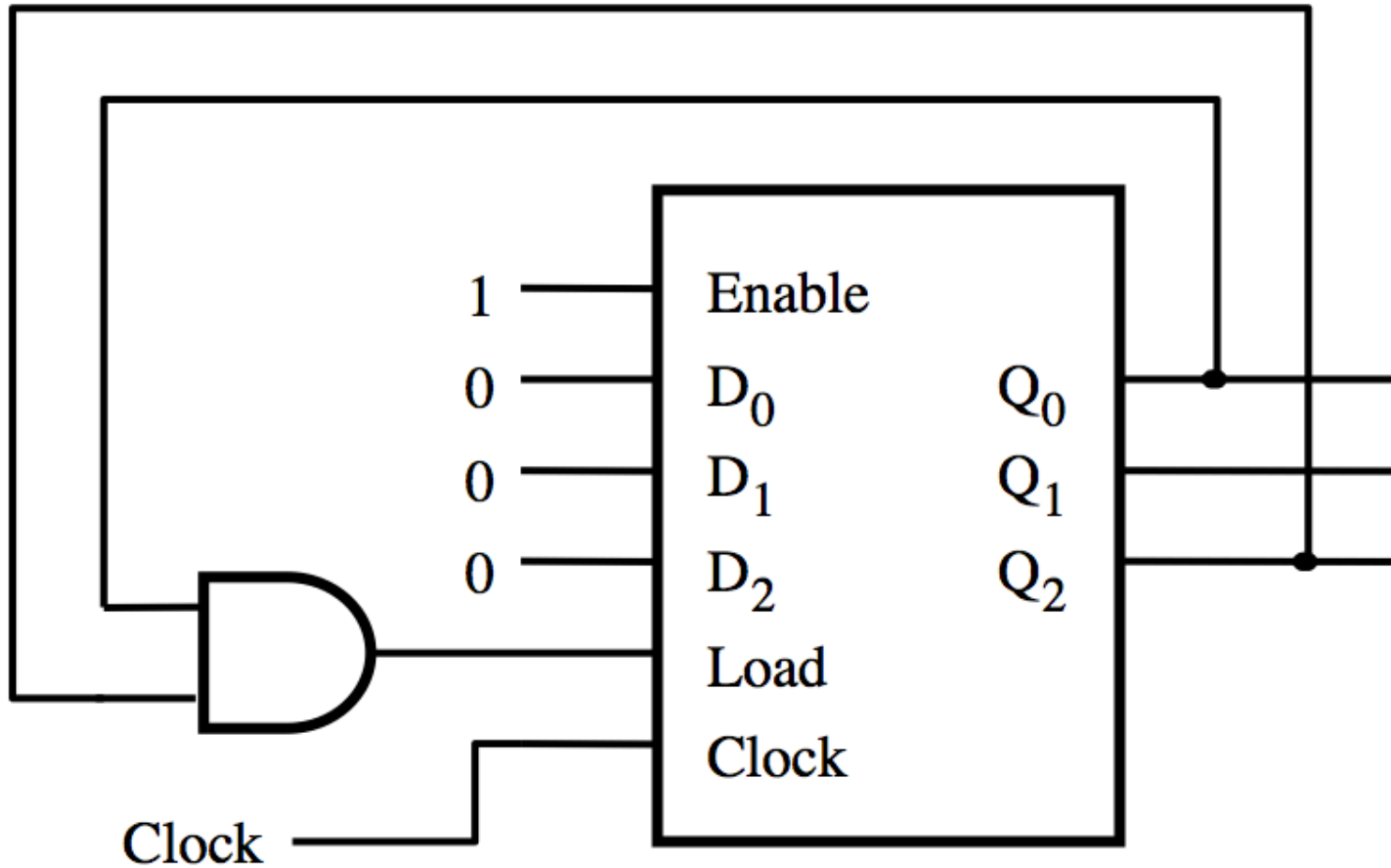
When the next positive edge of the clock arrives, the outputs of the flip-flops are updated.

Reset Synchronization

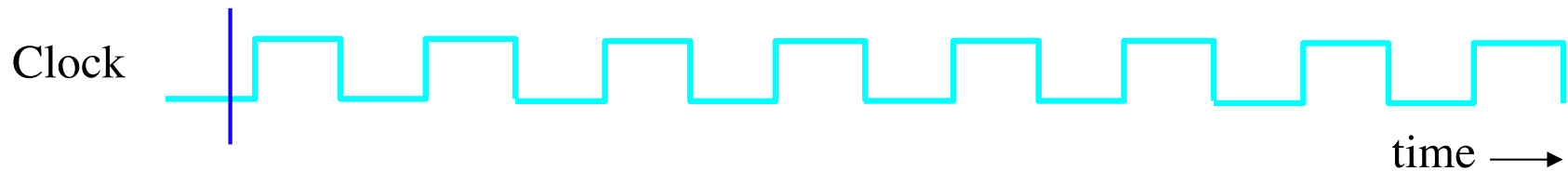
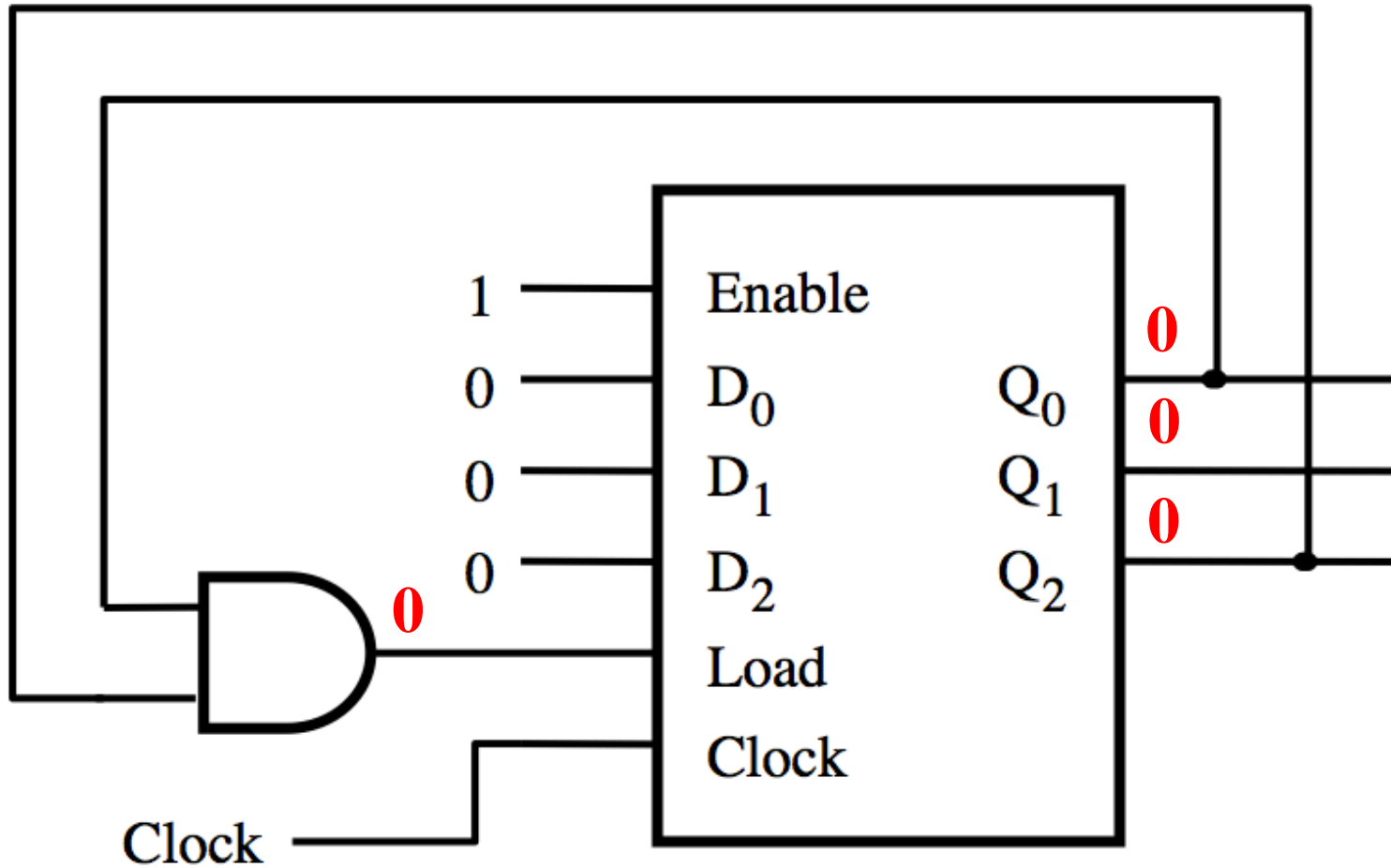
Motivation

- **An n-bit counter counts from 0, 1, ..., 2^n-1**
- **For example a 3-bit counter counts up as follow**
 - **0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, ...**
- **What if we want it to count like this**
 - **0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0, 1, ...**
- **In other words, what is the cycle is not a power of 2?**

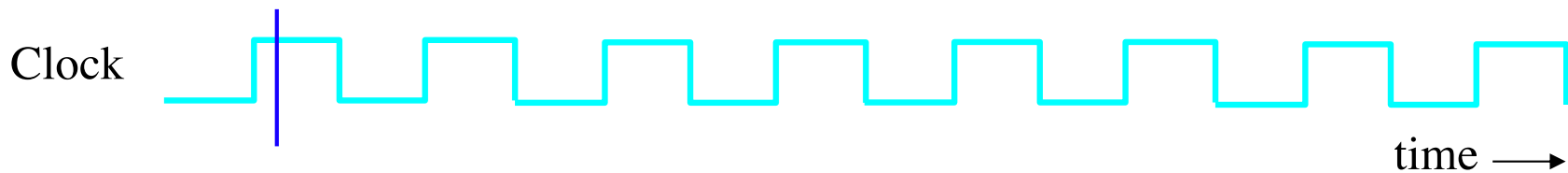
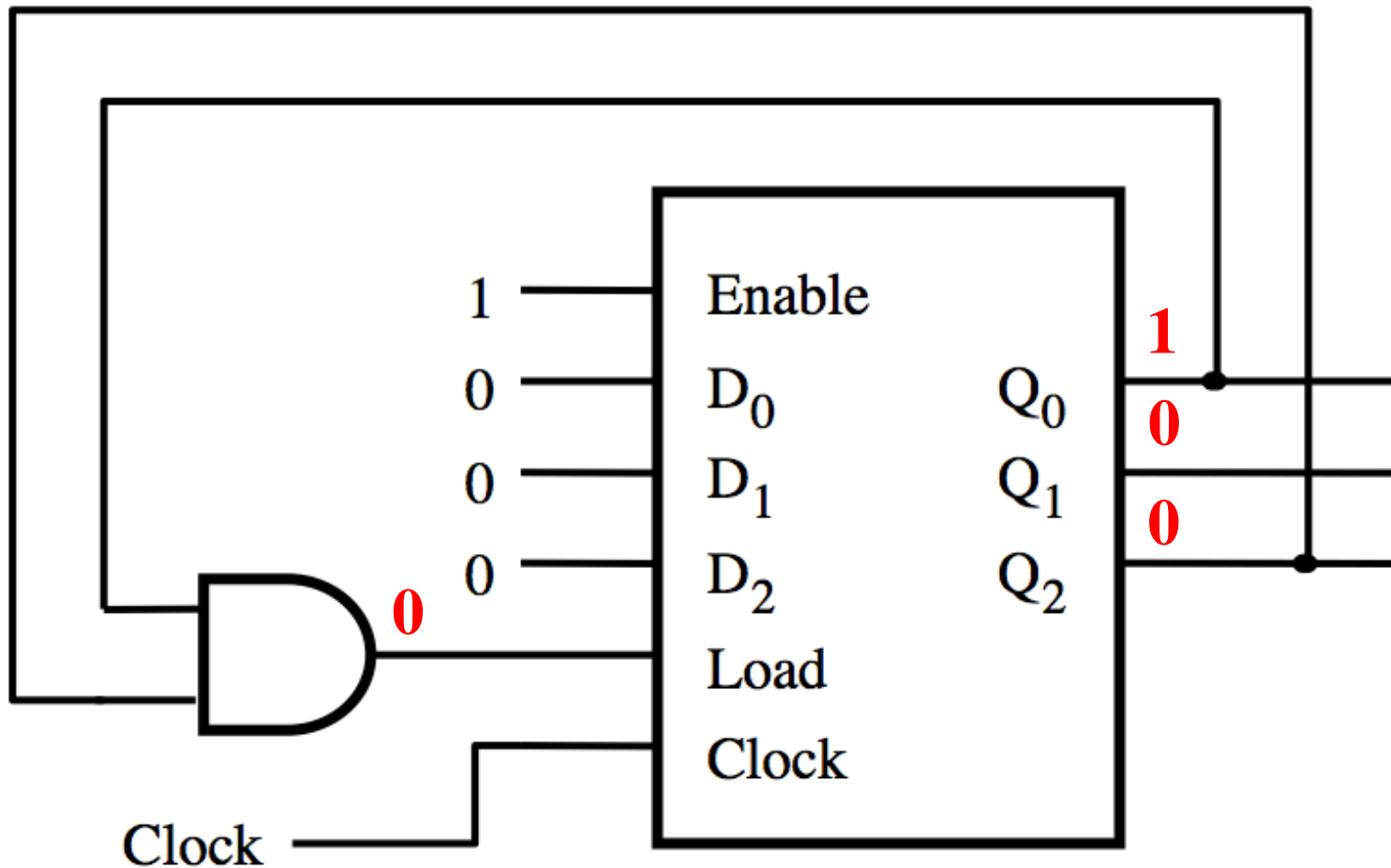
What does this circuit do?



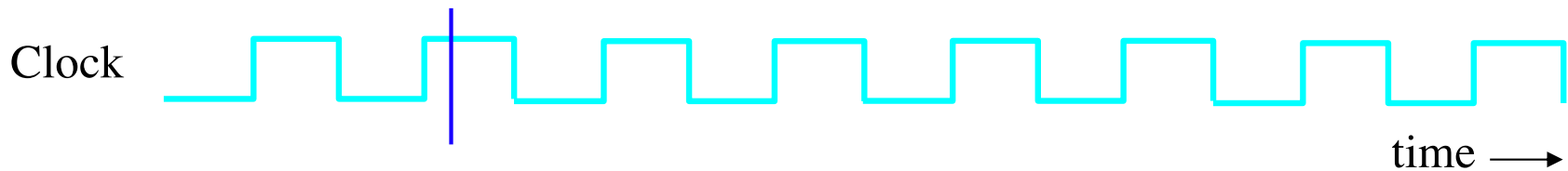
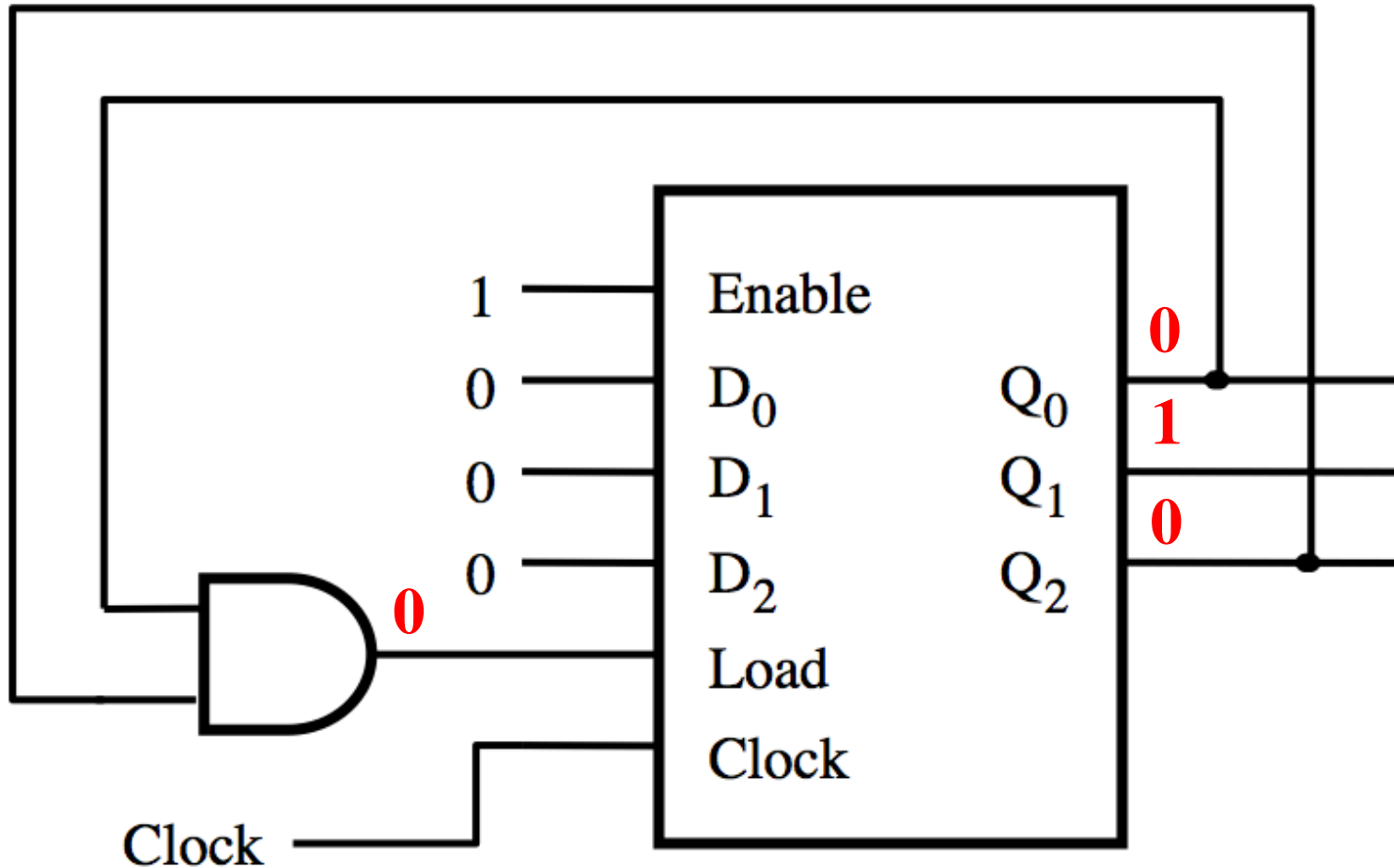
What does this circuit do?



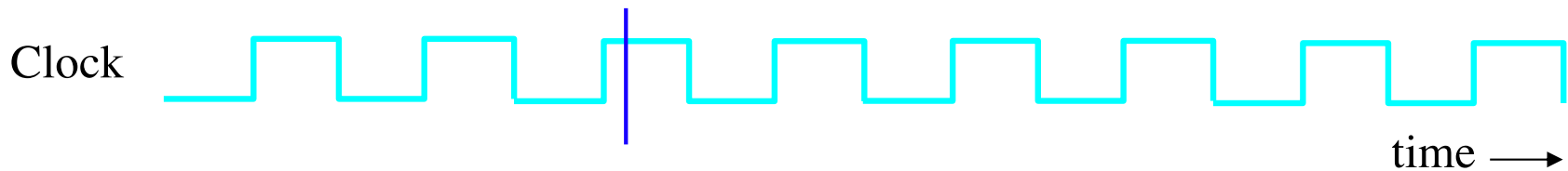
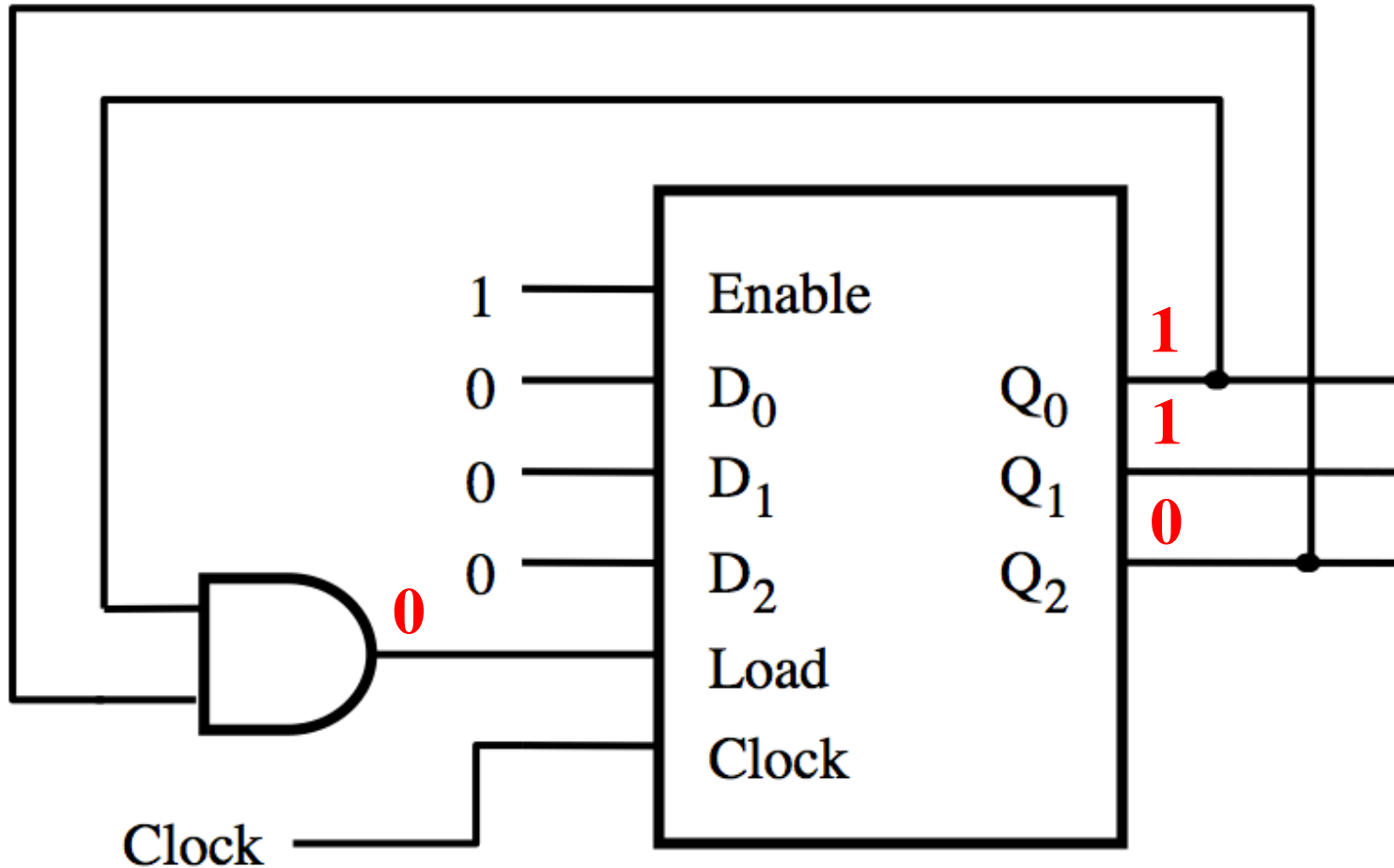
What does this circuit do?



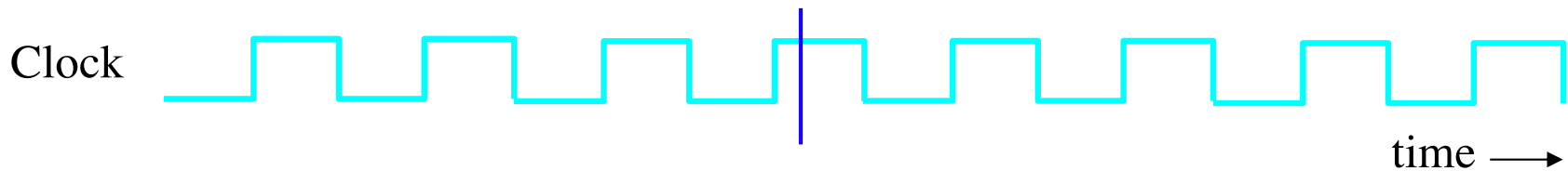
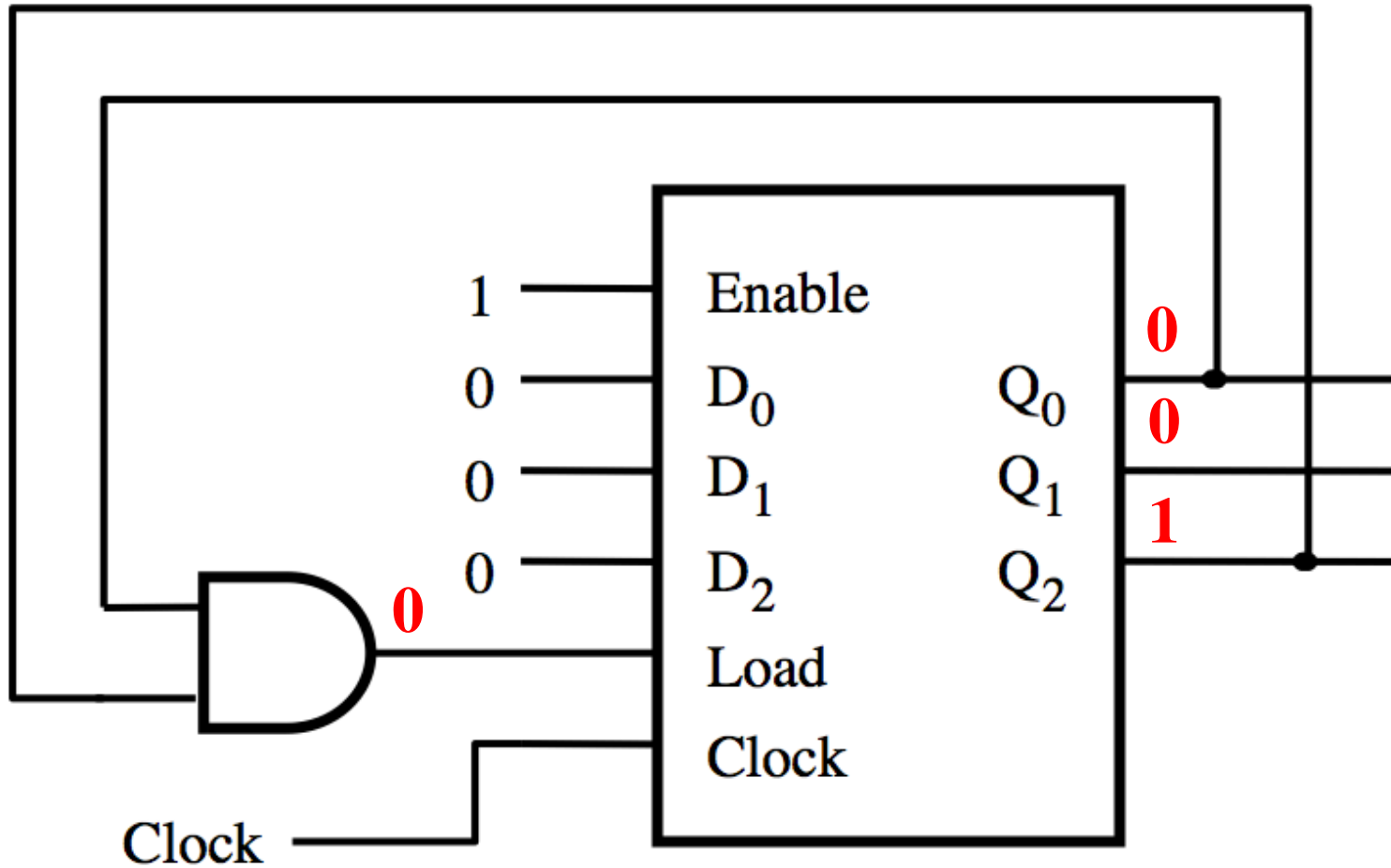
What does this circuit do?



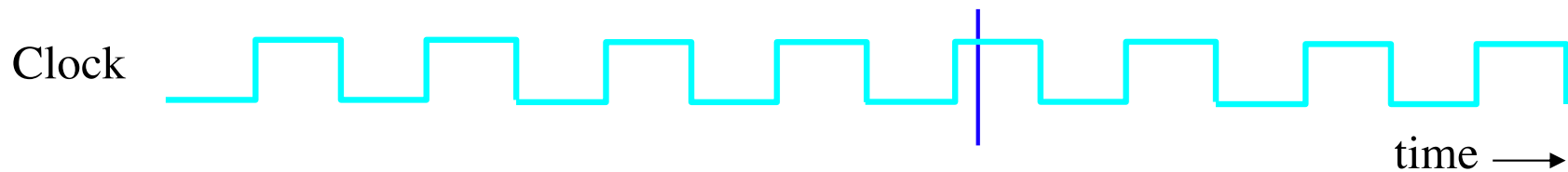
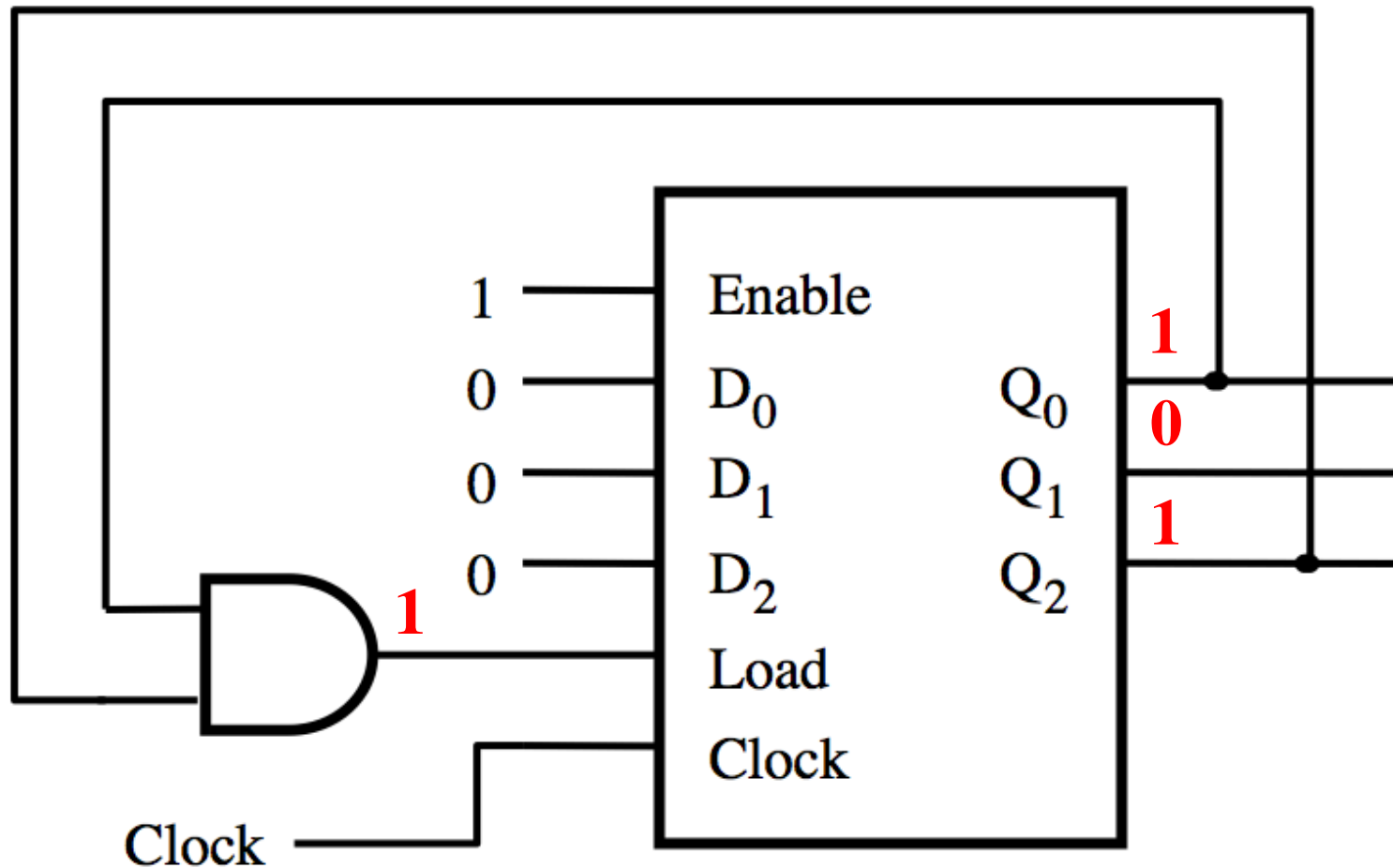
What does this circuit do?



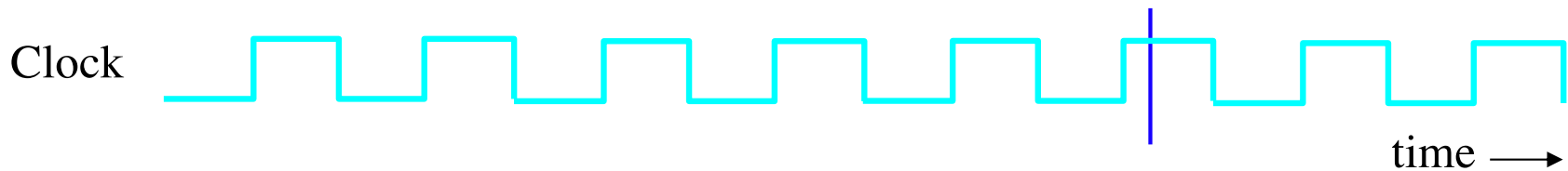
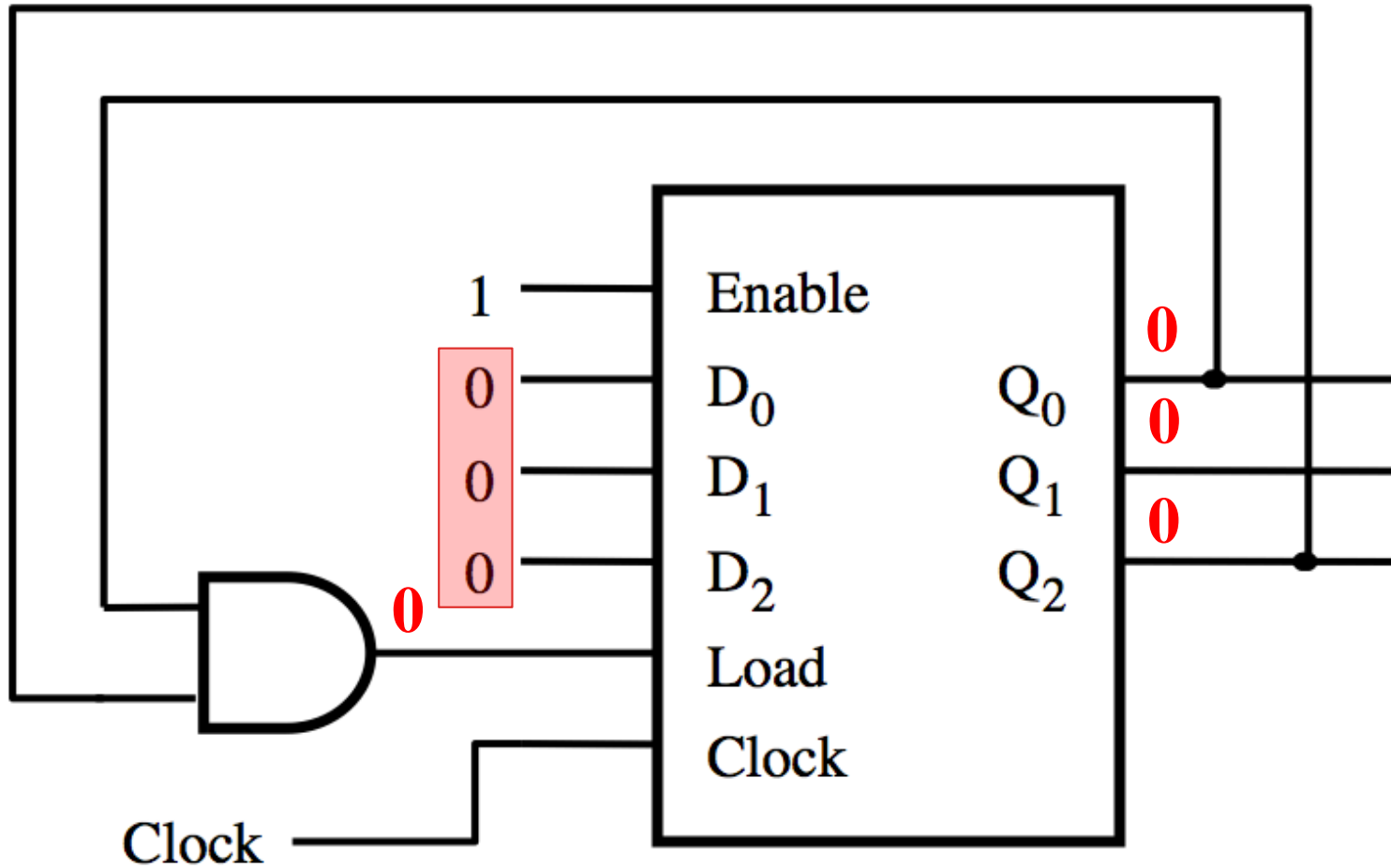
What does this circuit do?



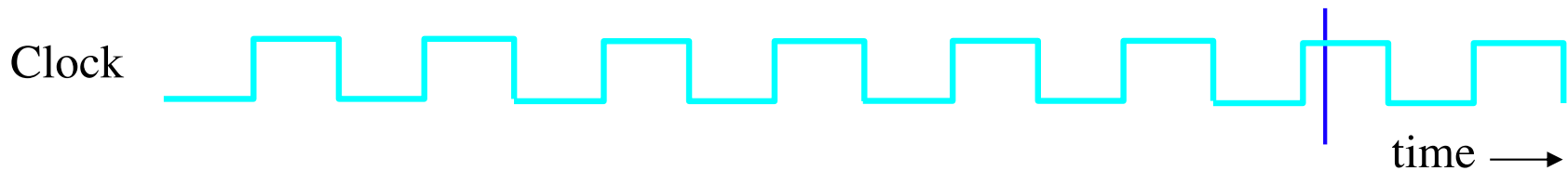
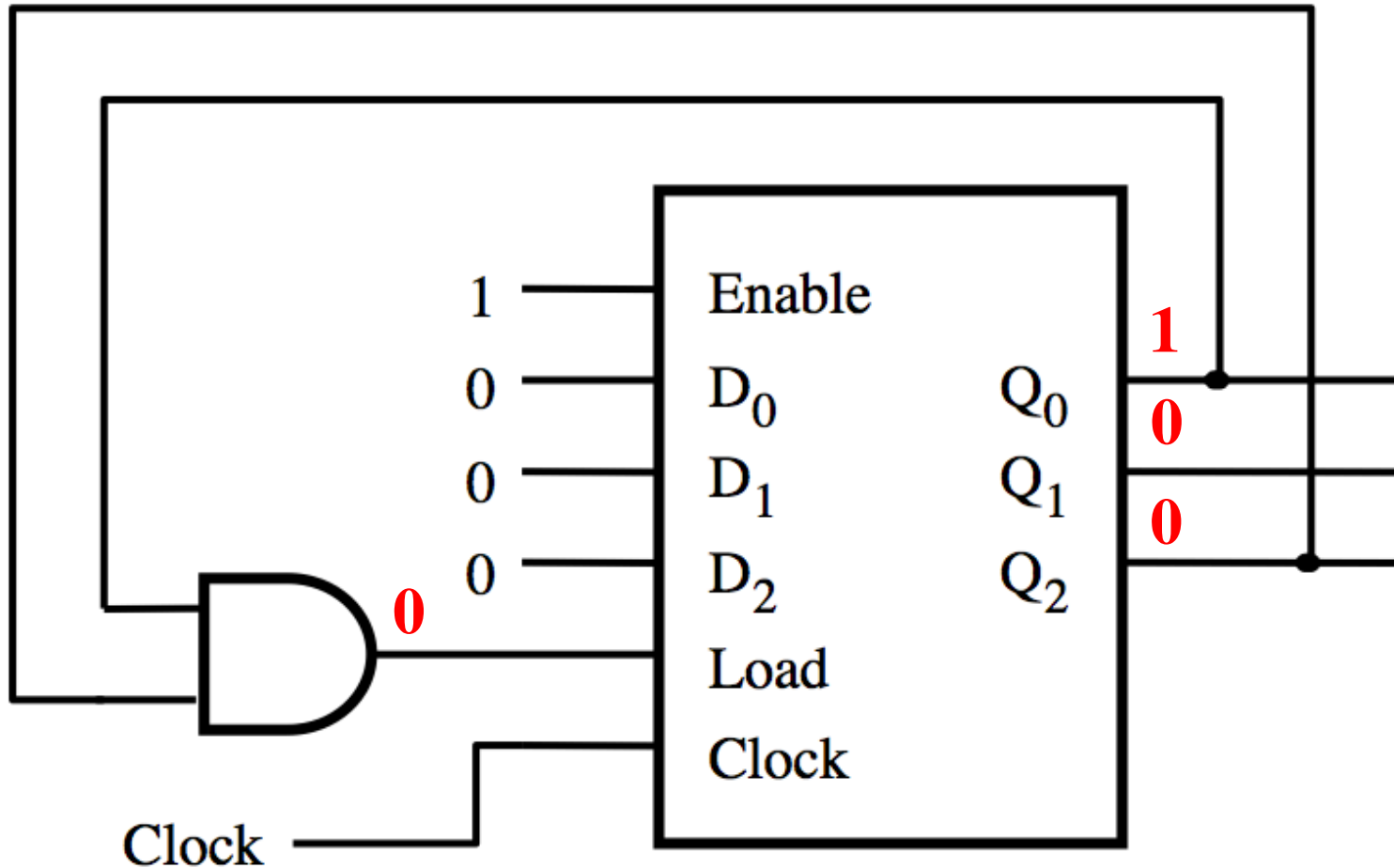
What does this circuit do?



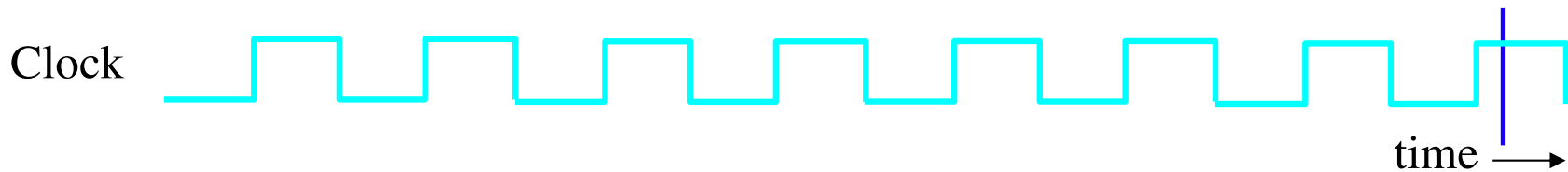
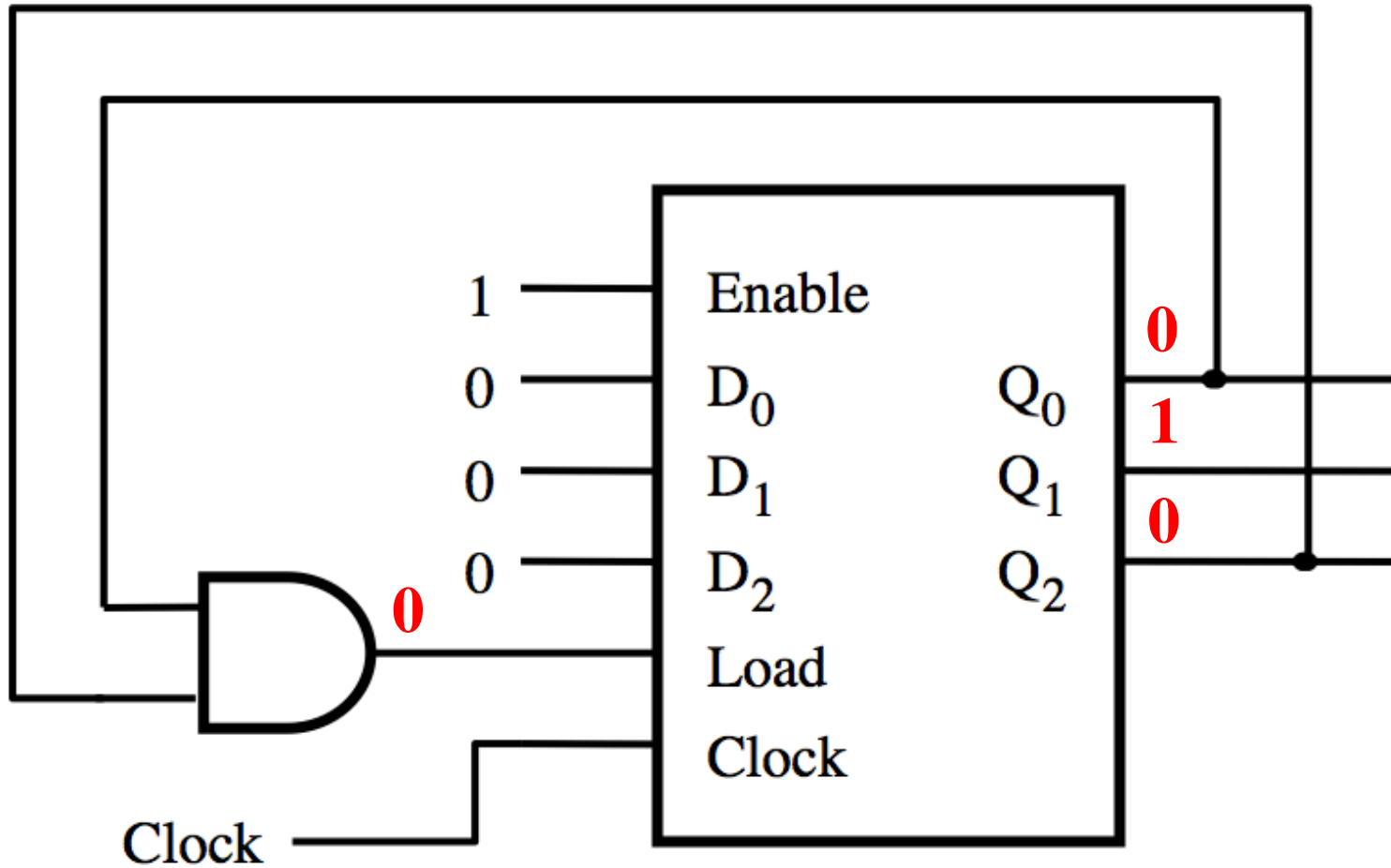
What does this circuit do?



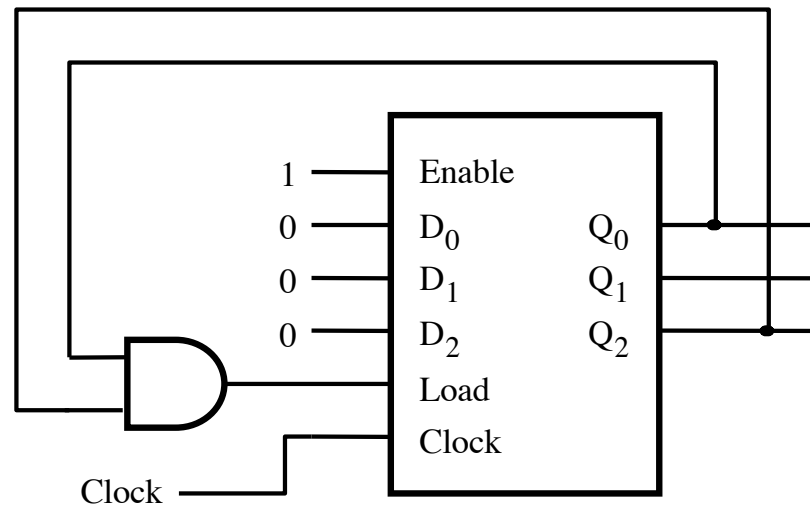
What does this circuit do?



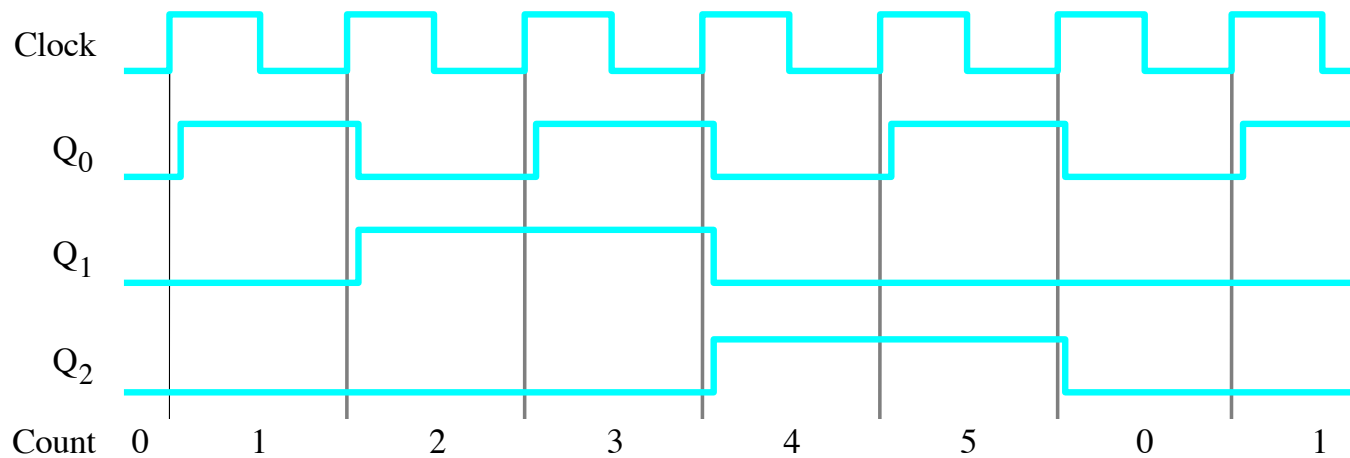
What does this circuit do?



A modulo-6 counter with synchronous reset

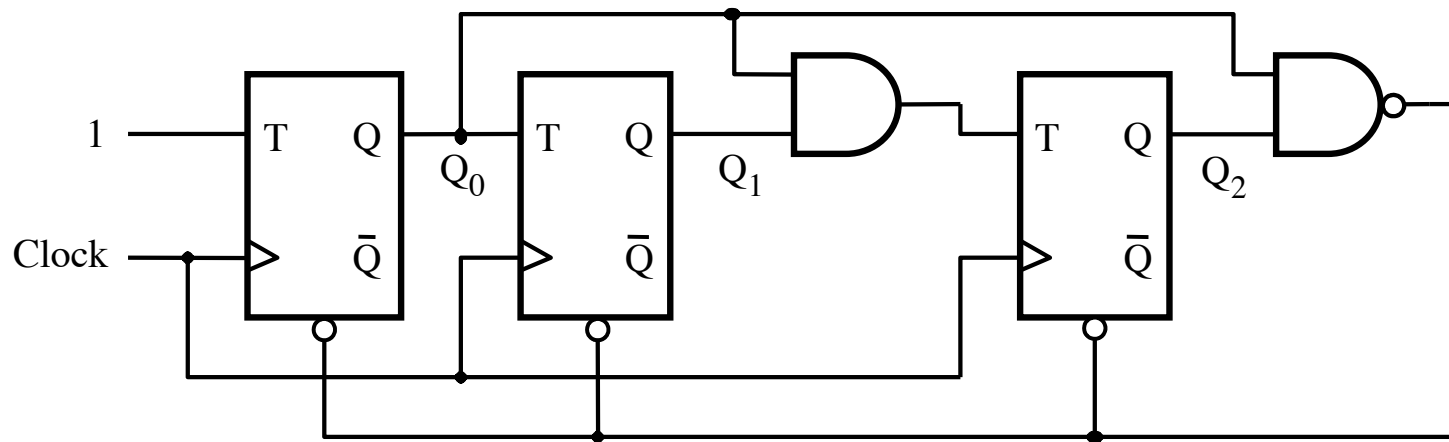


(a) Circuit

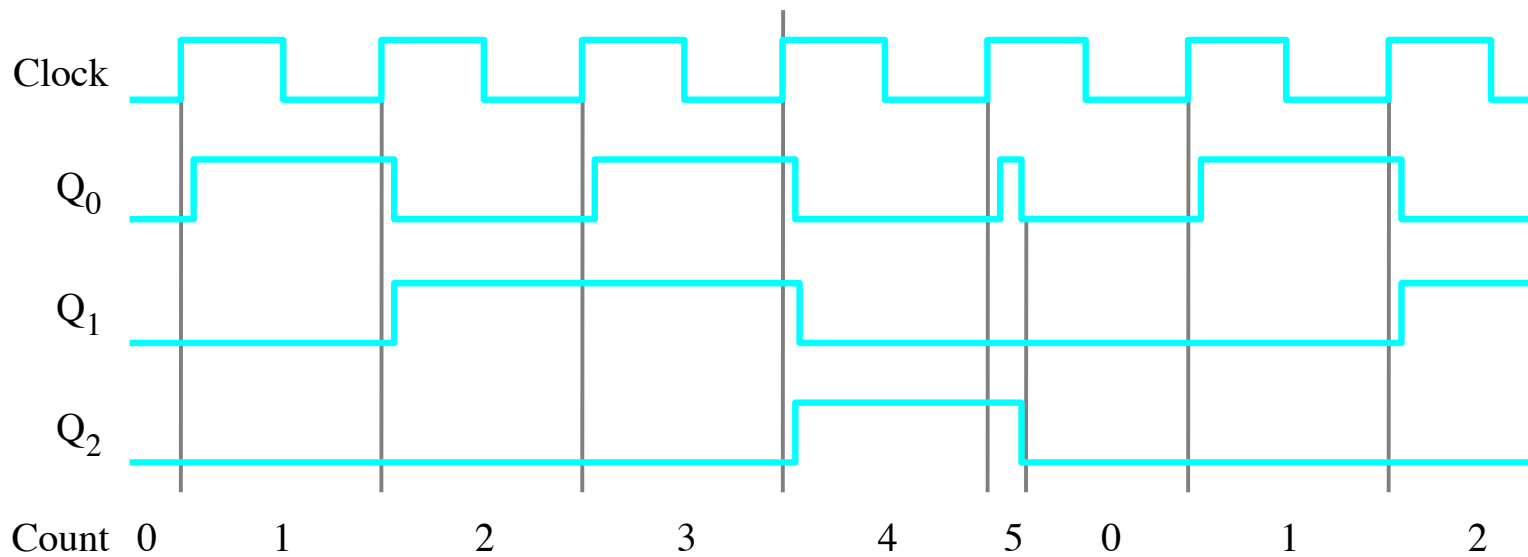


(b) Timing diagram

A modulo-6 counter with asynchronous reset

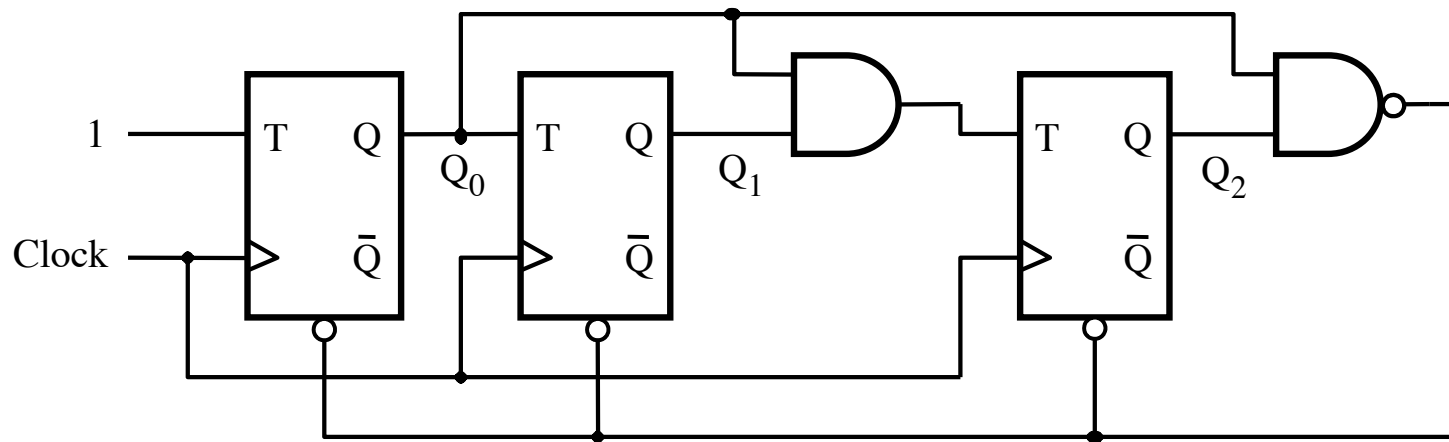


(a) Circuit



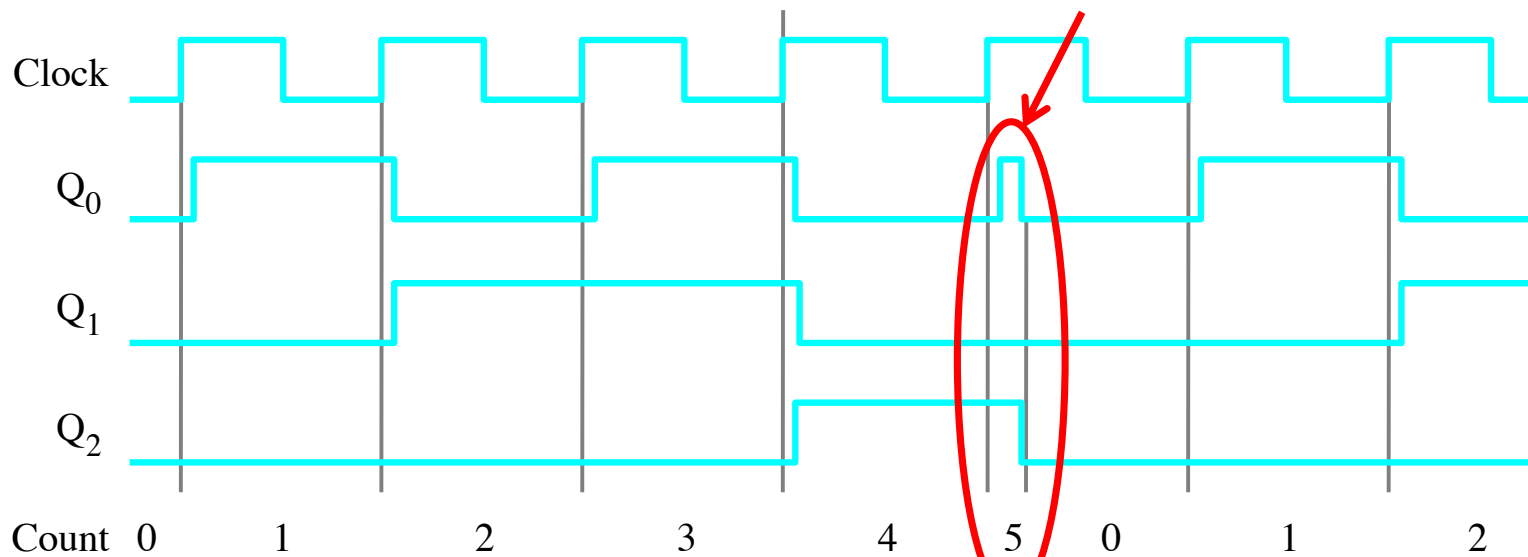
(b) Timing diagram

A modulo-6 counter with asynchronous reset



(a) Circuit

The number 5 is displayed for a very short amount of time



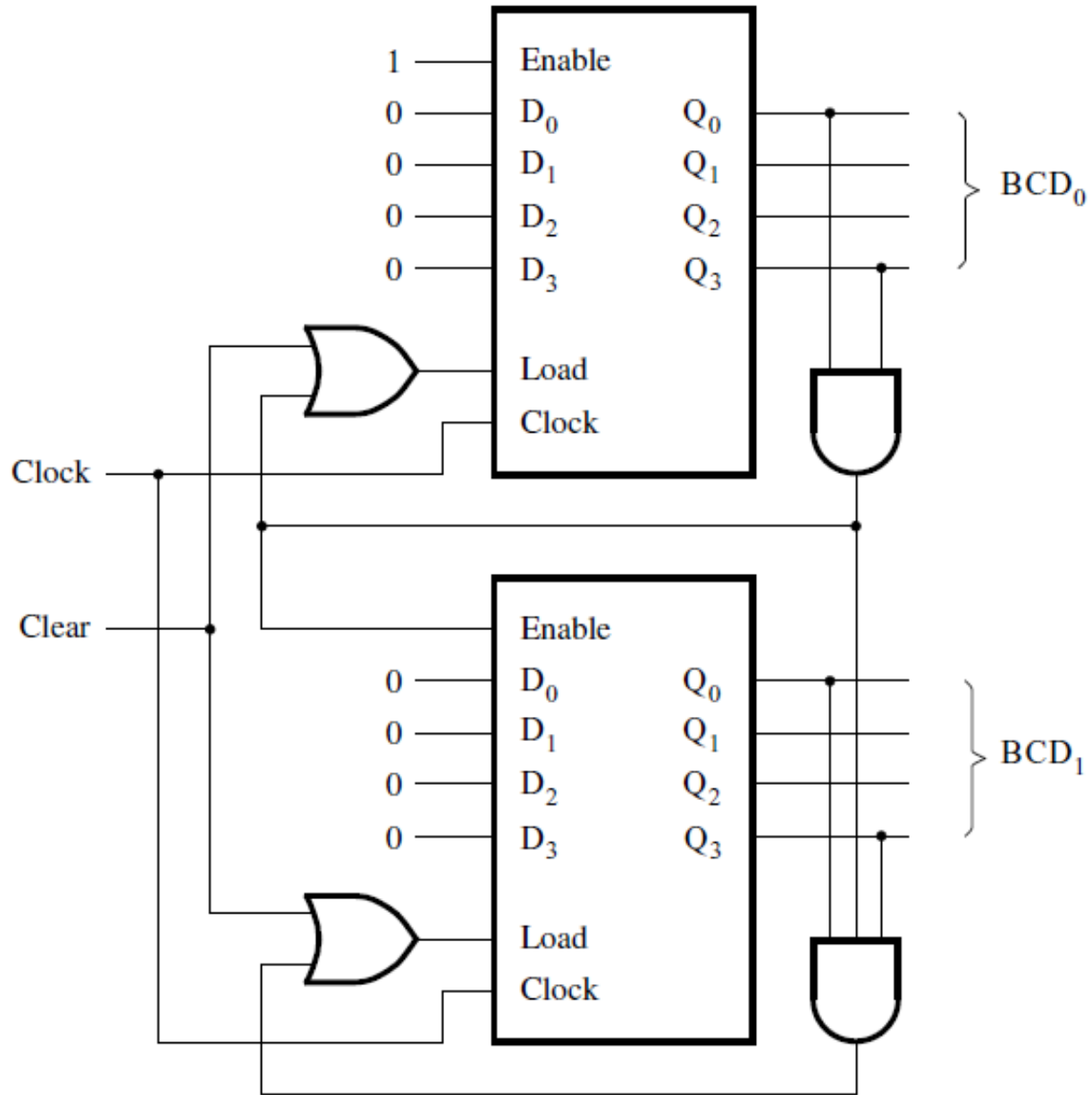
(b) Timing diagram

Other Types of Counters (Section 5.11)

A two-digit BCD counter

- **Use Two Parallel-load four-bit counters**
 - Figure 5.24
- **Each counts in binary**
 - 0-9
- **Resets generated on 9**
 - Reset by loading 0's
- **Second digit enabled by a 9 on first counter**

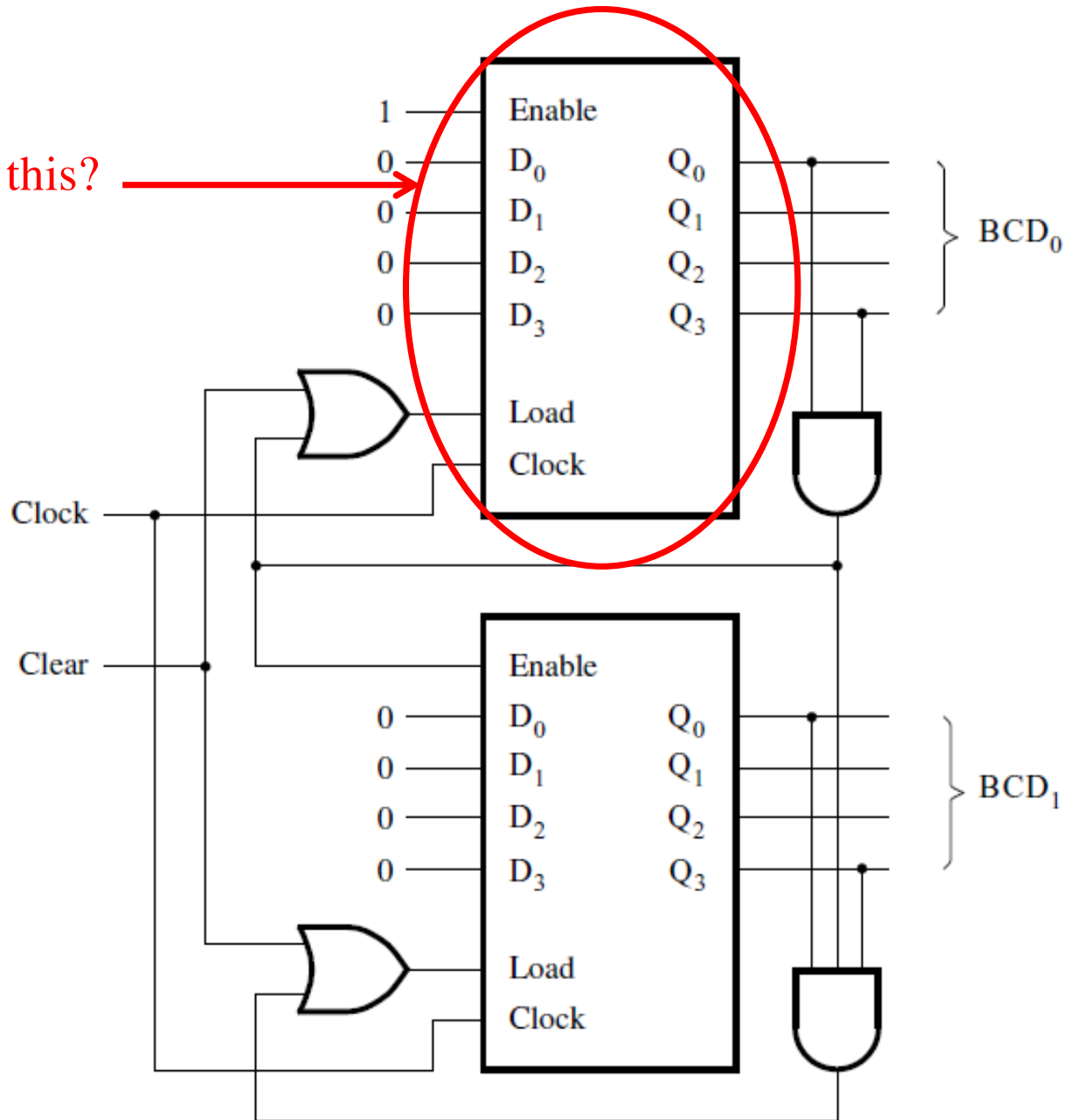
A two-digit BCD counter



[Figure 5.27 from the textbook]

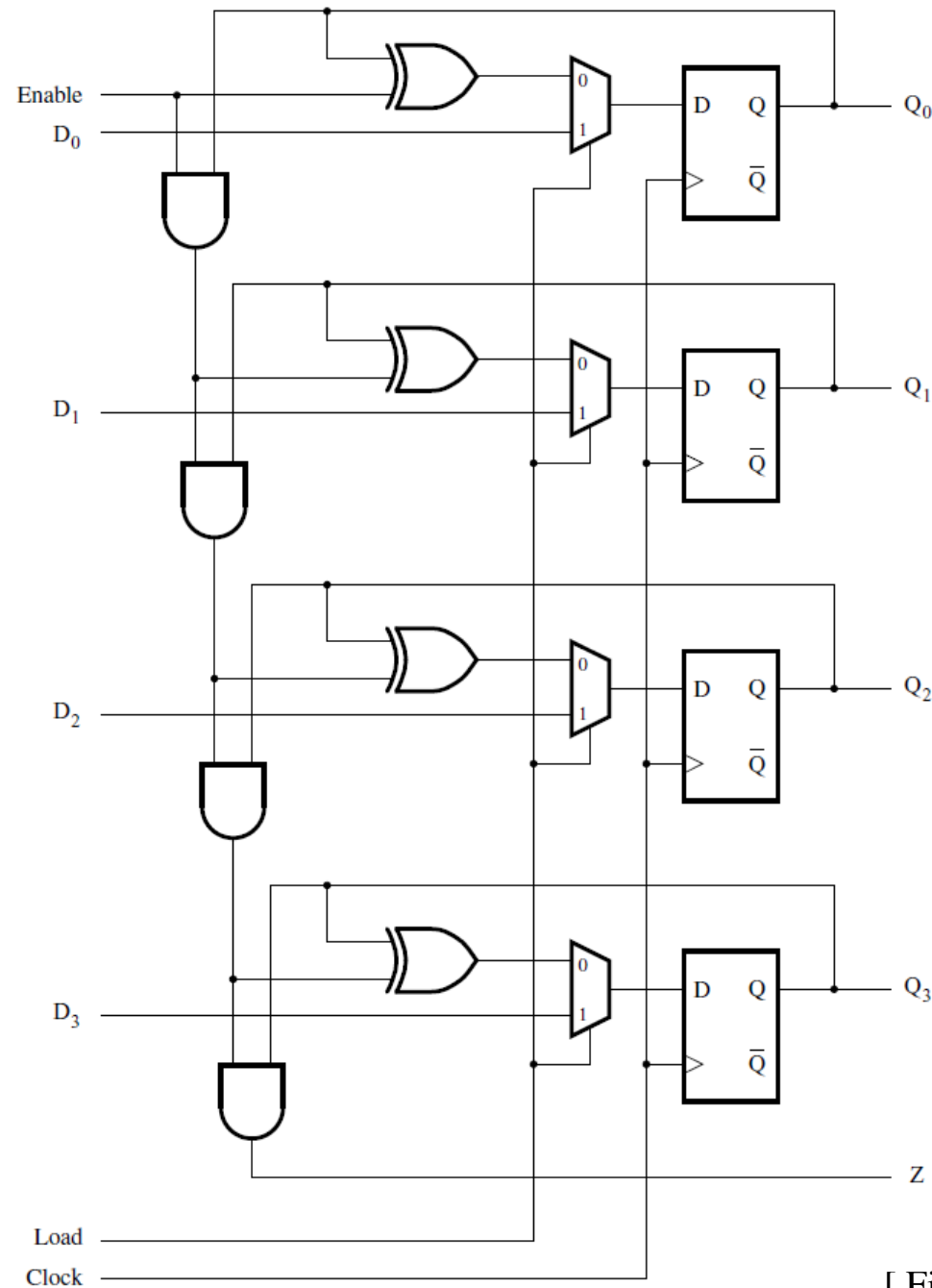
A two-digit BCD counter

What is this? →



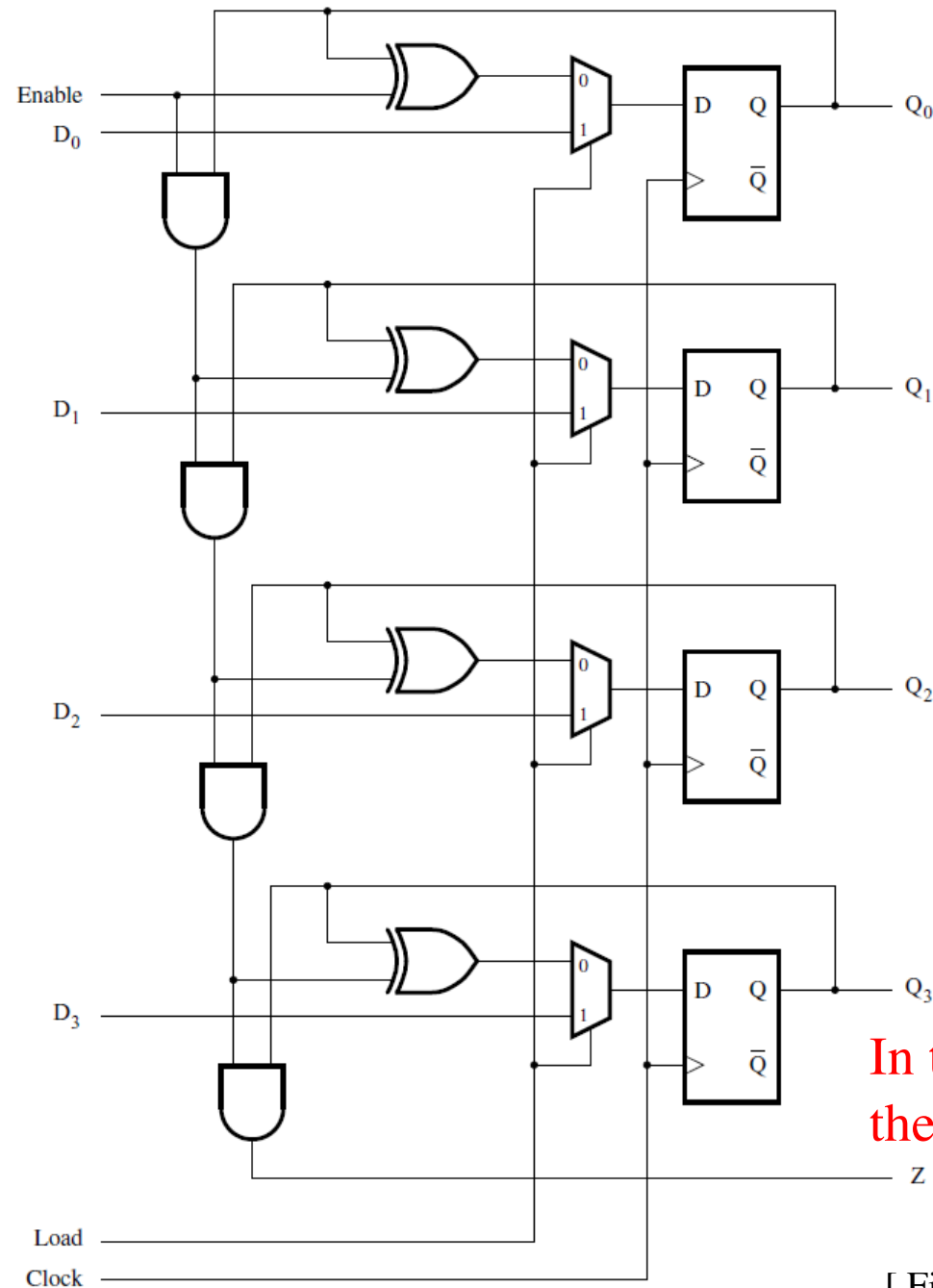
[Figure 5.27 from the textbook]

It is a counter with parallel-load capability



[Figure 5.24 from the textbook]

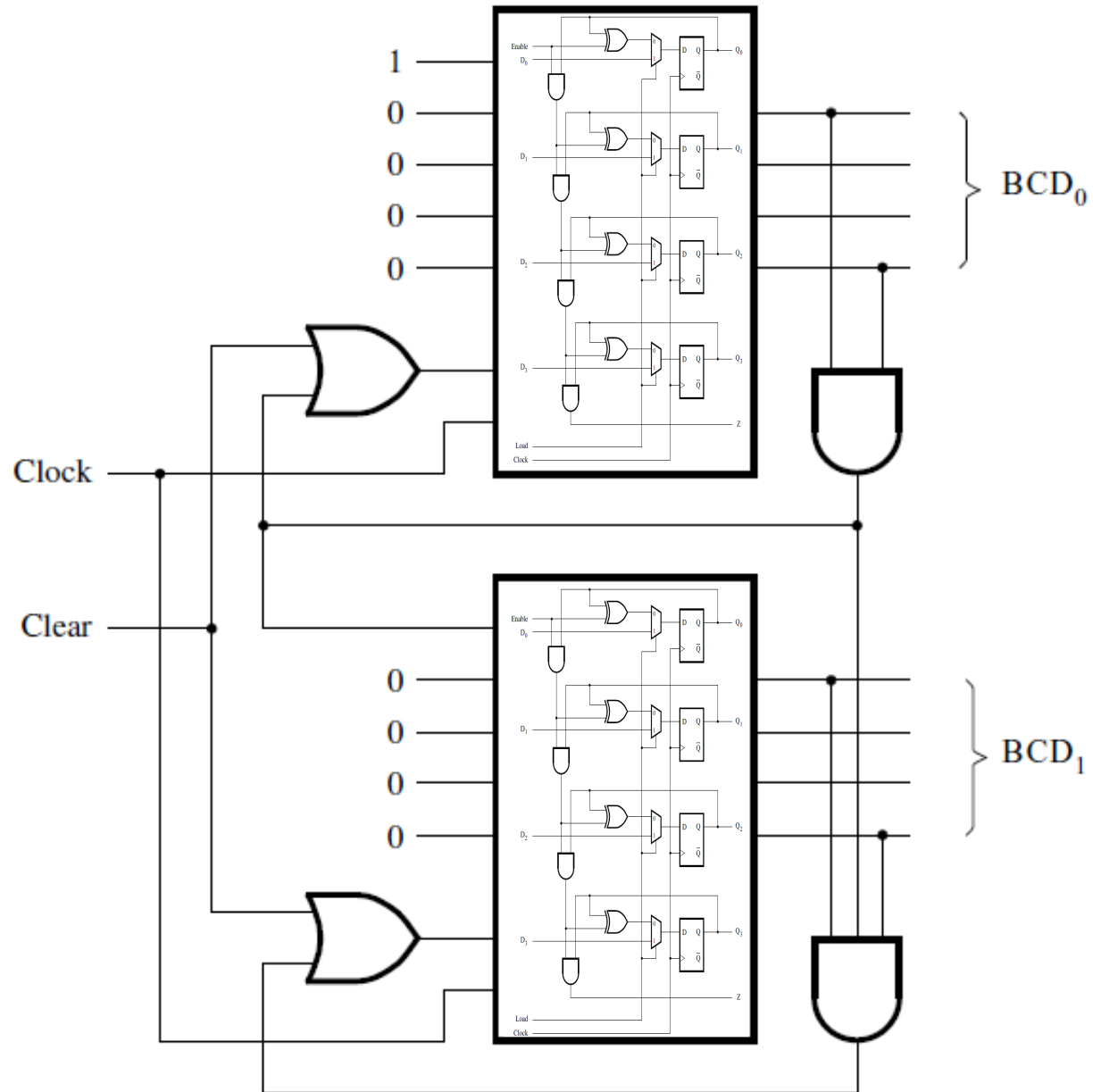
It is a counter with parallel-load capability



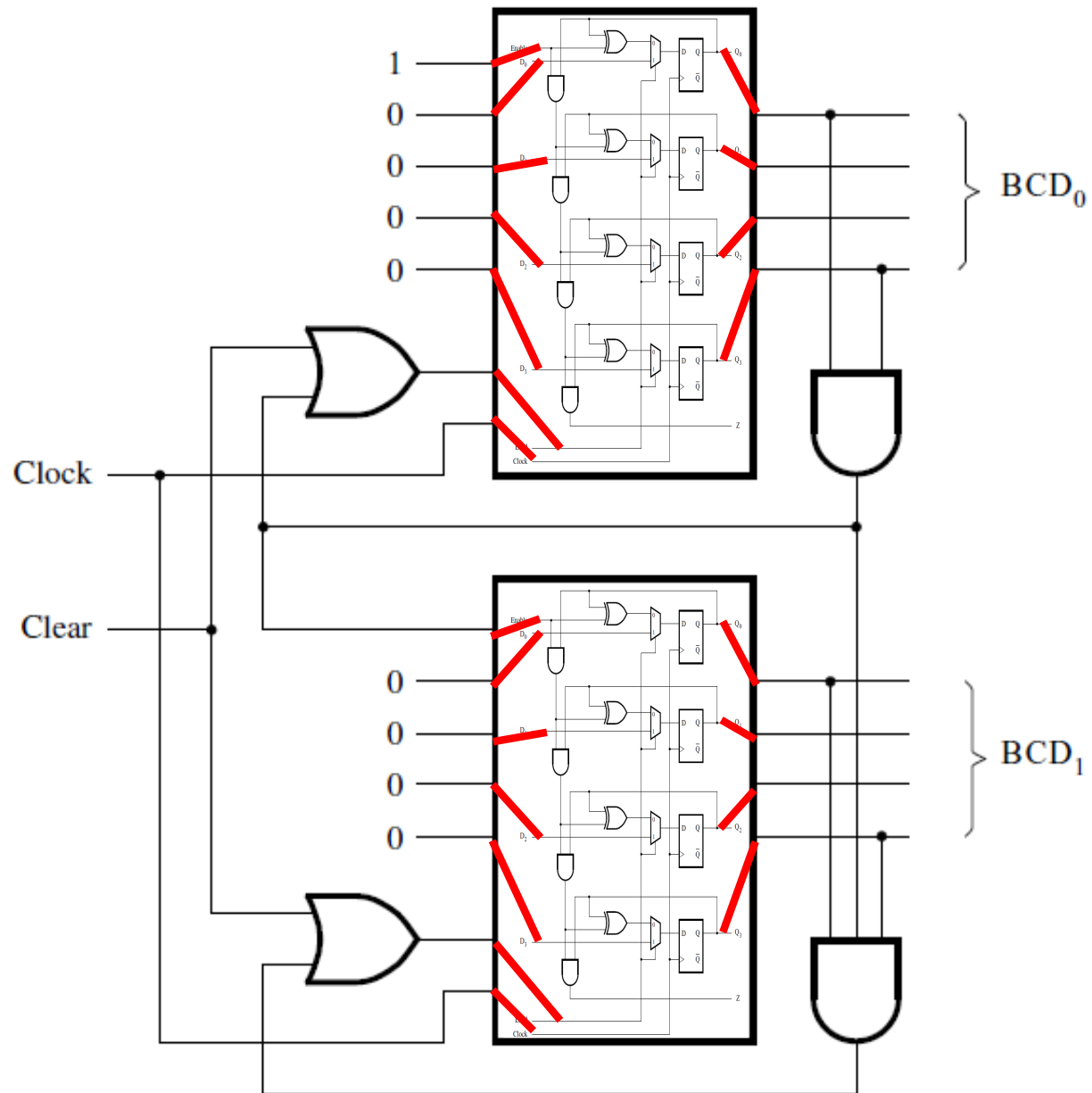
In this case,
the z output is ignored

[Figure 5.24 from the textbook]

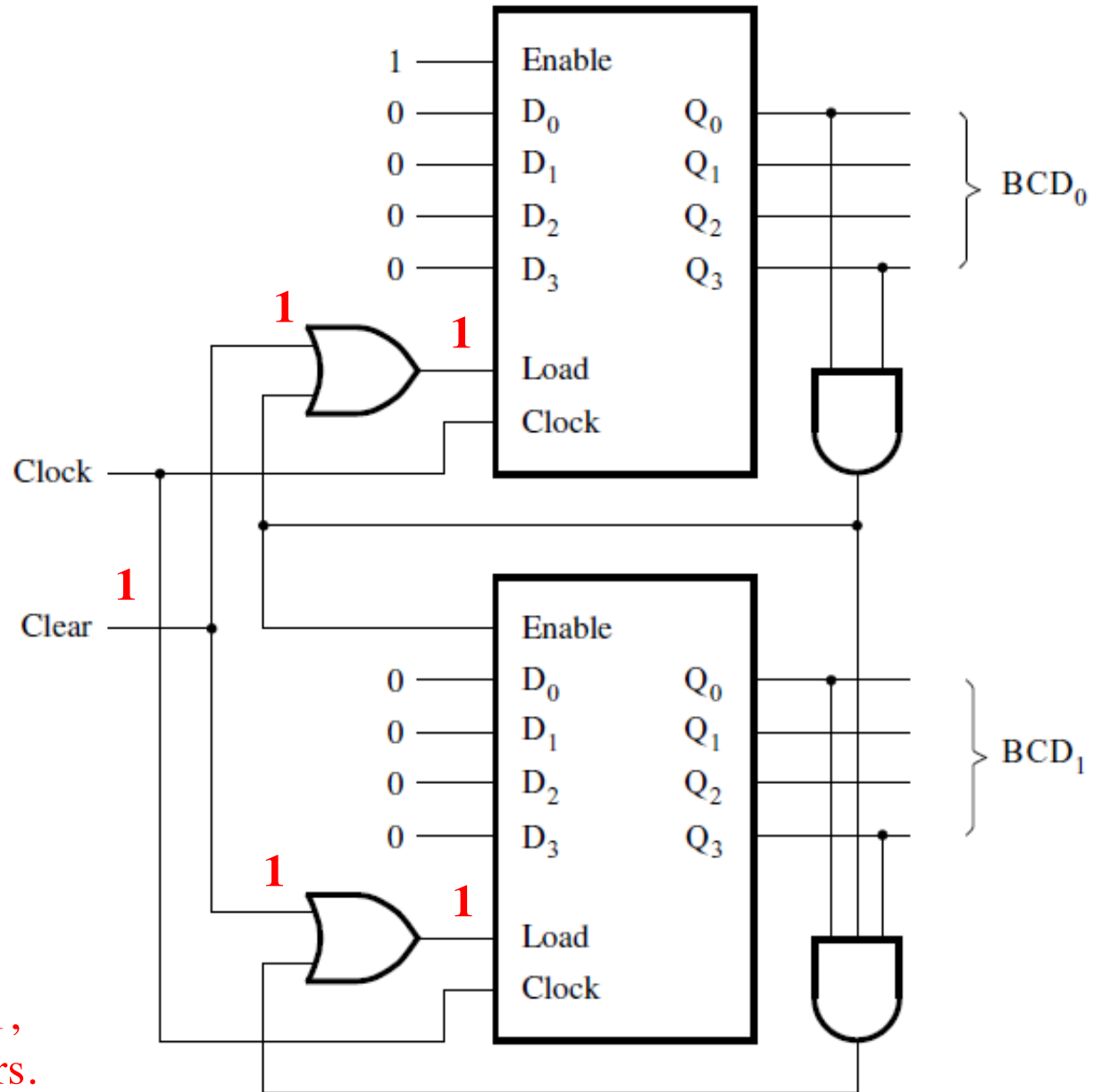
A two-digit BCD counter



A two-digit BCD counter

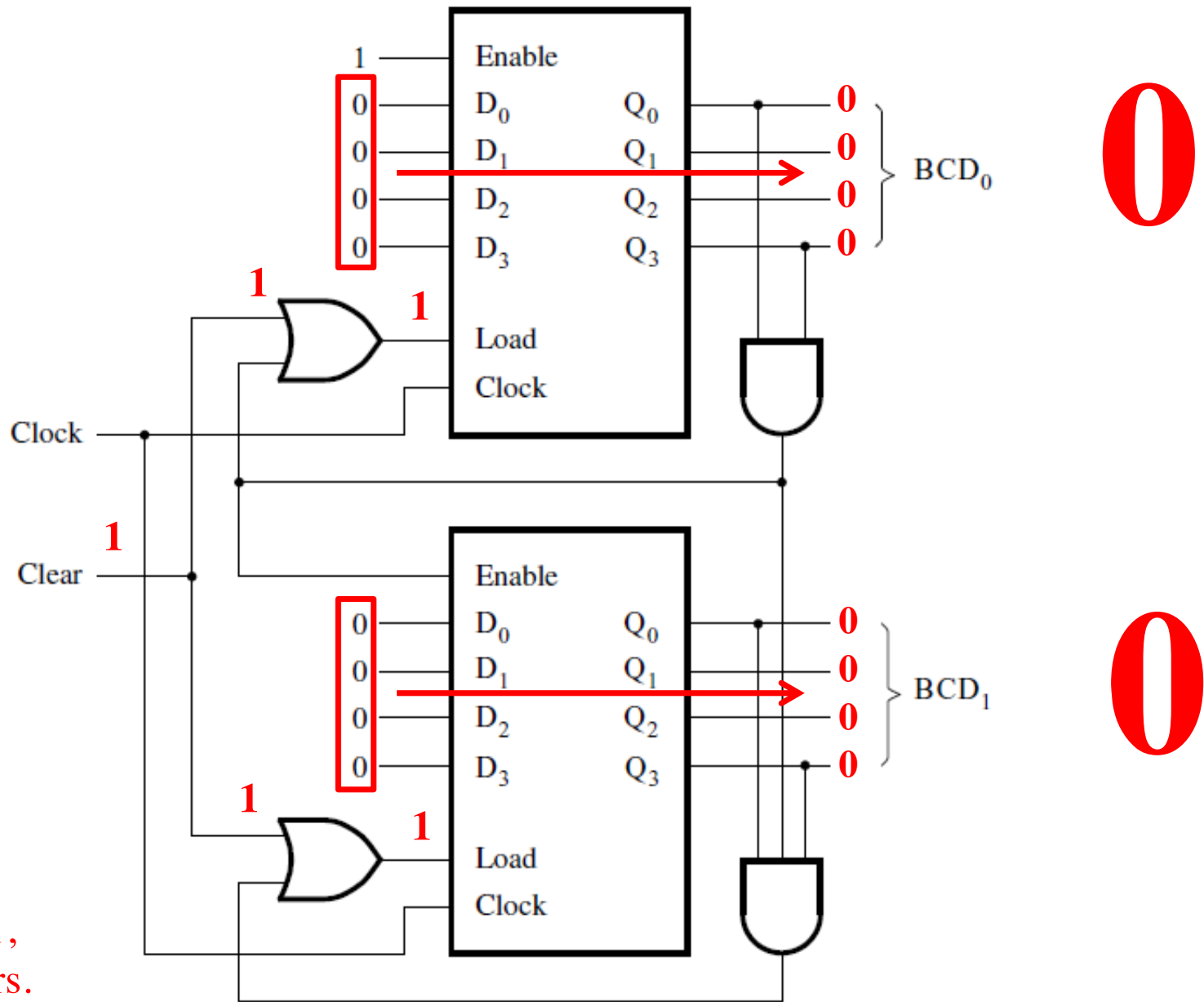


Zeroing the BCD counter



Setting "Clear" to 1,
zeroes both counters.

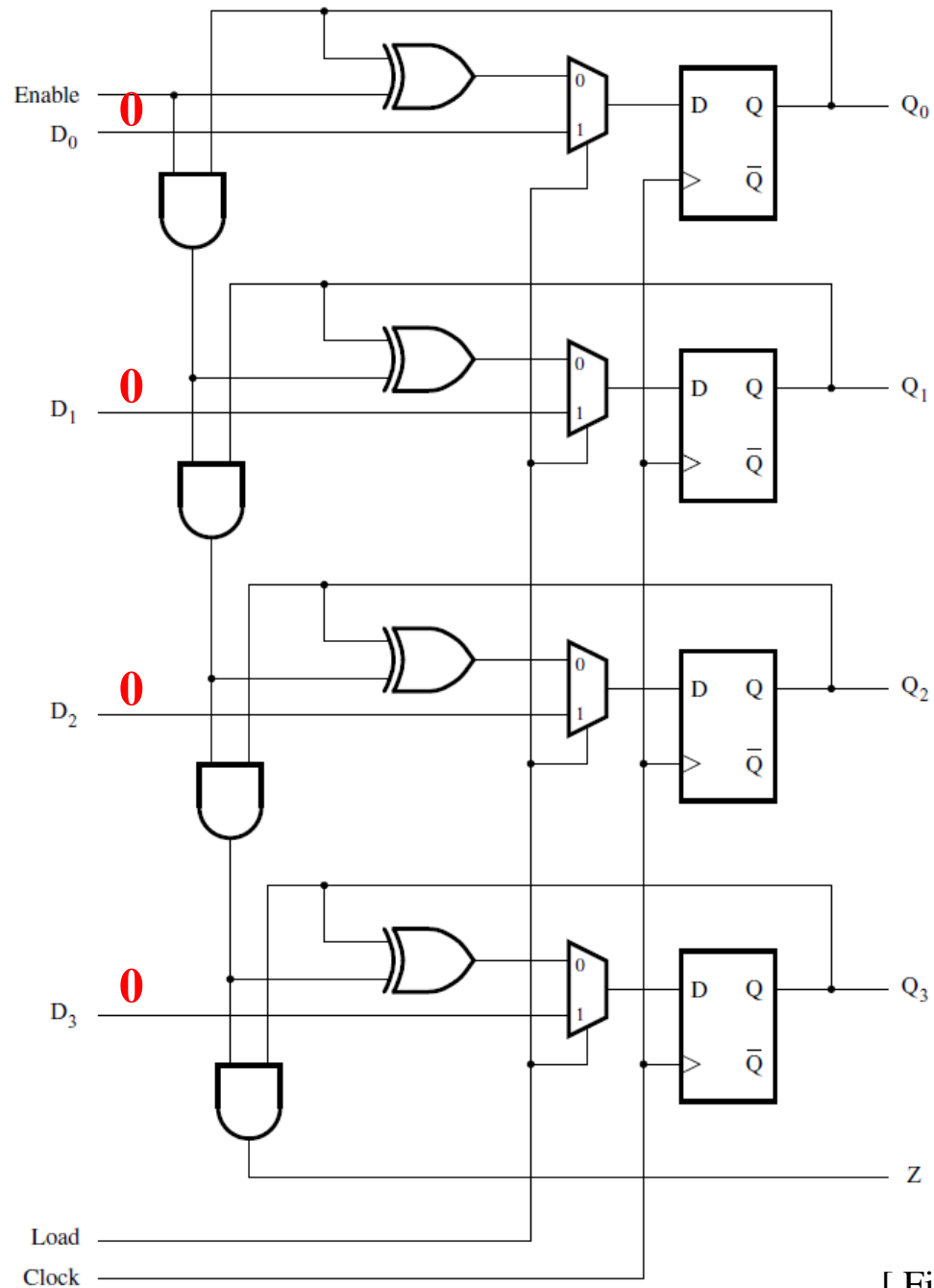
Zeroing the BCD counter



Setting "Clear" to 1,
zeroes both counters.

[Figure 5.27 from the textbook]

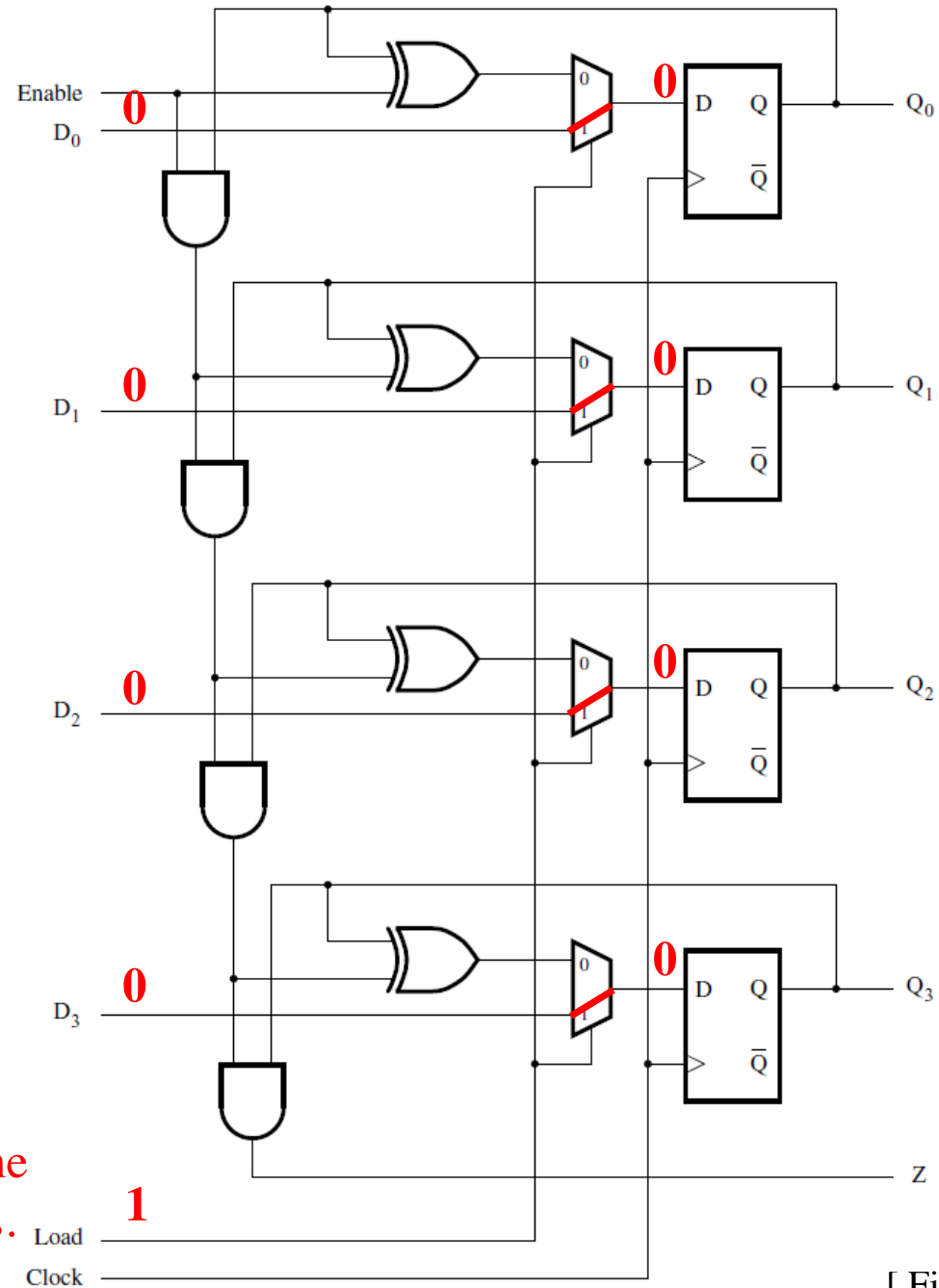
How to zero a counter



Set all parallel load input lines to zero.

[Figure 5.24 from the textbook]

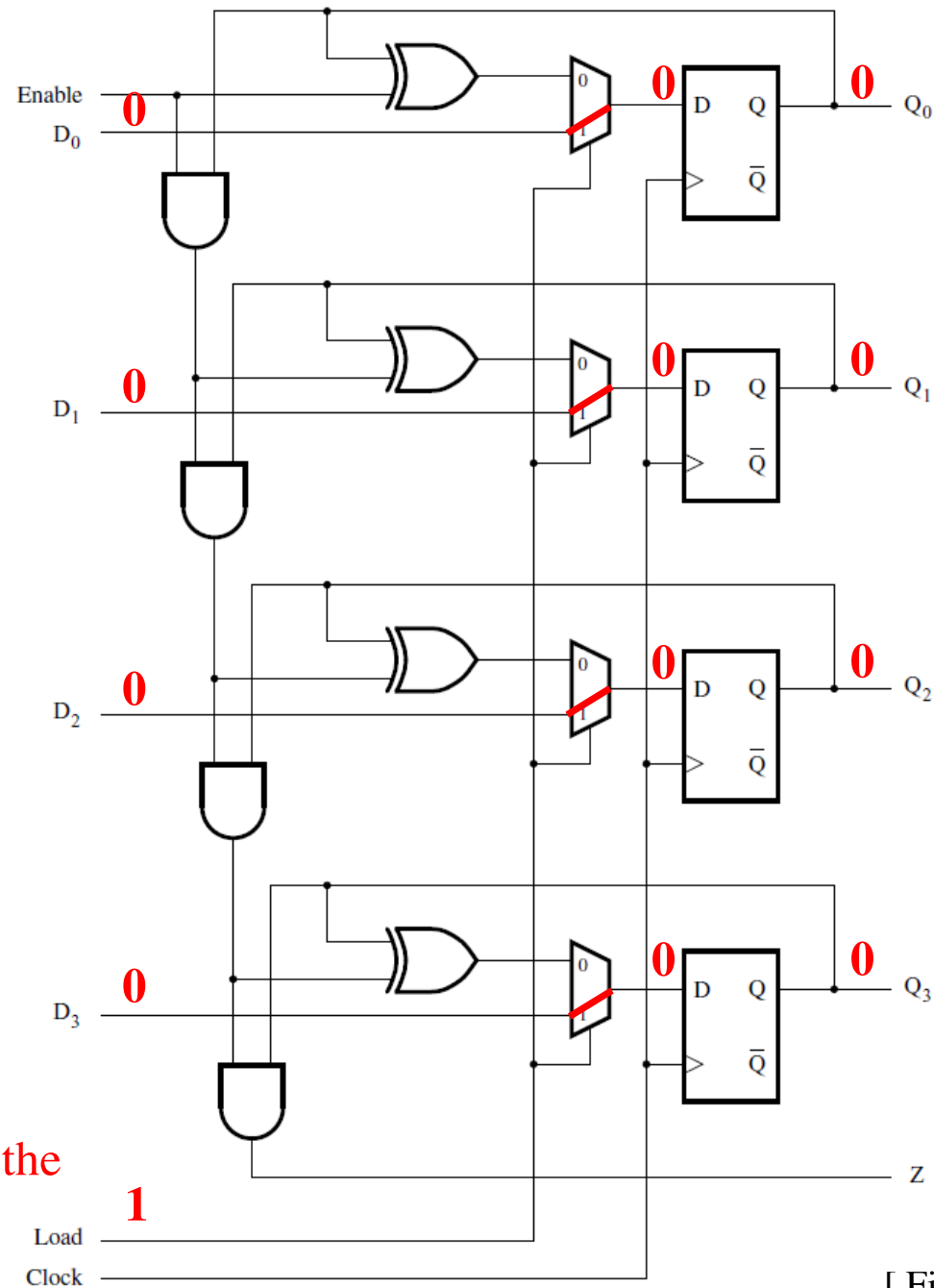
How to zero a counter



Set "Load" to 1, to open the "1" line of the multiplexers.

[Figure 5.24 from the textbook]

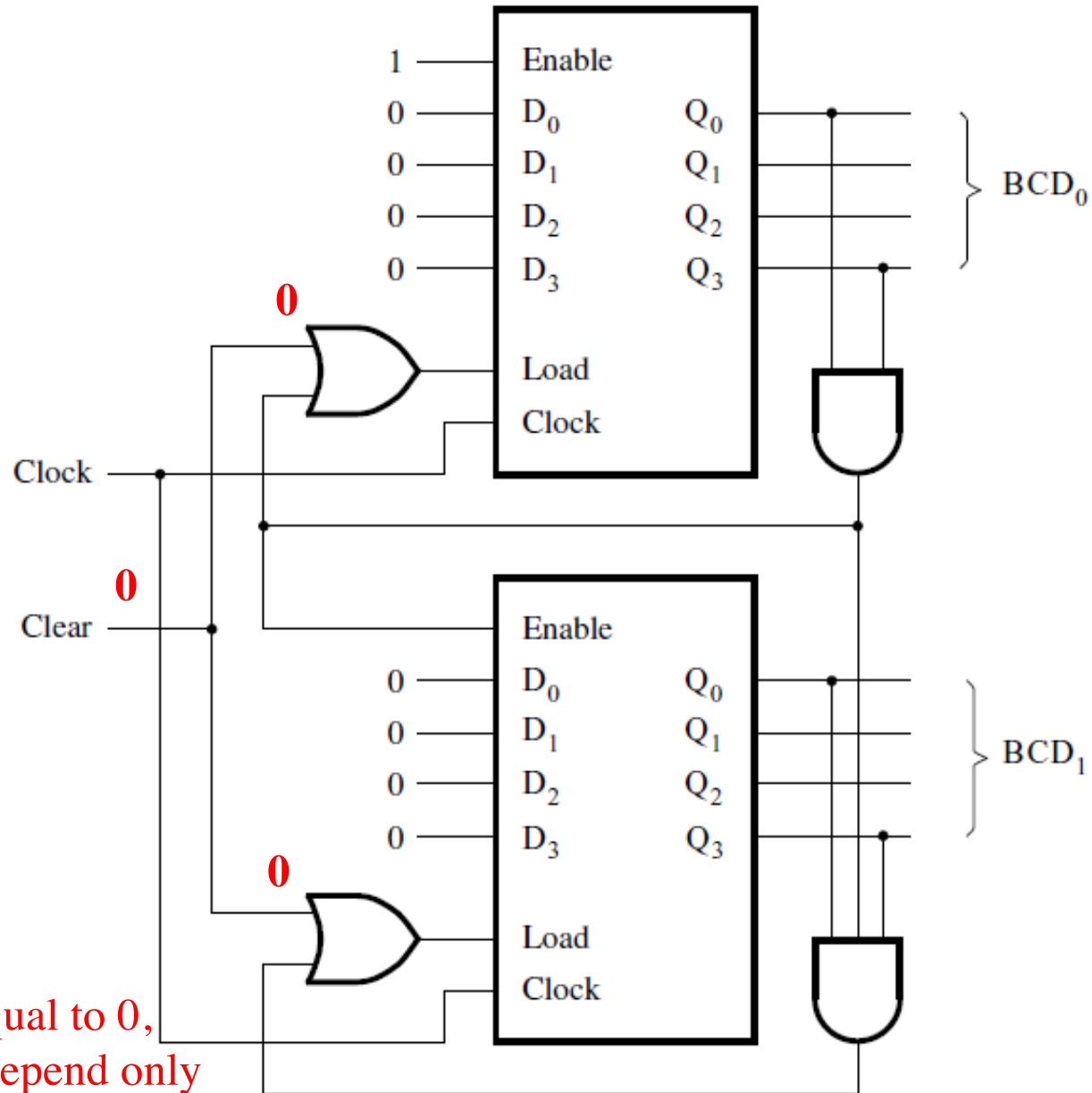
How to zero a counter



When the positive edge of the clock arrives, all outputs are set to zero together.

[Figure 5.24 from the textbook]

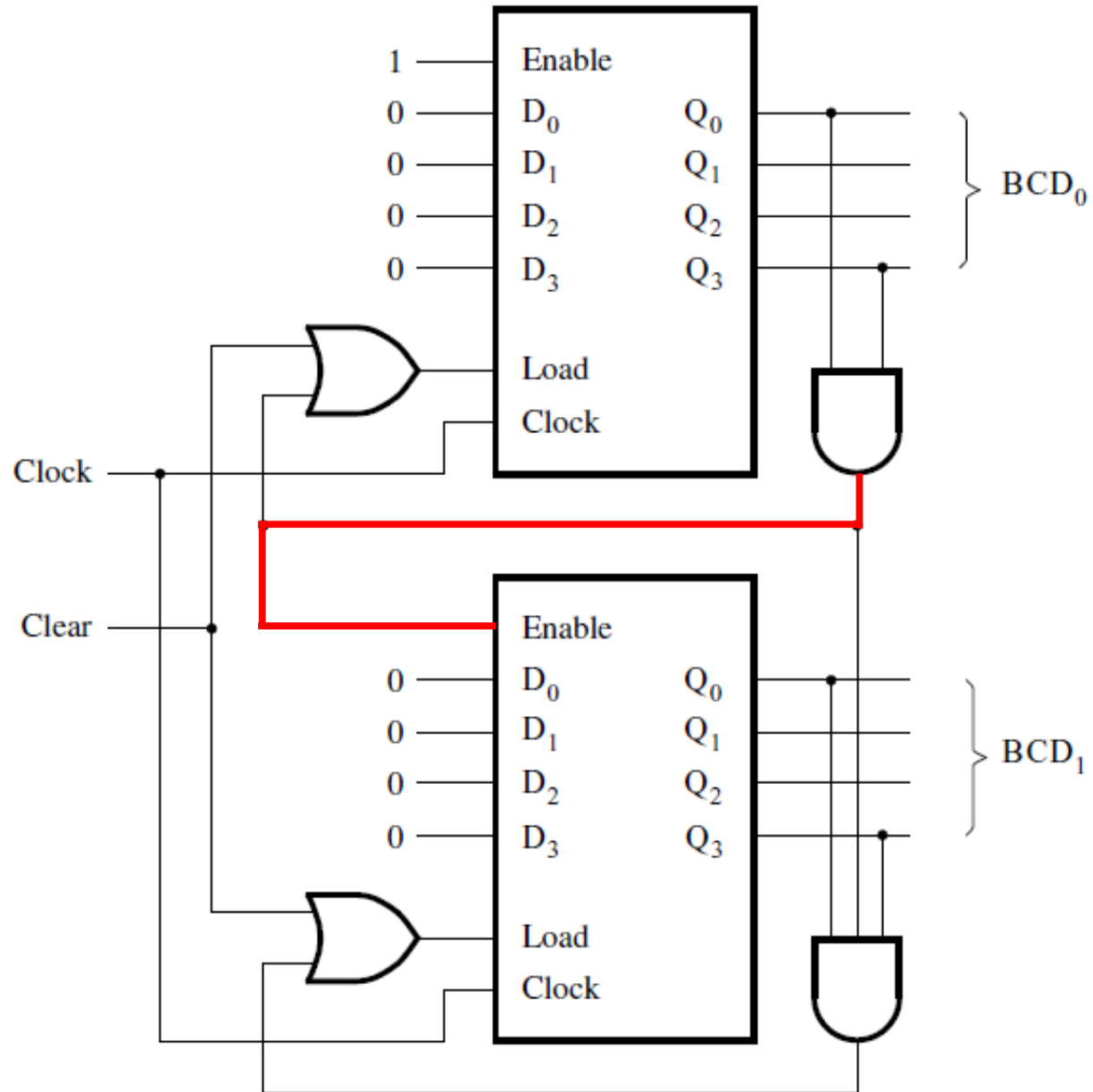
When Clear = 0



When "Clear" is equal to 0, the two OR gates depend only on the feedback connections.

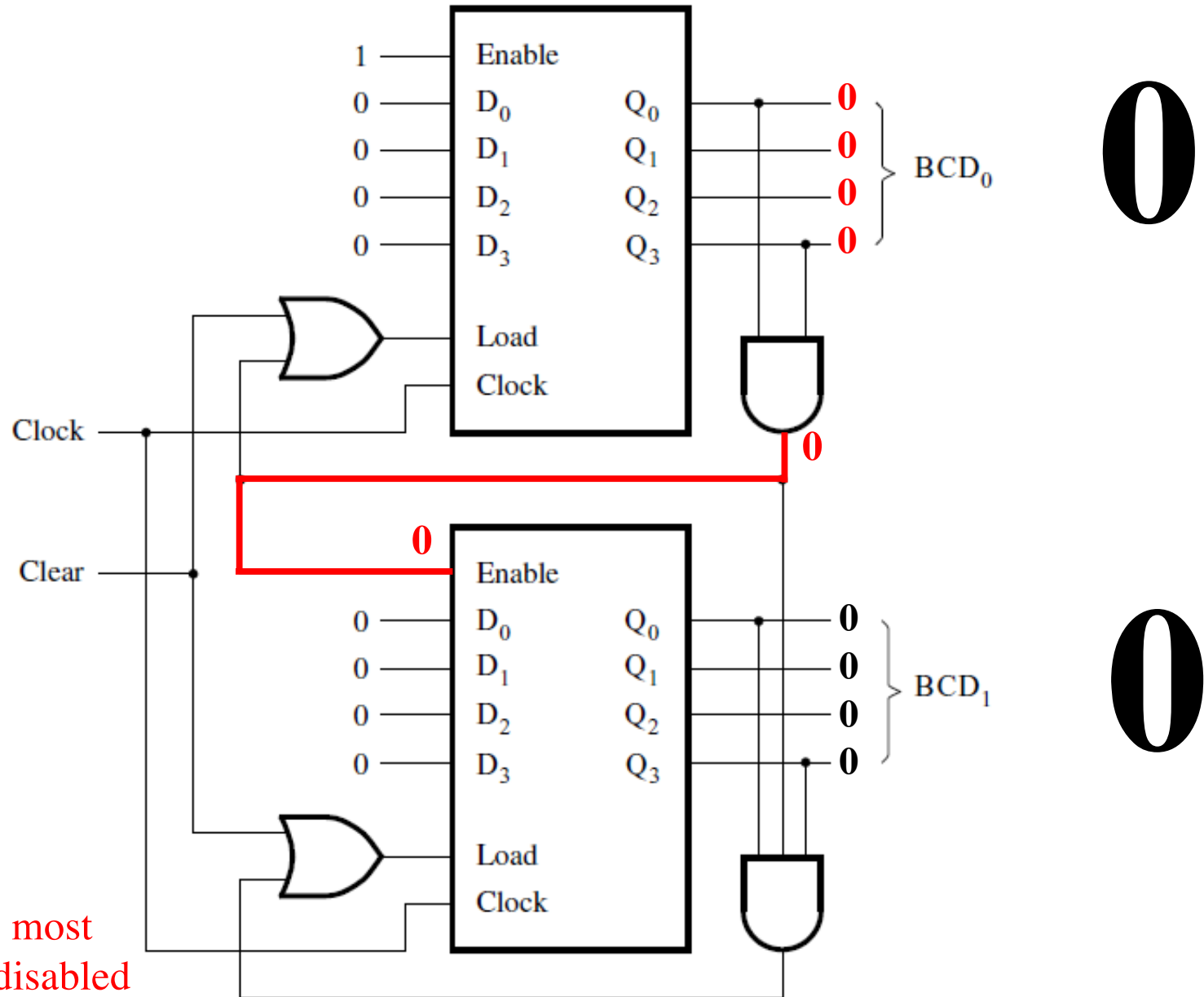
[Figure 5.27 from the textbook]

Enabling the second counter



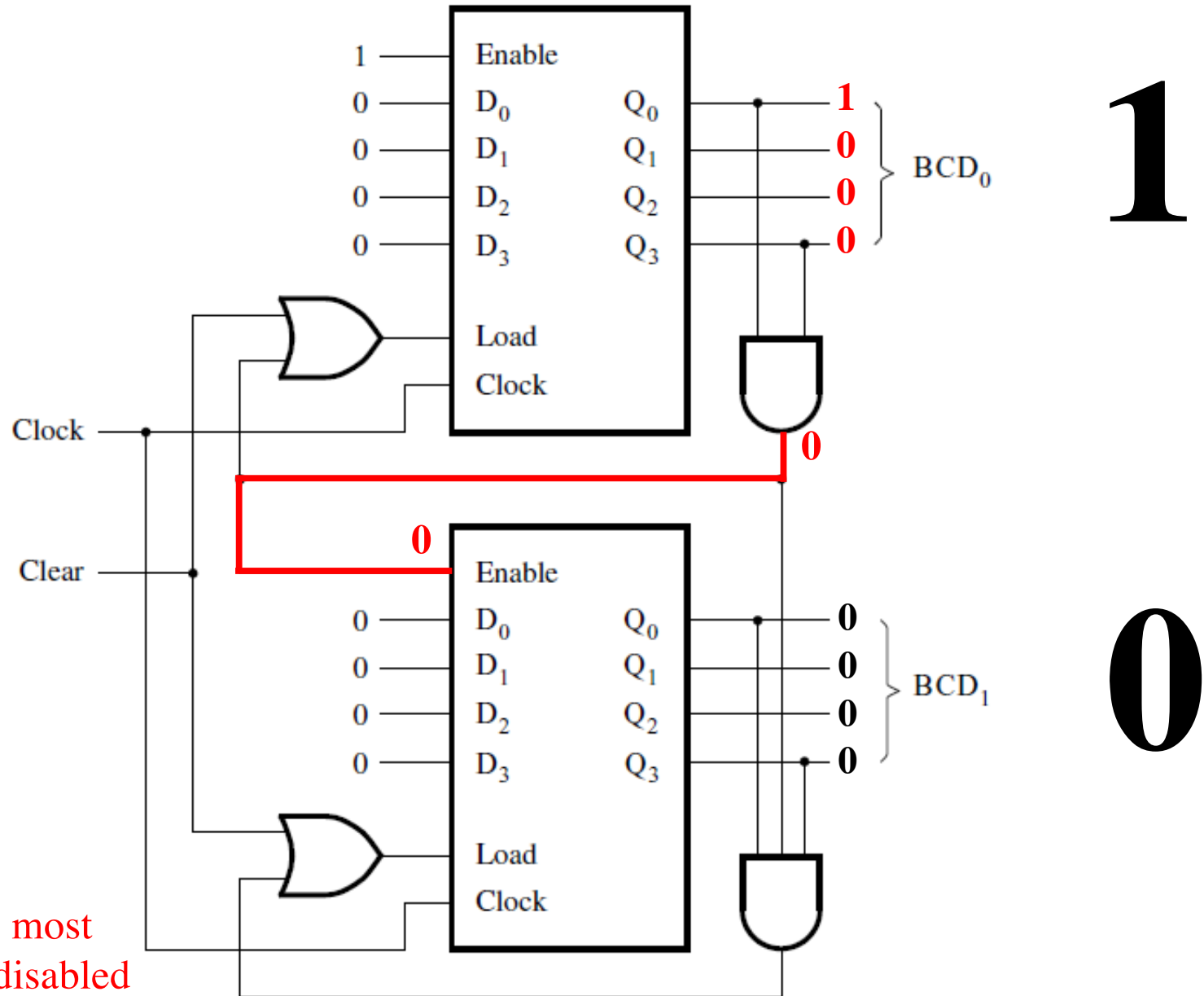
[Figure 5.27 from the textbook]

Enabling the second counter



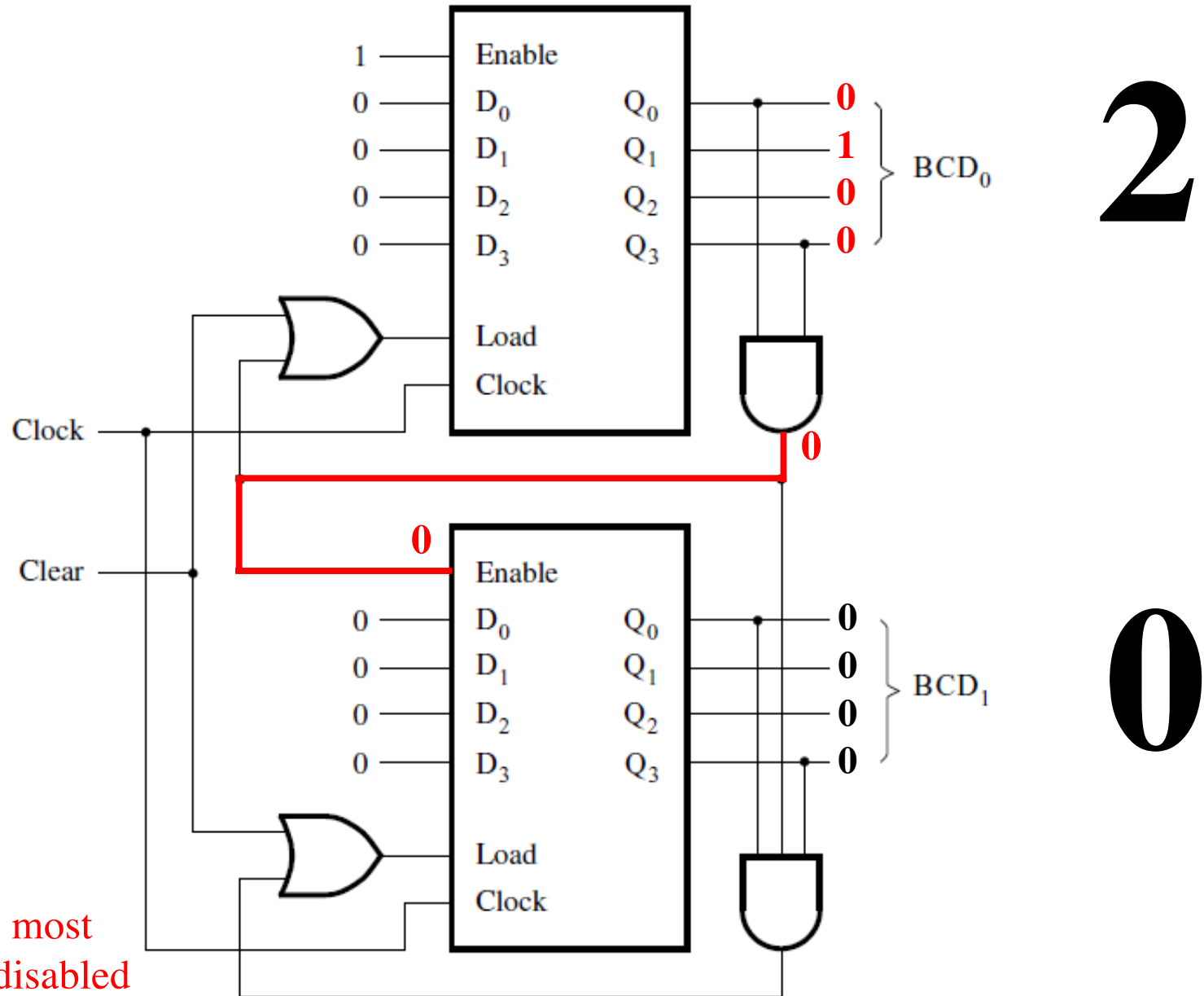
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



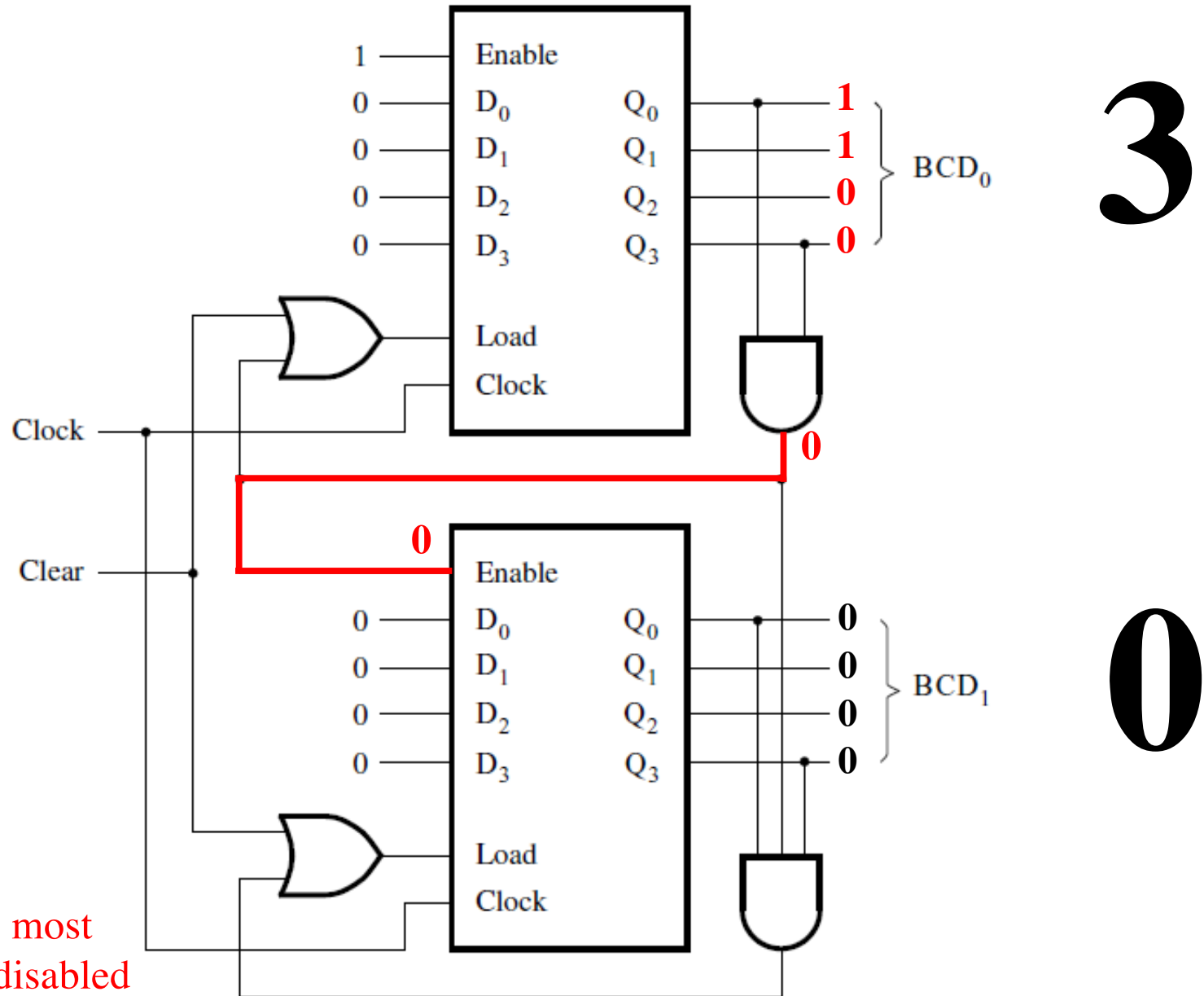
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



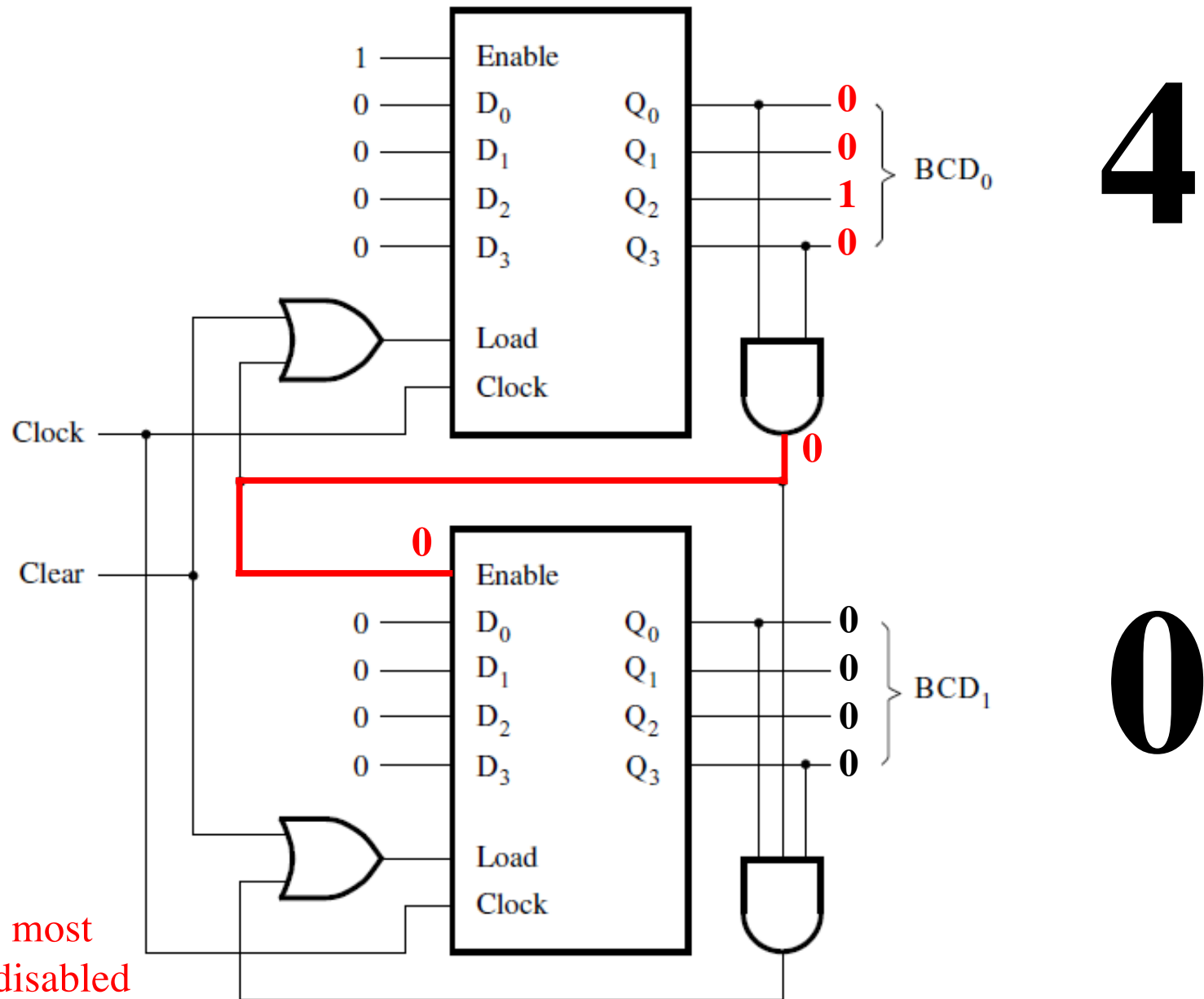
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



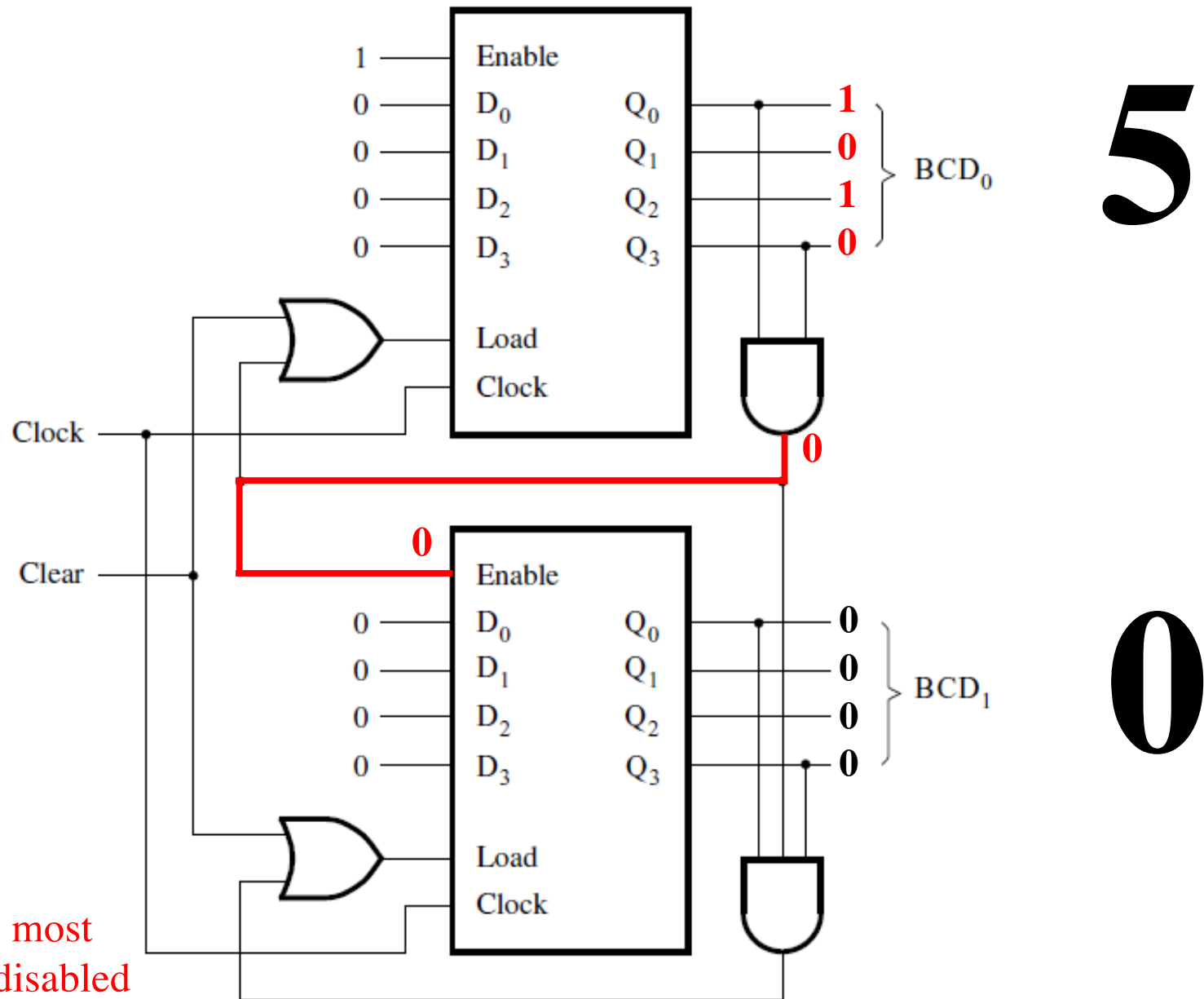
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



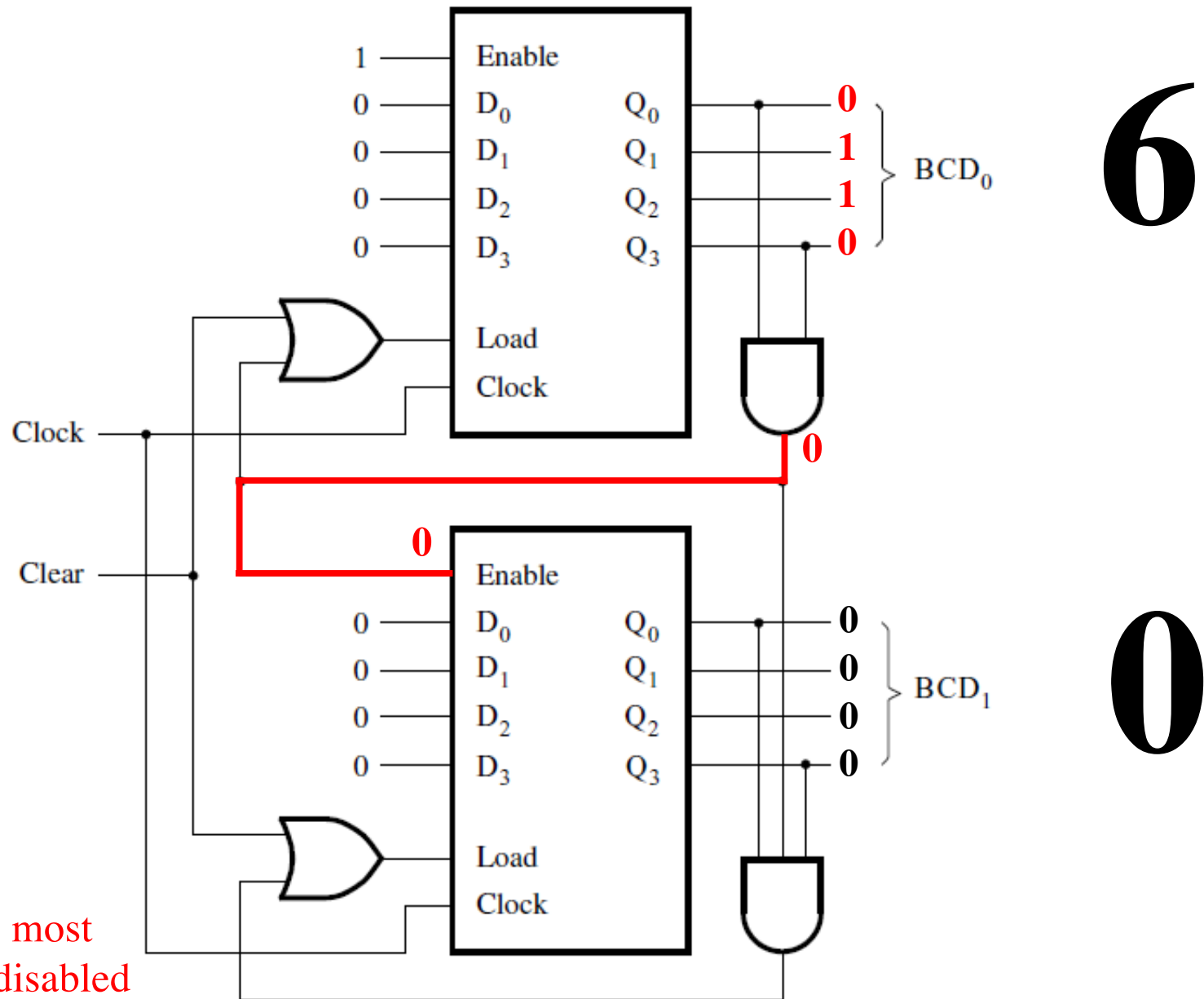
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



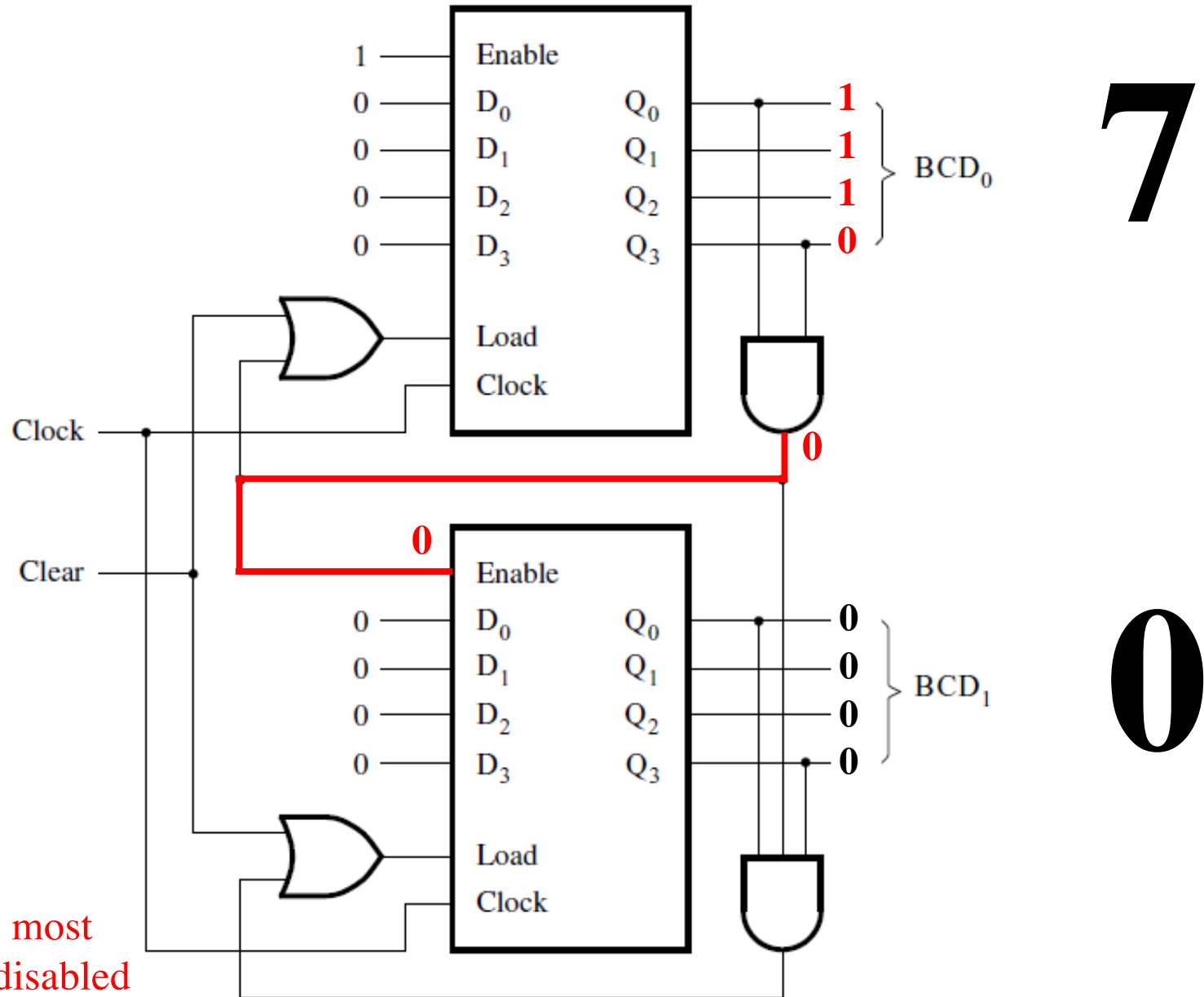
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



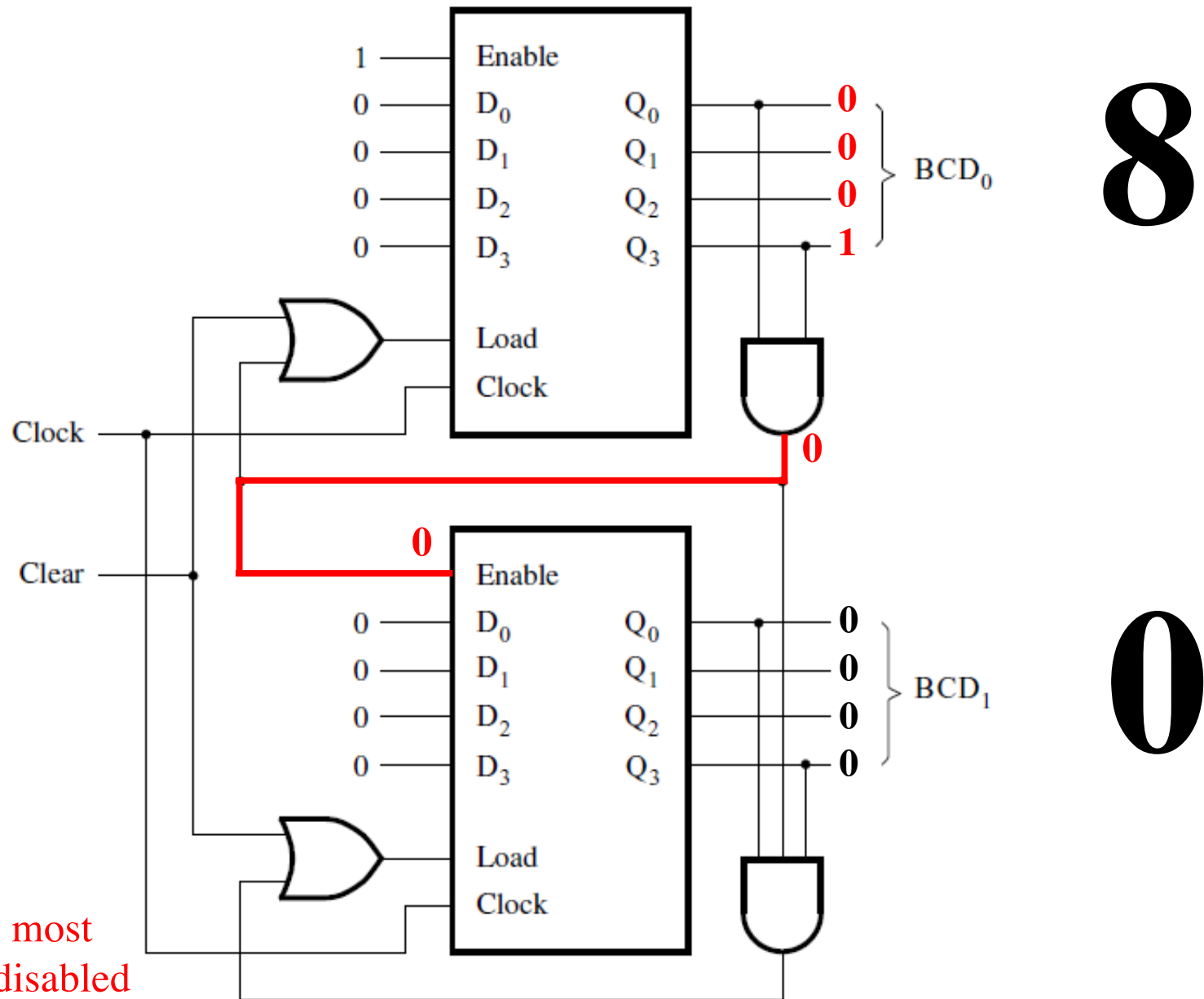
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



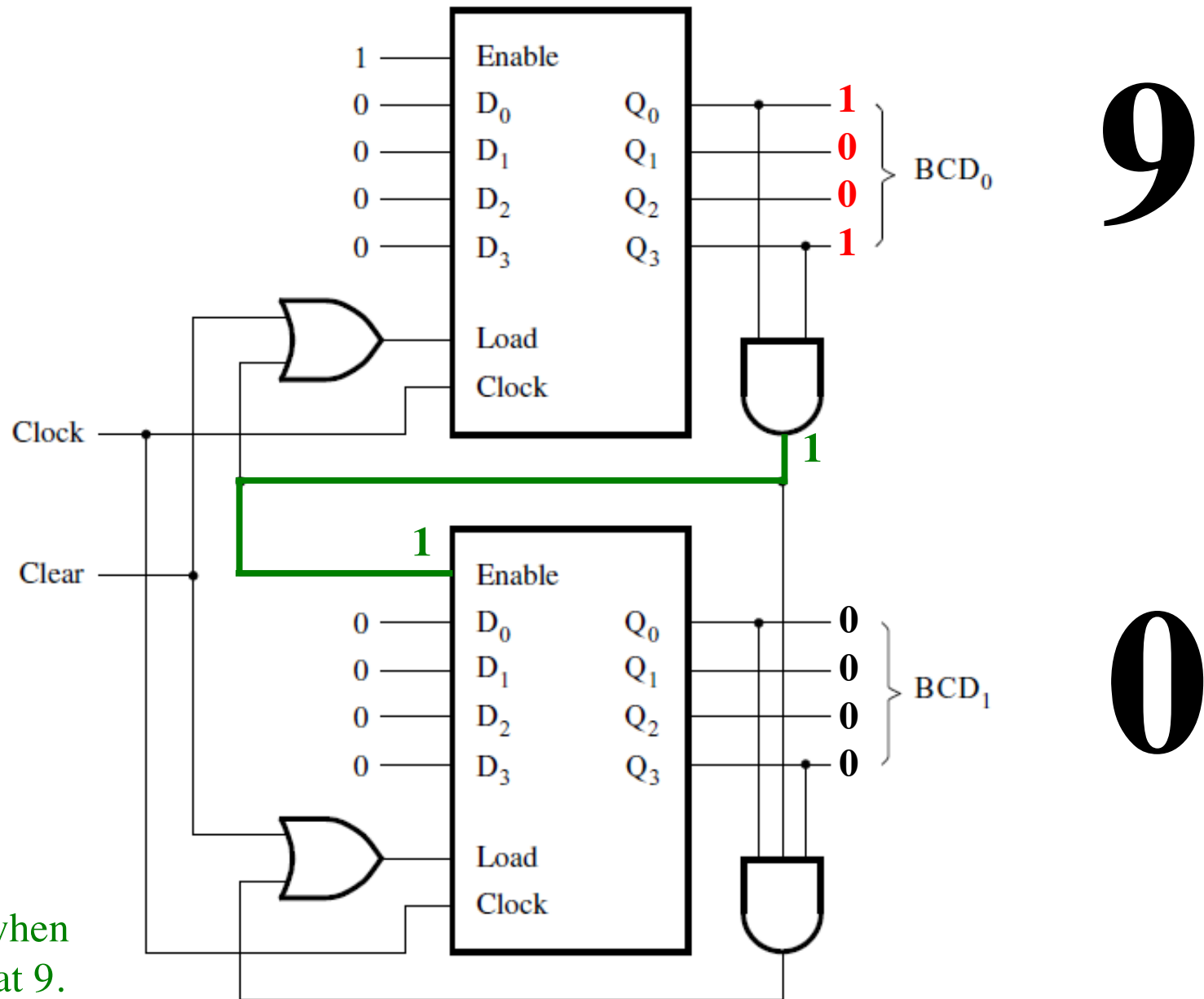
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



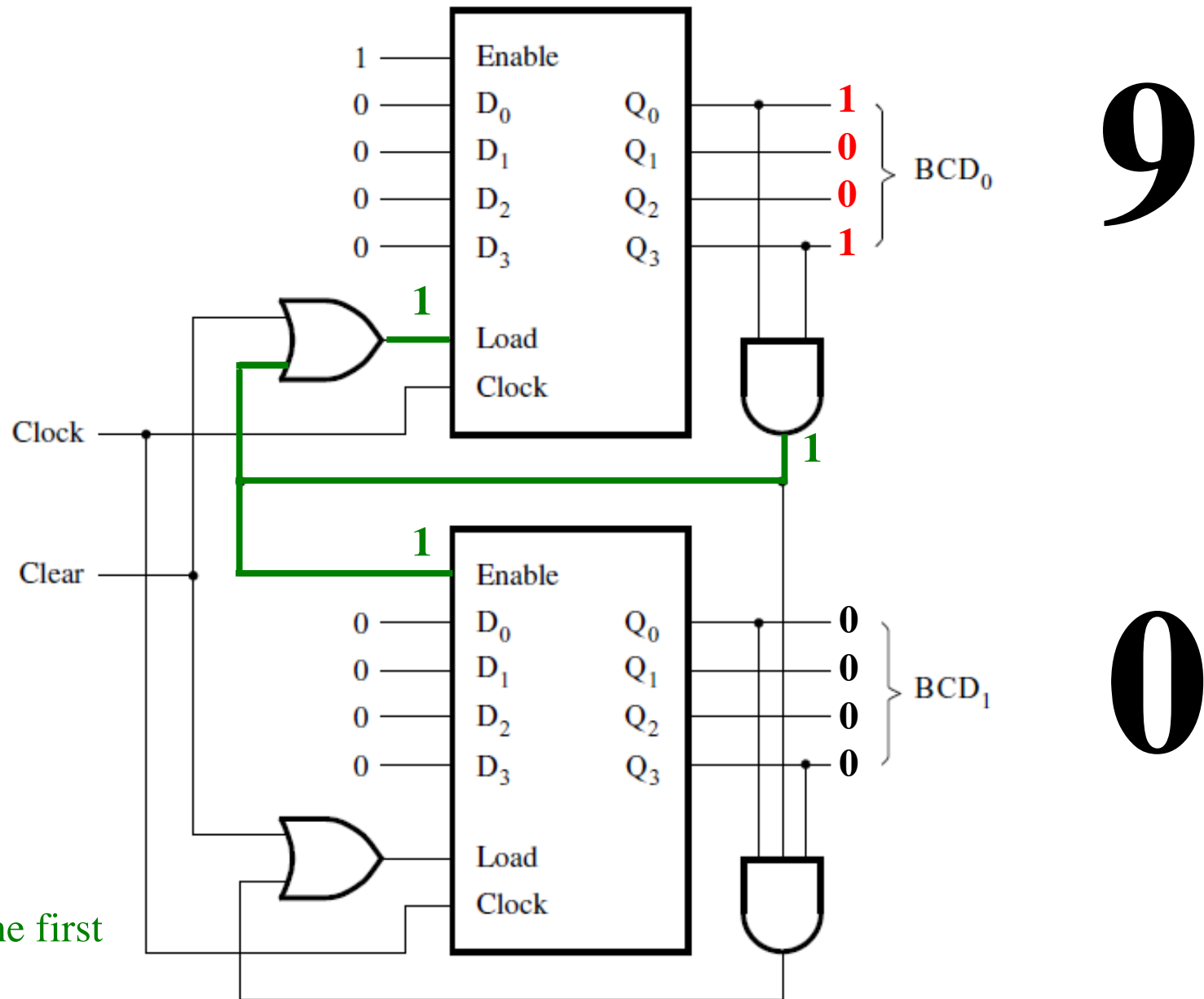
The counter for the most significant digit is disabled most of the time.

Enabling the second counter



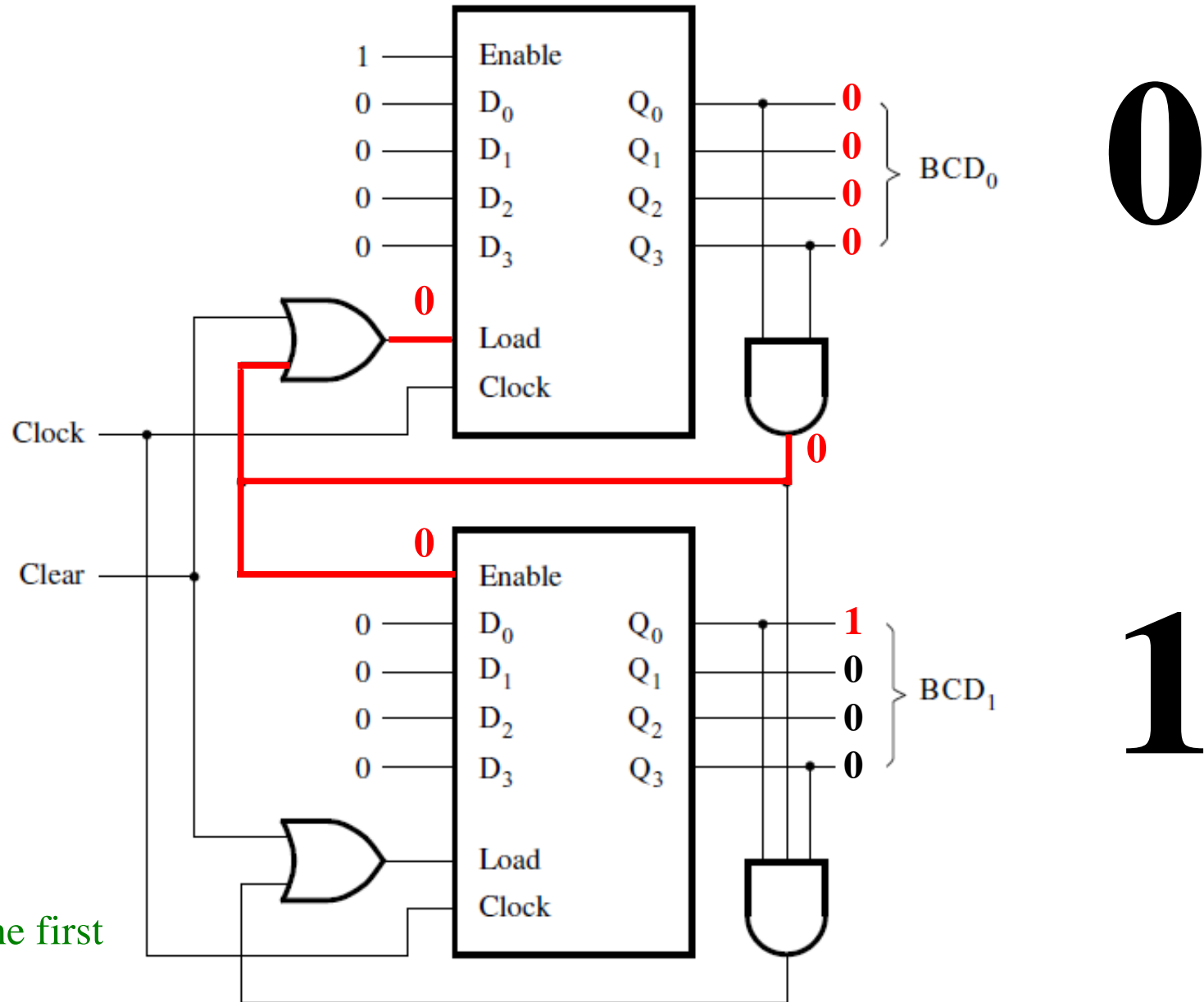
It is enabled only when the first counter is at 9.

Enabling the second counter



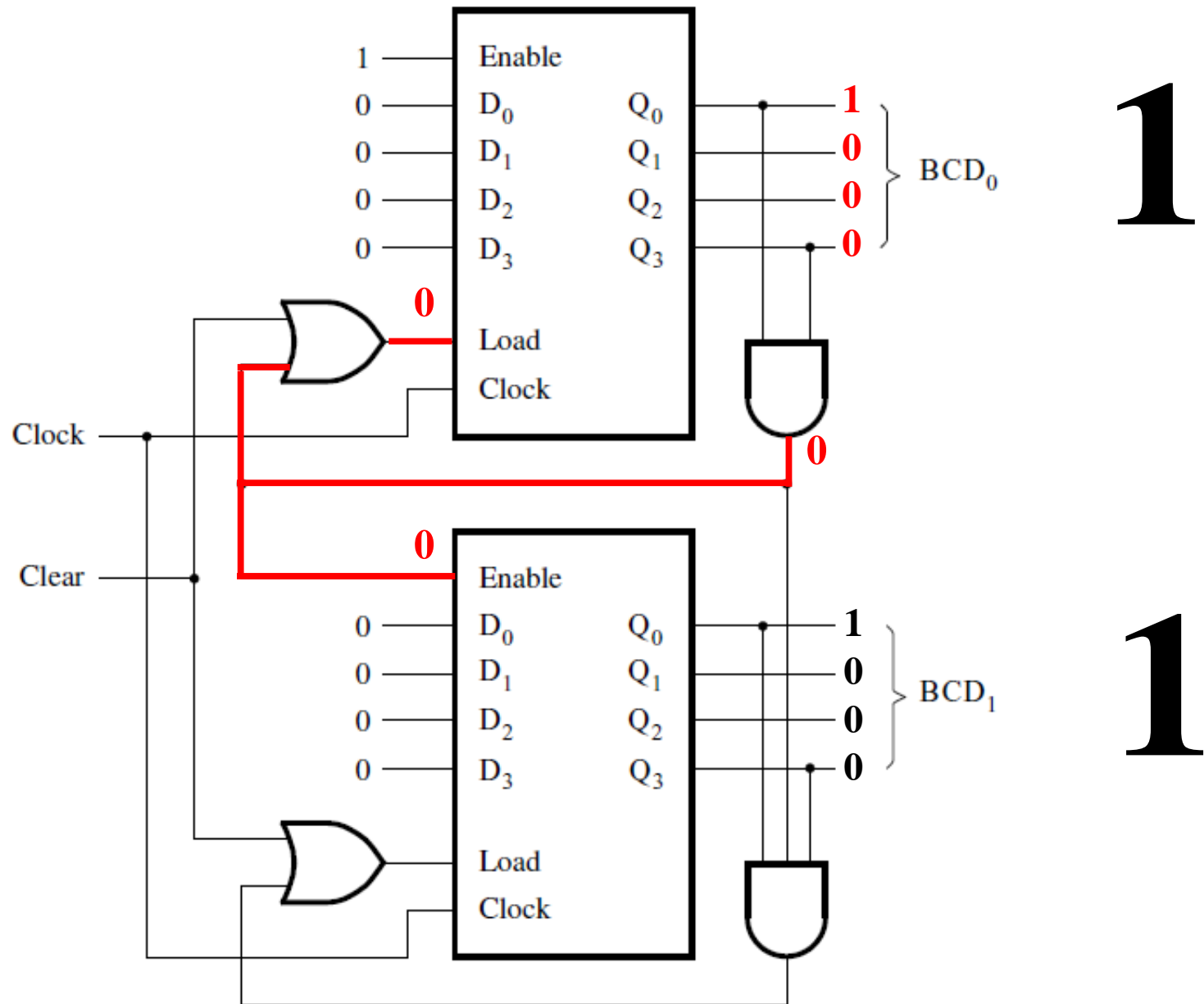
At the same time the first counter is reset.

Enabling the second counter

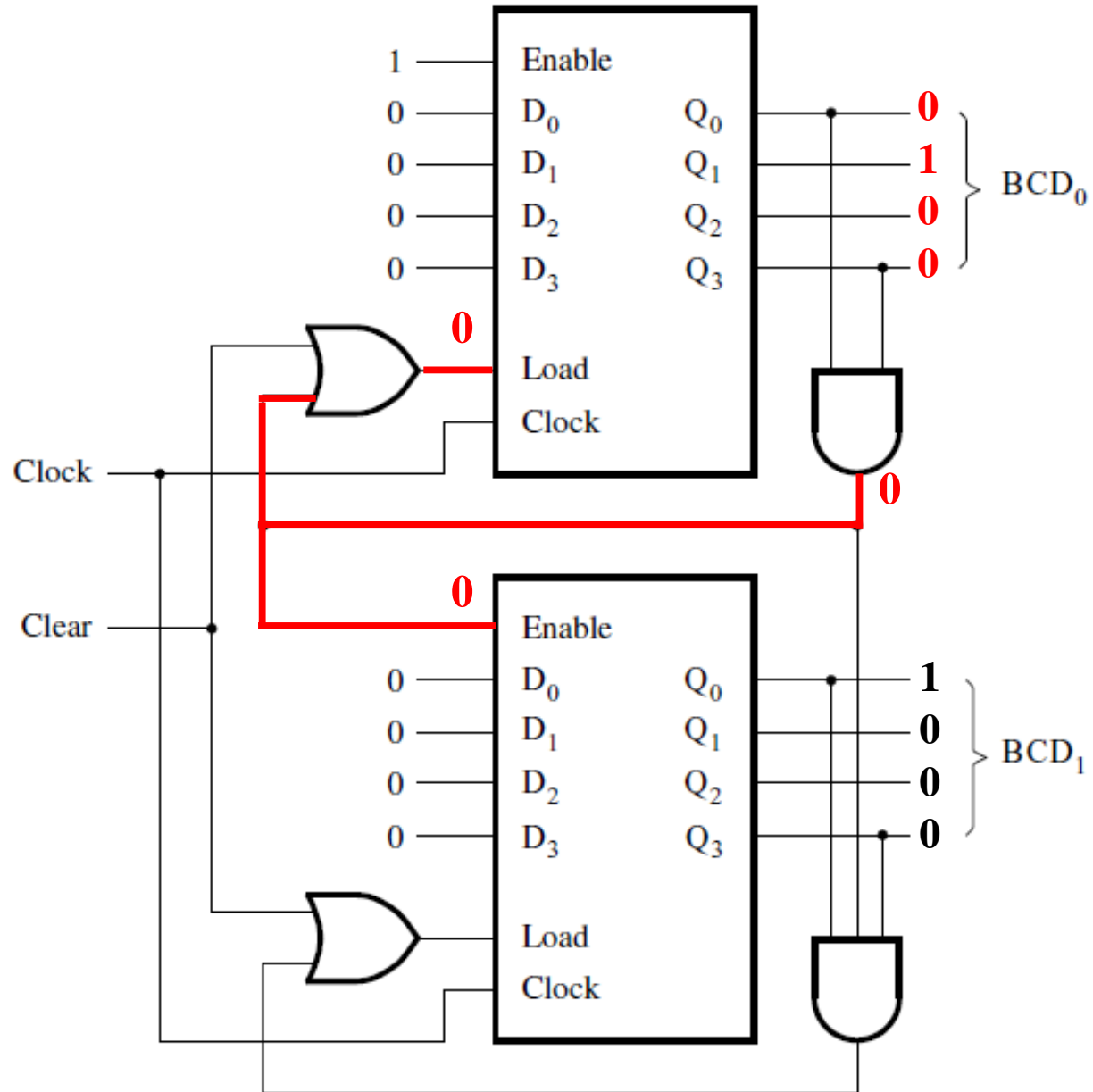


At the same time the first counter is reset.

Enabling the second counter



Enabling the second counter

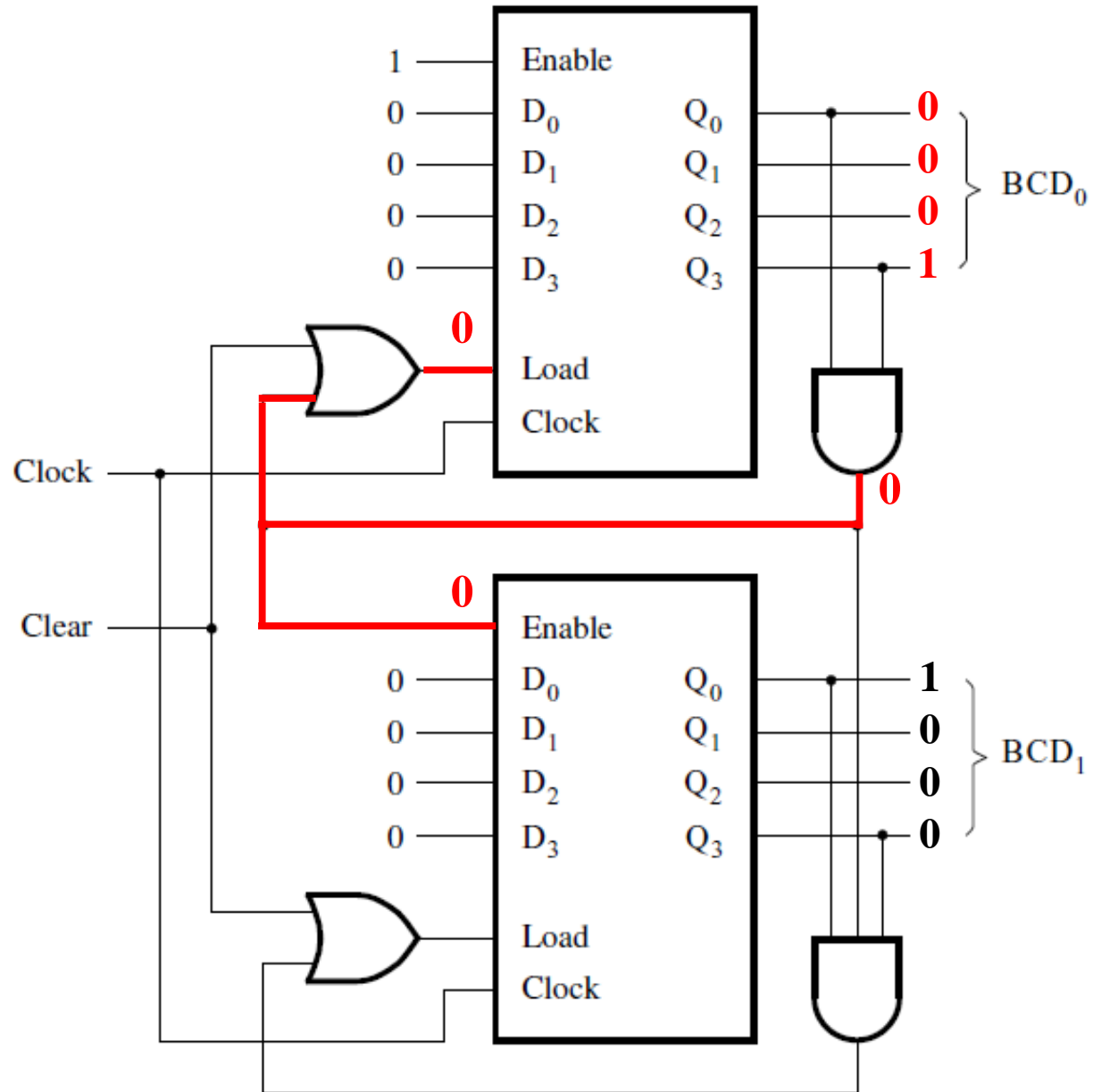


2

1



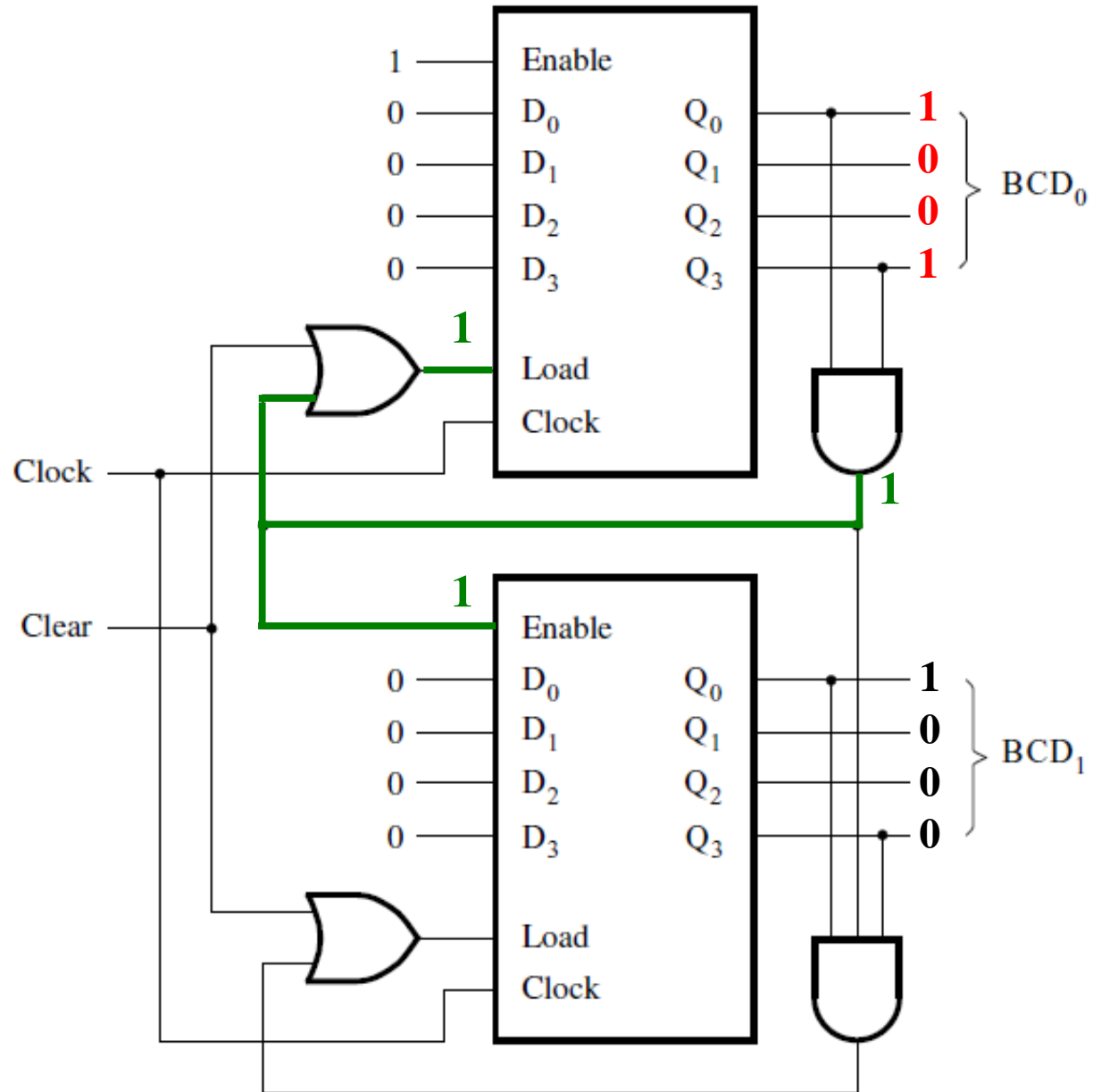
Enabling the second counter



8

1

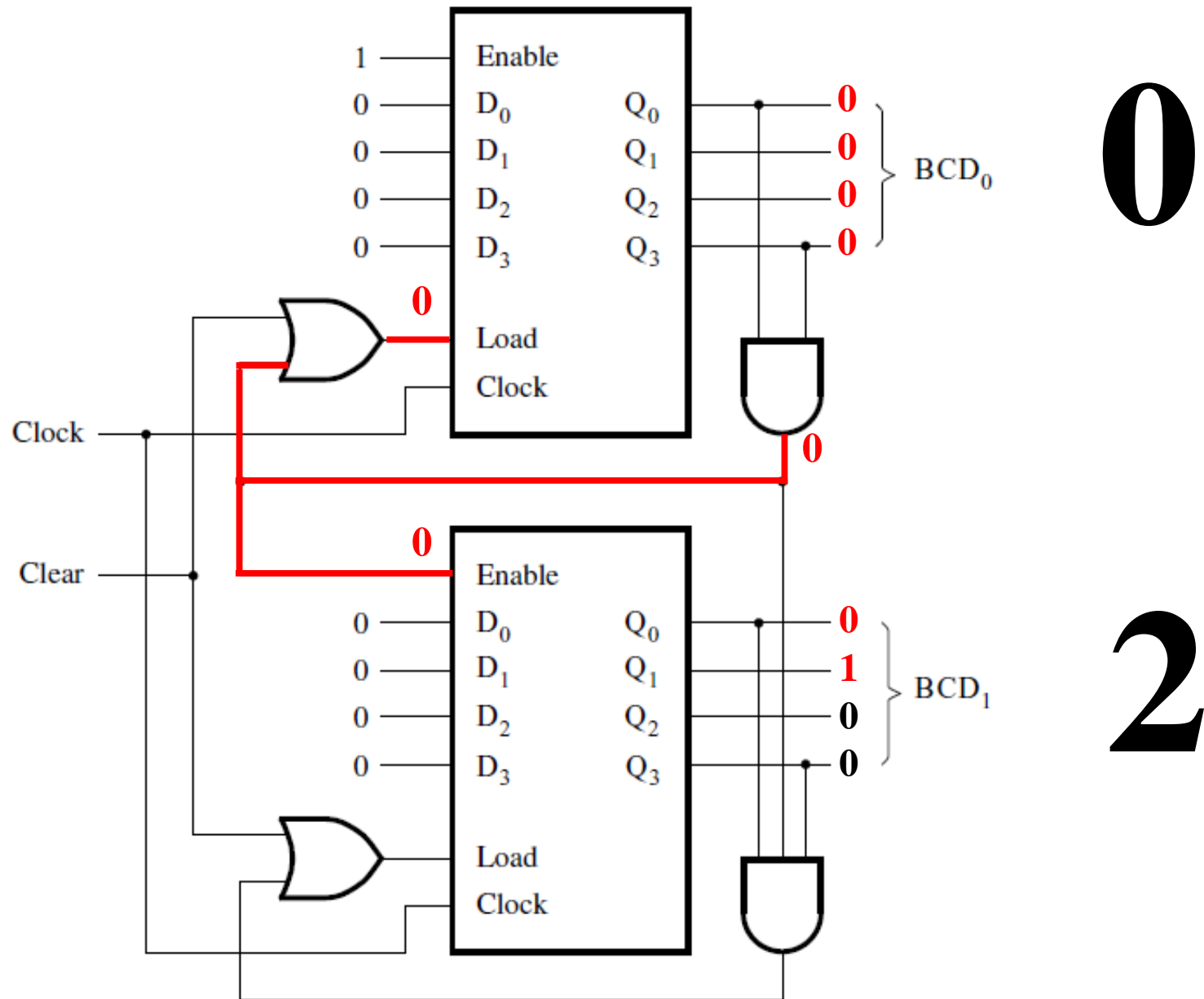
Enabling the second counter



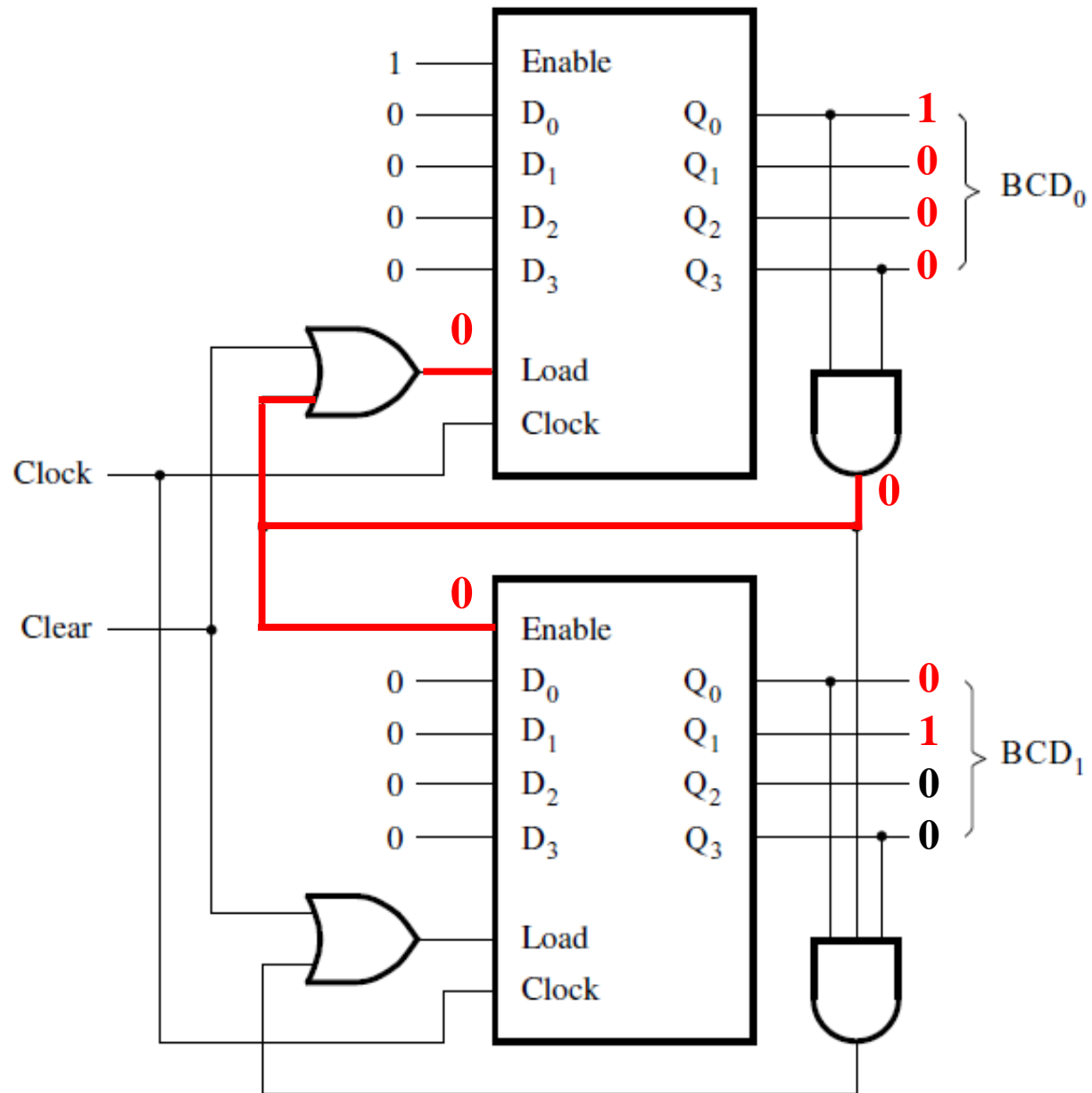
9

1

Enabling the second counter



Enabling the second counter

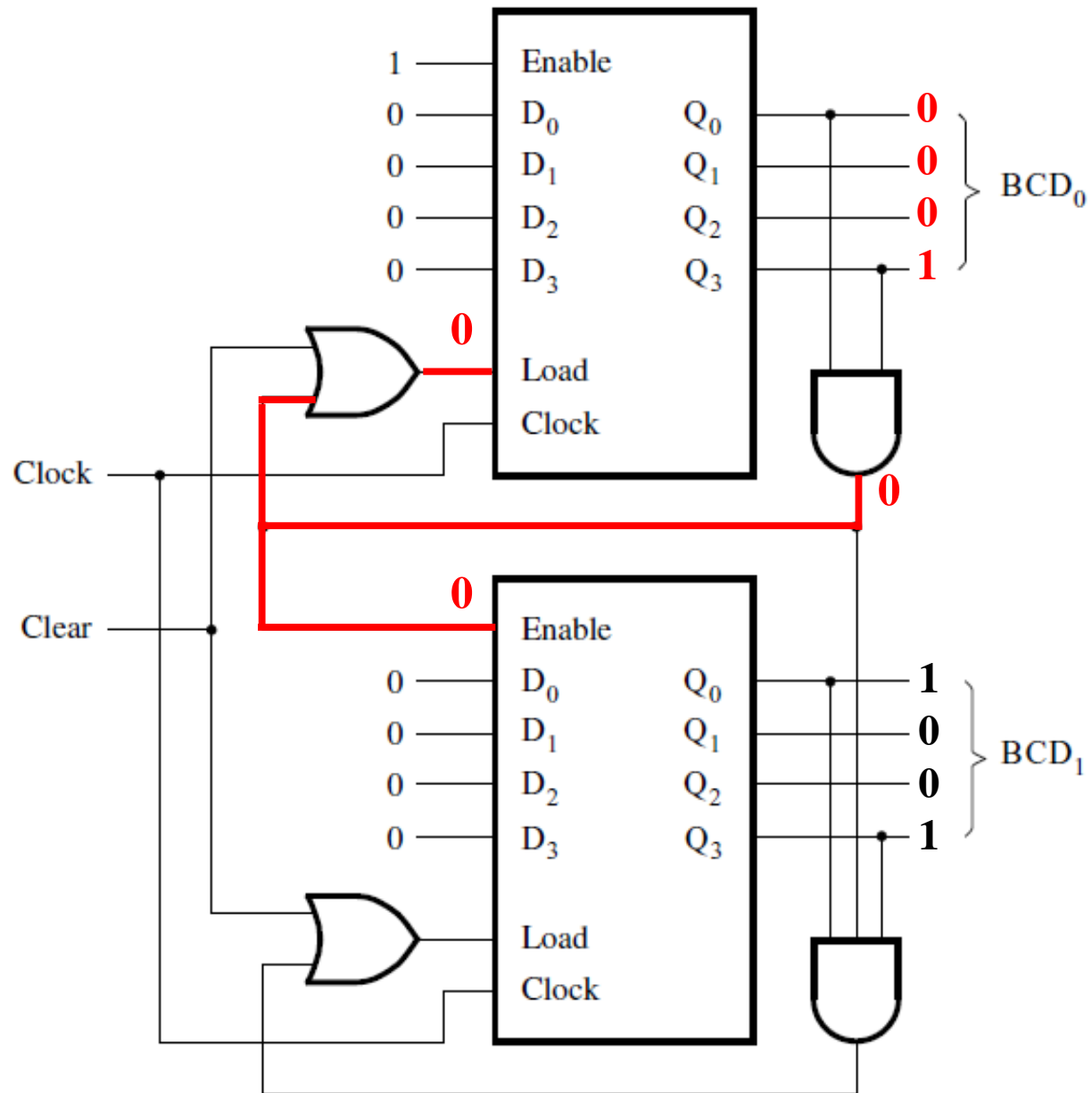


1

2



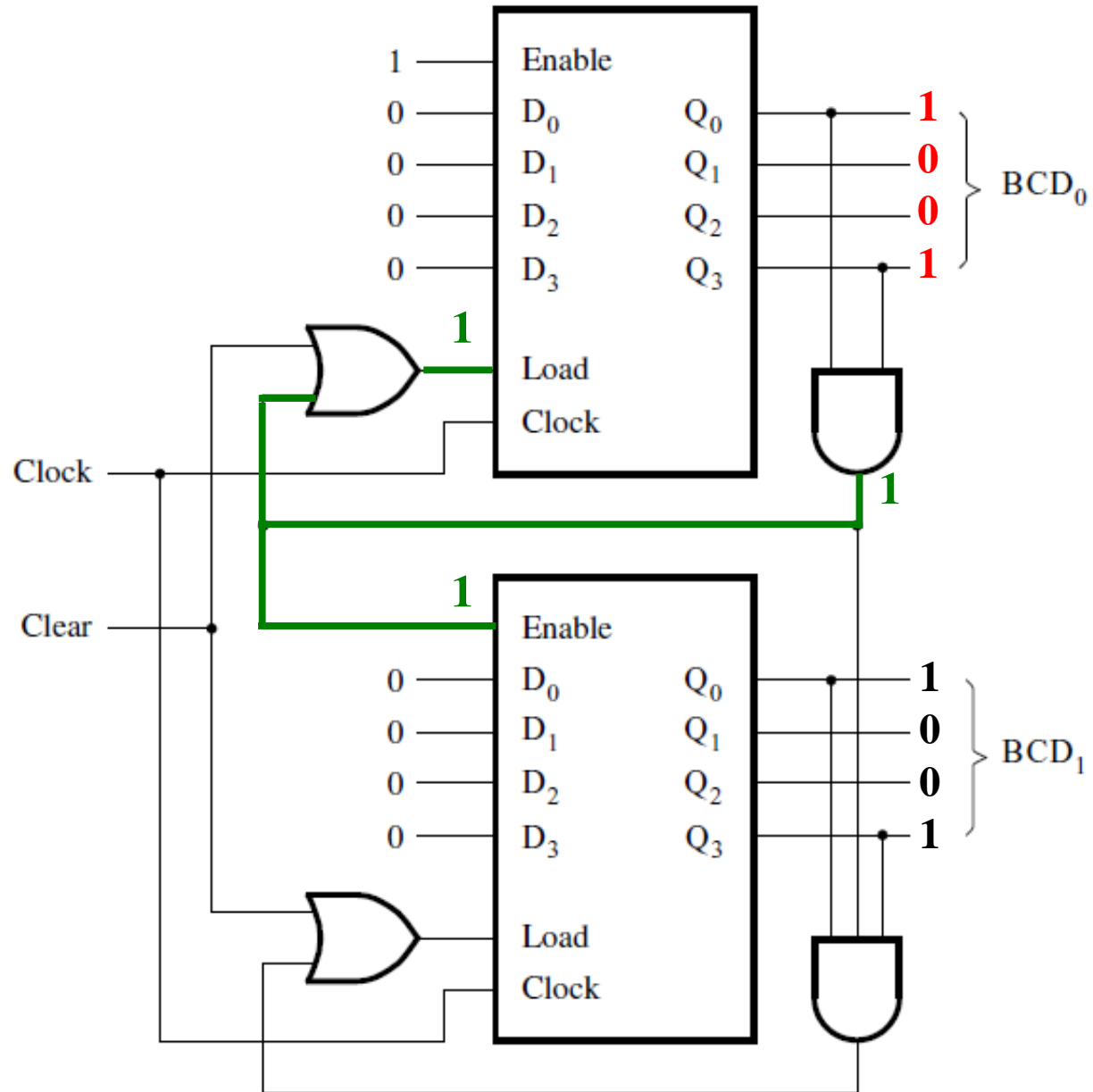
Enabling the second counter



8

9

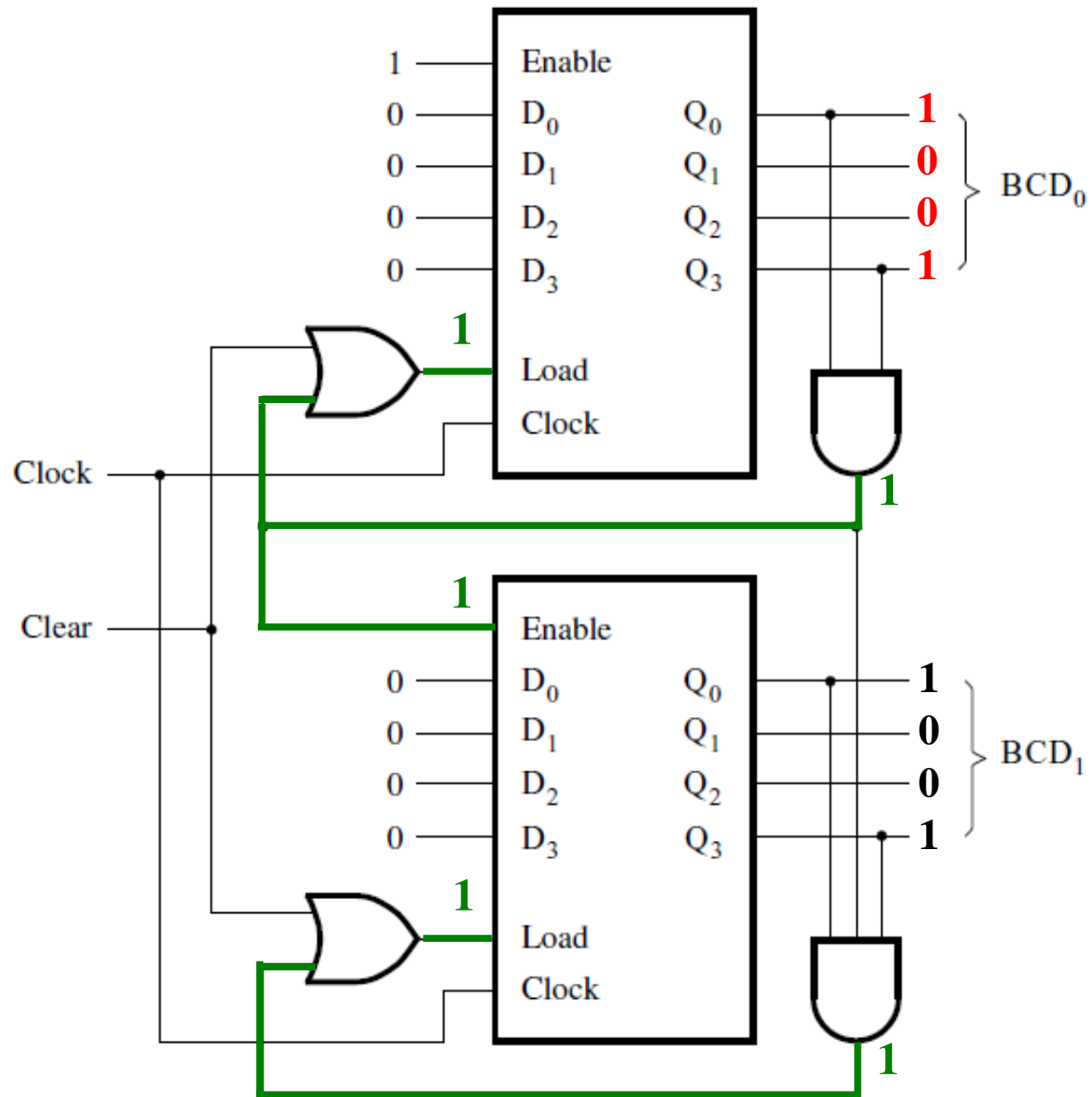
Enabling the second counter



9

9

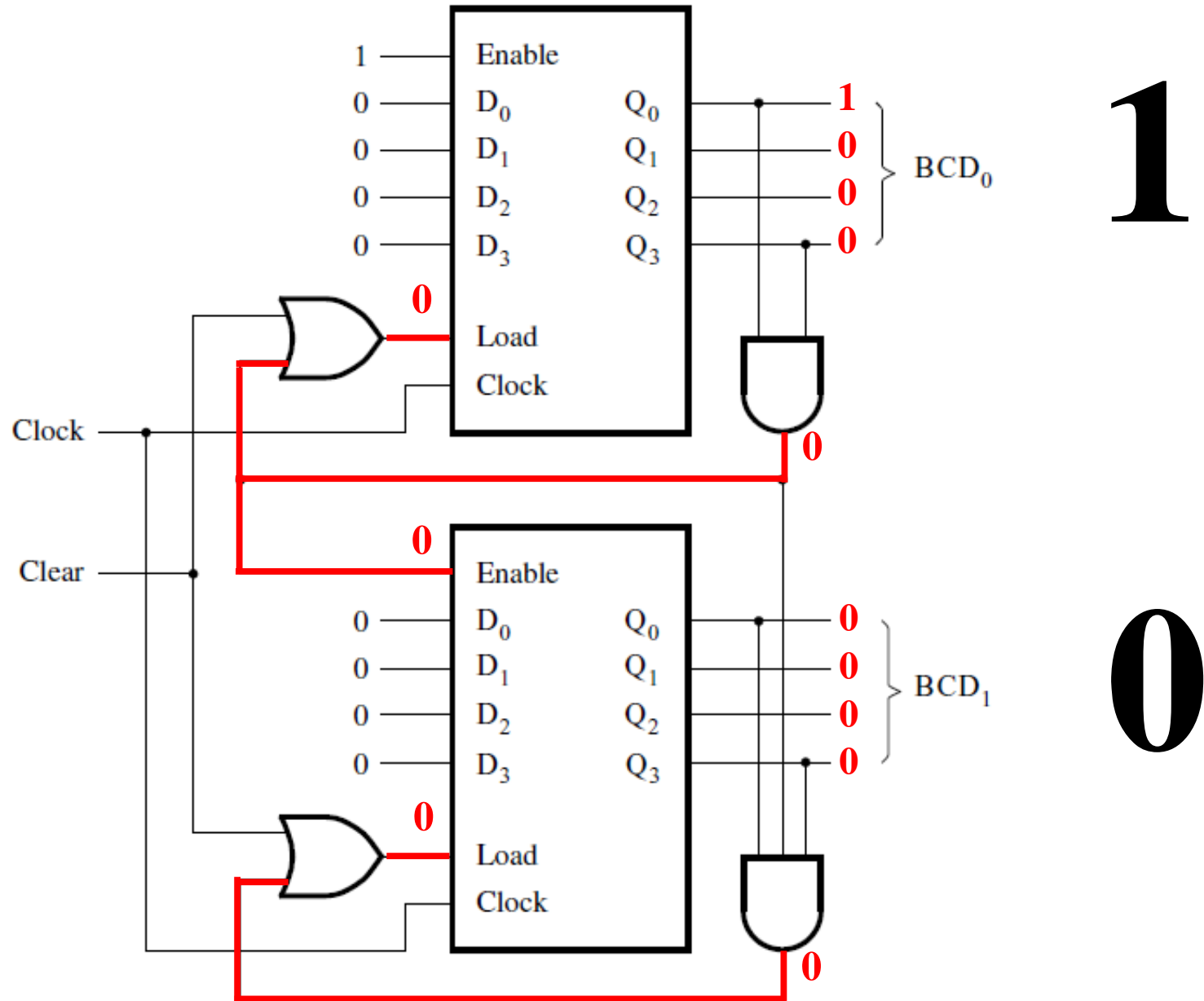
Enabling the second counter



9

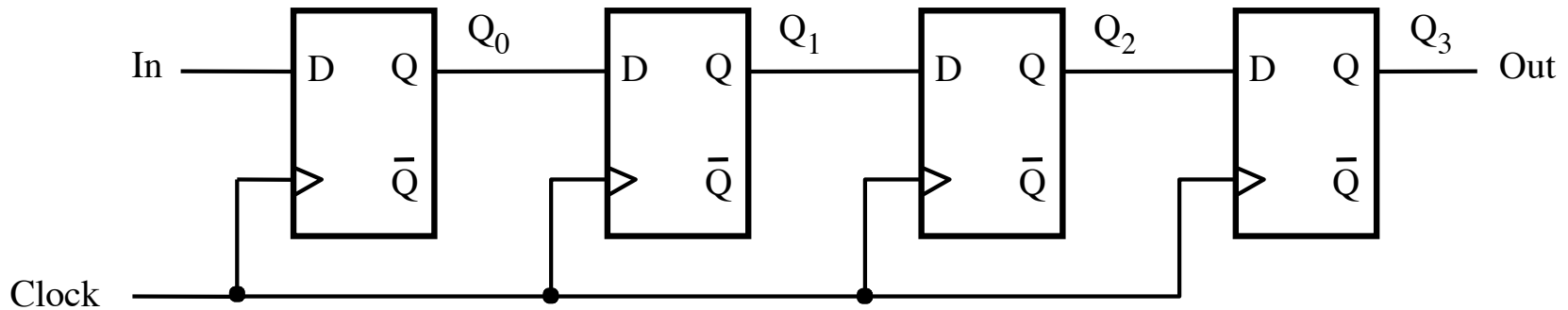
9

Enabling the second counter



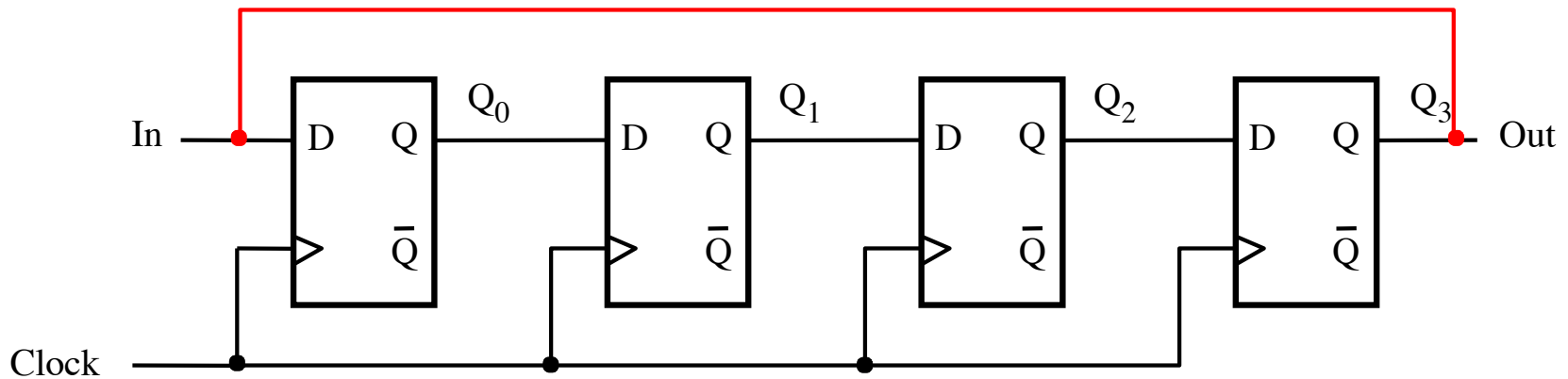
Ring Counter

How to build a 4-bit ring counter



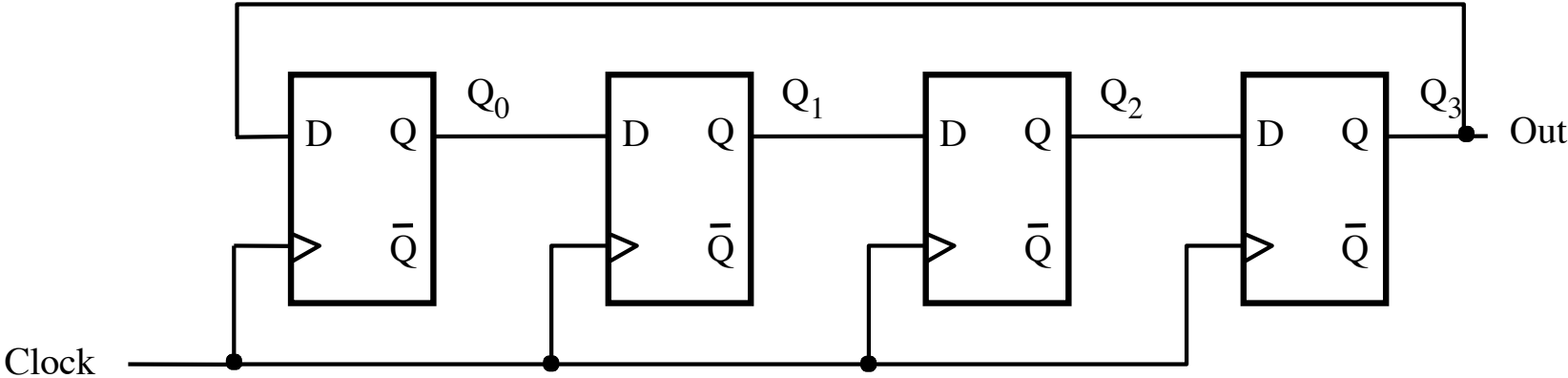
To build a ring counter we start with a shift register.

How to build a 4-bit ring counter



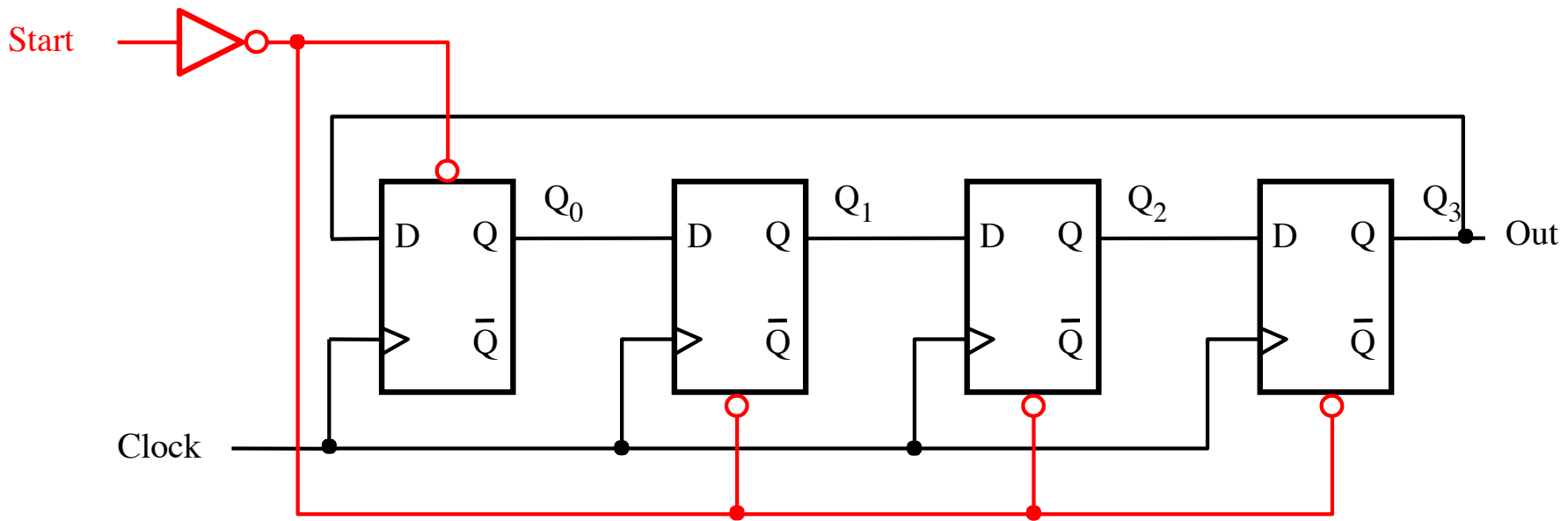
Next, add a loop from the last flip-flop to the first...

How to build a 4-bit ring counter



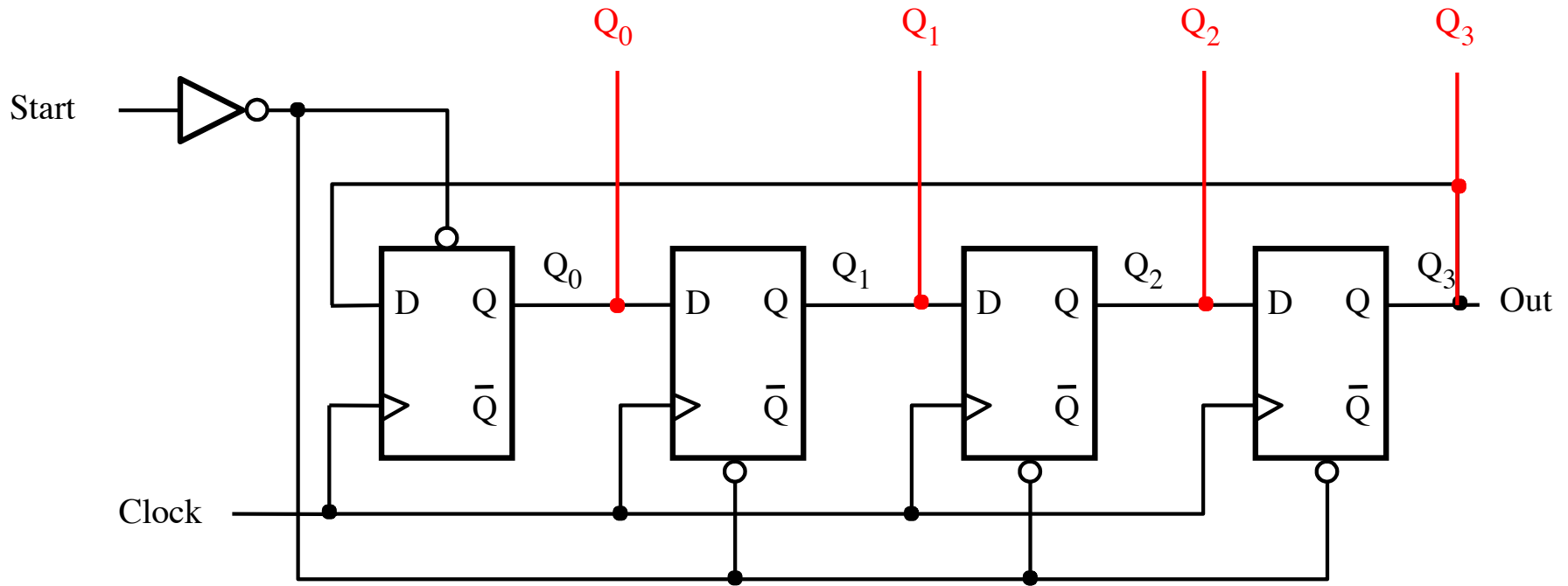
... and remove the In input line.

How to build a 4-bit ring counter



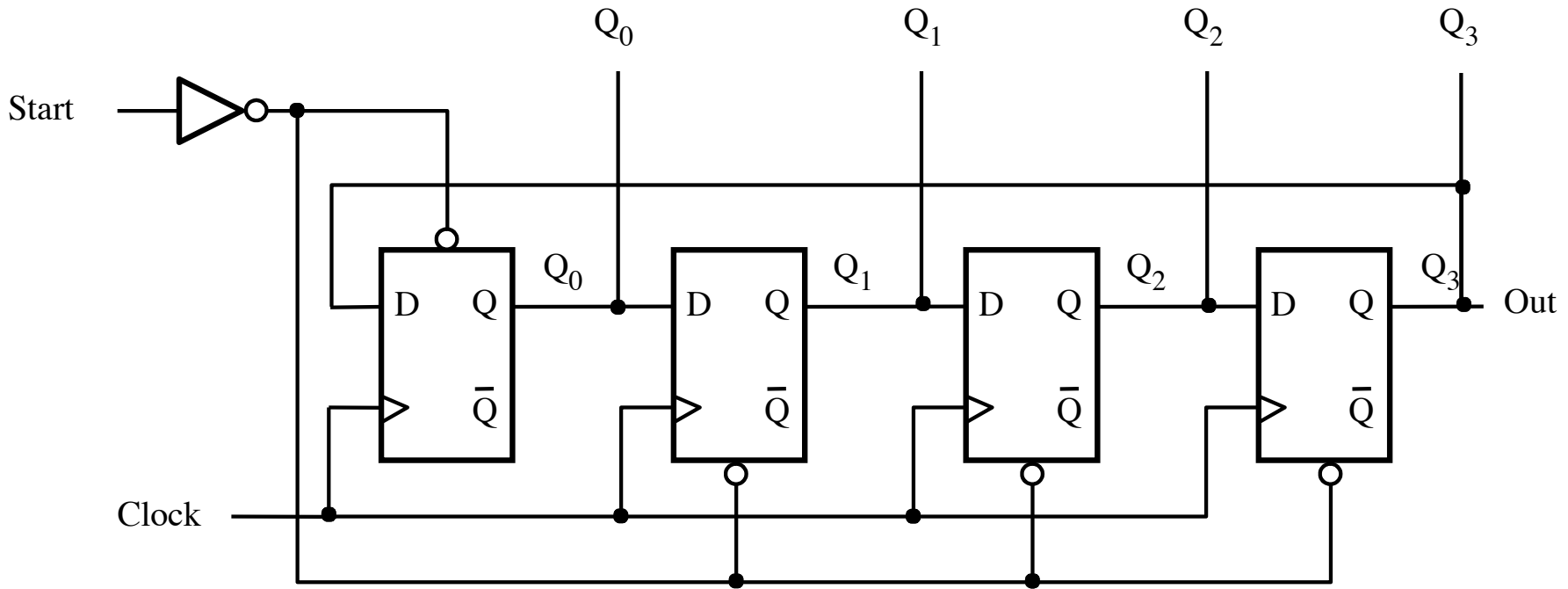
Also, add a start input that goes inverted to preset_n of the first flip=flop and to clear_n of all remaining flip-flops.

How to build a 4-bit ring counter



Finally, extend the output lines that form the count number.

How to build a 4-bit ring counter

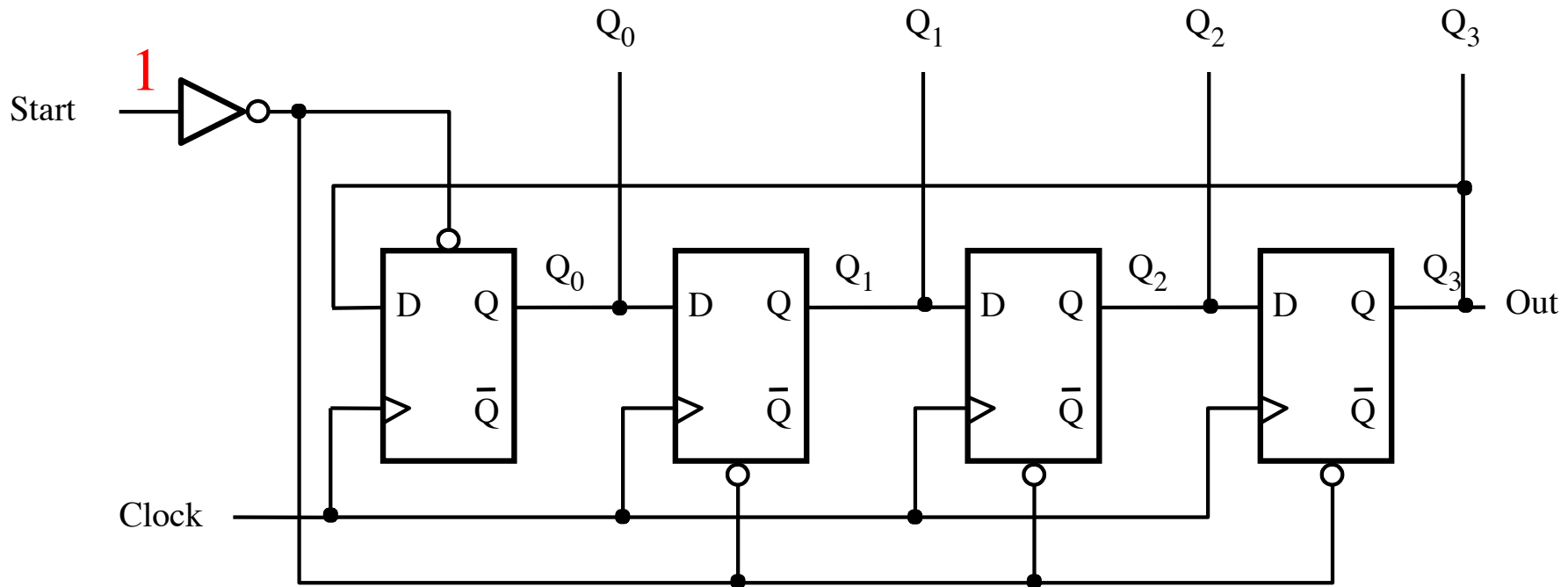


This is the final circuit diagram.

4-bit ring counter

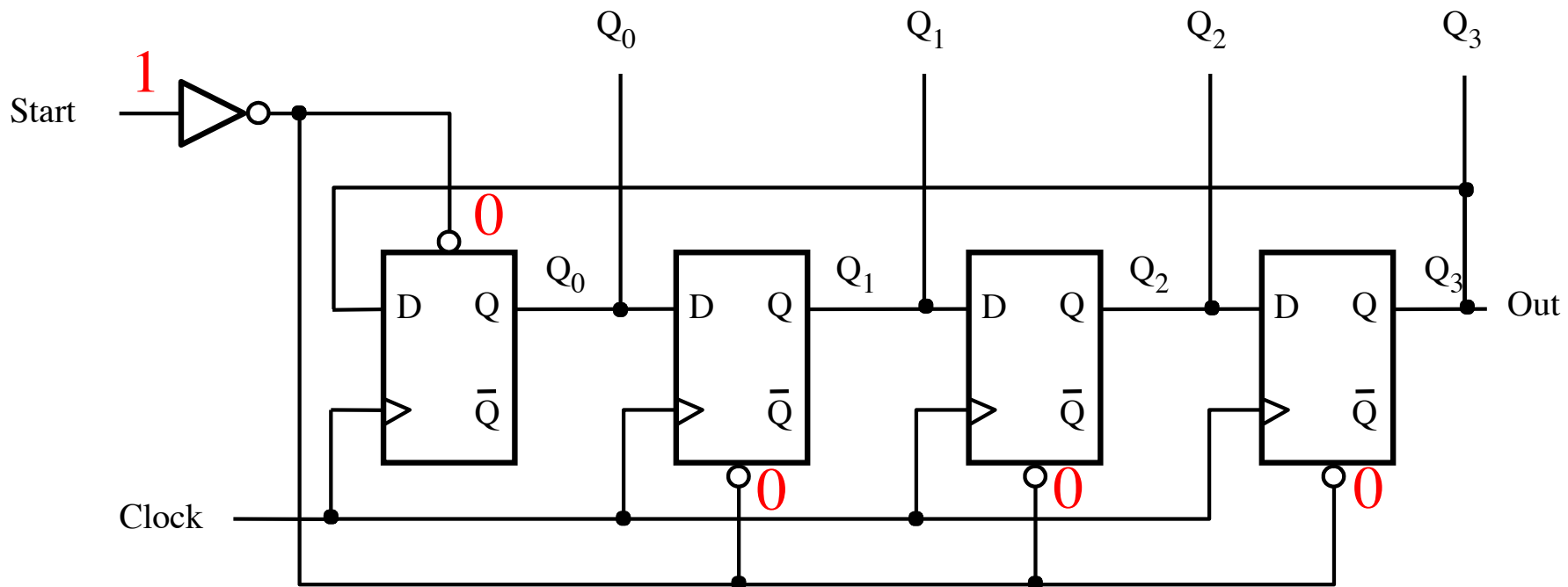
- There is only one 1 on the outputs of the four flip-flops
- The counting sequence is: 1000, 0100, 0010, 0001, 1000, ...
- To reset the counter
 - set start to 1 for a short period of time
 - This sets the four outputs to 1000

4-bit ring counter: How does it work



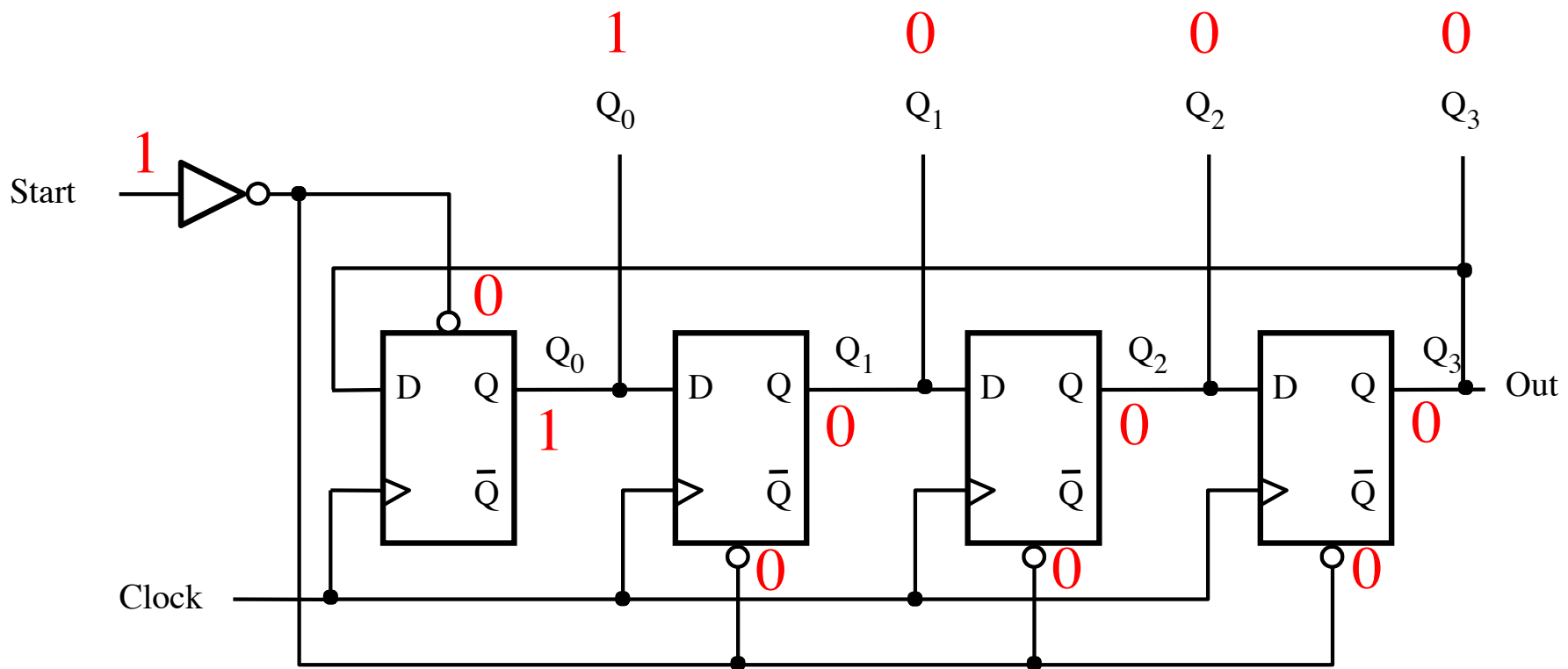
To initialize the counter set Start to 1.

4-bit ring counter: How does it work



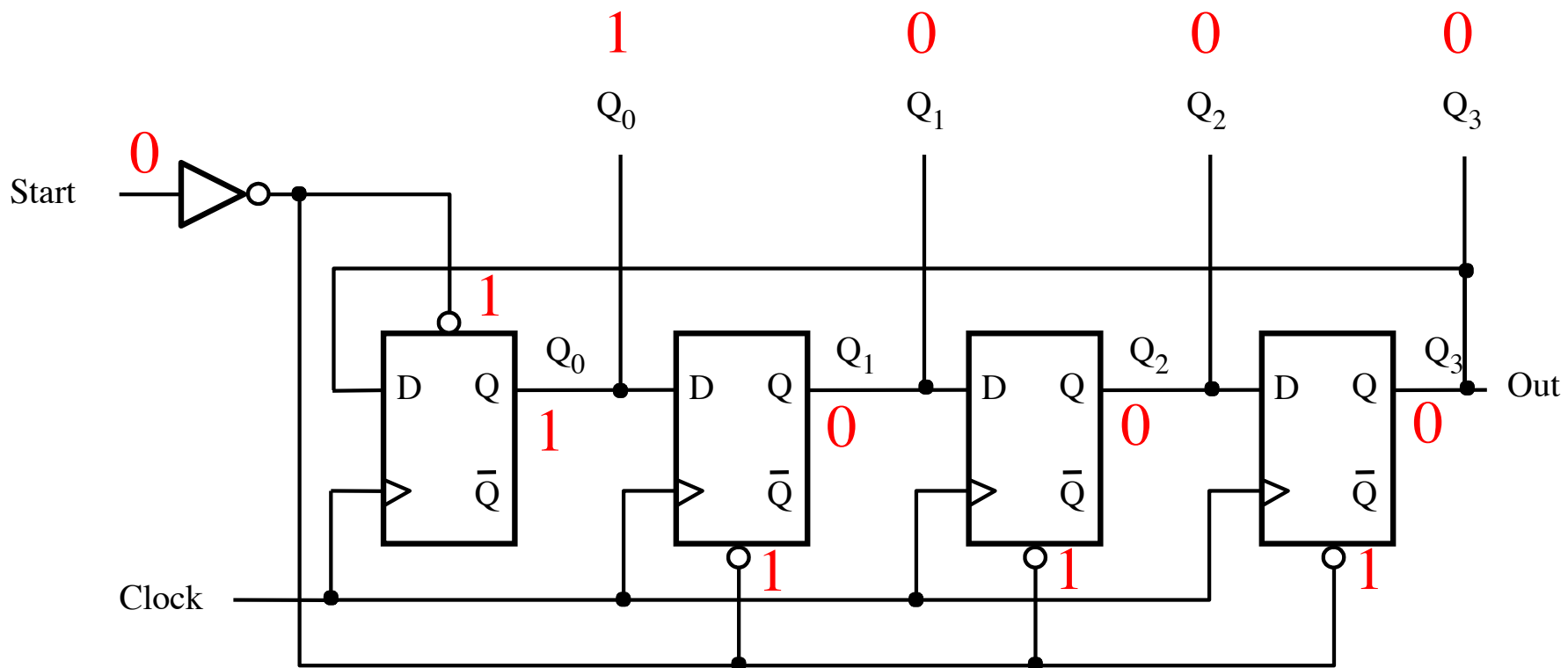
After the NOT gate, this 1 goes as 0 to preset_n of the first flip-flop and to clear_n of all remaining flip-flops.

4-bit ring counter: How does it work



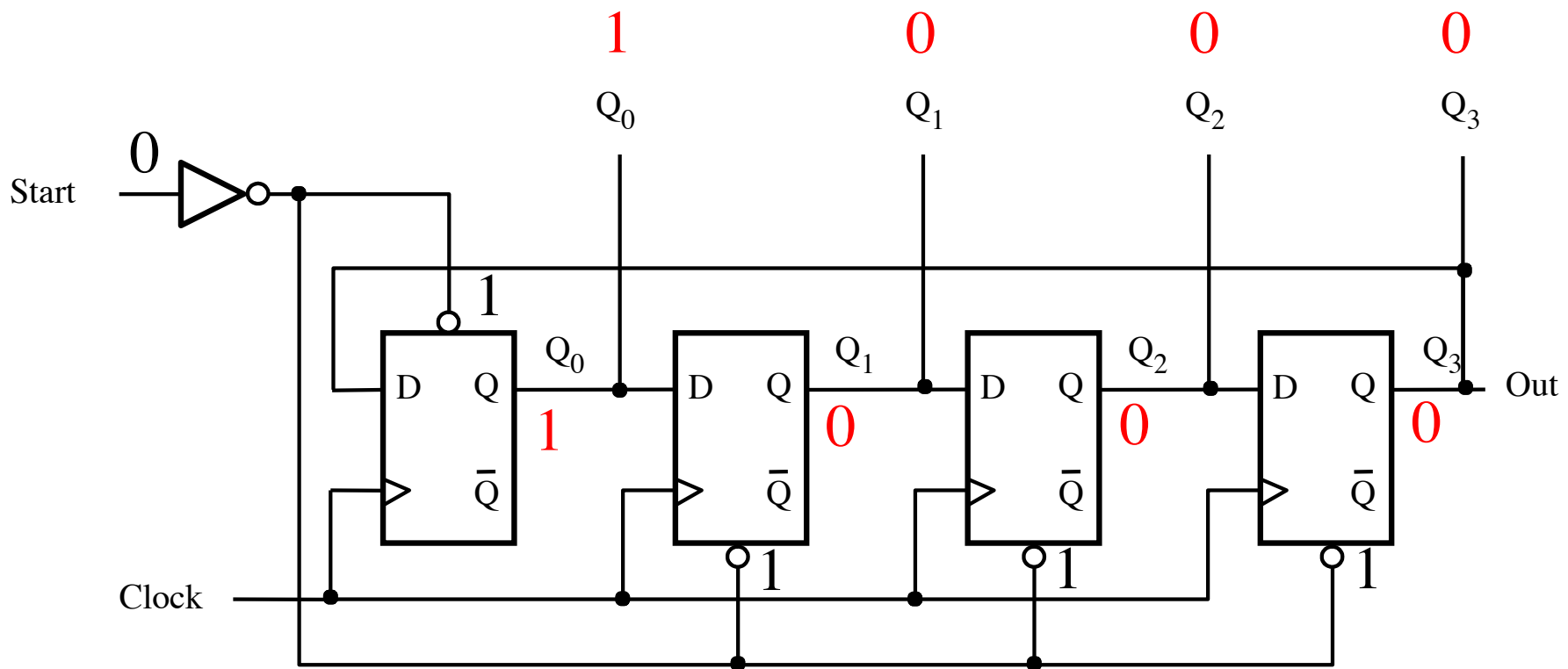
This sets the output pattern to 1000,
i.e., only the first bit is one and the rest are zeros.

4-bit ring counter: How does it work



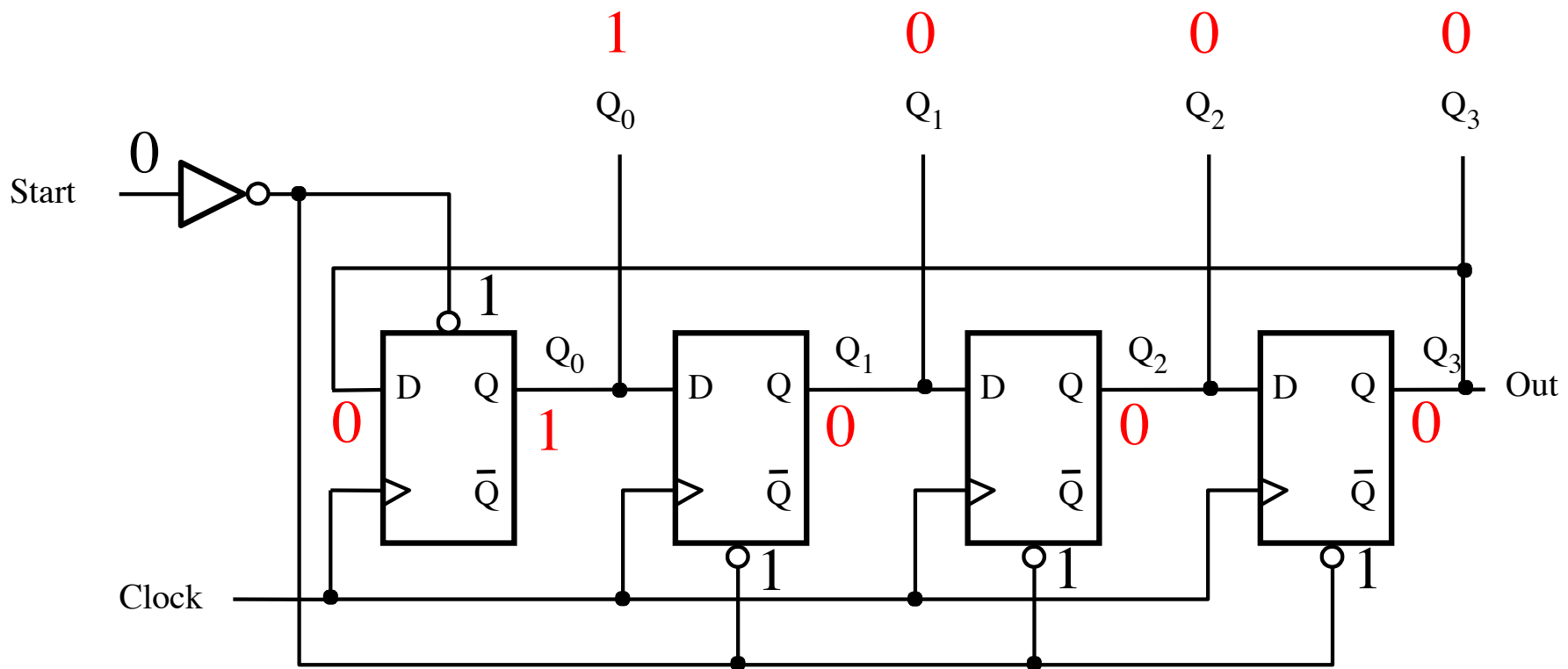
Setting Start to 0 has no effect on the outputs, because both `preset_n` and `clear_n` are sensitive only to 0.

4-bit ring counter: How does it work



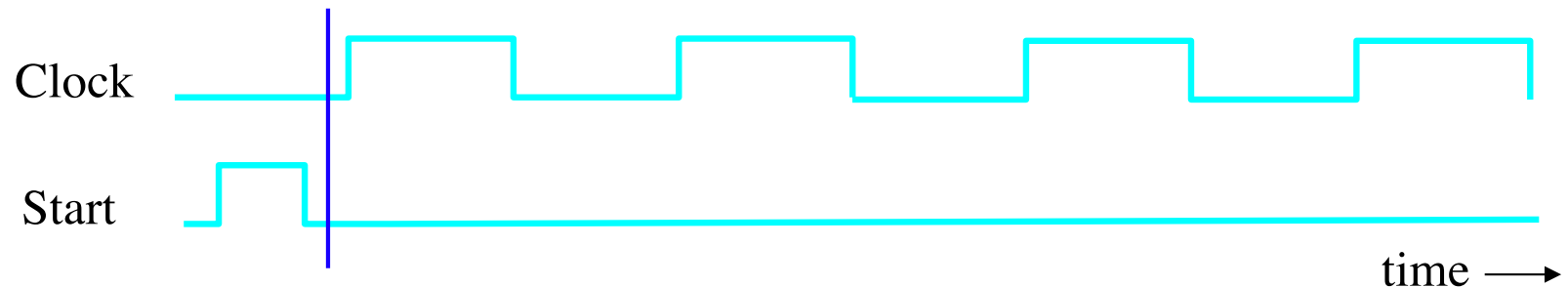
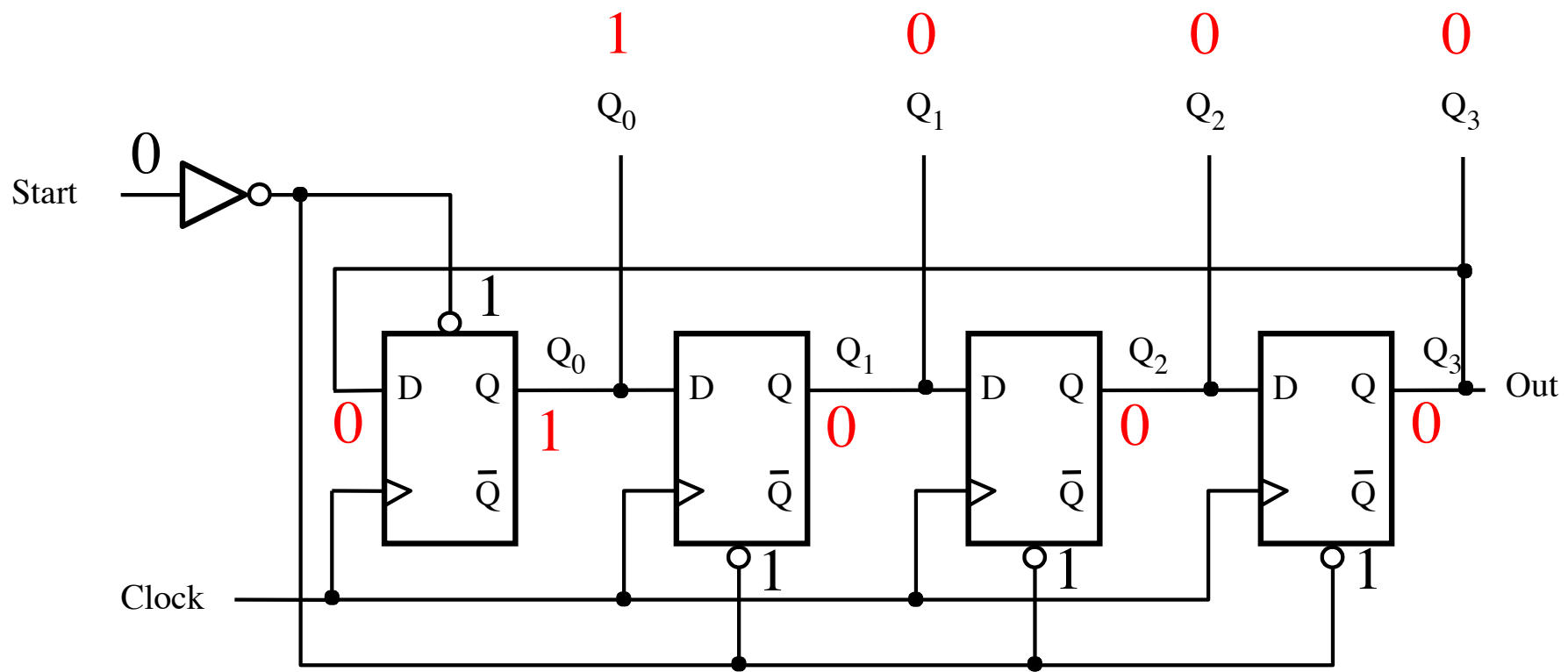
The initialization does not depend on the clock since both preset_n and clear_n bypass the gates of the latches in the flip-flops.

4-bit ring counter: How does it work

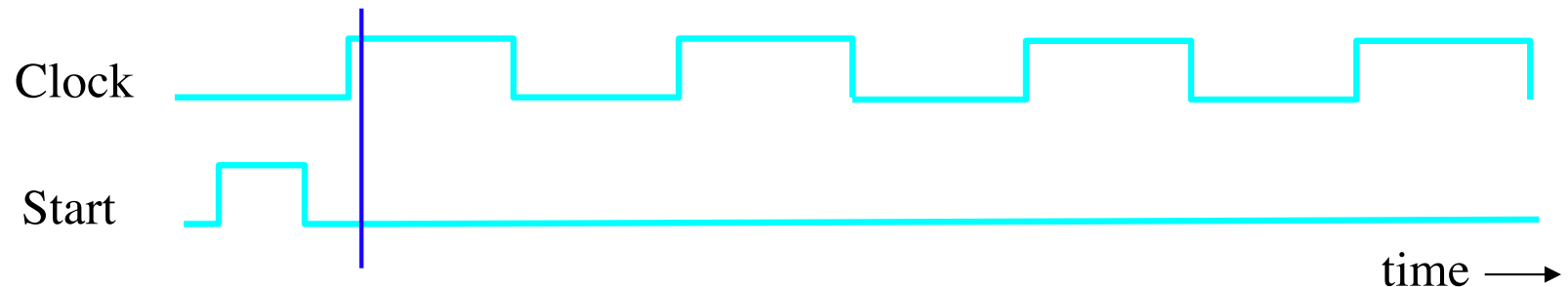
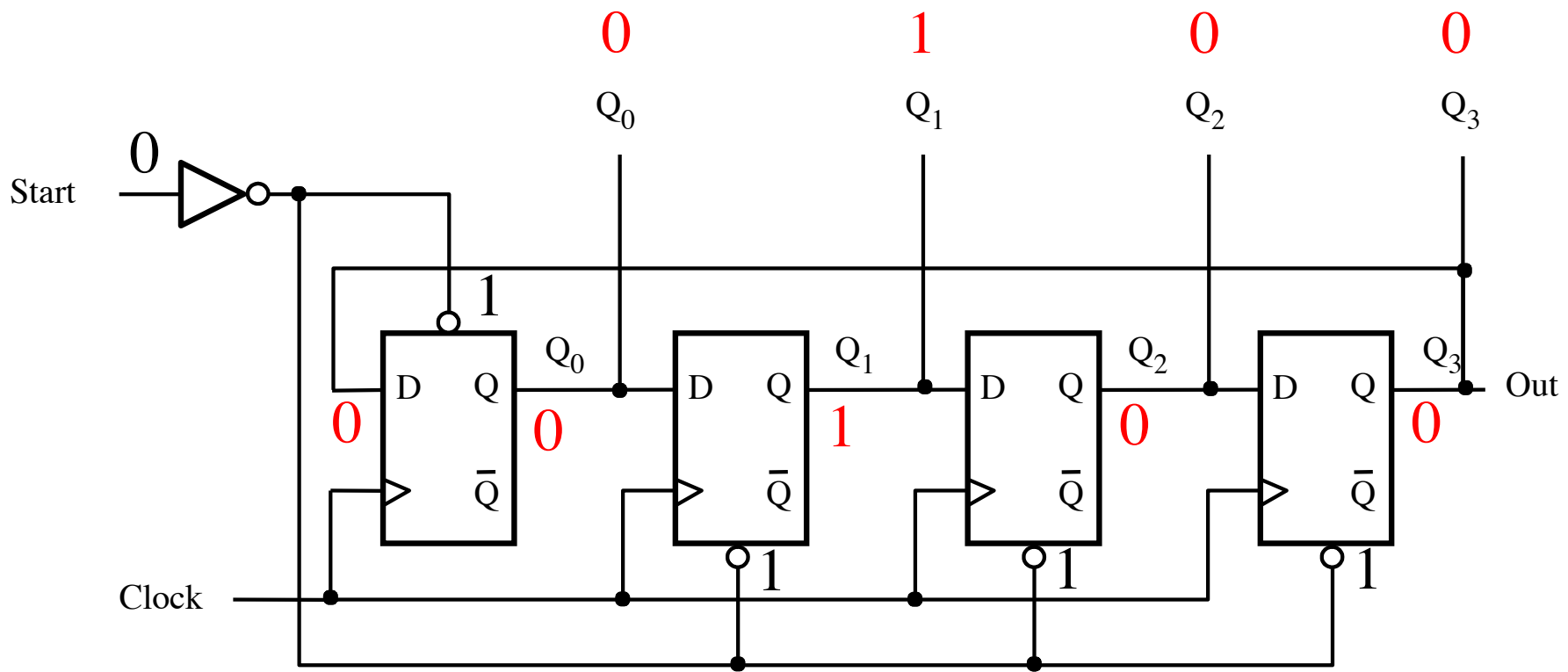


That last 0 loops back to the D input of the first flip-flop.

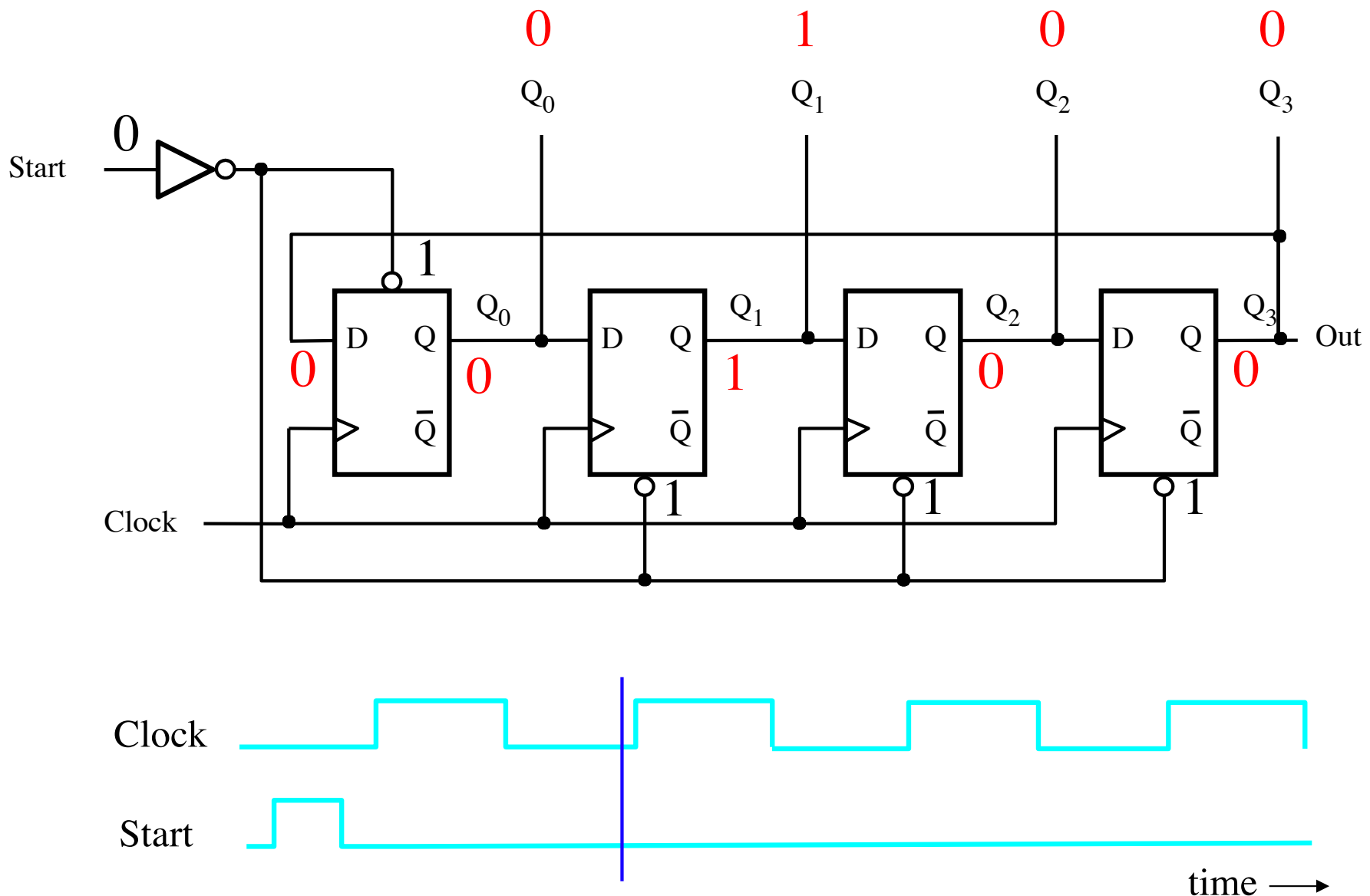
4-bit ring counter: How does it work



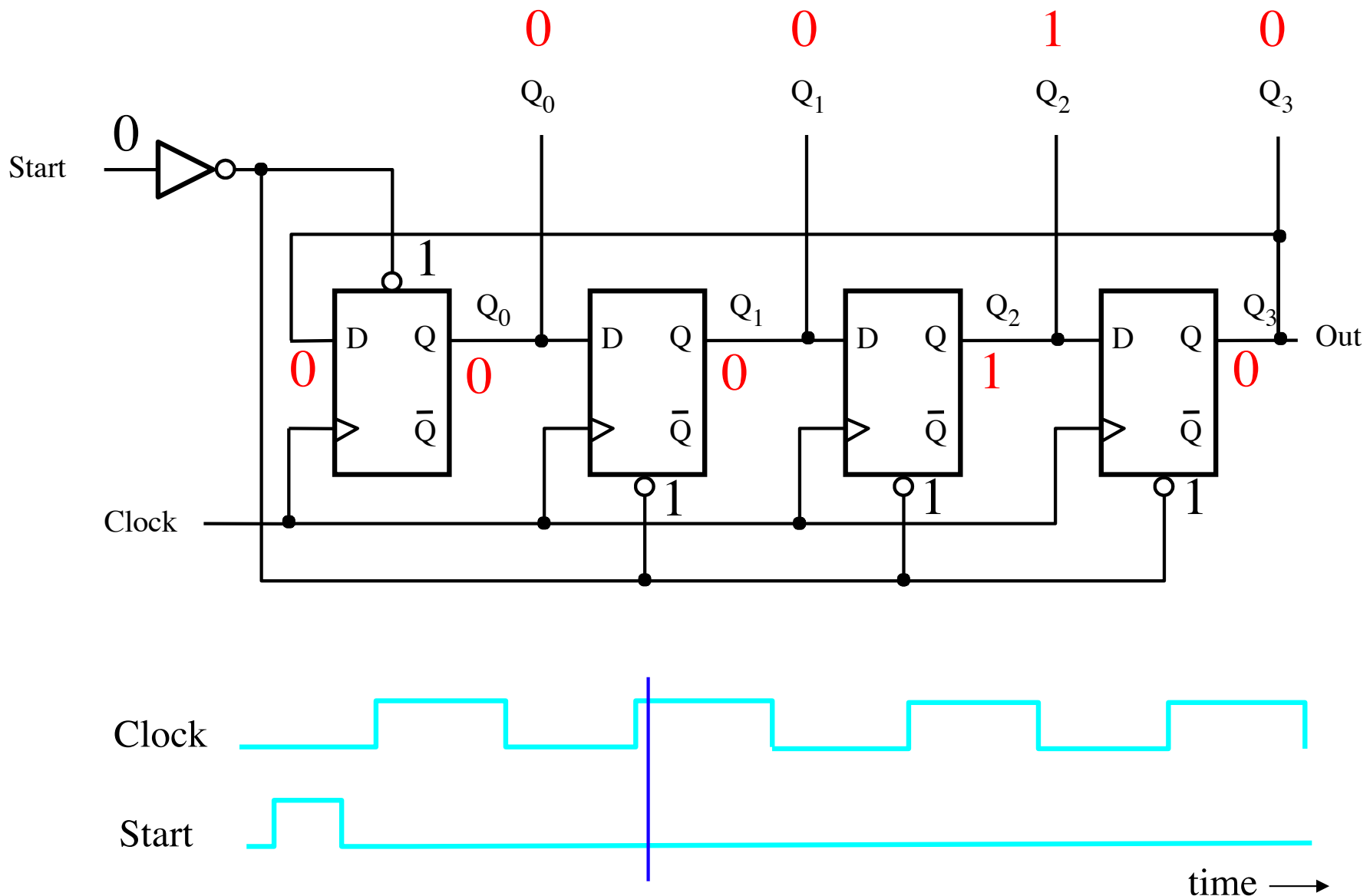
4-bit ring counter: How does it work



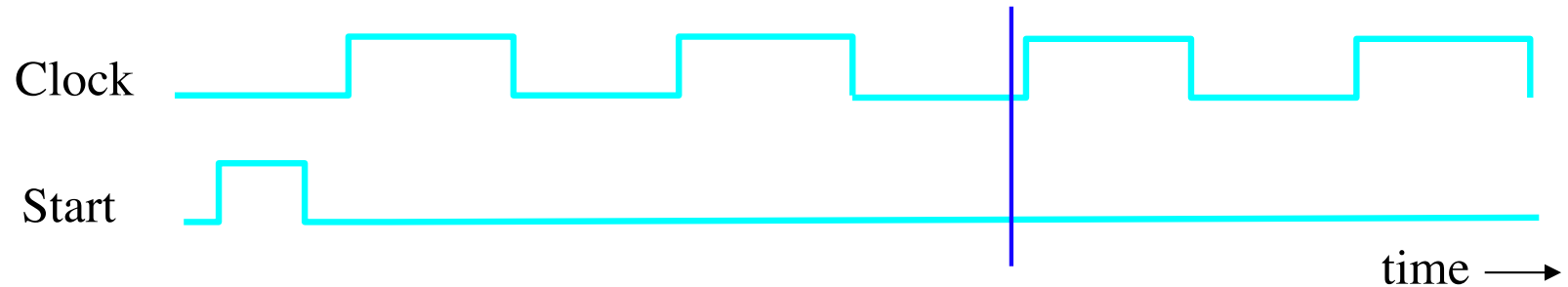
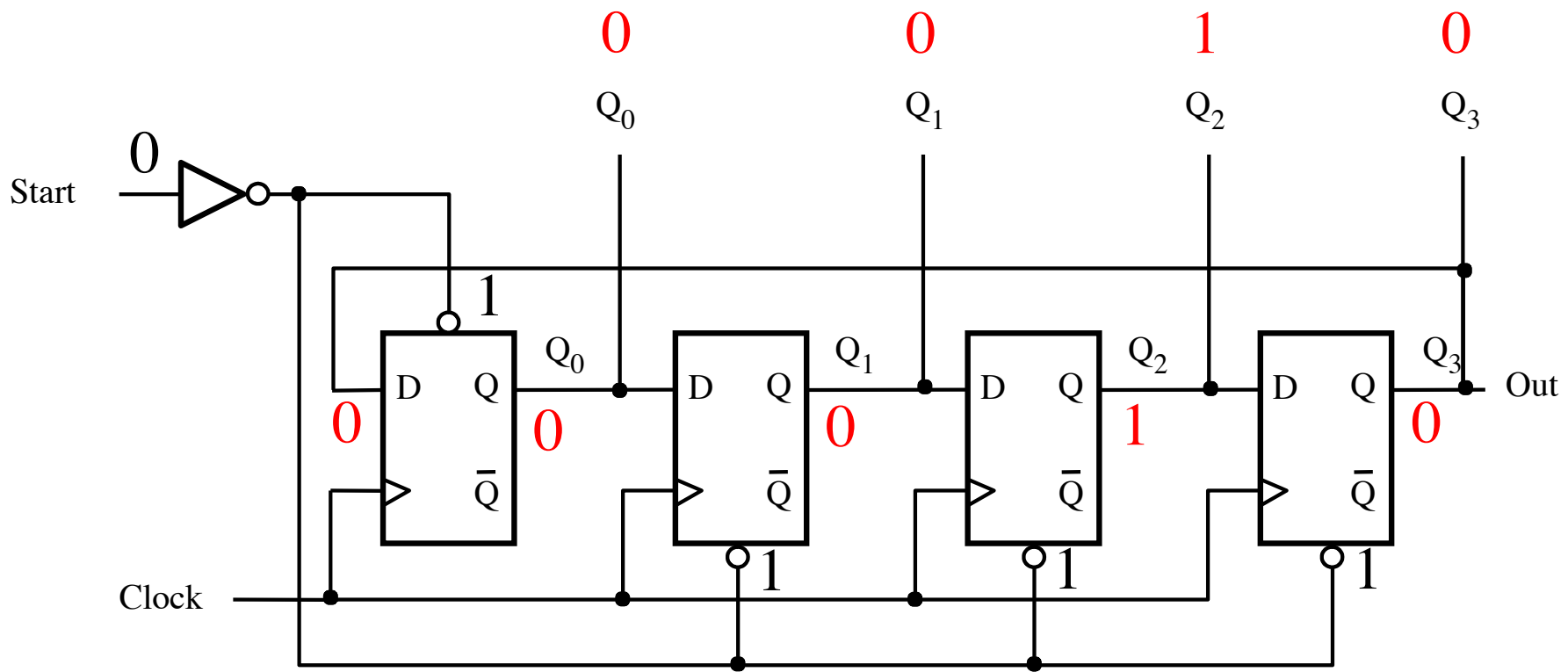
4-bit ring counter: How does it work



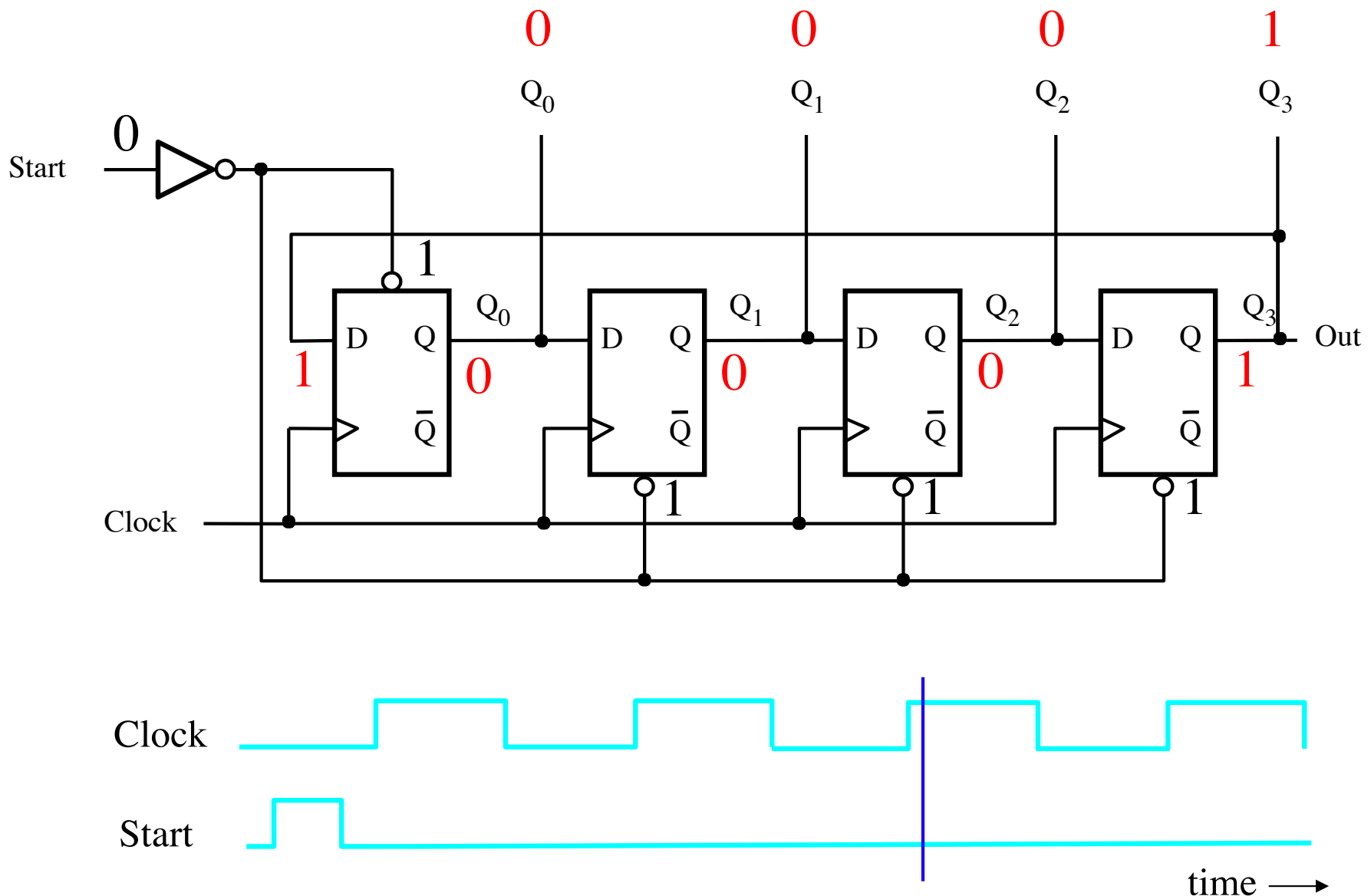
4-bit ring counter: How does it work



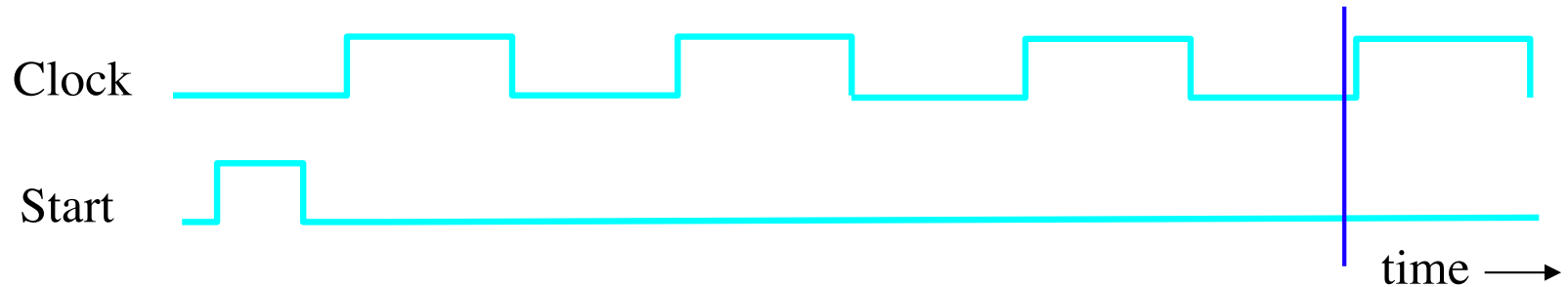
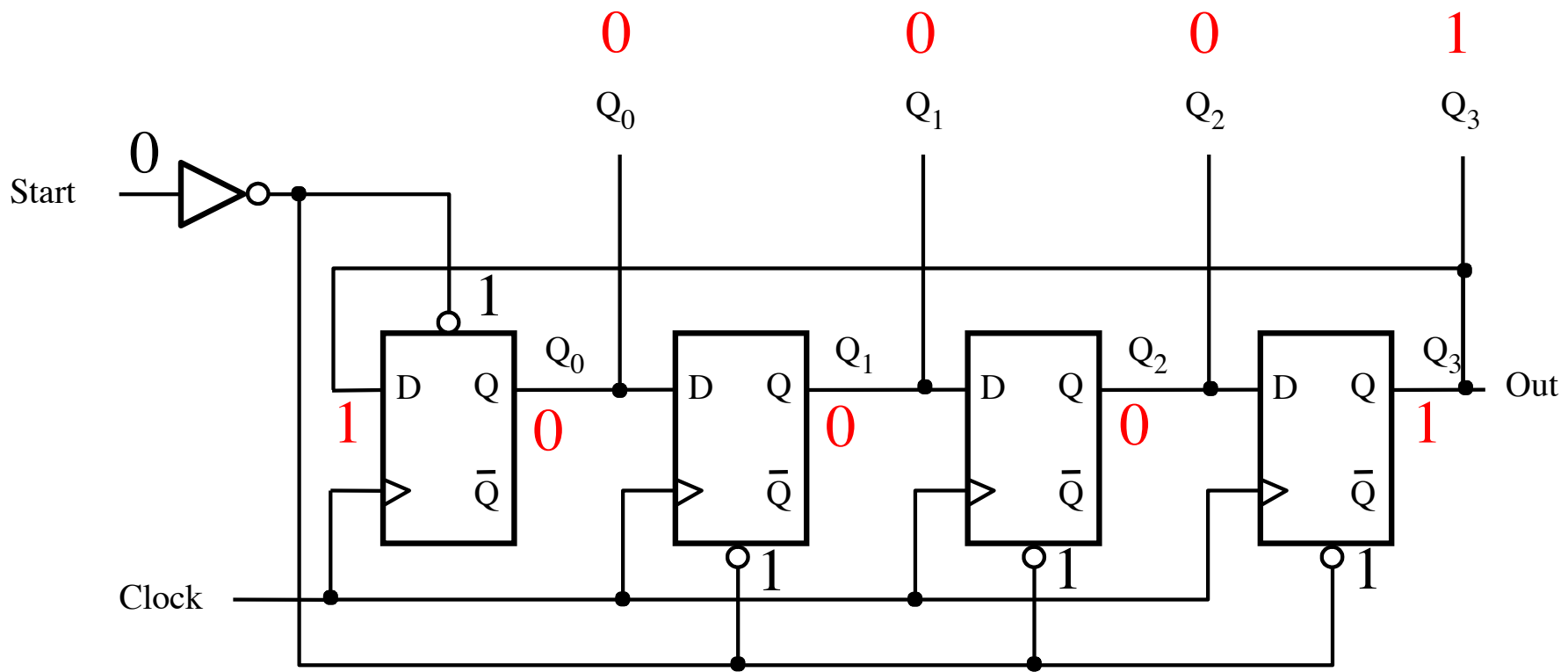
4-bit ring counter: How does it work



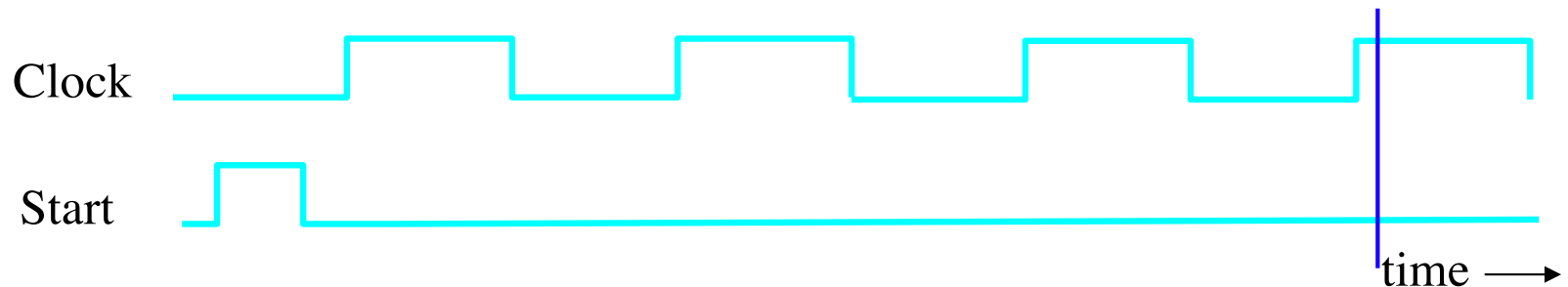
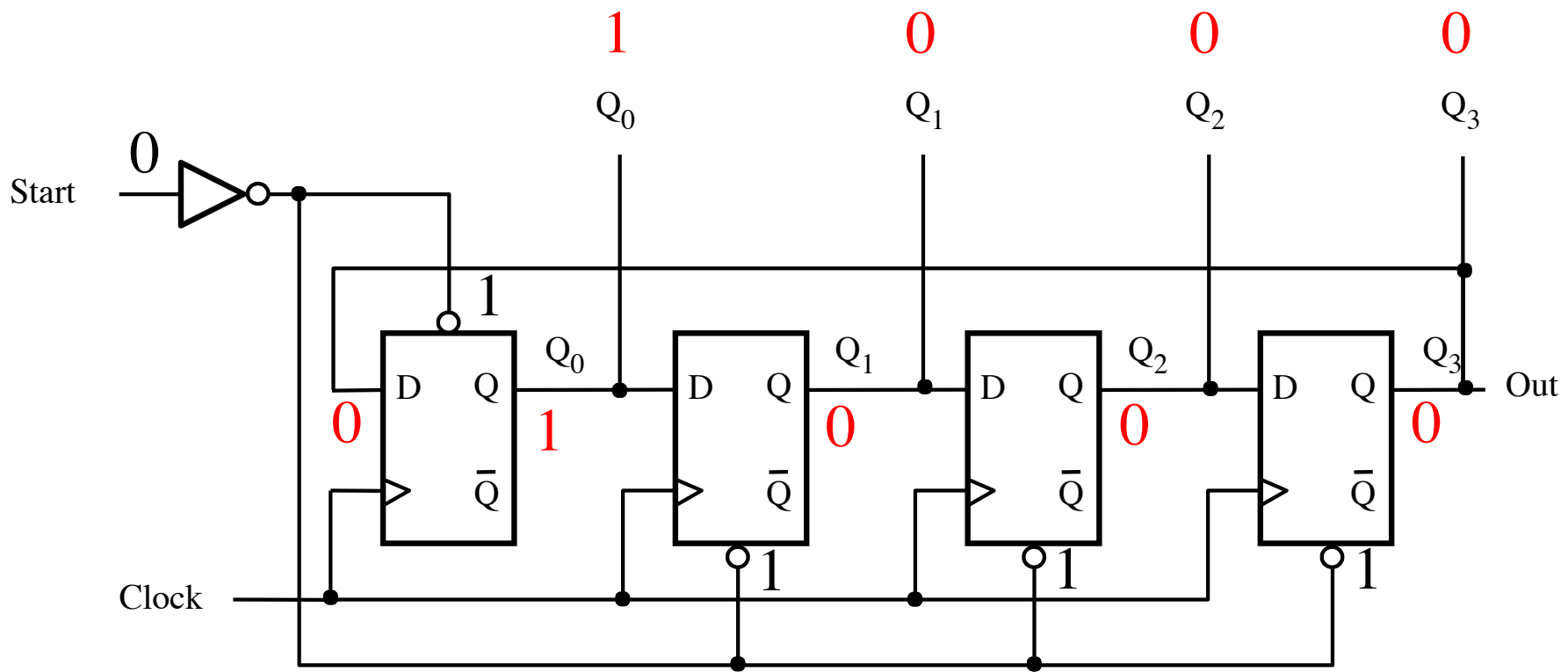
4-bit ring counter: How does it work



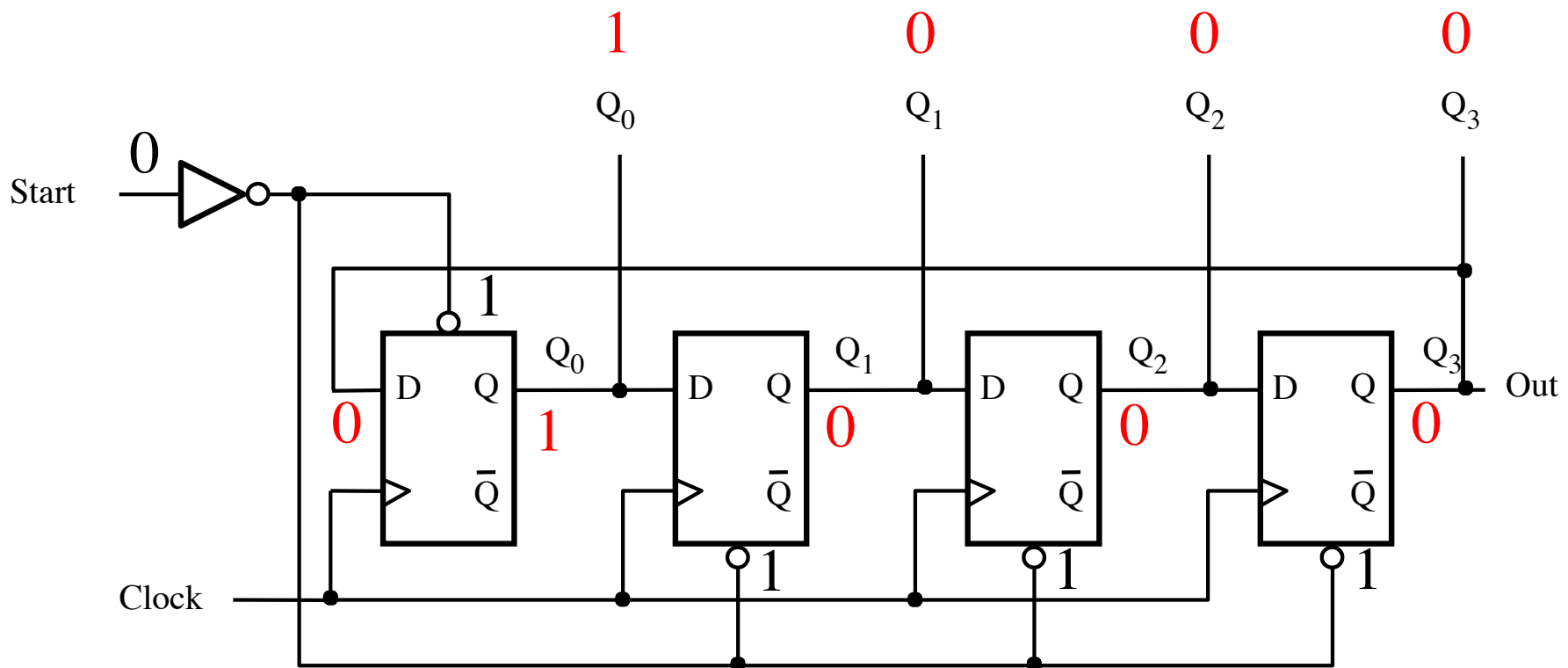
4-bit ring counter: How does it work



4-bit ring counter: How does it work

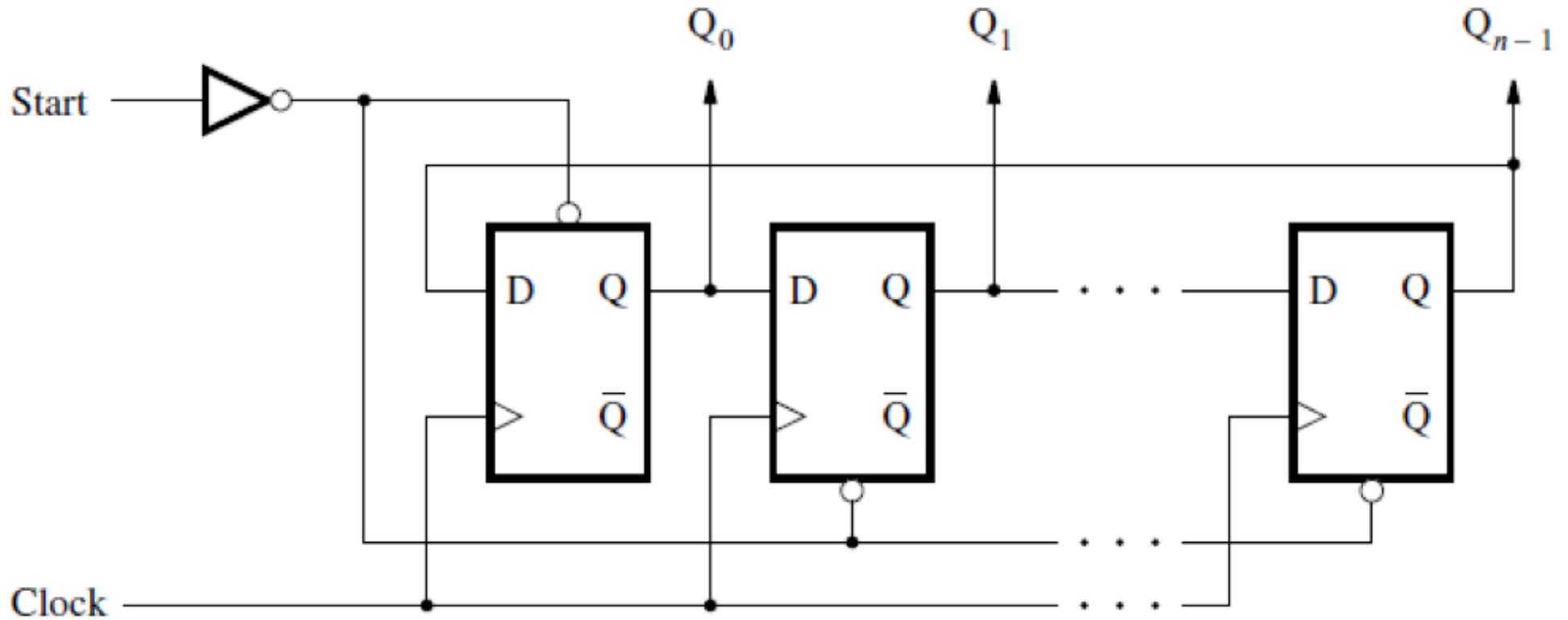


4-bit ring counter: How does it work



It is back to the start of the counting sequence,
which is: 1000, 0100, 0010, 0001.

n-bit ring counter



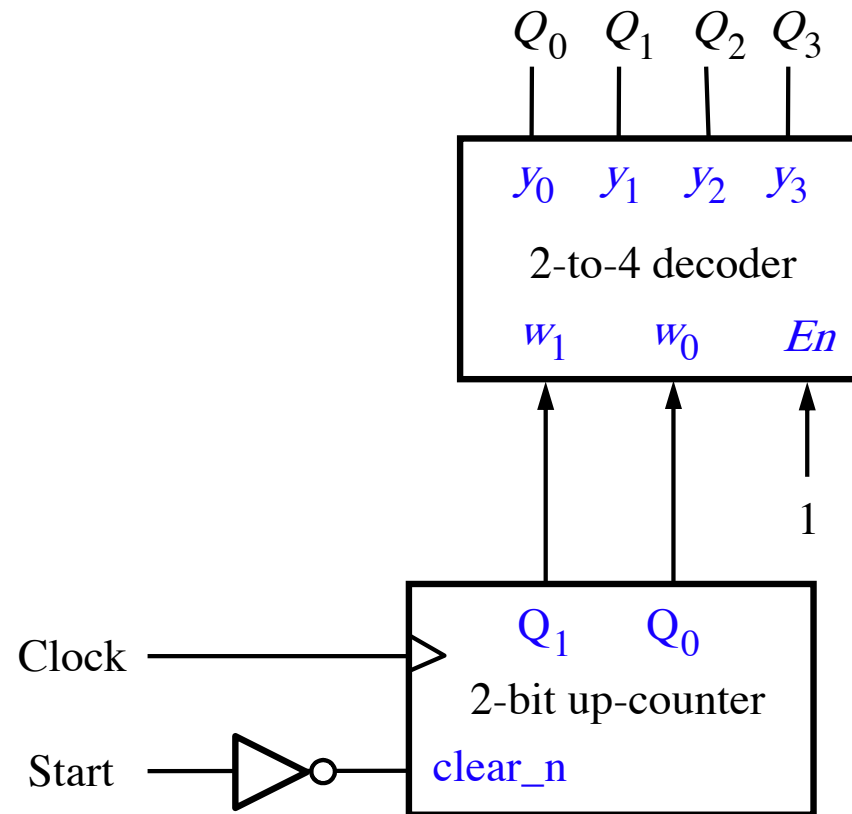
[Figure 5.28a from the textbook]

Ring Counter (alternative implementation)

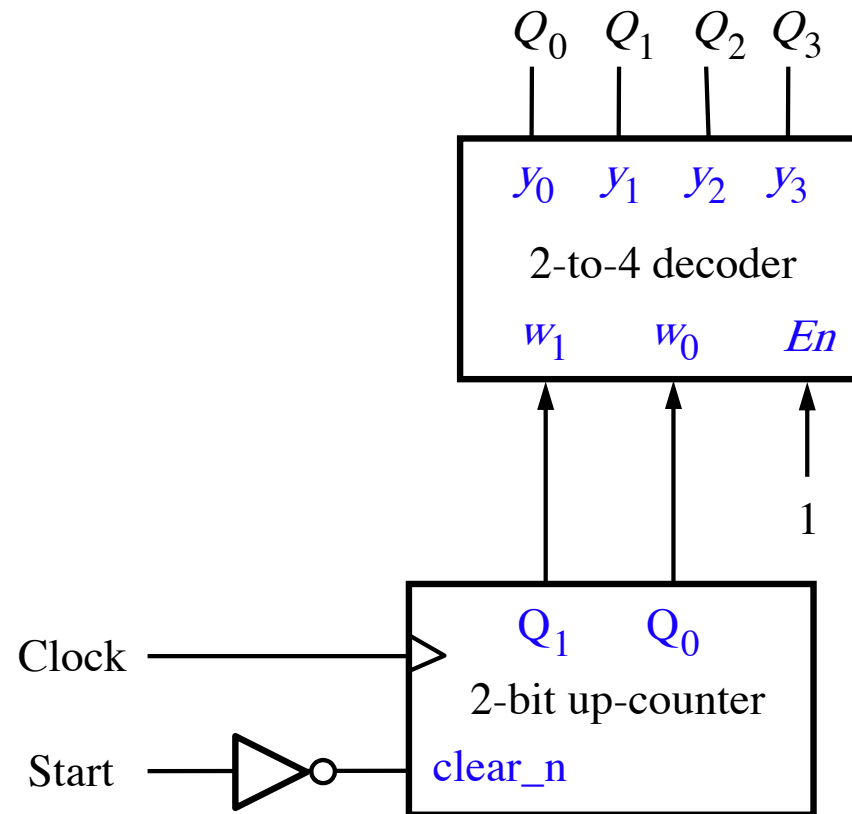
Alternative version of a 4-bit ring counter

- This implementation uses 2-bit up-counter followed by a 2-to-4 decoder.
- The counter cycles through 00, 01, 10, 11, 00, ...
- Recall that the outputs of the decoder are one-hot encoded. Thus, there is only one 1 on its outputs.
- Because the output of the counter is the input to the decoder, the outputs of the decoder cycle through: 1000, 0100, 0010, 0001, 1000, ...
- This is the counting sequence for a ring counter.

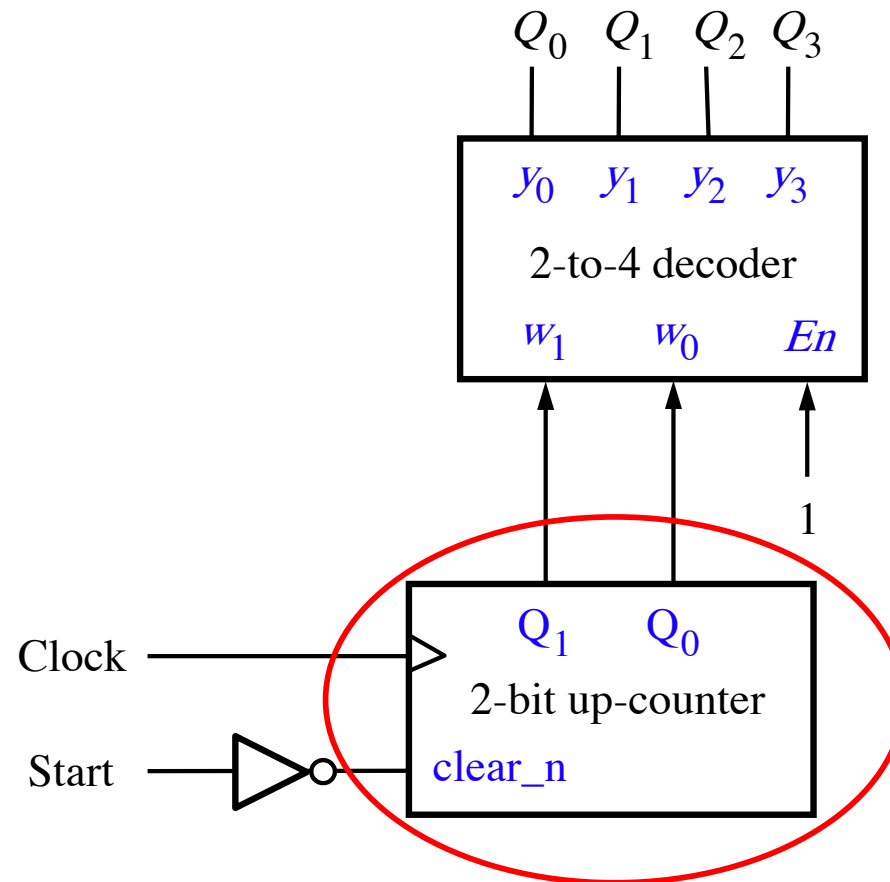
Alternative version of a 4-bit ring counter



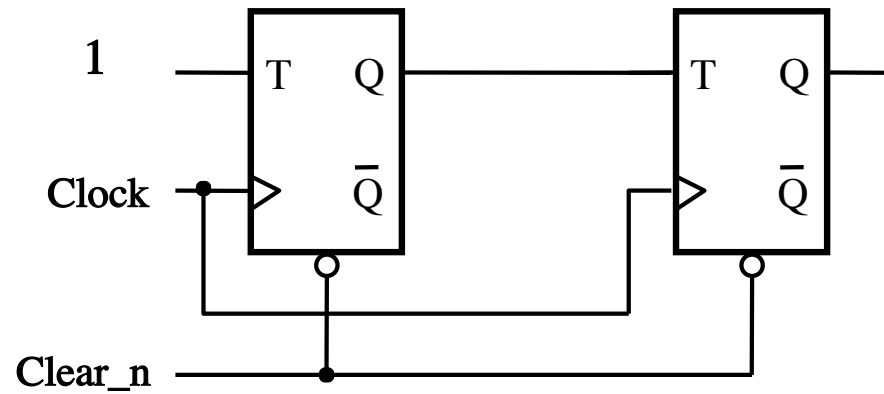
What are the components?



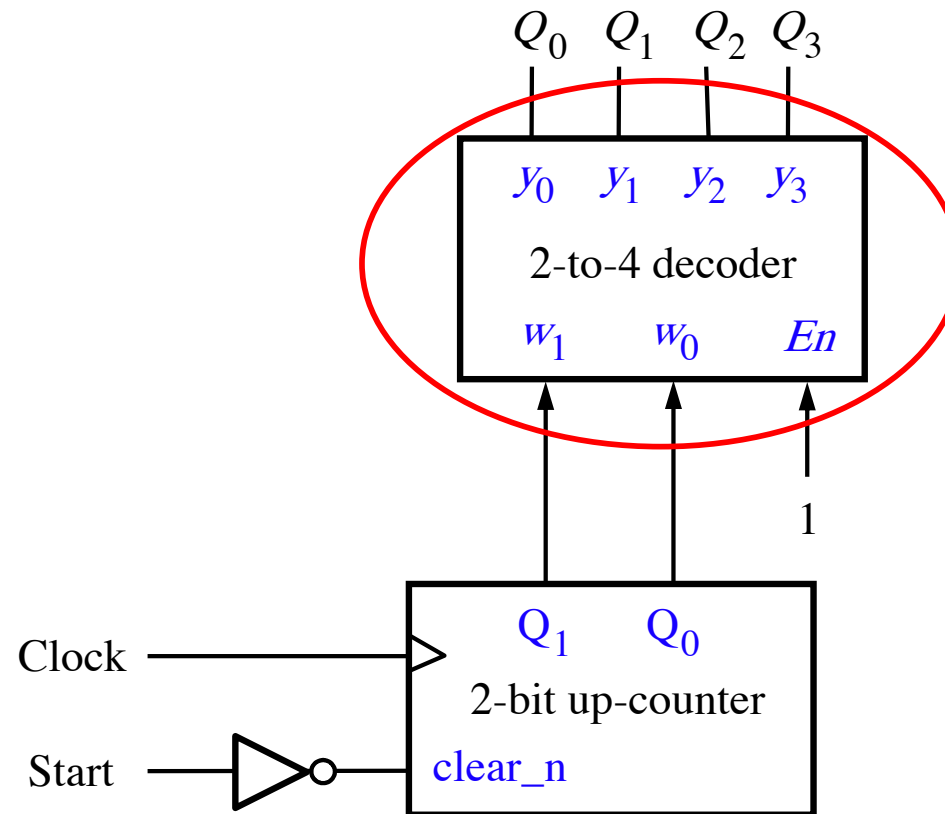
2-Bit Synchronous Up-Counter



2-Bit Synchronous Up-Counter

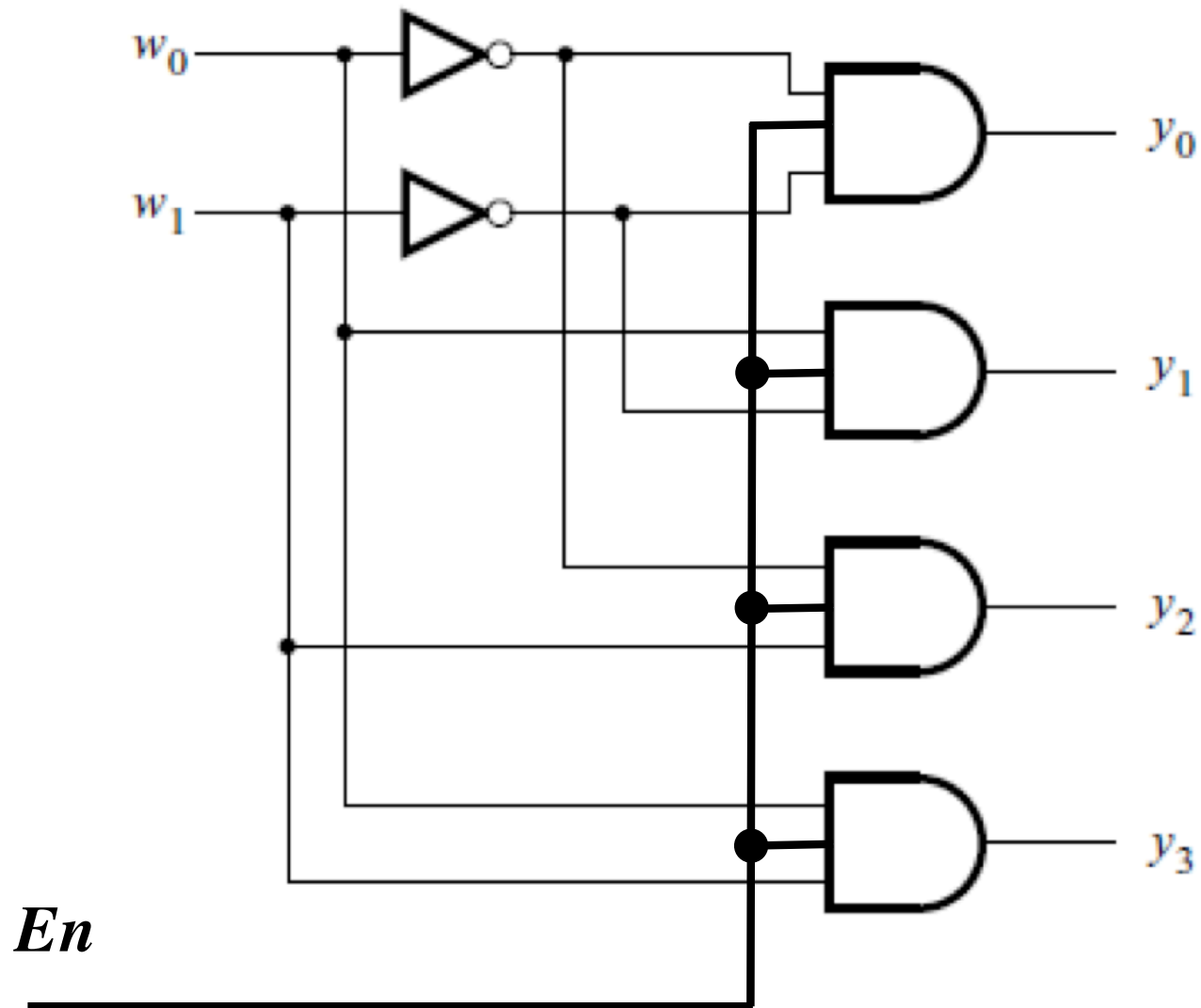


2-to-4 Decoder with Enable Input



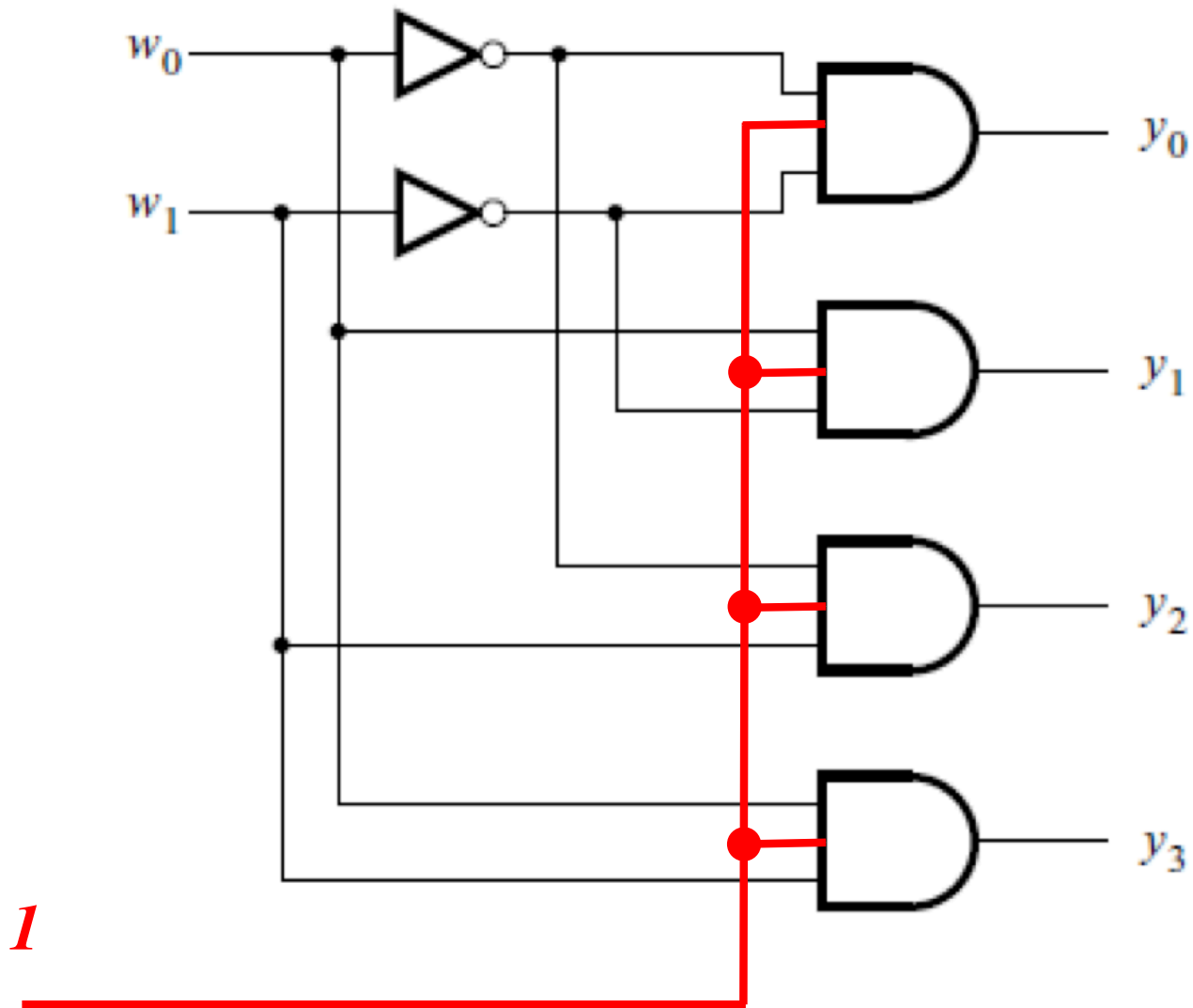
[Figure 5.28b from the textbook]

2-to-4 Decoder with Enable Input



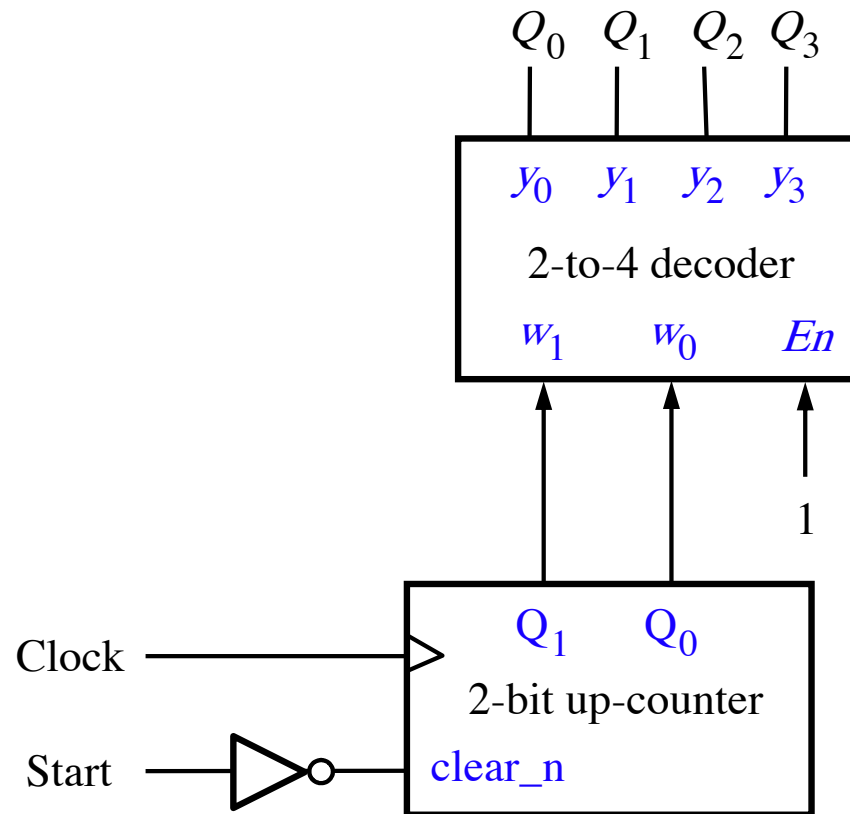
[Figure 4.14c from the textbook]

2-to-4 Decoder with Enable Input

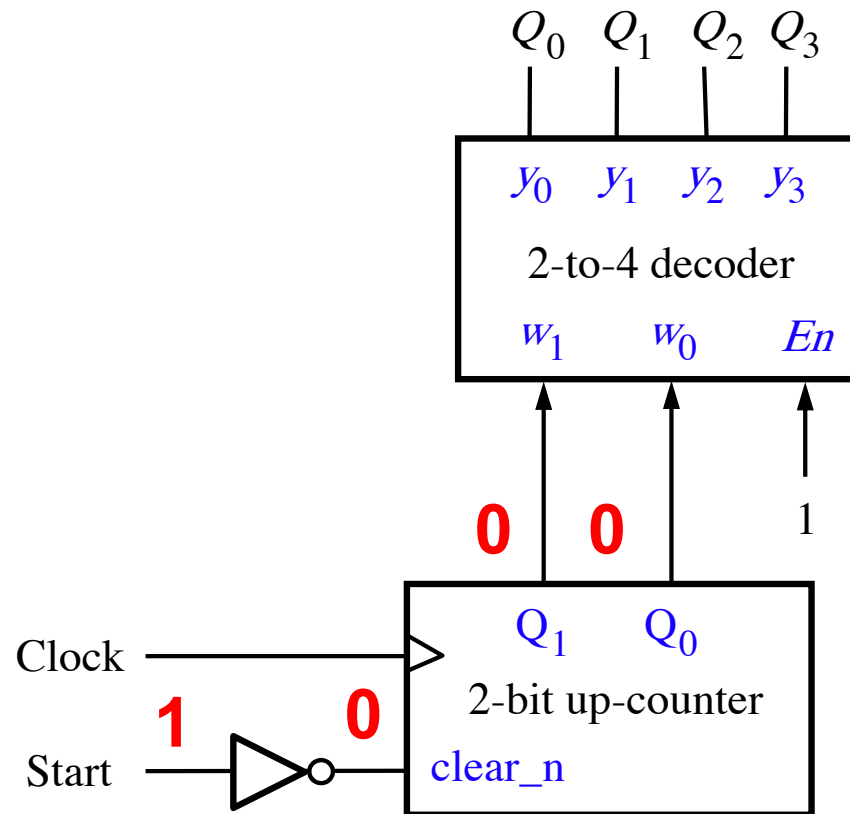


(always enabled in this example)

How Does It Work?

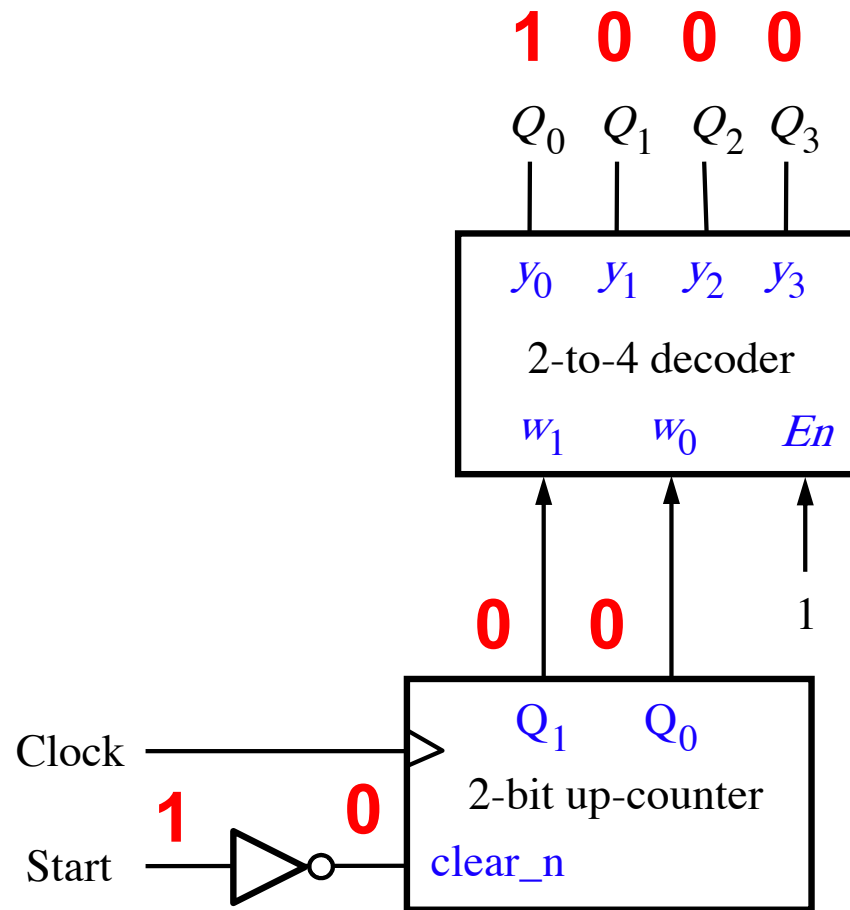


How Does It Work?



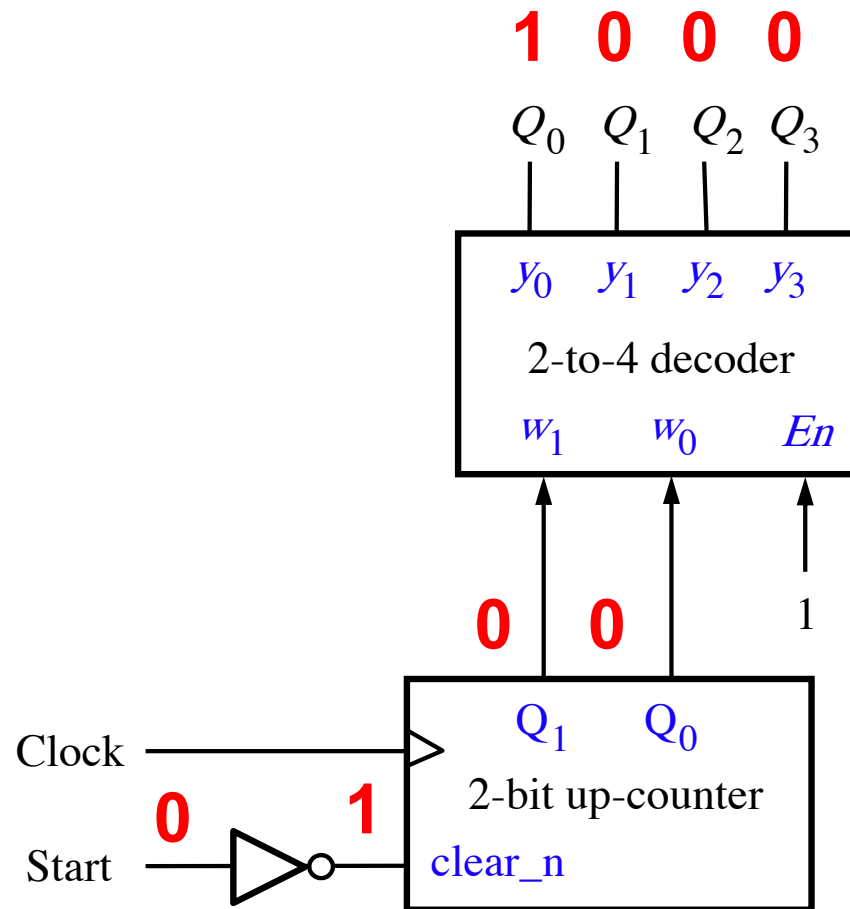
To initialize this circuit set Start to 1, which sets the counter to 00.

How Does It Work?



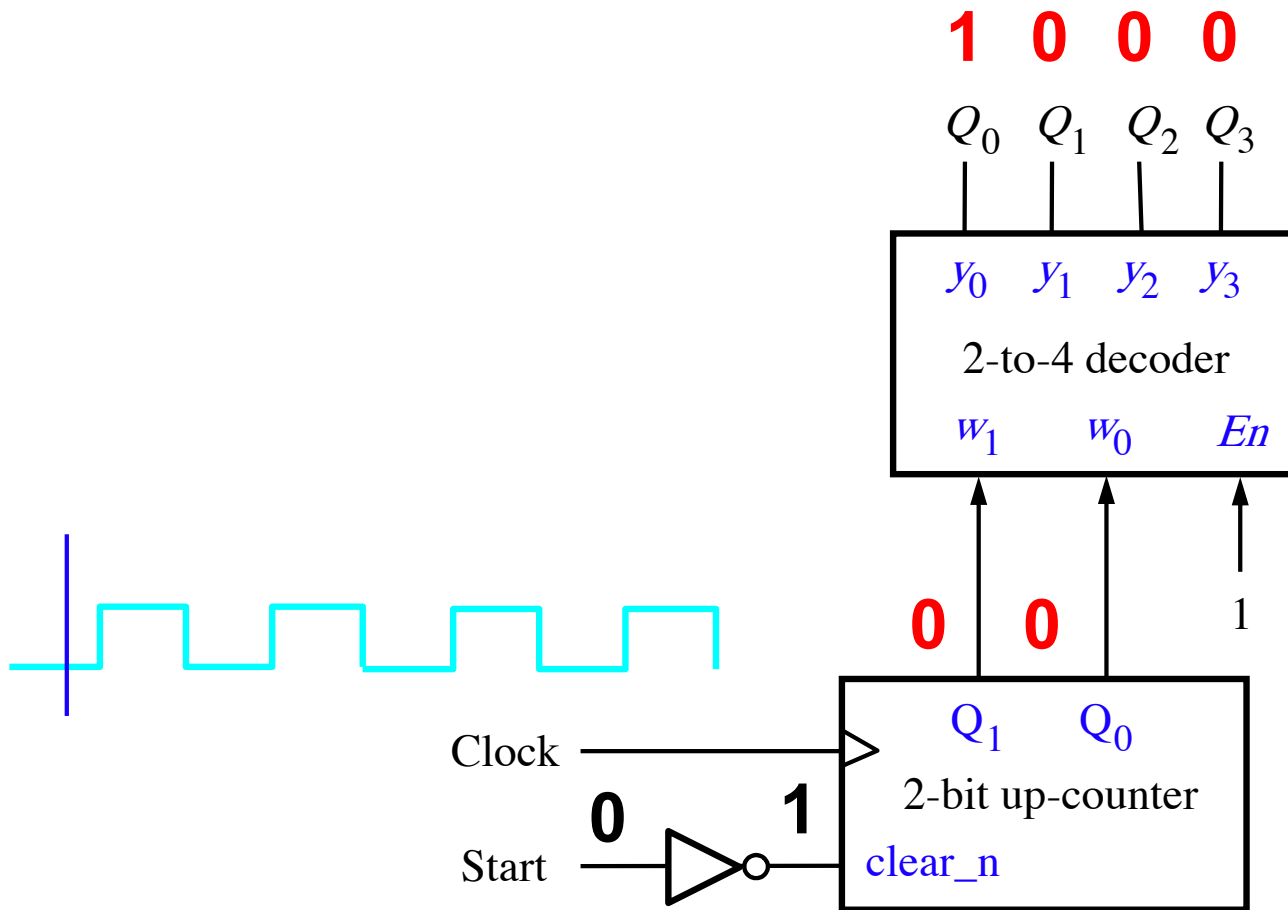
This sets the outputs of the decoder to the start of the counting sequence.

How Does It Work?

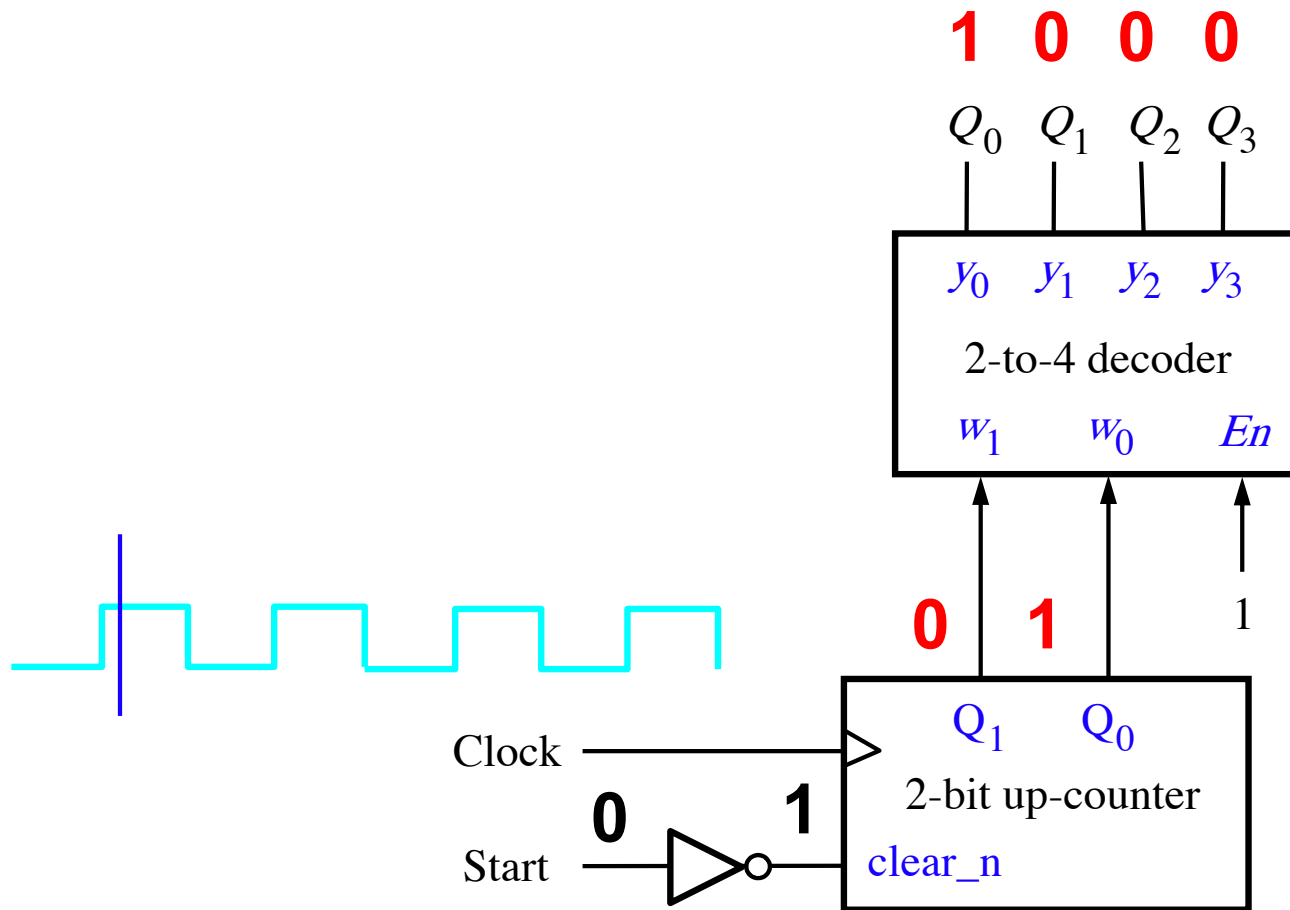


When Start is equal to 0, clear_n has no effect.

How Does It Work?

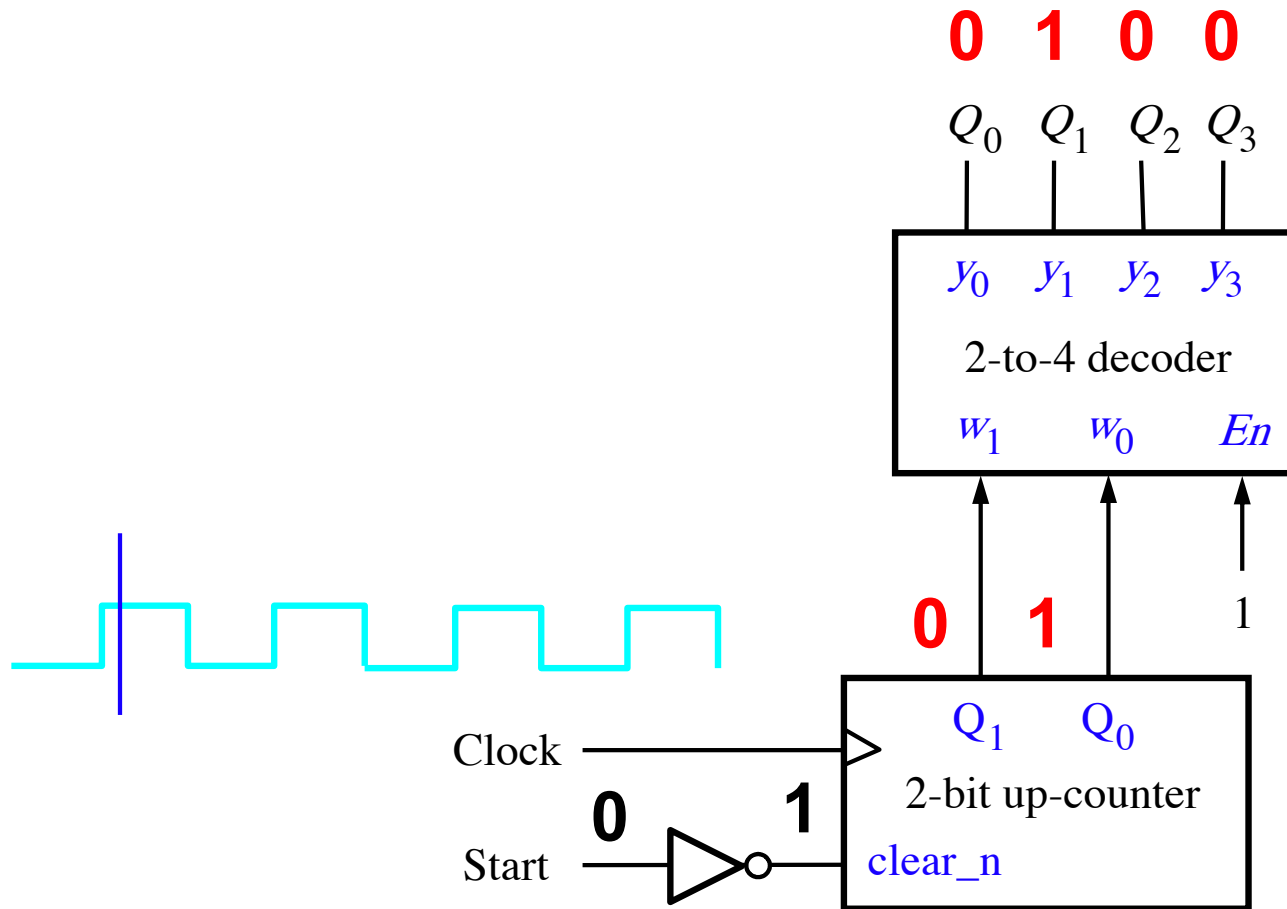


How Does It Work?



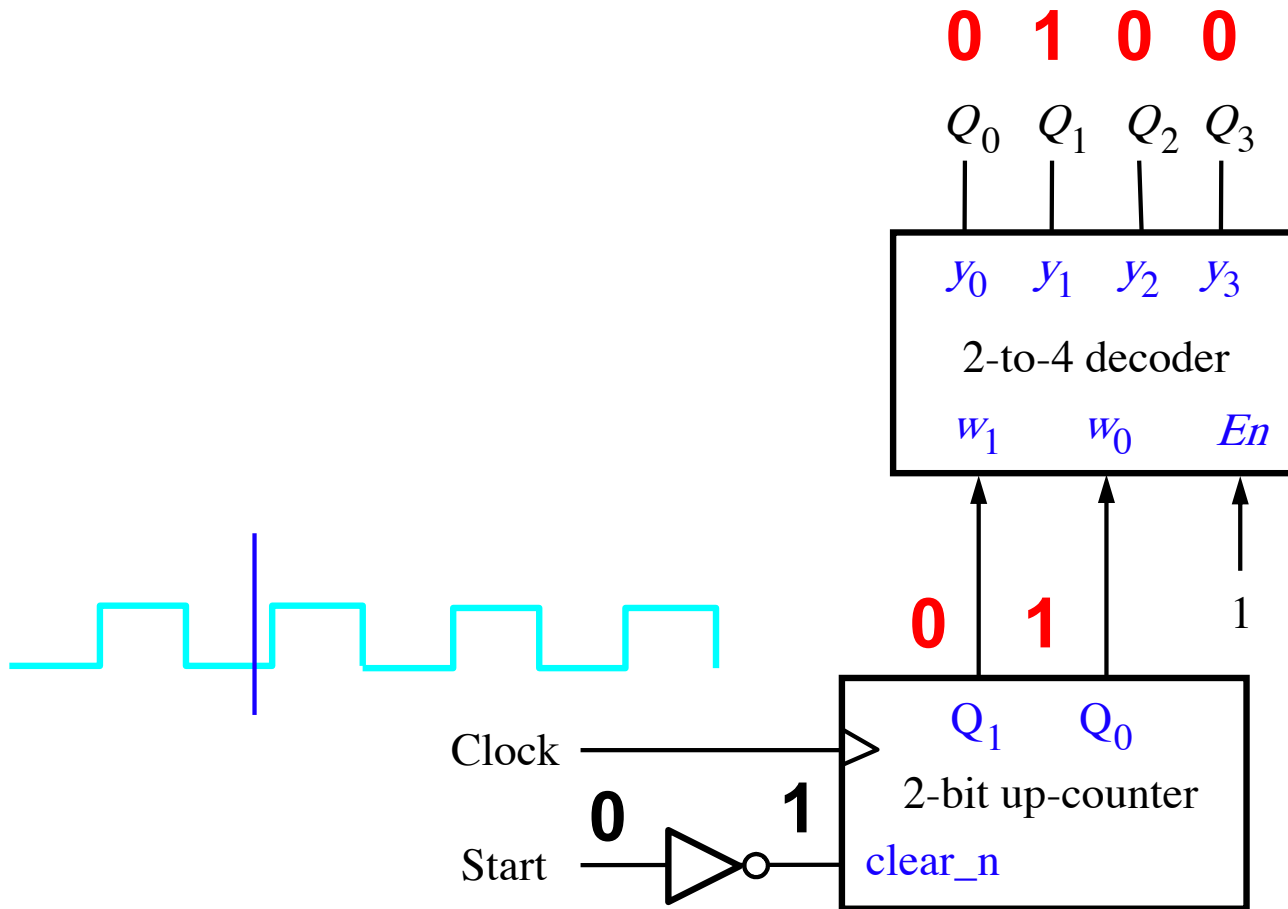
The counter increments the count on the positive clock edge ...

How Does It Work?

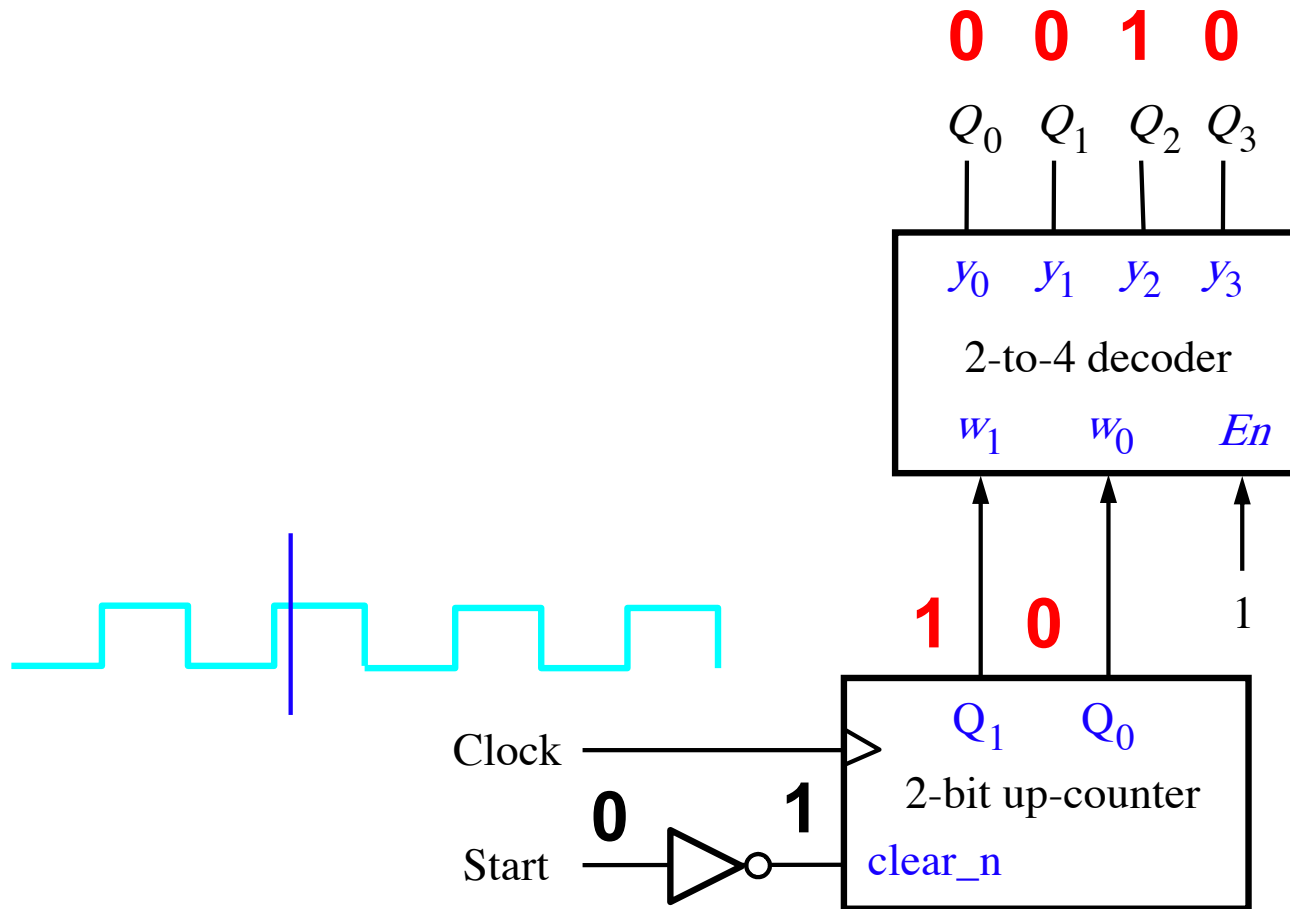


... this updates the outputs of the decoder (which are one hot encoded).

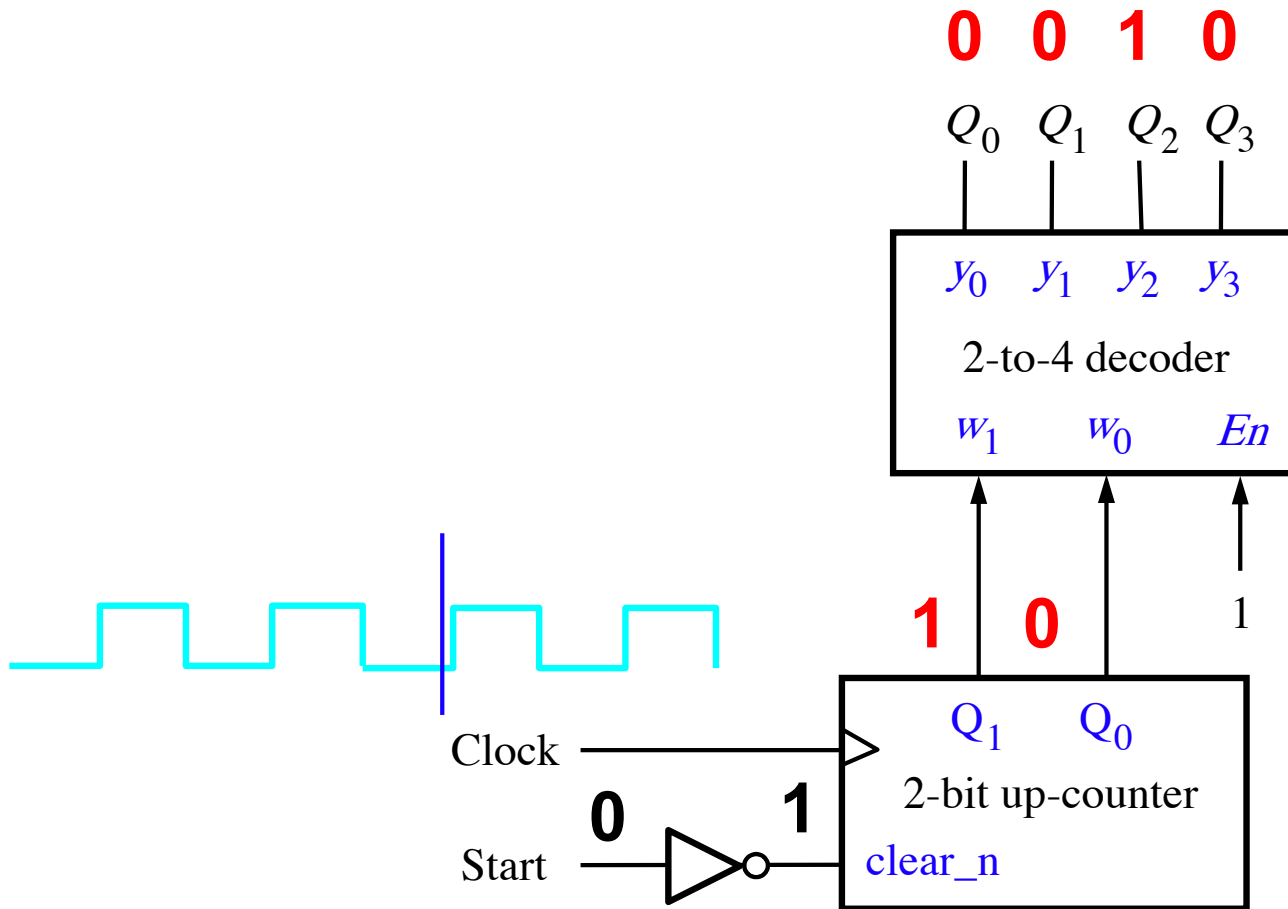
How Does It Work?



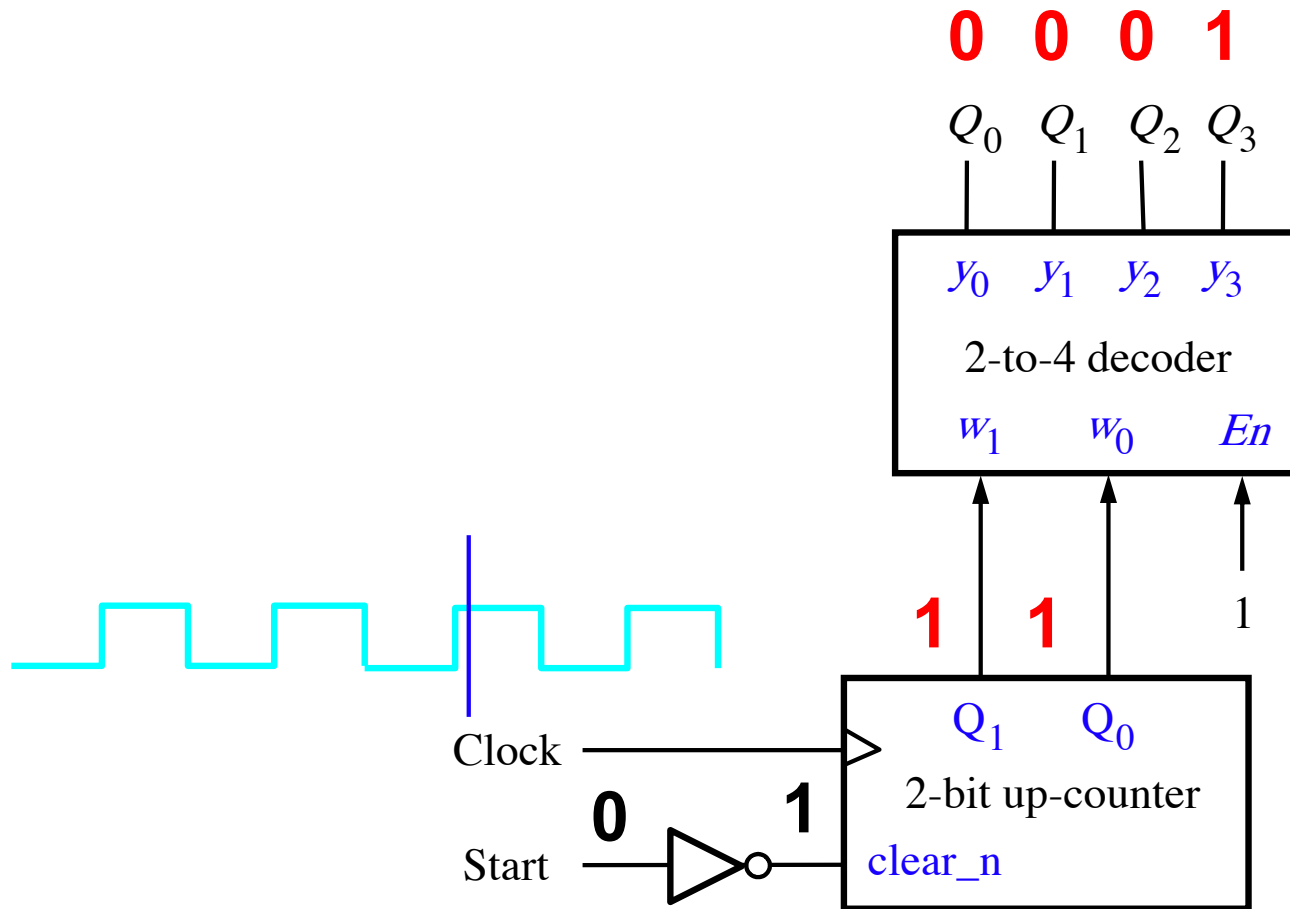
How Does It Work?



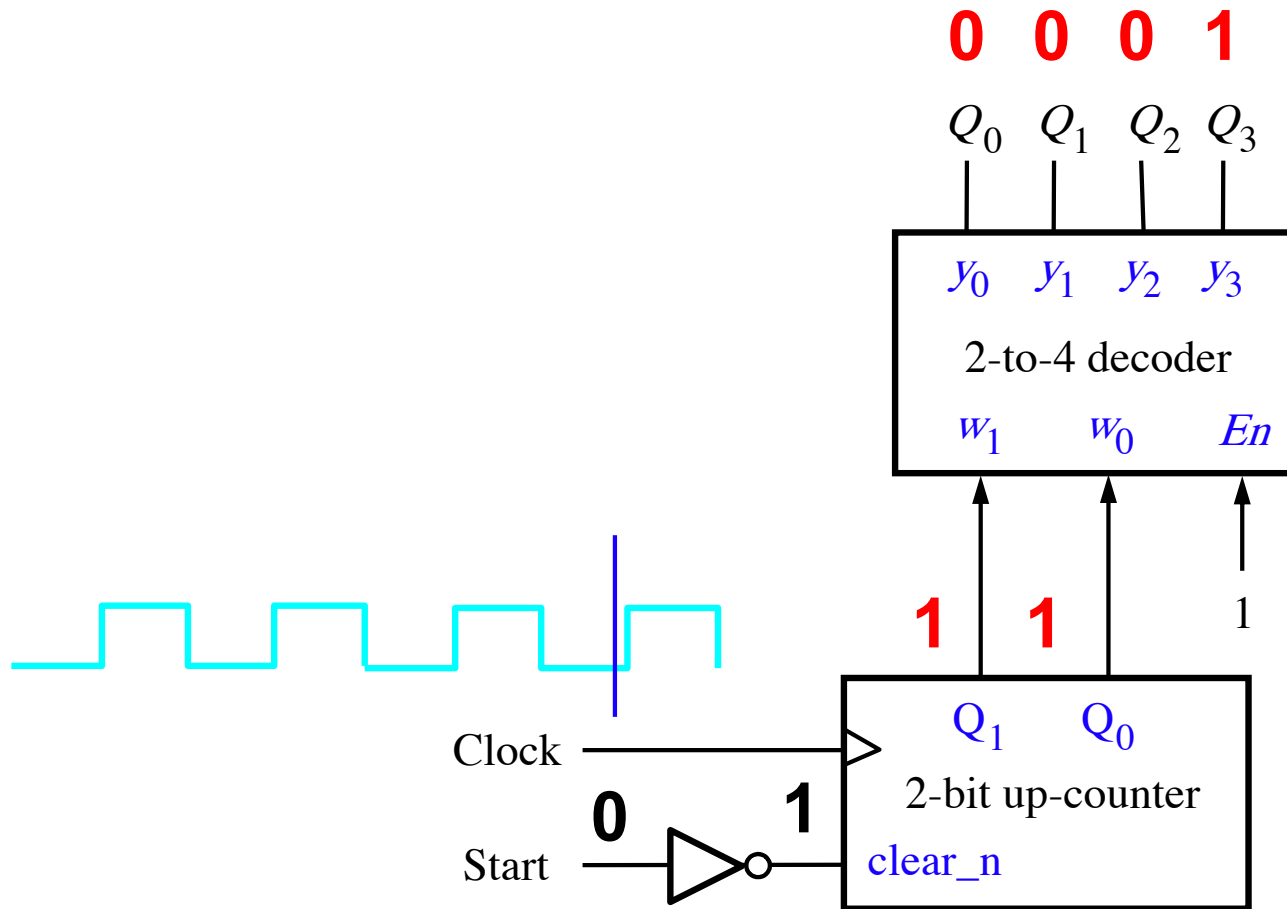
How Does It Work?



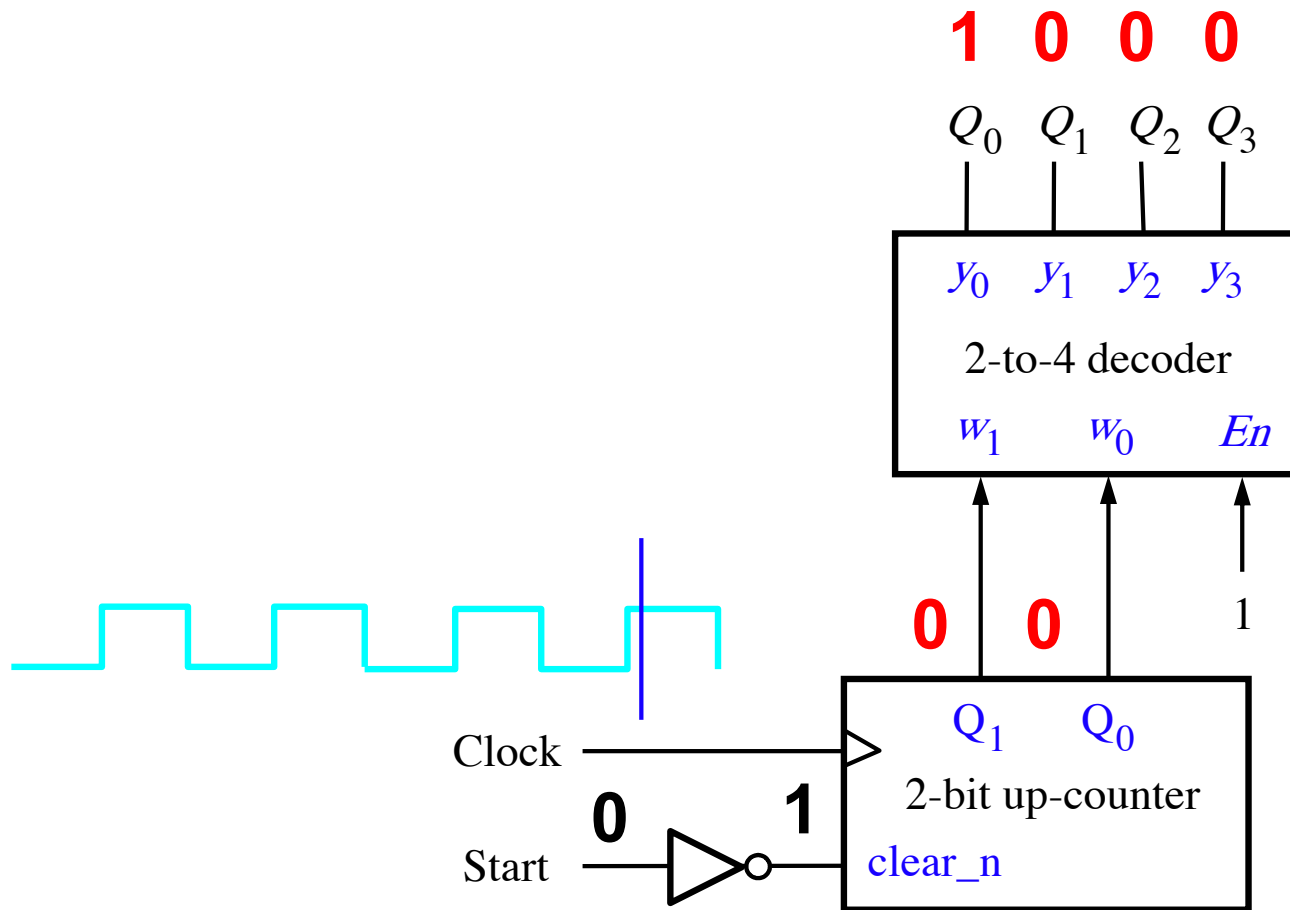
How Does It Work?



How Does It Work?



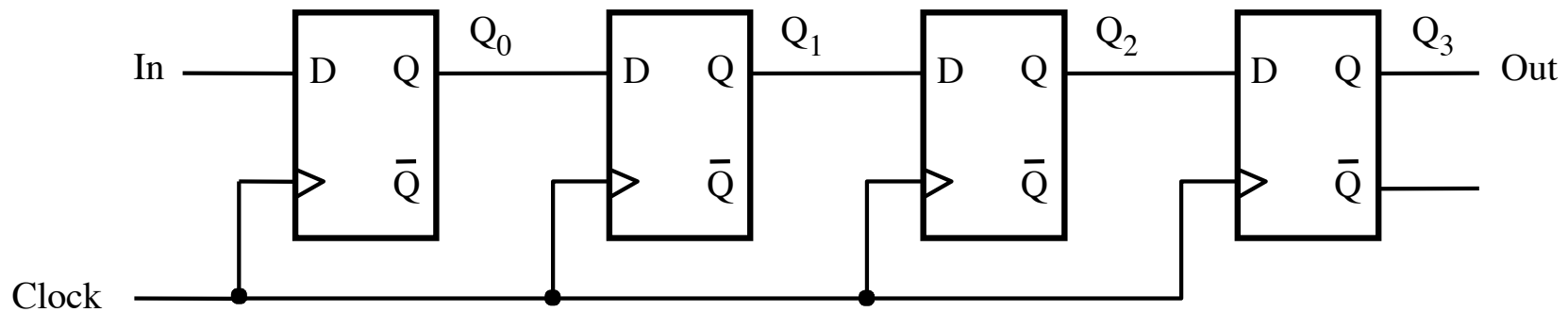
How Does It Work?



It is back to the start of the counting sequence,
which is: 1000, 0100, 0010, 0001.

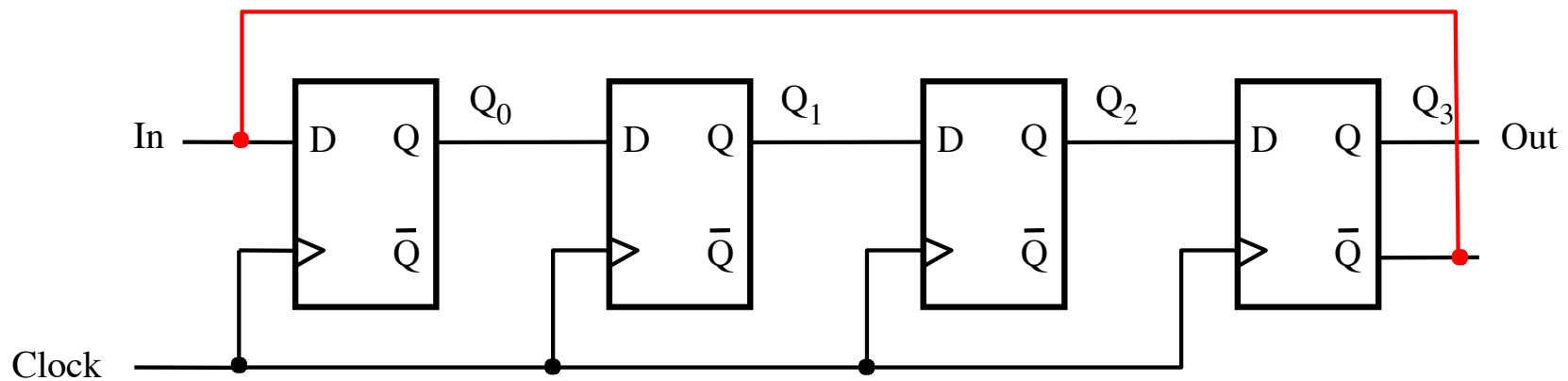
Johnson Counter

How to build a 4-bit Johnson counter



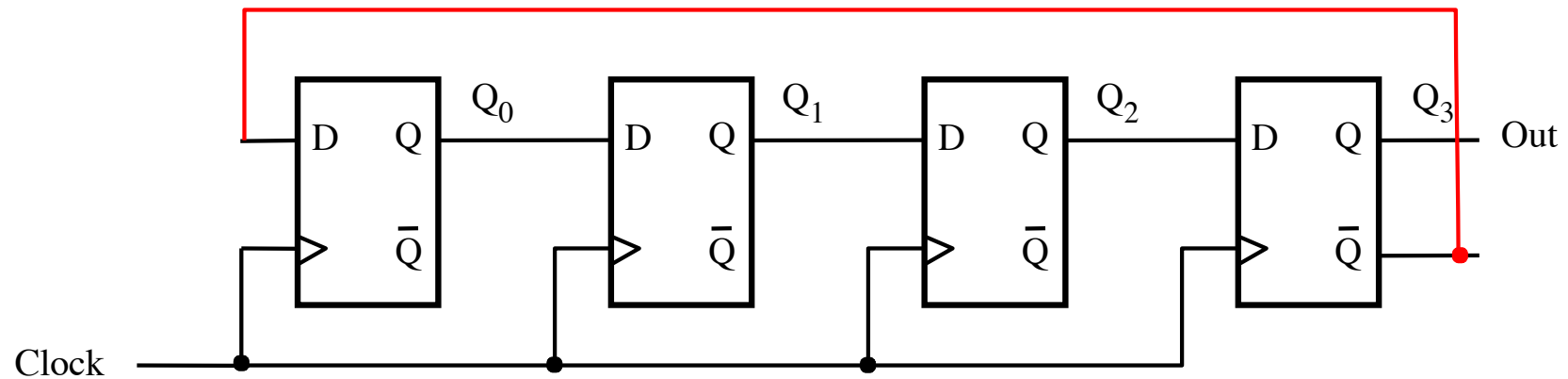
To build a Johnson counter start with a shift register.

How to build a 4-bit Johnson counter



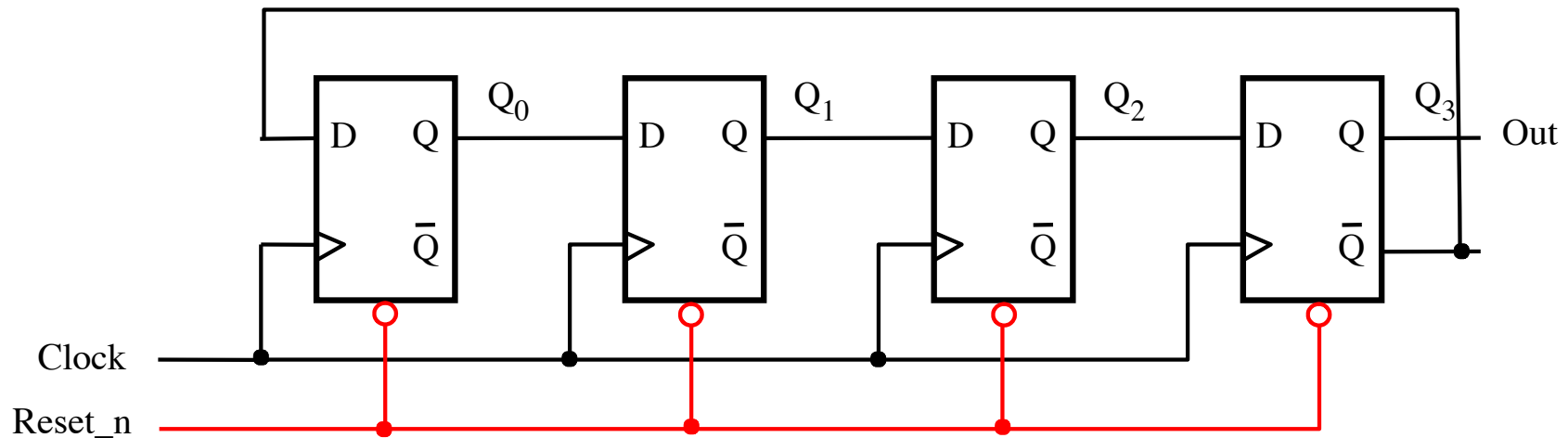
Next, add a loop from the \bar{Q} output of the last flip-flop to the first...

How to build a 4-bit Johnson counter



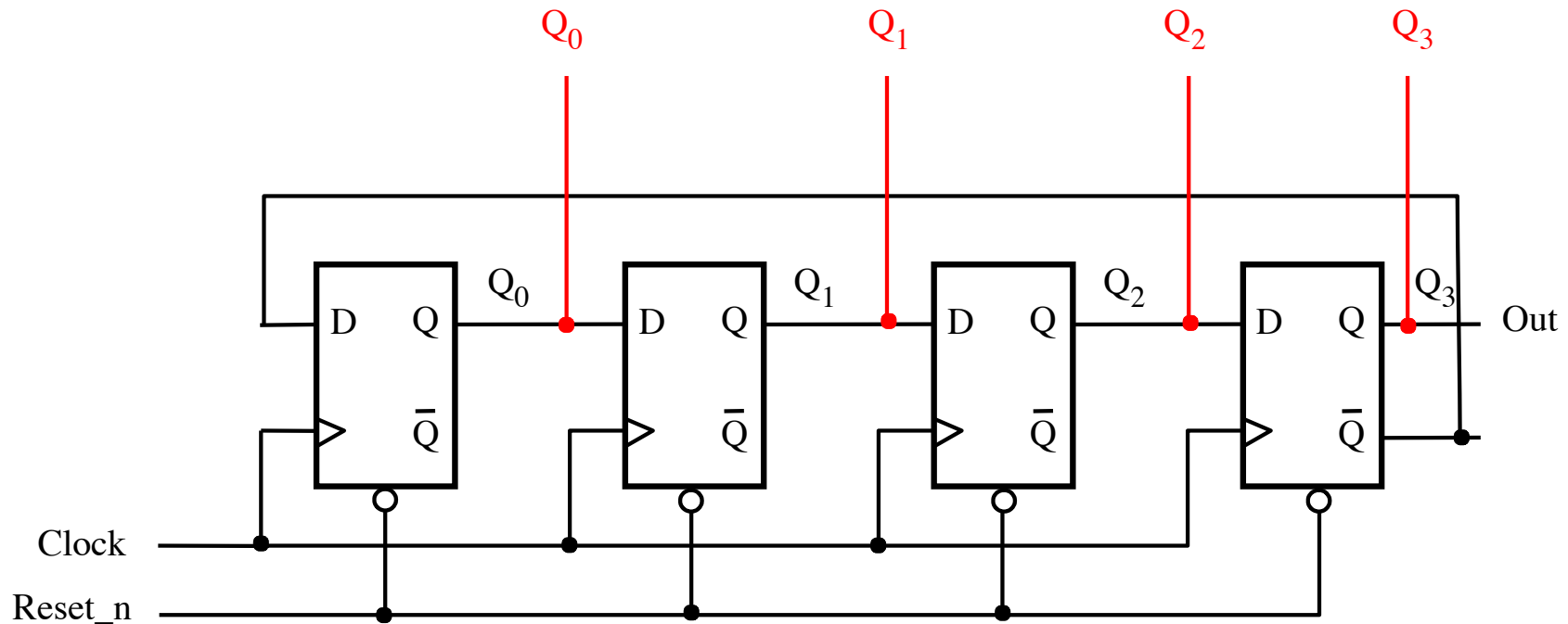
... and remove the In input line.

How to build a 4-bit Johnson counter



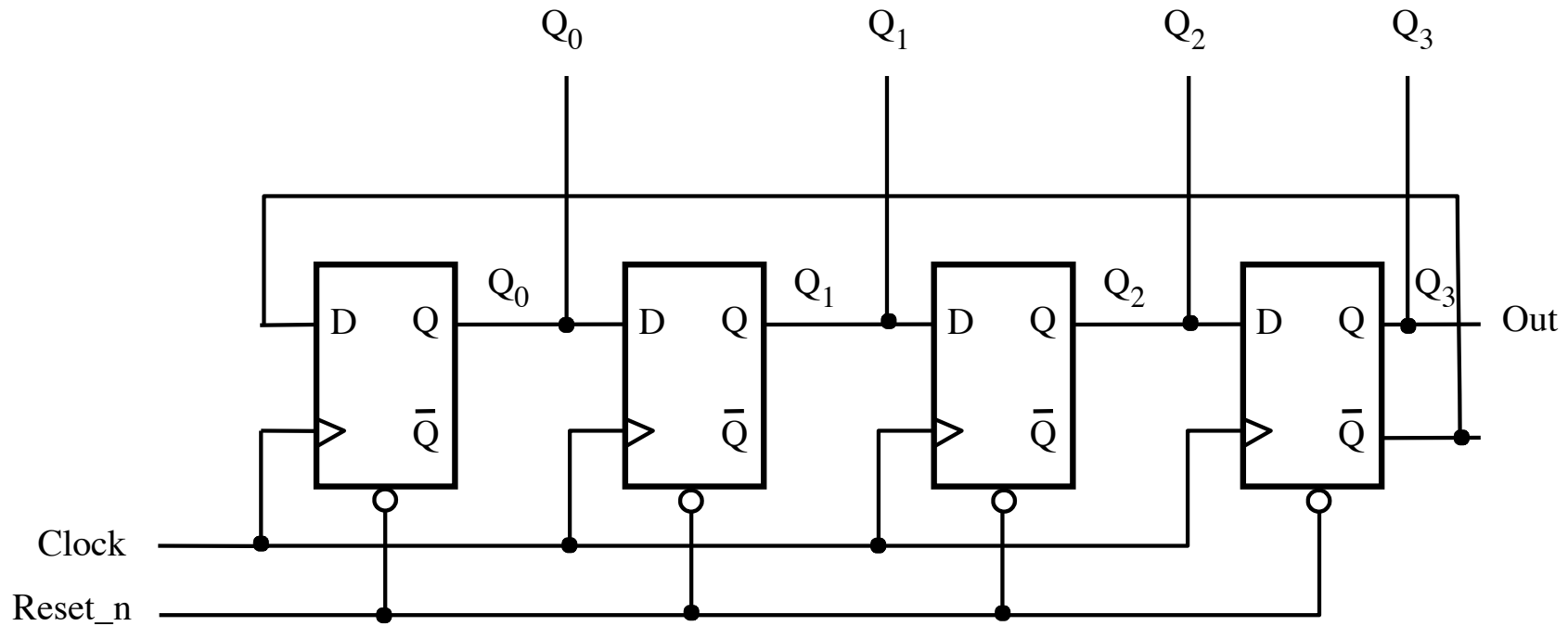
Also, add a Reset_n line that goes to clear_n of all flip-flops.

How to build a 4-bit Johnson counter



Finally, extend the output lines that form the count number.

How to build a 4-bit Johnson counter

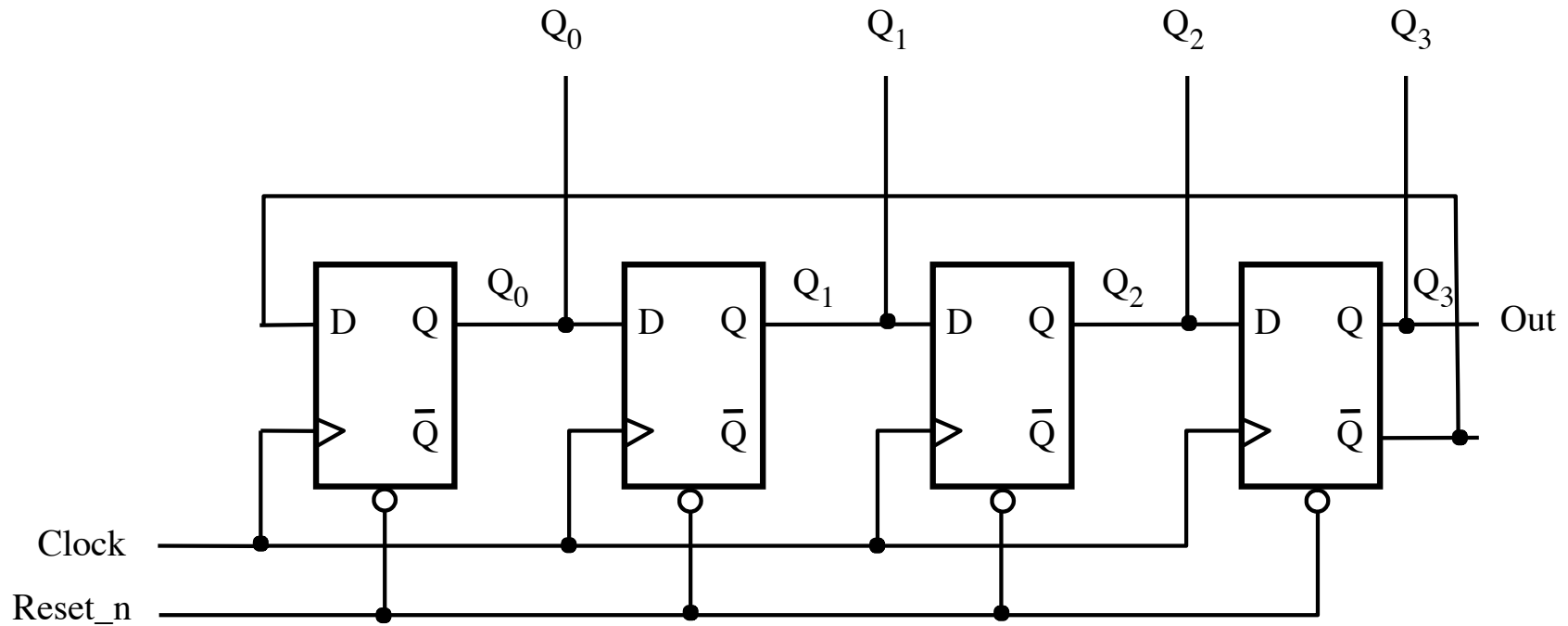


This is the final circuit diagram.

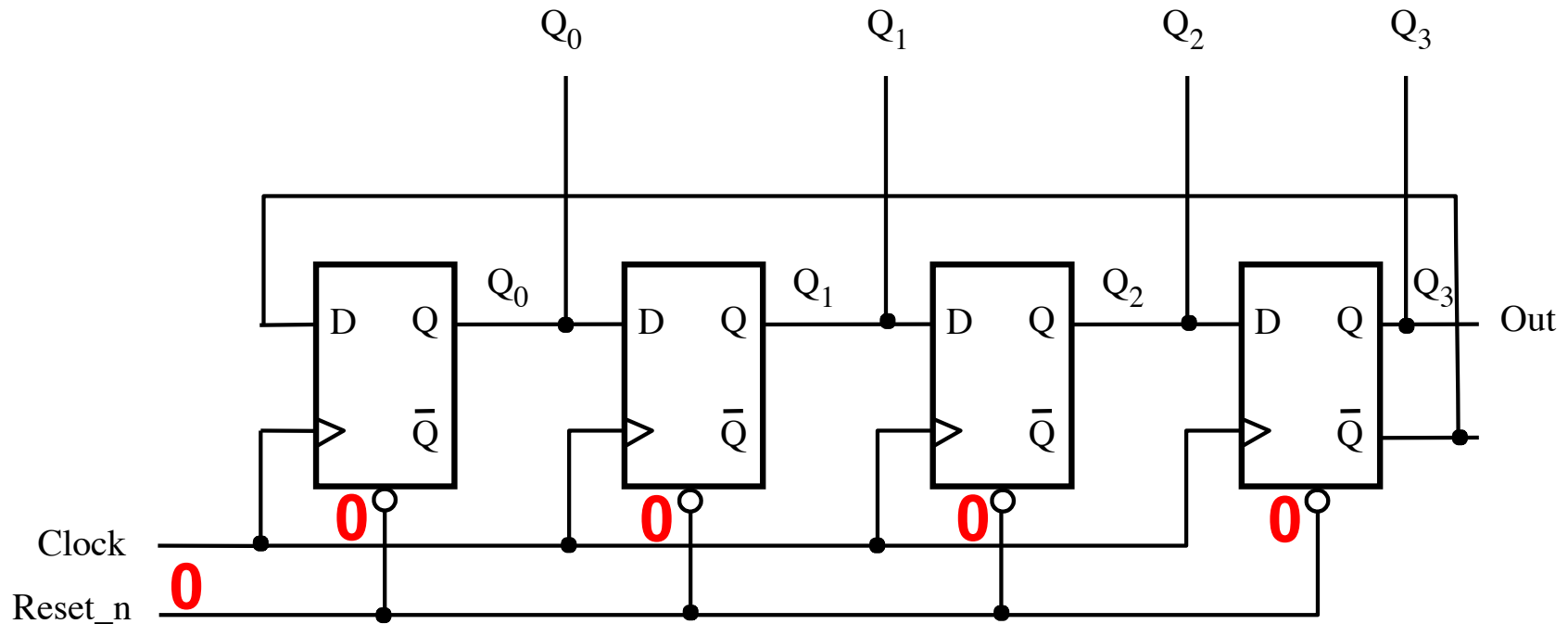
4-Bit Johnson Counter

- Only 1-bit changes at a time
- Start with a reset of all flip-flops
- The counting sequence is:
0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001, 0000
- An n-bit Johnson counter has a counting sequence of length $2n$

Initialization: How does it work?

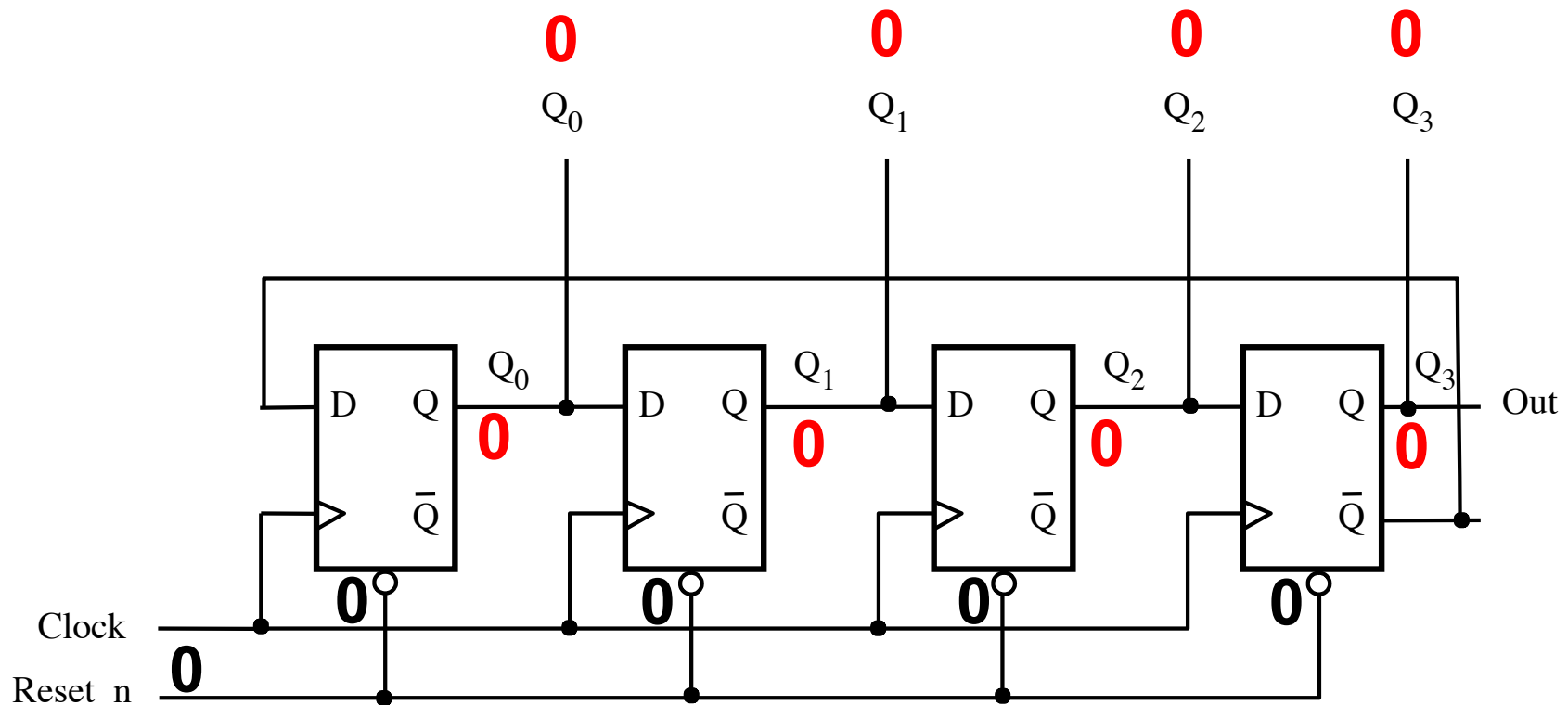


Initialization: How does it work?



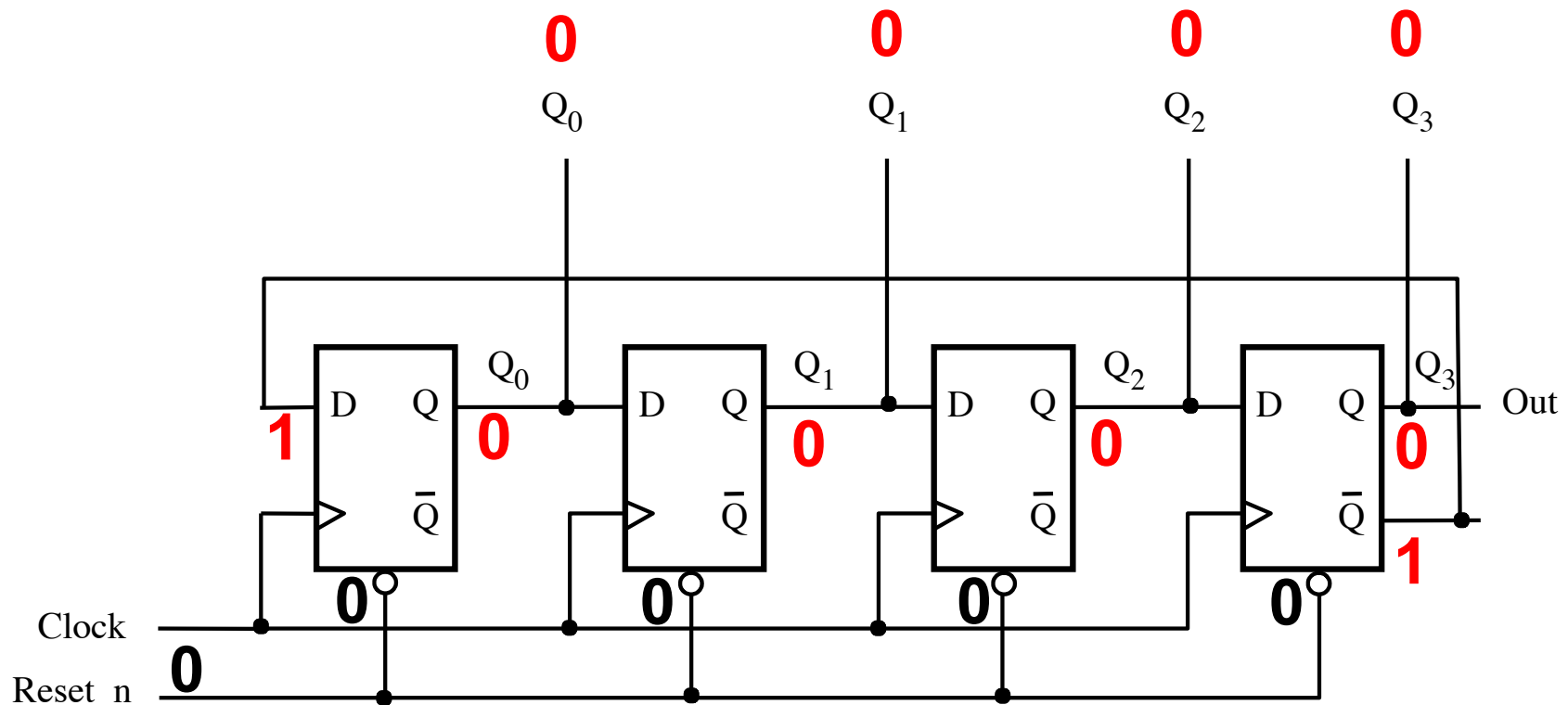
To initialize the Johnson counter set $Reset_n$ to 0 ...

Initialization: How does it work?



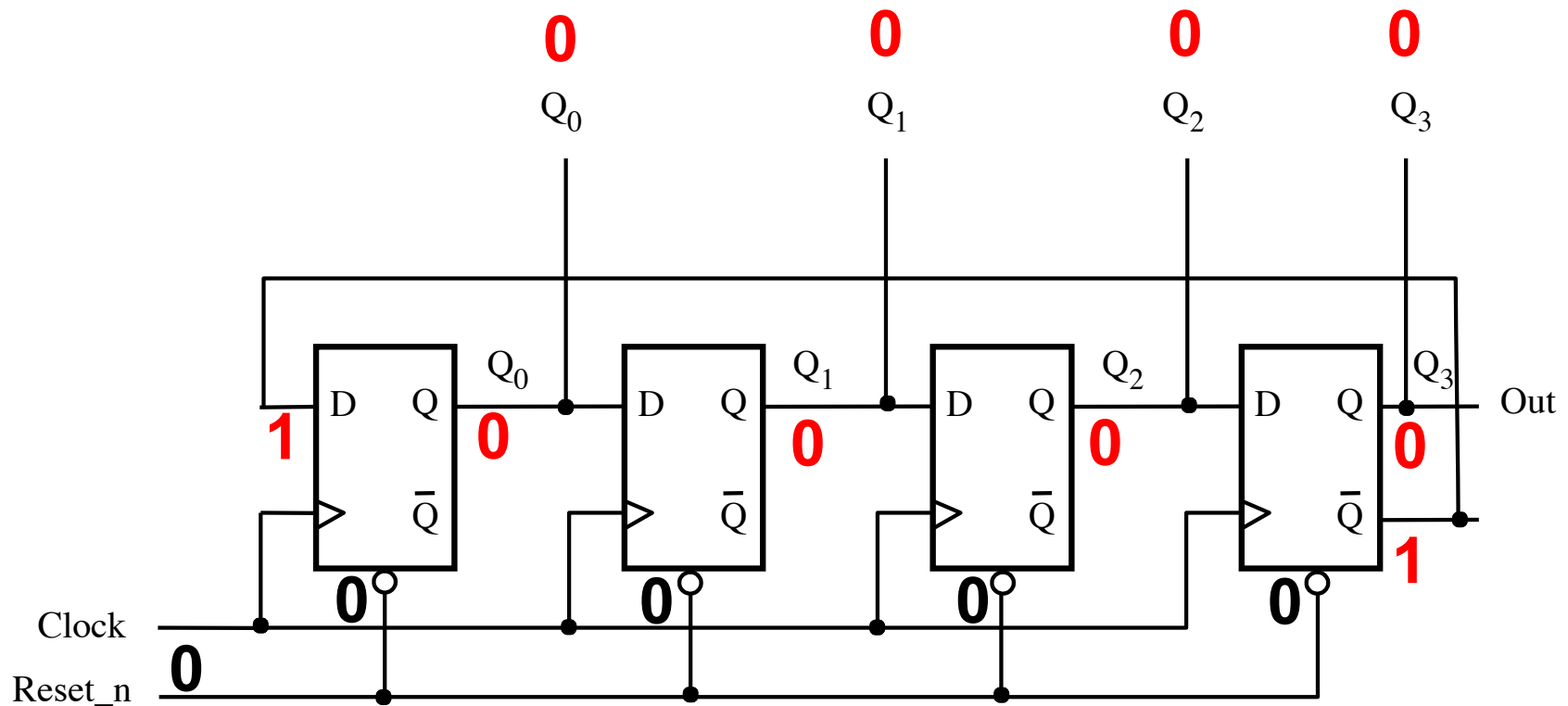
... this zeros the outputs of all flip-flops ...

Initialization: How does it work?

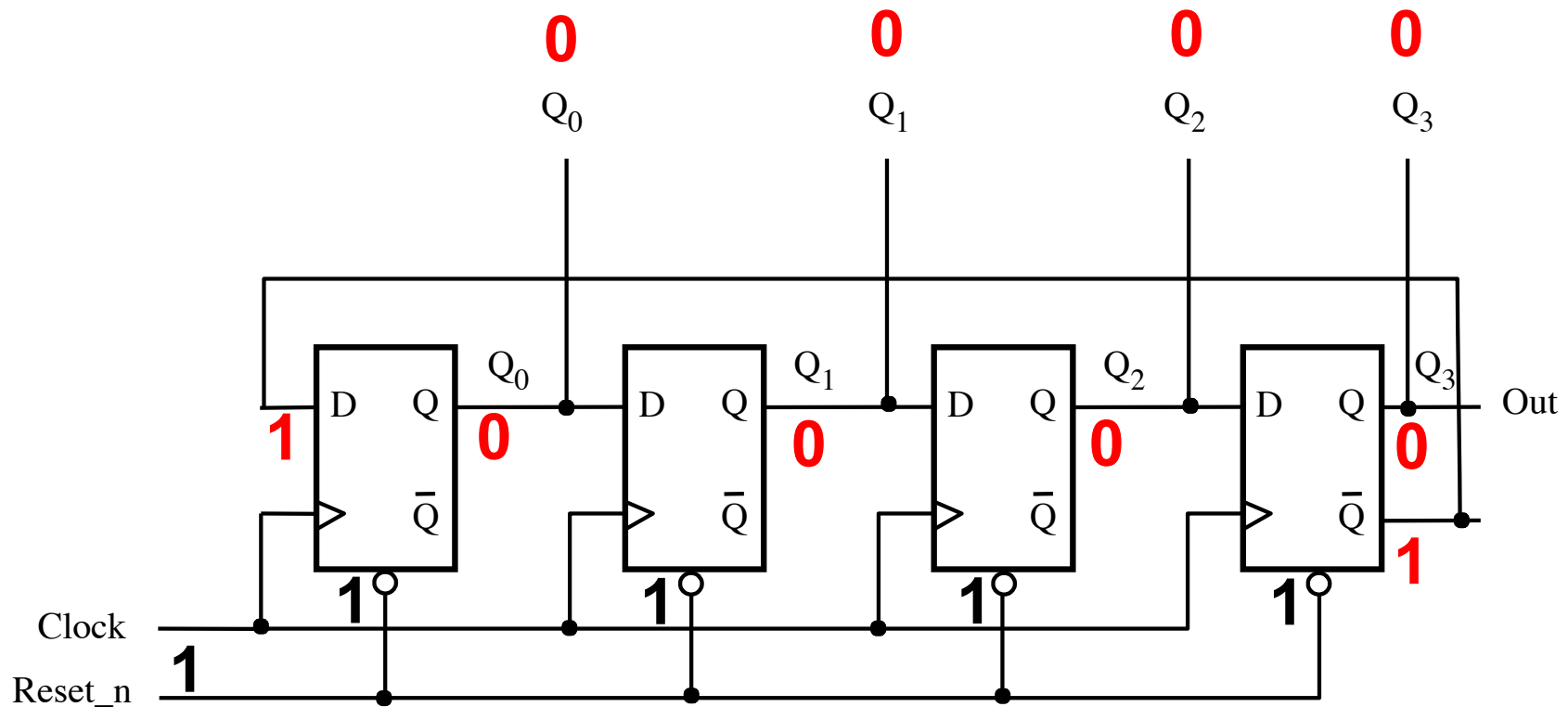


... and also sets the \bar{Q} output of the last flip-flop to 1.

Counting: How does it work?

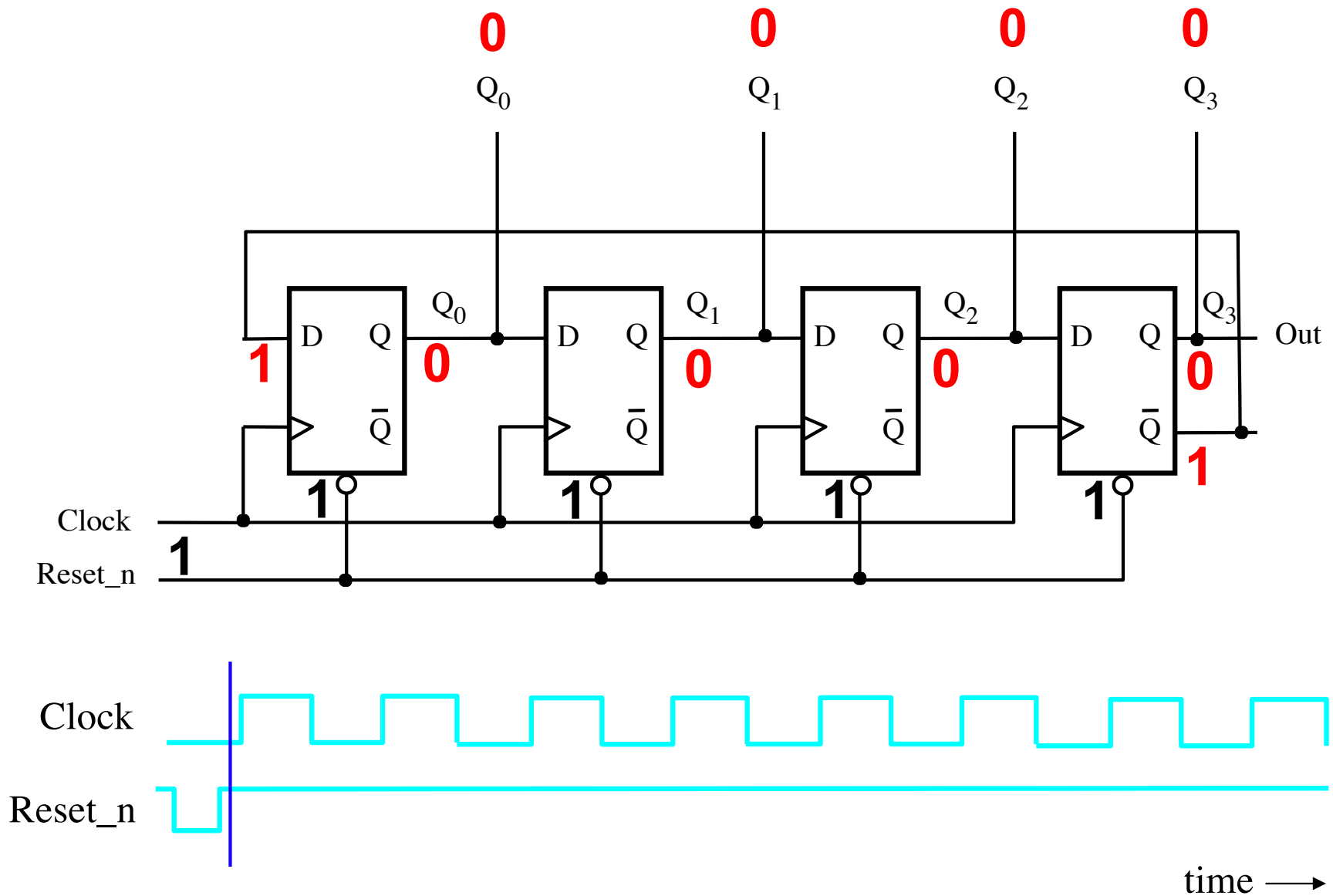


Counting: How does it work?

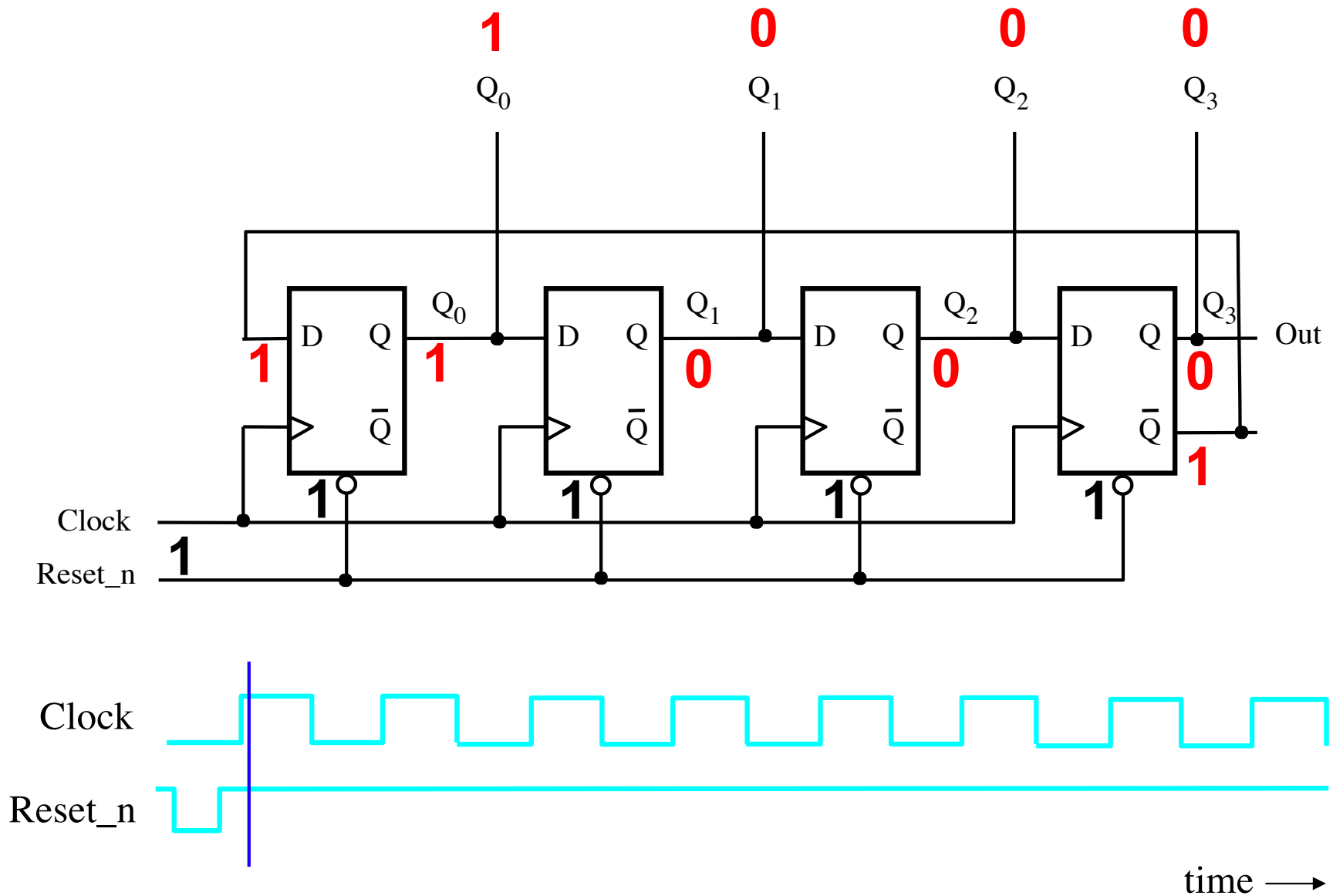


To start counting, Reset_n needs to be set to 1.

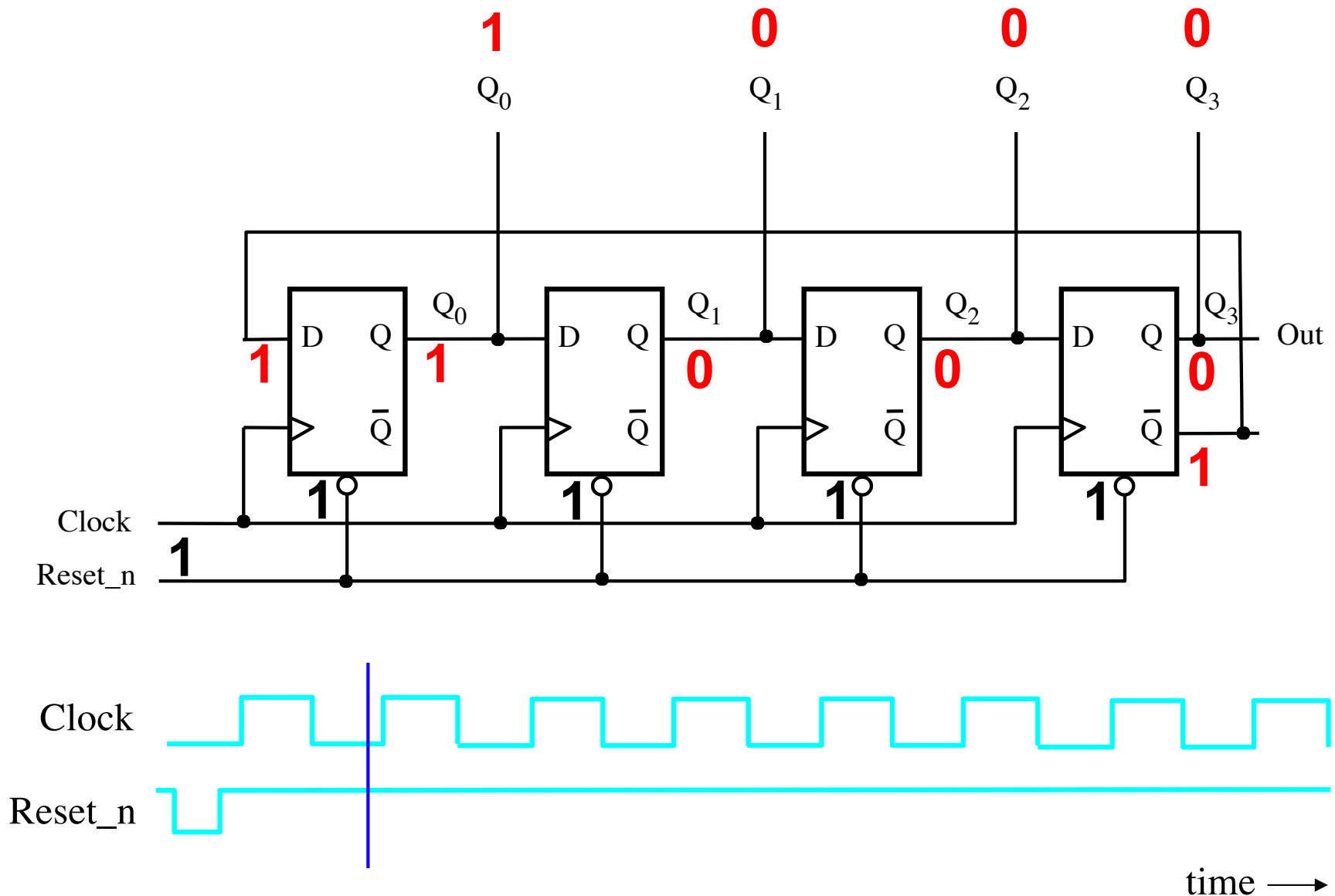
Counting: How does it work?



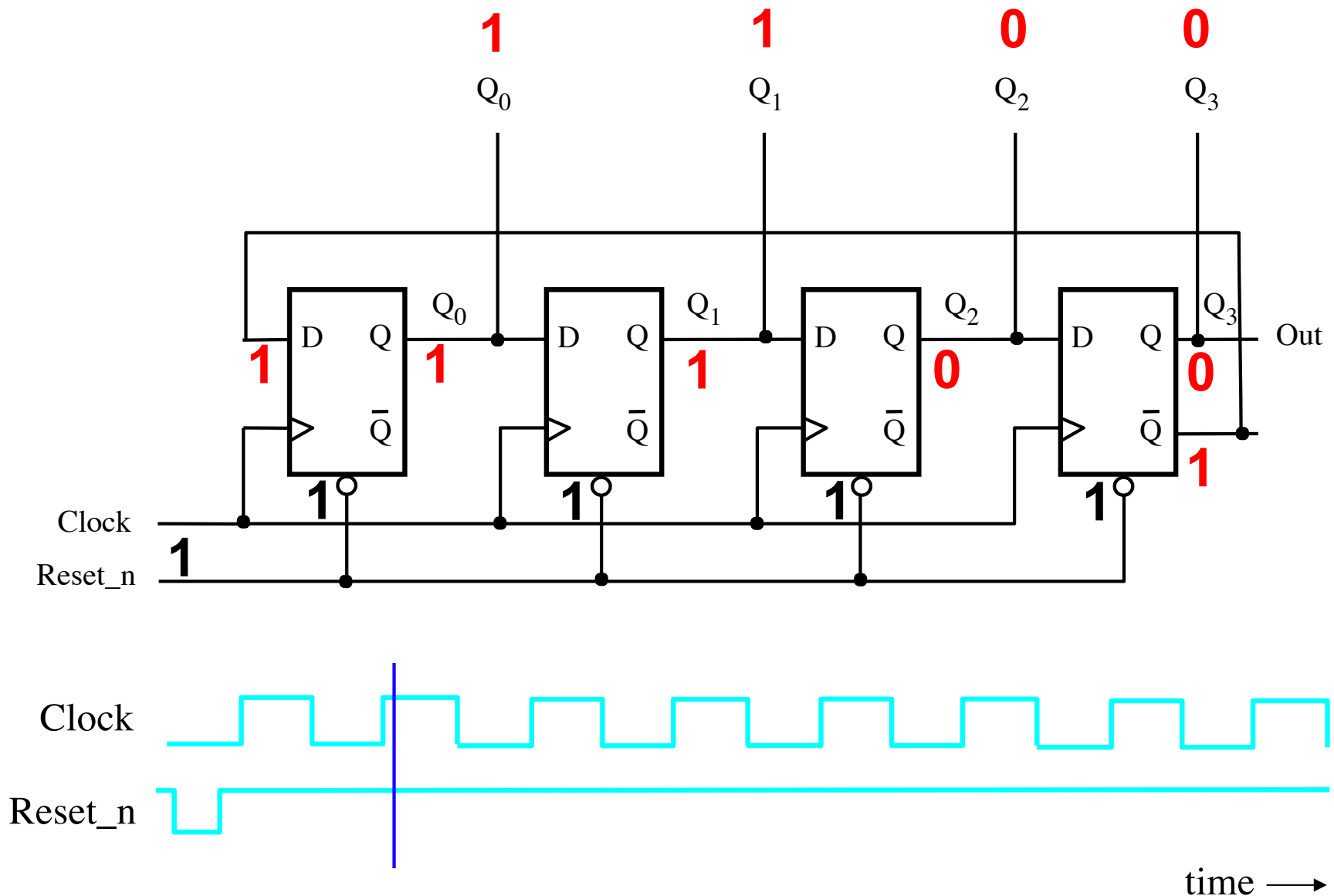
Counting: How does it work?



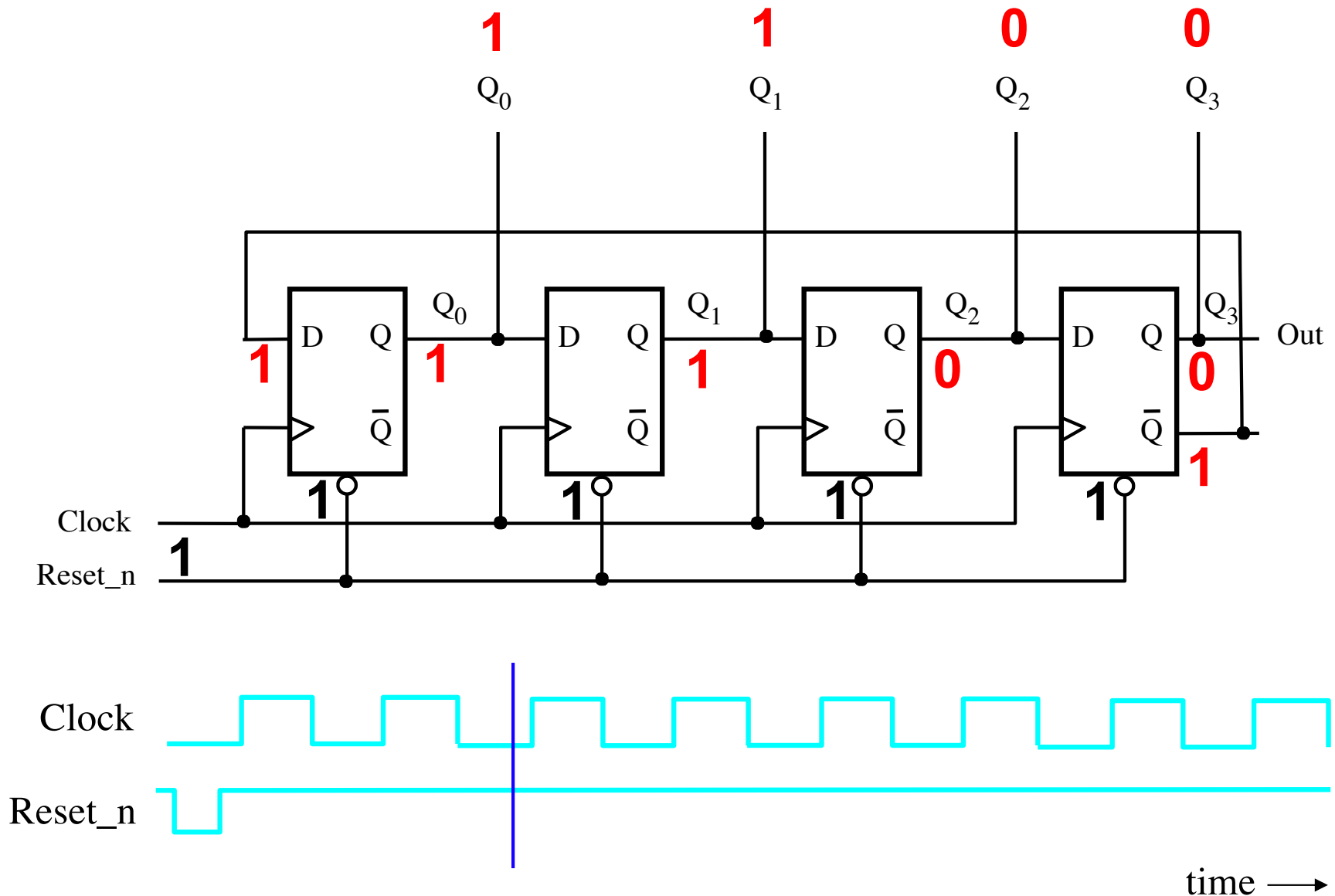
Counting: How does it work?



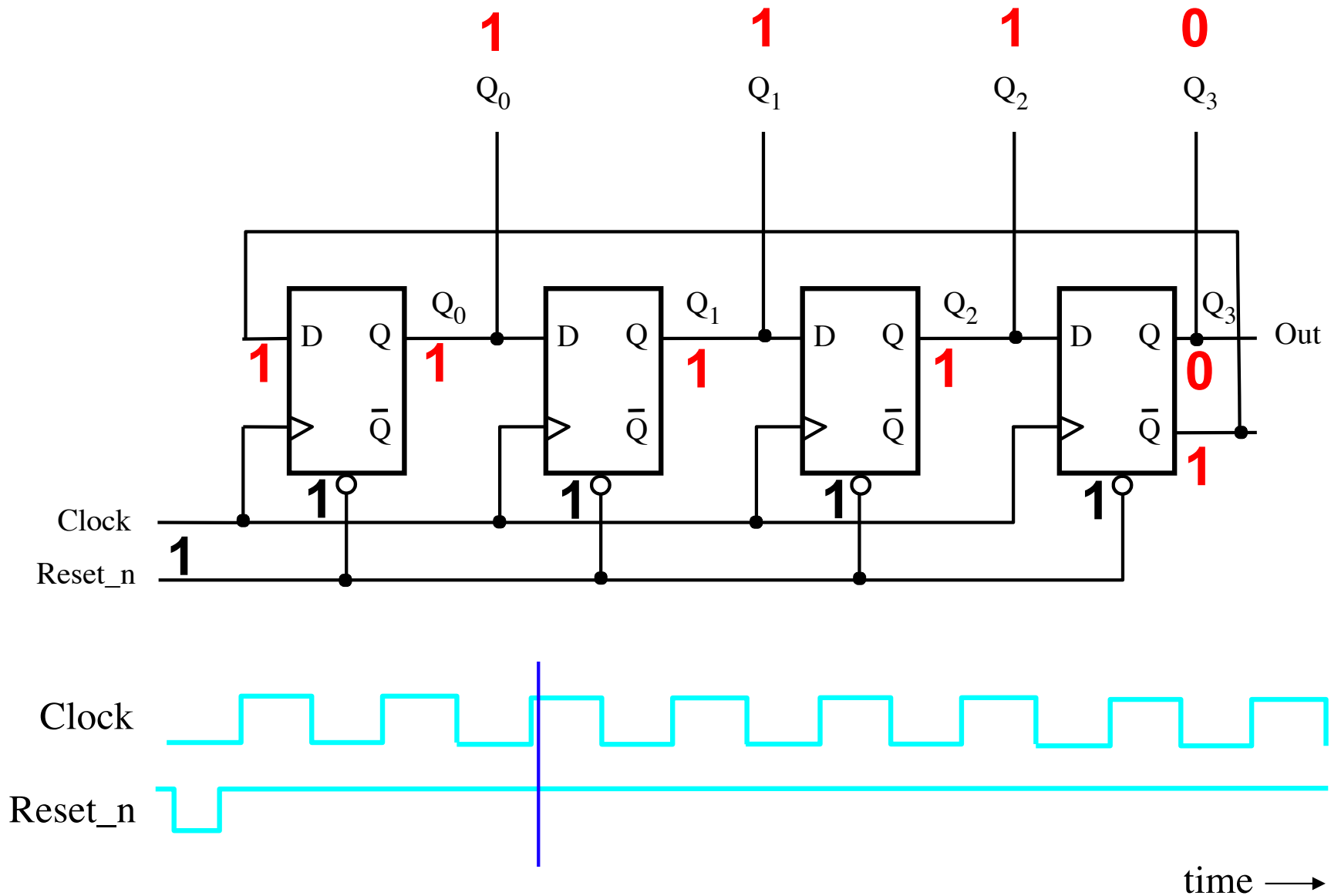
Counting: How does it work?



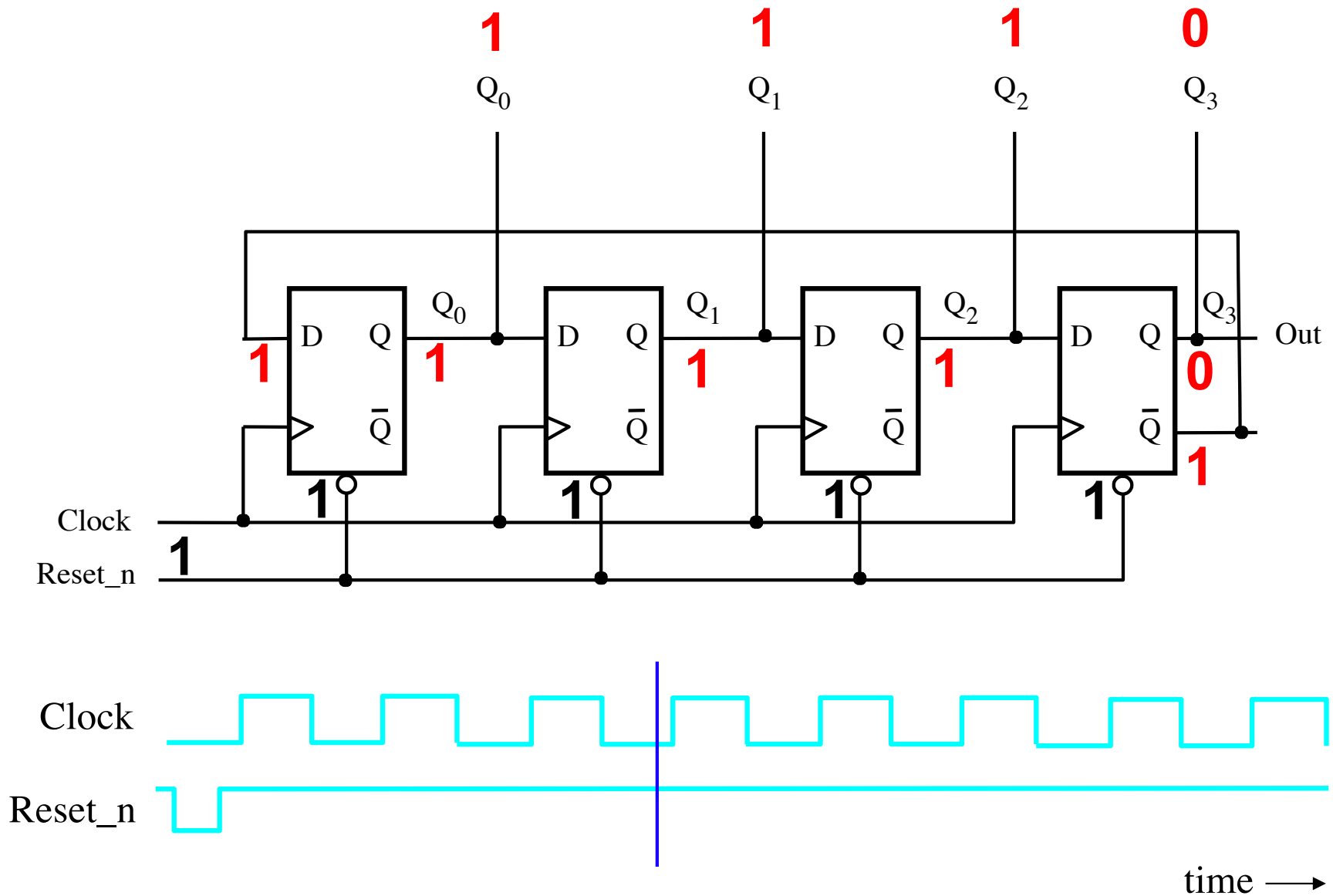
Counting: How does it work?



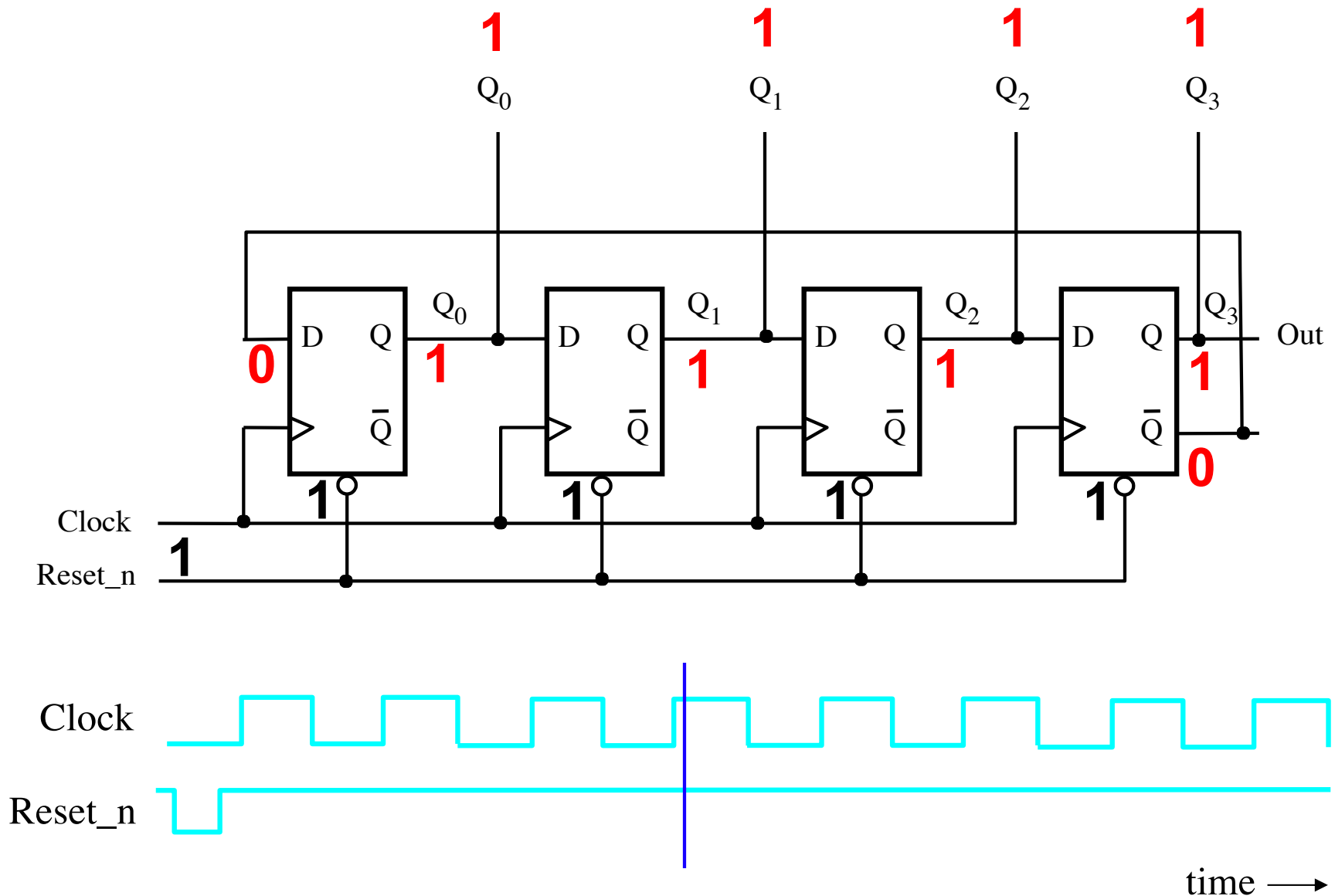
Counting: How does it work?



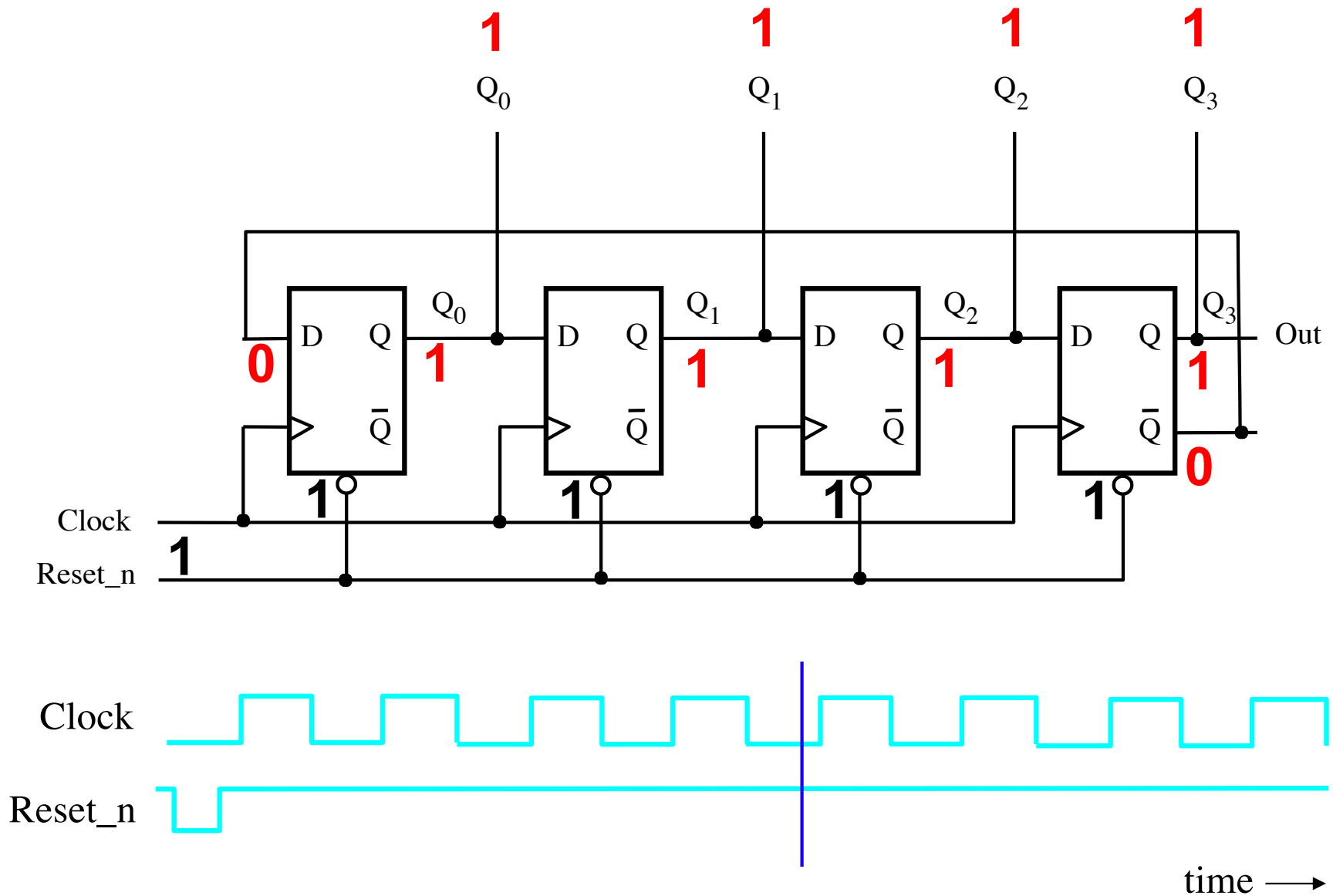
Counting: How does it work?



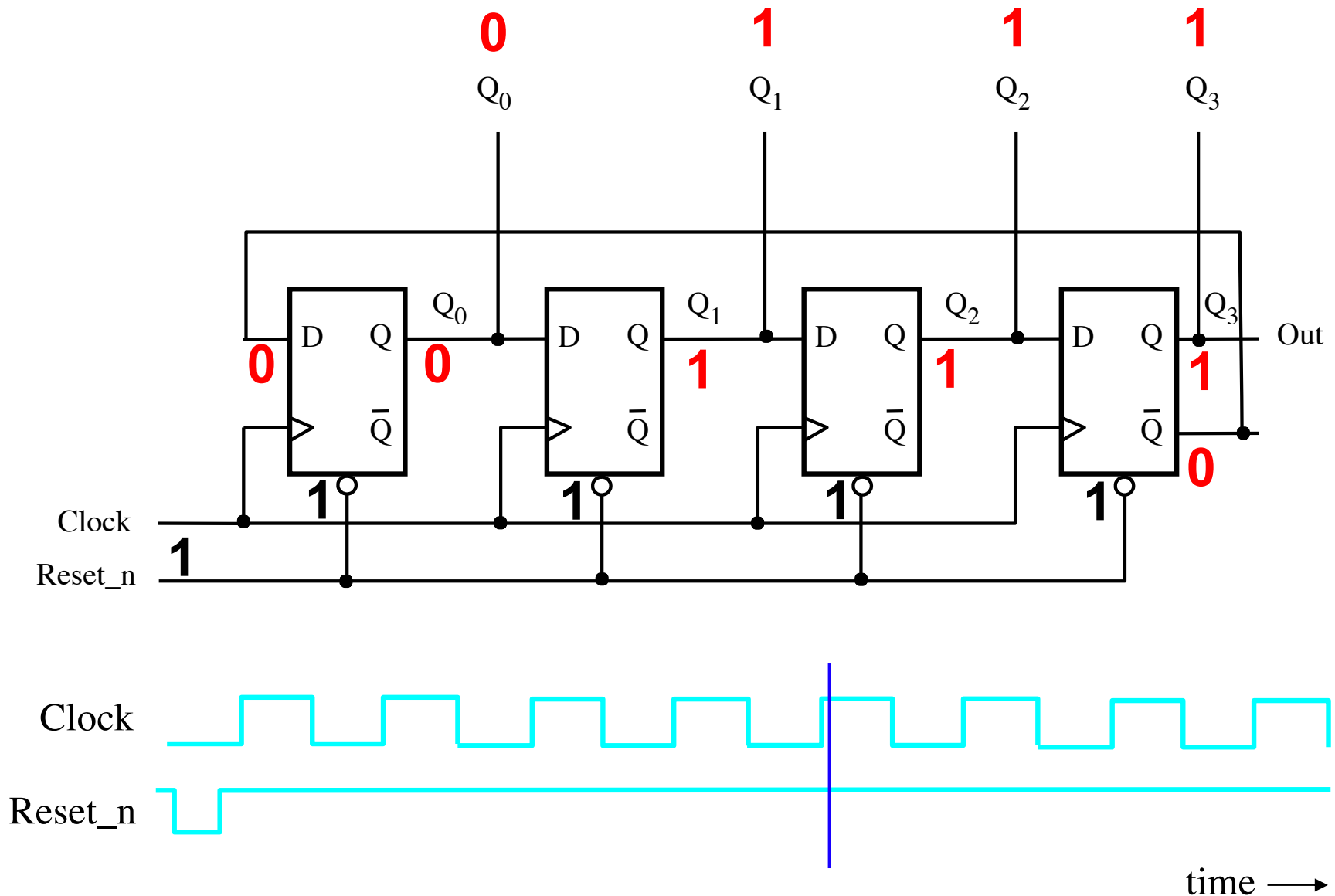
Counting: How does it work?



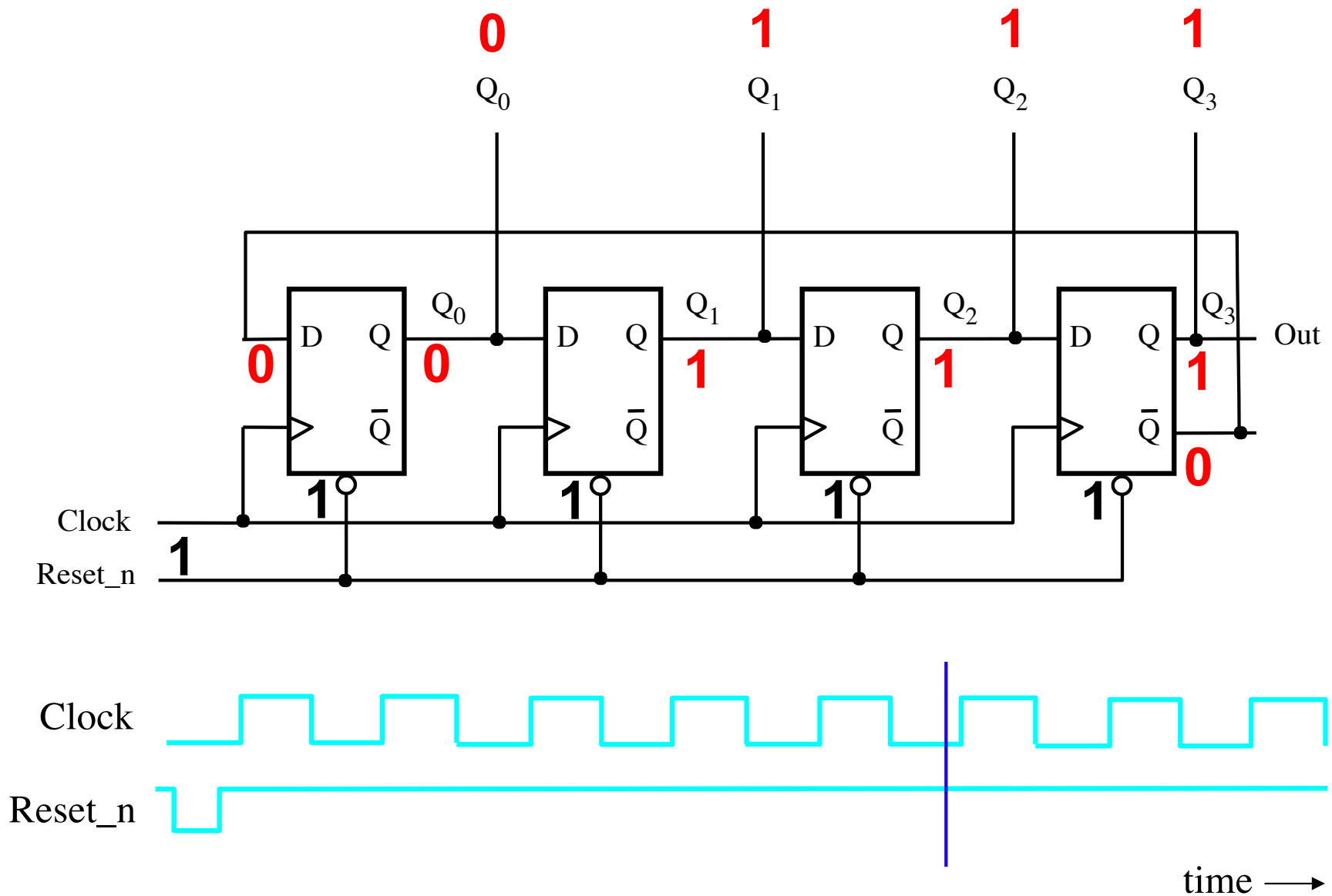
Counting: How does it work?



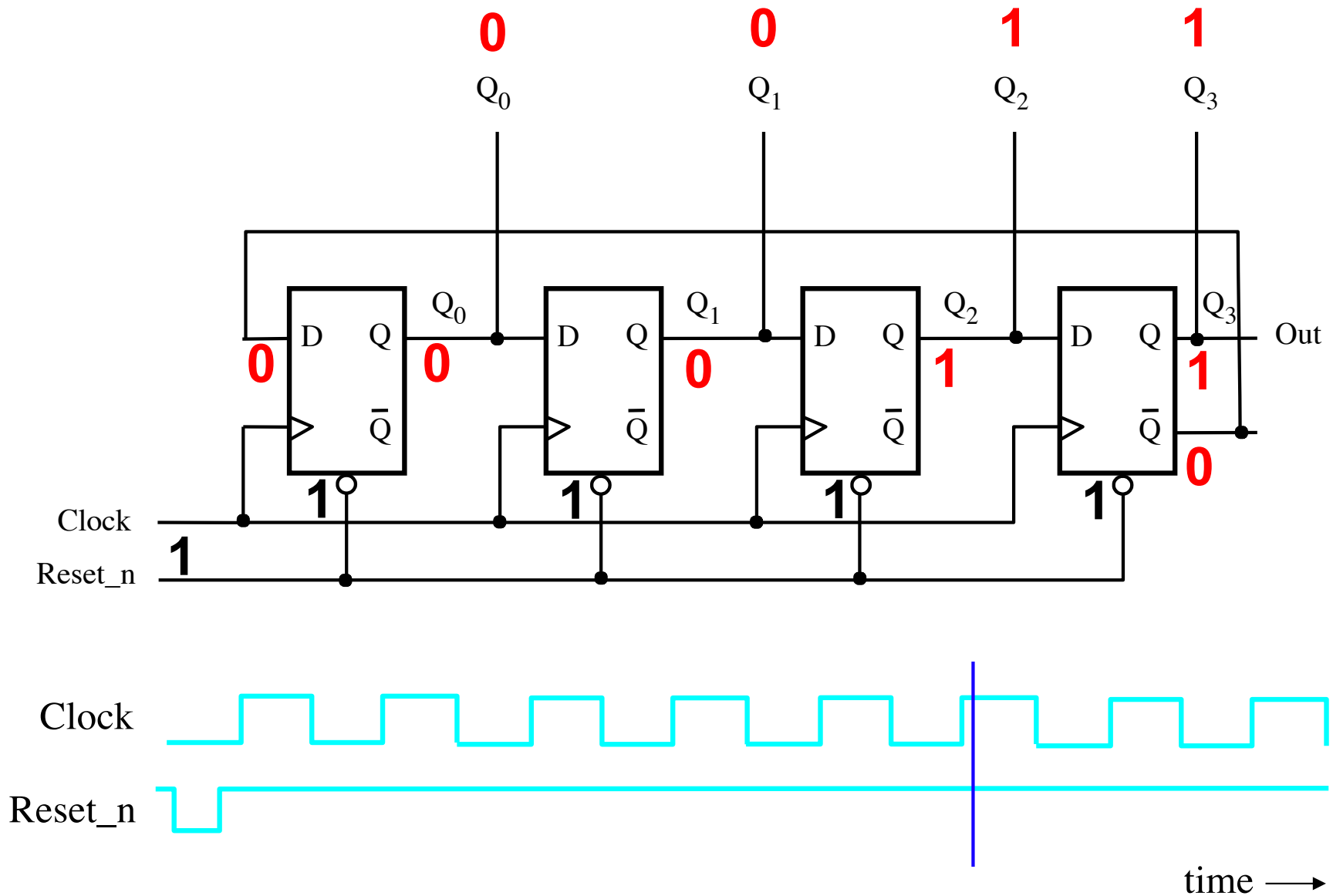
Counting: How does it work?



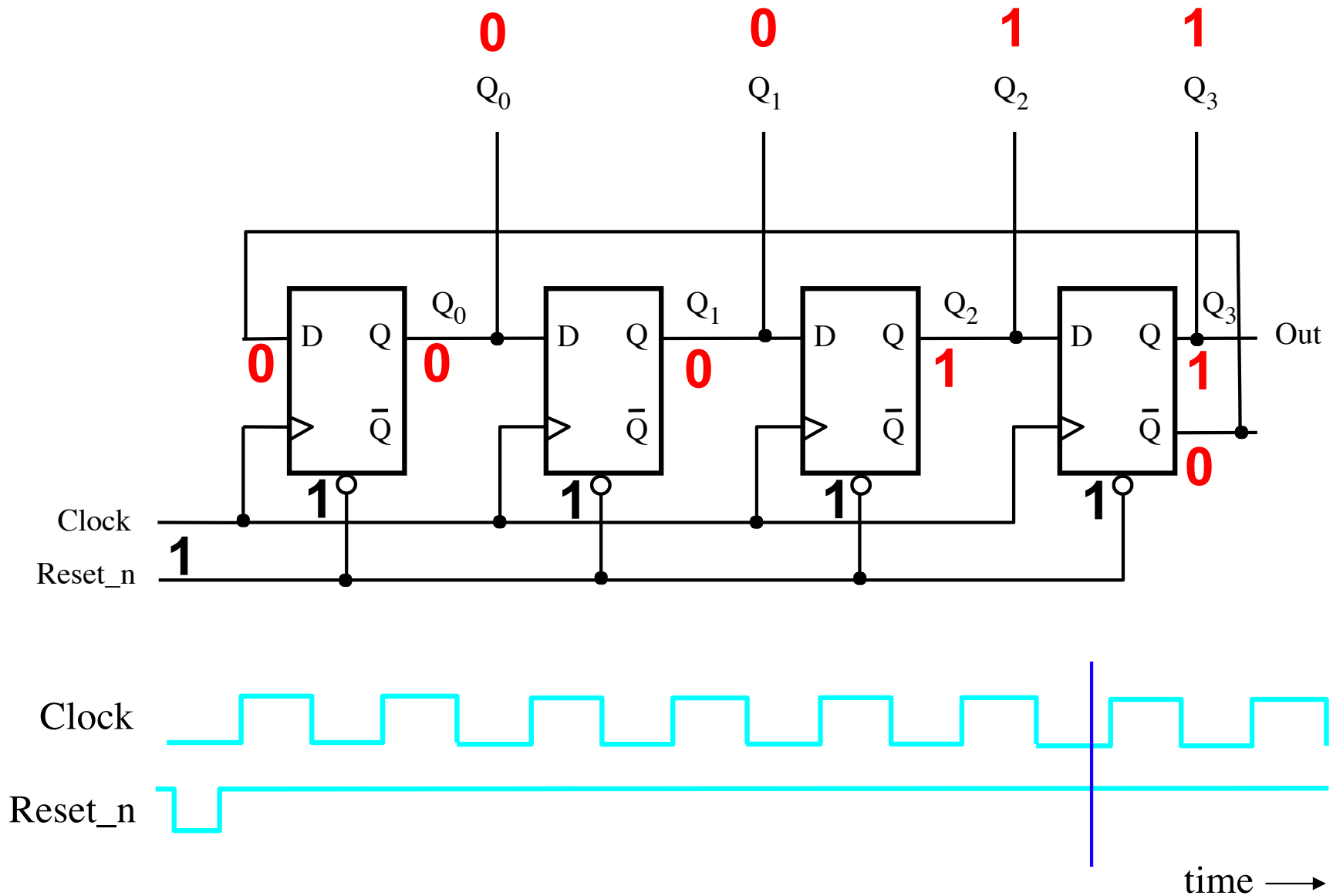
Counting: How does it work?



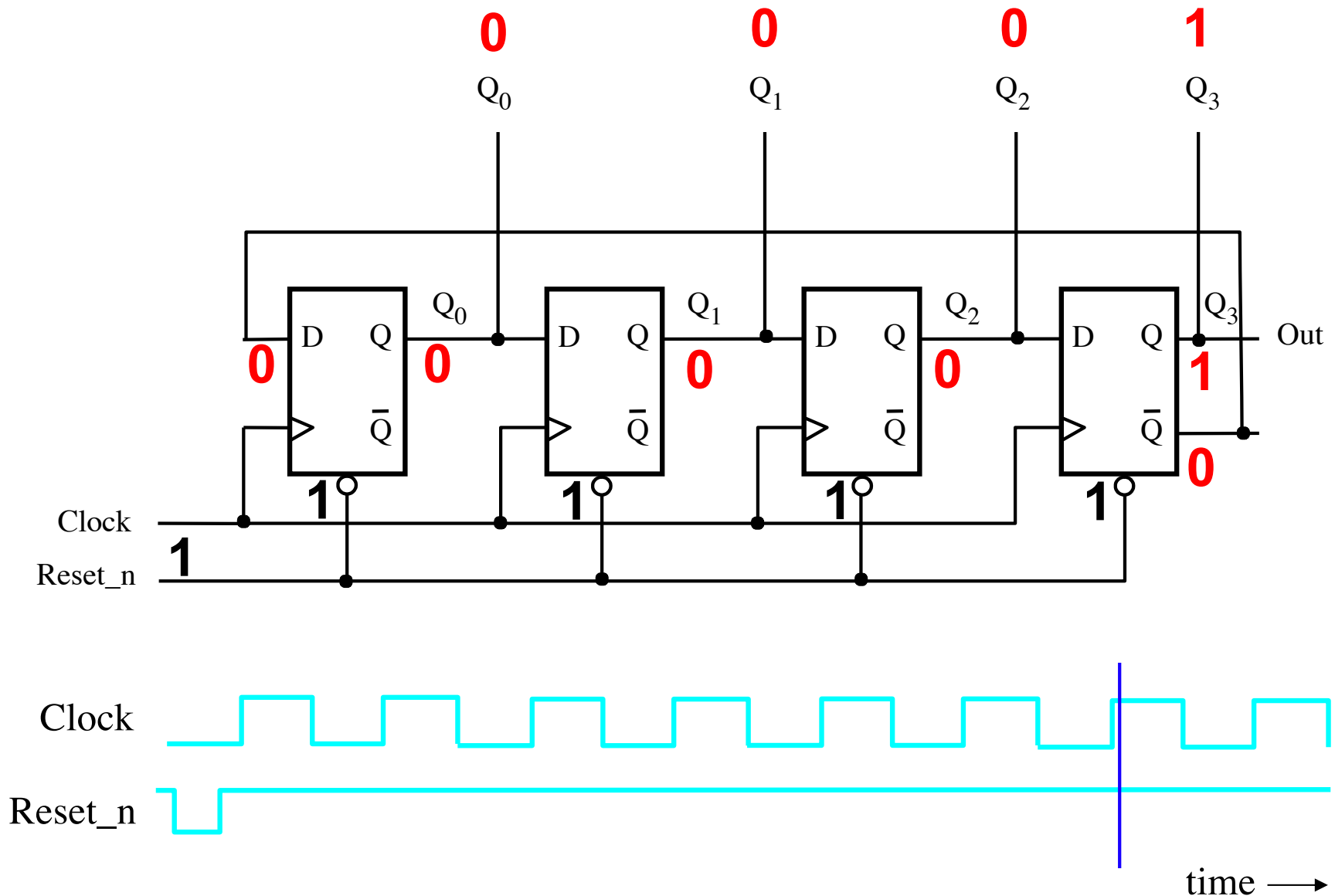
Counting: How does it work?



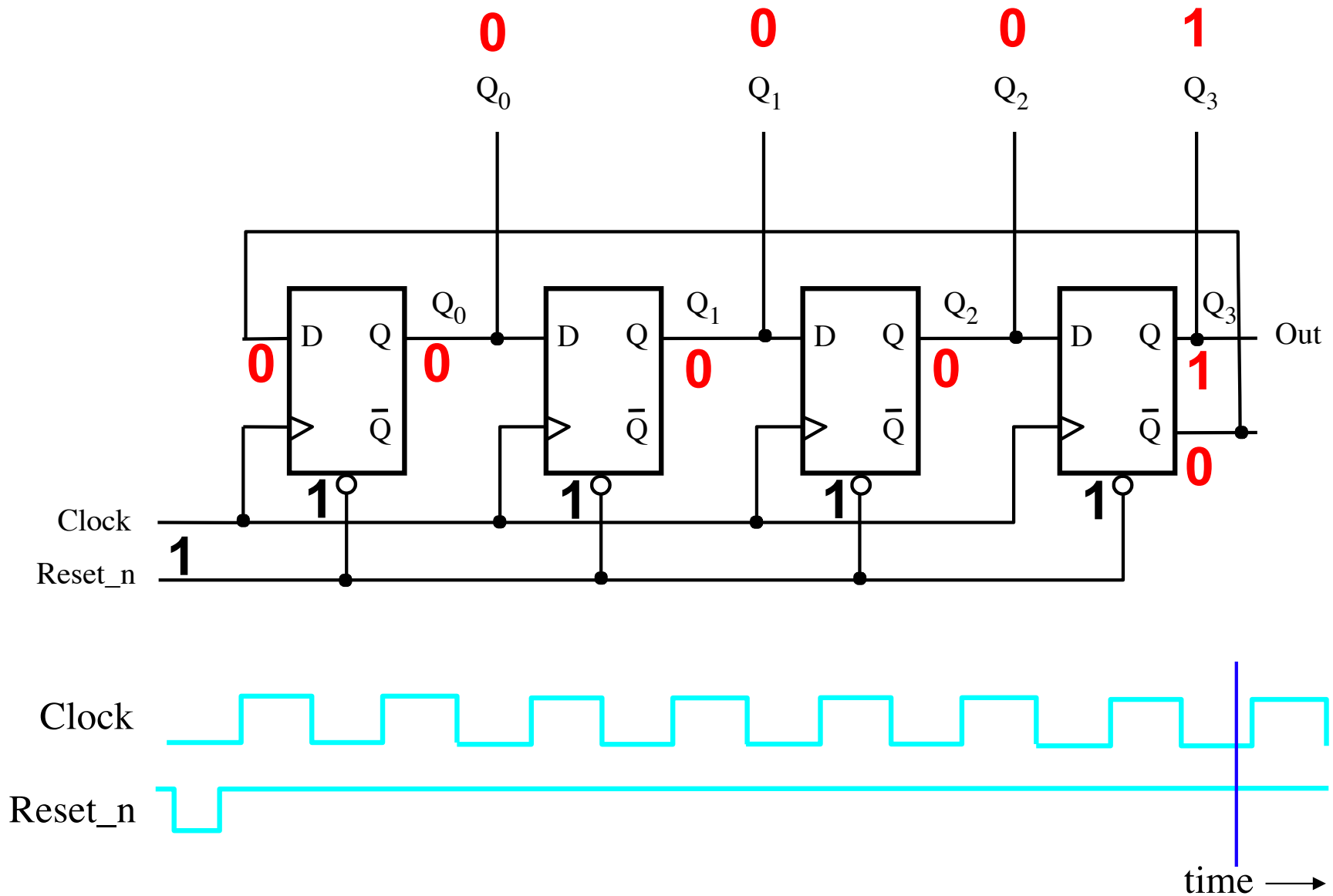
Counting: How does it work?



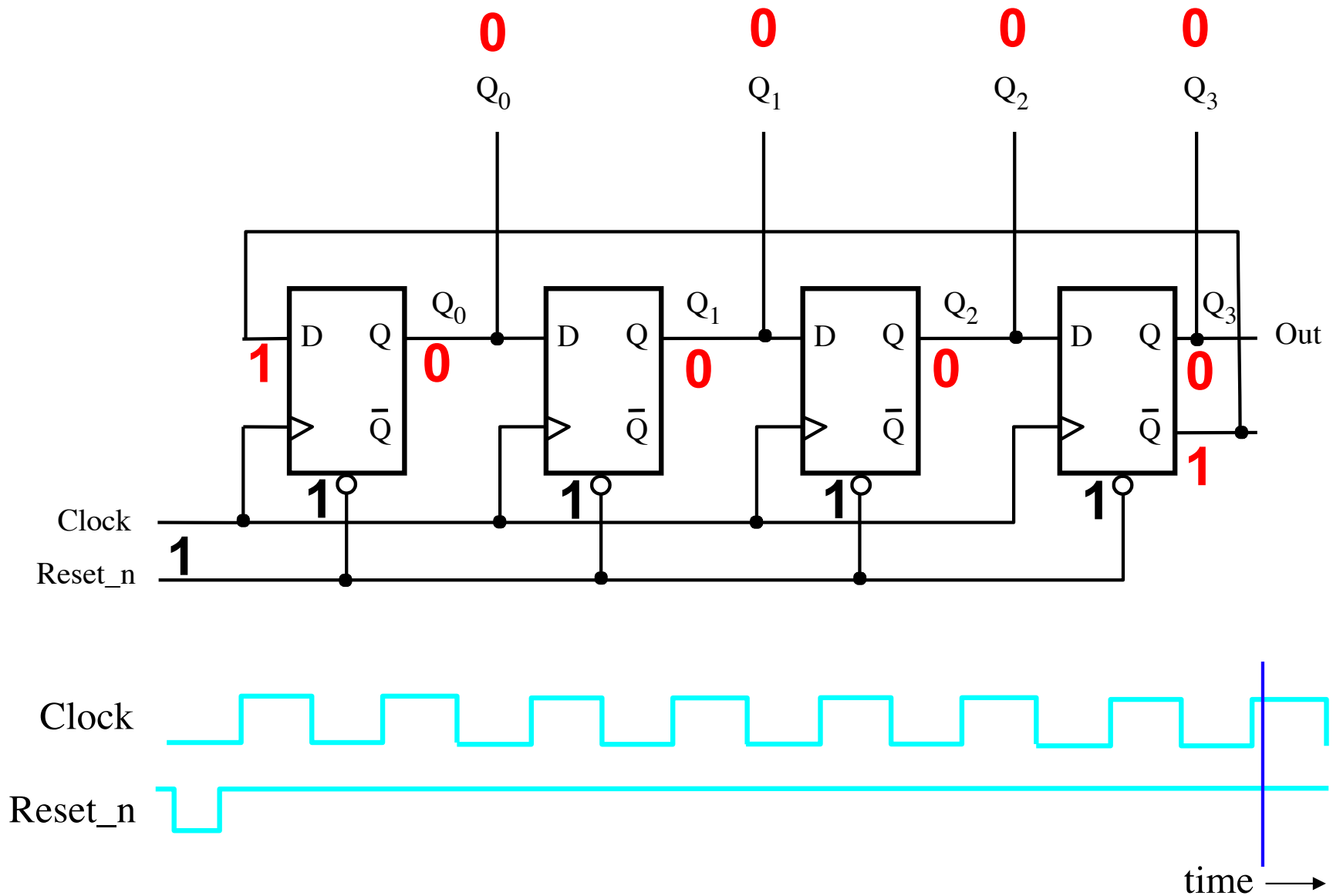
Counting: How does it work?



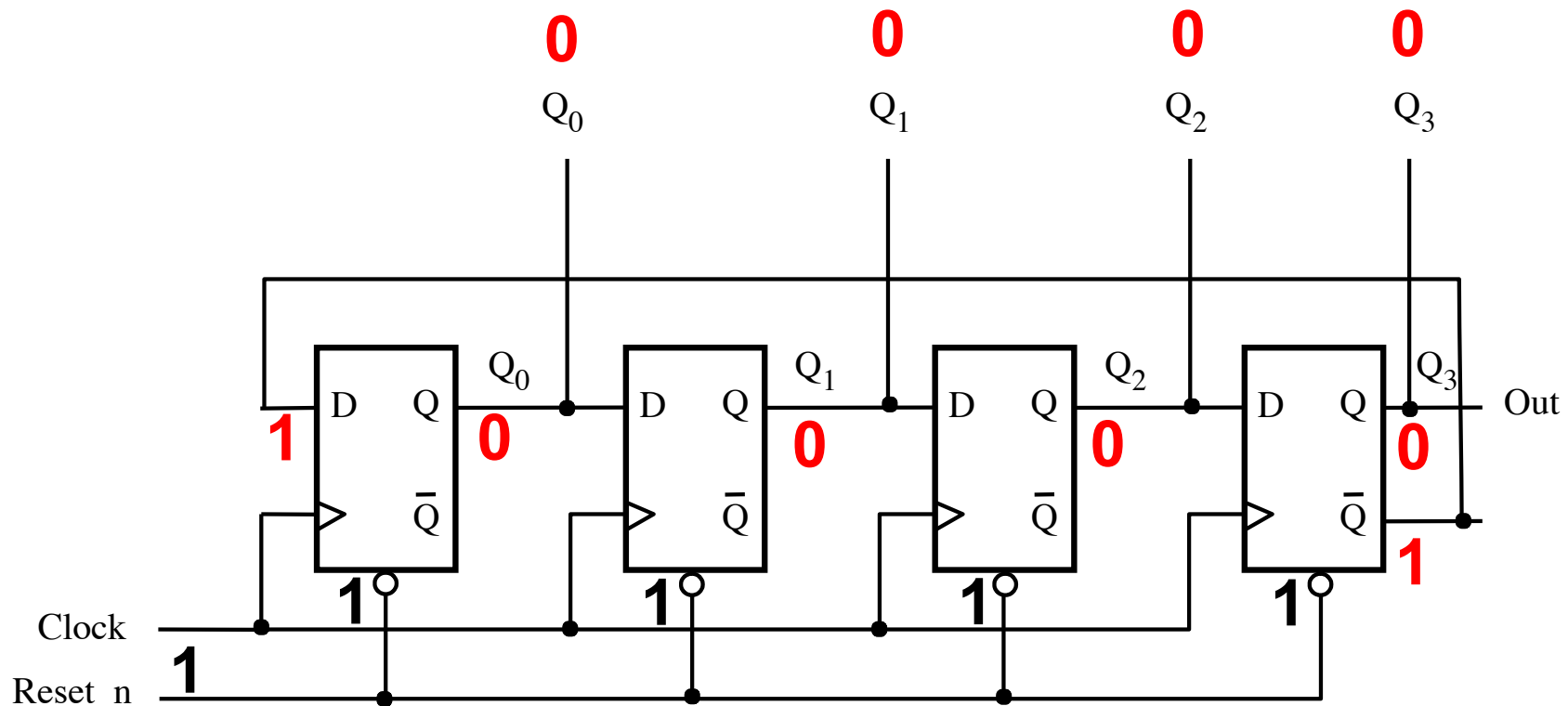
Counting: How does it work?



Counting: How does it work?

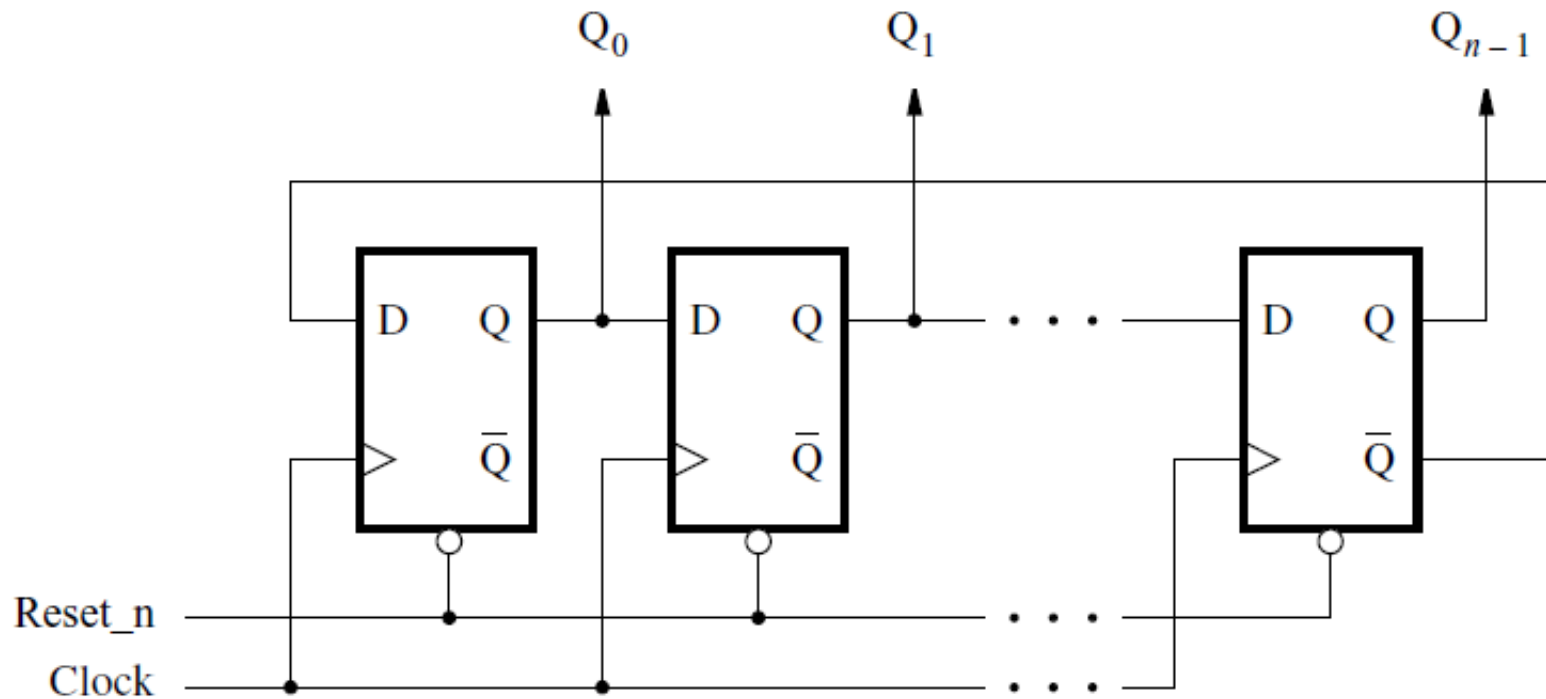


Counting: How does it work?



It is back to the start of the counting sequence, which is:
0000, 1000, 1100, 1110, 1111, 0111, 0011, 0001.

n-bit Johnson Counter



[Figure 5.29 from the textbook]

Questions?

THE END