



# **CprE 281: Digital Logic**

**Instructor: Alexander Stoytchev**

**<http://www.ece.iastate.edu/~alexs/classes/>**

# Review for the Final Exam

*CprE 281: Digital Logic*  
*Iowa State University, Ames, IA*  
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# **Administrative Stuff**

- **The FINAL exam is scheduled for**
- **Saturday Nov 21 @ 2:15 – 4:15 PM**

# Final Exam Format

- **The exam will cover: Chapter 1 to Chapter 6, and Sections 7.1-7.2, register machines, and i281 CPU**
- **Emphasis will be on Chapter 5, 6, and 7**
- **The exam will be closed book but open notes.**
- **You can bring up to 5 pages of handwritten or typed notes.**



# Final Exam Format

- **The exam will be out of 135 points**
- **You need 95 points to get an A on this exam**
- **It will be great if you can score more than 100 points.**
  - **but you can't roll over your extra points 😞**

# Topics for the Final Exam

- **K-maps for 2, 3, and 4 variables**
- **Multiplexers (circuits and function)**
- **Synthesis of logic functions using multiplexers**
- **Shannon's Expansion Theorem**
- **1's complement and 2's complement representation**
- **Addition and subtraction of binary numbers**
- **Circuits for adding and subtracting**
- **Serial adder**
- **Latches (circuits, behavior, timing diagrams)**
- **Flip-Flops (circuits, behavior, timing diagrams)**
- **Counters (up, down, synchronous, asynchronous)**
- **Registers and Register Files**

# Topics for the Final Exam

- **Synchronous Sequential Circuits**
- **FSMs**
- **Moore Machines**
- **Mealy Machines**
- **State diagrams, state tables, state-assigned tables**
- **State minimization**
- **Designing a counter**
- **Arbiter Circuits**
- **Reverse engineering a circuit**
- **ASM Charts**
- **Register Machines and programs for them**
- **ALU, PC, and control for a simple processor (i281 CPU)**
- **Assembly and machine language (i281 assembly)**
- **Something from Star Wars and/or the Matrix**

# **How to Study for the Final Exam**

- **Form a study group**
- **Go over the slides for this class**
- **Go over the homeworks again**
- **Go over the problems at the end of Ch 5 & 6**
- **Exercise**
- **Get some sleep**

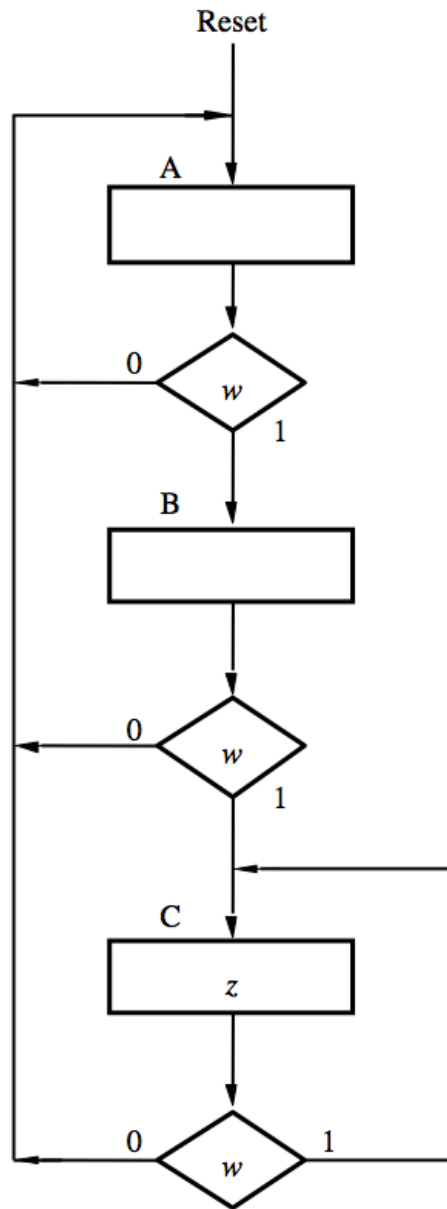
# **Administrative Stuff**

- **Please check your grades on Canvas**
- **Let me know if something is wrong or missing**

# **Sample Problems**

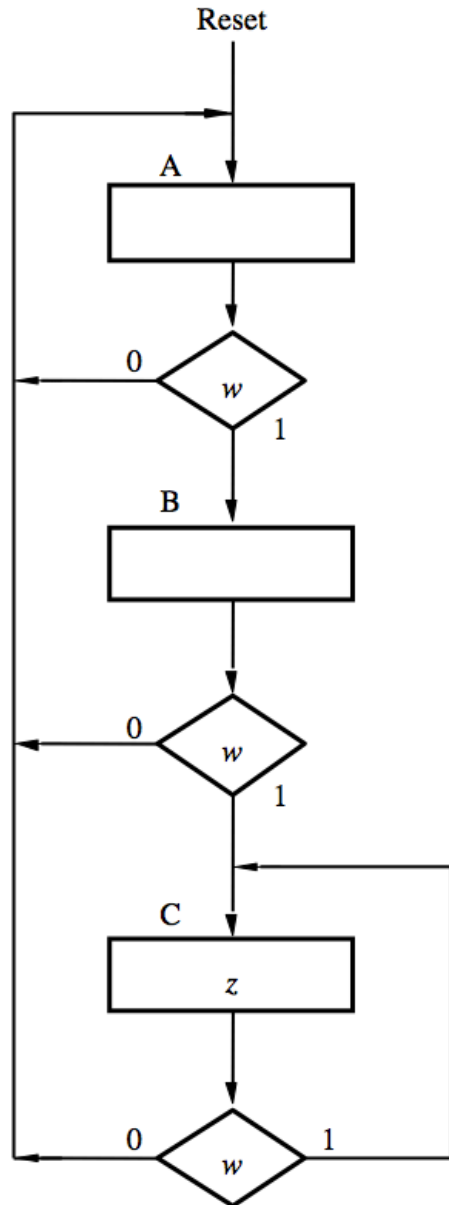
# ASM Charts

Given an ASM chart draw the corresponding FSM

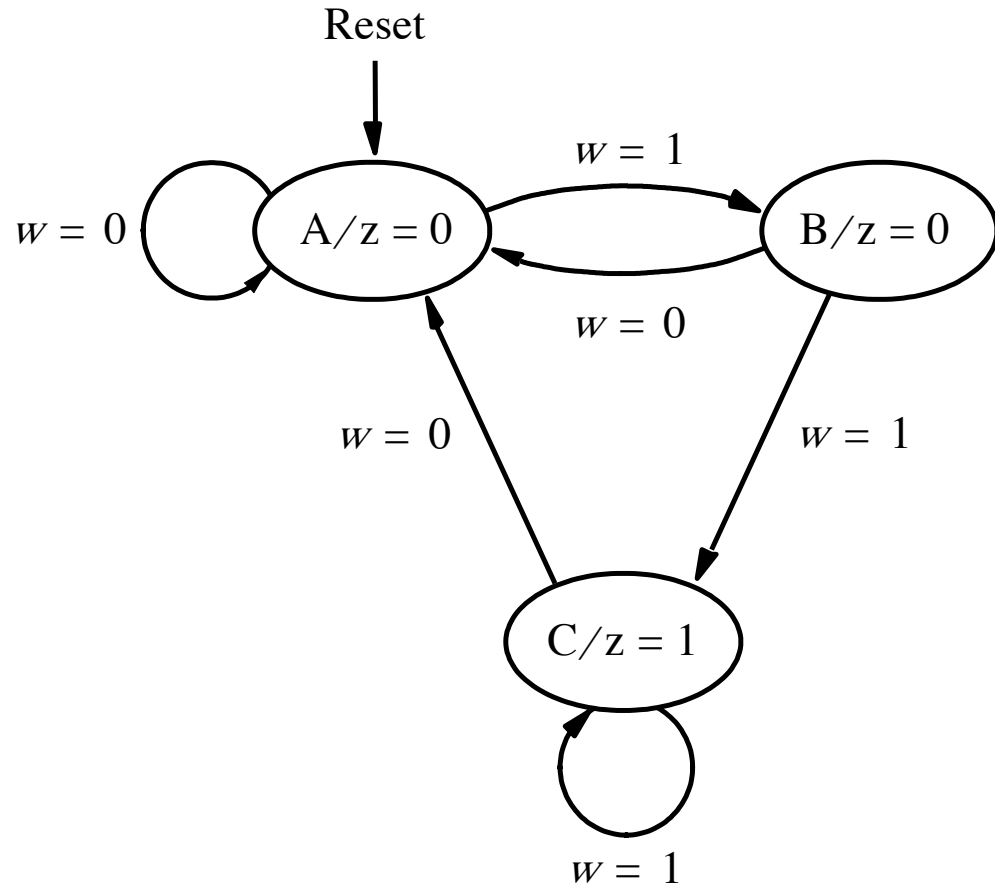


# ASM Charts

Given an ASM chart draw the corresponding FSM



[ Figure 6.82 from the textbook ]



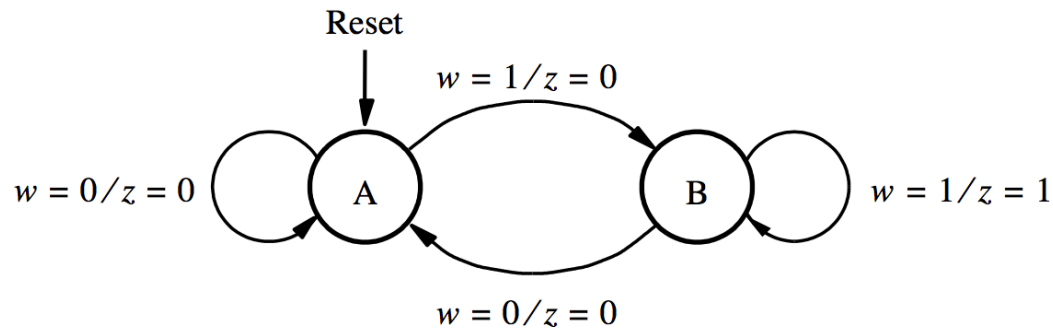
[ Figure 6.3 from the textbook ]





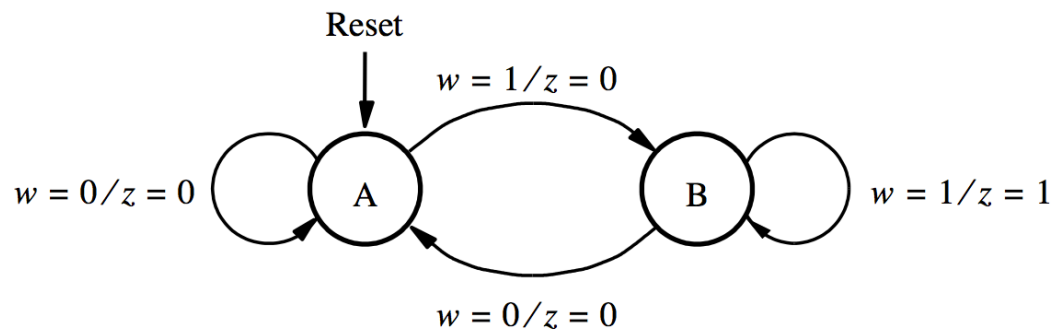
# ASM Charts

Given an FSM draw the corresponding ASM Chart

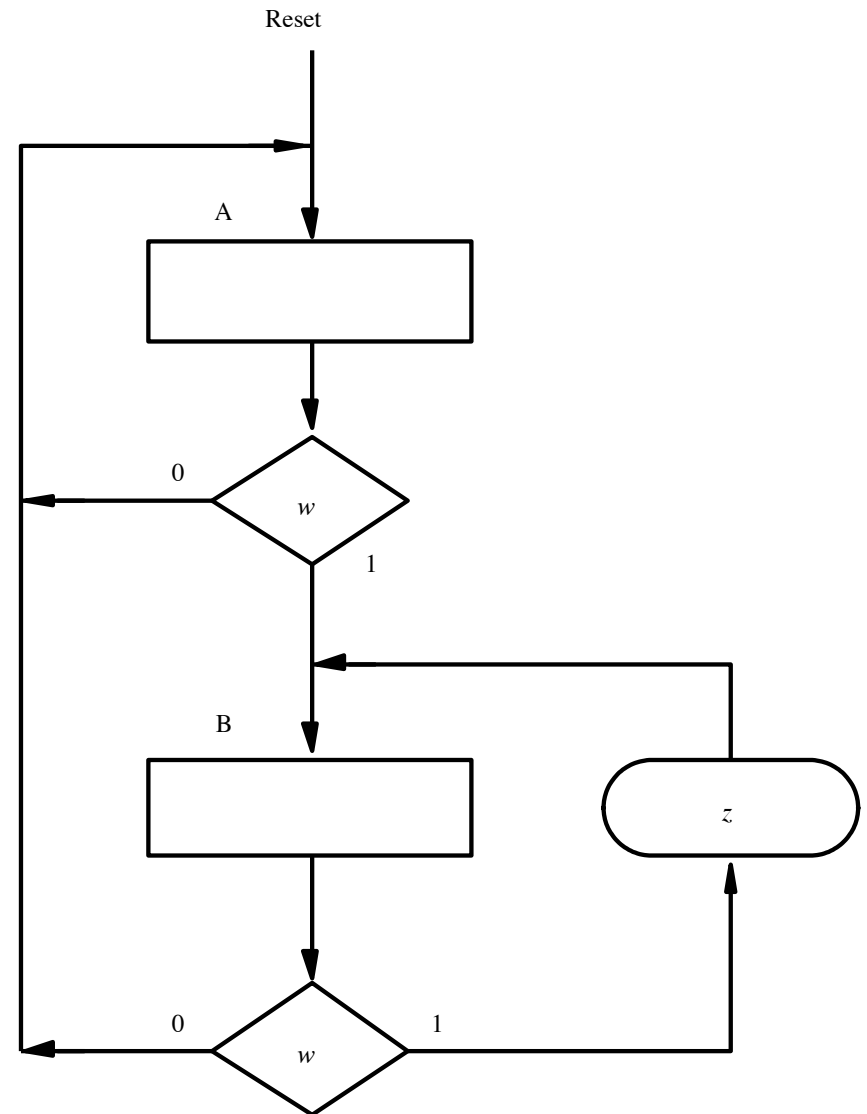


# ASM Charts

Given an FSM draw the corresponding ASM Chart



[ Figure 6.23 from the textbook ]



[ Figure 6.83 from the textbook ]



# Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

	Present state $y_3y_2y_1$	Next state		Output $z$
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

# Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

$$J_1 = wy_2 + \bar{w}y_3\bar{y}_2$$

$$K_1 = \bar{w}y_2 + wy_1\bar{y}_2$$

$$J_2 = w$$

$$K_2 = \bar{w}$$

$$J_3 = 1$$

$$K_3 = 0$$

$$z = y_1$$

	Present state $y_3y_2y_1$	Next state		Output $z$
		$w = 0$	$w = 1$	
		$Y_3Y_2Y_1$	$Y_3Y_2Y_1$	
A	000	100	110	0
B	100	101	110	0
C	101	101	110	1
D	110	100	111	0
E	111	100	111	1

# Circuit Implementation of FSMs

Implement this state-assigned Table using JK flip-flops

	Present state $y_3y_2y_1$	Flip-flop inputs								Output $z$
		$w = 0$				$w = 1$				
		$Y_3Y_2Y_1$	$J_3K_3$	$J_2K_2$	$J_1K_1$	$Y_3Y_2Y_1$	$J_3K_3$	$J_2K_2$	$J_1K_1$	
A	000	100	$1d$	$0d$	$0d$	110	$1d$	$1d$	$0d$	0
B	100	101	$d0$	$0d$	$1d$	110	$d0$	$1d$	$0d$	0
C	101	101	$d0$	$0d$	$d0$	110	$d0$	$1d$	$d1$	1
D	110	100	$d0$	$d1$	$0d$	111	$d0$	$d0$	$1d$	0
E	111	100	$d0$	$d1$	$d1$	111	$d0$	$d0$	$d0$	1

**Excitation table with JK flip-flops**





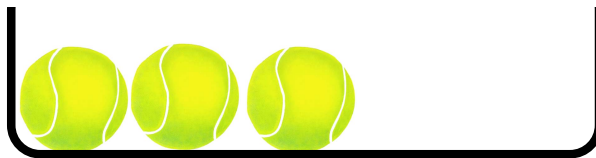
# Register Machines:

What does this program do?

How many balls are left in each register at the end of the program?



Register 1



Register 2



Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	3	4
3.	Inc	3	2	
4.	End			

# Register Machines:

Move the contents of register 2 to register 3



Register 1



Register 2



Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	3	4
3.	Inc	3	2	
4.	End			



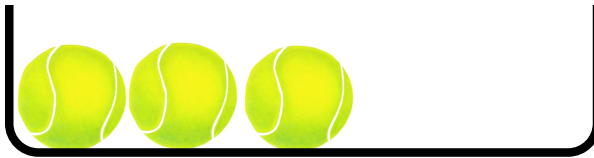
# Register Machines:

What does this program do?

How many balls are left in each register at the end of the program?



Register 1



Register 2



Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	2	3
3.	Deb	1	4	6
4.	Inc	3	5	
5.	Inc	2	3	
6.	Deb	2	7	8
7.	Inc	1	6	
8.	End			

# Register Machines:

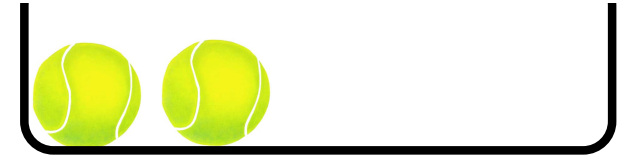
Copy the contents of register 1 to register 3  
using register 2 as a temporary storage



Register 1



Register 2



Register 3

STEP	INSTRUCTION	REGISTER	GO TO STEP	[BRANCH TO STEP]
1.	Deb	3	1	2
2.	Deb	2	2	3
3.	Deb	1	4	6
4.	Inc	3	5	
5.	Inc	2	3	
6.	Deb	2	7	8
7.	Inc	1	6	
8.	End			

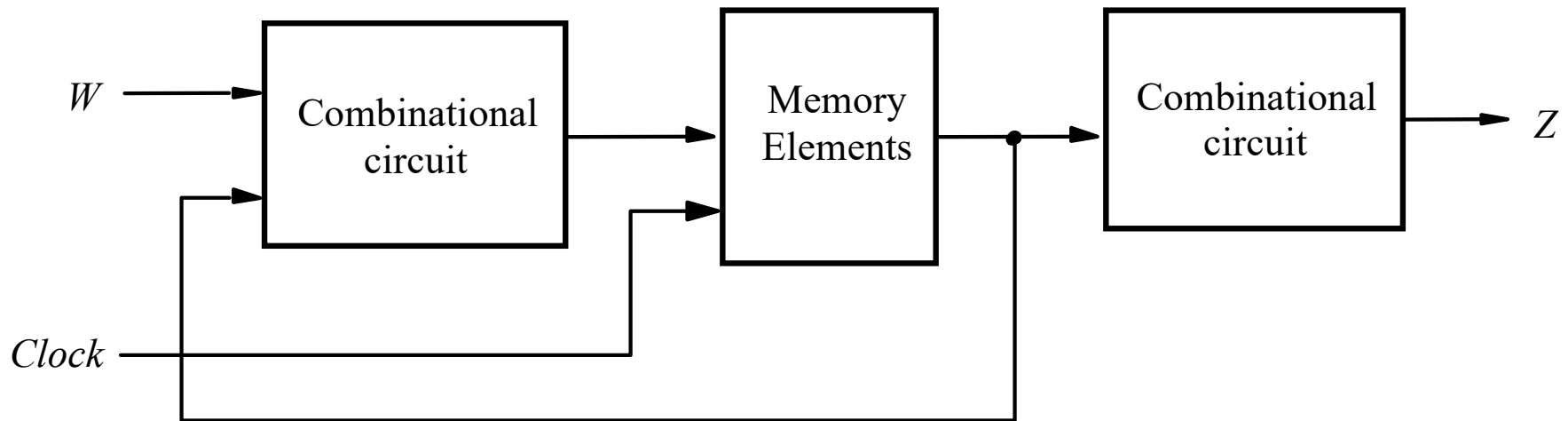


# Moore Machine Implementation

**The state diagram is just an illustration to help us describe and reason about how the FSM will behave in each of its states.**

**So, how do we turn it into a circuit?**

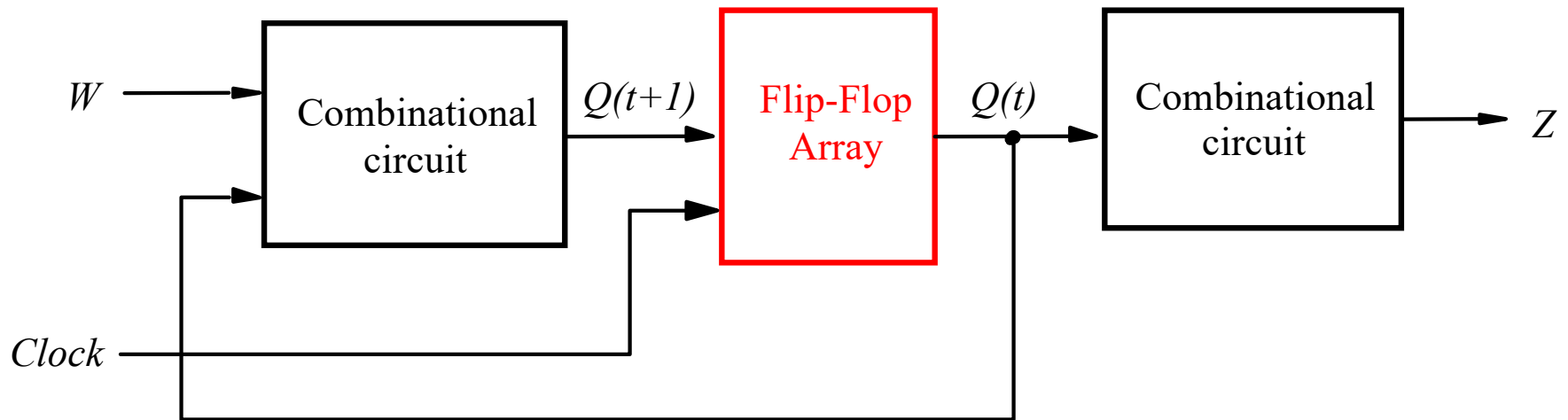
# Moore Machine Implementation



**Note: The  $W$  and  $Z$  lines need not be wires. They can be buses.**



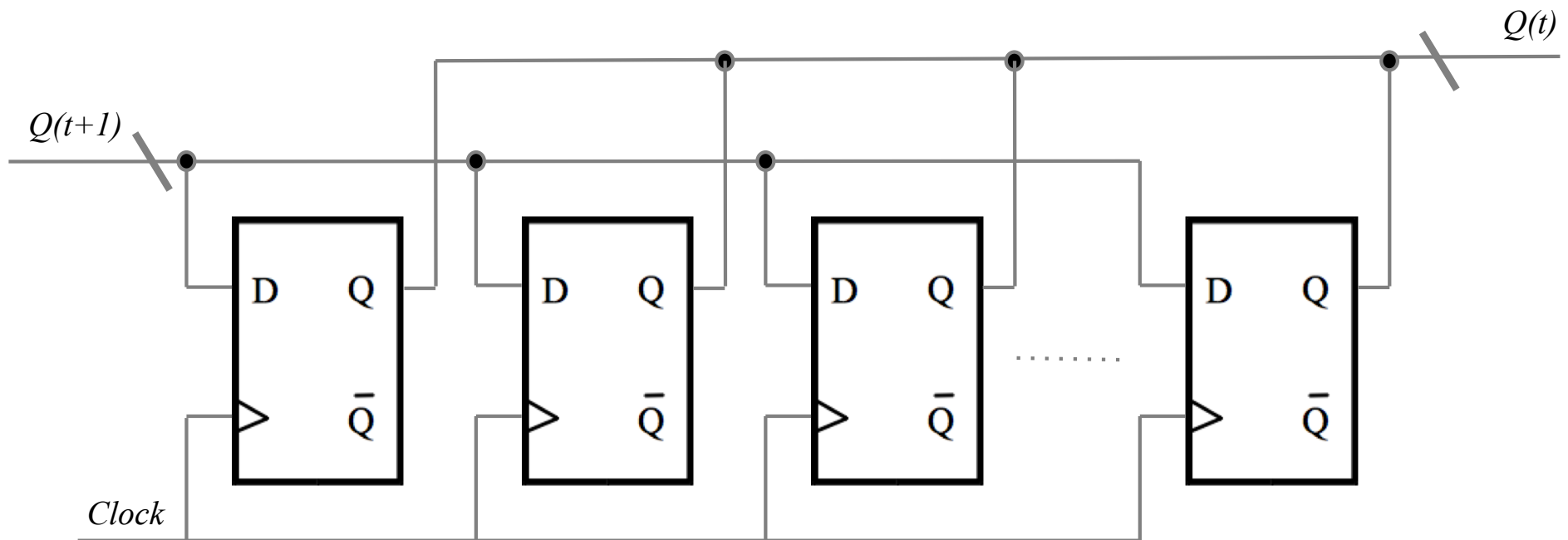
# State Storage



**Any usable “memory” of the preceding input sequence is encoded in the flip-flop array.**

# FSM States

**The Flip-Flop array stores an encoding of the current state.**



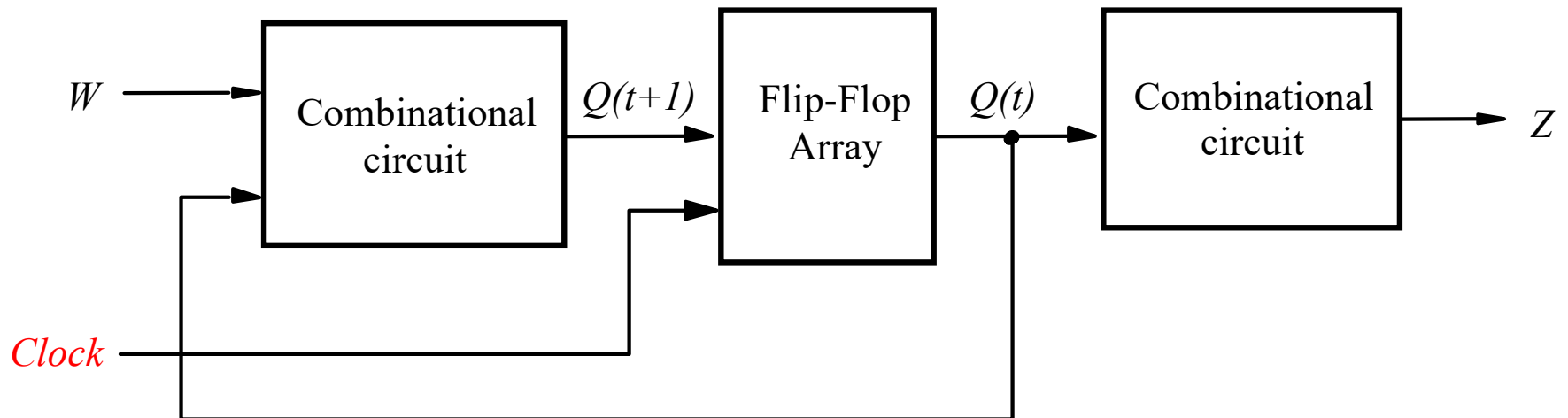
# State Encoding

**Each of the states in our design is identified by a distinct code.**

**If we use 3 flip-flops, then the FSM can have up to  $2^3 = 8$  distinct states.**

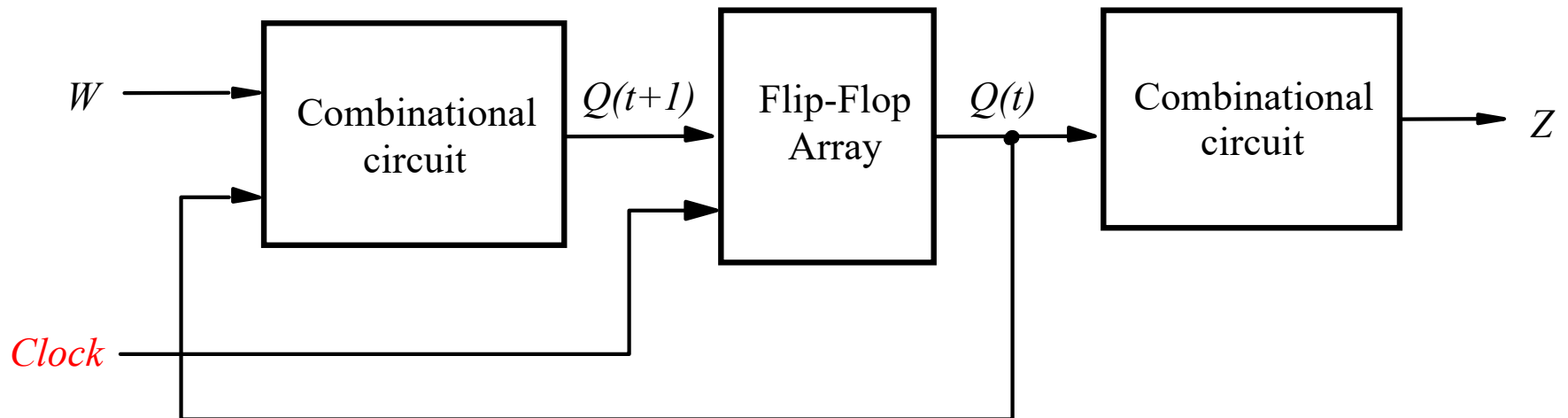
**So, when the flip-flop array contains the code *011*, we say that the machine is in state *011*.**

# Synchronous Design



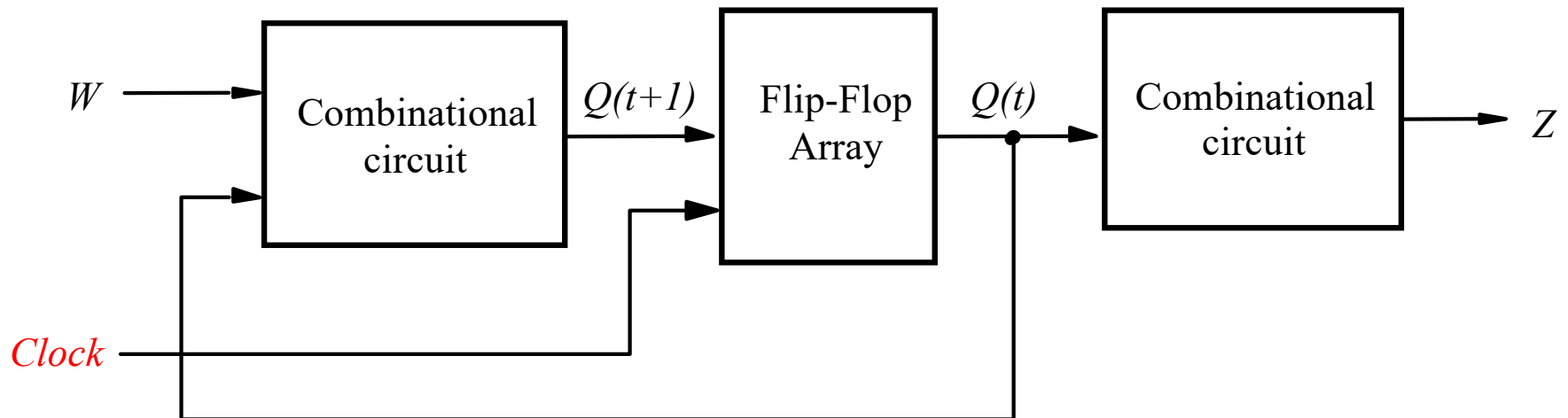
**Every *active clock edge* causes a state transition.**

# Synchronous Design



**We expect the input signals to be stable  
before the *active clock edge* occurs.**

# Synchronous Design



**There is a whole other class of sequential circuits that are asynchronous, but we will not study them in this course.**

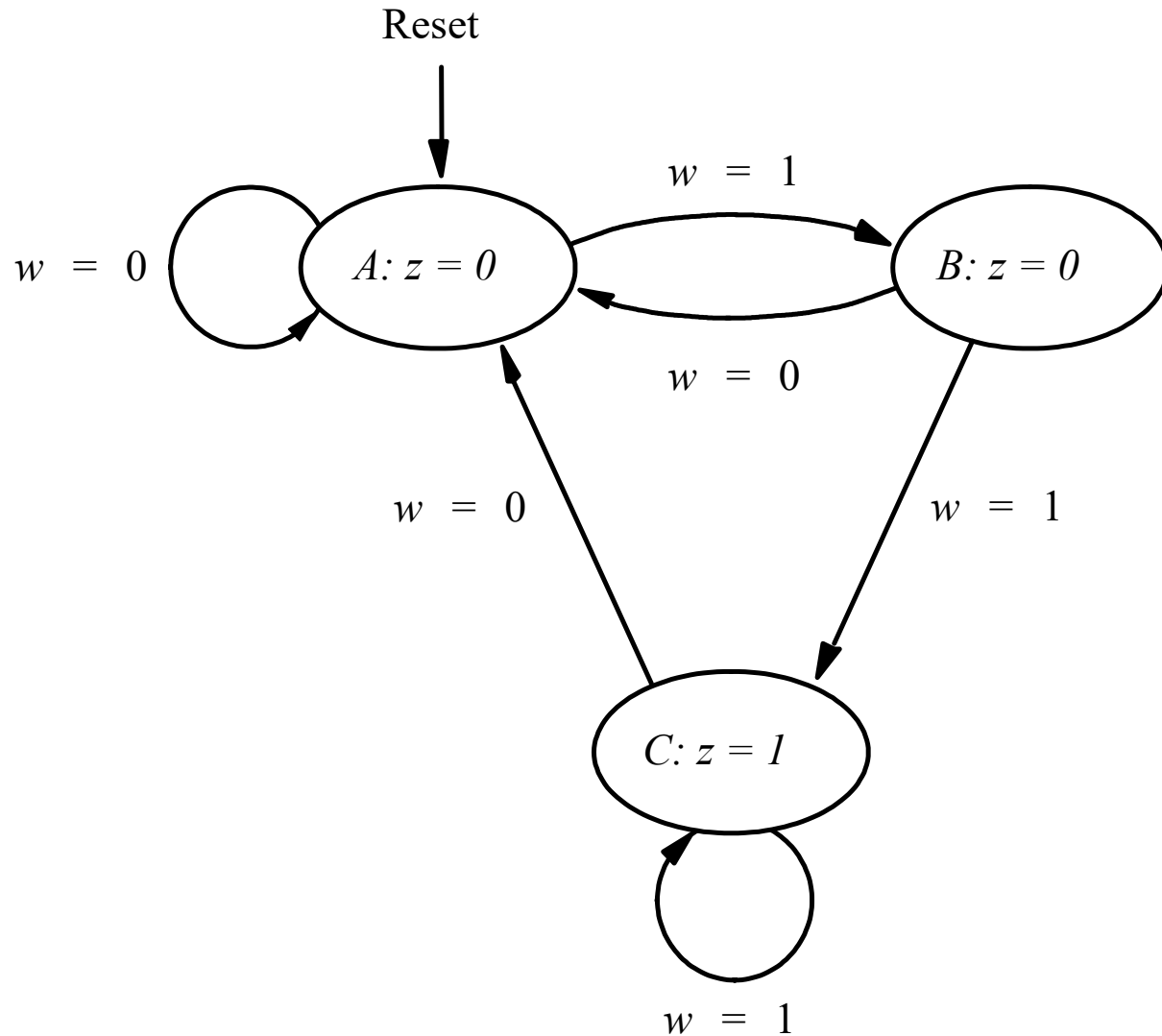
# Sequential Circuits: Key Ideas

**The current output depends on something about the preceding sequence of inputs (and maybe the current output).**

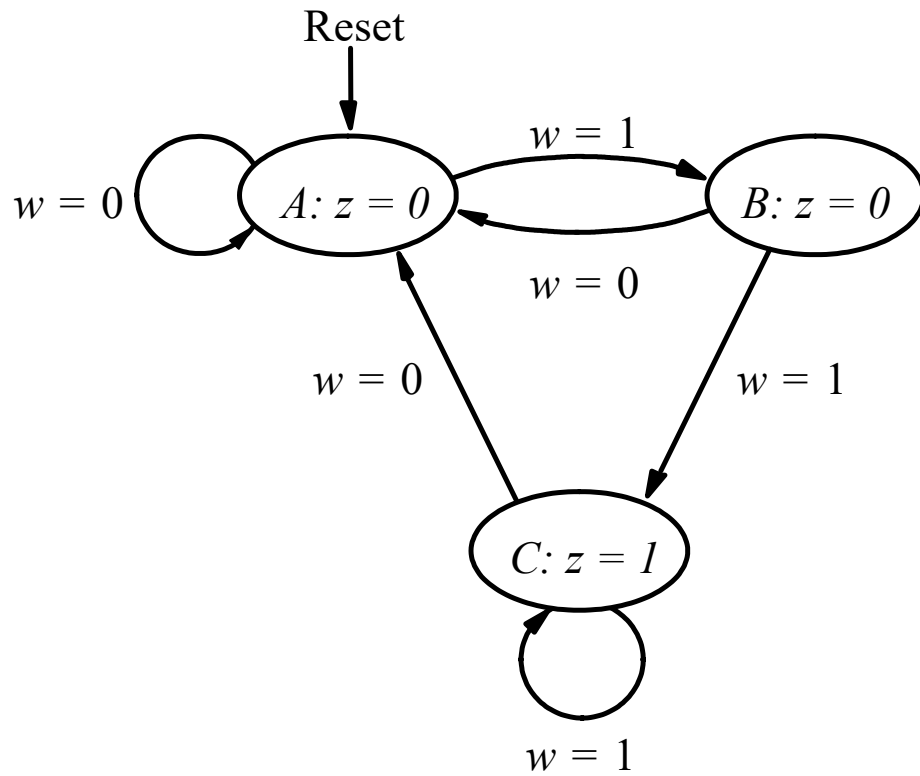
**Using *memory elements* (i.e., flip-flops), we design the circuit to remember some *relevant* information about the prior inputs.**

**Example**

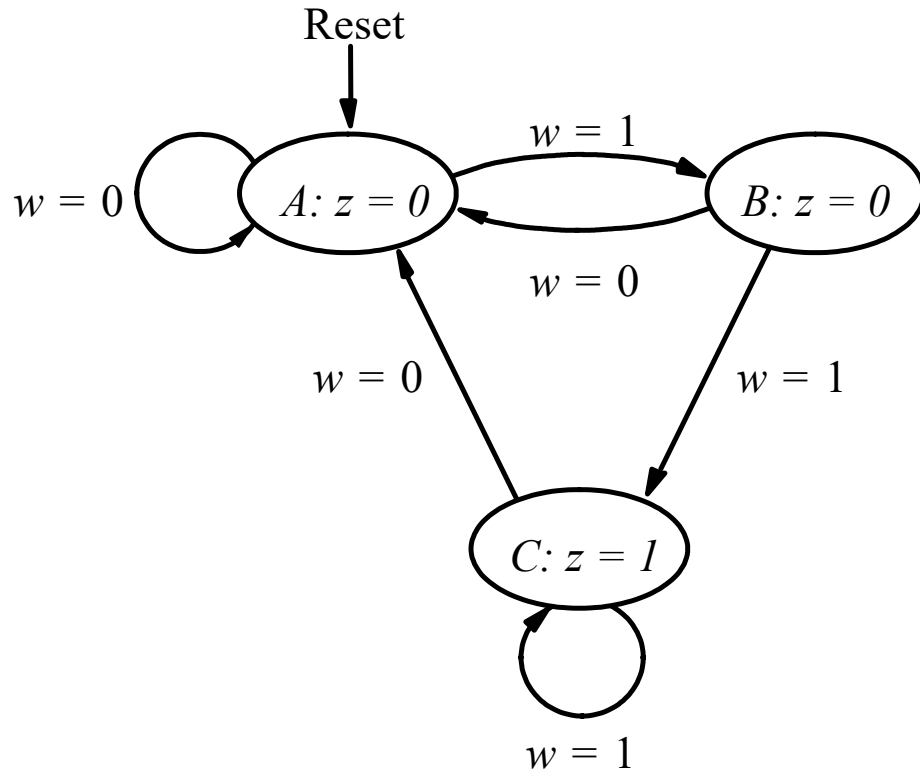




**We need to find both the *next state logic* and the *output logic* implied by this machine.**



Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A			
B			
C			



Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

[ Figure 6.4 from the textbook ]

# How to represent the States?

One way is to encode each state with a 2-bit binary number

A ~ 00

B ~ 01

C ~ 10

# How to represent the states?

One way is to encode each state with a 2-bit binary number

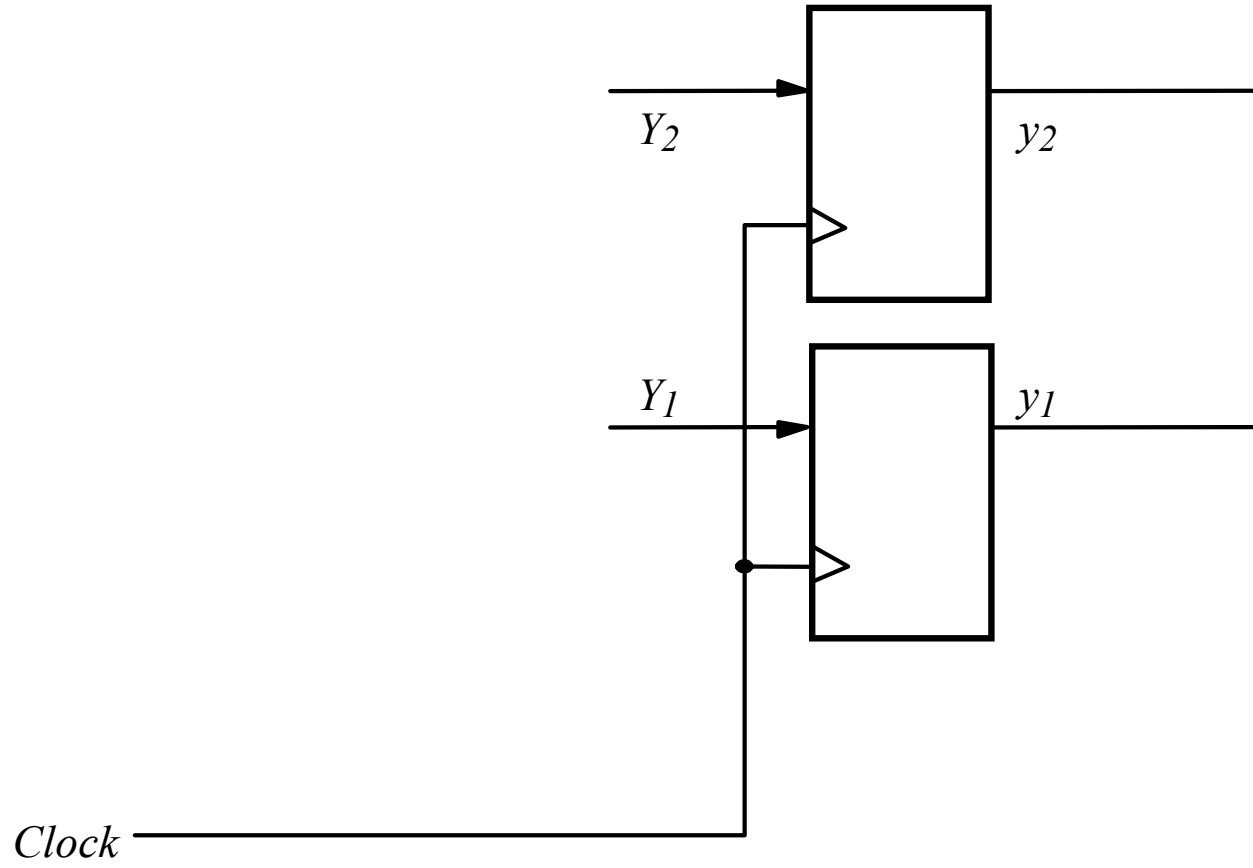
A ~ 00

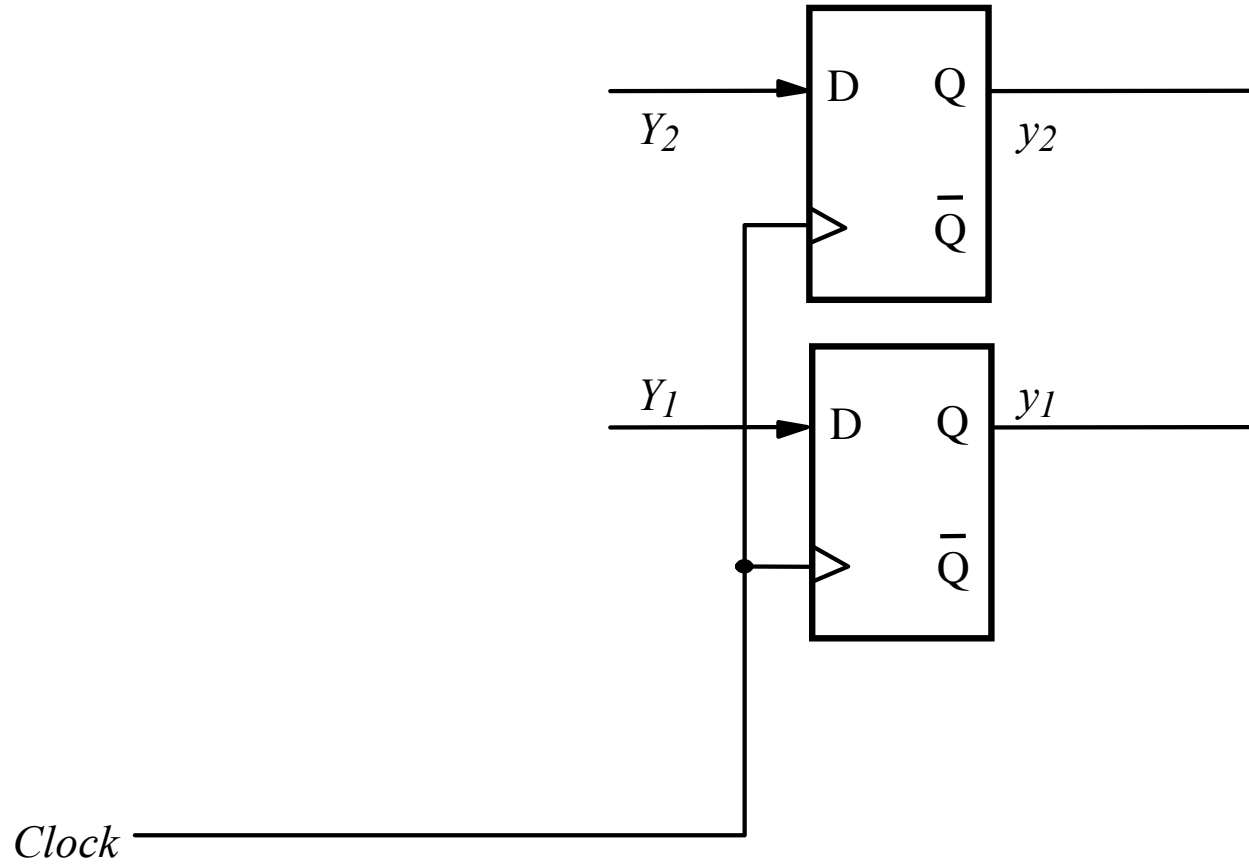
B ~ 01

C ~ 10

How many flip-flops do we need?

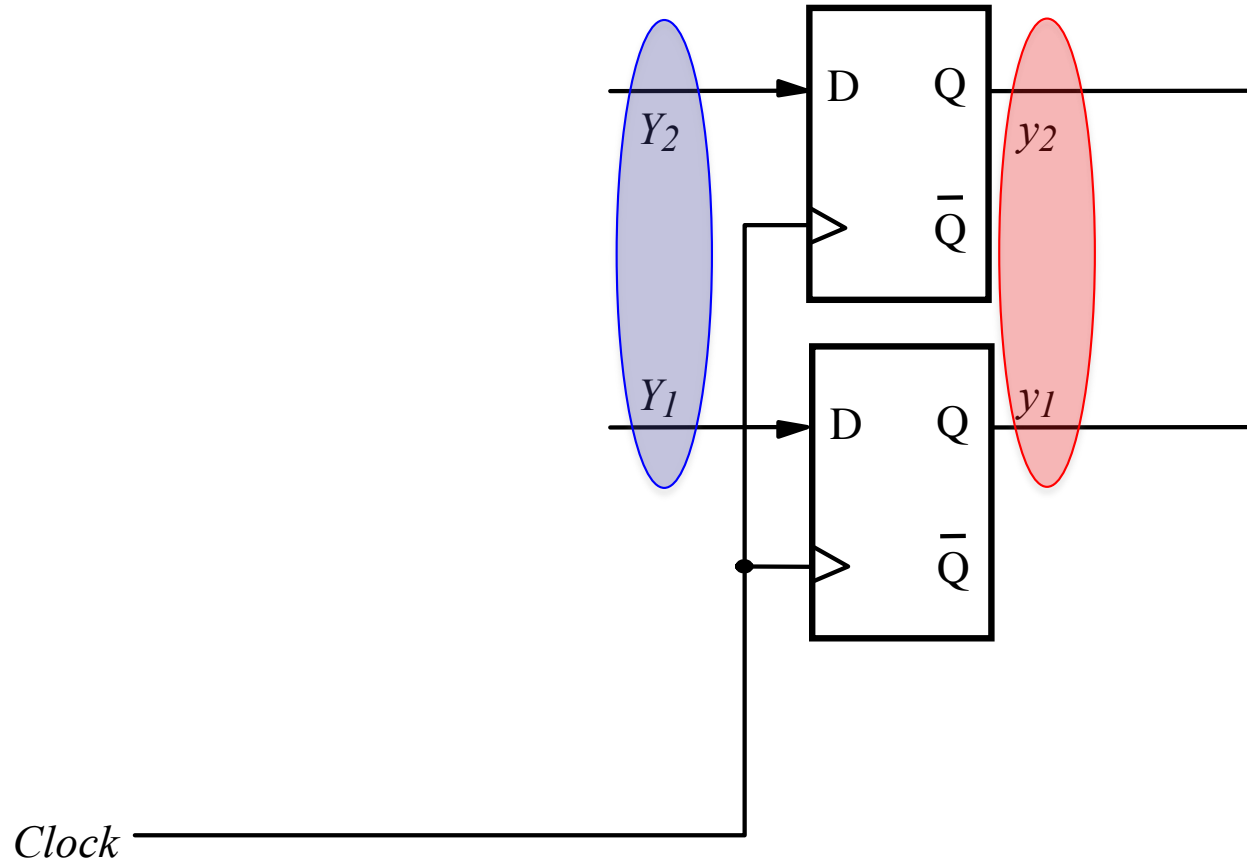
**Let's use **two** flip-flops  
to hold the machine's state**





Let's pick D Flip-Flops.

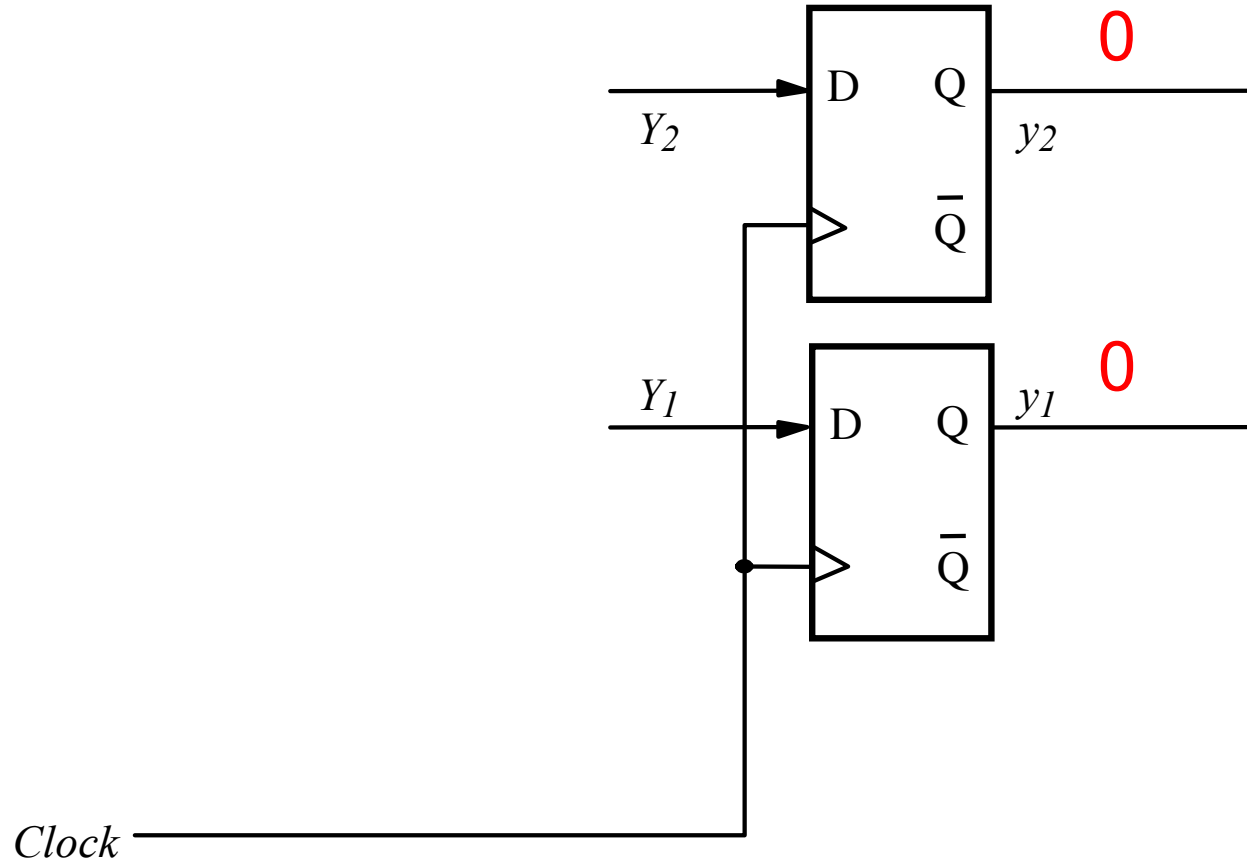




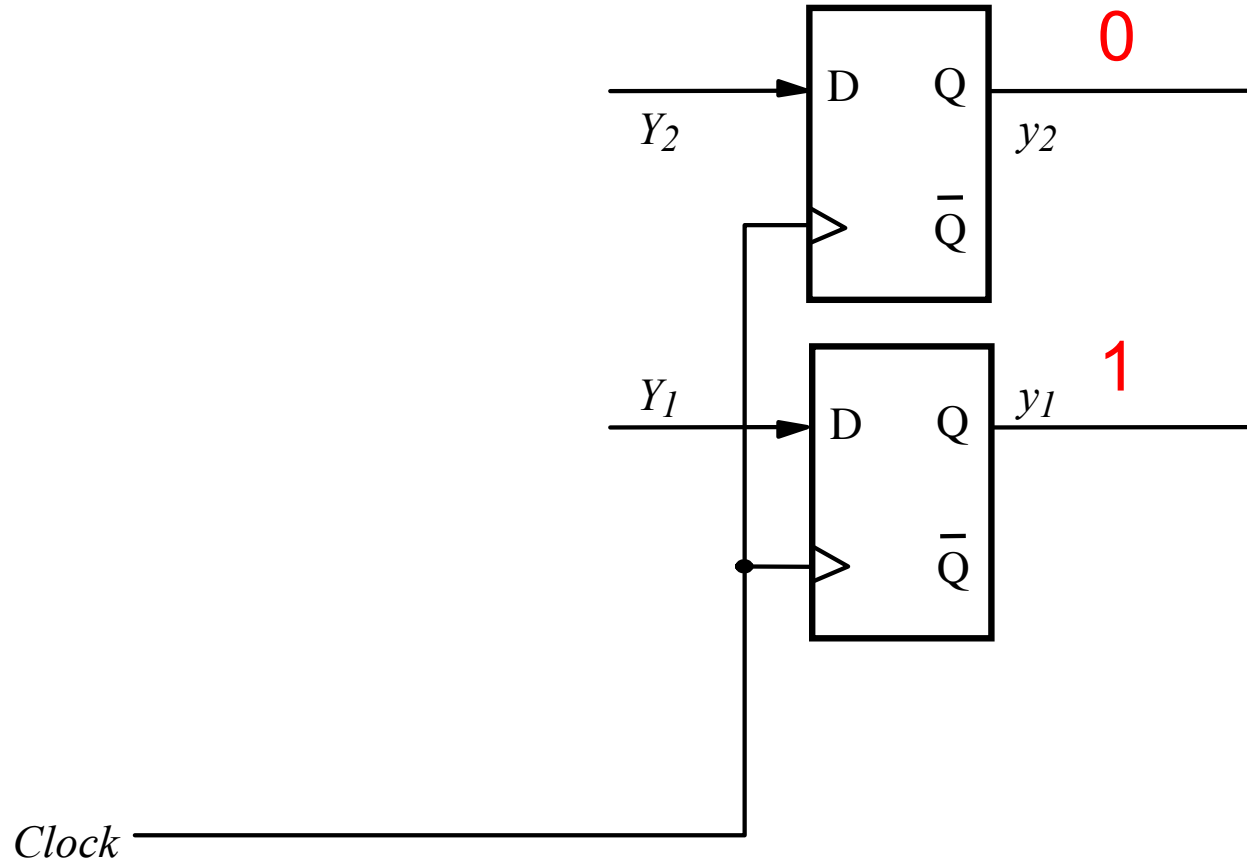
• We will call  $y_1$  and  $y_2$  the *present state variables*.

• We will call  $Y_1$  and  $Y_2$  the *next state variables*.

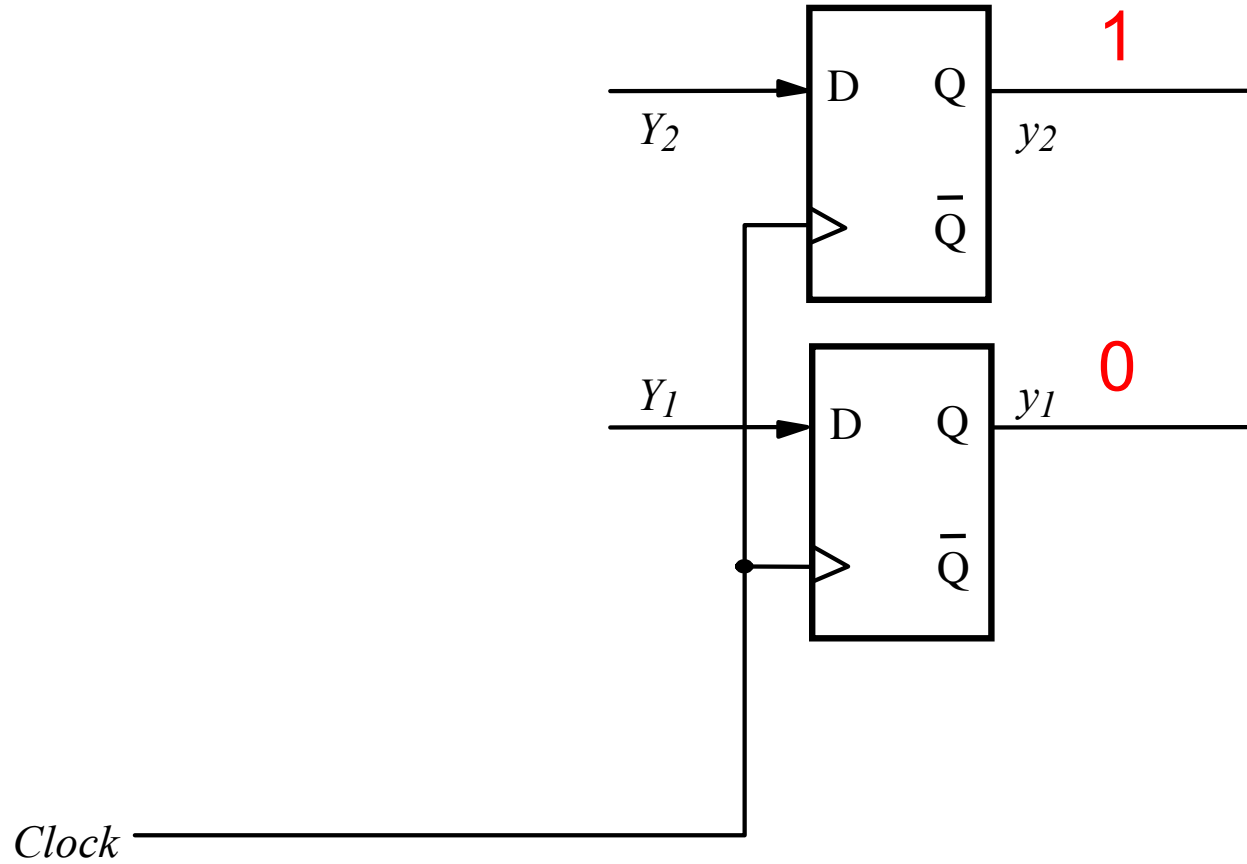
[ Figure 6.5 from the textbook ]



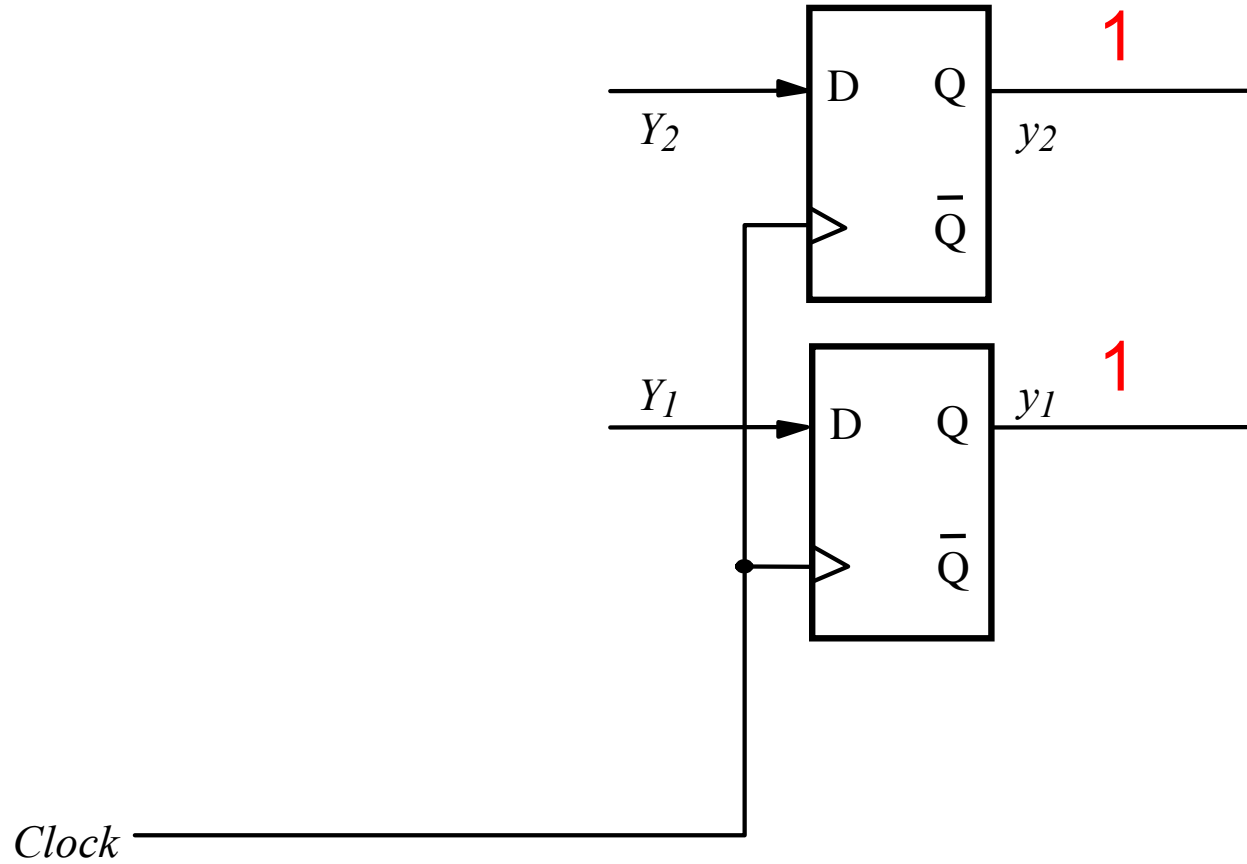
- **Two zeros on the output JOINTLY represent state A.**



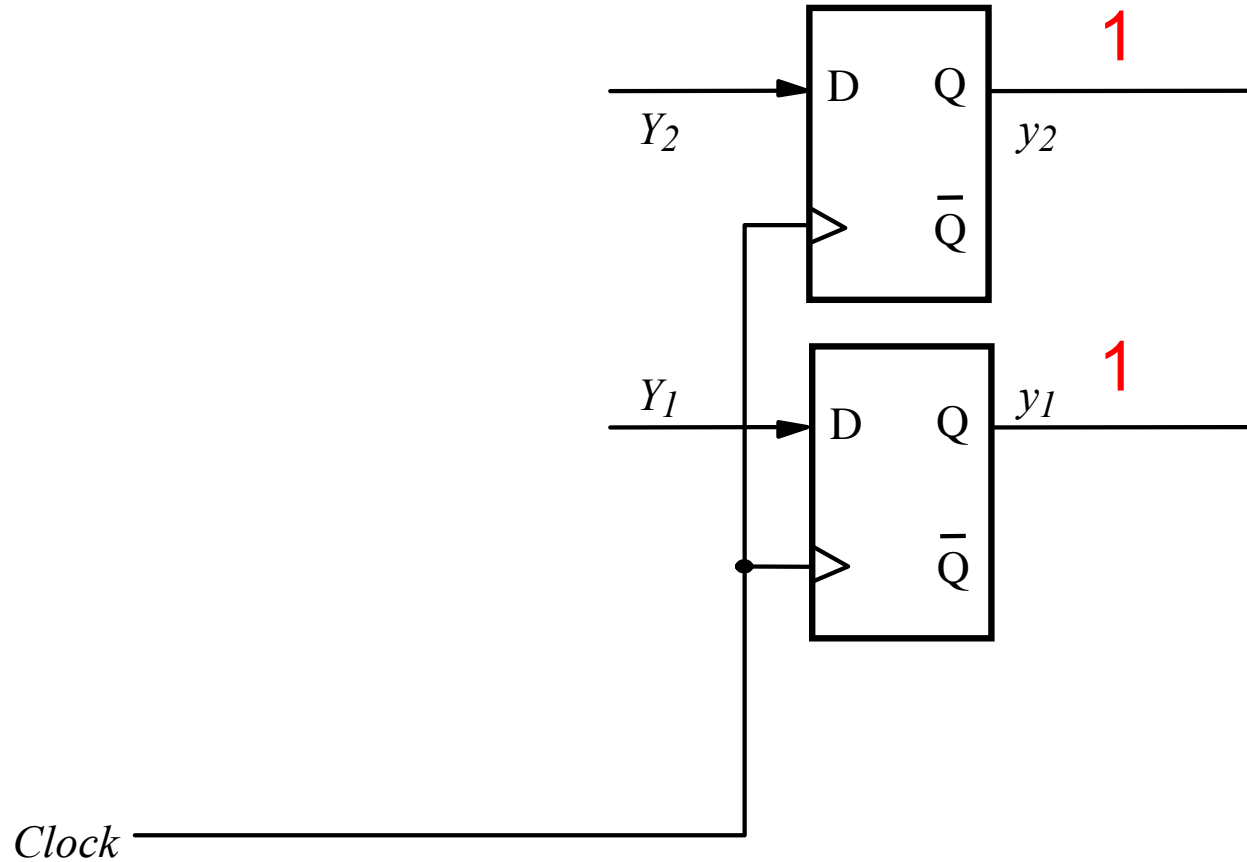
**•This flip-flop output pattern represents state B.**



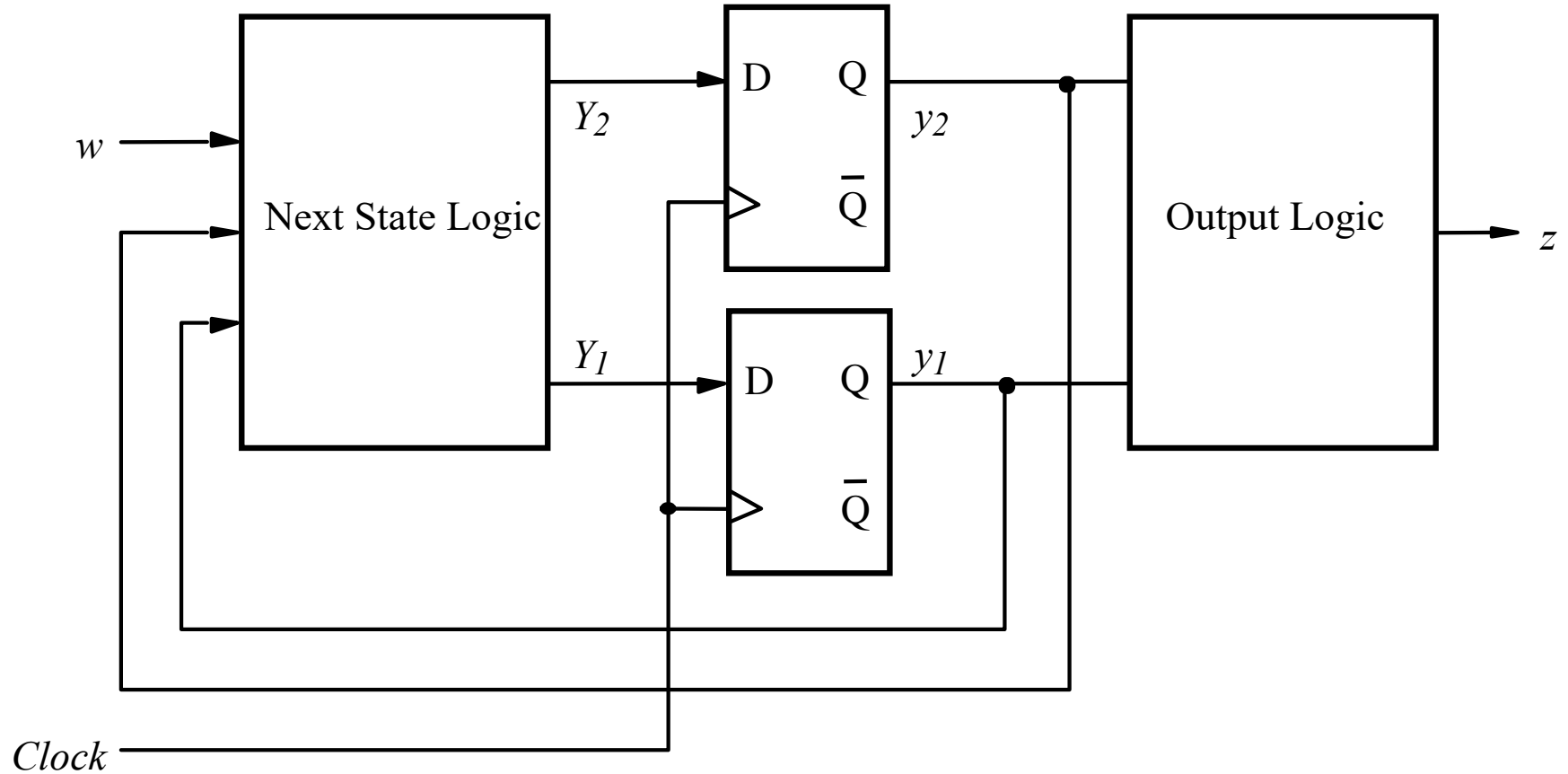
- This flip-flop output pattern represents state C.



**•What does this flip-flop output pattern represent?**

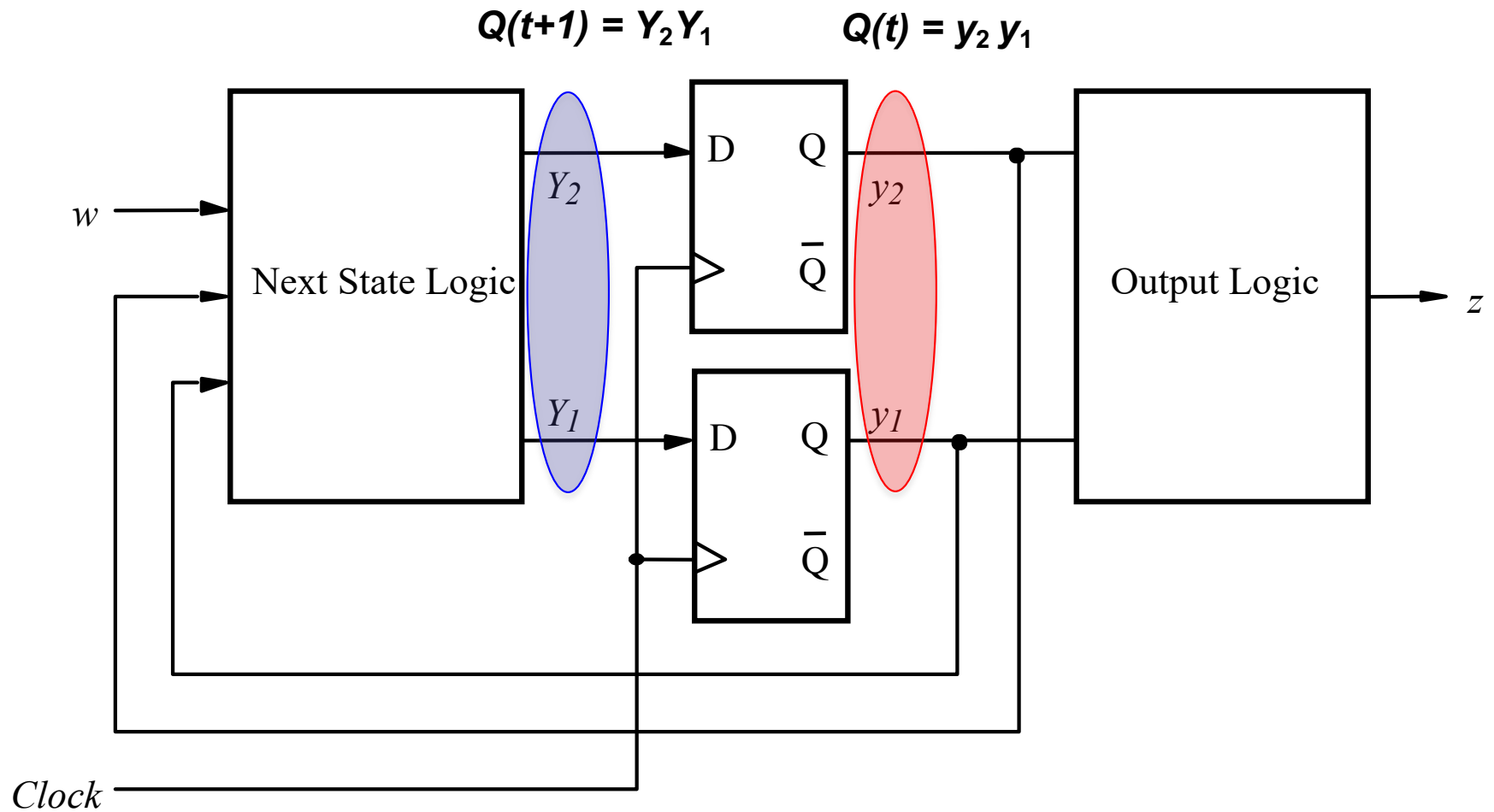


- This would be state D, but we don't have one
- in this example. So this is an impossible state.



• We will call  $y_1$  and  $y_2$  the *present state variables*.

• We will call  $Y_1$  and  $Y_2$  the *next state variables*. [ Figure 6.5 from the textbook ]

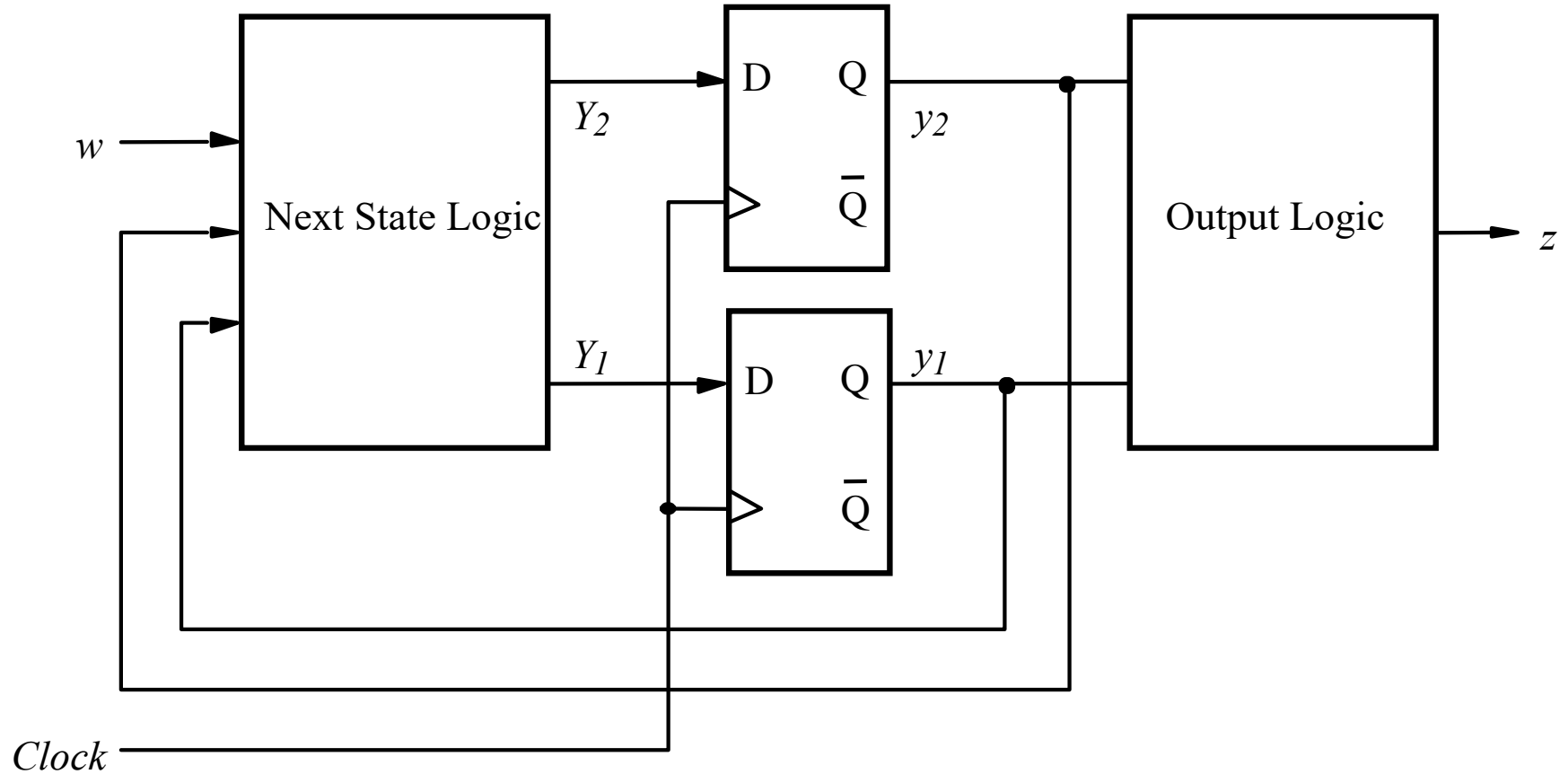


• We will call  $y_1$  and  $y_2$  the *present state variables*.

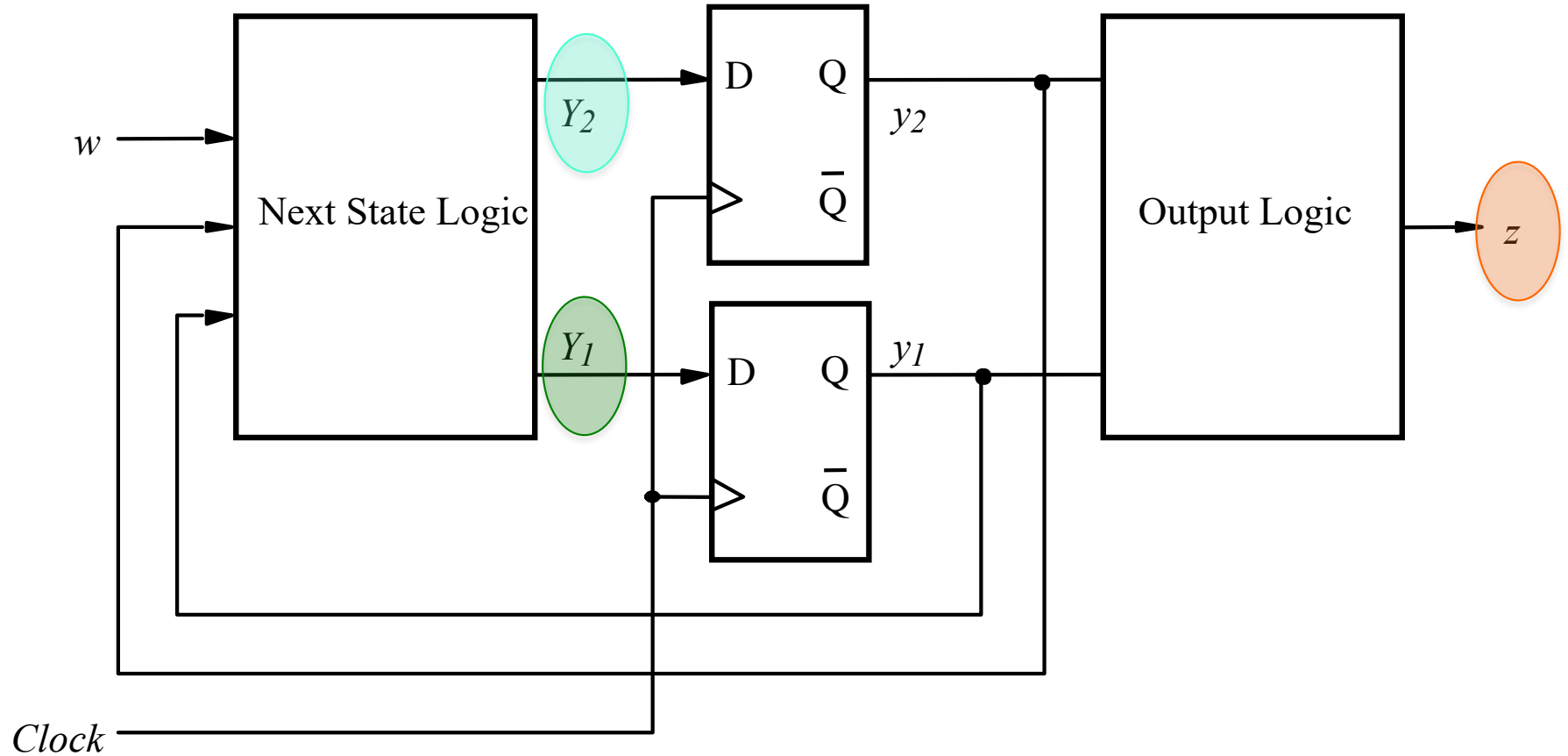
• We will call  $Y_1$  and  $Y_2$  the *next state variables*.

[ Figure 6.5 from the textbook ]





- We need to find logic expressions for
- $Y_1(w, y_1, y_2)$ ,  $Y_2(w, y_1, y_2)$ , and  $z(y_1, y_2)$ .



- We need to find logic expressions for  $Y_1(w, y_1, y_2)$ ,  $Y_2(w, y_1, y_2)$ , and  $z(y_1, y_2)$ .

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

• **Suppose we encoded our states in the same order in which they were labeled:**

• **A ~ 00**

• **B ~ 01**

• **C ~ 10**

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	00		
B	01		
C	10		
	11		

The finite state machine will never reach a state encoded as 11.

[ Figure 6.6 from the textbook ]

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

	Present state	Next state		Output $z$
		$w = 0$	$w = 1$	
	$y_2 y_1$	$Y_2 Y_1$	$Y_2 Y_1$	
A	00	00	01	0
B	01	00	10	0
C	10	00	10	1
	11	<i>dd</i>	<i>dd</i>	<i>d</i>

We arbitrarily chose these as our state encodings. We could have used others.

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	
0	1	
1	0	
1	1	

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	<i>d</i>

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	<i>d</i>

[ Figure 6.6 from the textbook ]



$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	dd	dd	d

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	d	
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	d	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	d	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	d	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	d	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	d	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

Note that the textbook draws these K-Maps differently from all previous K-maps (the most significant bit indexes the rows).

$Y_1$

$y_2 y_1$

$w$		00	01	11	10
0		0	0	d	0
1		1	0	d	0

$Y_2$

$y_2 y_1$

$w$		00	01	11	10
0		0	0	d	0
1		0	1	d	1

$z$

$y_1$

$y_2$

		0	1
0		0	0
1		1	d

$$Q(t) = y_2 y_1 \text{ and } Q(t+1) = Y_2 Y_1$$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

# •Don't care conditions simplify the combinatorial logic

$Y_1$

$w$	$y_2 y_1$	00	01	11	10
0		0	0	d	0
1		1	0	d	0

$Y_2$

$w$	$y_2 y_1$	00	01	11	10
0		0	0	d	0
1		0	1	d	1

$z$

$y_2$	$y_1$	0	1
0		0	0
1		1	d

Ignoring don't cares

$$Y_1 = w\bar{y}_1\bar{y}_2$$

$$Y_2 = wy_1\bar{y}_2 + \bar{w}y_1y_2$$

$$z = \bar{y}_1y_2$$

Using don't cares

$$Y_1 = w\bar{y}_1\bar{y}_2$$

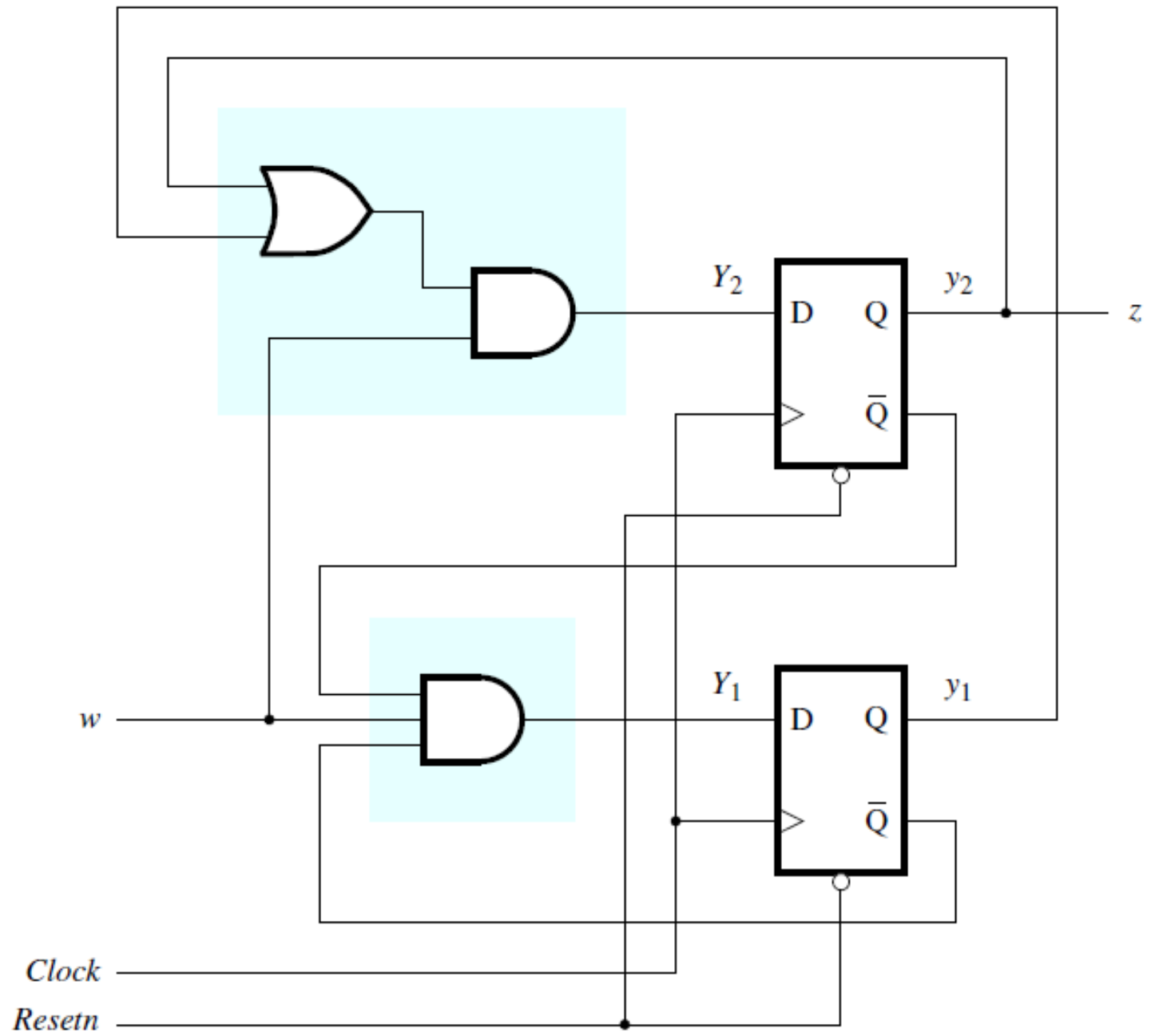
$$Y_2 = wy_1 + wy_2$$

$$= w(y_1 + y_2)$$

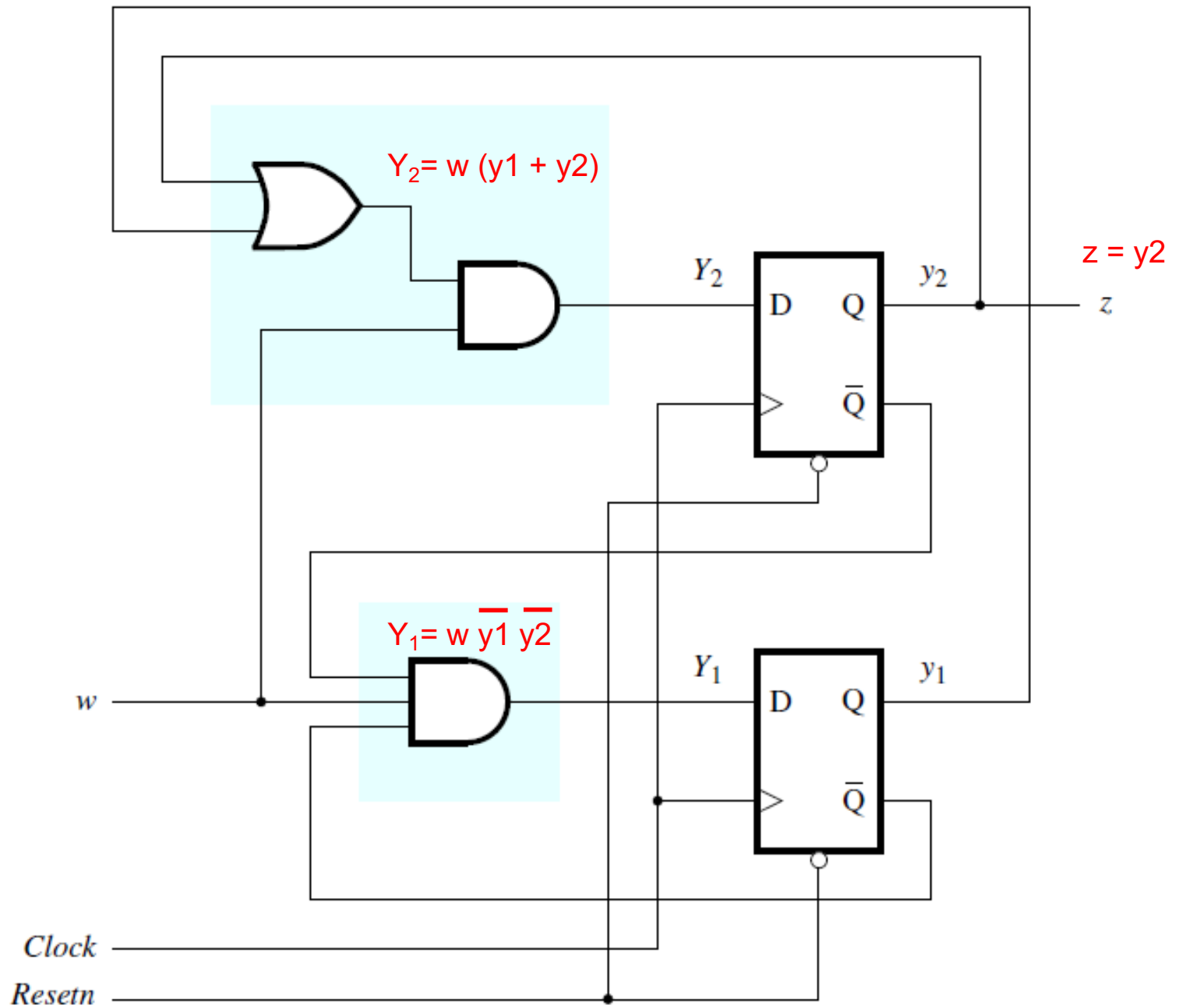
$$z = y_2$$

[ Figure 6.7 from the textbook ]



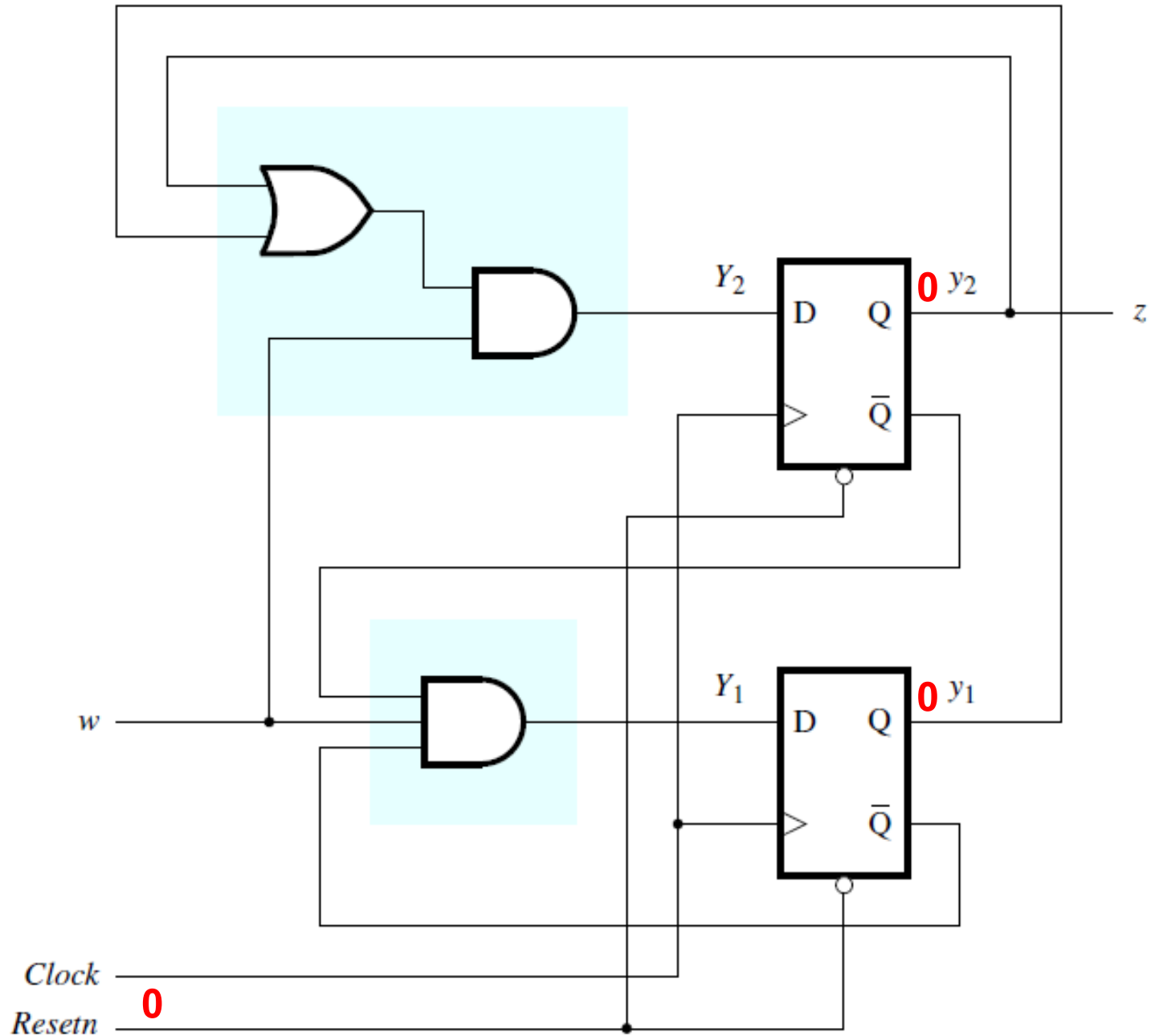


[ Figure 6.8 from the textbook ]



[ Figure 6.8 from the textbook ]

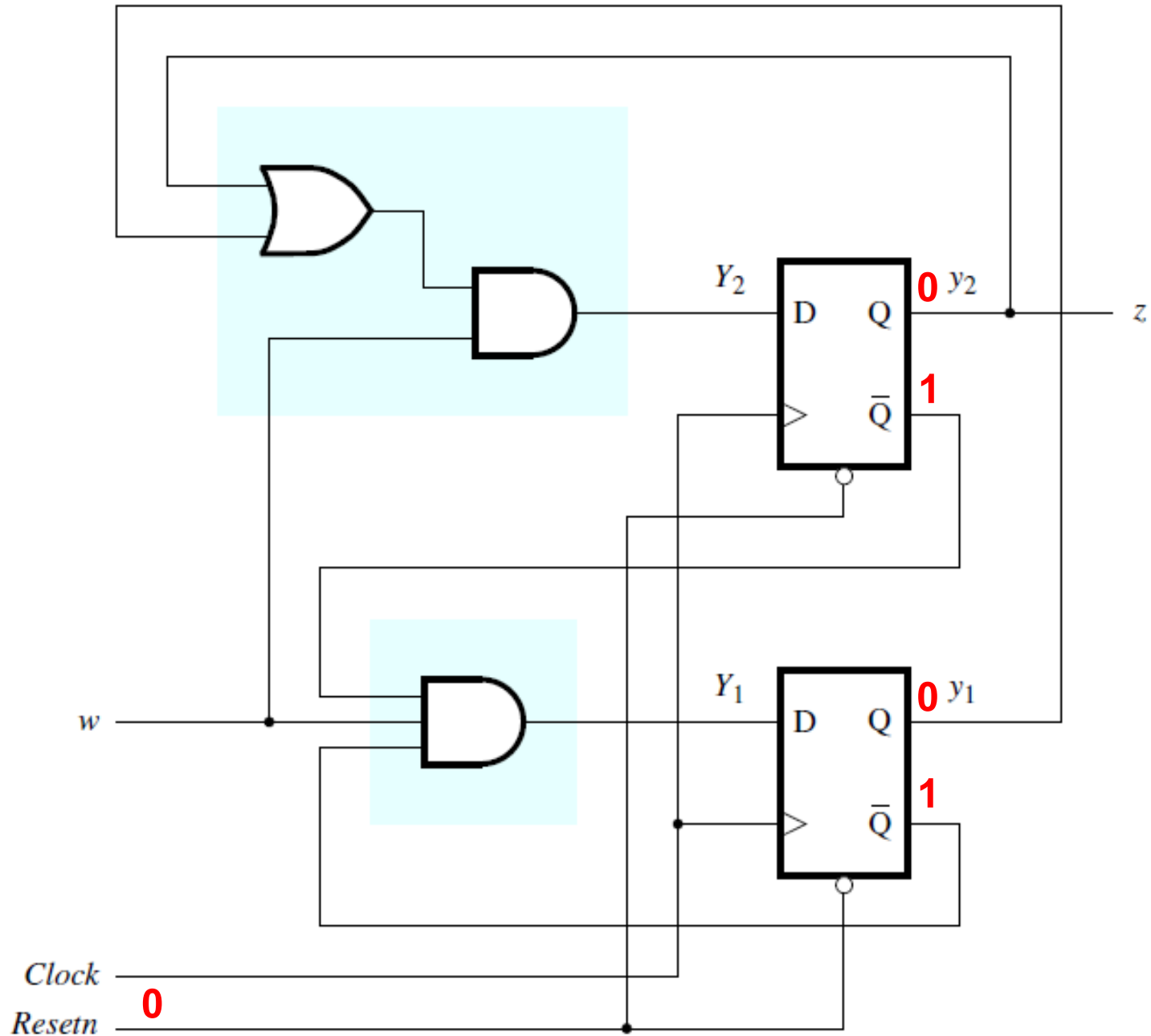
State A=00



Finally, we add a reset signal. When it is equal to zero it puts the machine back to its start state, which is state 00 in this case.

[ Figure 6.8 from the textbook ]

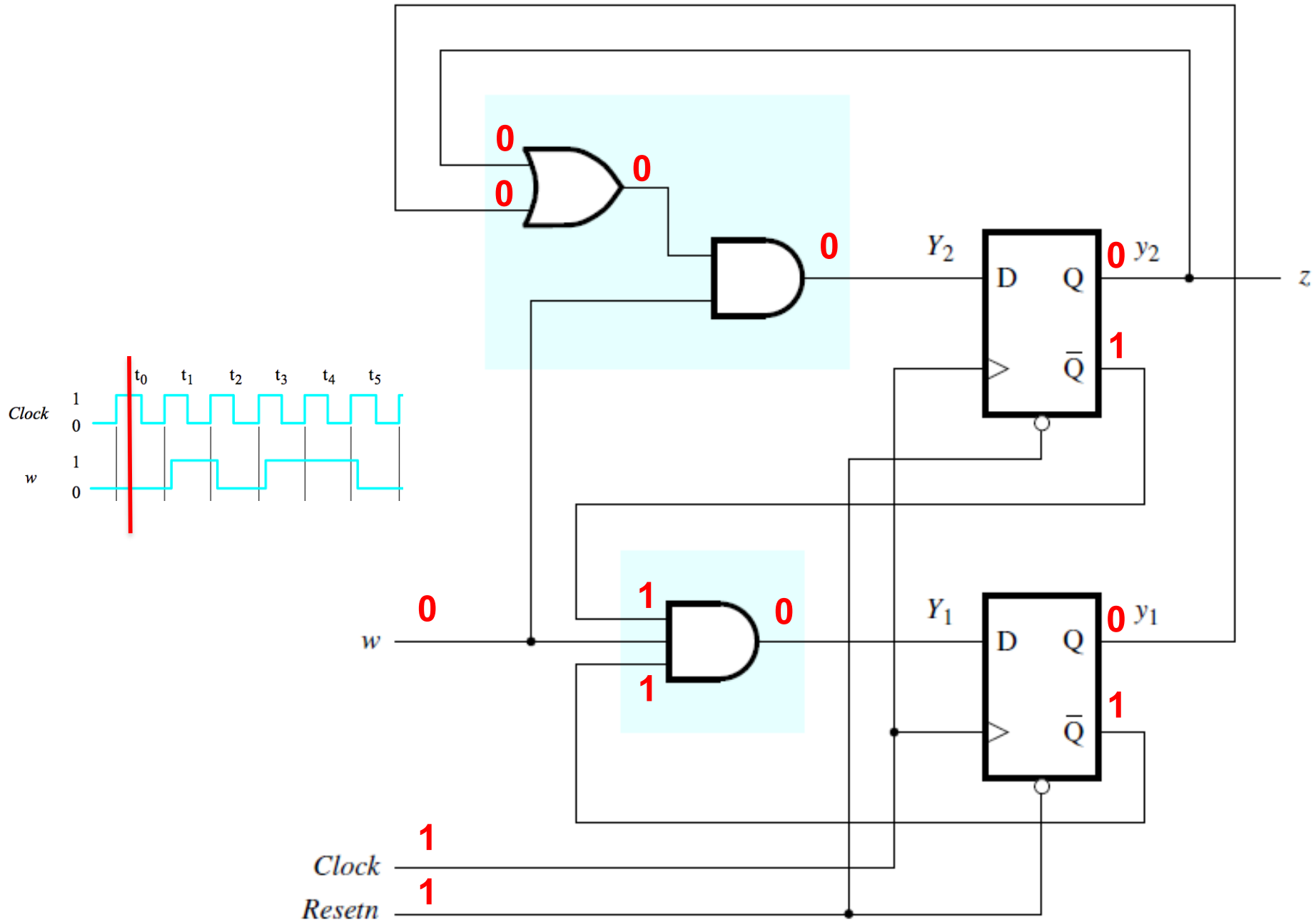
State A=00



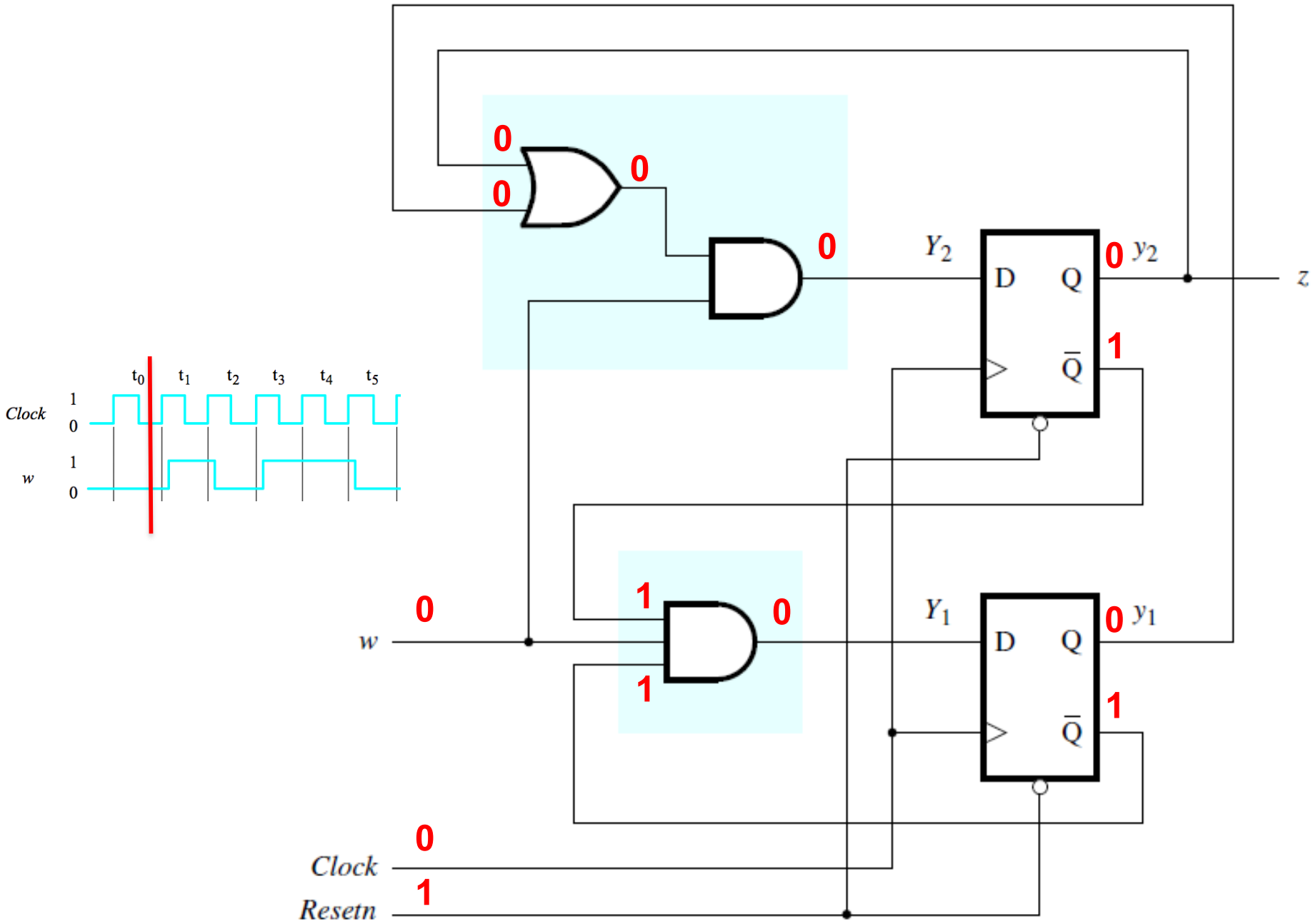
Finally, we add a reset signal. When it is equal to zero it puts the machine back to its start state, which is state 00 in this case.

[ Figure 6.8 from the textbook ]

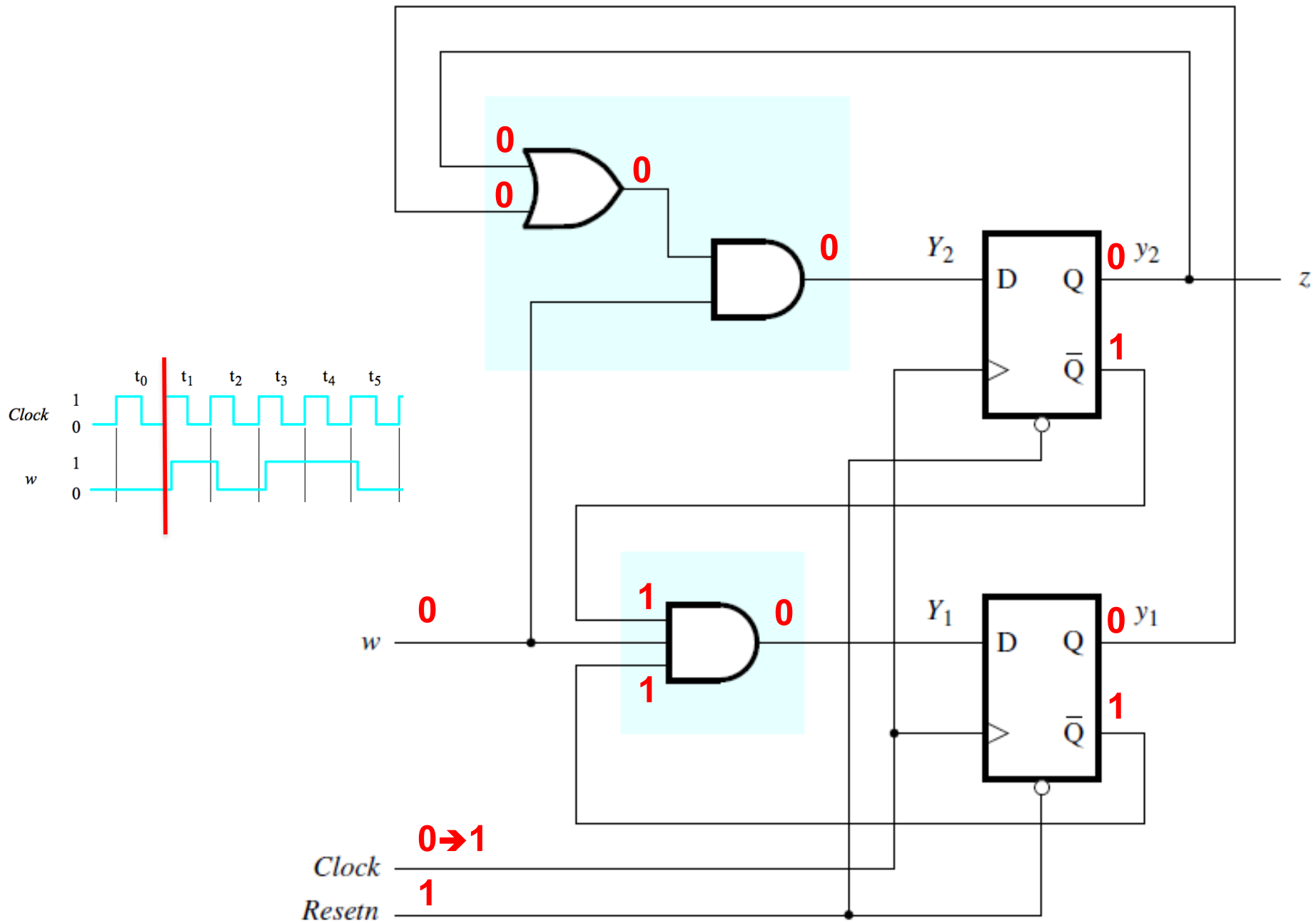
State A=00



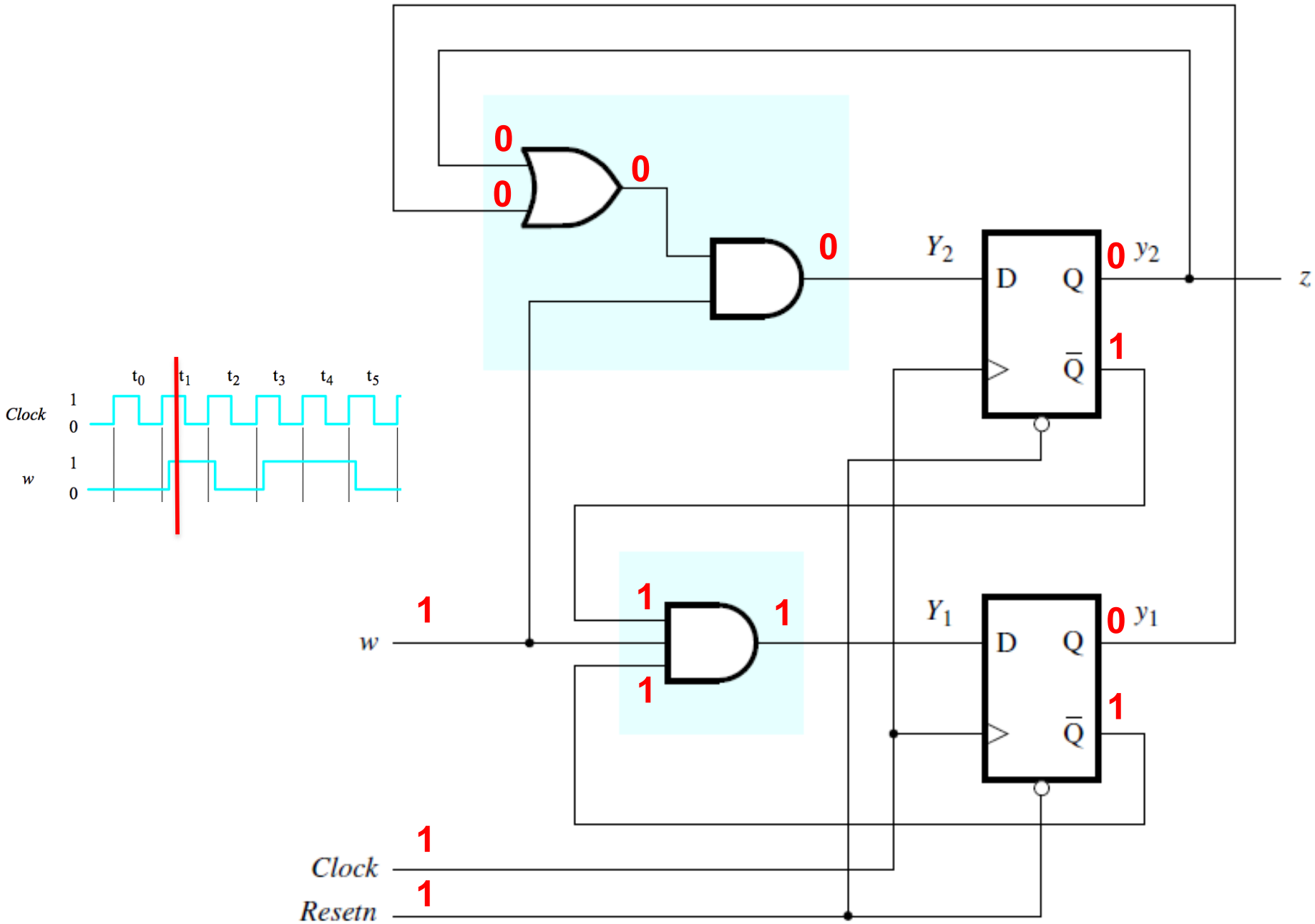
State A=00



State A=00

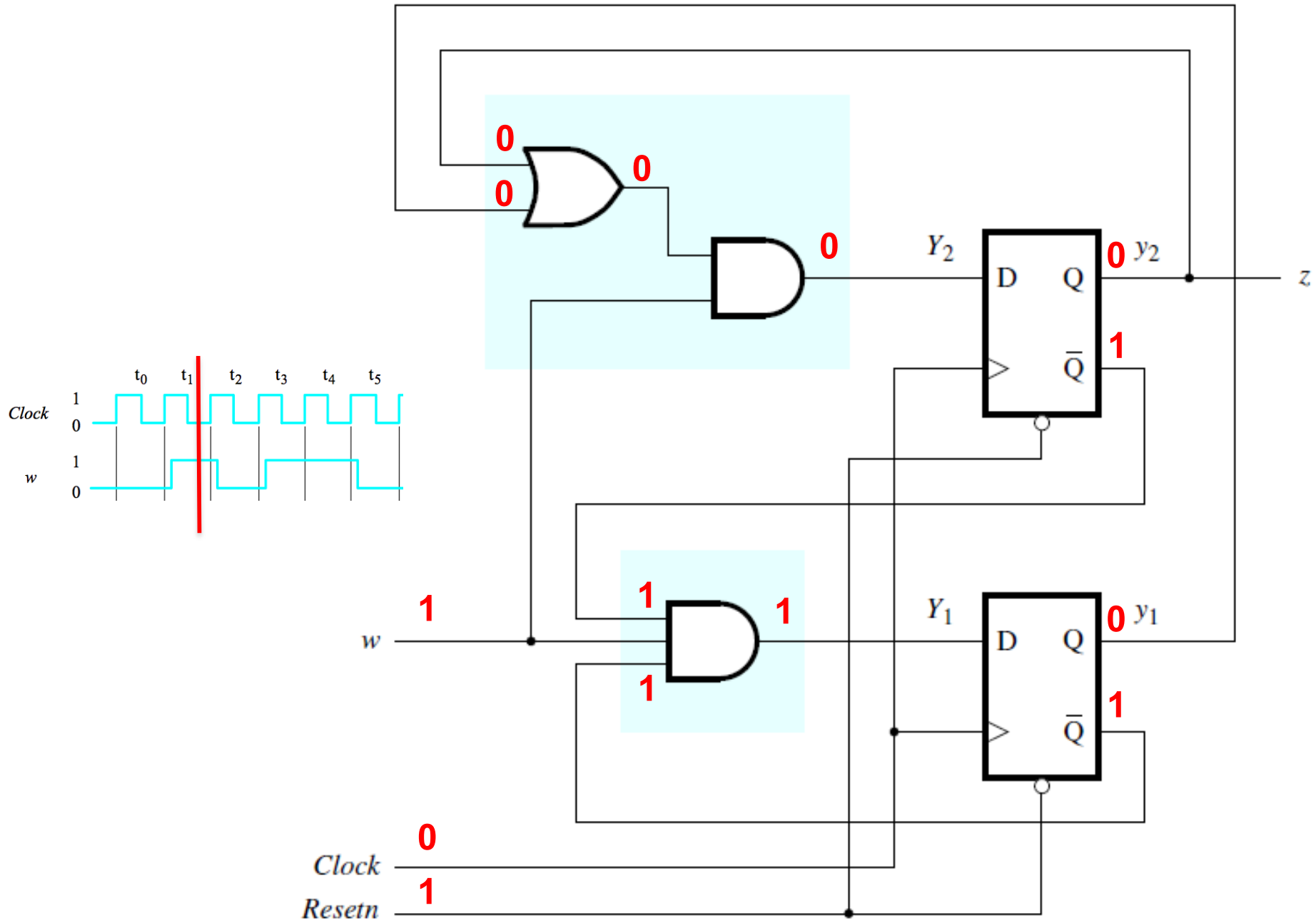


State A=00

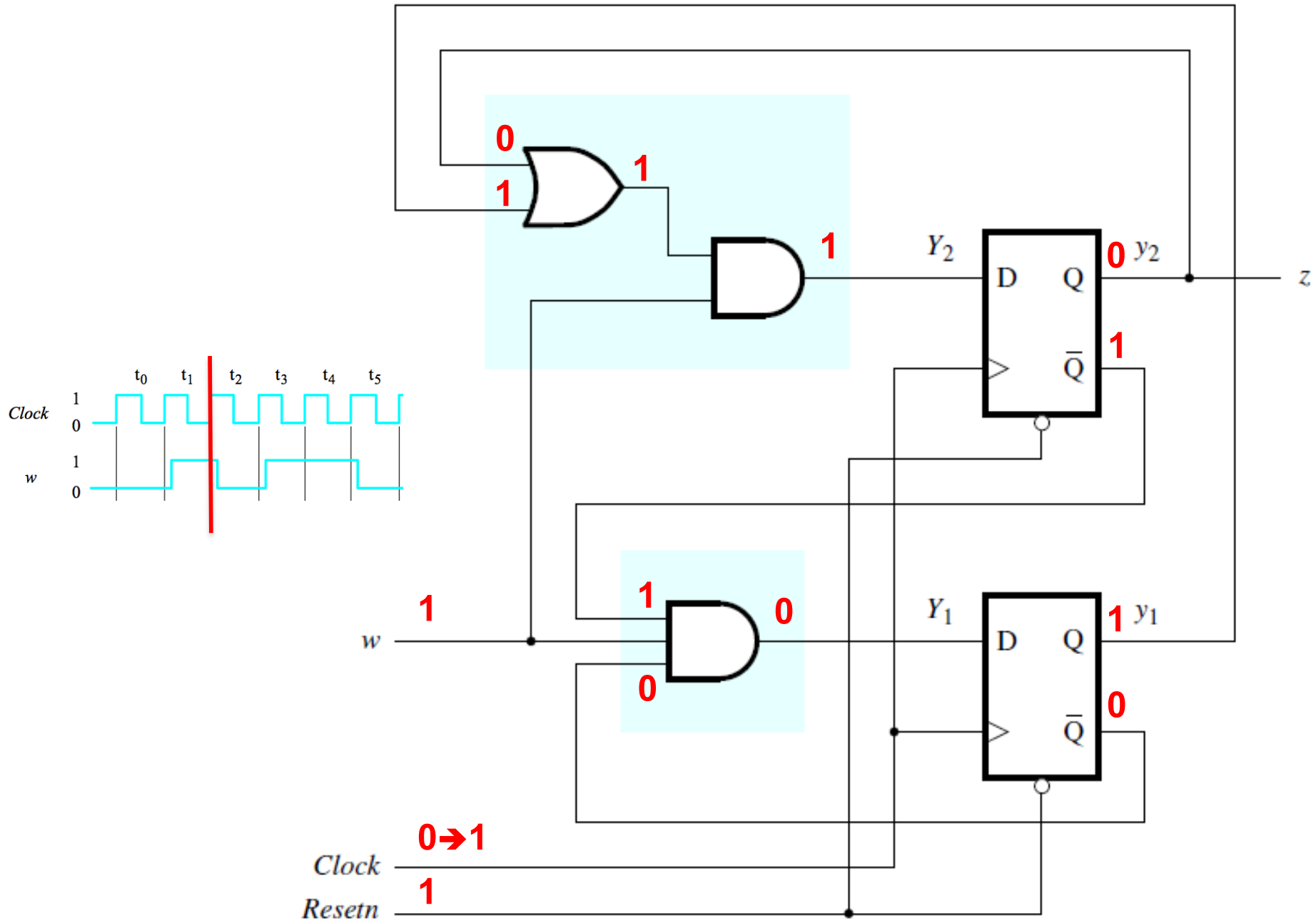




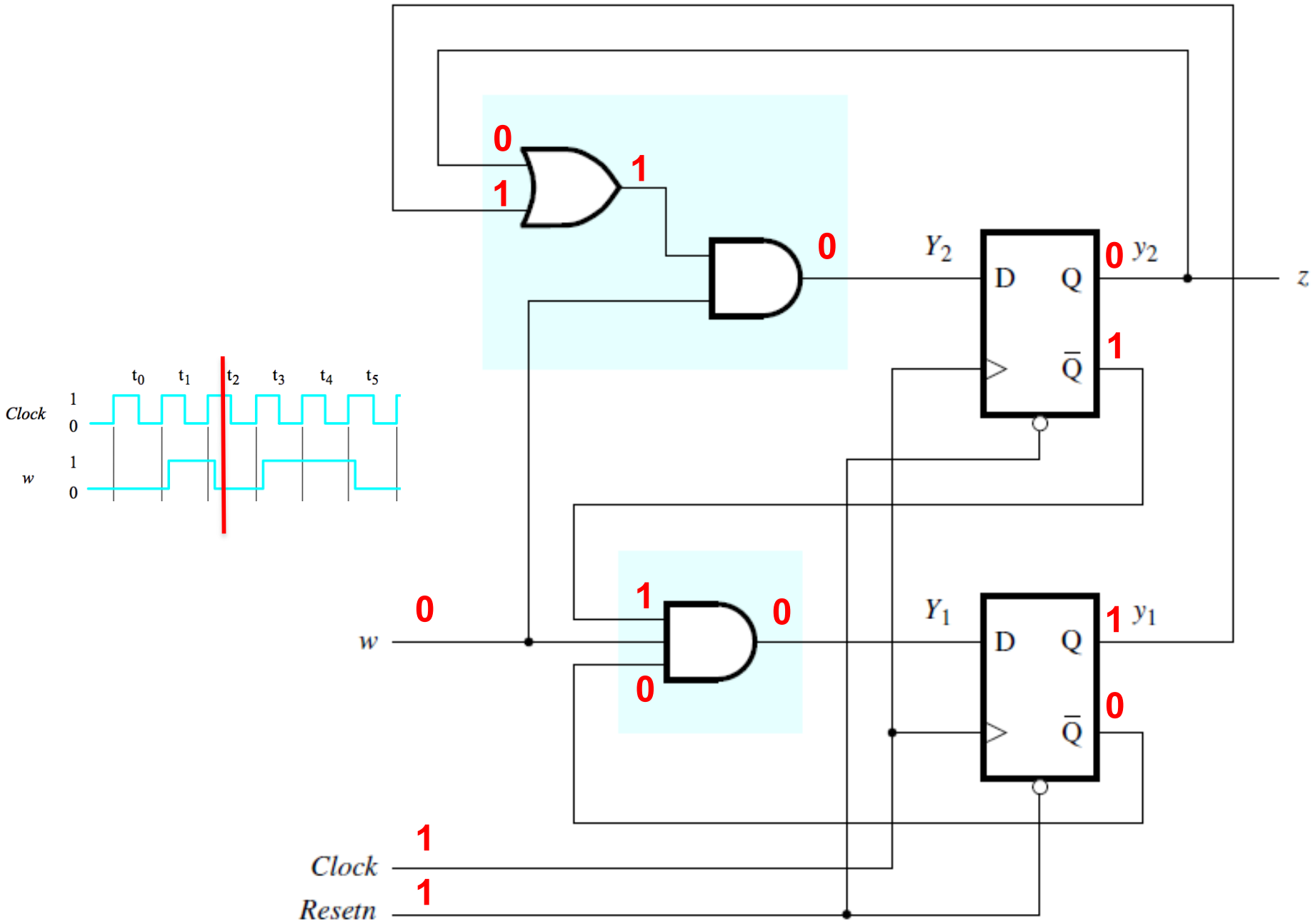
State A=00



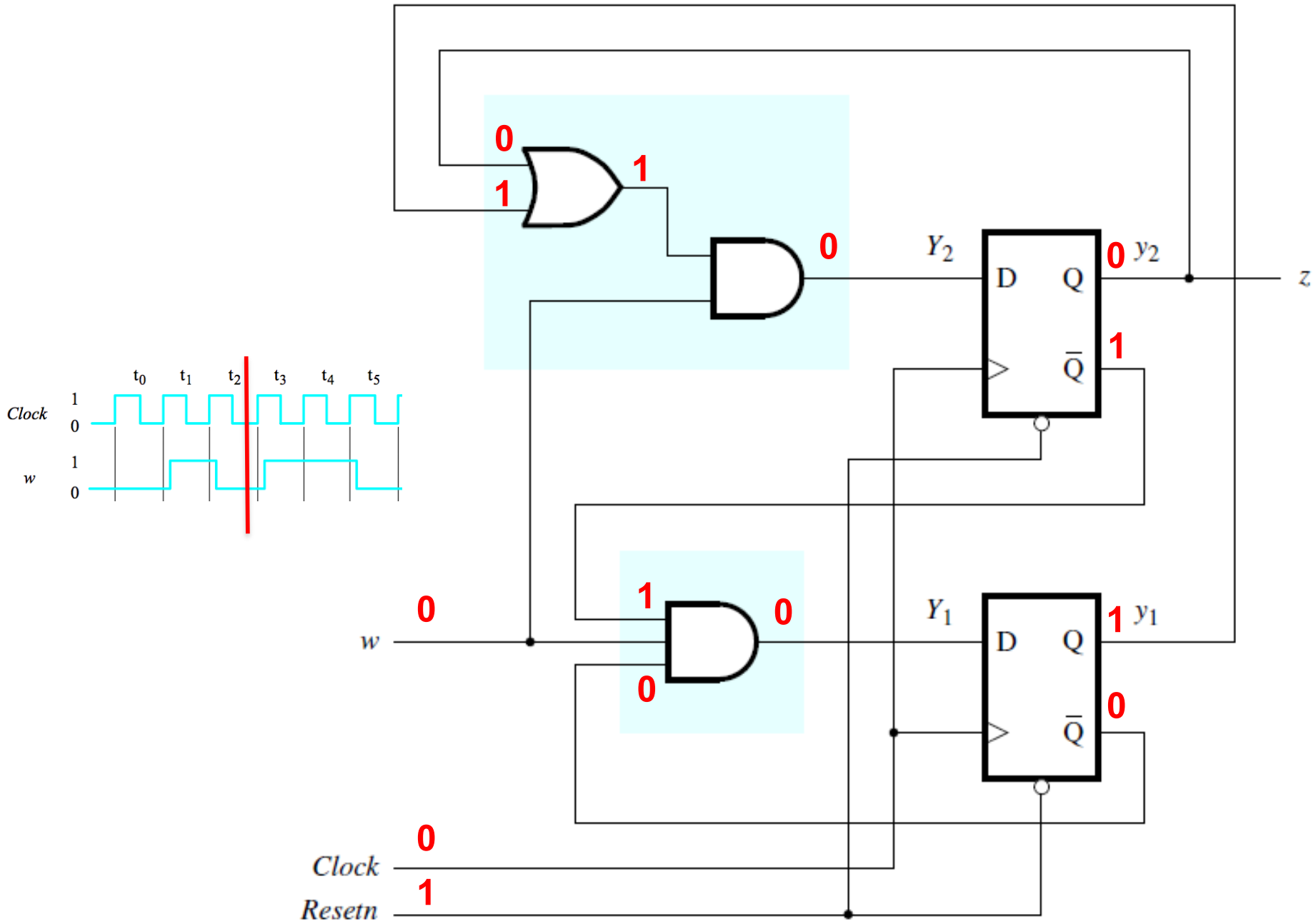
State B=01



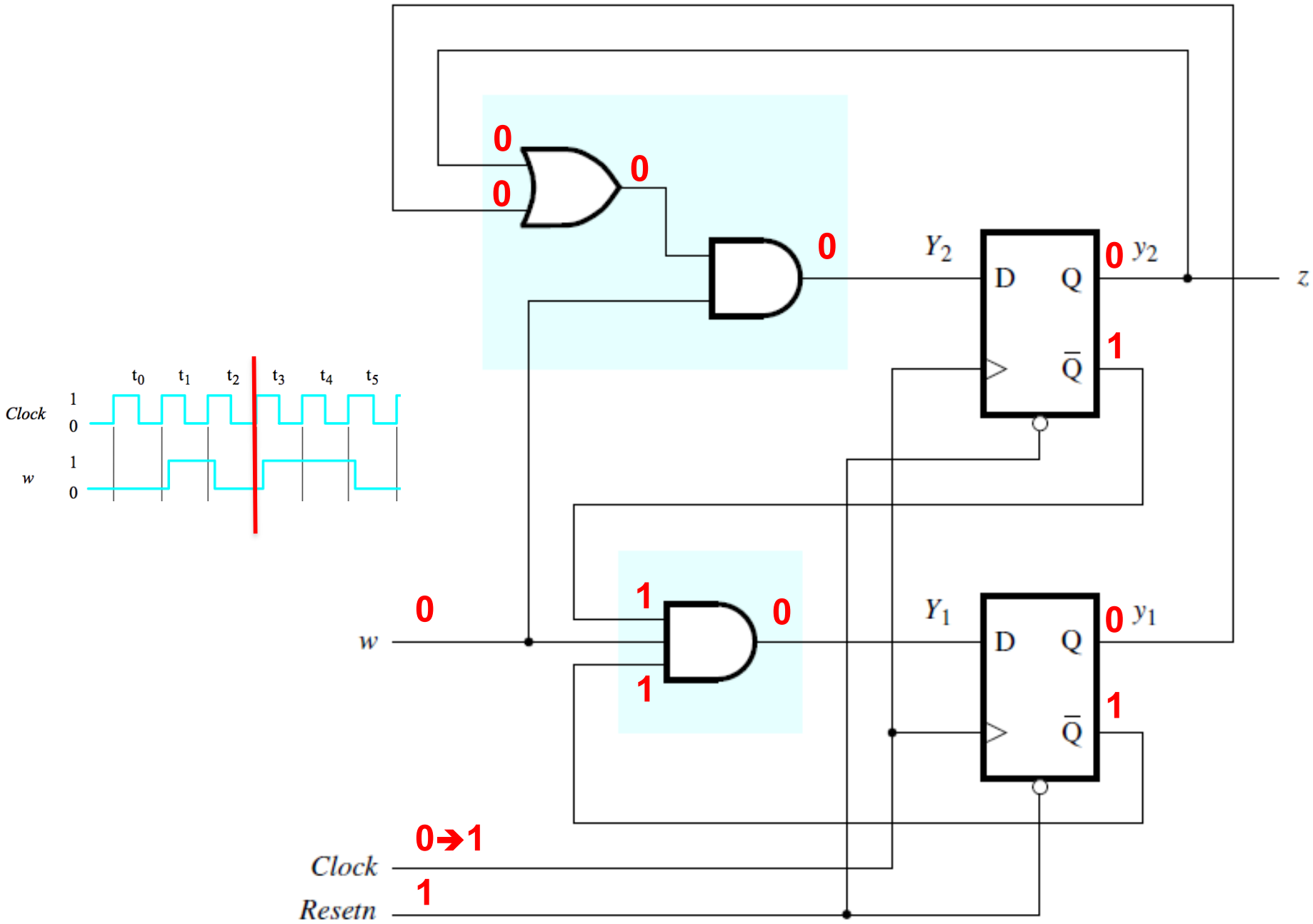
State B=01



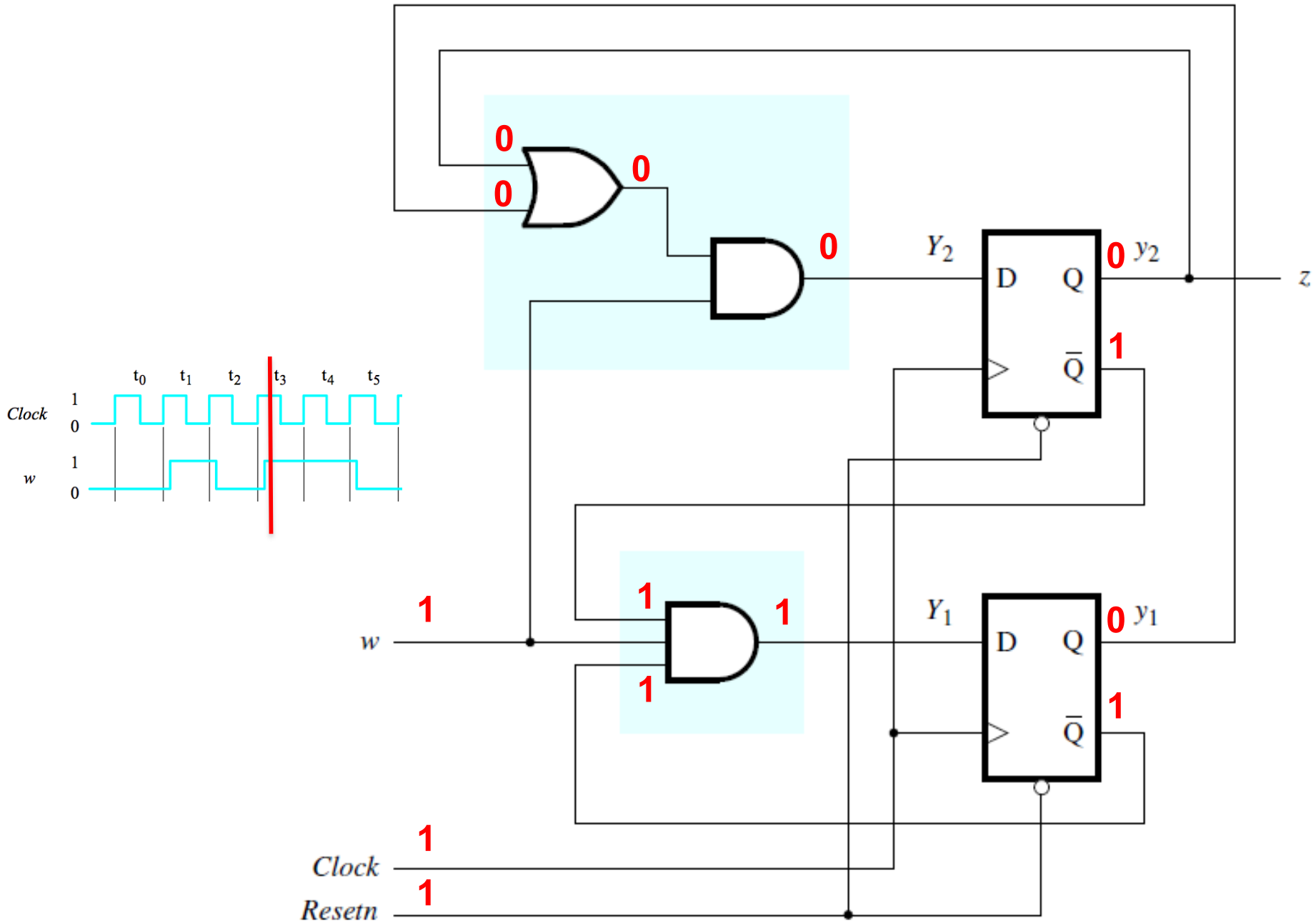
State B=01



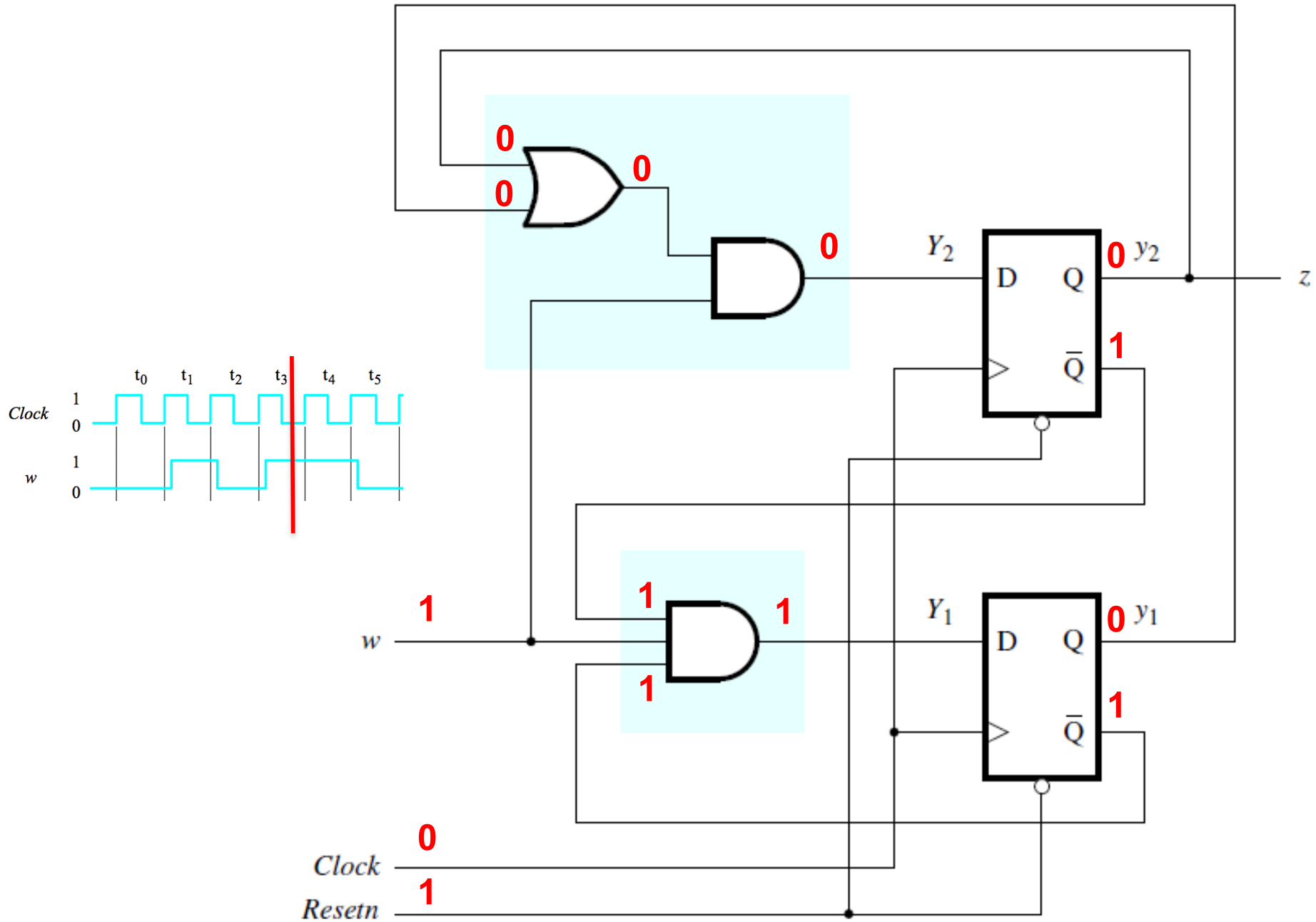
State A=00



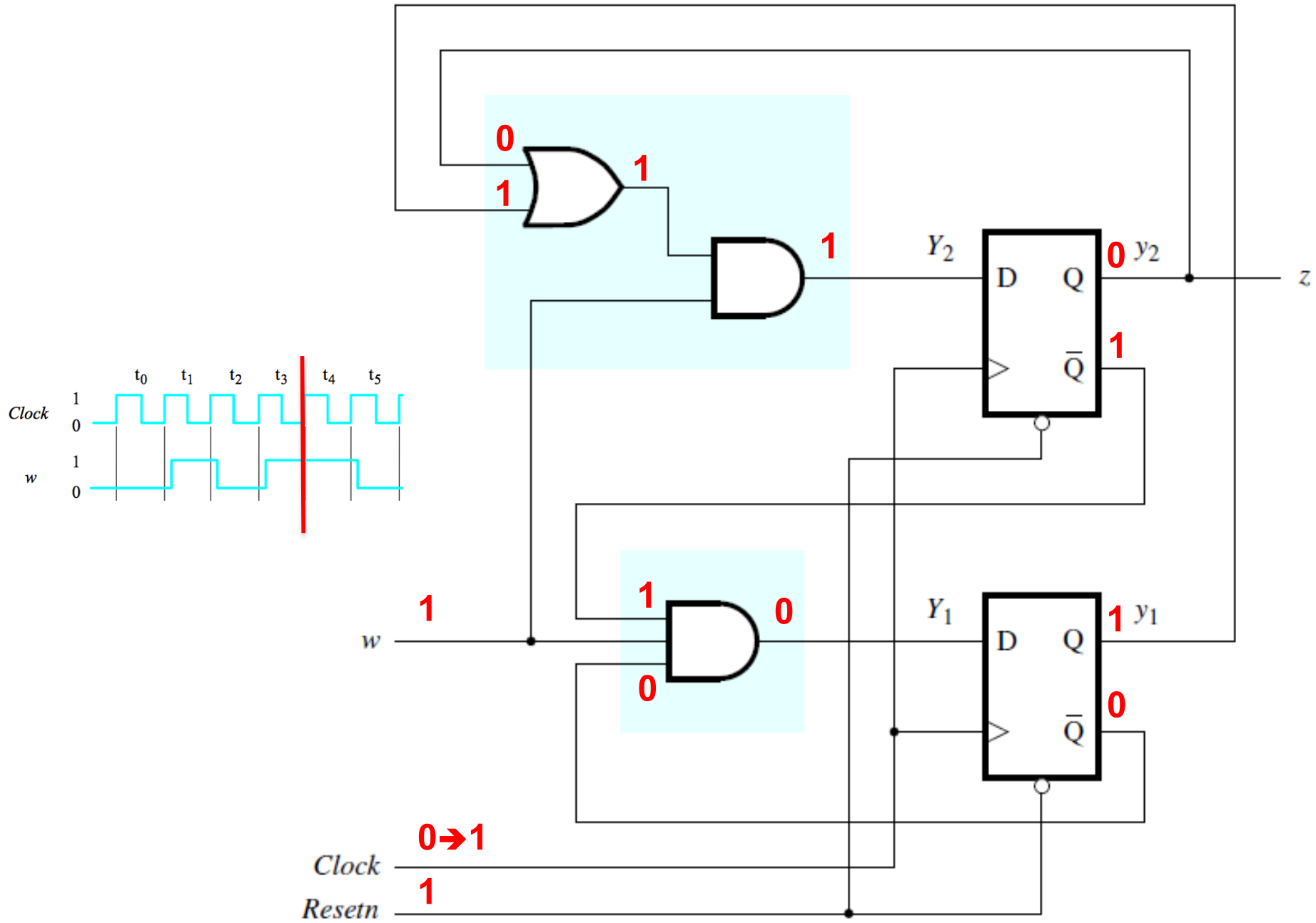
State A=00



State A=00

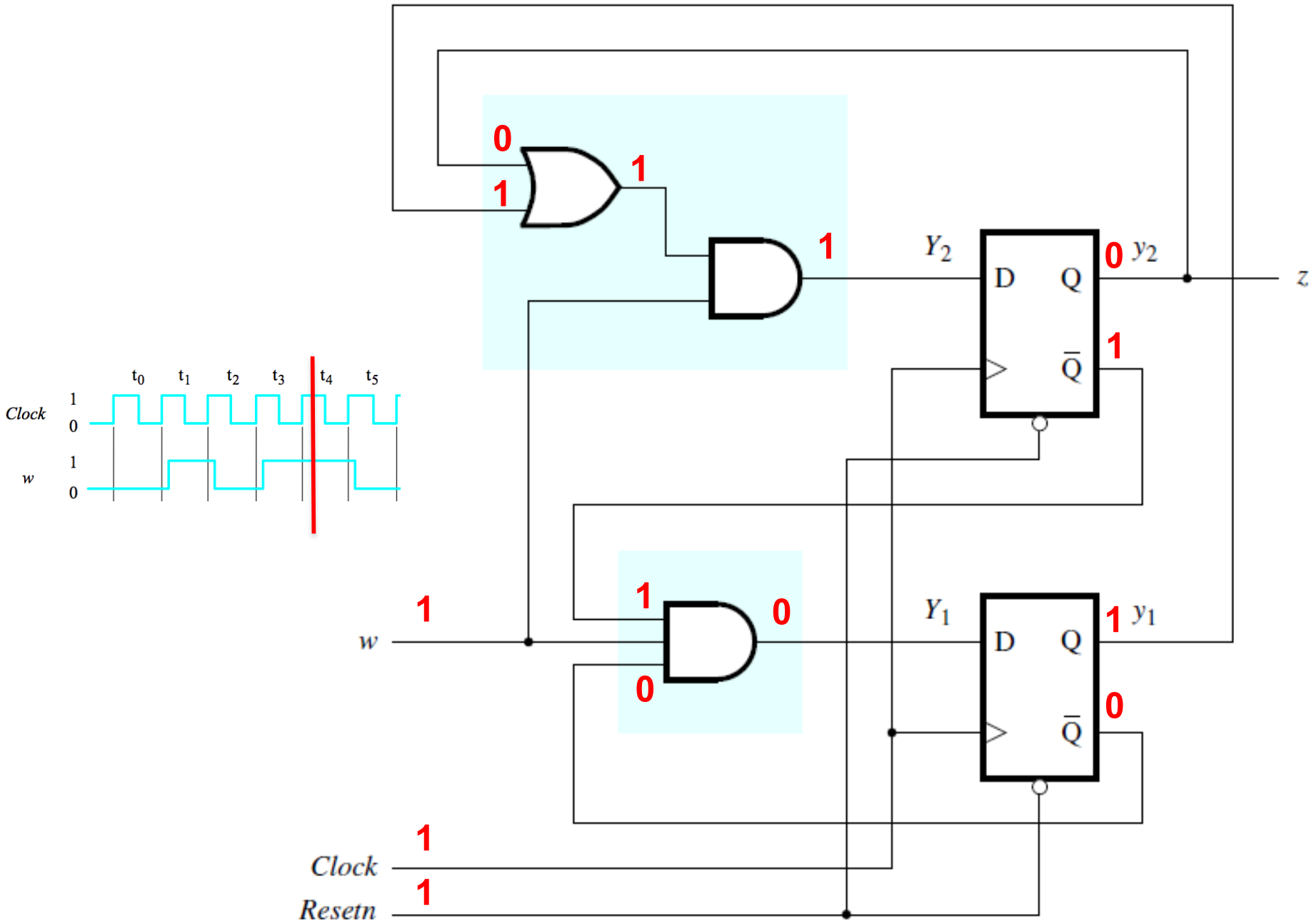


State B=01

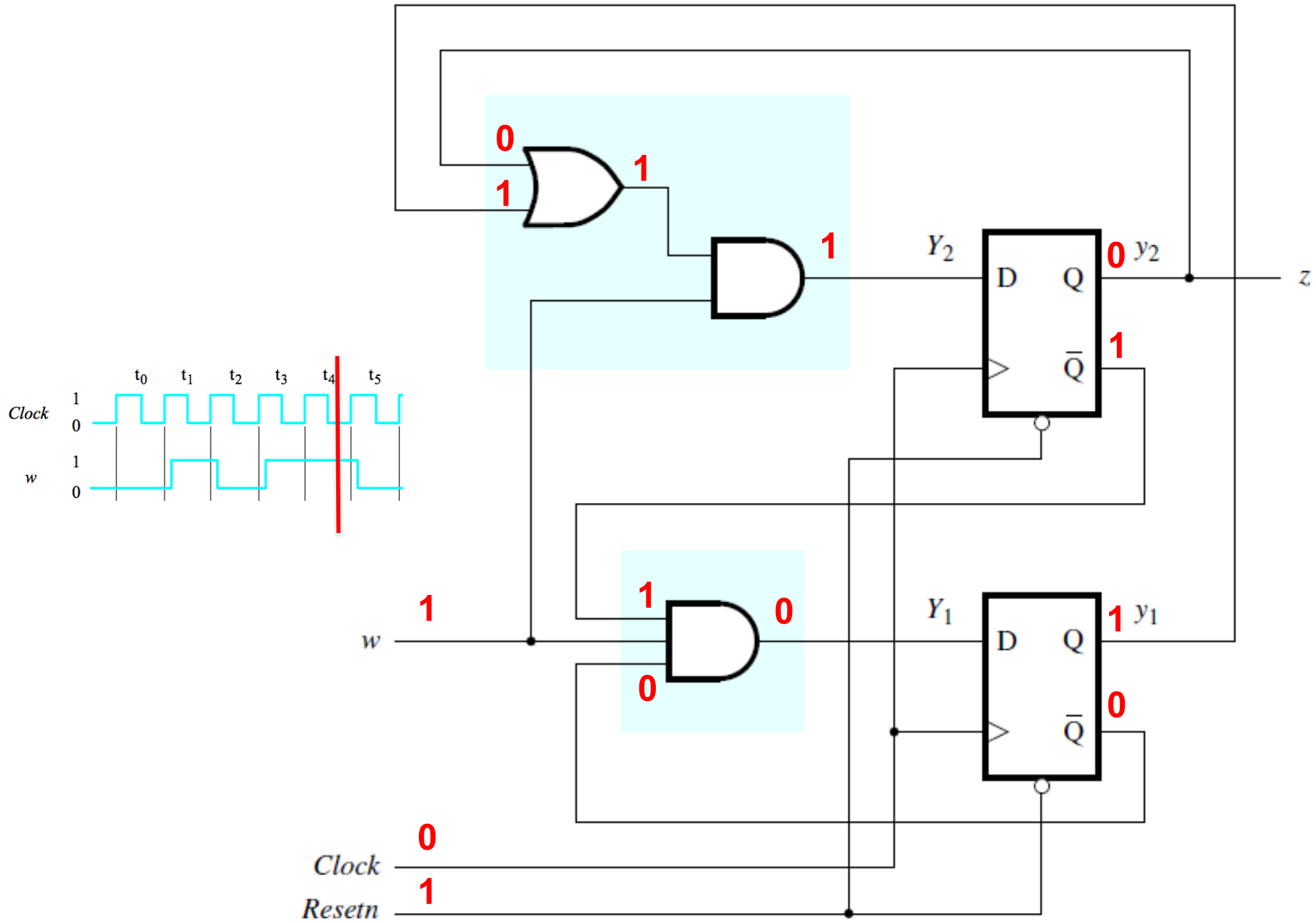




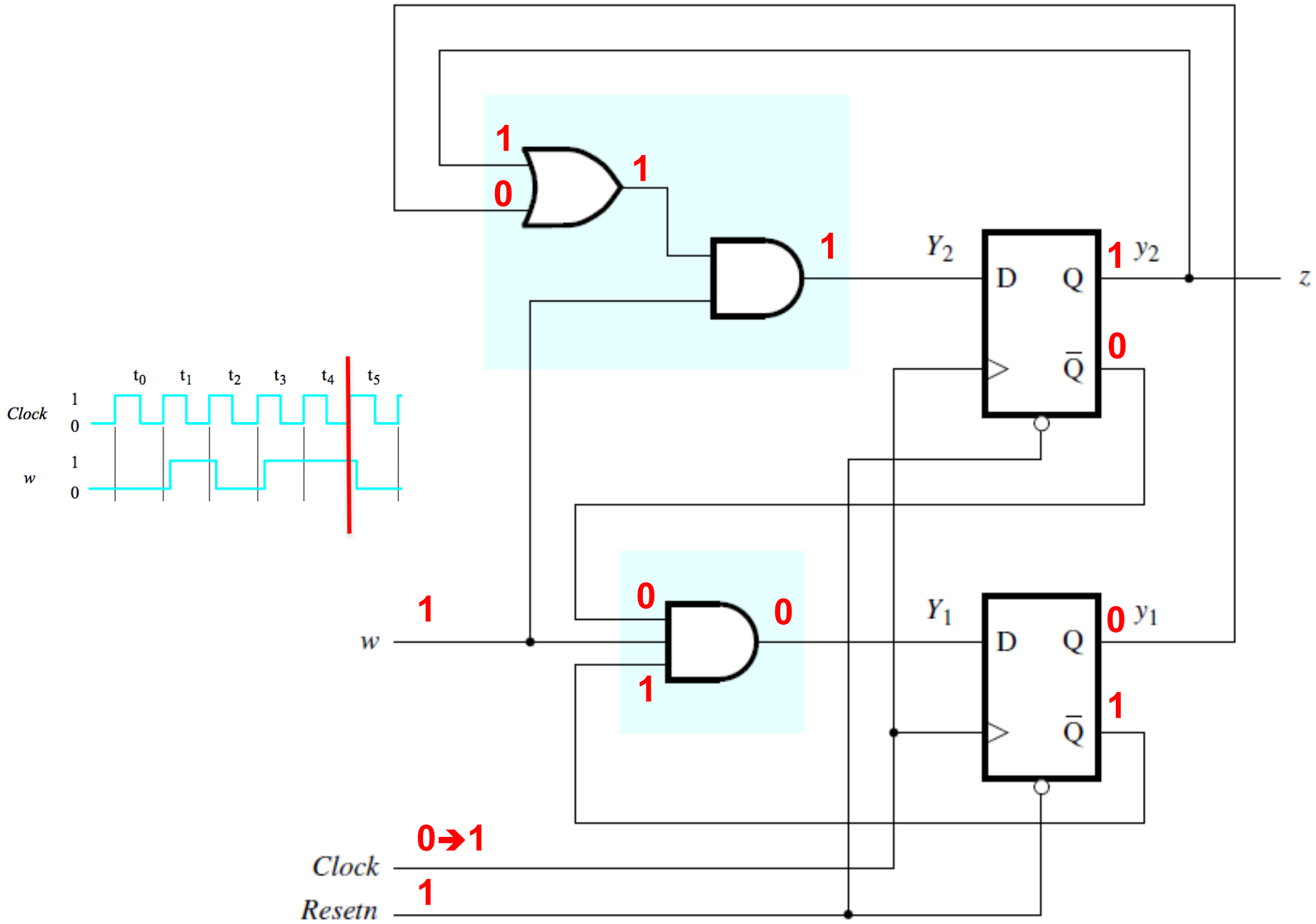
State B=01



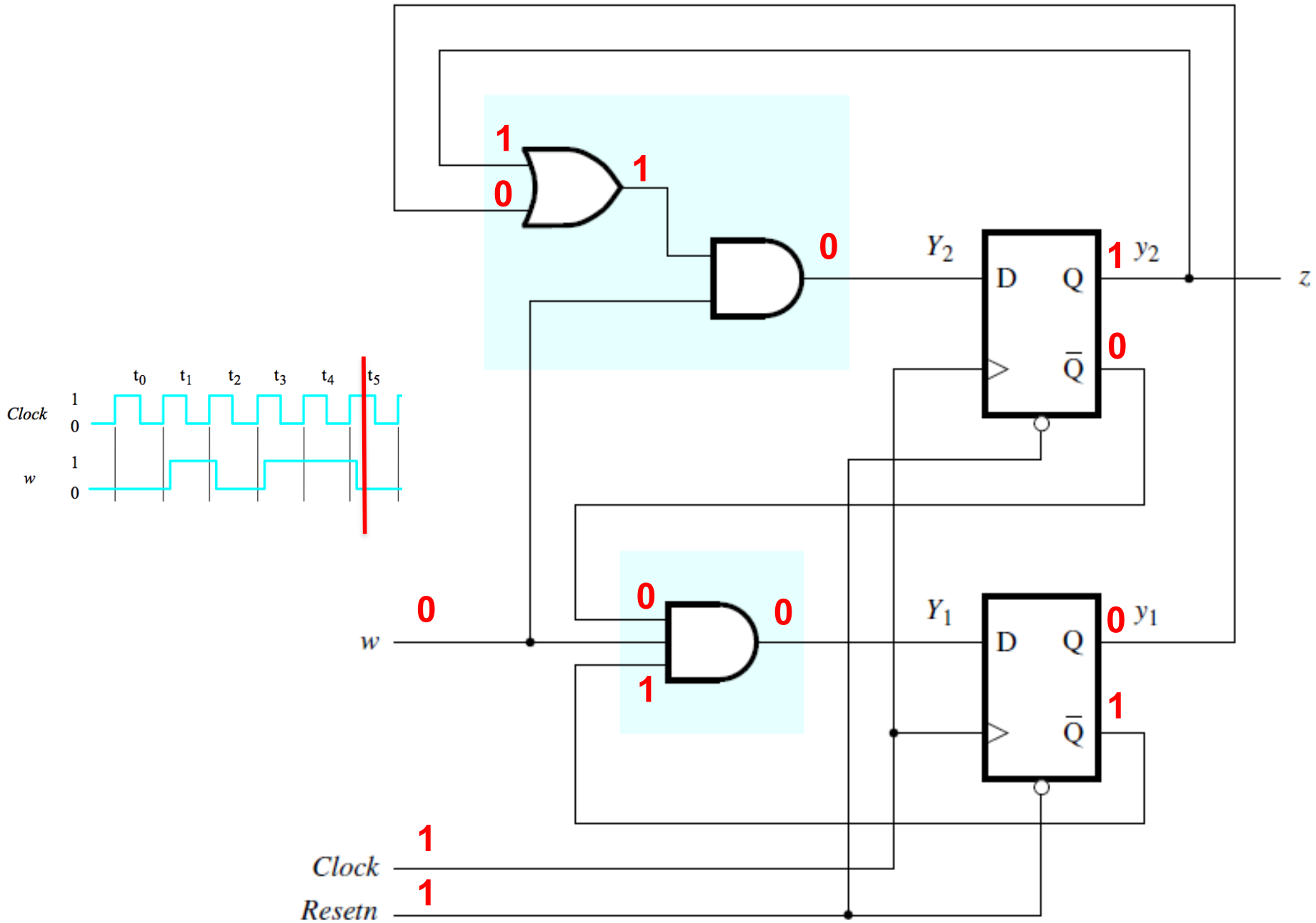
State B=01



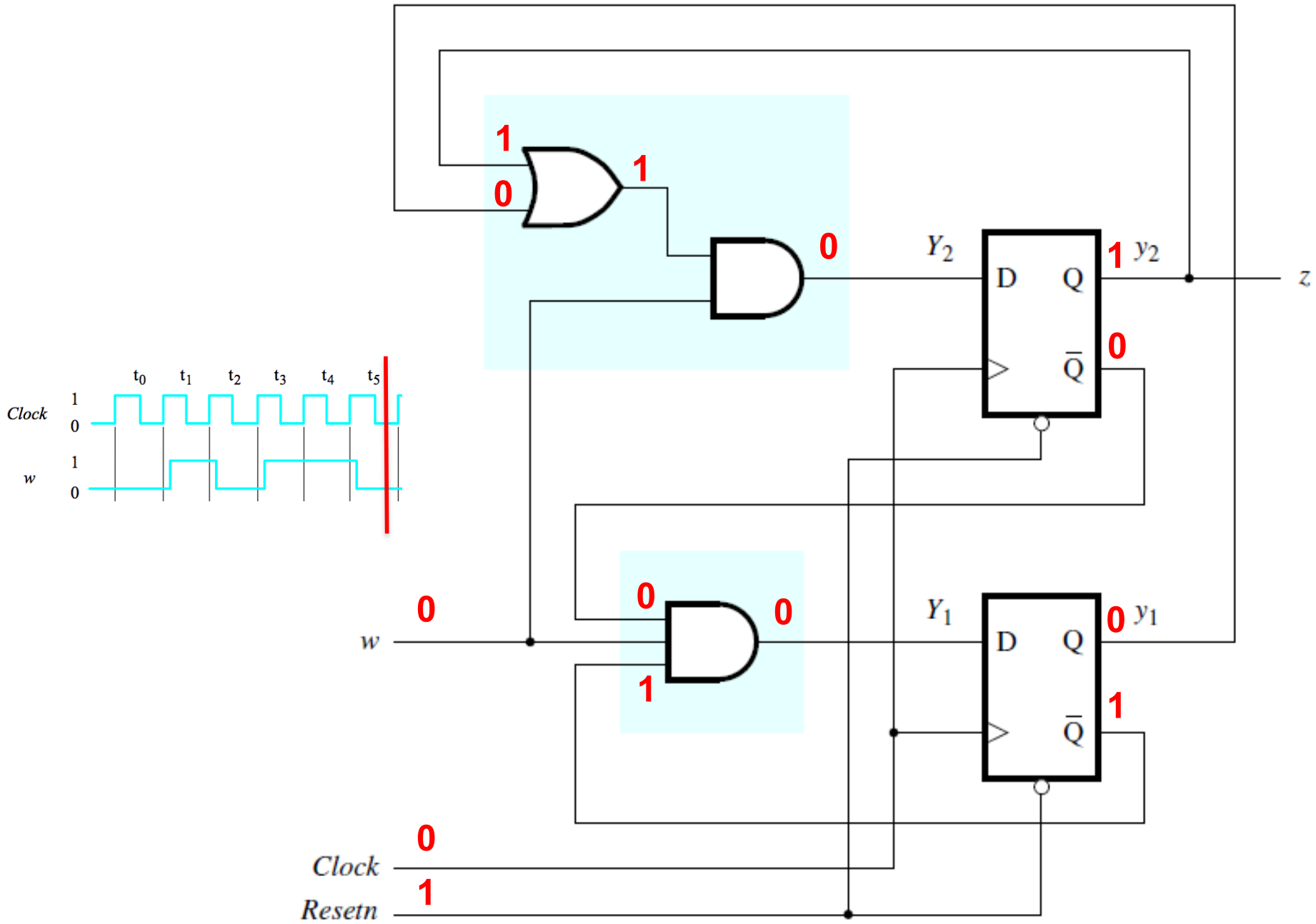
State C=10



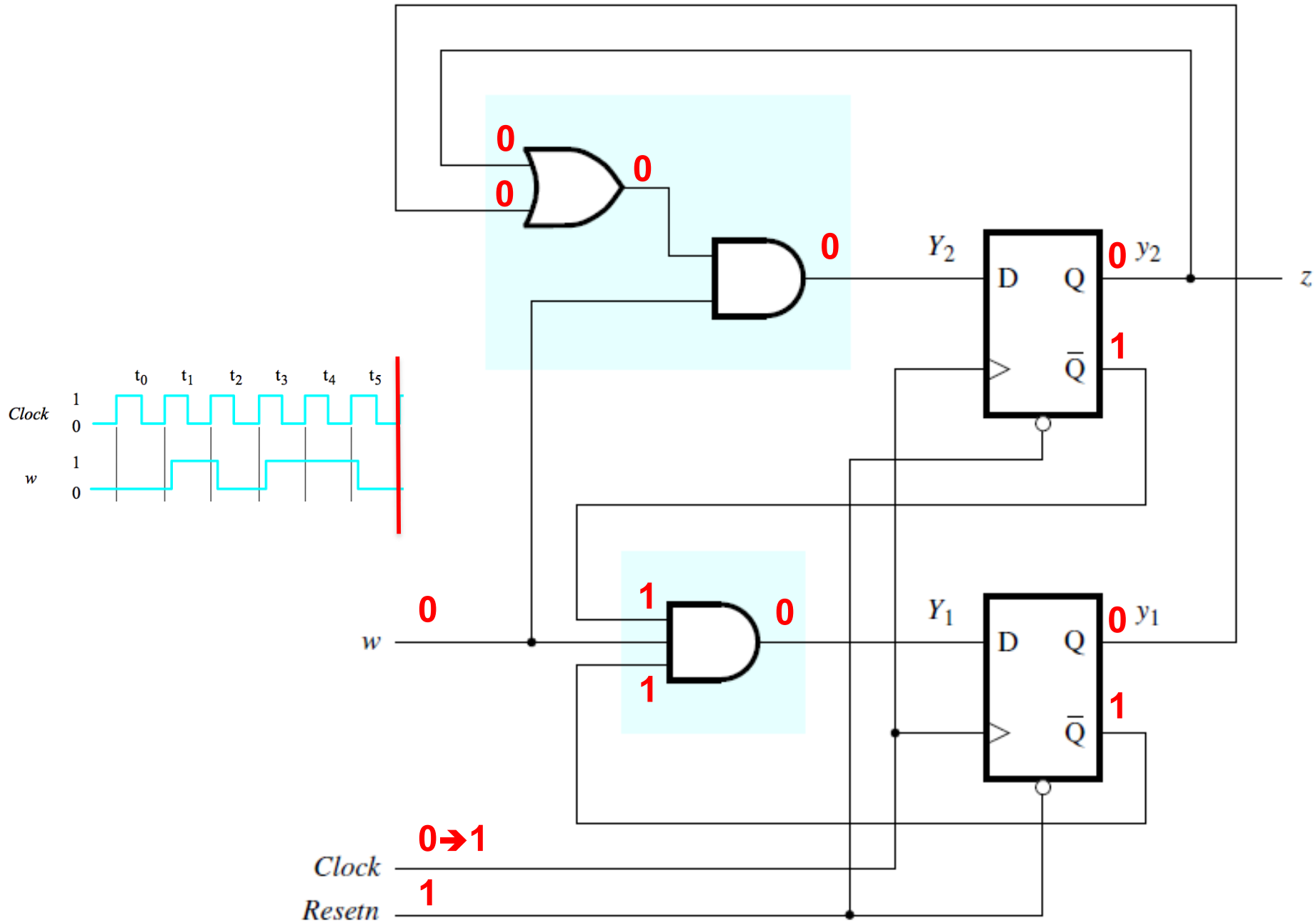
State C=10



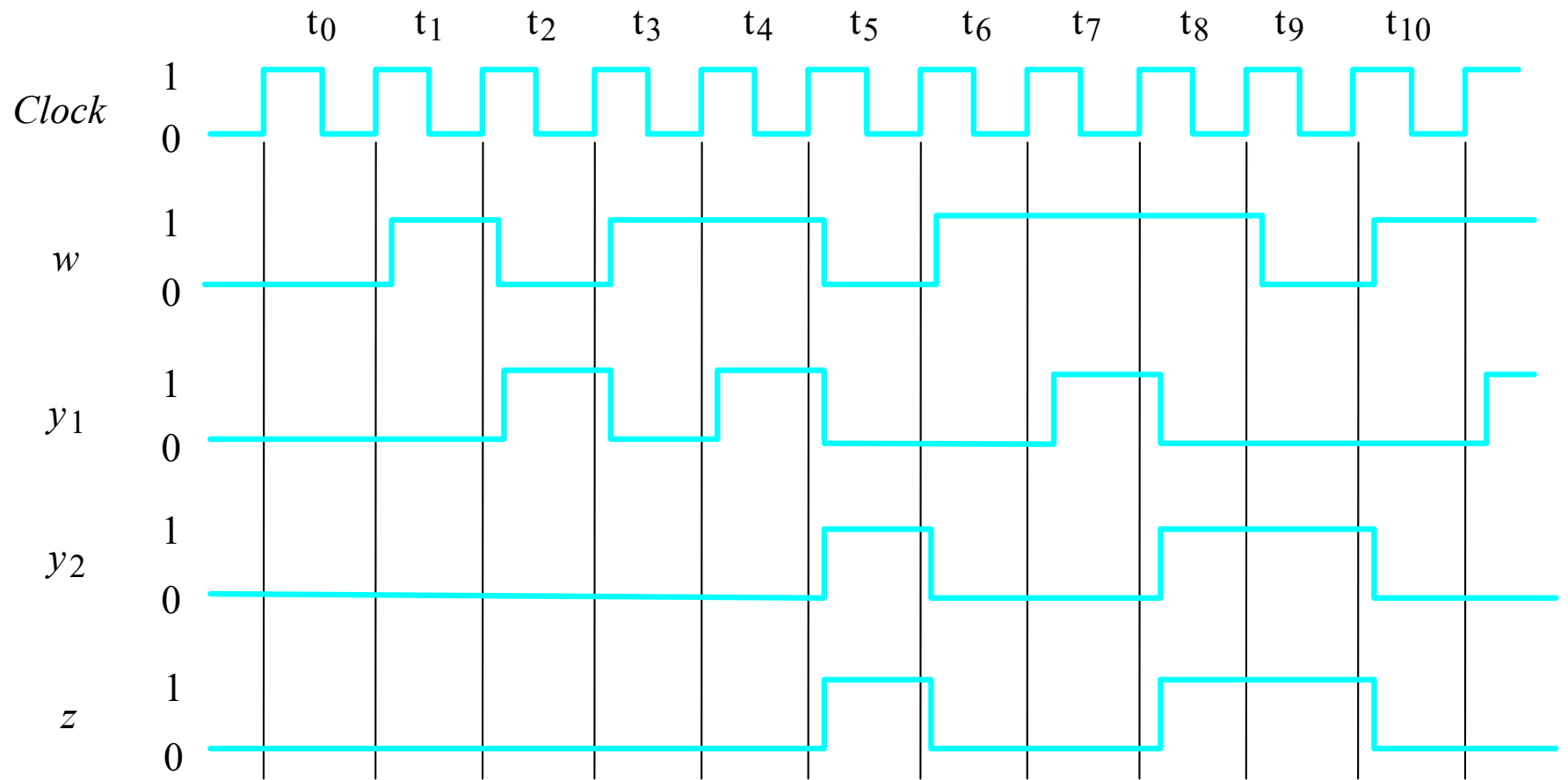
State C=10



State A=00

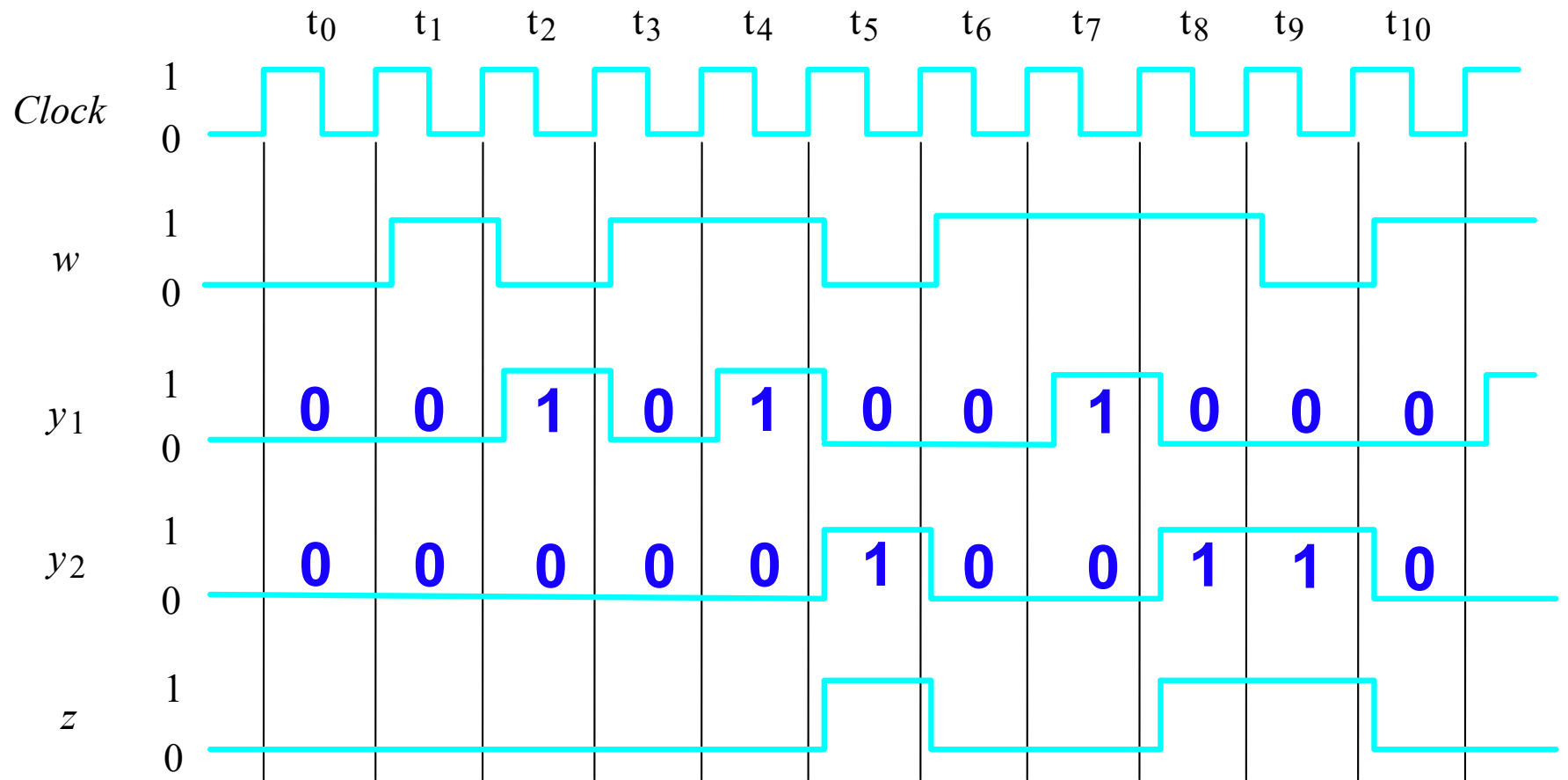


<b>Clockcycle:</b>	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



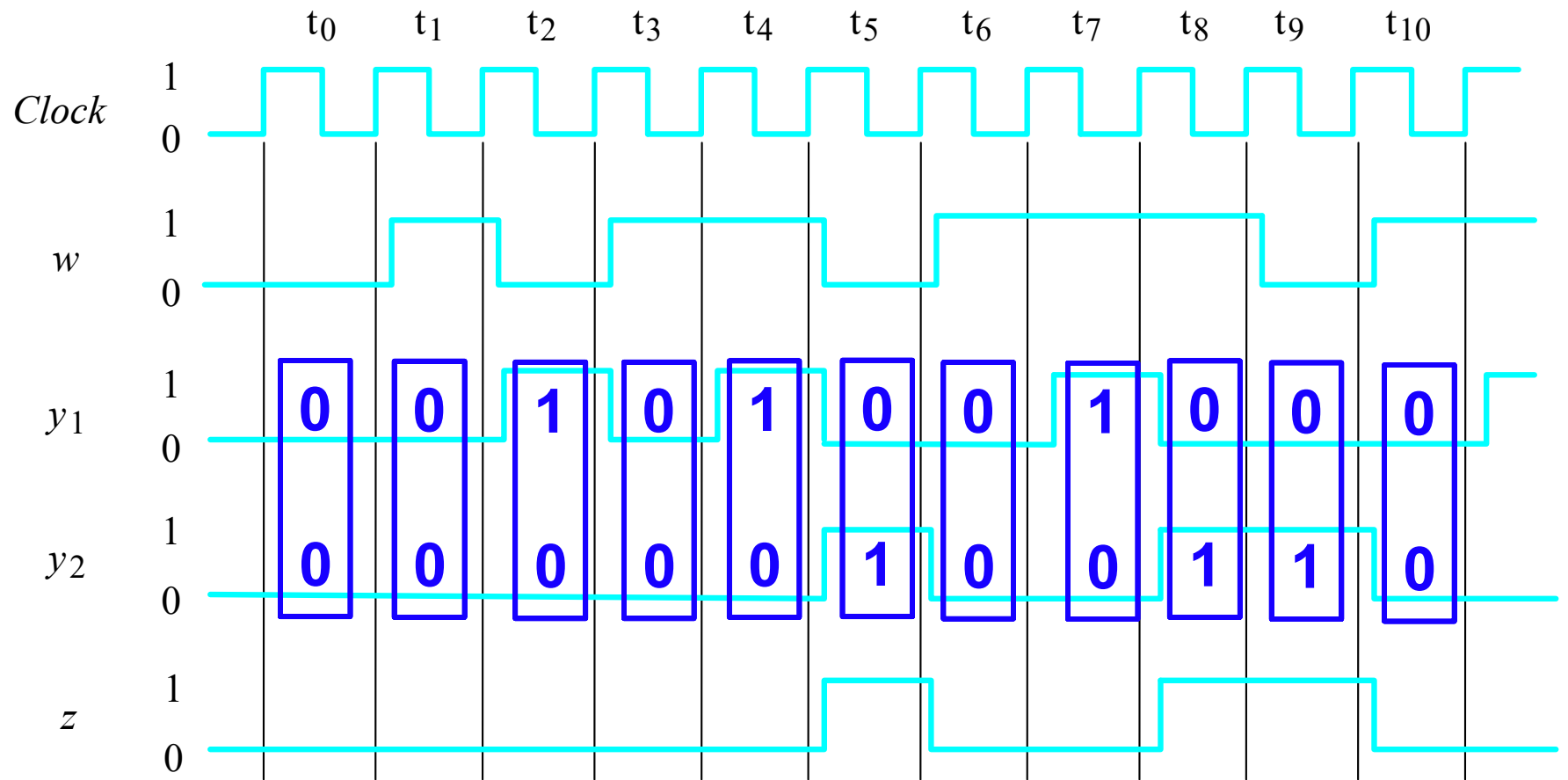
[ Figure 6.9 from the textbook ]

Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

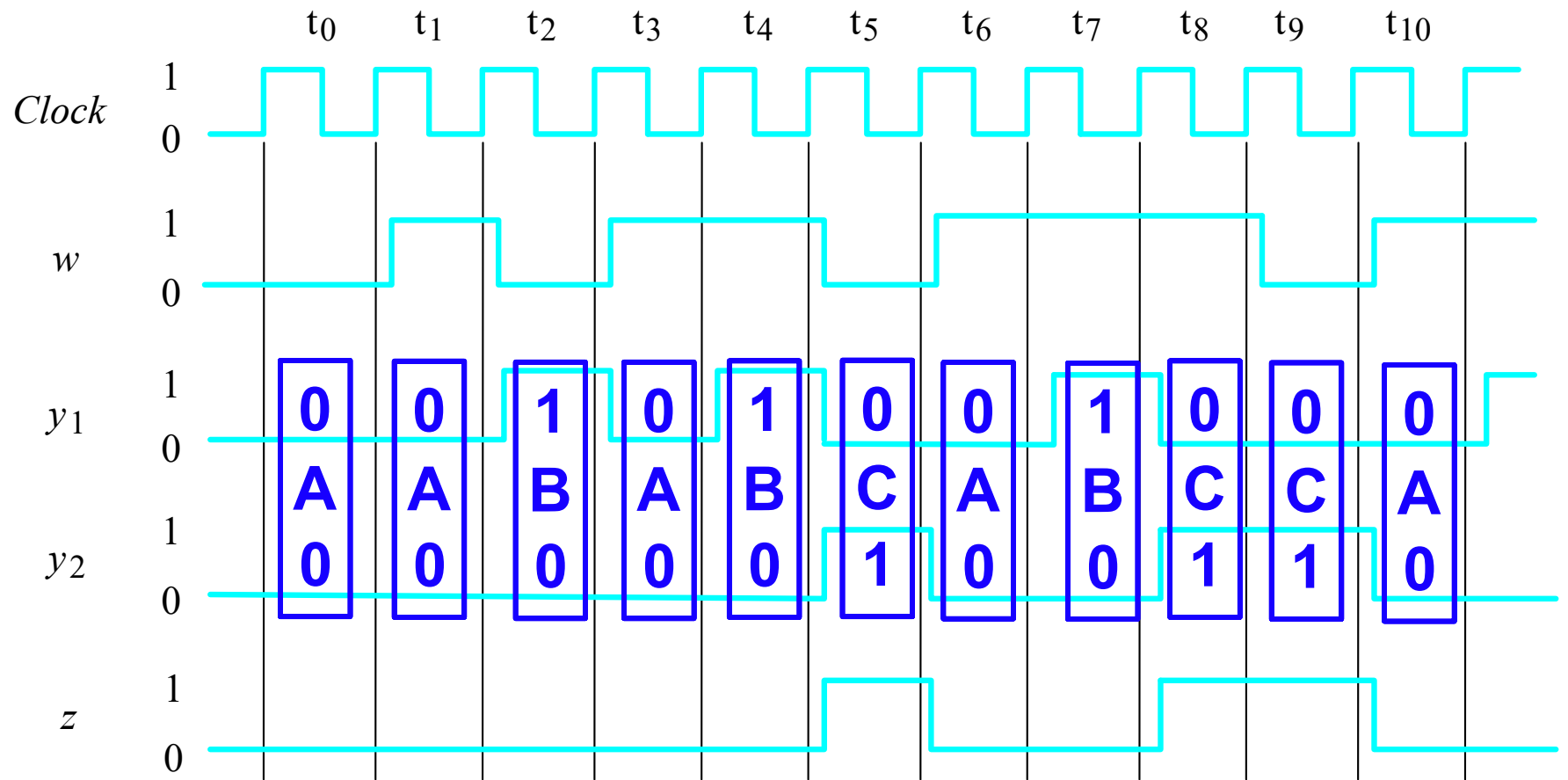




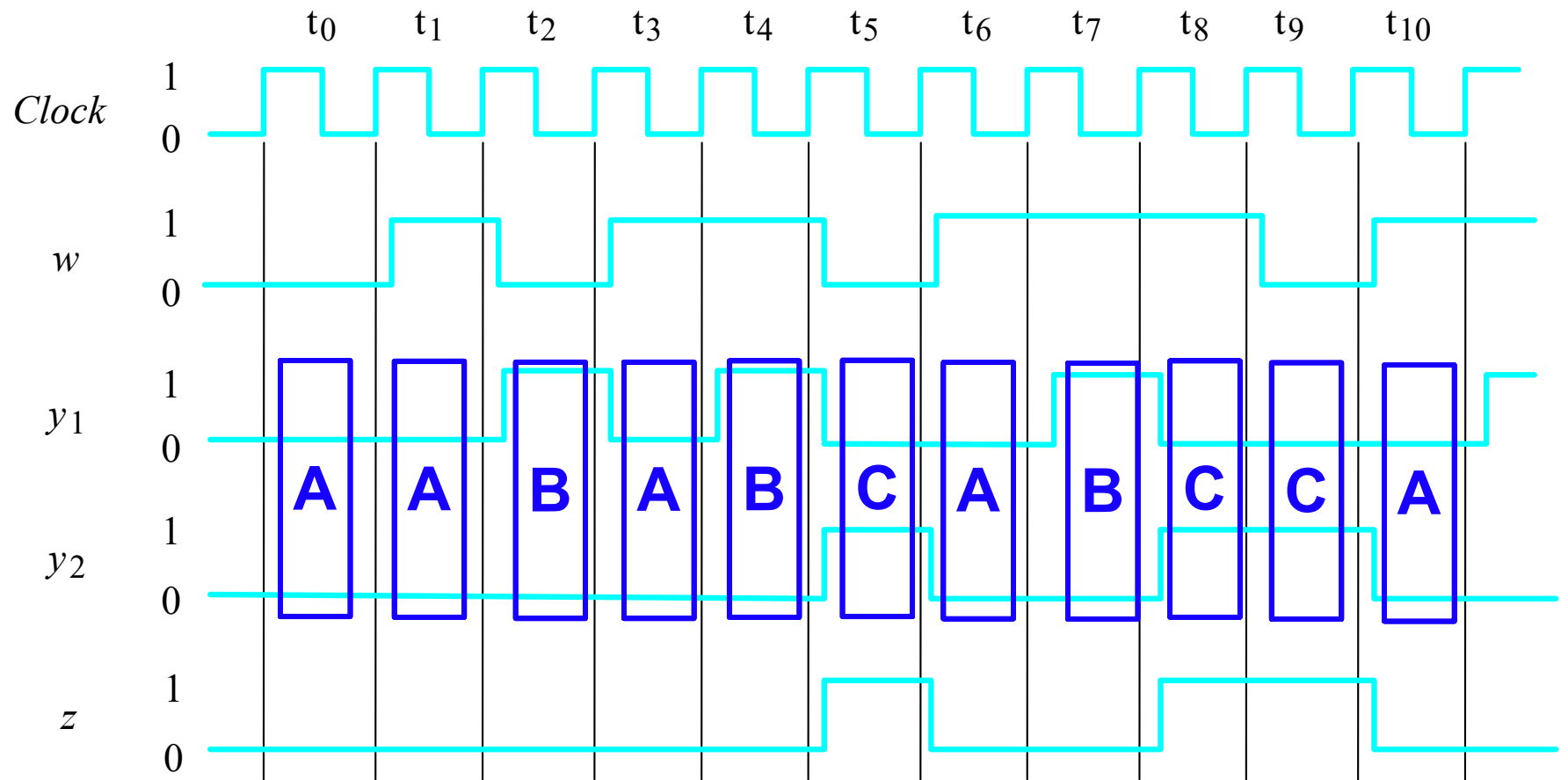
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

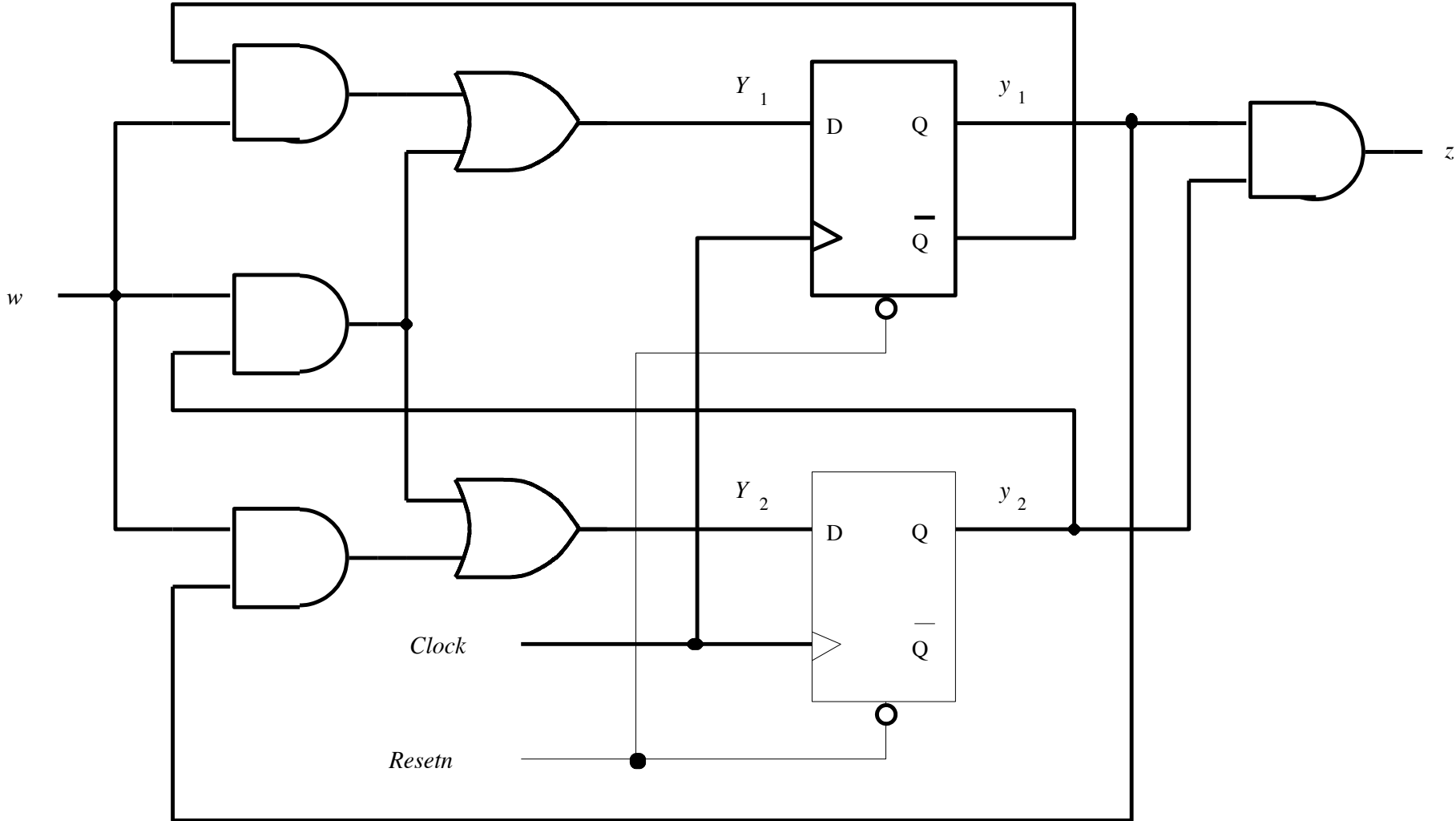


# Summary: Designing a Moore Machine

- Obtain the circuit specification.
- Derive a state diagram.
- Derive the state table.
- Decide on a state encoding.
- Encode the state table.
- Derive the output logic and next-state logic.
- Add a reset signal.

# **Reverse Engineering**

# What does this circuit do?



[ Figure 6.75 from the textbook ]

# Approach

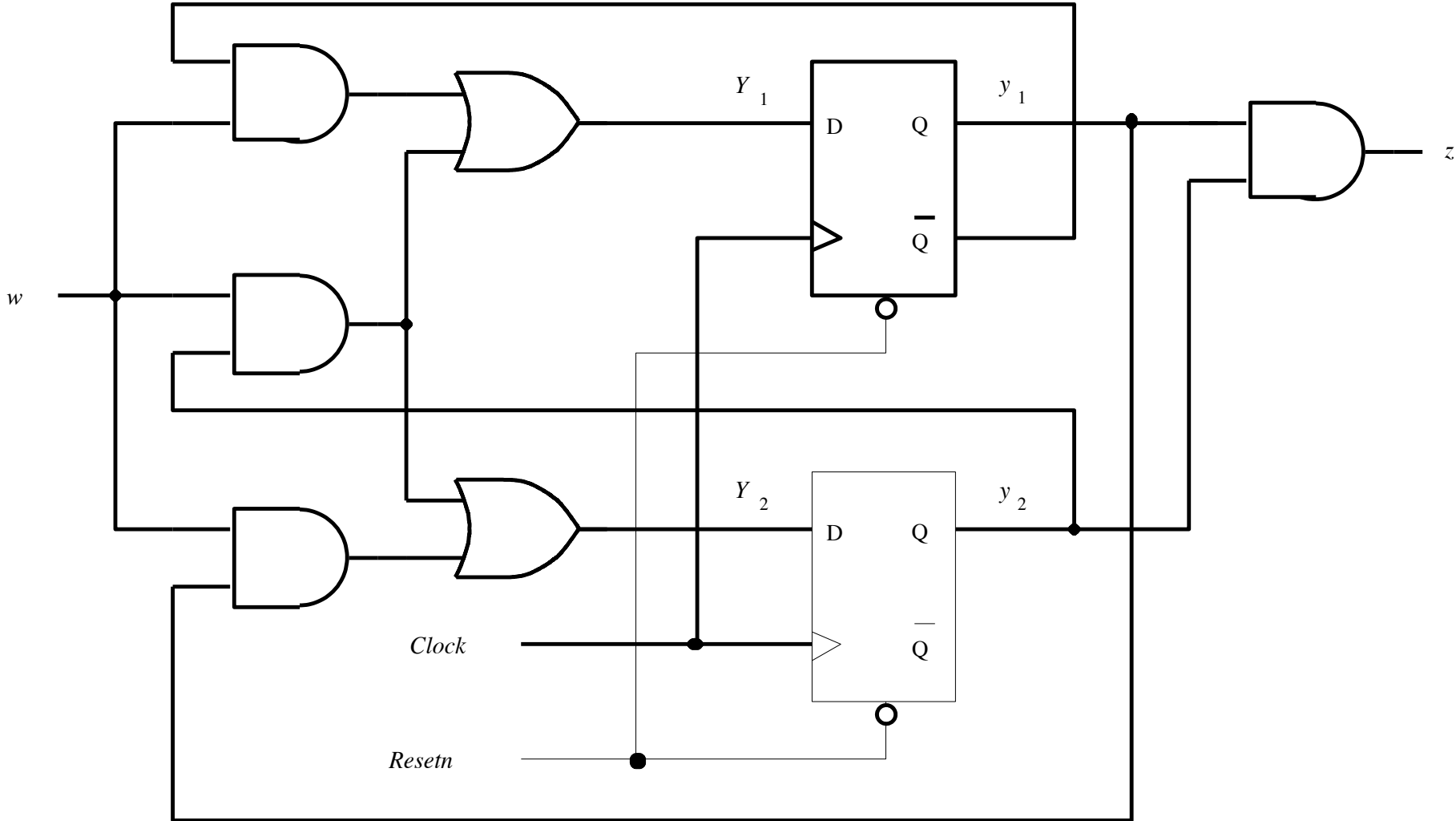
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

# Goal

- **Given a circuit diagram for a synchronous sequential circuit, the goal is to figure out the FSM**
- **Figure out the present state variables, the next state variables, the state-assigned table, the state table, and finally the state diagram.**
- **In other words, the goal is to reverse engineer the circuit.**



# What does this circuit do?

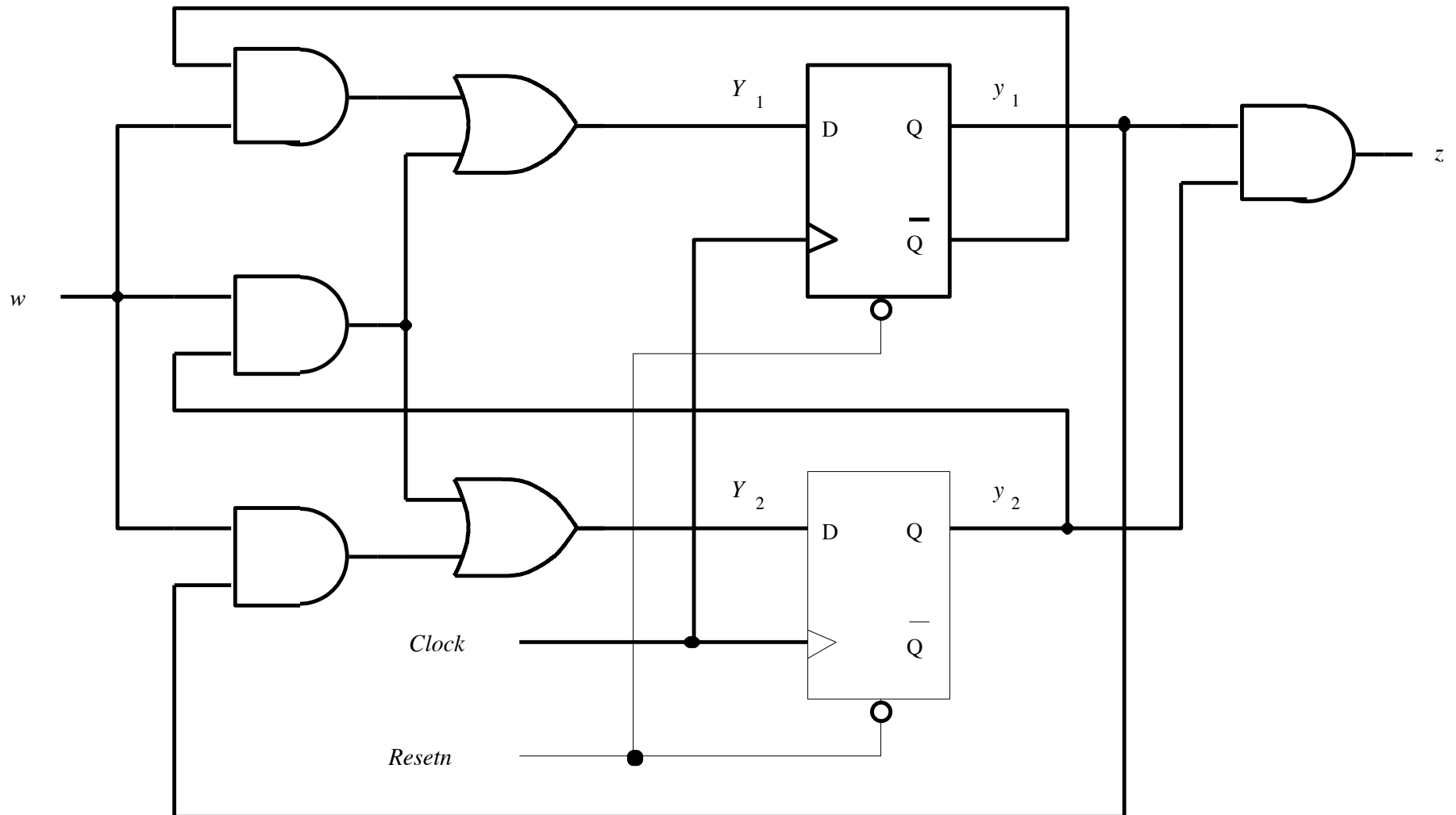


[ Figure 6.75 from the textbook ]

# Approach

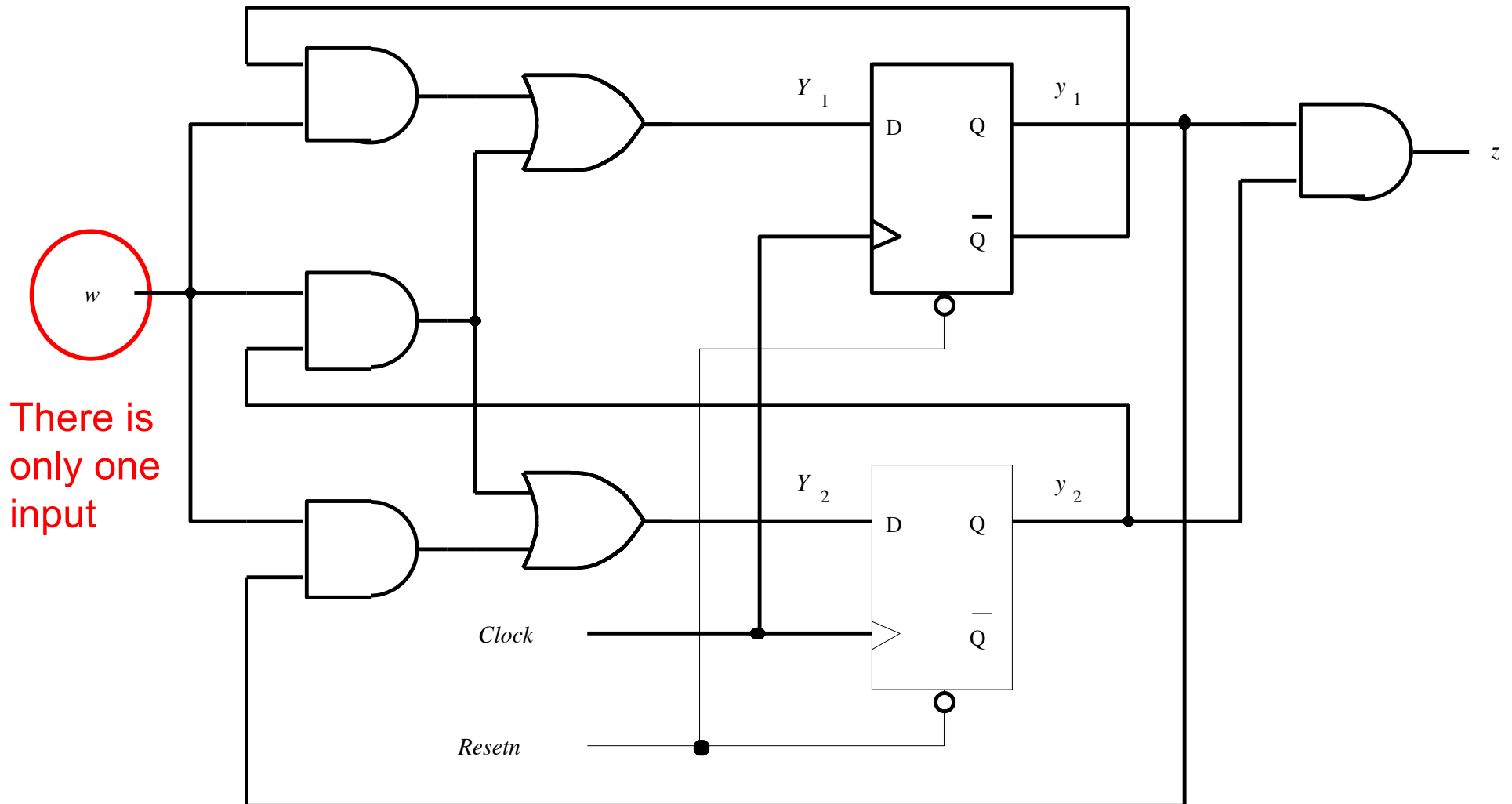
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

# Where are the inputs?



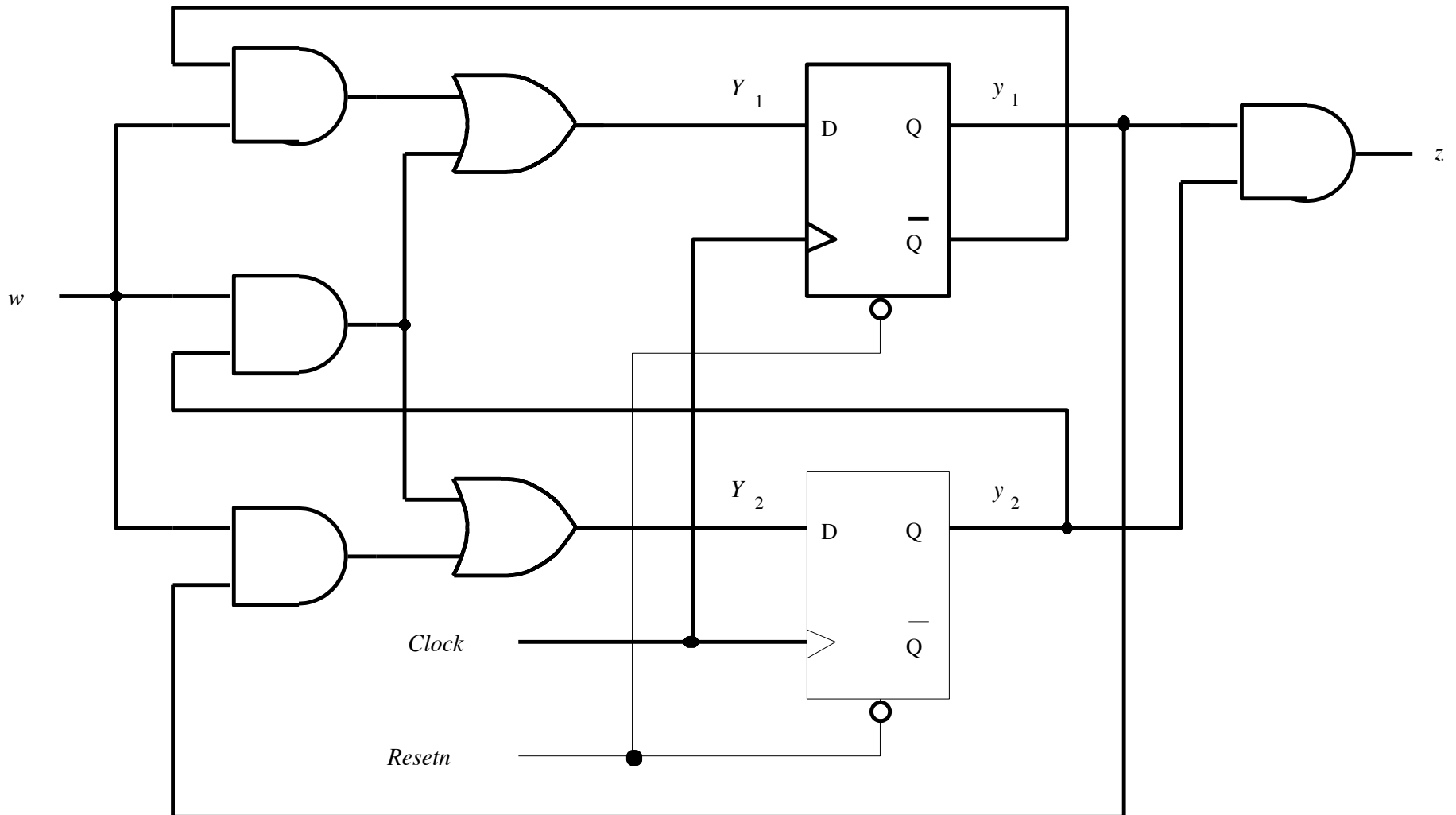
[ Figure 6.75 from the textbook ]

# Where are the inputs?



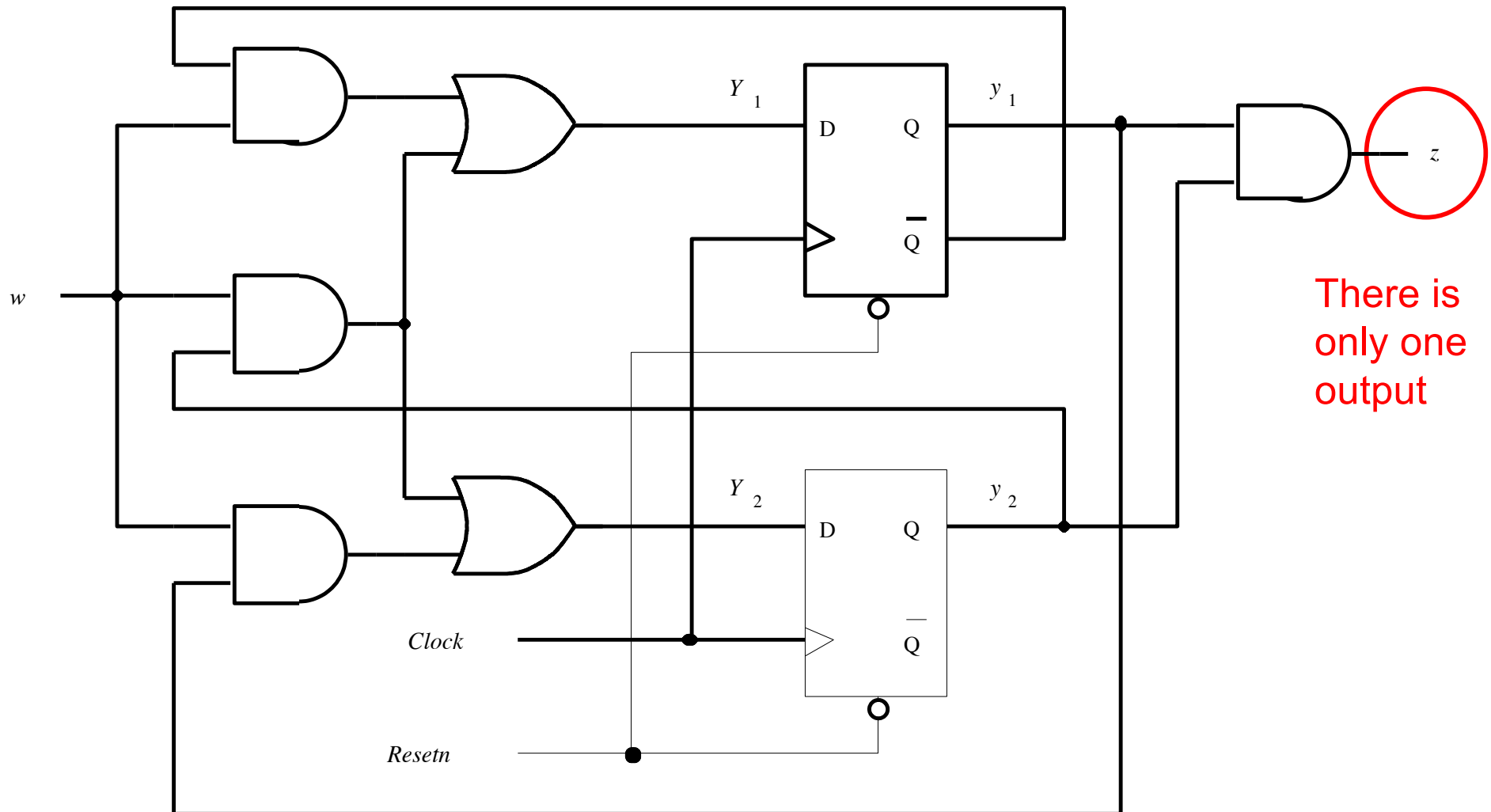
[ Figure 6.75 from the textbook ]

# Where are the outputs?



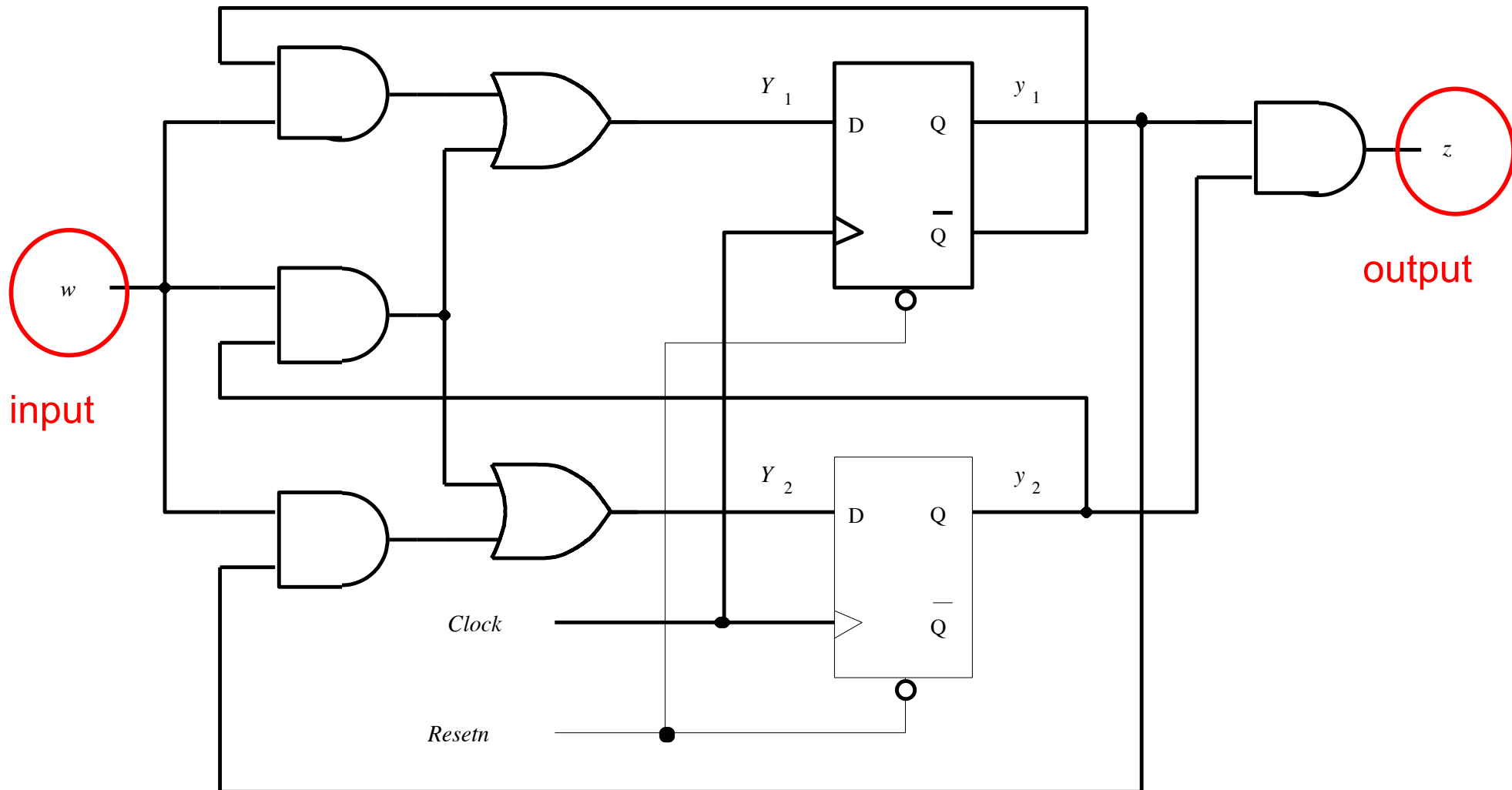
[ Figure 6.75 from the textbook ]

# Where are the outputs?

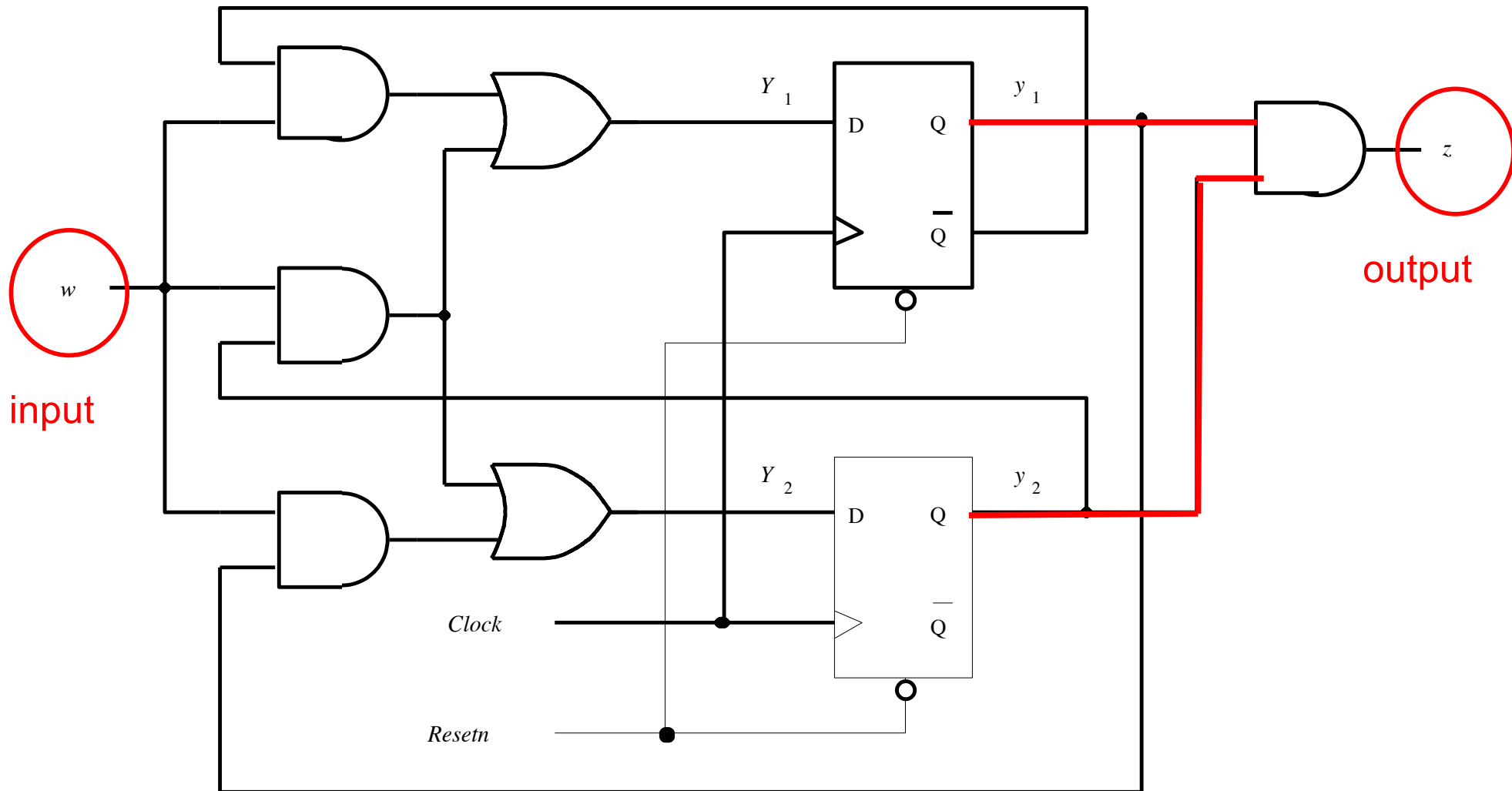


[ Figure 6.75 from the textbook ]

# Where kind of machine is this? Moore or Mealy?

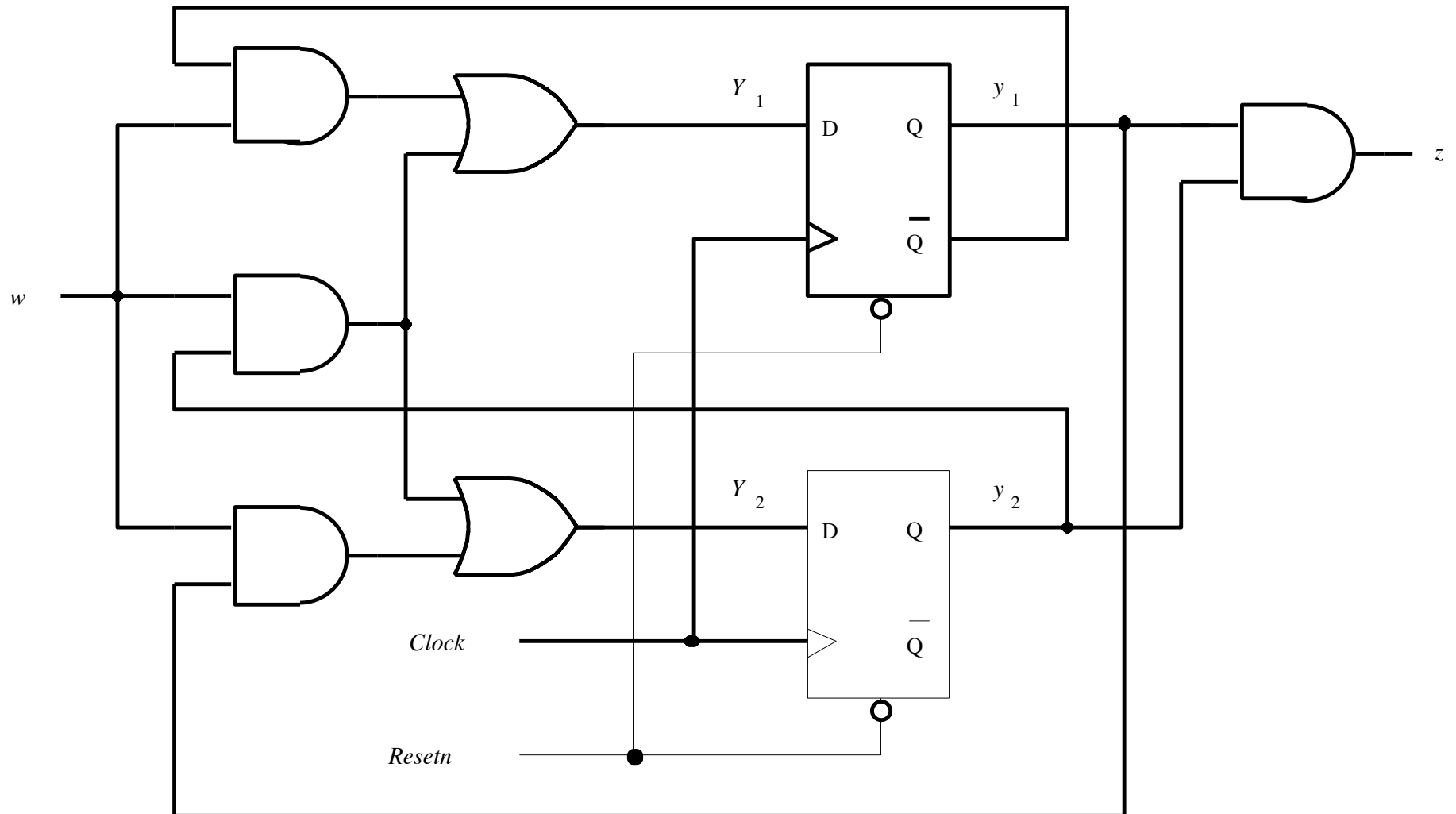


# Moore: because the output does not depend directly on the primary input

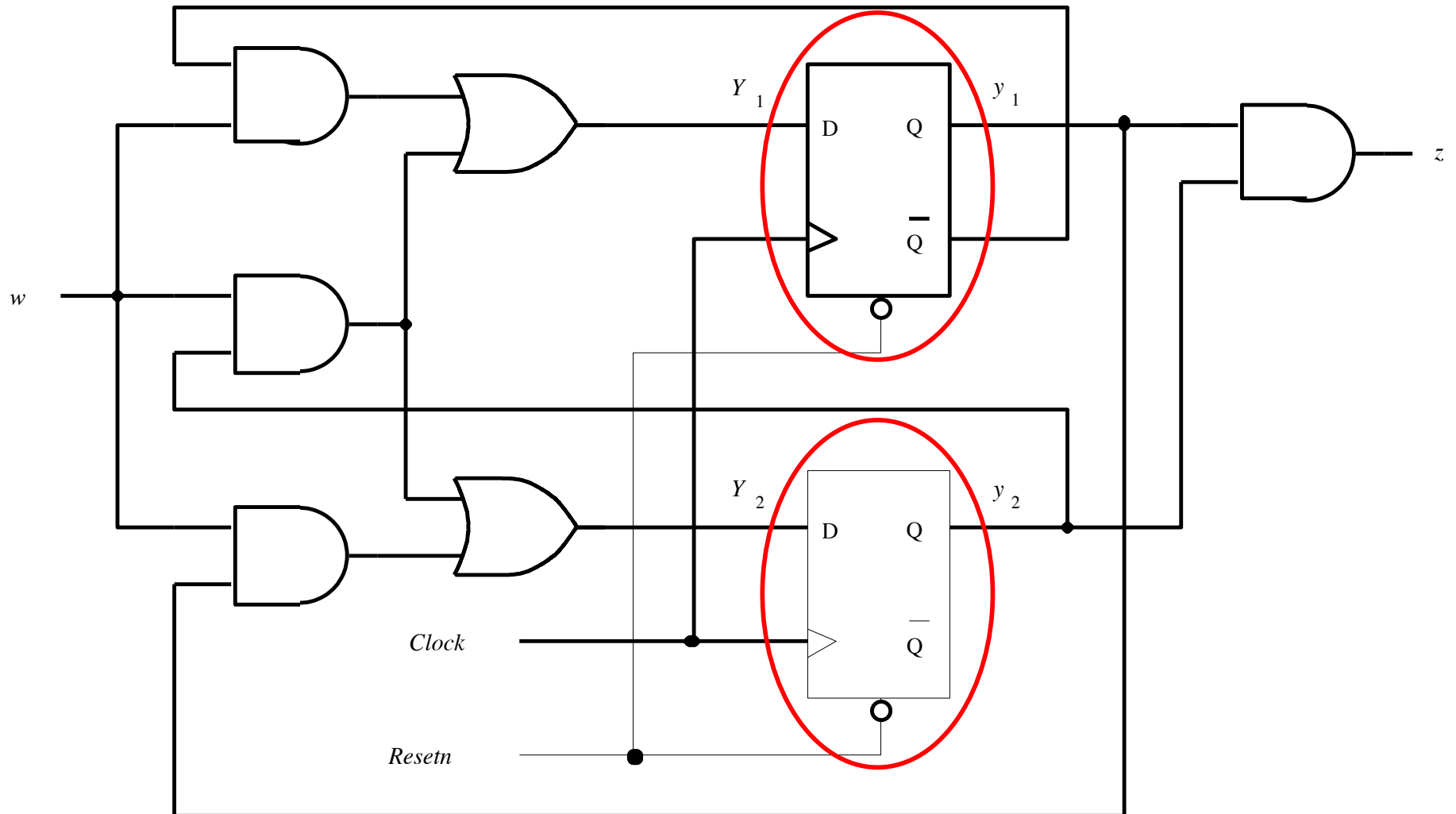




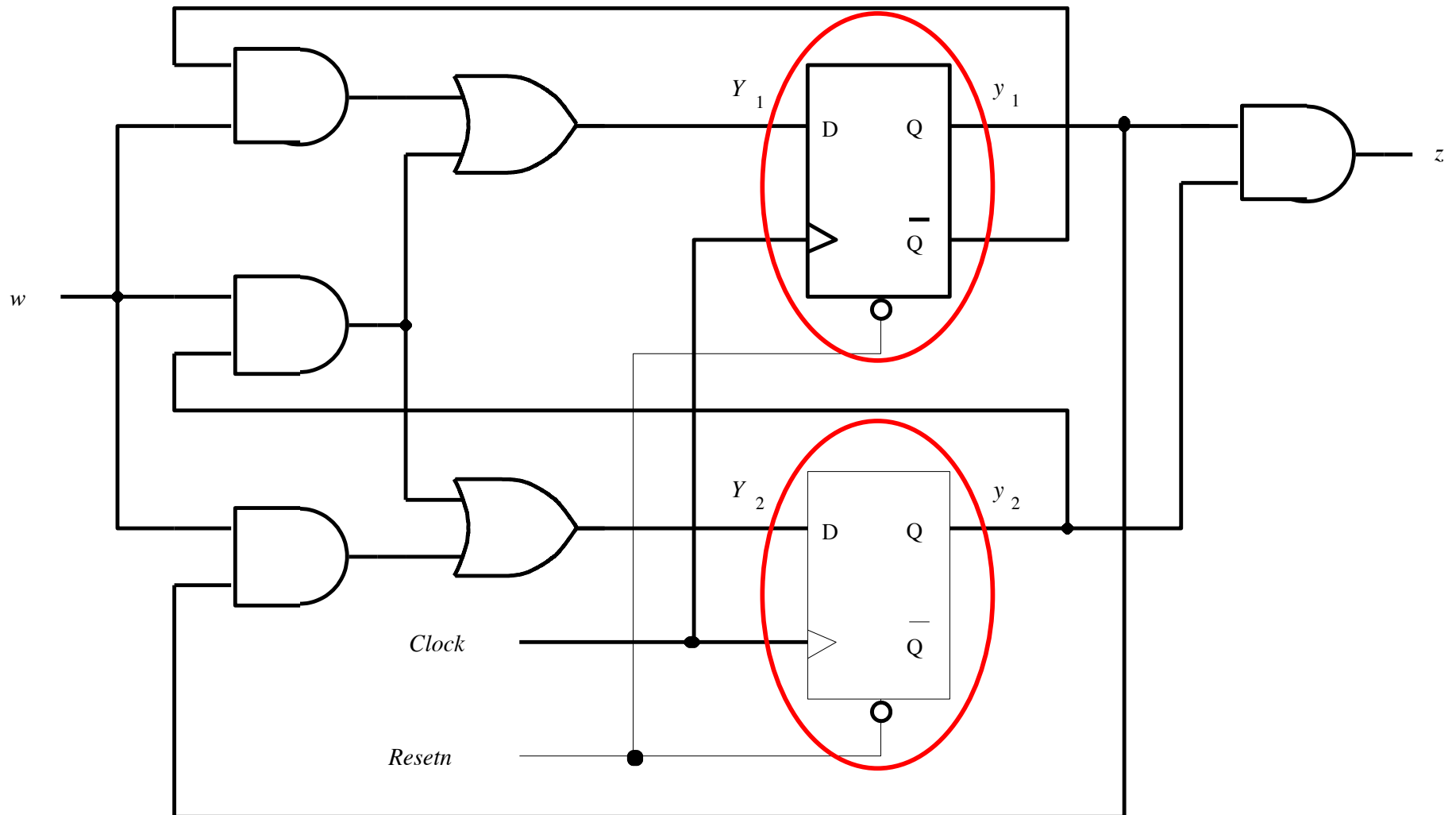
# Where are the memory elements?



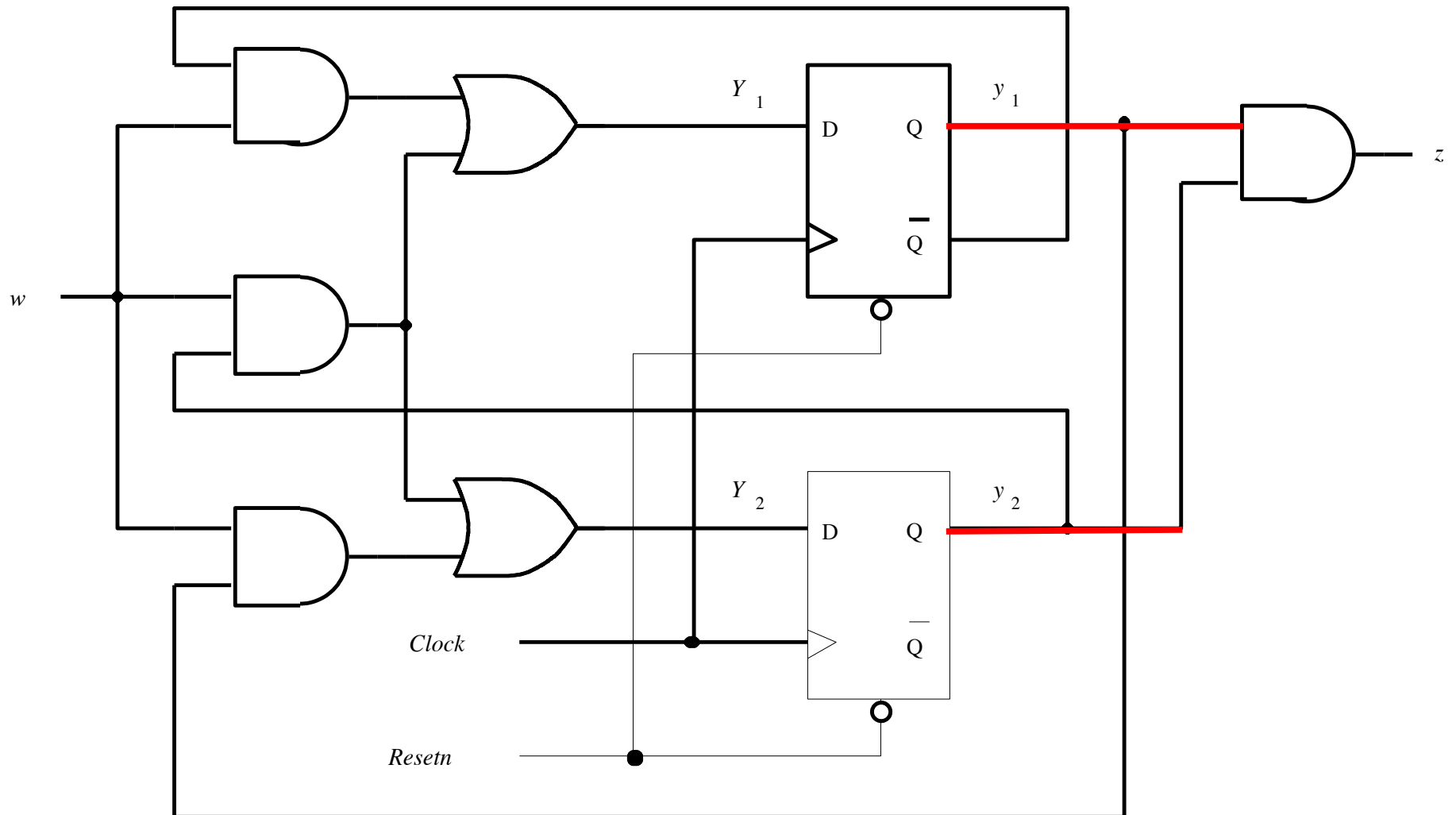
# Where are the memory elements?



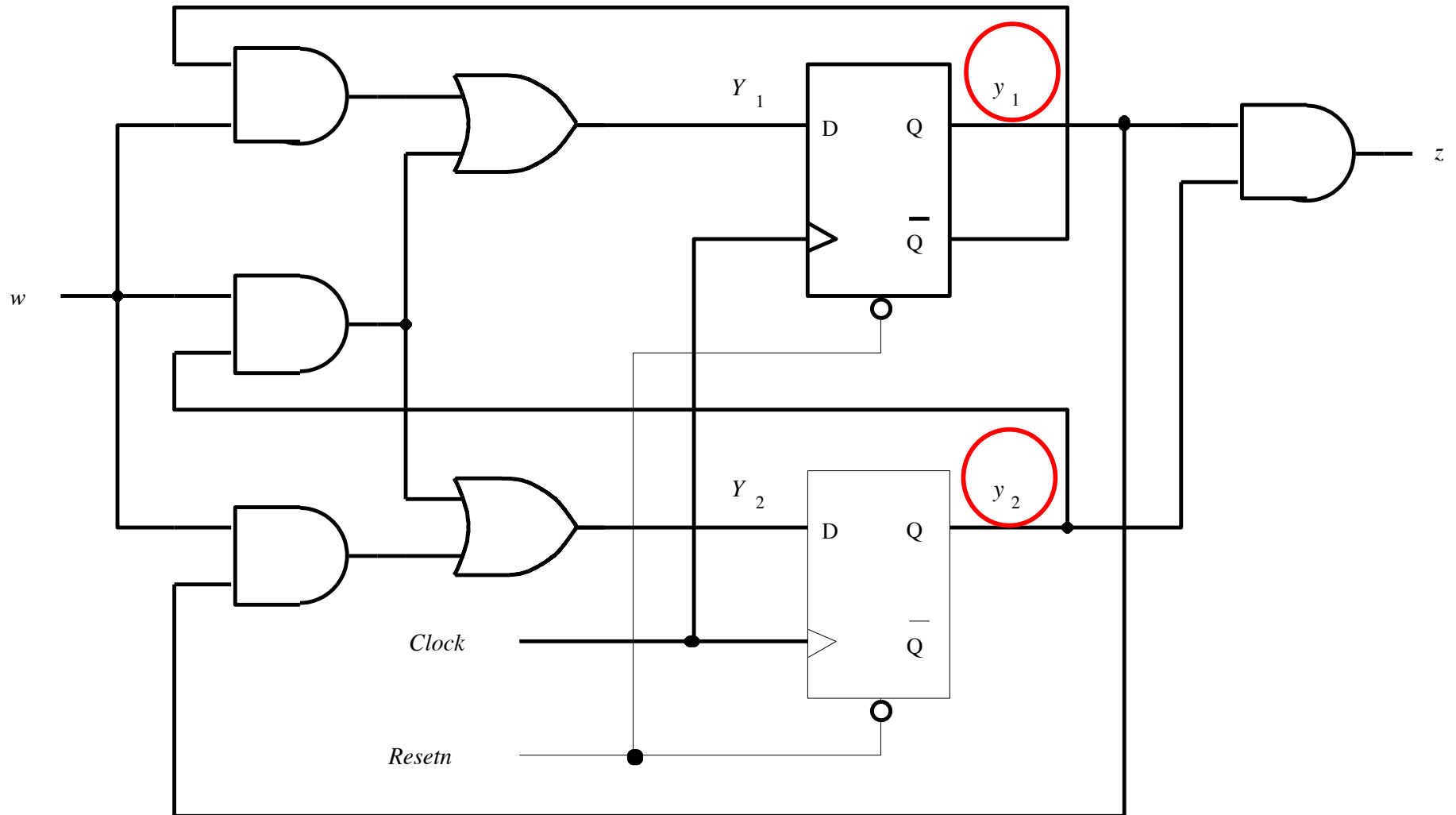
# Where are the outputs of the flip-flops?



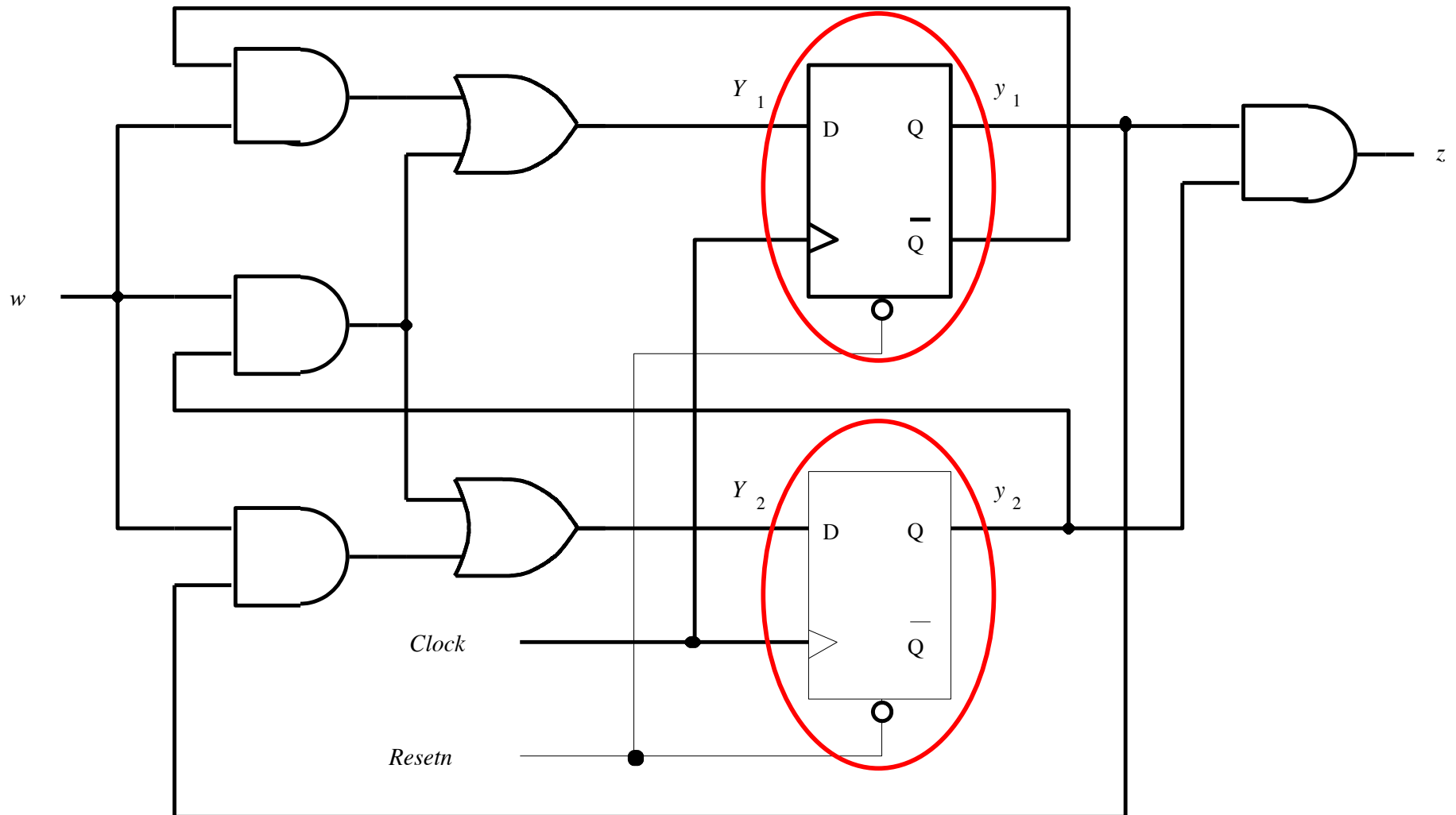
# Where are the outputs of the flip-flops?



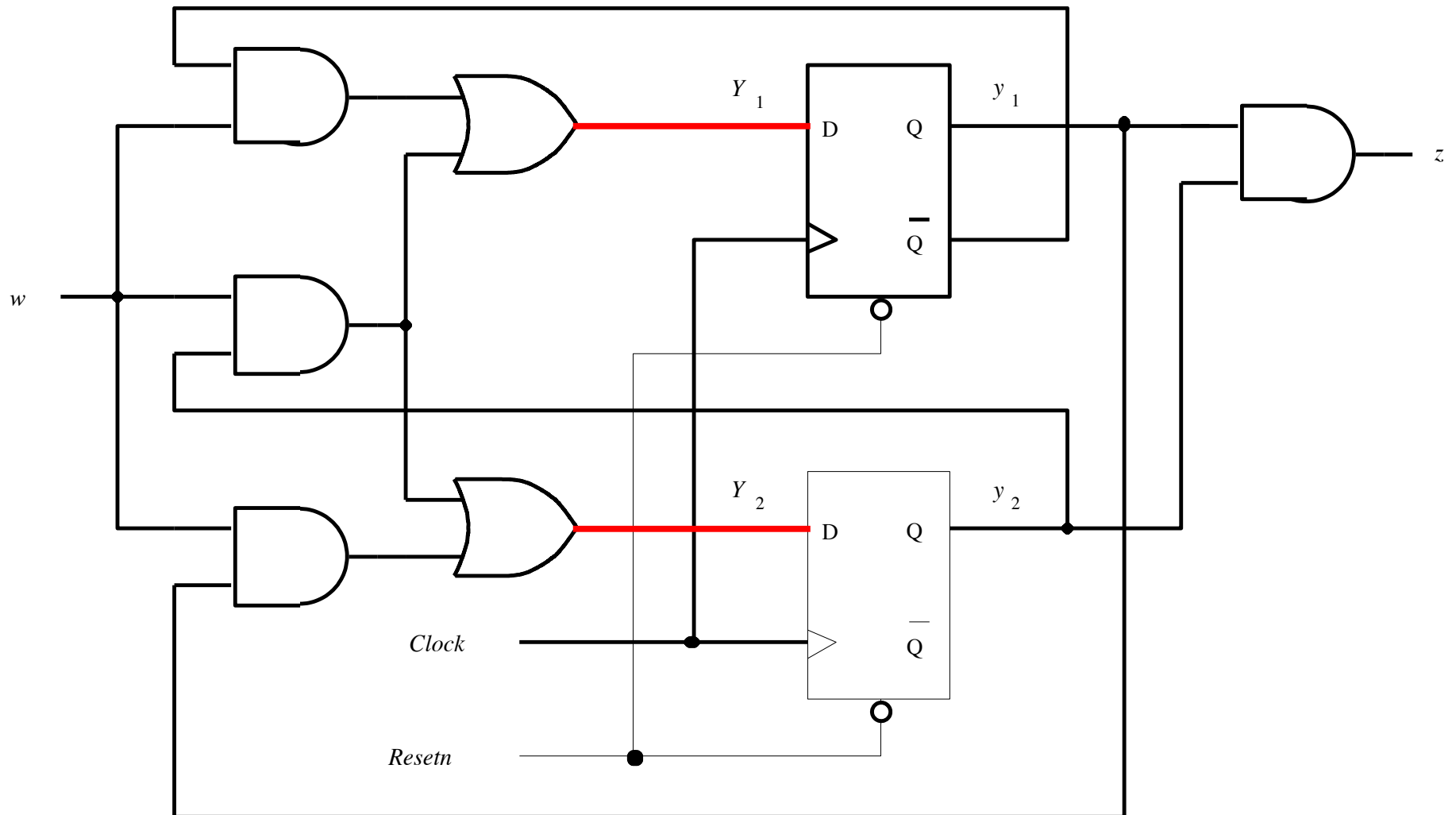
# These are the present-state variables



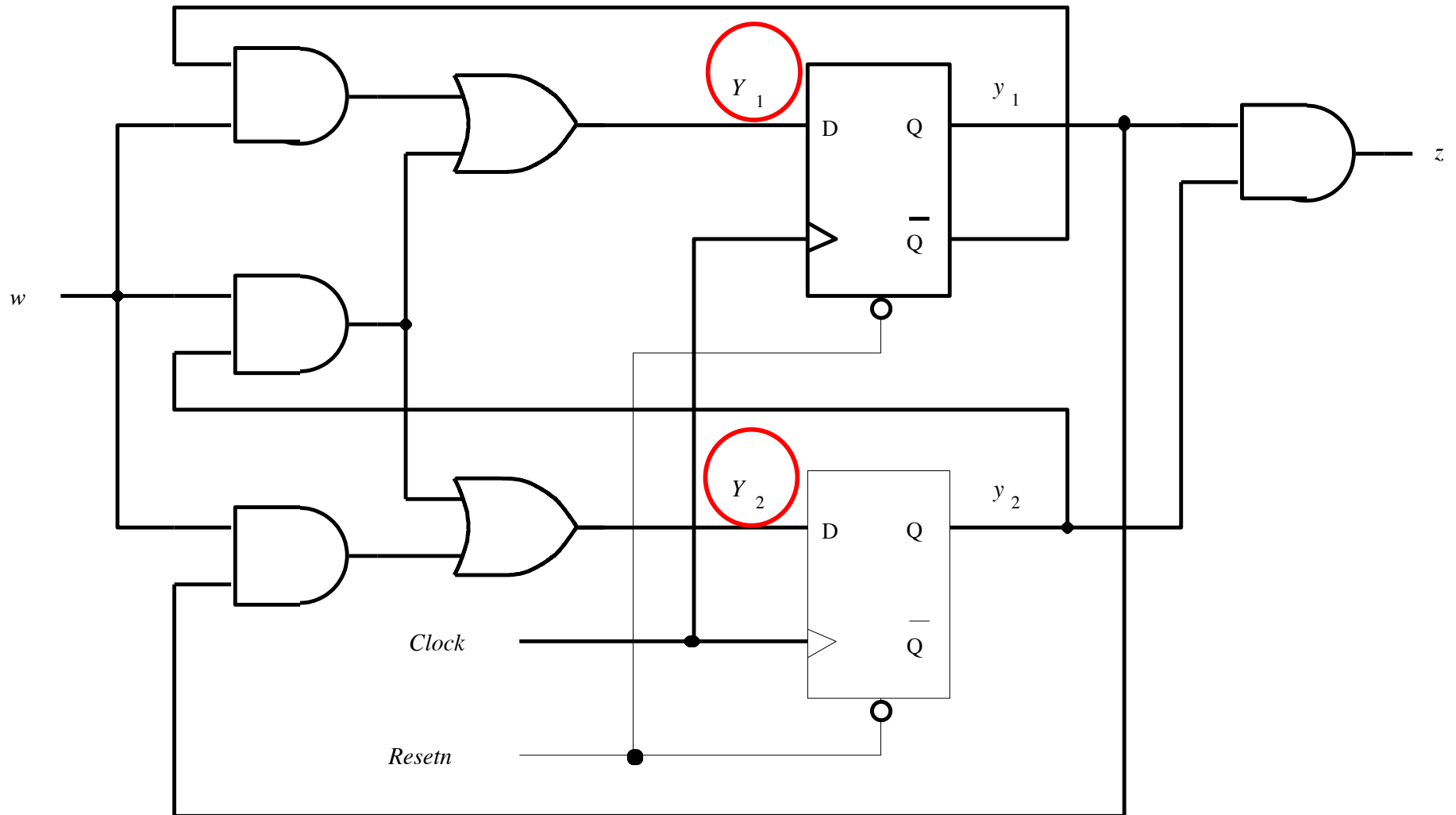
# Where are the inputs of the flip-flops?



# Where are the inputs of the flip-flops?

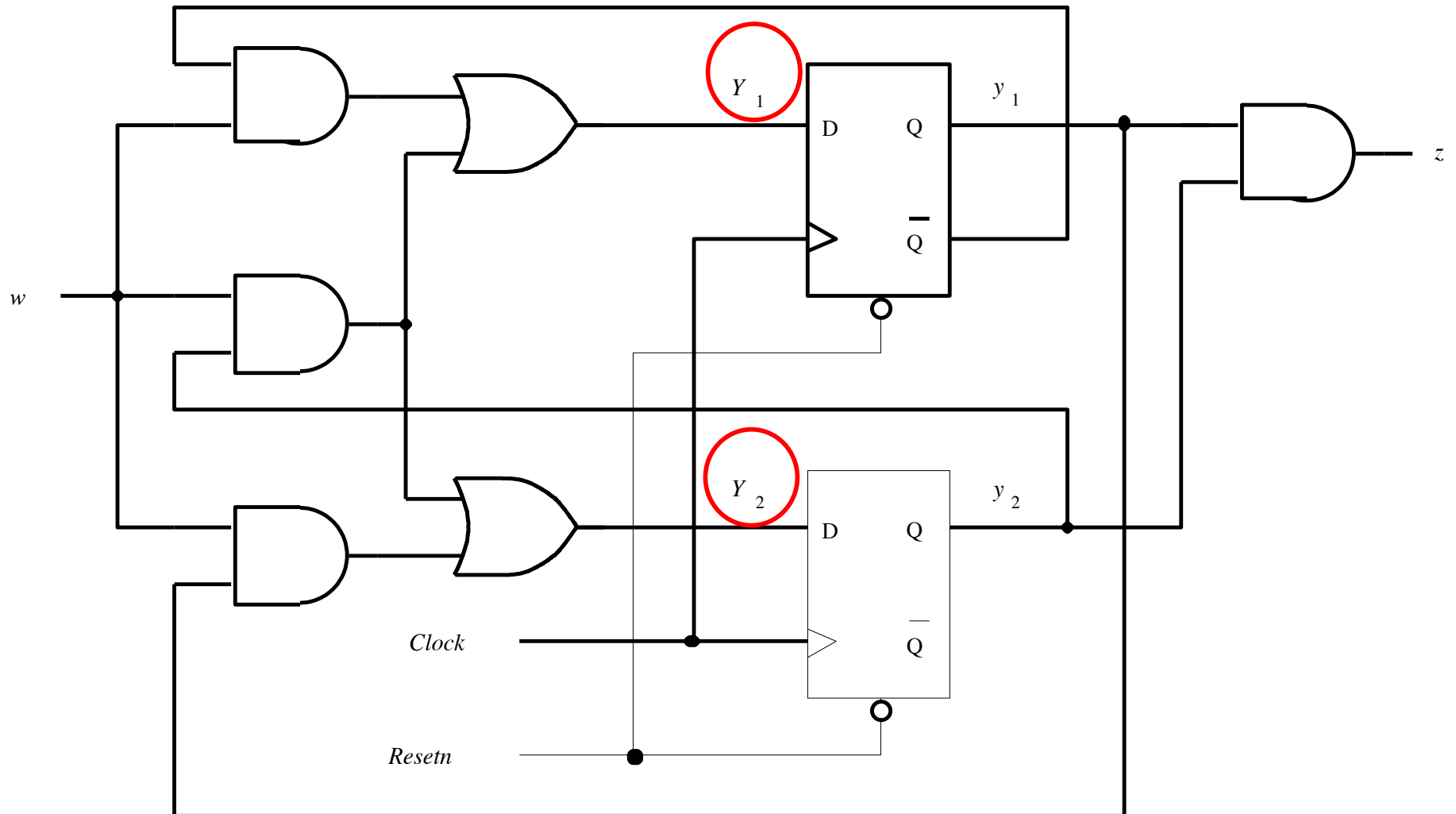


# These are the next-state variables



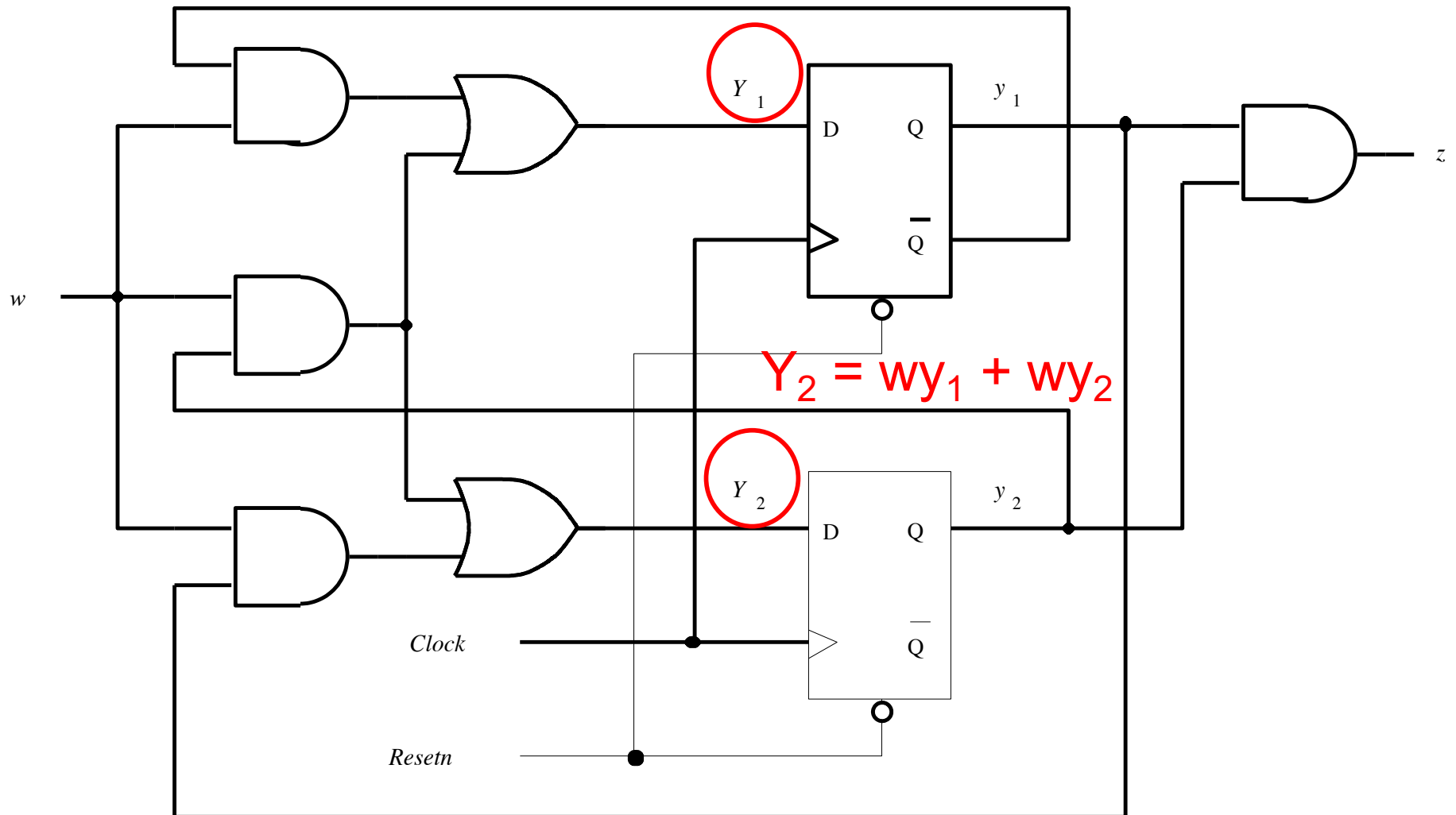


# What are their logic expressions?



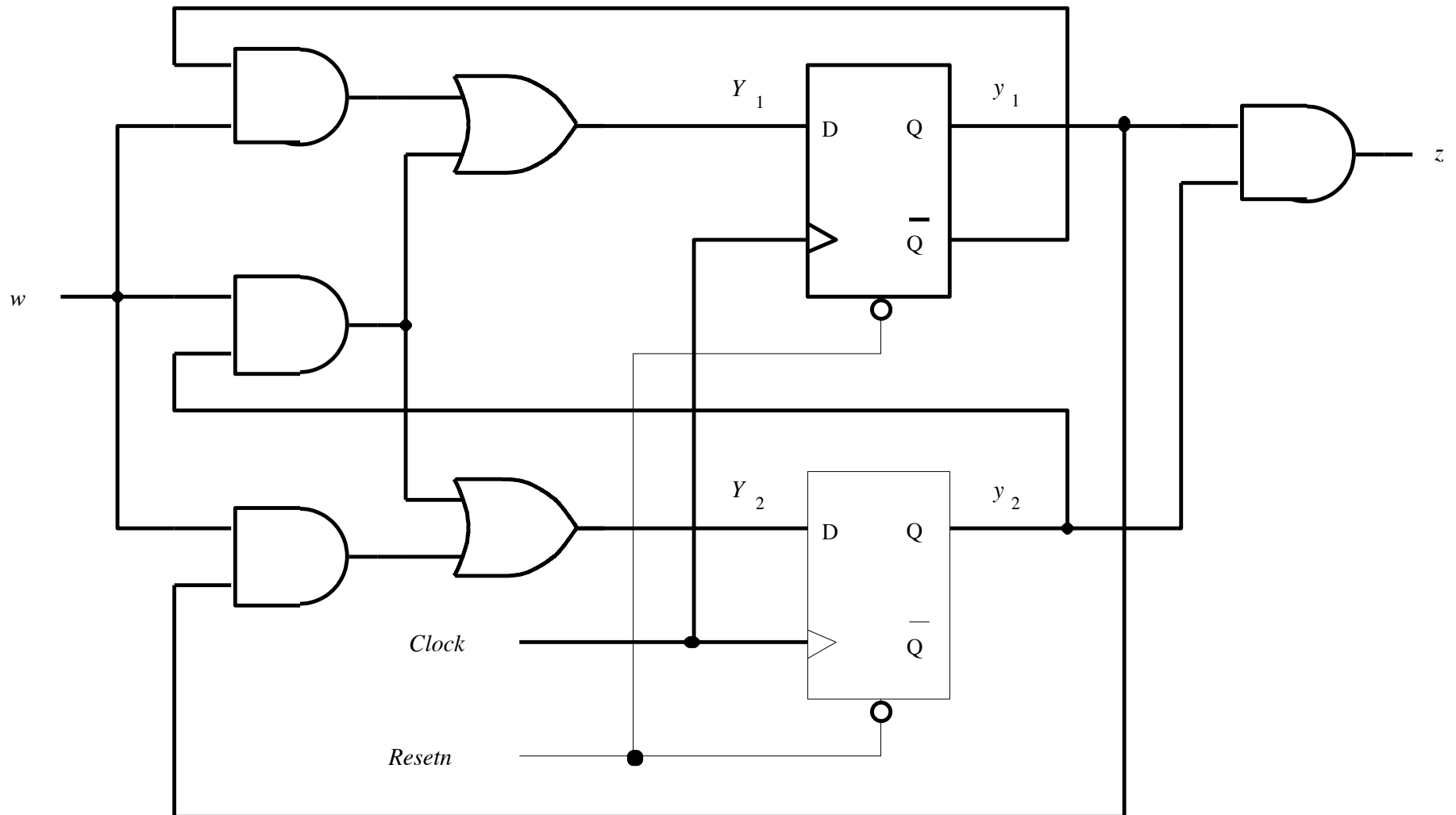
# What are their logic expressions?

$$Y_1 = w\bar{y}_1 + wy_2$$

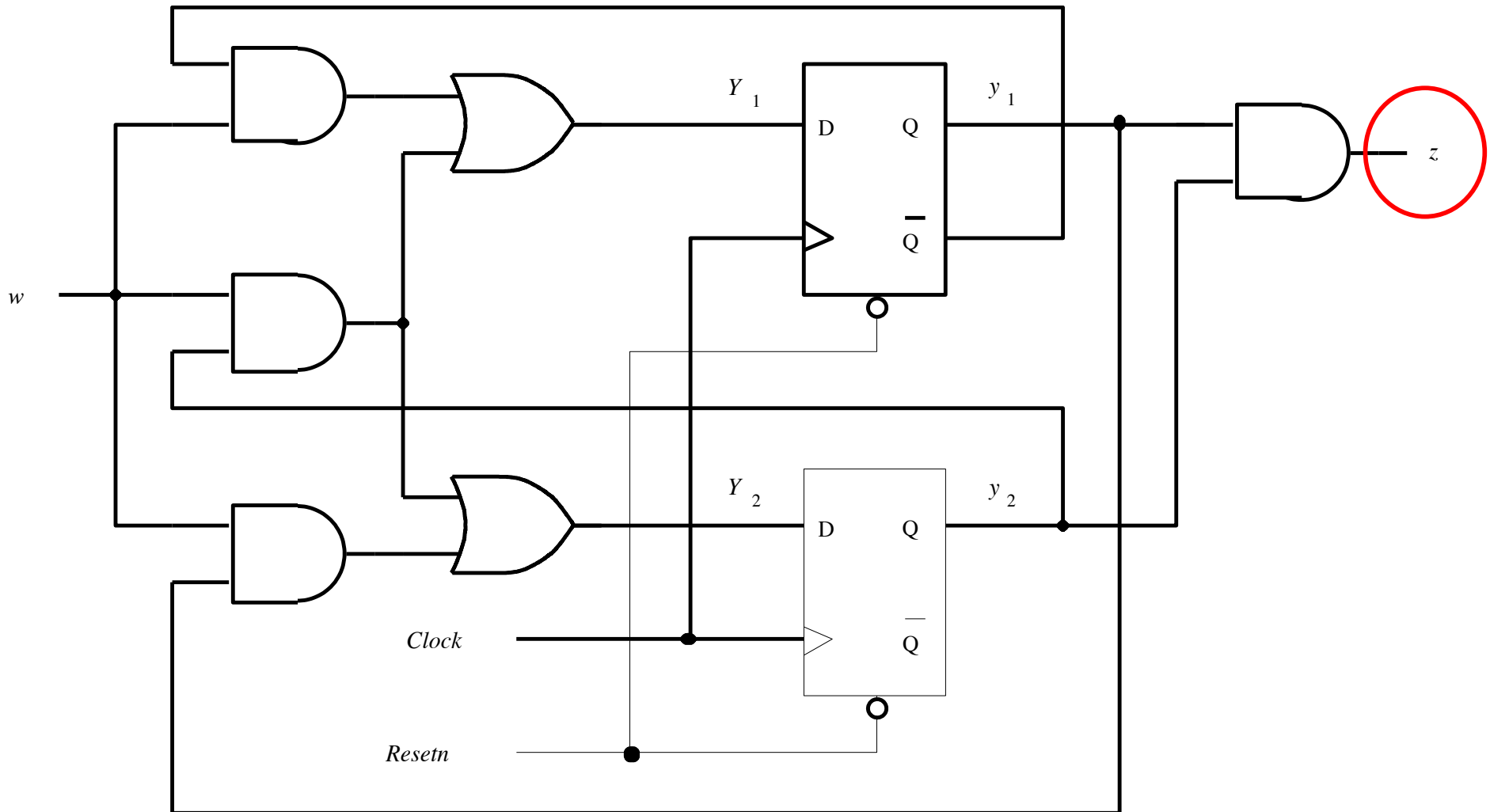


$$Y_2 = wy_1 + wy_2$$

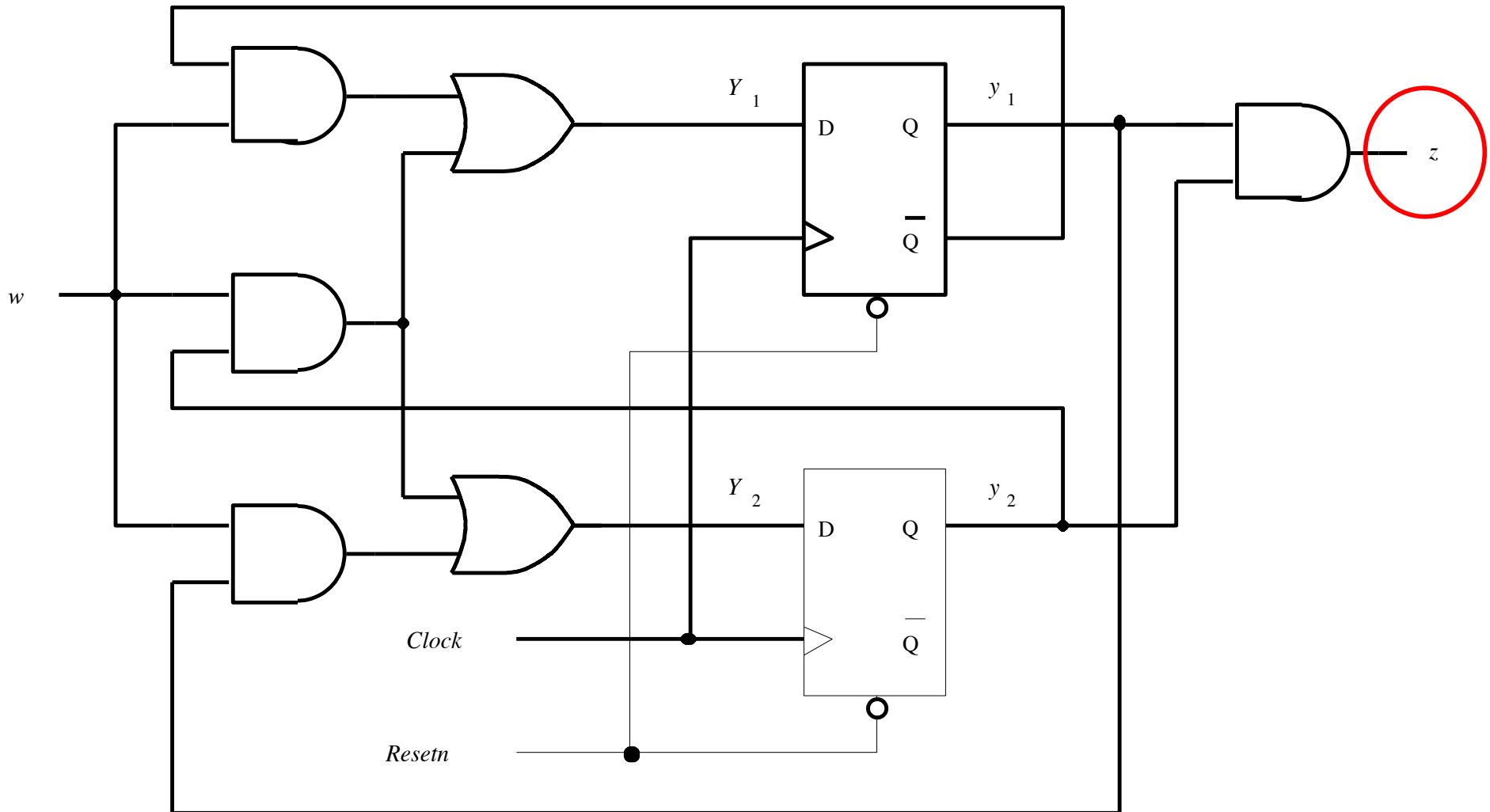
# Where is the output, again?



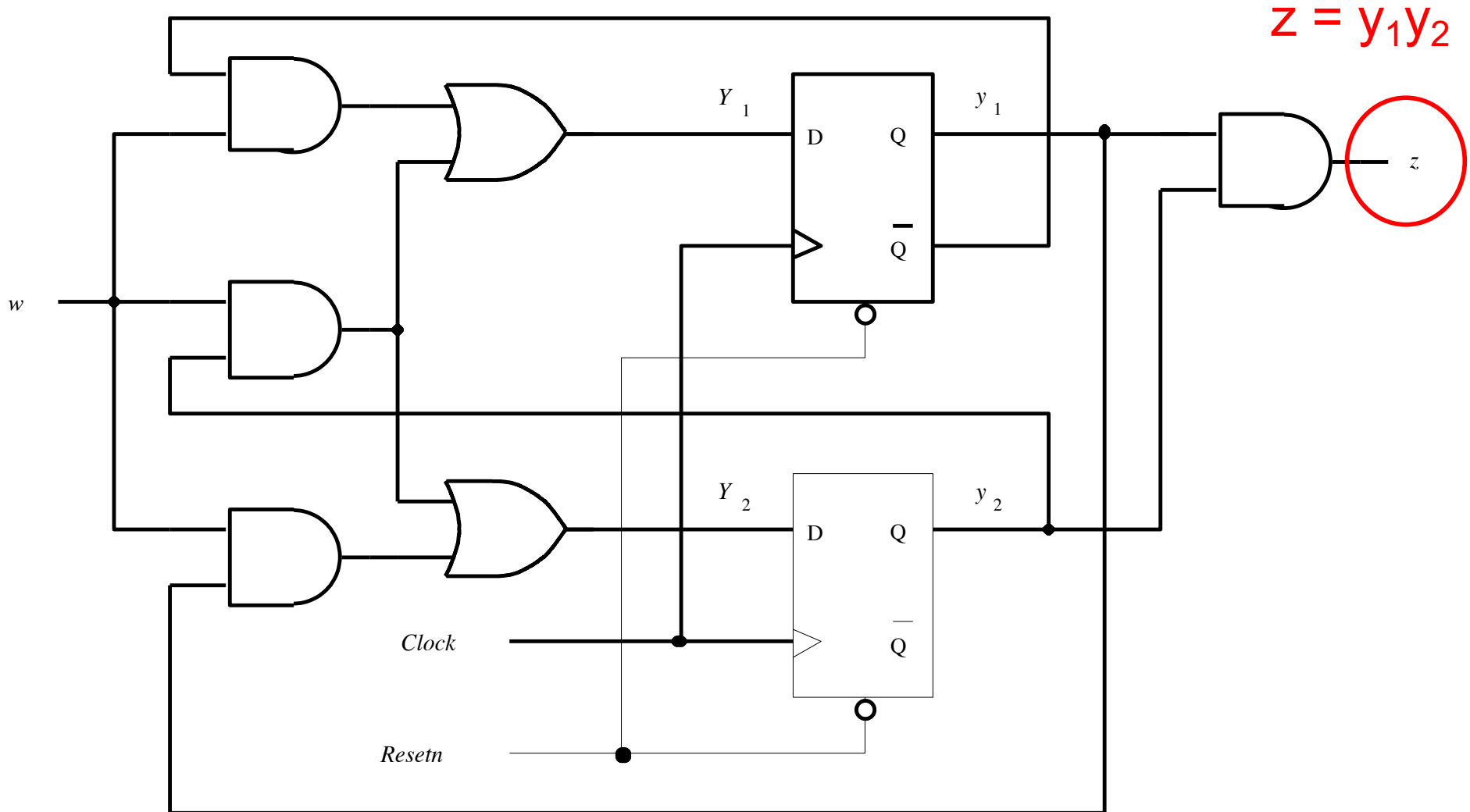
# Where is the output, again?



# What is its logic expression?



# What is its logic expression?



**This is what we have to work with now  
(we don't need the circuit anymore)**

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00			
01			
10			
11			



# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
00			
01			
10			
11			

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00			0
01			0
10			0
11			1

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
		$Y_2Y_1$	$Y_2Y_1$
00			0
01			0
10			0
11			1

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	0	1	0
01	0	0	0
10	0	1	0
11	0	1	1

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	0	1	0
01	0	0	0
10	0	1	0
11	0	1	1

# Let's derive the state-assigned table

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

# We don't need the logic expressions anymore

$$Y_1 = w\bar{y}_1 + wy_2$$

$$Y_2 = wy_1 + wy_2$$

$$z = y_1y_2$$

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

# We don't need the logic expressions anymore

Present state $y_2y_1$	Next State		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state $y_2y_1$	Next State		Output z
	w = 0	w = 1	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state $y_2y_1$	Next State		Output z
	w = 0	w = 1	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

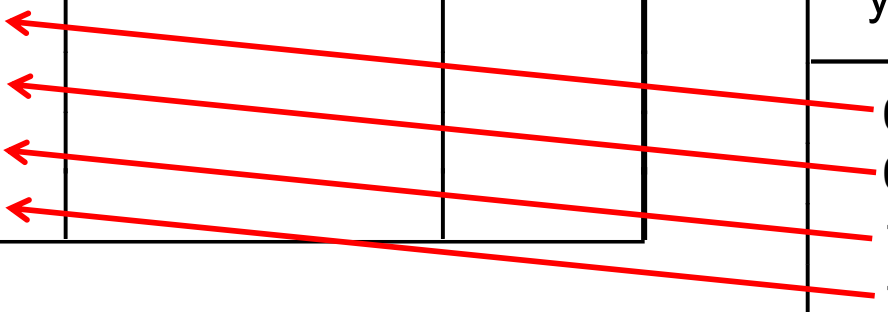
# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

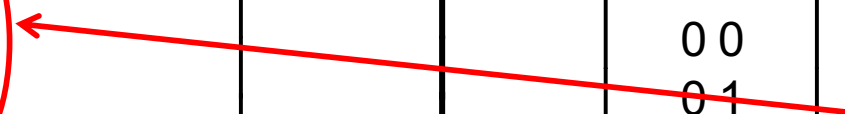
# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		
B	A		
C	A		
D	A		

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state	Next State		Output z
	w = 0	w = 1	
$y_2y_1$	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state $y_2y_1$	Next State		Output z
	w = 0	w = 1	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	
B	A	C	
C	A	D	
D	A	D	

State table

Present state $y_2y_1$	Next State		Output z
	w = 0	w = 1	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

The output is the same in both tables

# The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

# We don't need the state-assigned table anymore

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state $y_2y_1$	Next State		Output z
	w = 0 $Y_2Y_1$	w = 1 $Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	11	0
11	00	11	1

State-assigned table

# We don't need the state-assigned table anymore

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

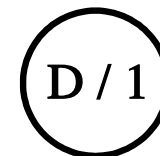
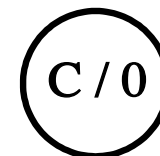
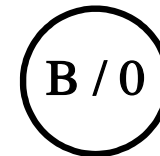
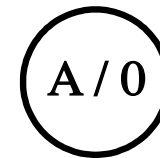
State table

# Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

# Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1



Because this is a Moore machine the output is tied to the state

# Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

A / 0

B / 0

C / 0

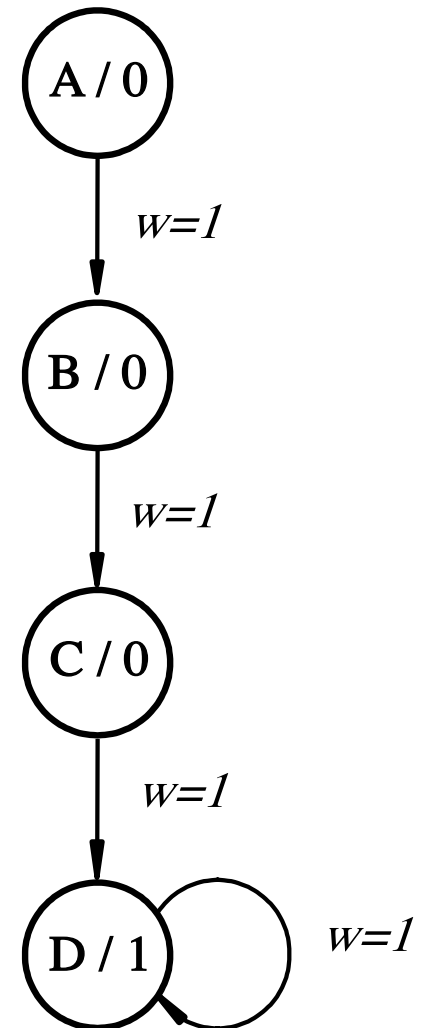
D / 1

All transitions when the input w is equal to 1

# Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

All transitions when the input w is equal to 1

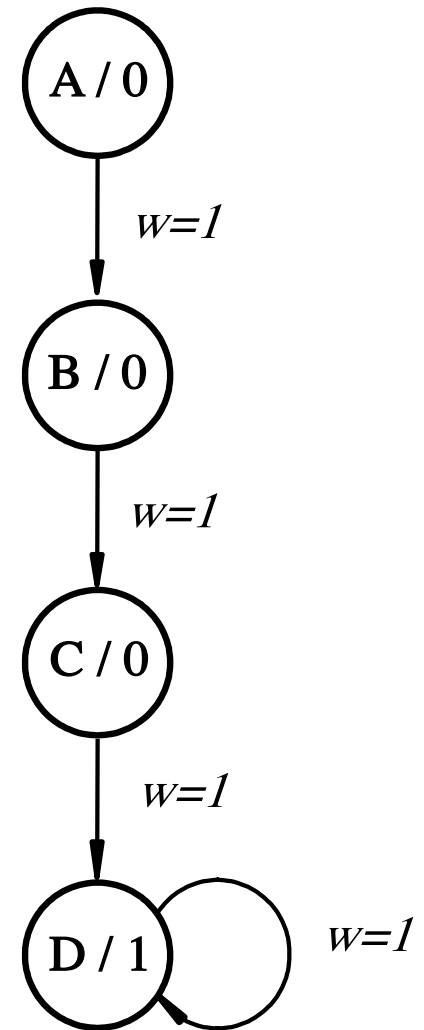




# Let's Draw the State Diagram

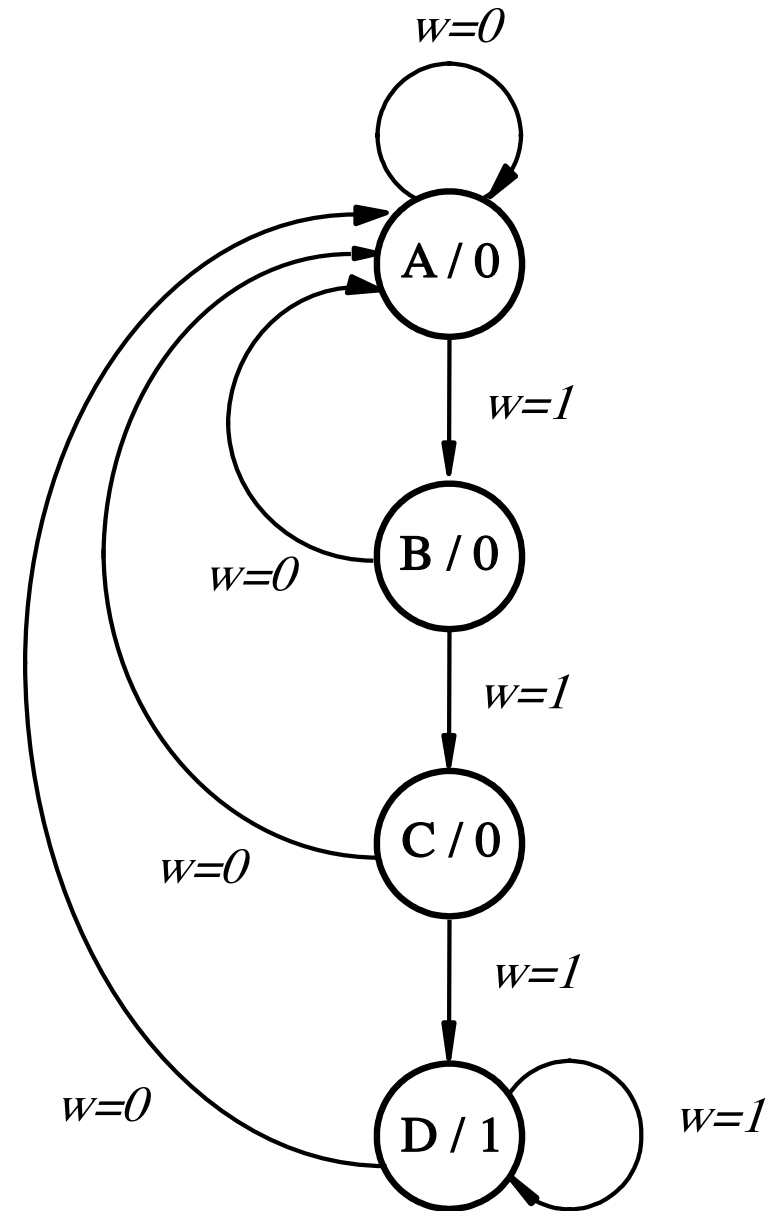
Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

All transitions when the input w is equal to 0



# Let's Draw the State Diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

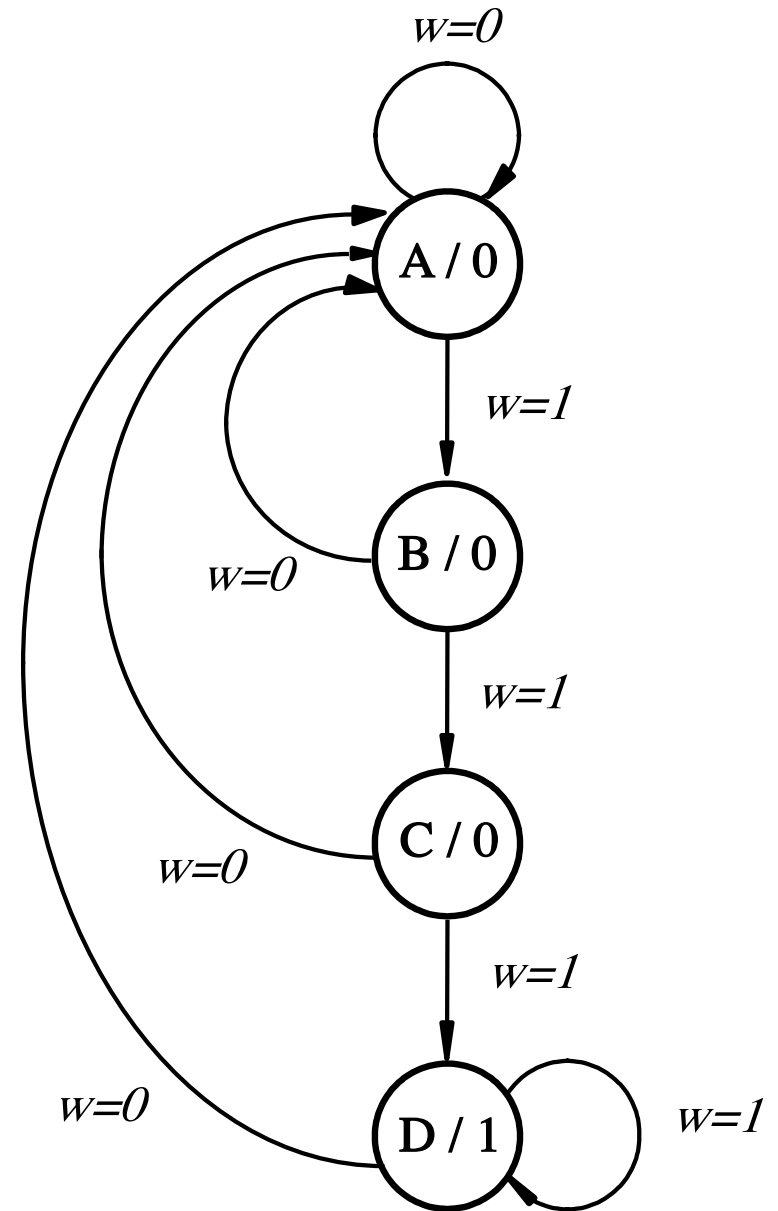


All transitions when the input  $w$  is equal to 0

# We are done!

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

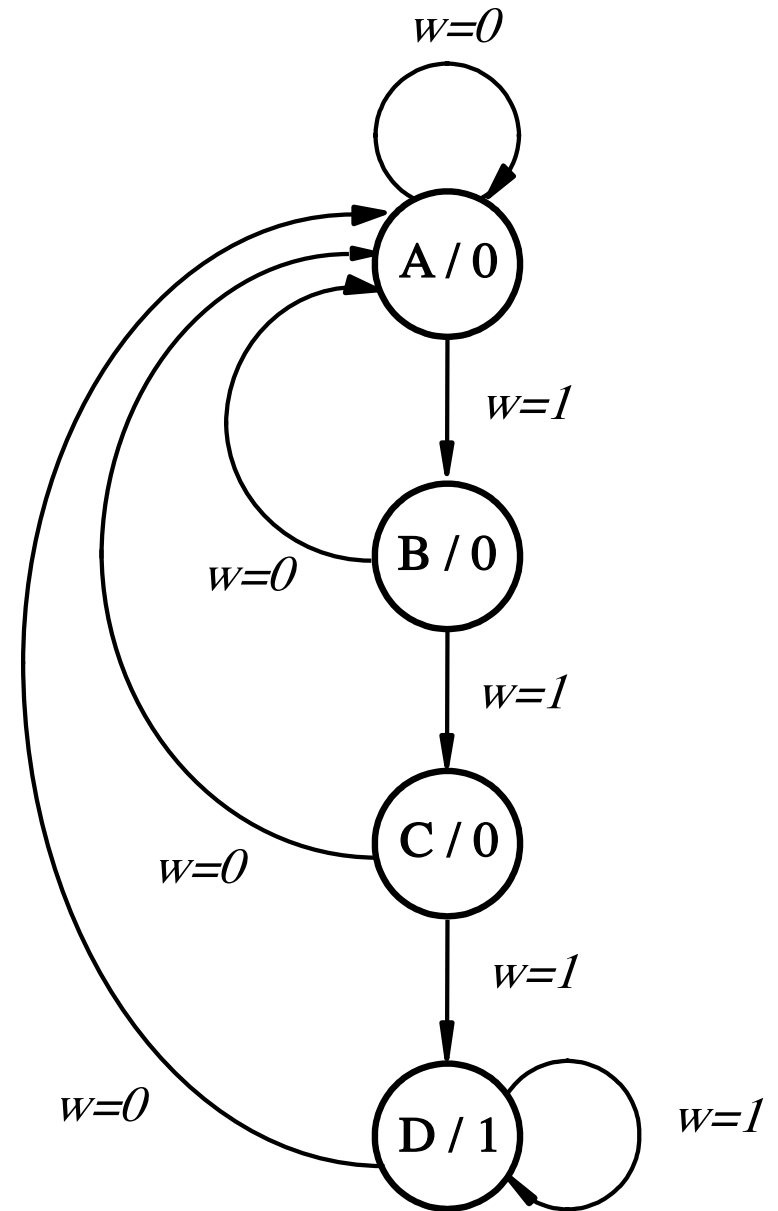


State diagram

# Almost done. What does this FSM do?

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



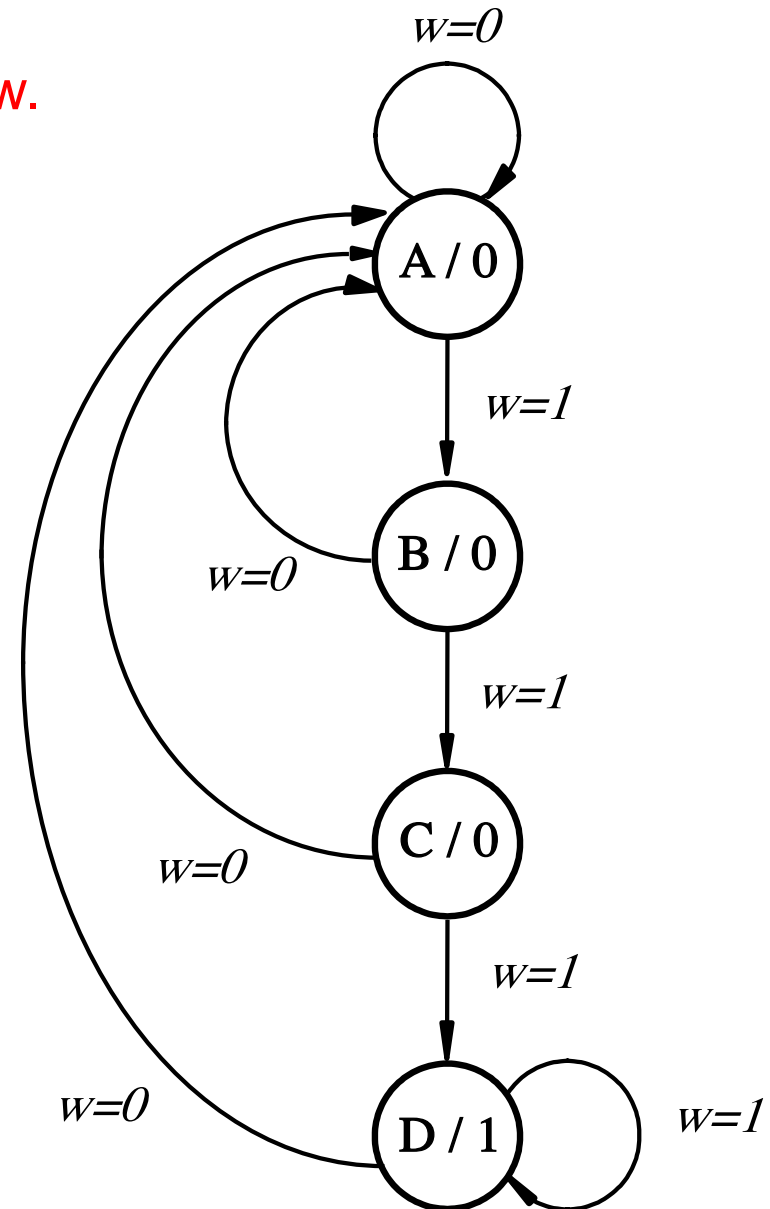
State diagram

# Almost done. What does this FSM do?

It sets the output  $z$  to 1 when three consecutive 1's occur on the input  $w$ . In other words, it is a sequence detector for the input pattern 111.

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

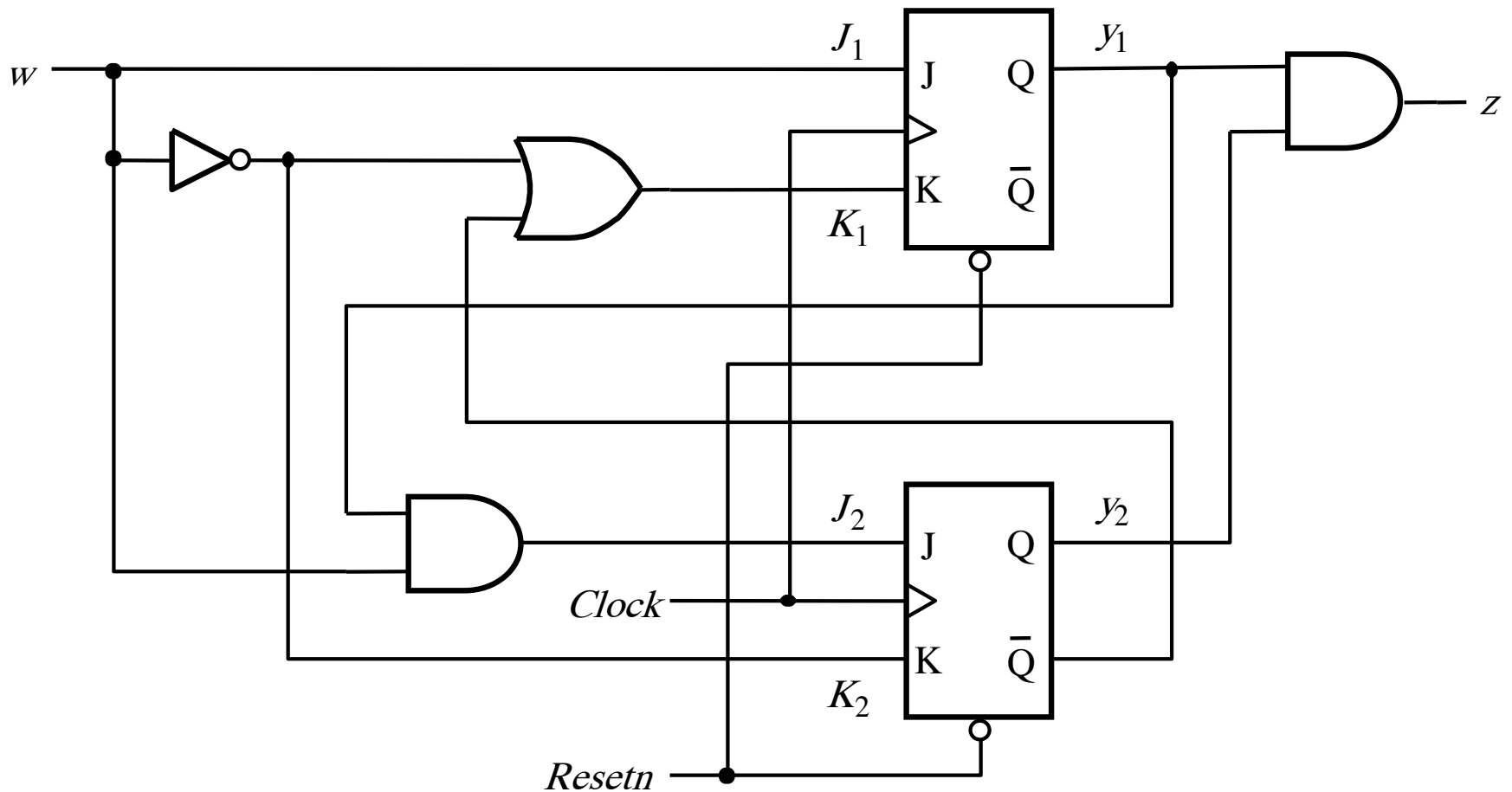
State table



State diagram

# **Another Example (with JK flip-flops)**

# What does this circuit do?



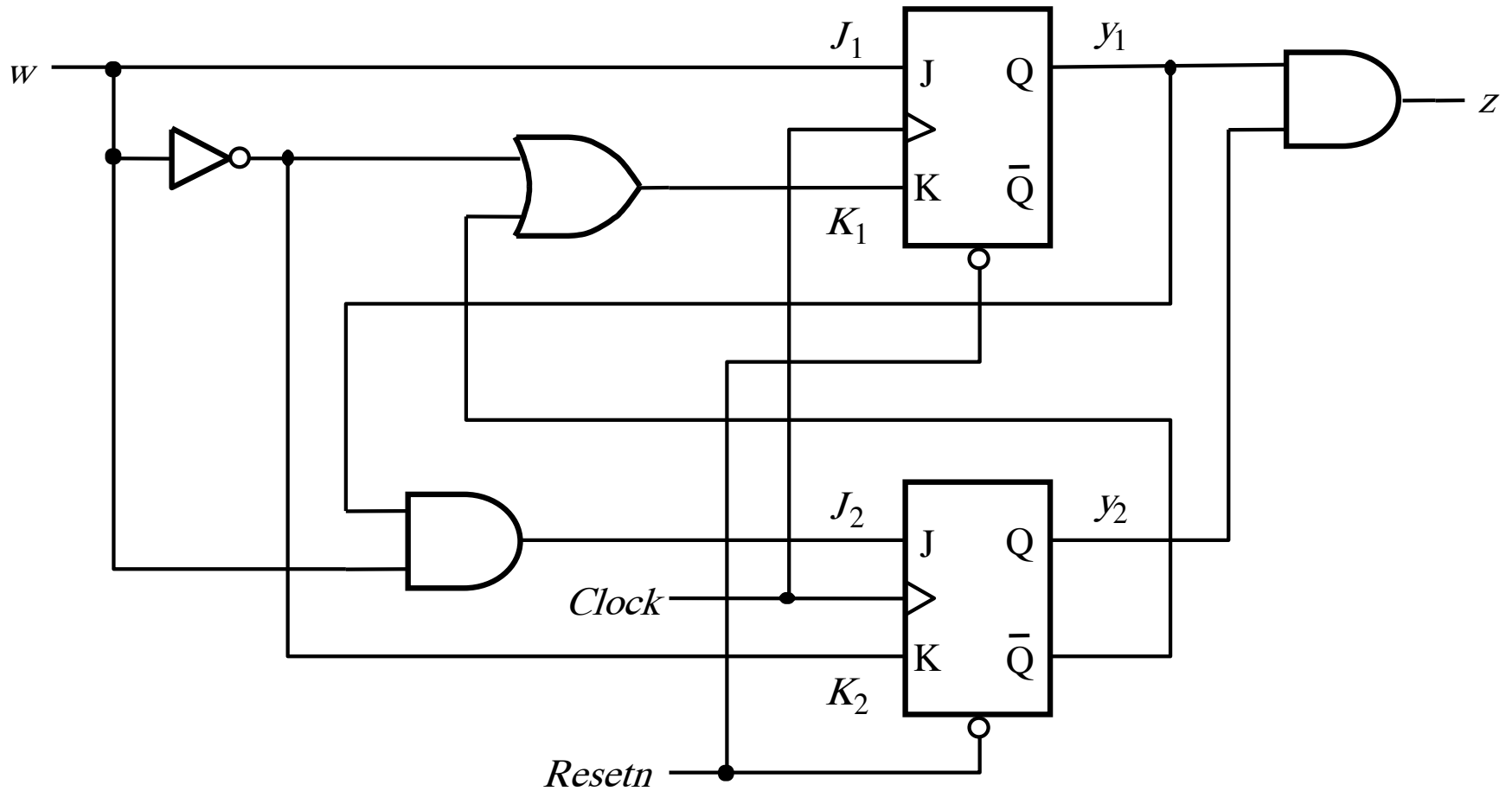
[ Figure 6.77 from the textbook ]

# Approach

- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

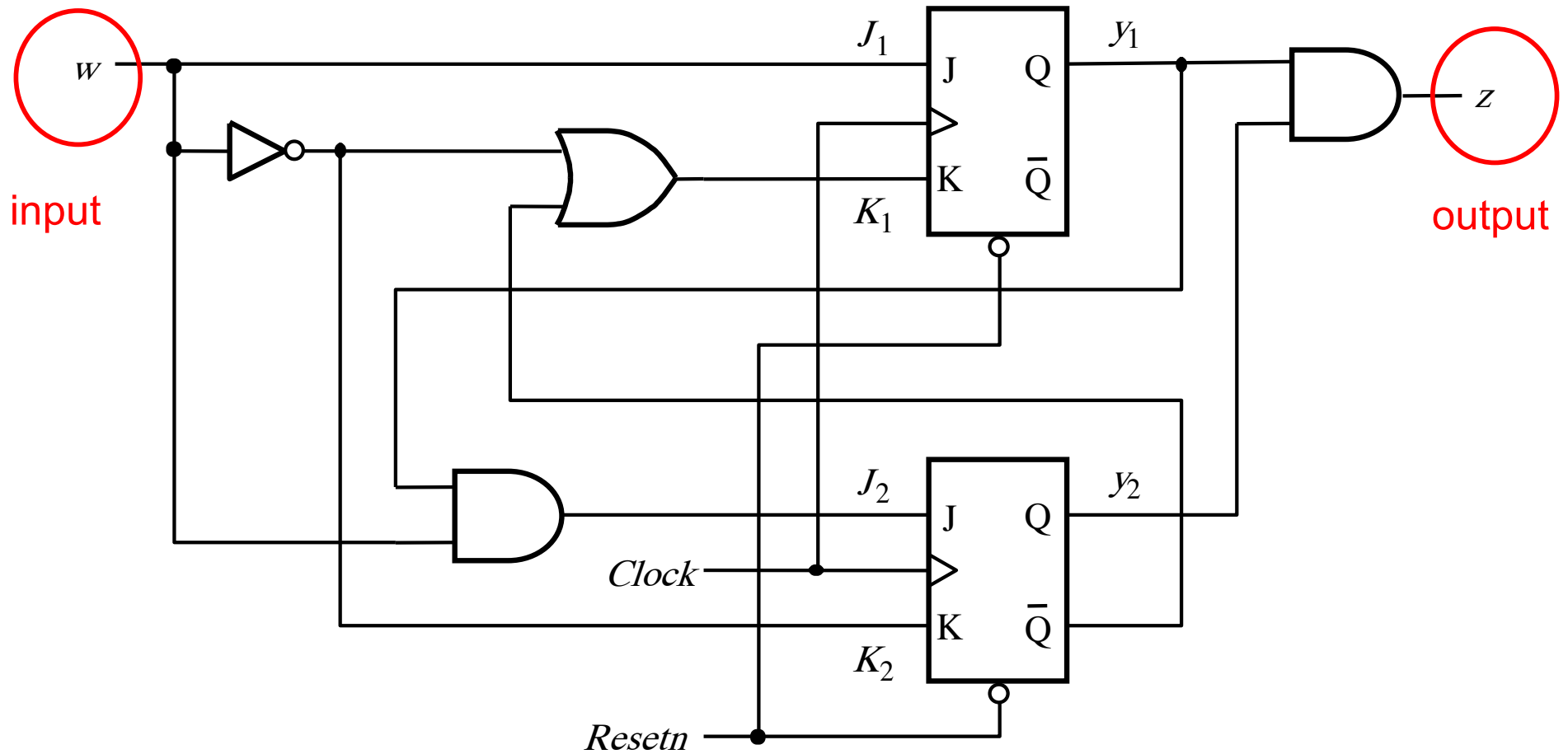


# Where are the inputs and outputs?

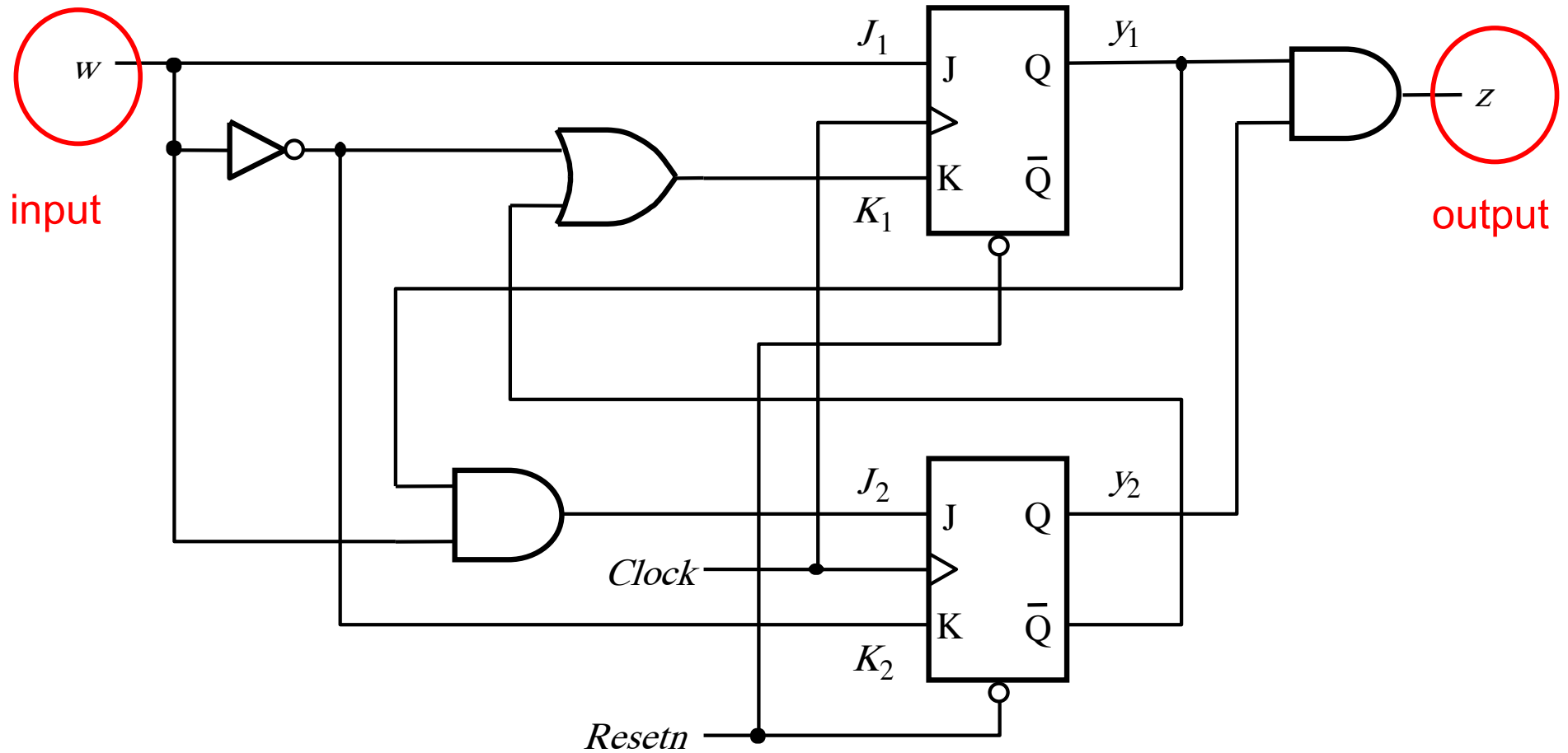


[ Figure 6.77 from the textbook ]

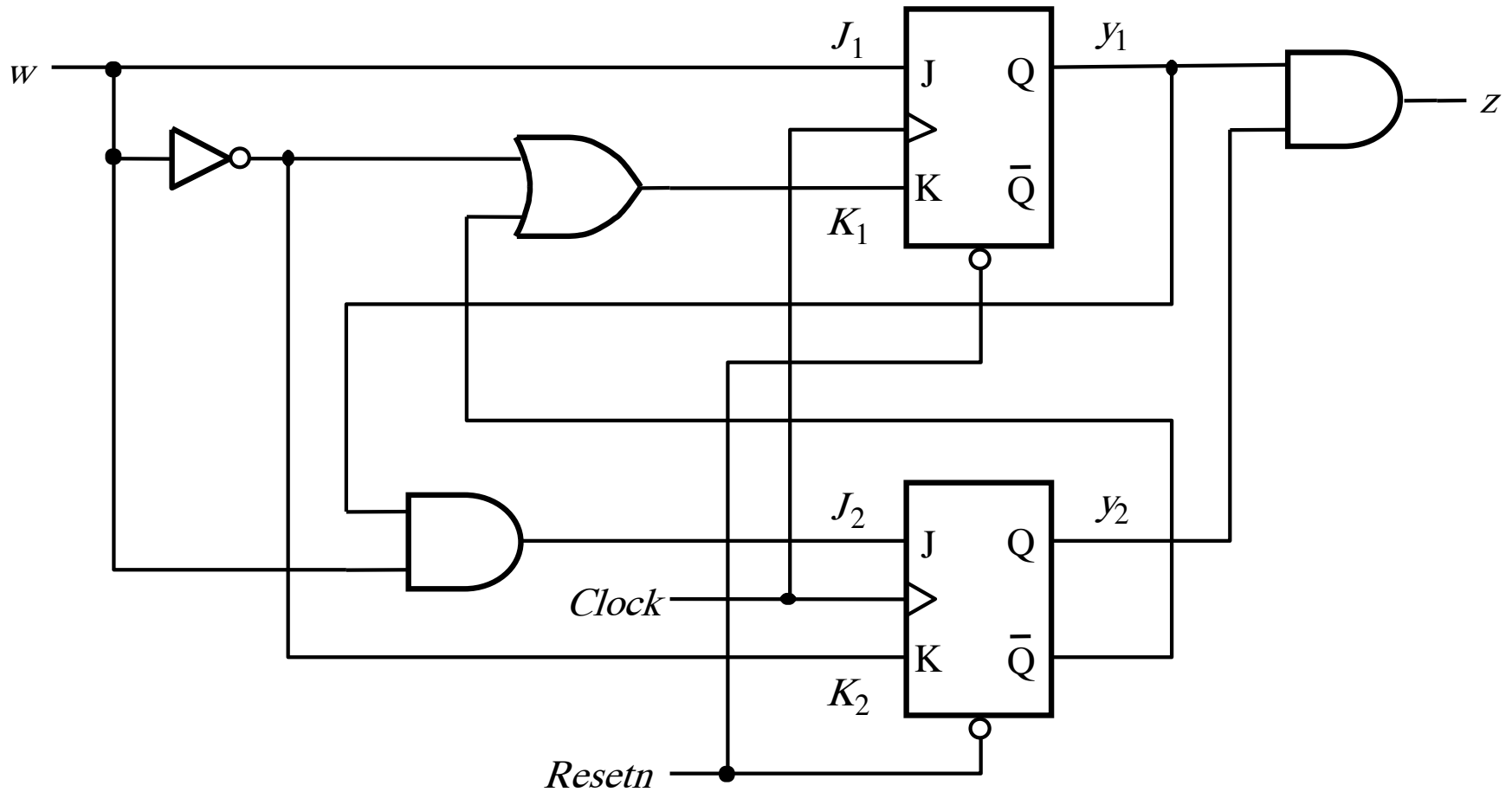
# Where are the inputs and outputs?



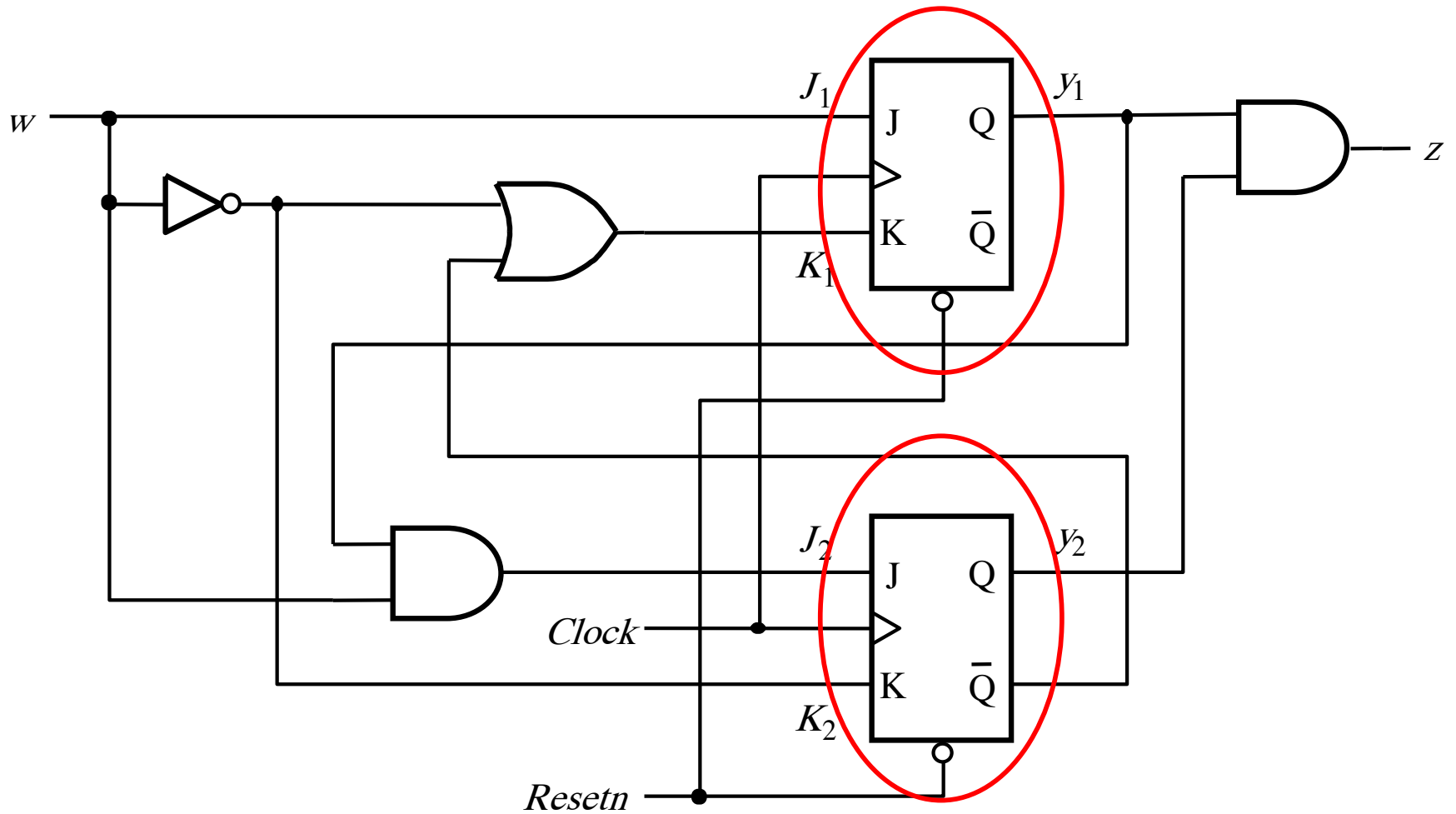
# What kind of machine is this?



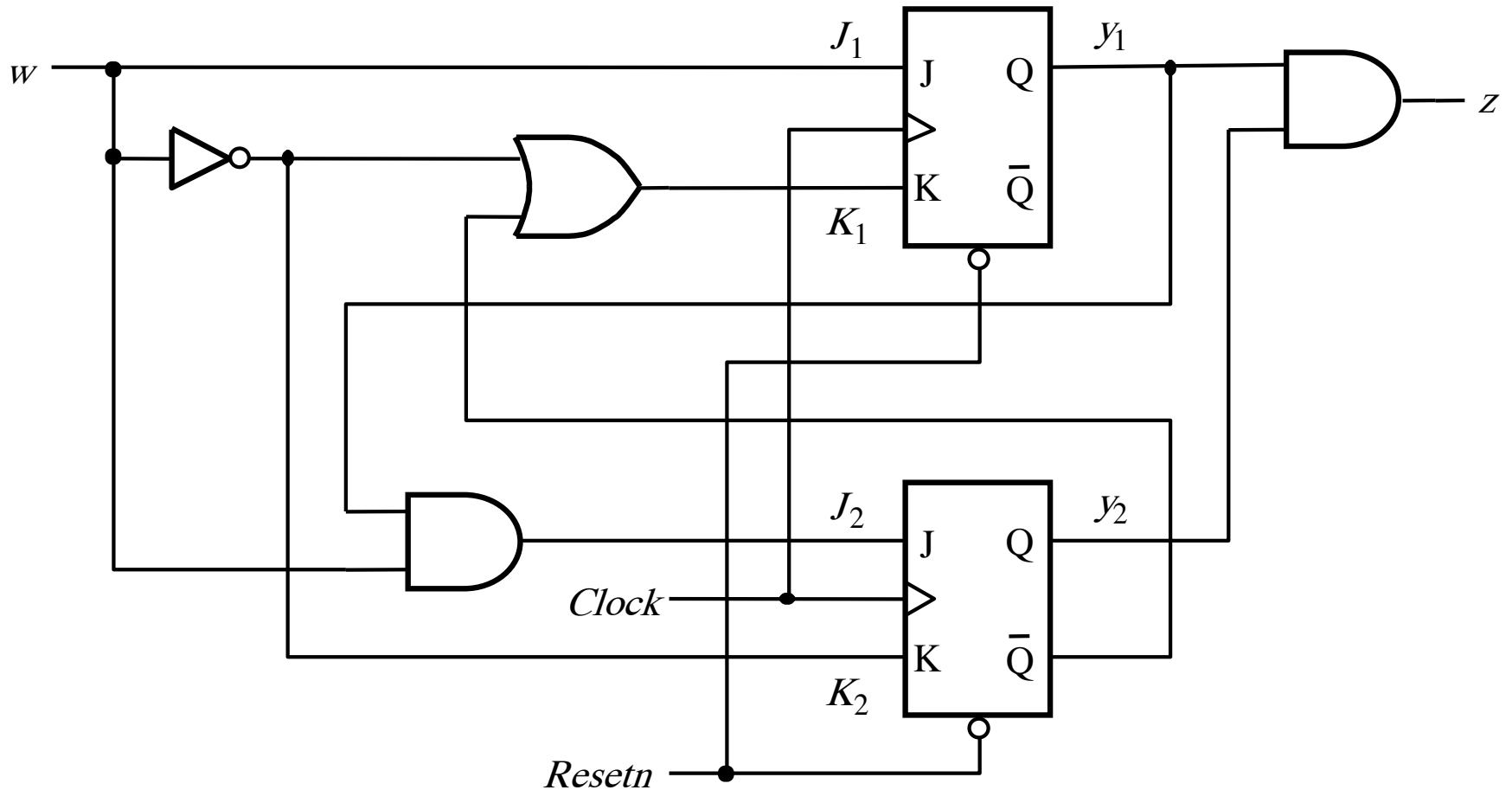
# Where are the flip-flops?



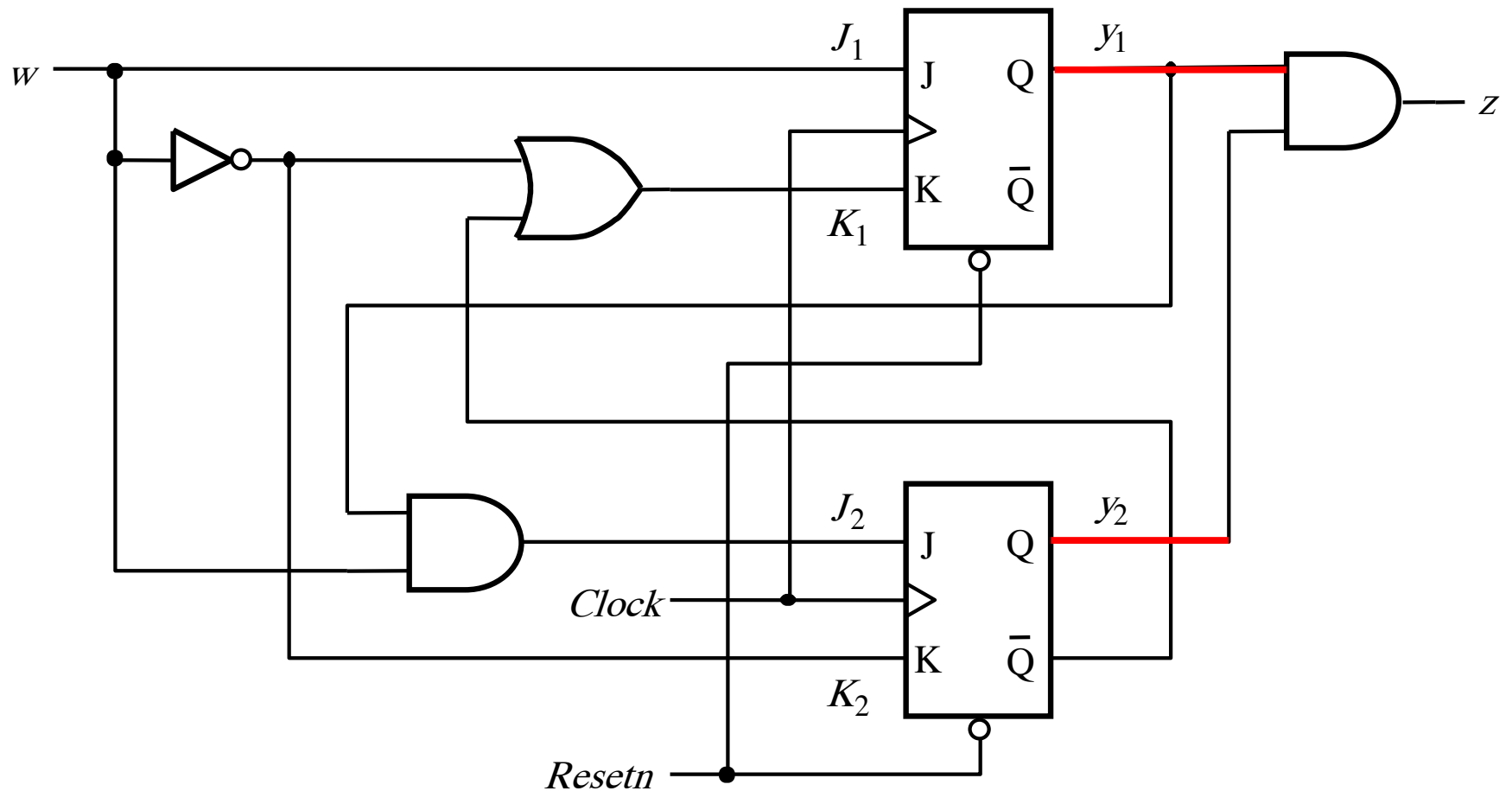
# Where are the flip-flops?



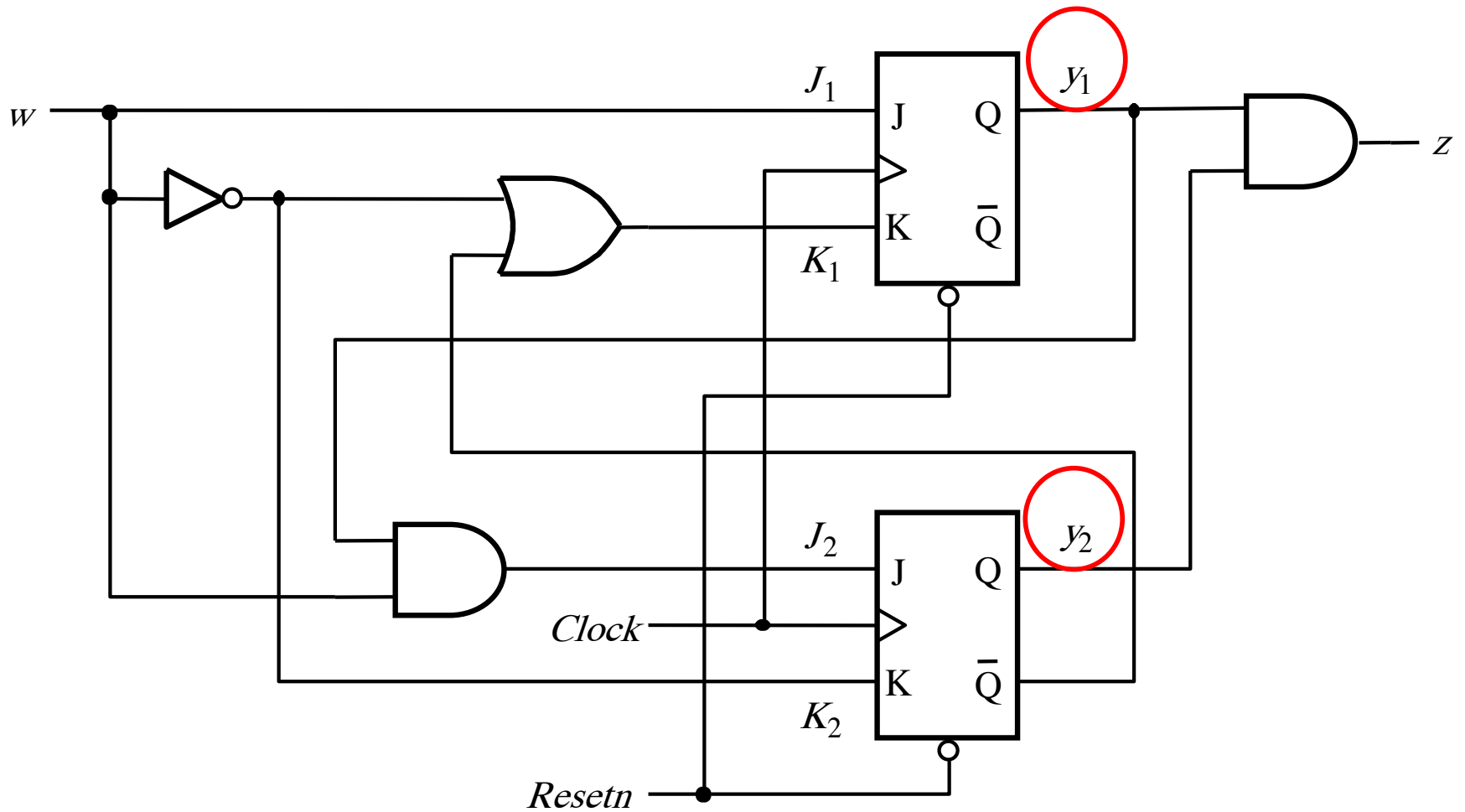
# Where are the outputs of the flip-flops?



# Where are the outputs of the flip-flops?

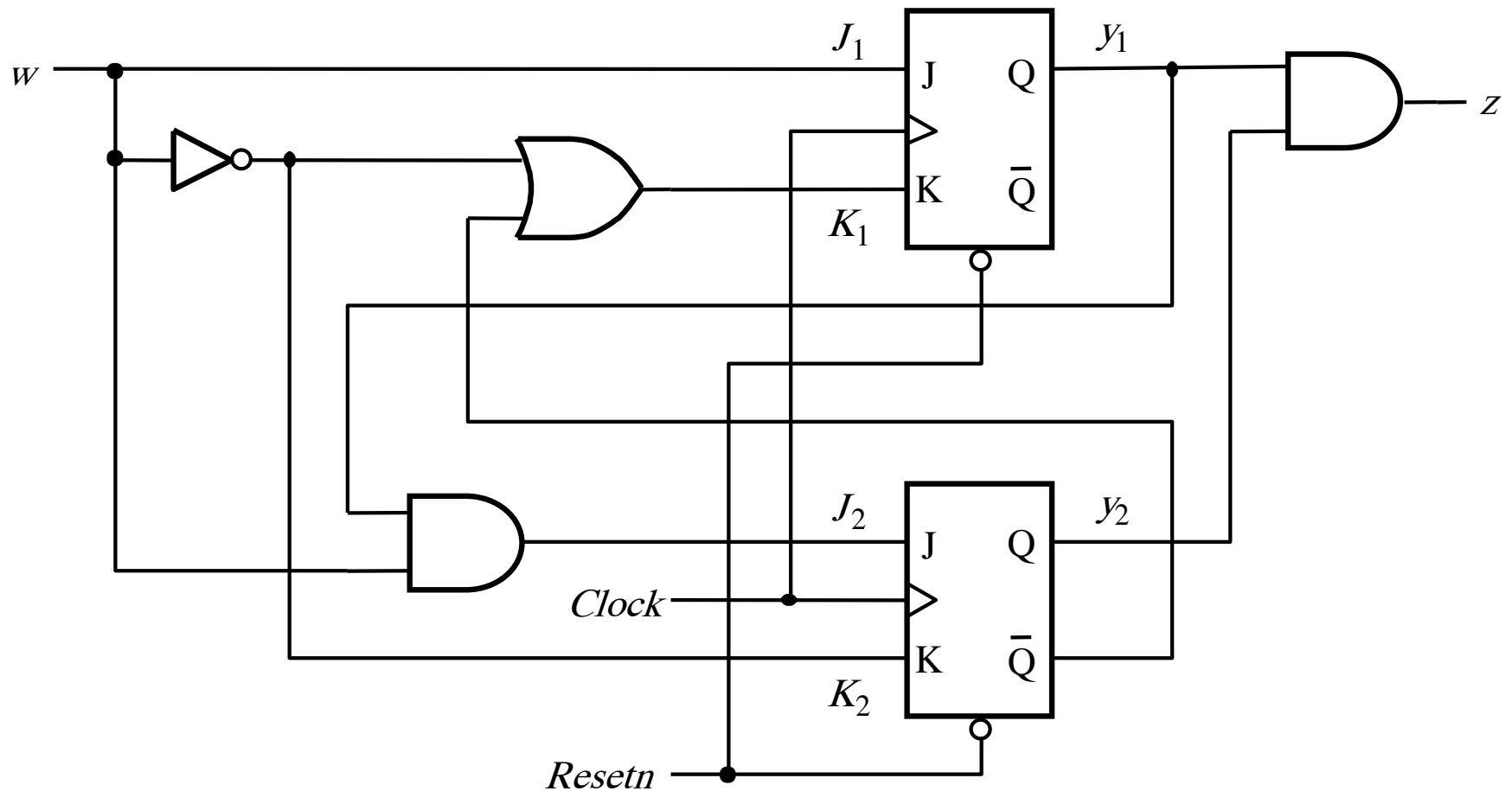


# These are the next-state variables

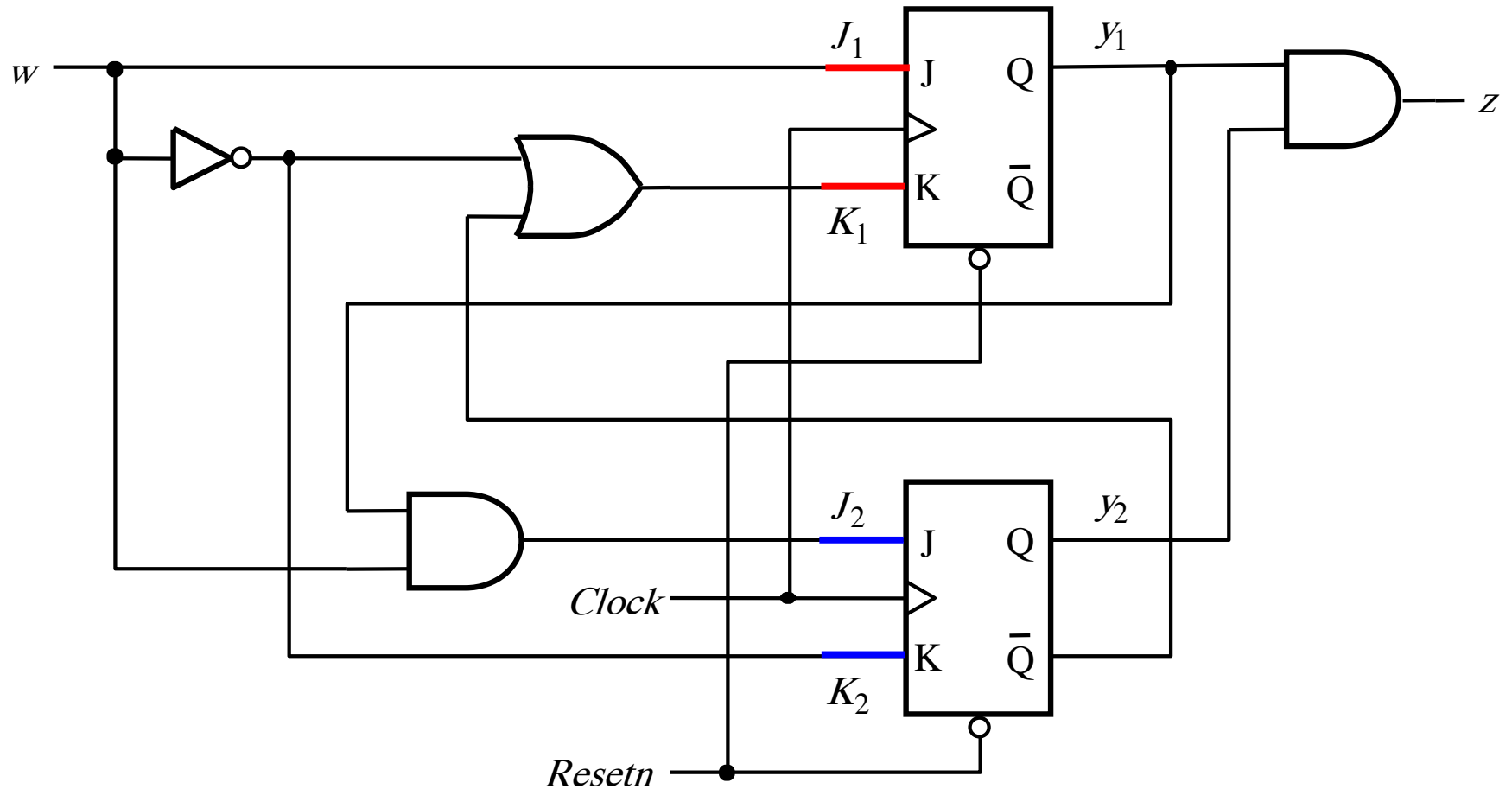




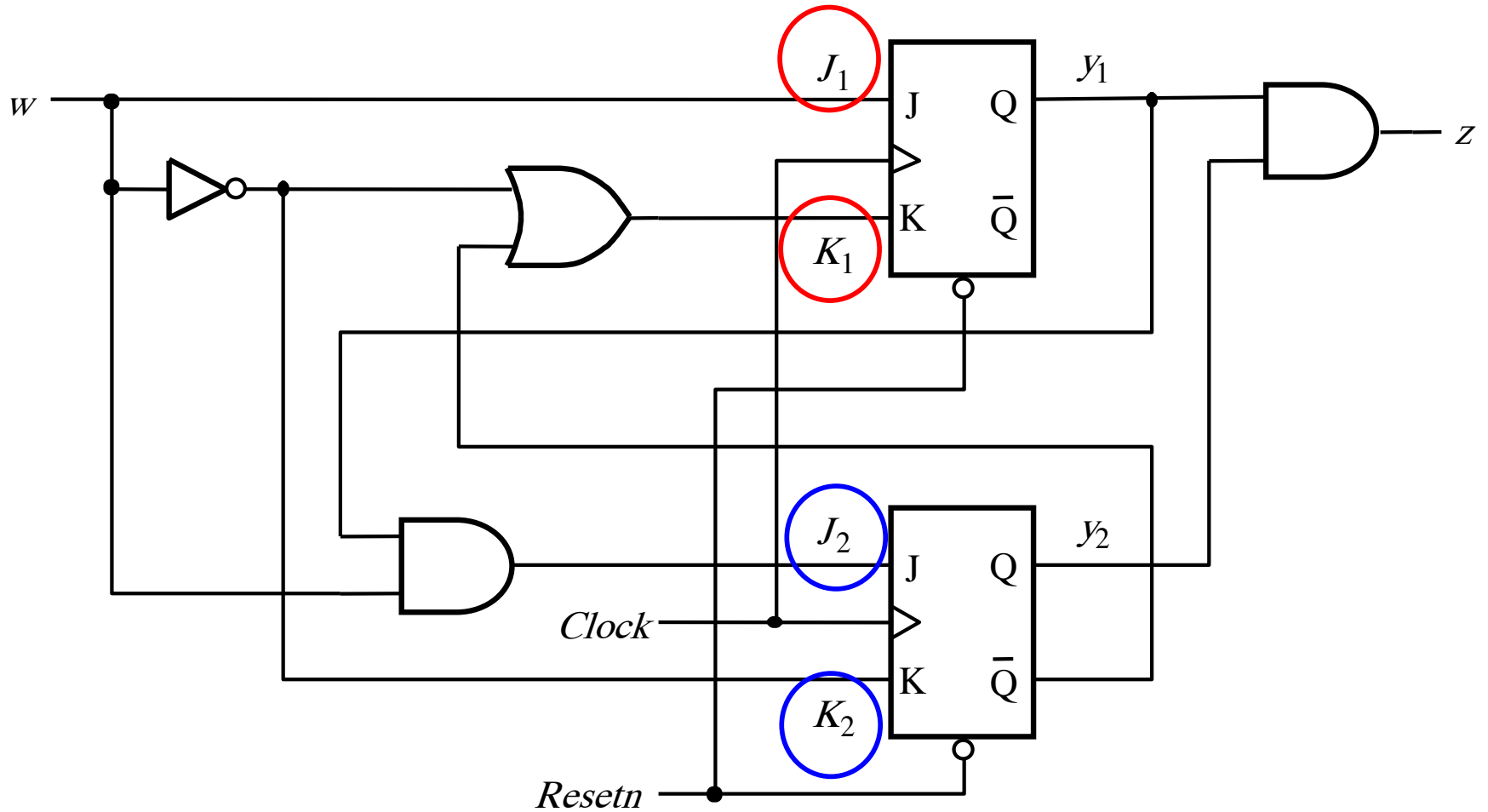
# Where are the inputs of the flip-flops?



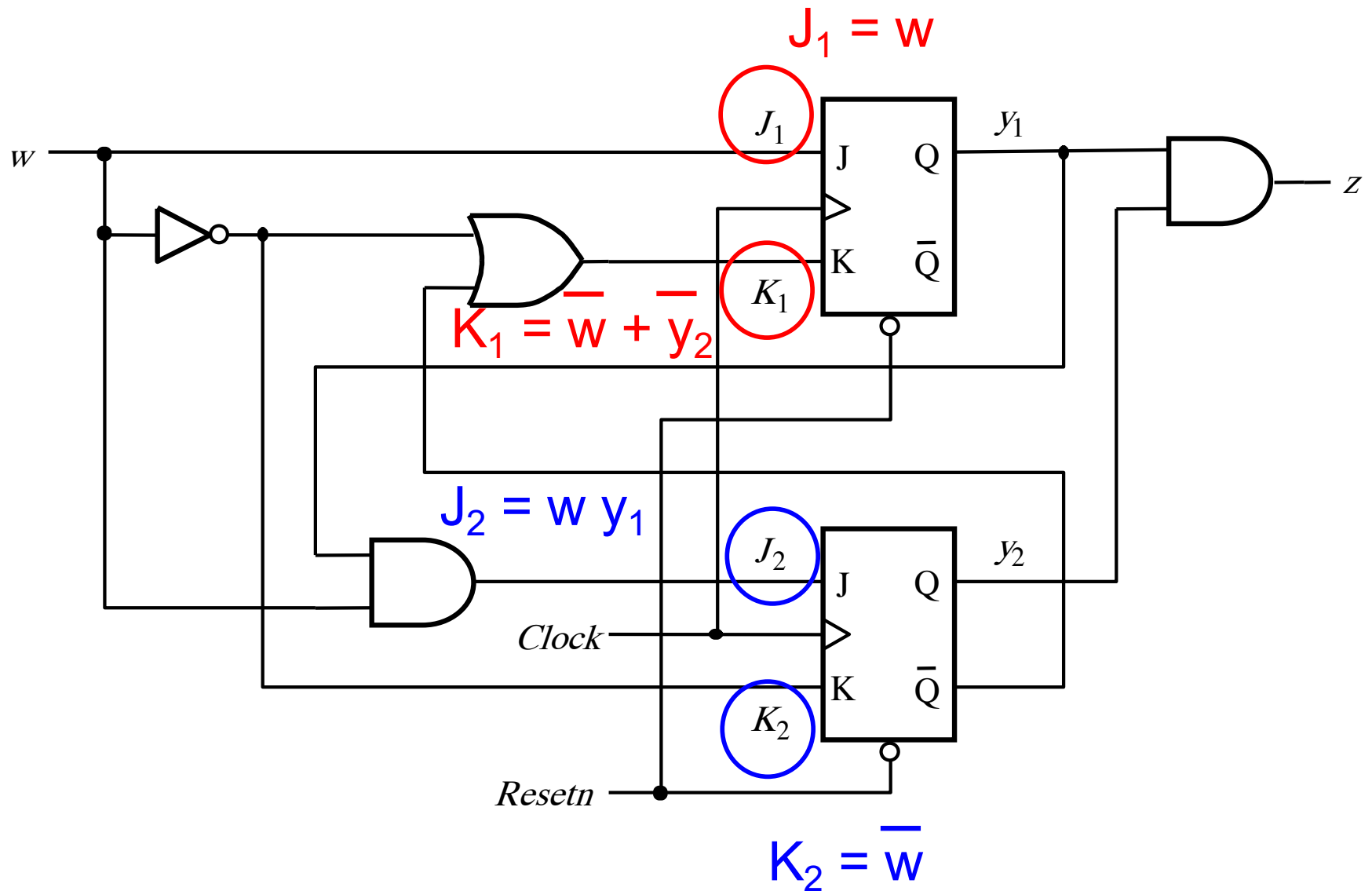
# Where are the inputs of the flip-flops?



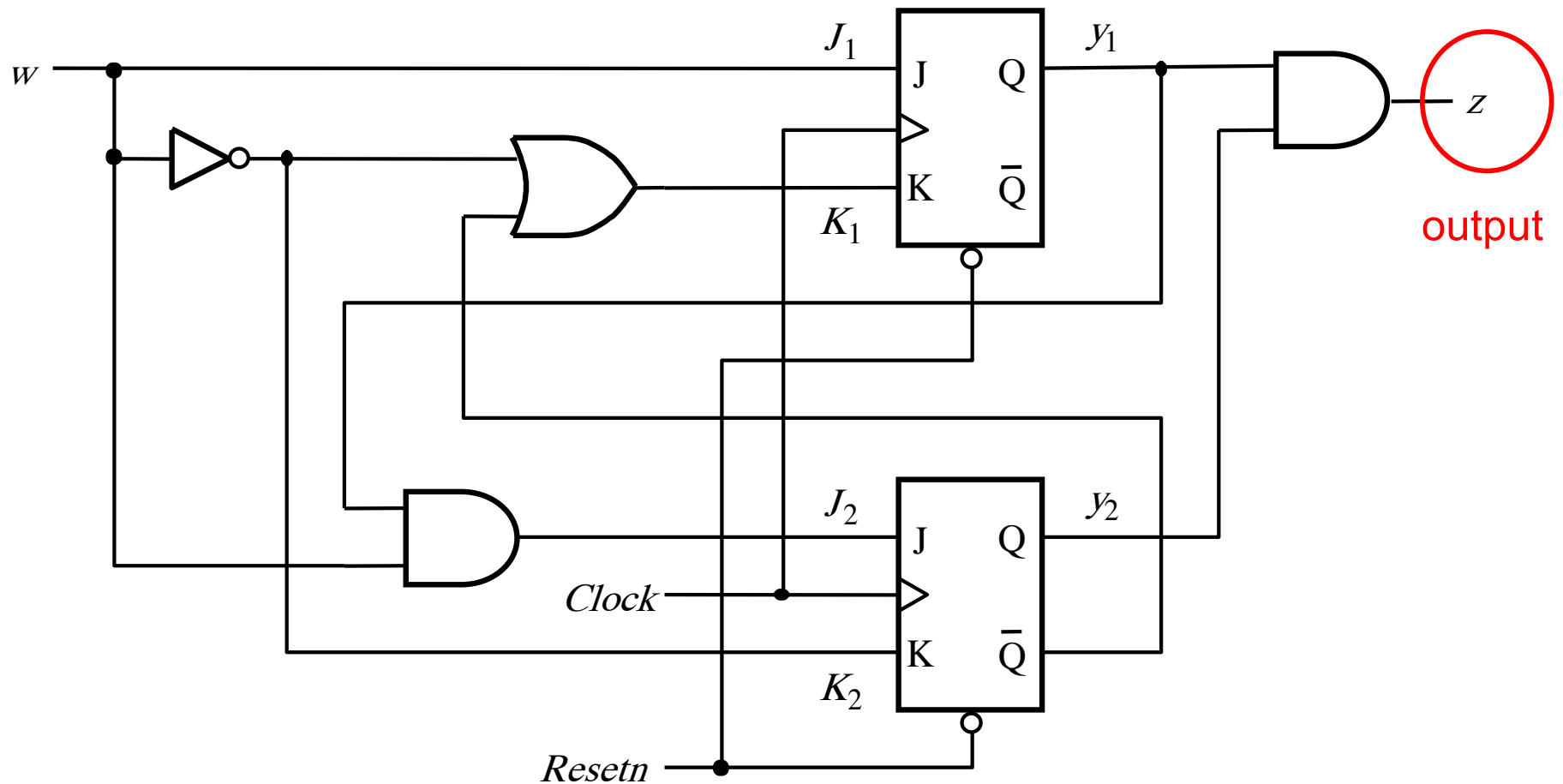
# What are their logic expressions?



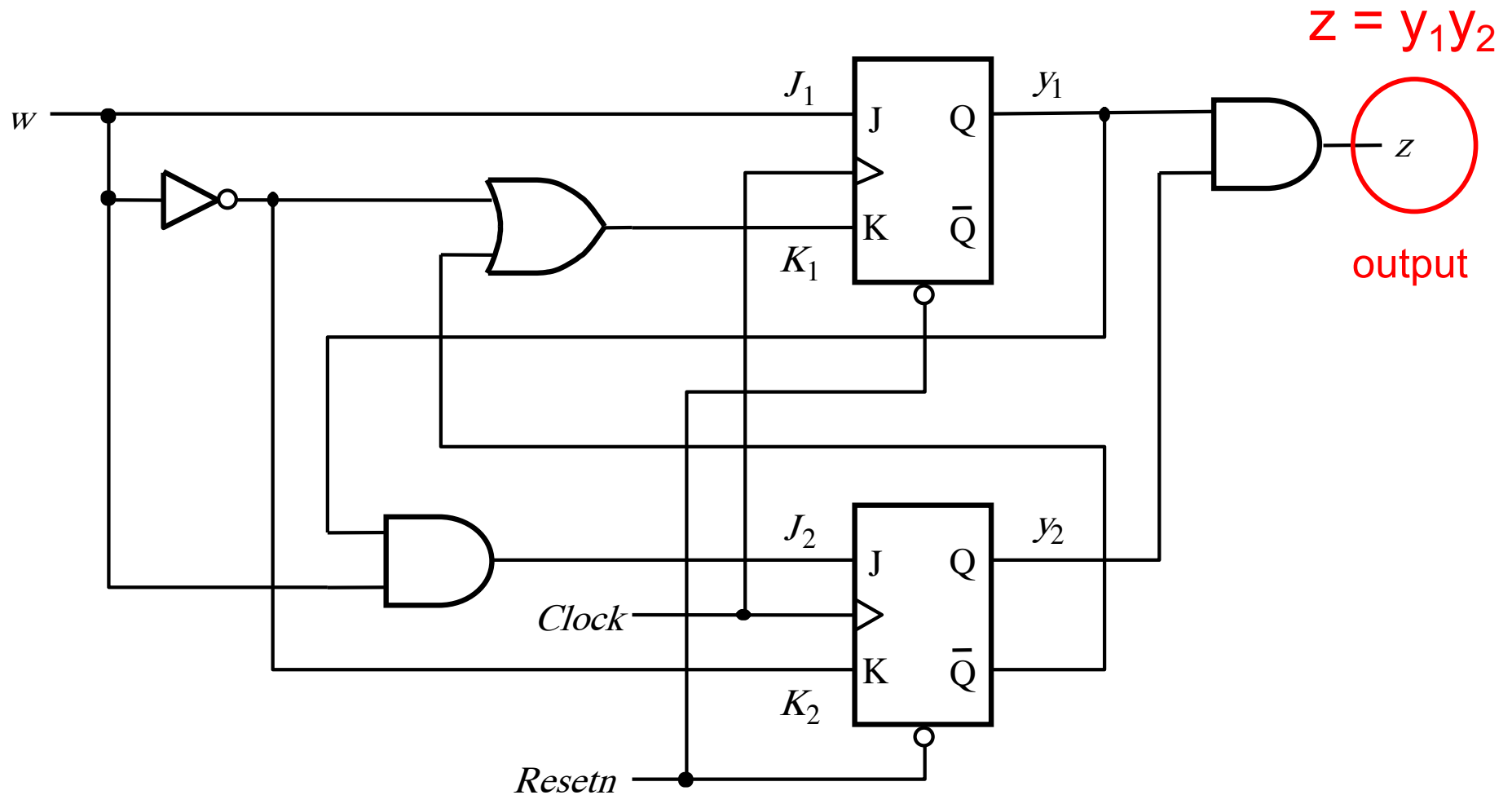
# What are their logic expressions?



# What is the logic expression of the output?



# What is the logic expression of the output?



**This is what we have to work with now  
(we don't need the circuit anymore)**

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00					
01					
10					
11					



# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

Present state $y_2y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00					
01					
10					
11					

$$z = y_1y_2$$

# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

Present state $y_2y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00					0
01					0
10					0
11					1

$$z = y_1y_2$$

# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00					0
01					0
10					0
11					1

# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00		01		11	0
01		01		11	0
10		01		10	0
11		01		10	1

# Let's derive the excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

Present state $y_2y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00		01		11	0
01		01		11	0
10		01		10	0
11		01		10	1

$$z = y_1y_2$$

# The excitation table

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

# We don't need the logic expressions anymore

$$J_1 = w$$

$$K_1 = \bar{w} + \bar{y}_2$$

$$J_2 = w y_1$$

$$K_2 = \bar{w}$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2 K_2$	$J_1 K_1$	$J_2 K_2$	$J_1 K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

# We don't need the logic expressions anymore

Present state $y_2y_1$	Flip-flop inputs				Output $z$
	$w = 0$		$w = 1$		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

State table

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

State table

Excitation table

This step is easy  
(map 2-bit numbers to 4 letters)

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

State table

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

This step is easy too  
(the outputs are the same in both tables)

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

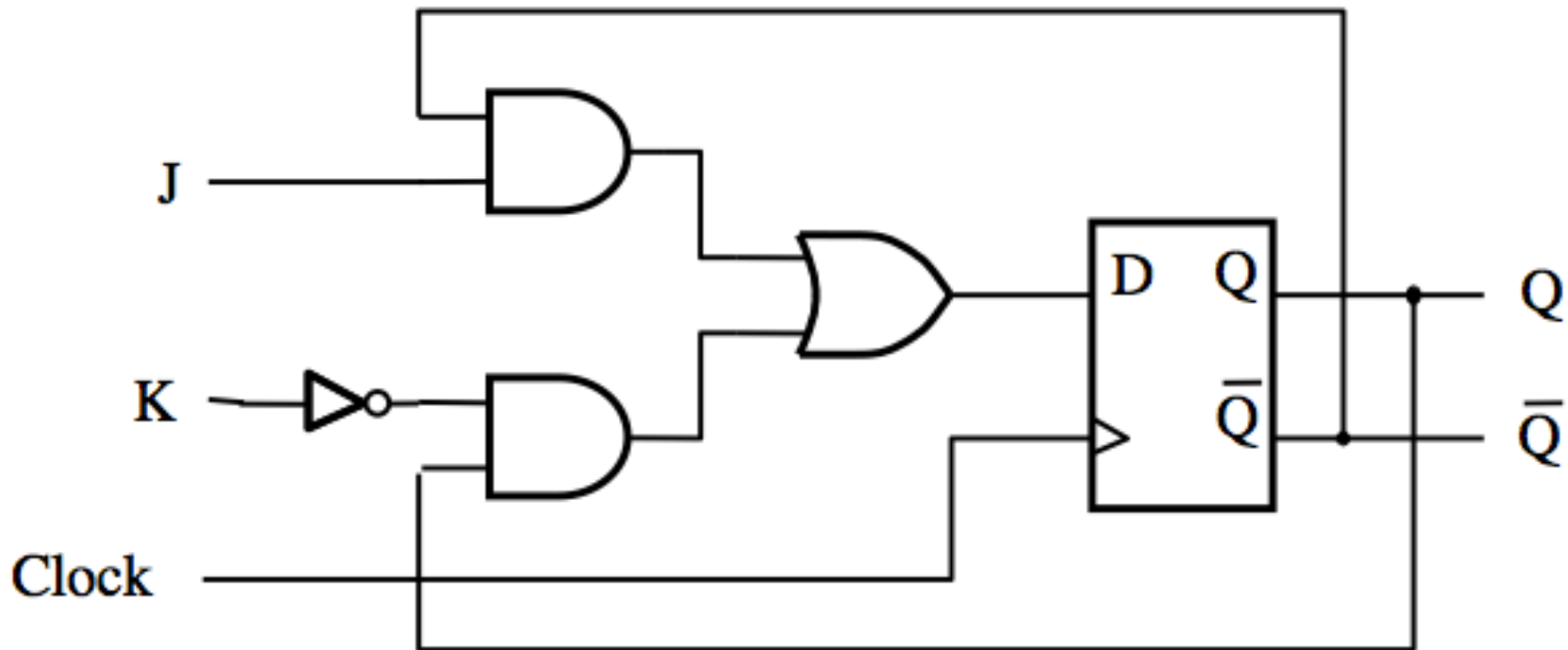
State table

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

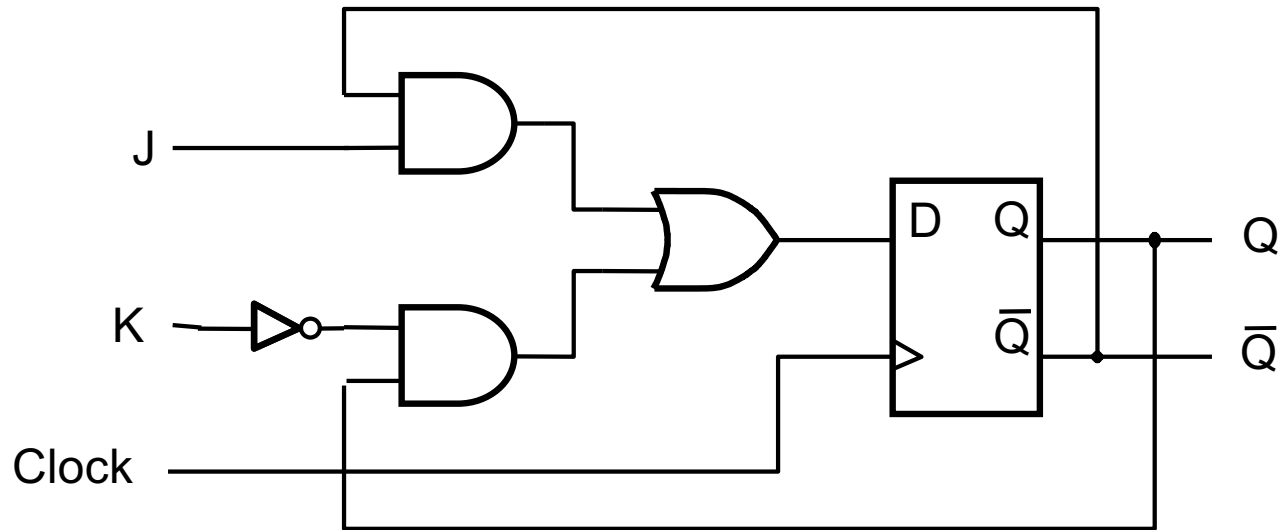
How should we do this?

# JK Flip-Flop Refresher



$$D = \overline{J}Q + \overline{K}Q$$

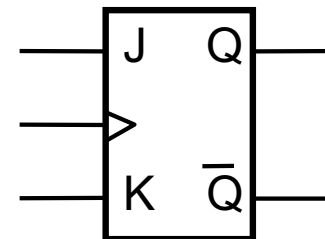
# JK Flip-Flop Refresher



(a) Circuit

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

(b) Truth table



(c) Graphical symbol

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

State table

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

How should we do this?

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J K	Q(t+1)
0 0	Q(t)
0 1	0
1 0	1
1 1	$\bar{Q}(t)$



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that A = 00

J	K	Q(t+1)	J	K	Q(t+1)
0	0	Q(t)	0	0	Q(t)
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	<span style="border: 1px solid red; padding: 2px;">?</span>	0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$

01

10

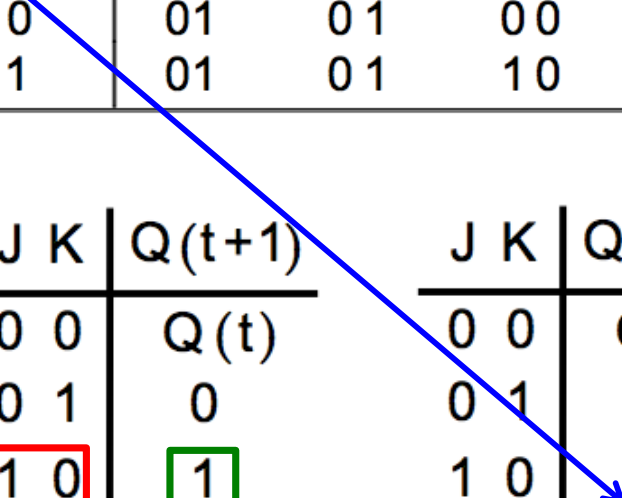
11

10

1

11

$\bar{Q}(t)$



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

J	K	$Q(t+1)$	J	K	$Q(t+1)$
0	0	$Q(t)$	0	0	$Q(t)$
0	1	0	0	1	0
1	0	1	1	0	1
1	1	$\bar{Q}(t)$	1	1	$\bar{Q}(t)$ = $\overline{1} = 0$

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B		C	0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Note that C = 10

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$ = 0



# The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

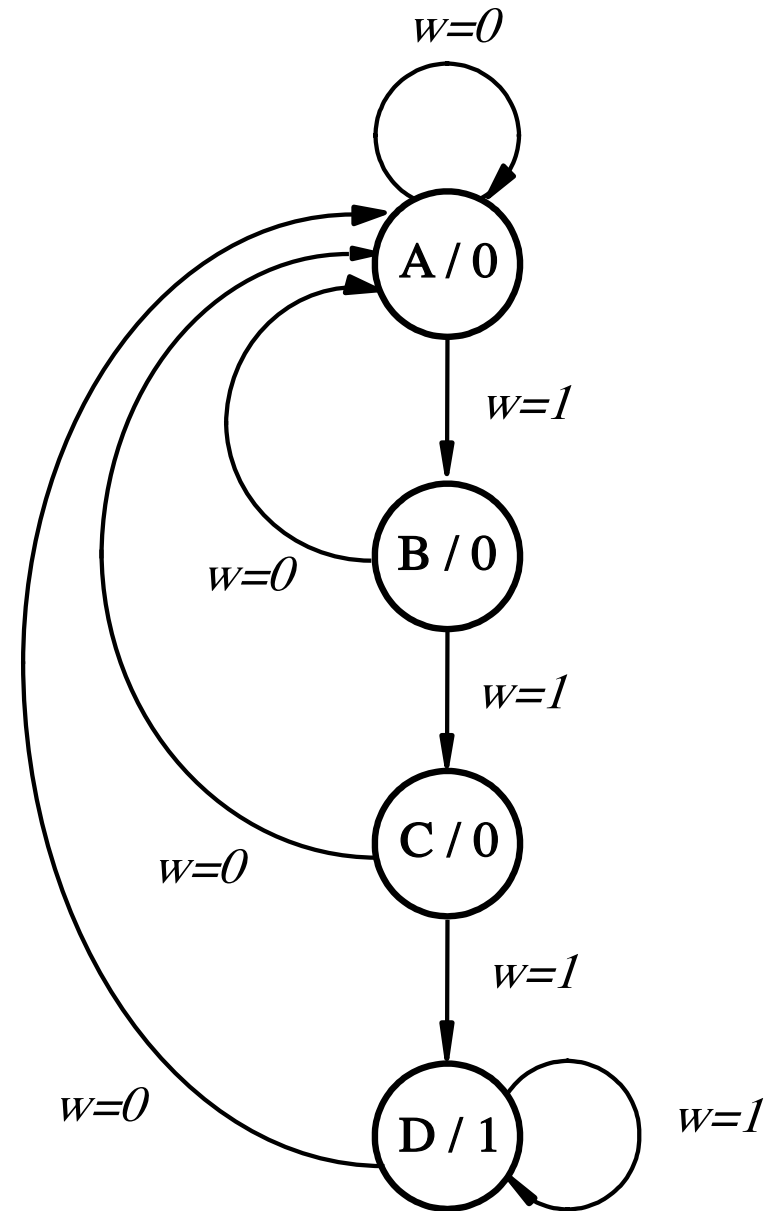
Present state $y_2y_1$	Flip-flop inputs				Output z
	w = 0		w = 1		
	$J_2K_2$	$J_1K_1$	$J_2K_2$	$J_1K_1$	
00	01	01	00	11	0
01	01	01	10	11	0
10	01	01	00	10	0
11	01	01	10	10	1

Excitation table

# The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



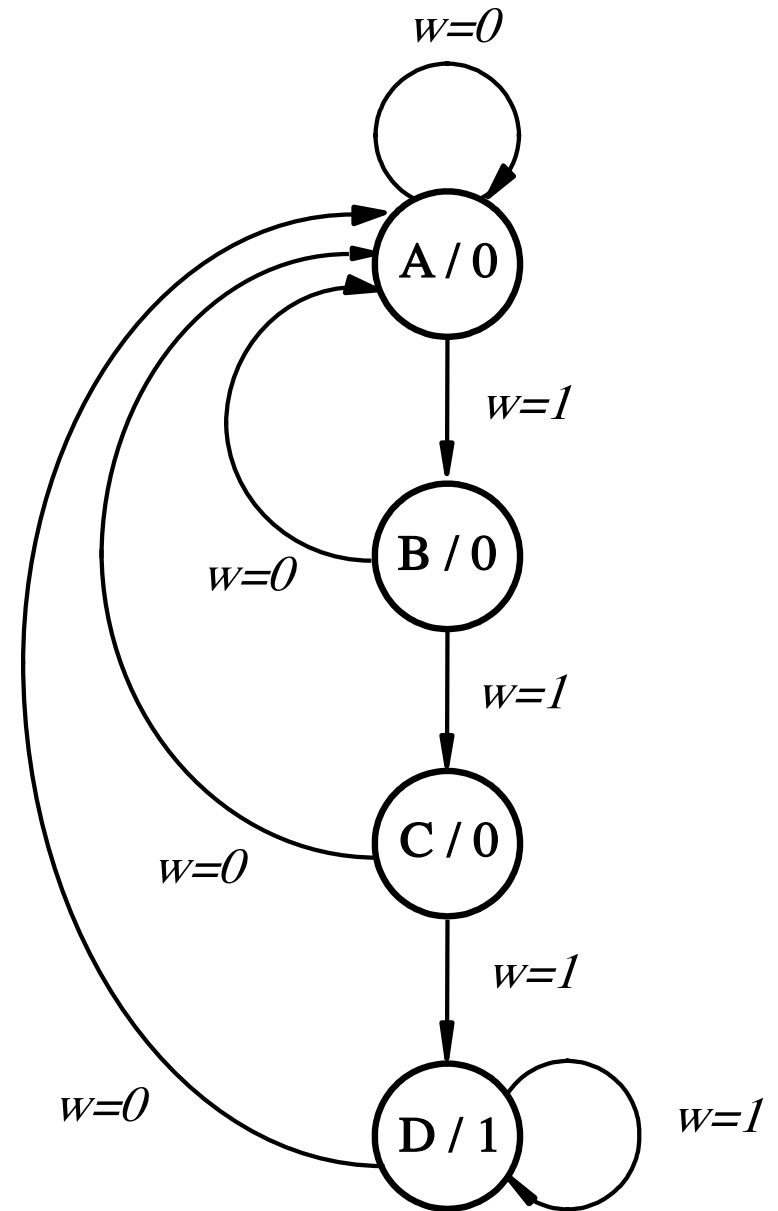
State diagram

# The state diagram

Thus, this FSM is identical to the one in the previous example, even though the circuit uses JK flip-flops.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

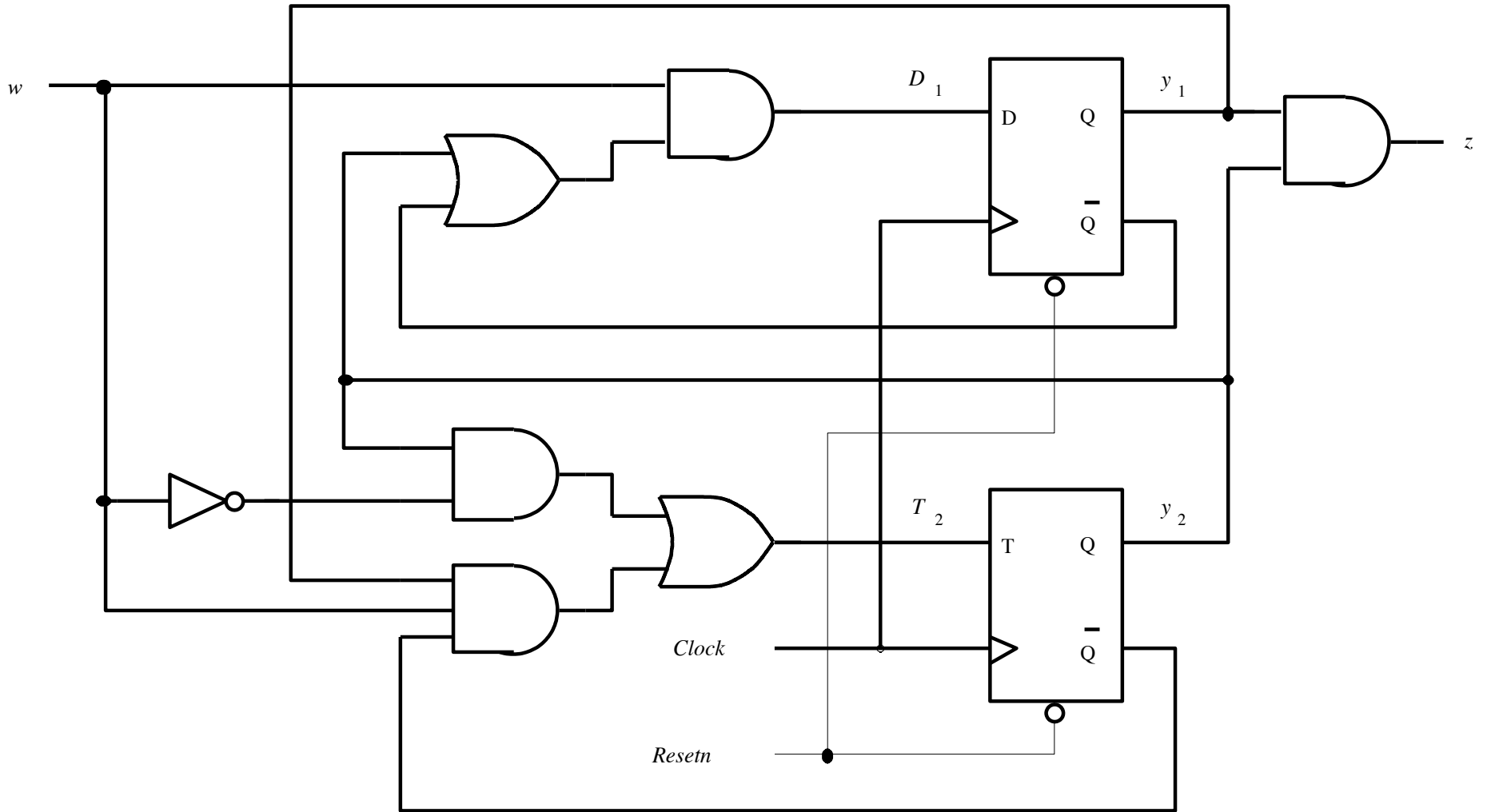
State table



State diagram

# **Yet Another Example (with mixed flip-flops)**

# What does this circuit do?

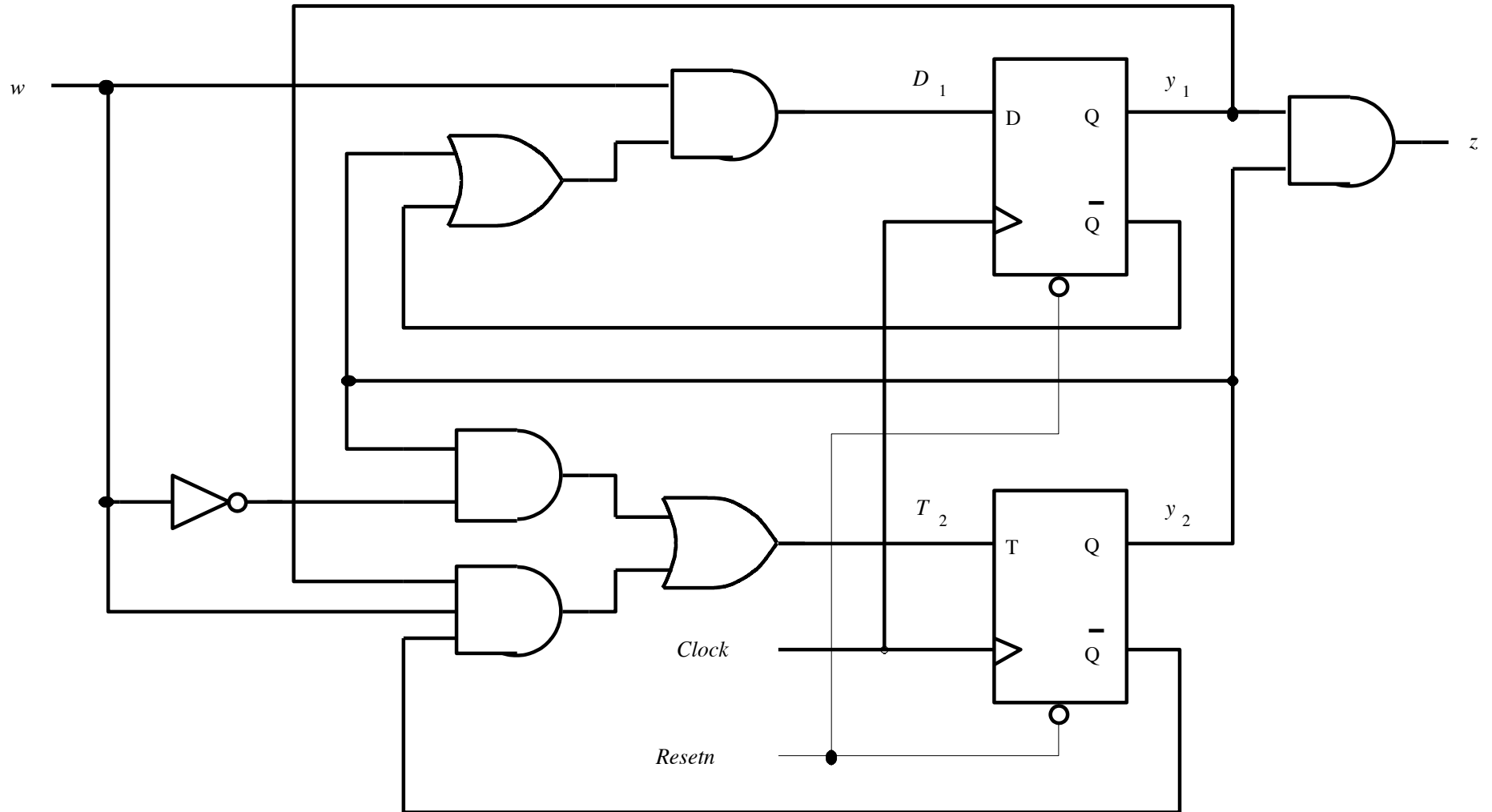


[ Figure 6.79 from the textbook ]

# Approach

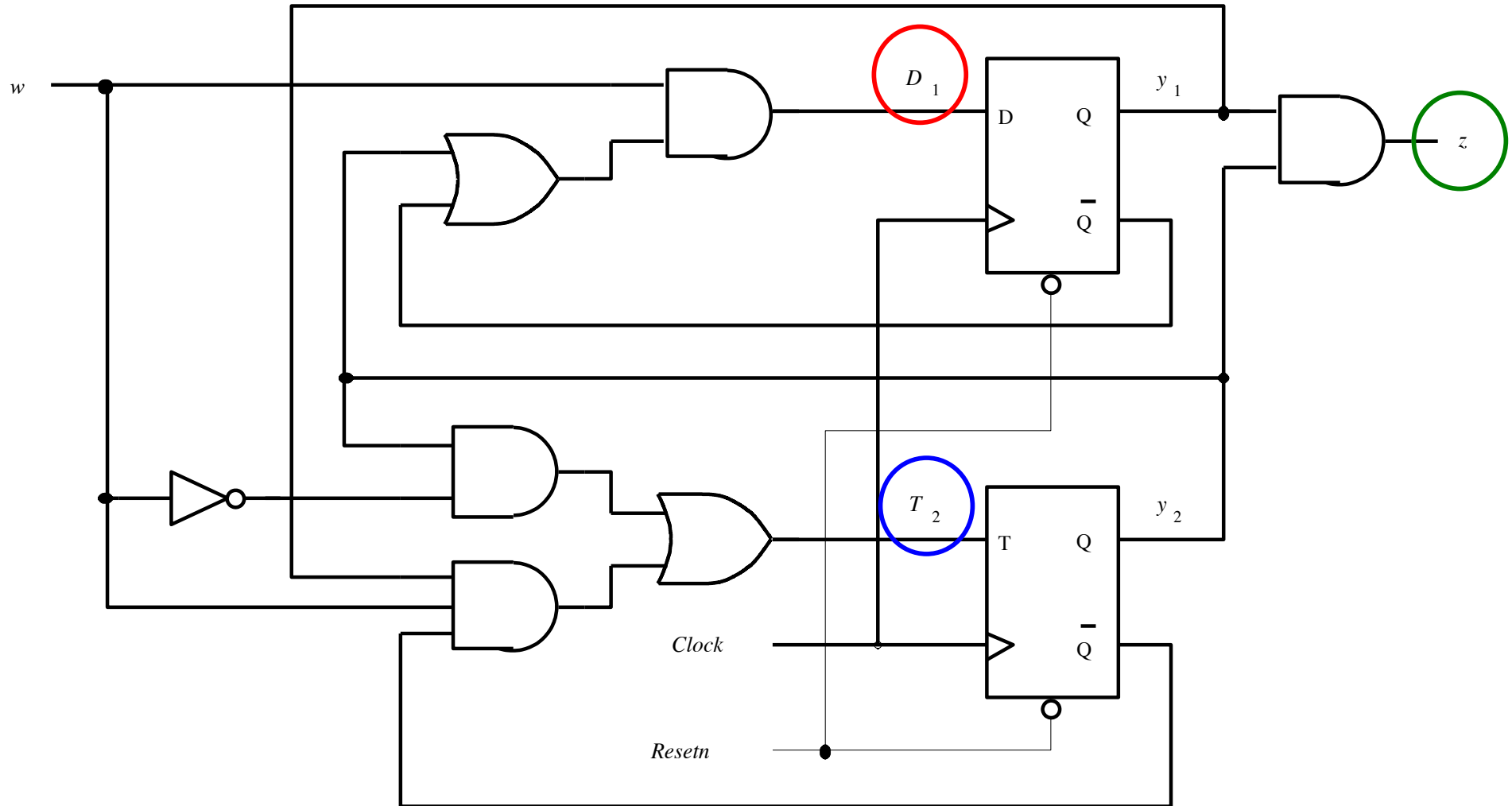
- **Find the flip-flops**
- **Outputs of the flip-flops = present state variables**
- **Inputs of the flip-flops determine the next state variables**
- **Determine the logical expressions for the outputs**
- **Given this info it is easy to do the state-assigned table**
- **Next do the state table**
- **Finally, draw the state diagram.**

# What are the logic expressions?



[ Figure 6.79 from the textbook ]

# What are the logic expressions?

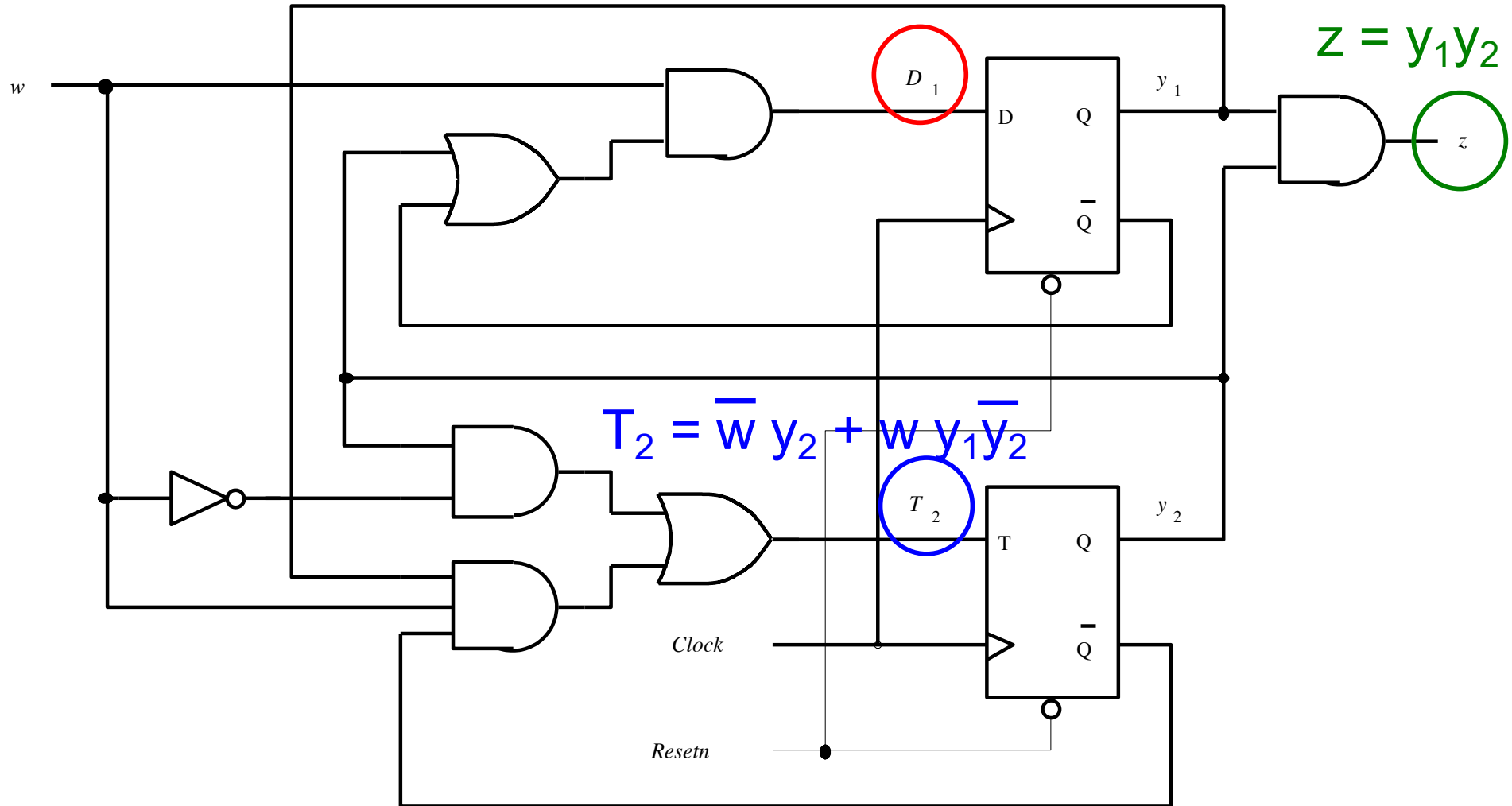




# What are the logic expressions?

$$D_1 = w (\bar{y}_1 + y_2)$$

$$z = y_1 y_2$$



# The Excitation Table

$$D_1 = w (\bar{y}_1 + y_2)$$

$$T_2 = \bar{w} y_2 + w y_1 \bar{y}_2$$

$$z = y_1 y_2$$

Present state $y_2 y_1$	Flip-flop inputs		Output $z$
	$w = 0$	$w = 1$	
	$T_2 D_1$	$T_2 D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Excitation table

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			
B			
C			
D			

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

This step is easy  
(map 2-bit numbers to 4 letters)

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

This step is easy too  
(the outputs are the same in both tables)

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	?		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A			0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Note that A = 00

T	$Q(t+1)$	D	$Q(t+1)$
0	0	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C		?	0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

What should we do here?

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1



# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B	A		0
C	A		0
D	A		1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q}(t)$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q(t)}$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C			0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\overline{Q}(t)$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A		0
B			0
C		D	0
D			1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

Note that D = 11

T	$Q(t+1)$	D	$Q(t+1)$
0	1	0	0
1	$\bar{Q}(t)$	1	1

# Let's derive the state table

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

T	$Q(t+1)$	D	$Q(t+1)$
0	$Q(t)$	0	0
1	$\overline{Q(t)}$	1	1

# The two tables for the initial circuit

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table

Present state $y_2y_1$	Flip-flop inputs		Output z
	w = 0	w = 1	
	$T_2D_1$	$T_2D_1$	
00	00	01	0
01	00	10	0
10	10	01	0
11	10	01	1

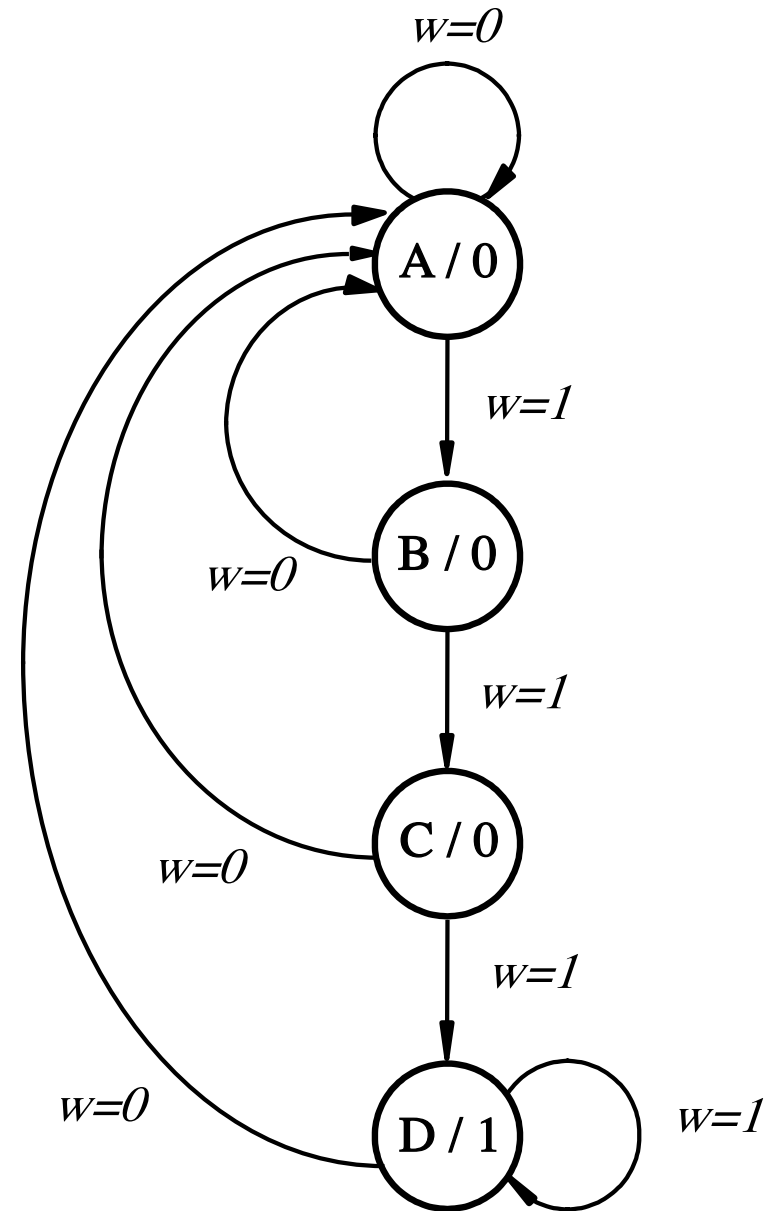
Excitation table



# The state diagram

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



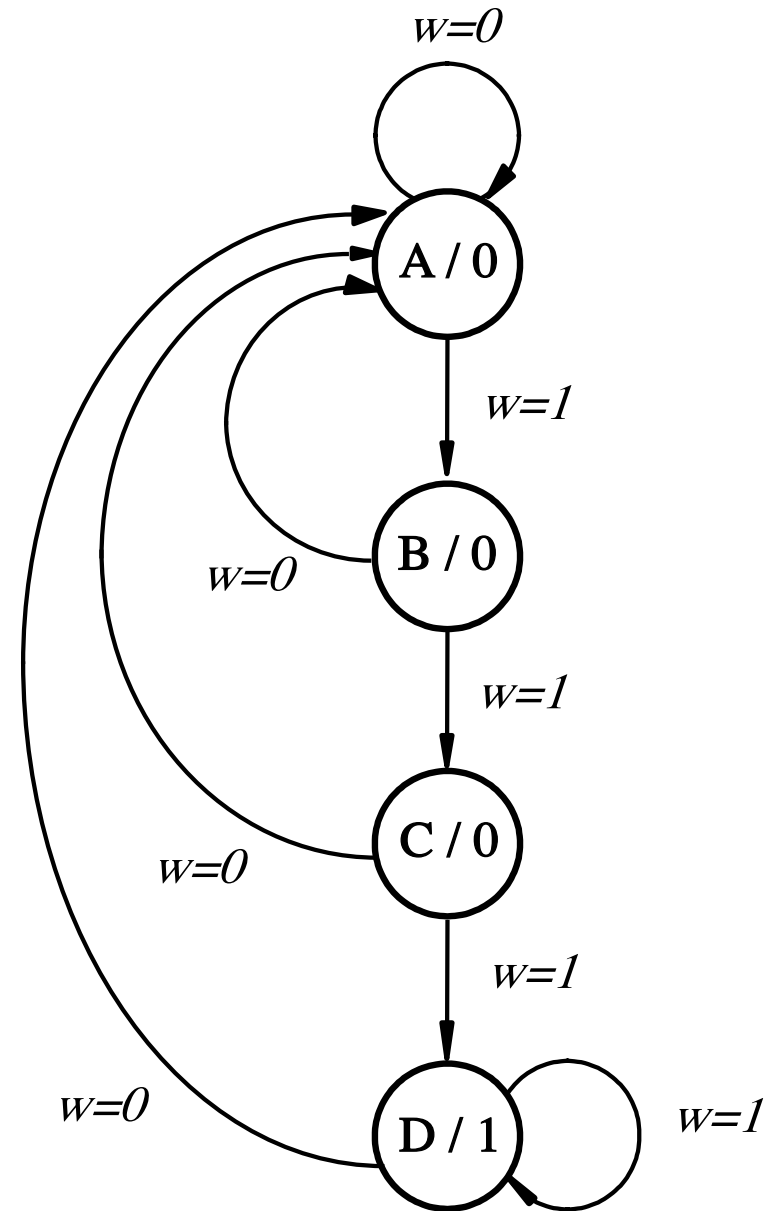
State diagram

# The state diagram

Thus, this FSM is identical to the ones in the previous examples, even though the circuit uses JK flip-flops.

Present state	Next state		Output z
	w = 0	w = 1	
A	A	B	0
B	A	C	0
C	A	D	0
D	A	D	1

State table



State diagram



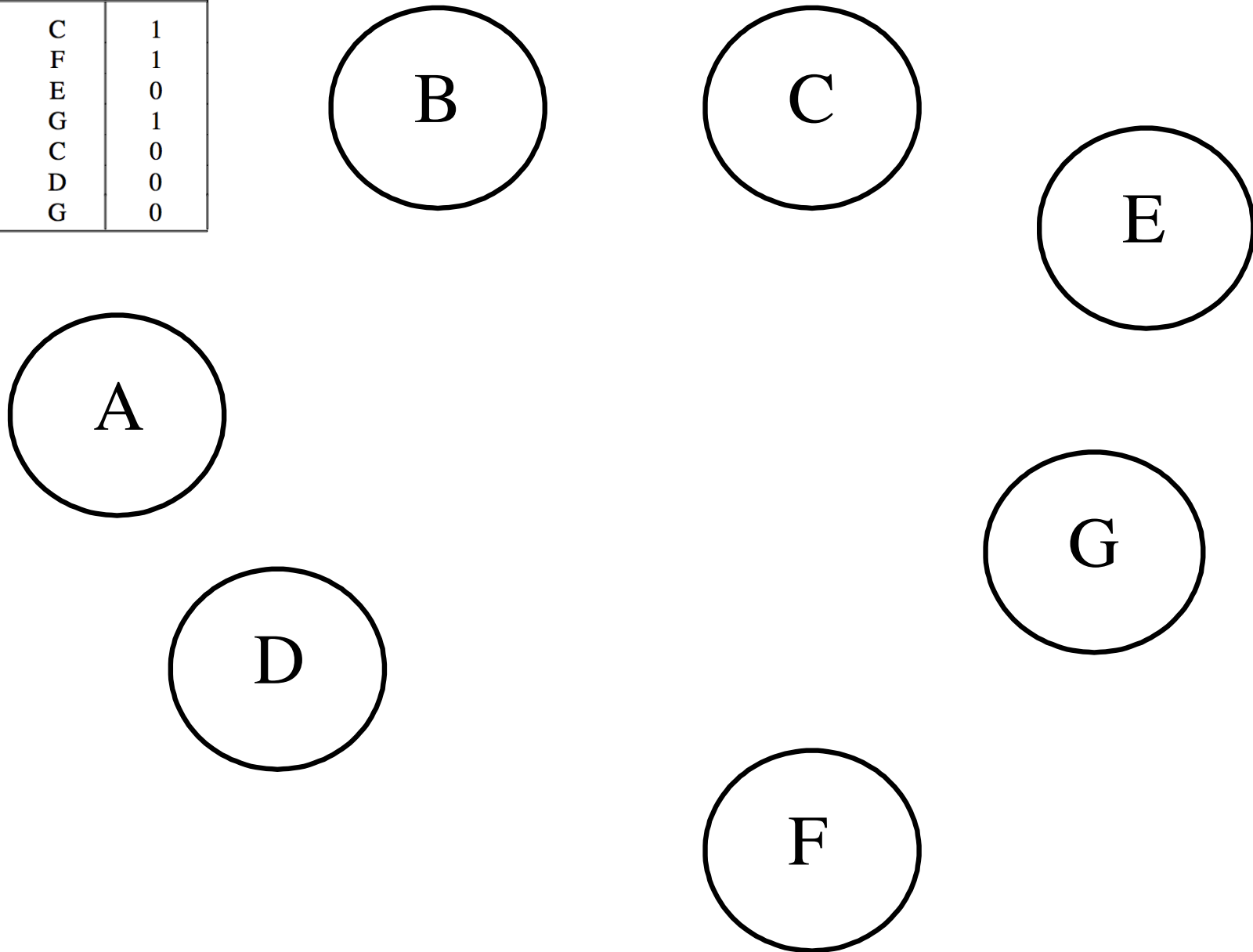
# **State Minimization**

# State Table for This Example

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0

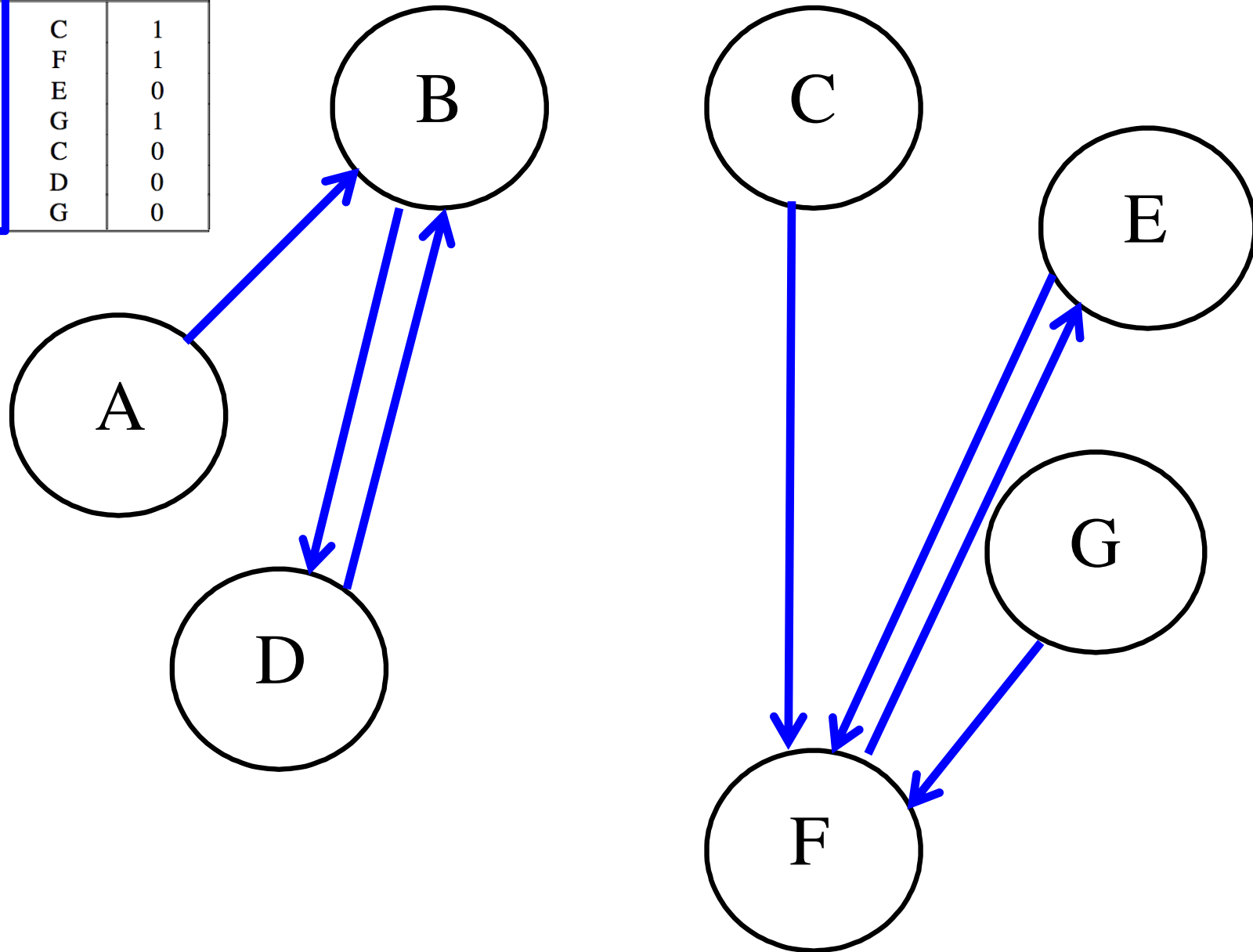
# State Diagram (just the states)

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



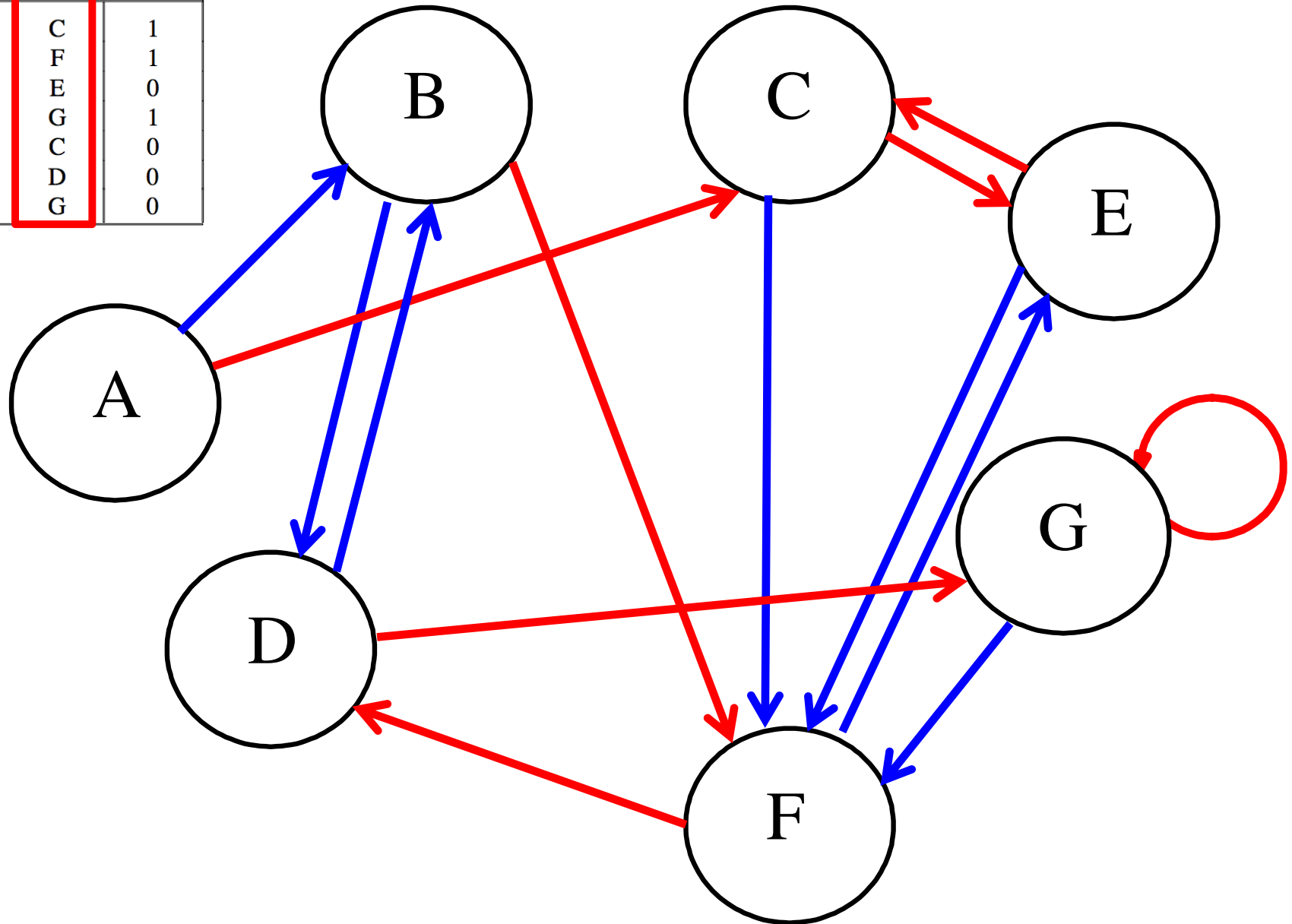
# State Diagram (transitions when $w=0$ )

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



# State Diagram (transitions when $w=1$ )

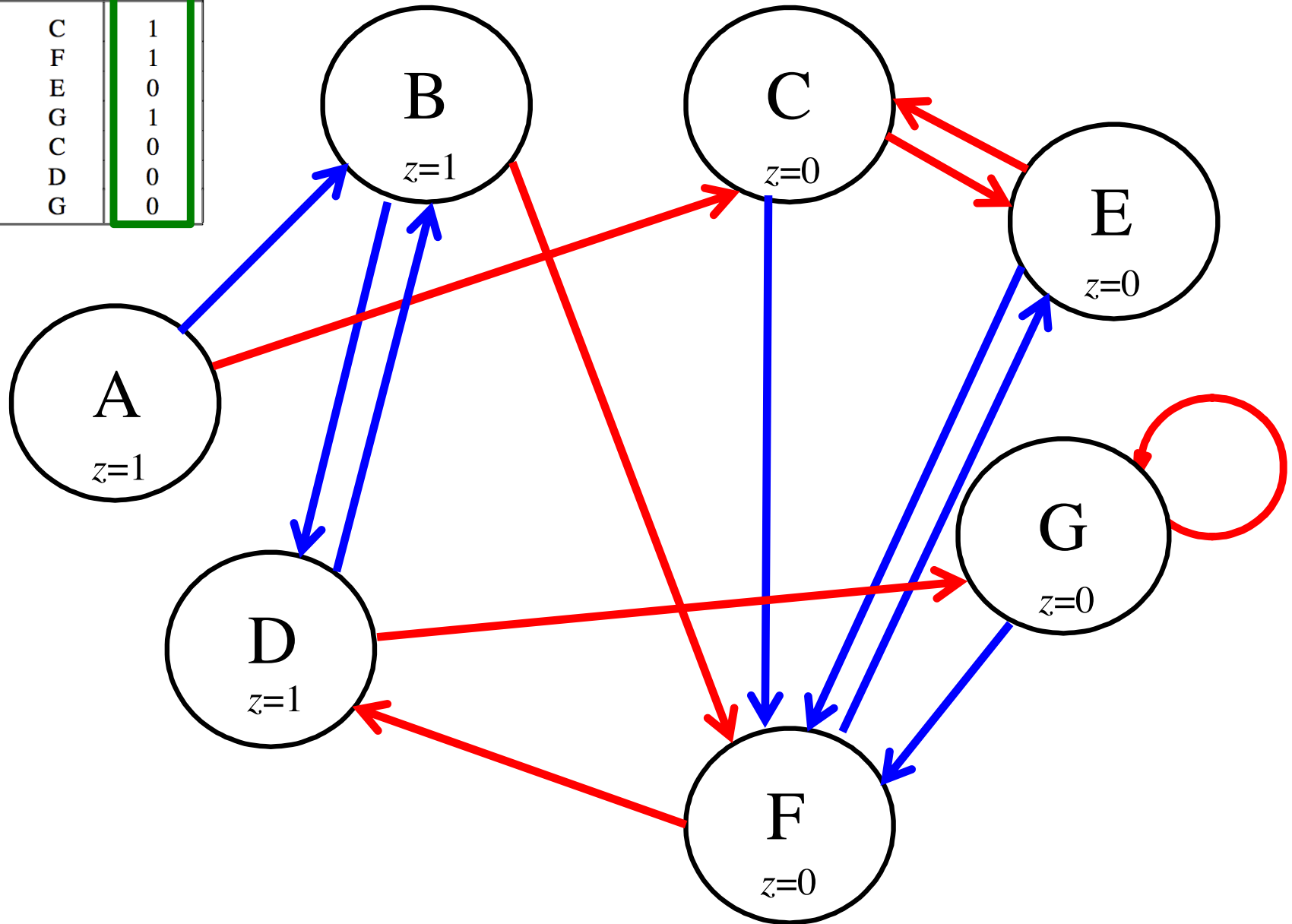
Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0





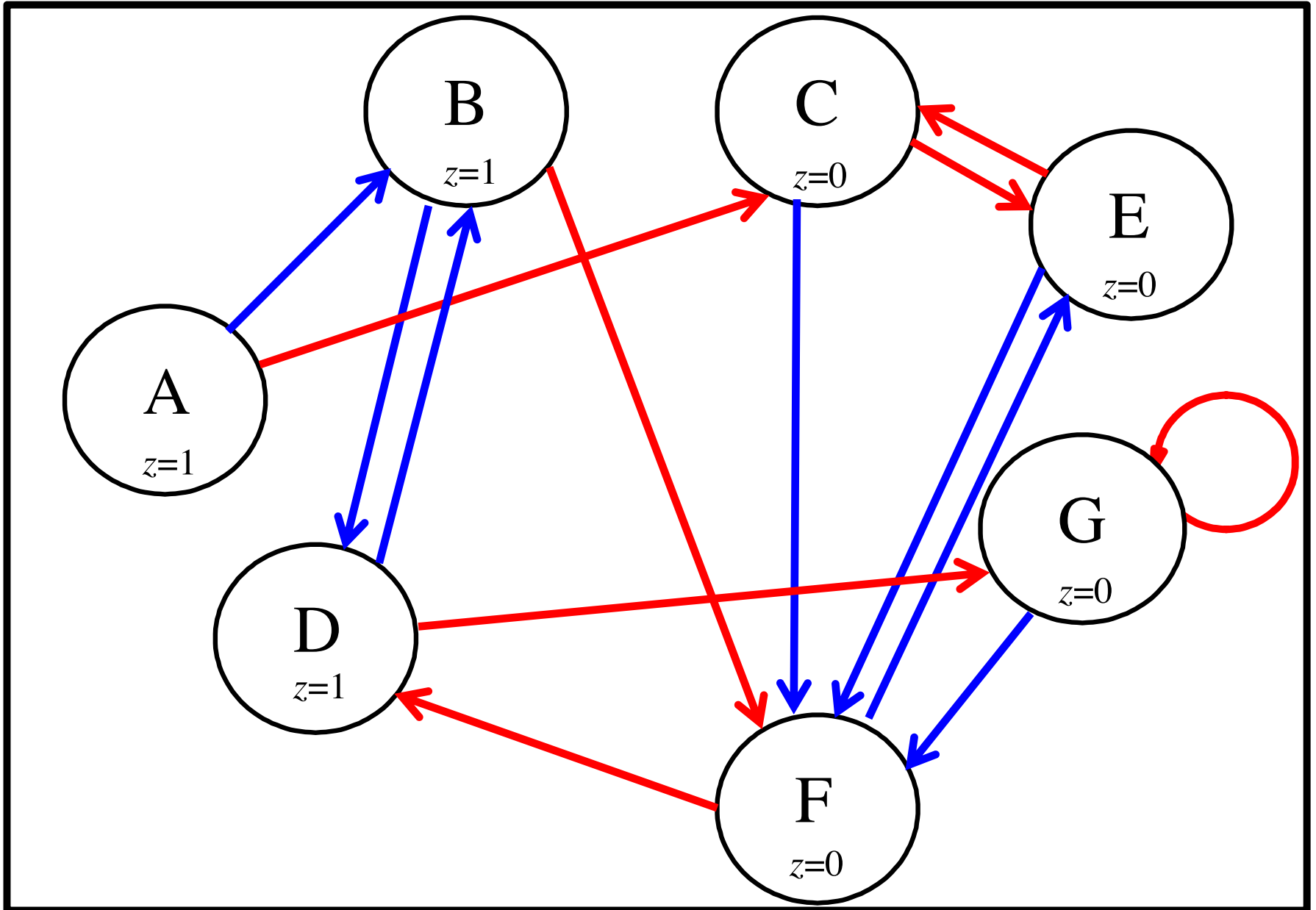
# Outputs

Present state	Next state		Output
	$w = 0$	$w = 1$	$z$
A	B	C	1
B	D	F	1
C	F	E	0
D	B	G	1
E	F	C	0
F	E	D	0
G	F	G	0



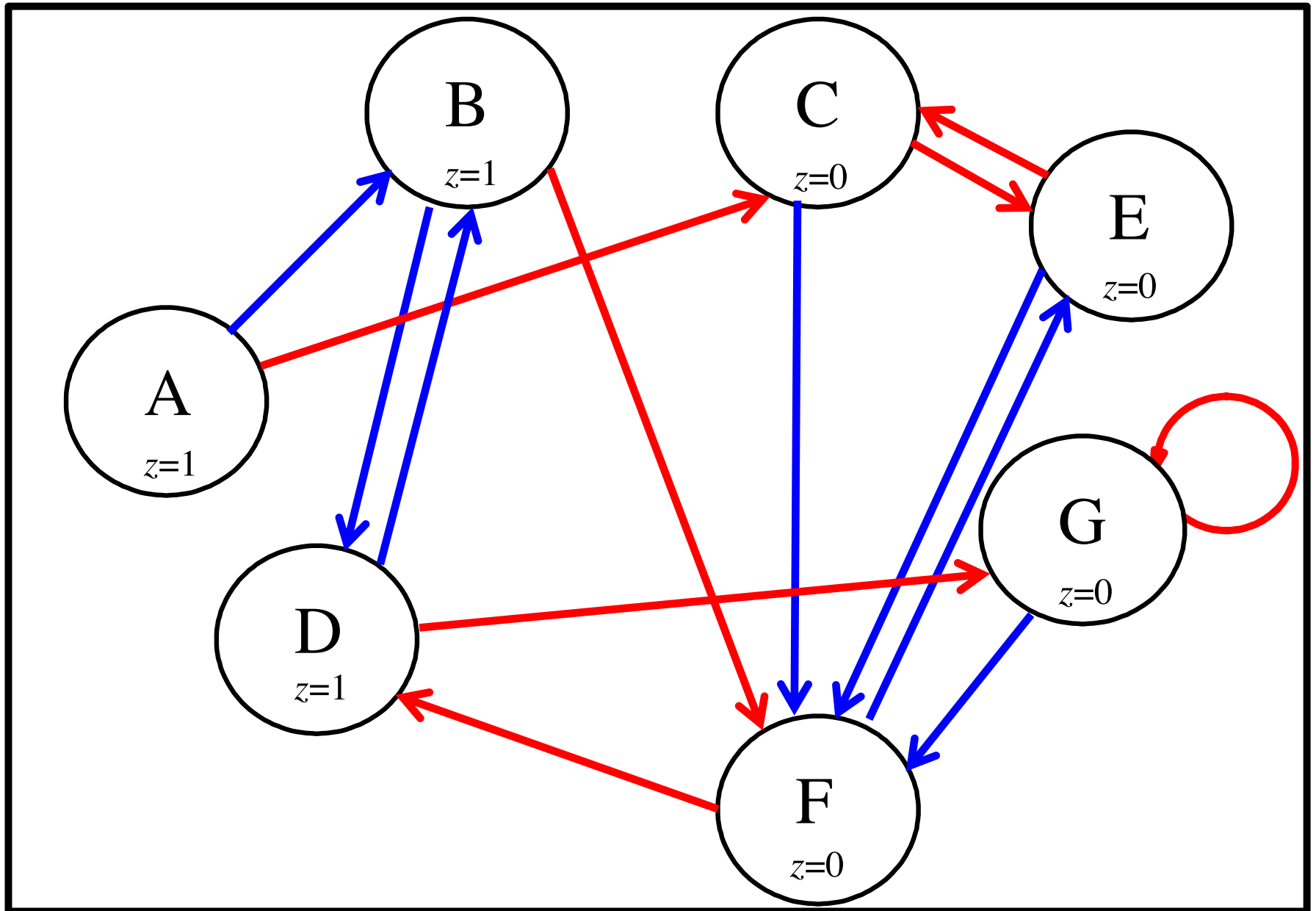
# Partition #1

(All states in the same partition)



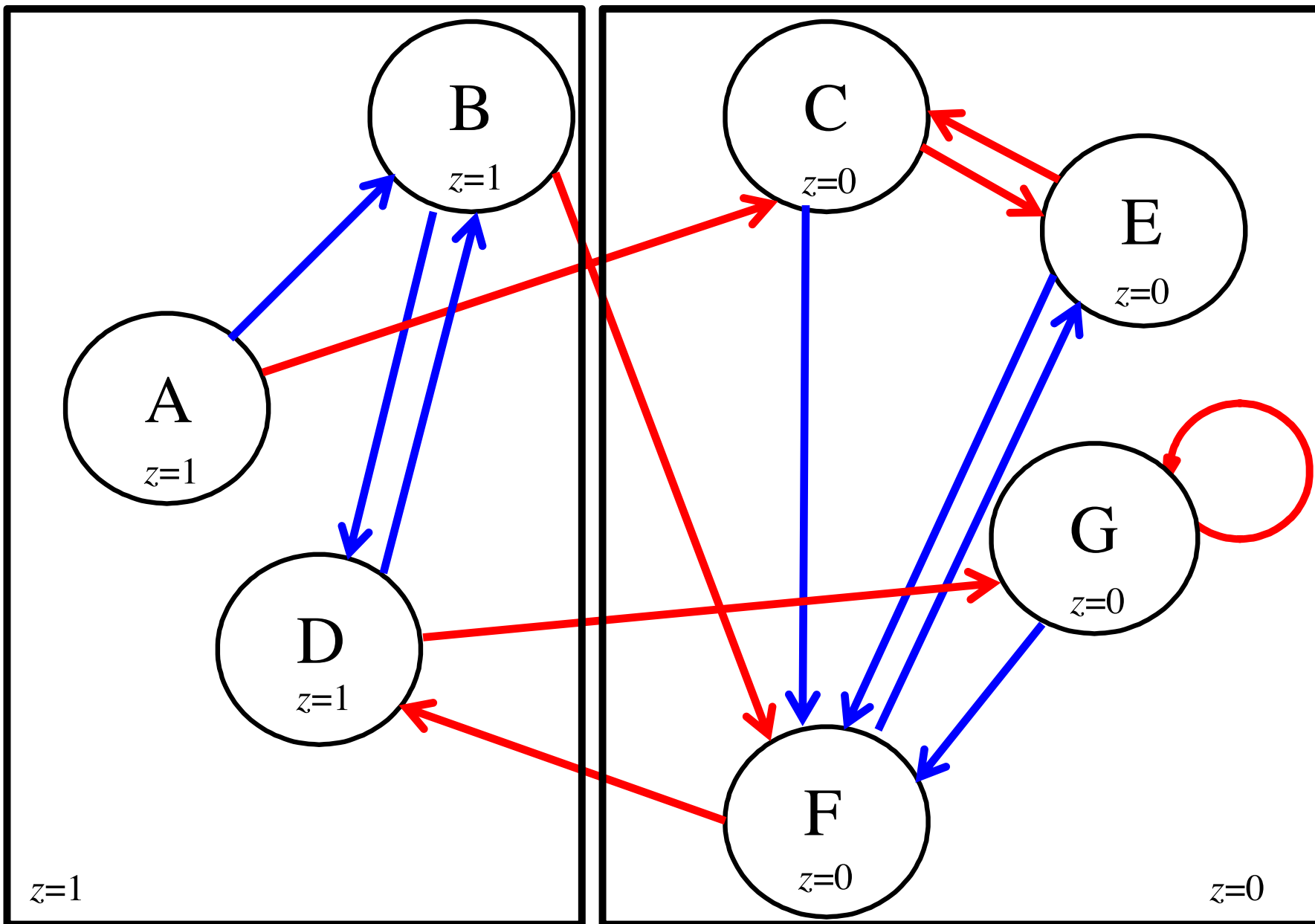
# Partition #1

(ABCDEFGG)



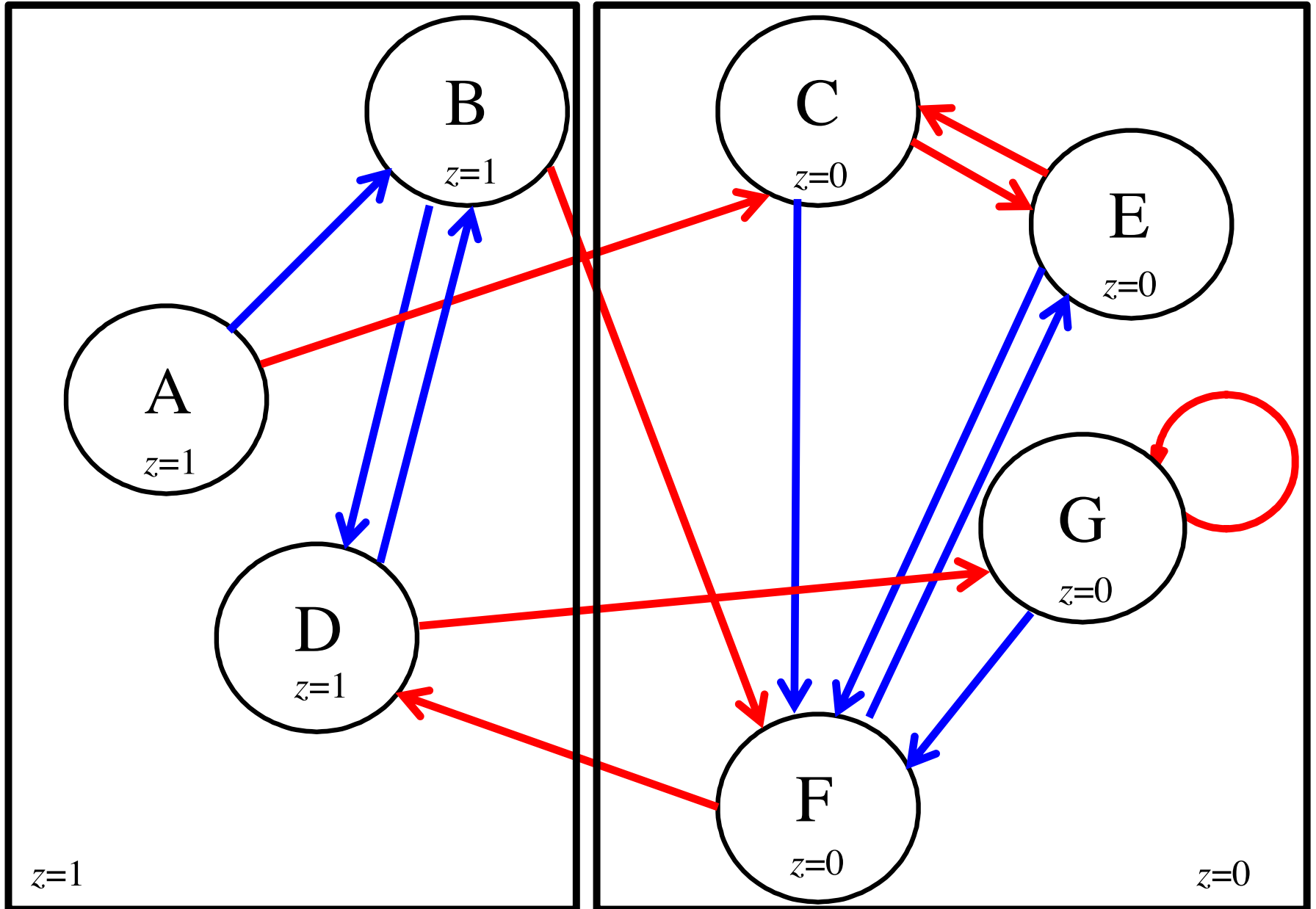
# Partition #2

(based on outputs)



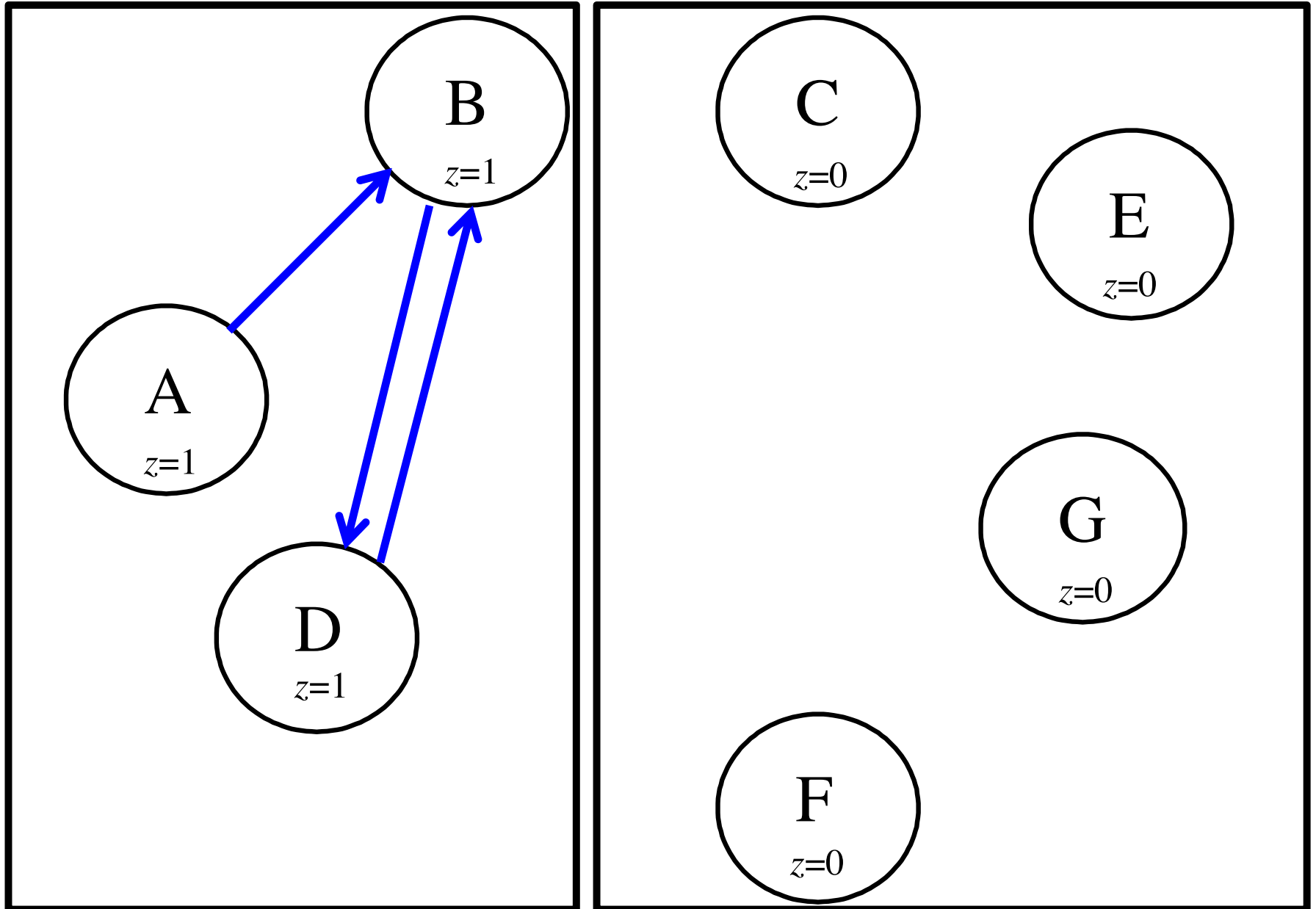
# Partition #2

(ABD)(CEFG)



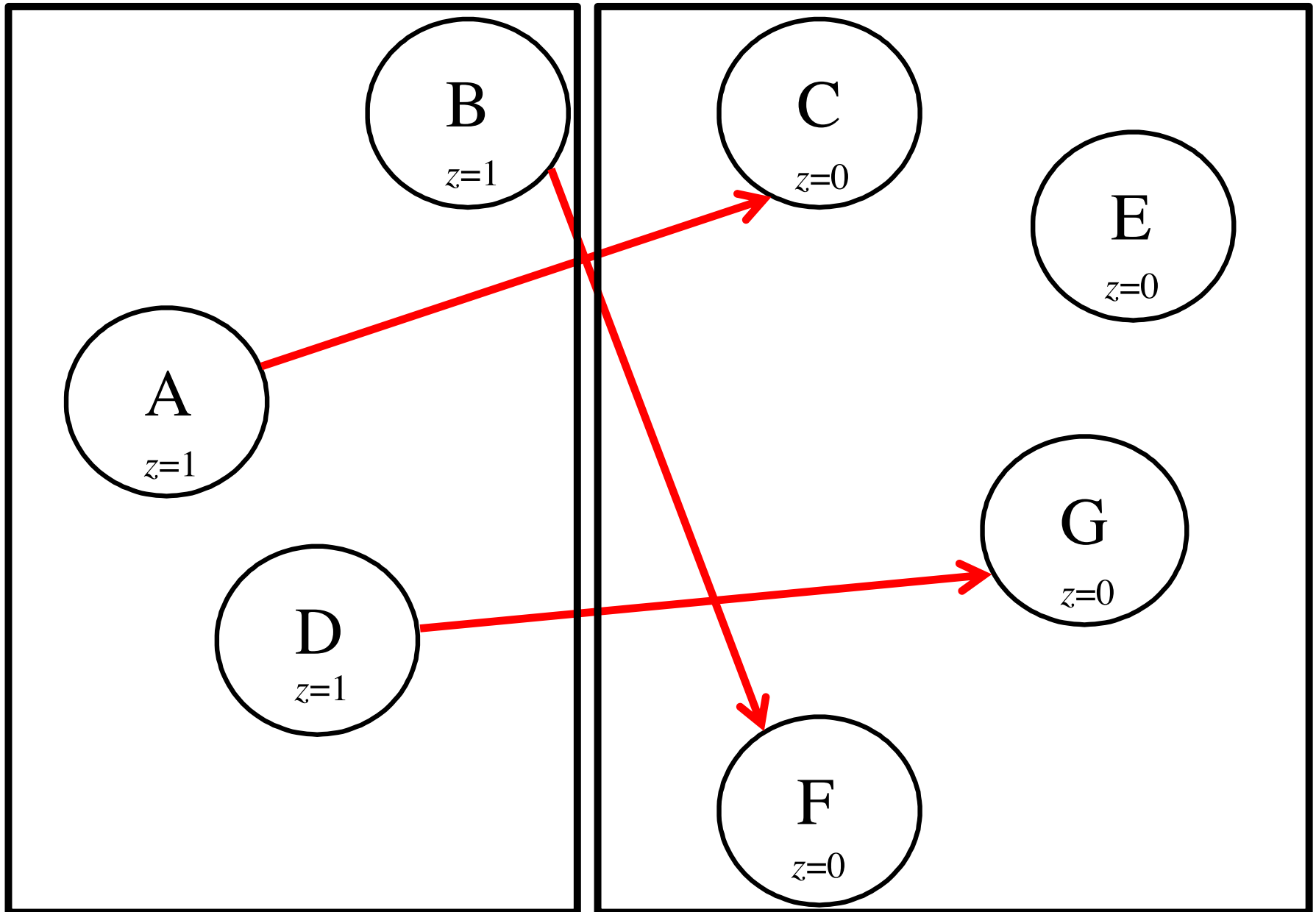
# Partition #3.1

(Examine the 0-successors of ABD)



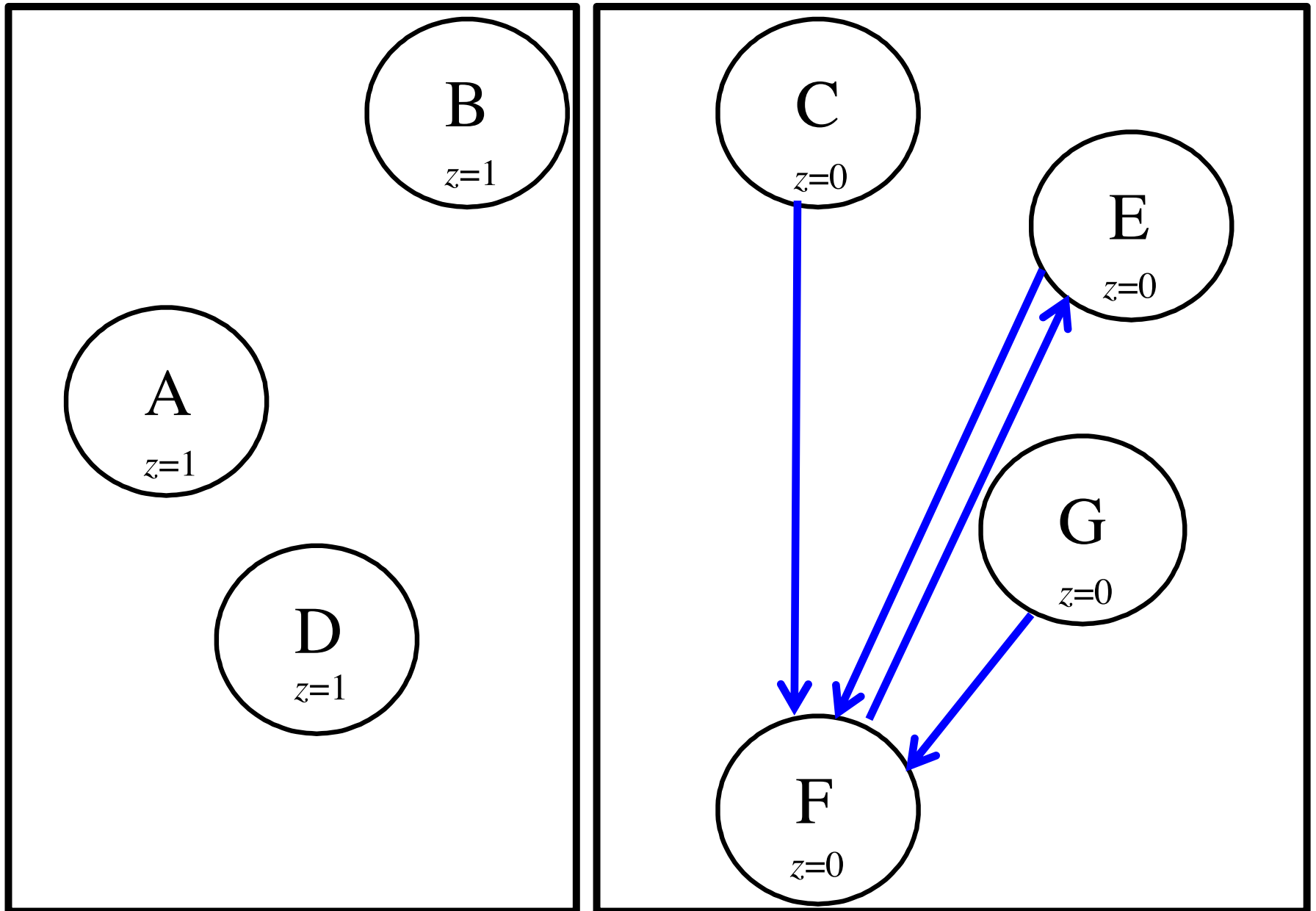
# Partition #3.1

(Examine the 1-successors of ABD)



# Partition #3.2

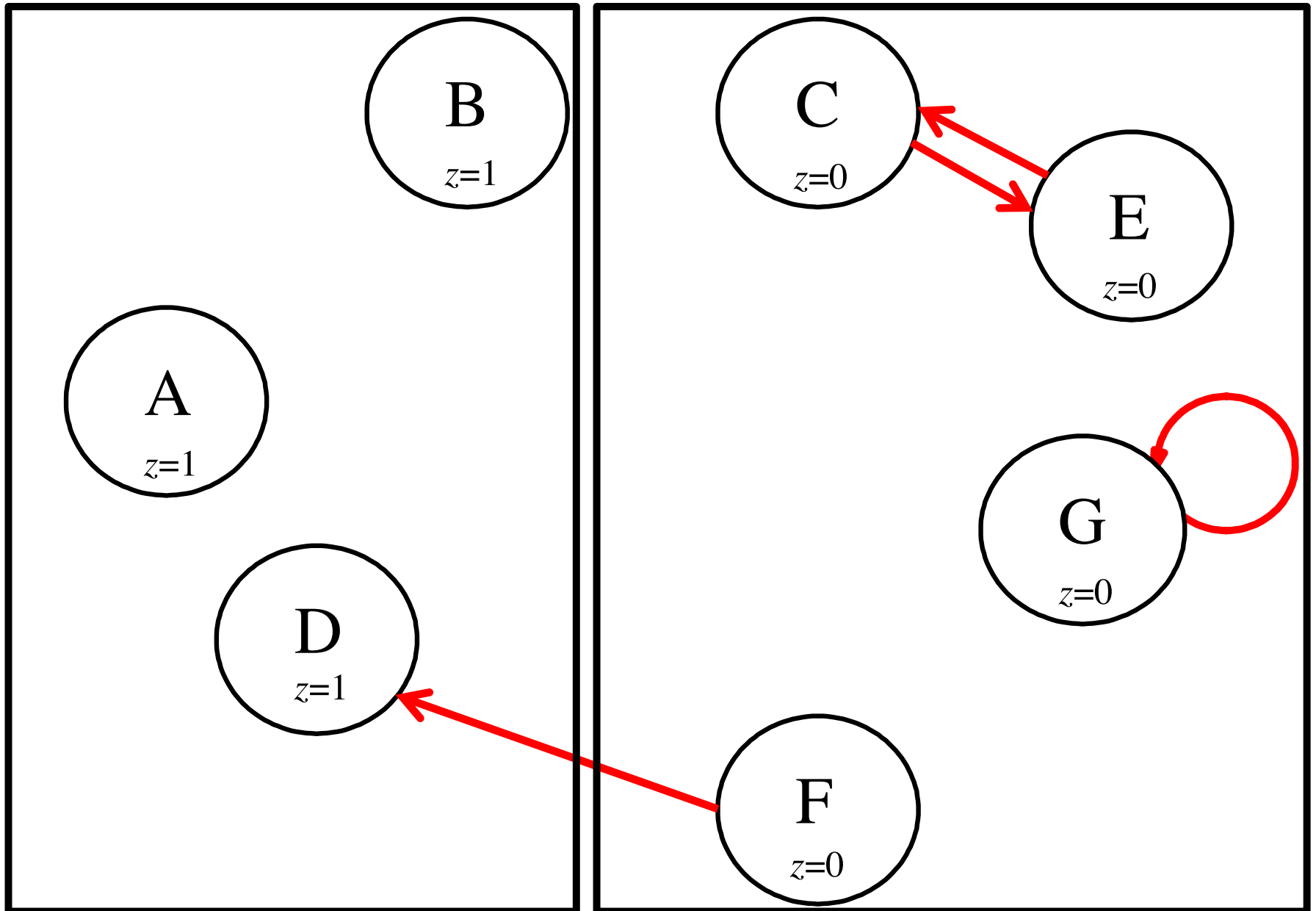
(Examine the 0-successors of CEEFG)





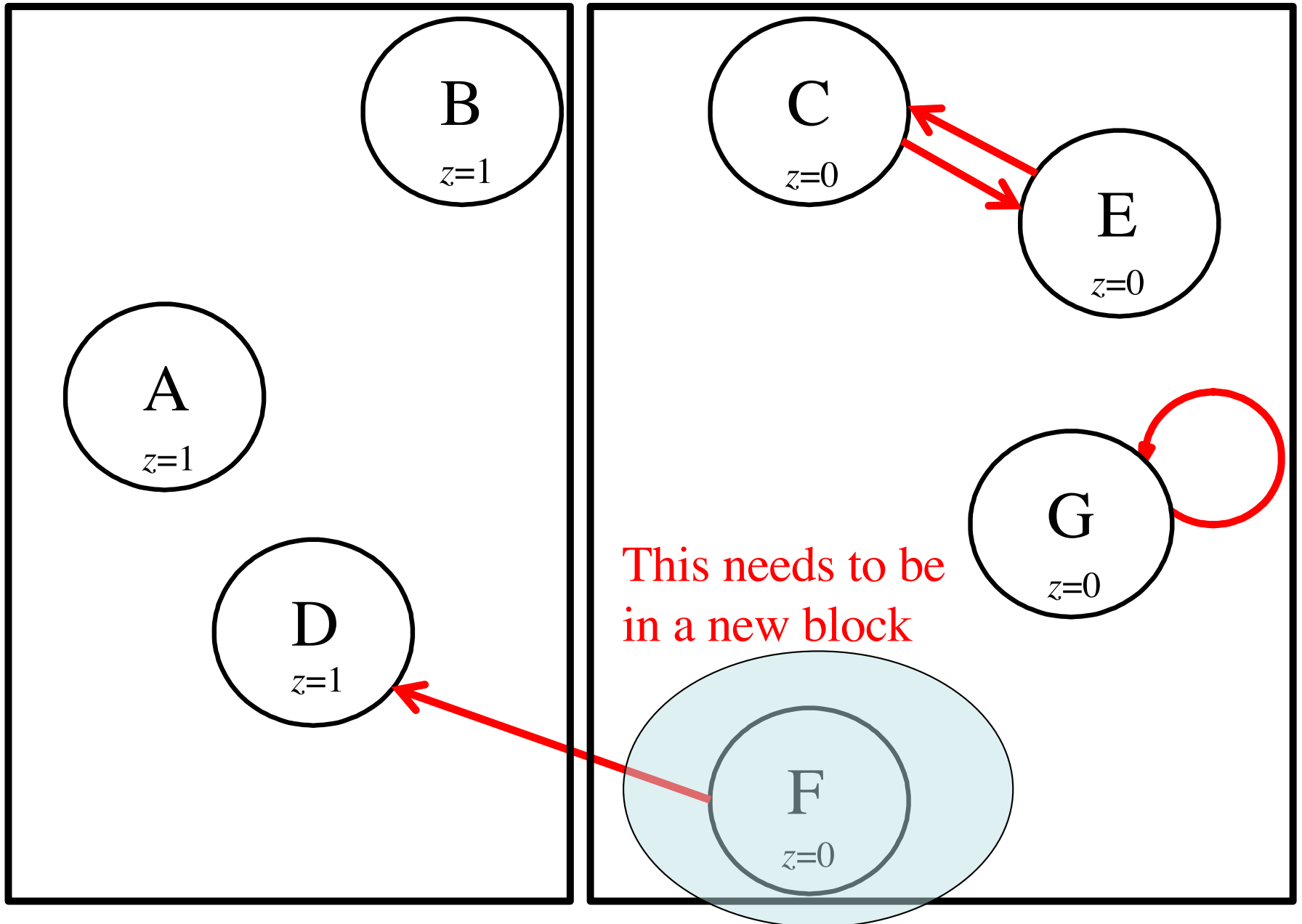
# Partition #3.2

(Examine the 1-successors of CEFG)



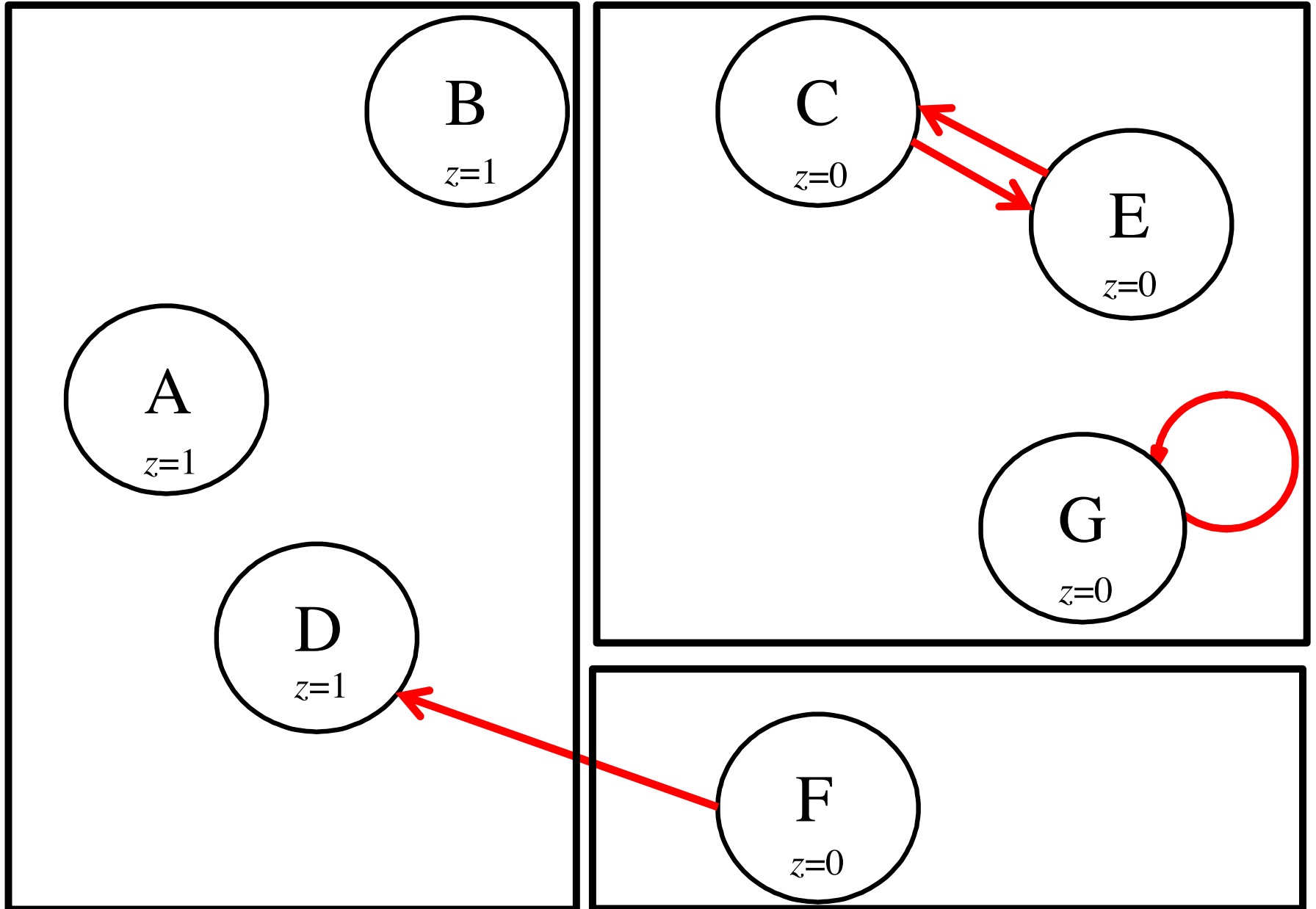
# Partition #3.2

(Examine the 1-successors of CEFG)



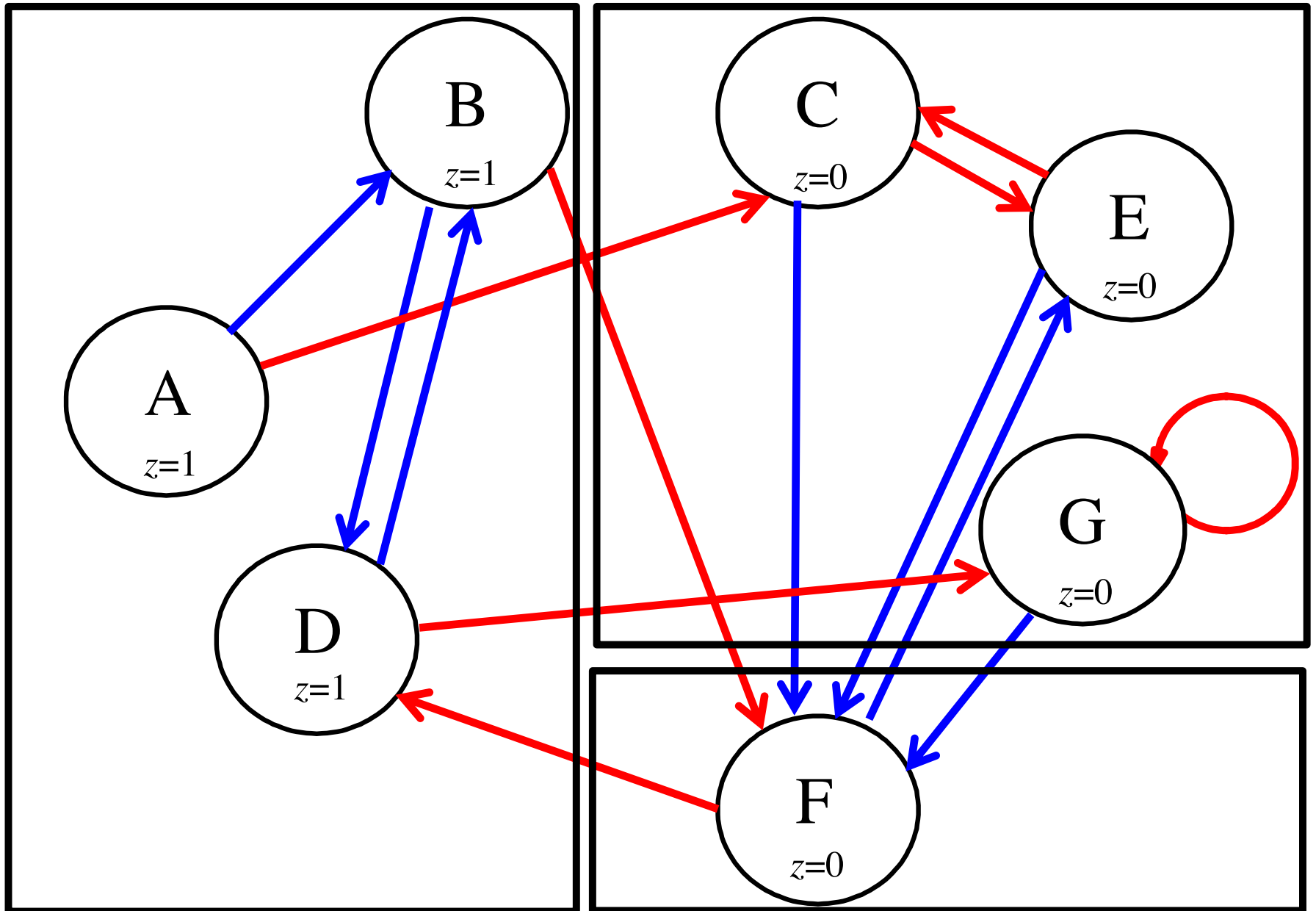
# Partition #3

(ABD)(CEG)(F)



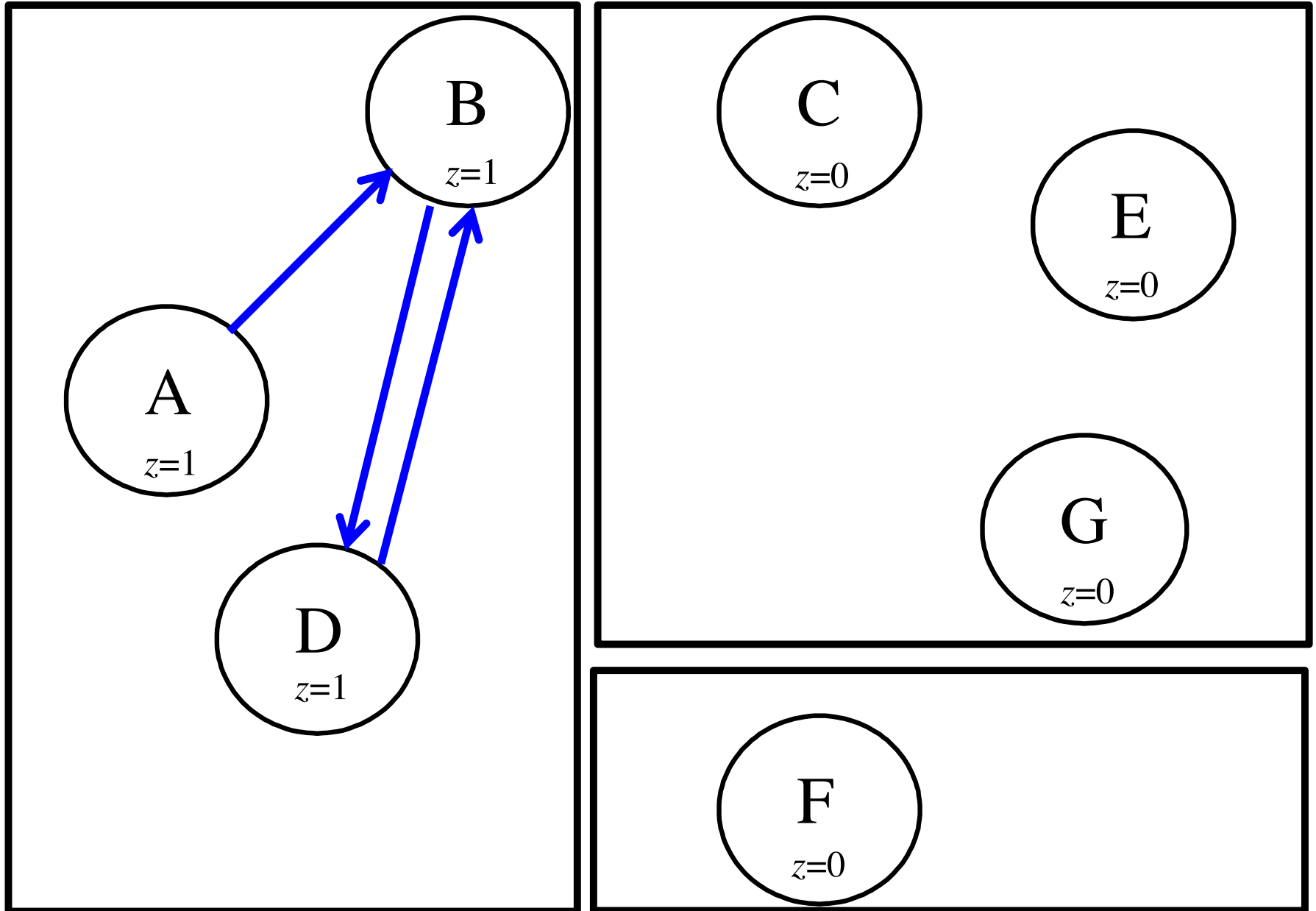
# Partition #3

(ABD)(CEG)(F)



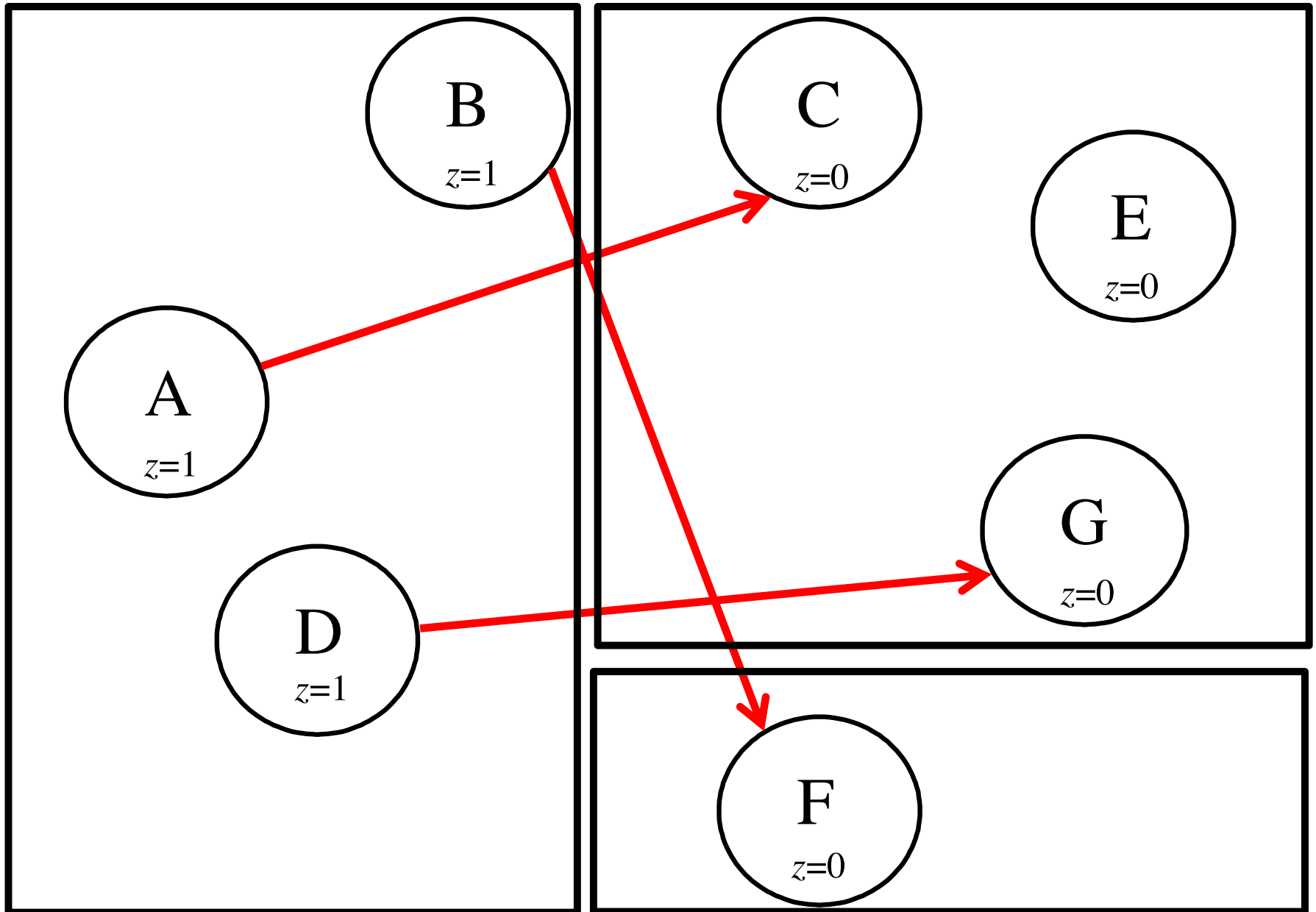
# Partition #4.1

(Examine the 0-successors of ABD)



# Partition #4.1

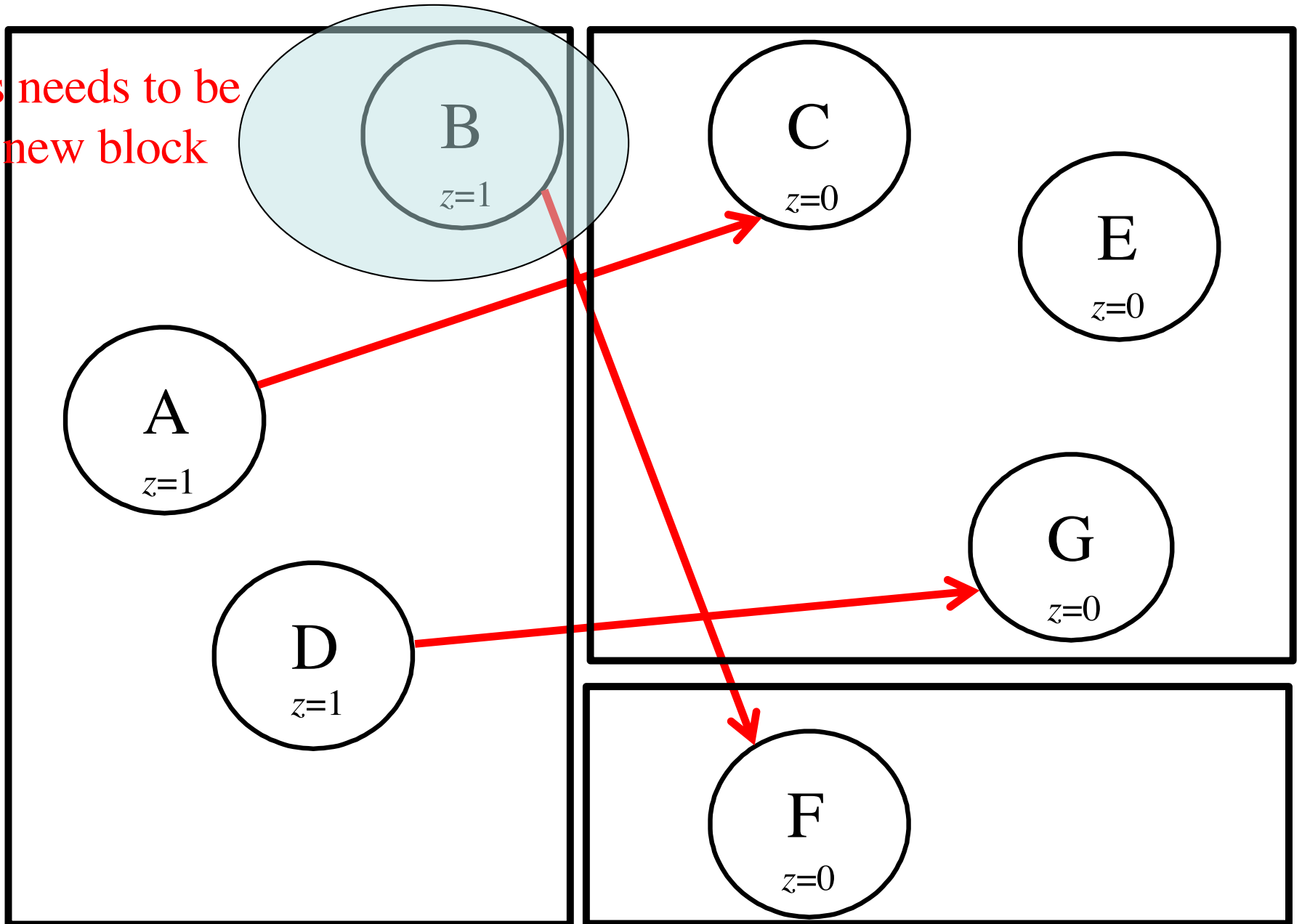
(Examine the 1-successors of ABD)



# Partition #4.1

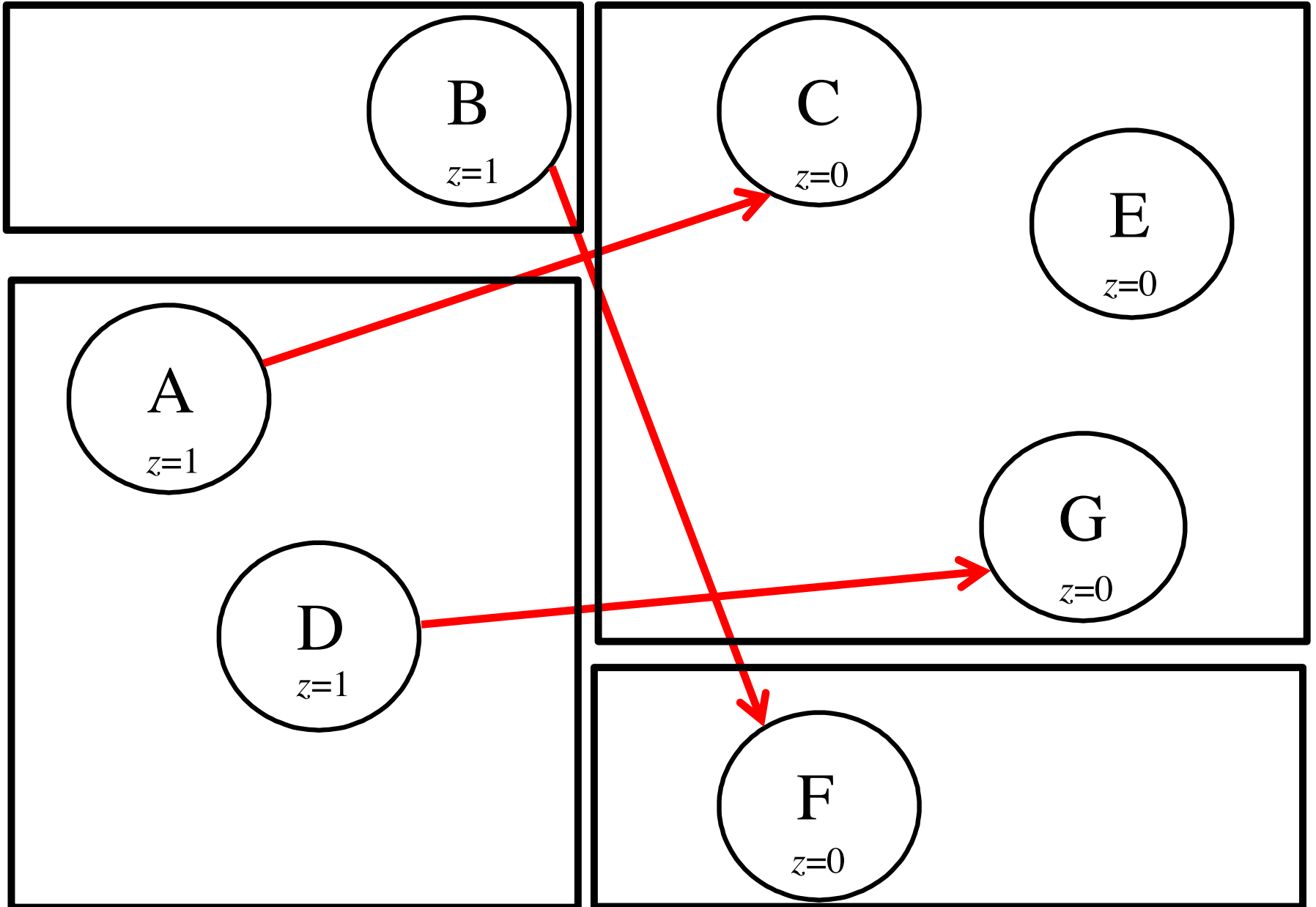
(Examine the 1-successors of ABD)

This needs to be  
in a new block



# Partition #4

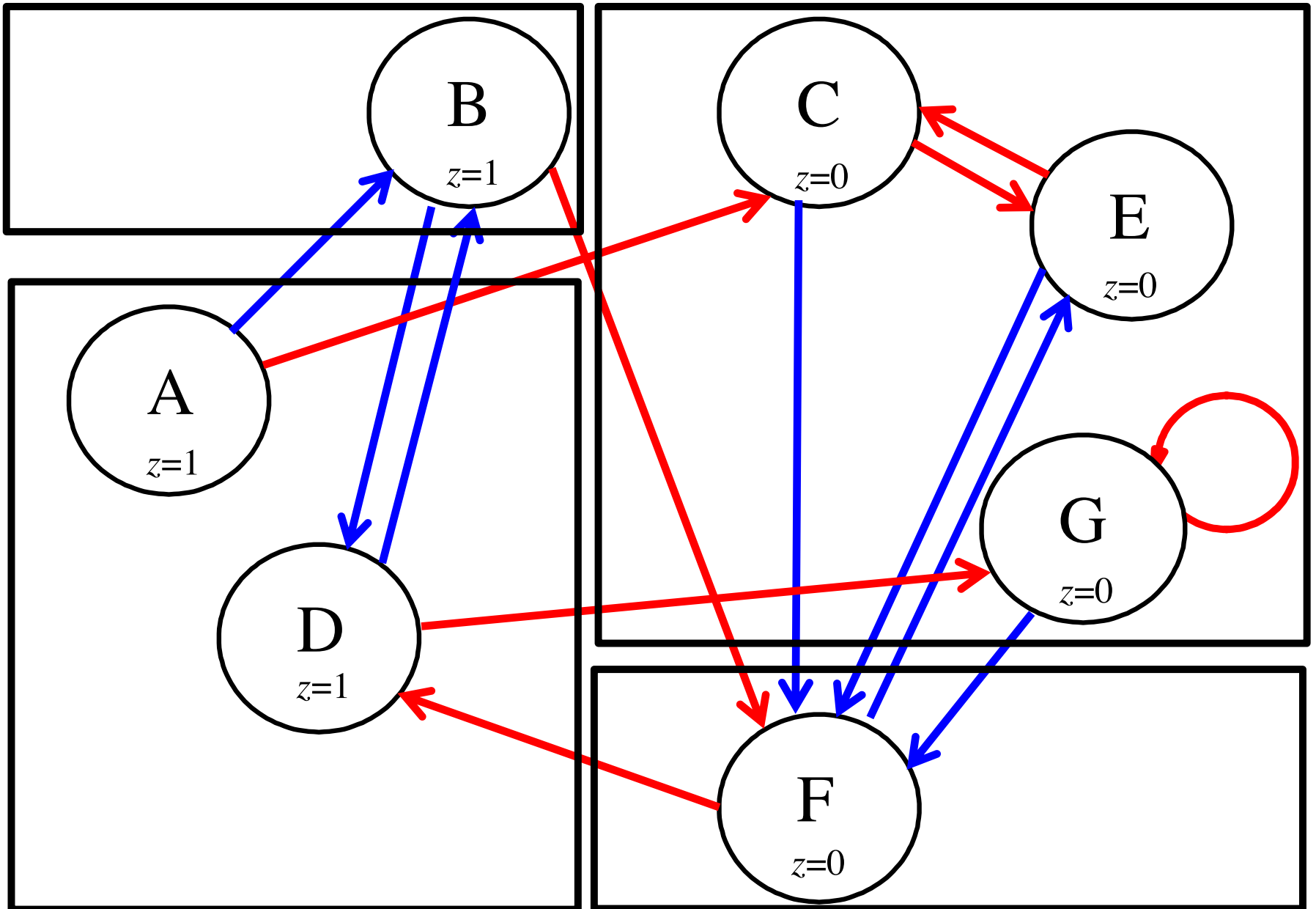
(AD)(B)(CEG)(F)





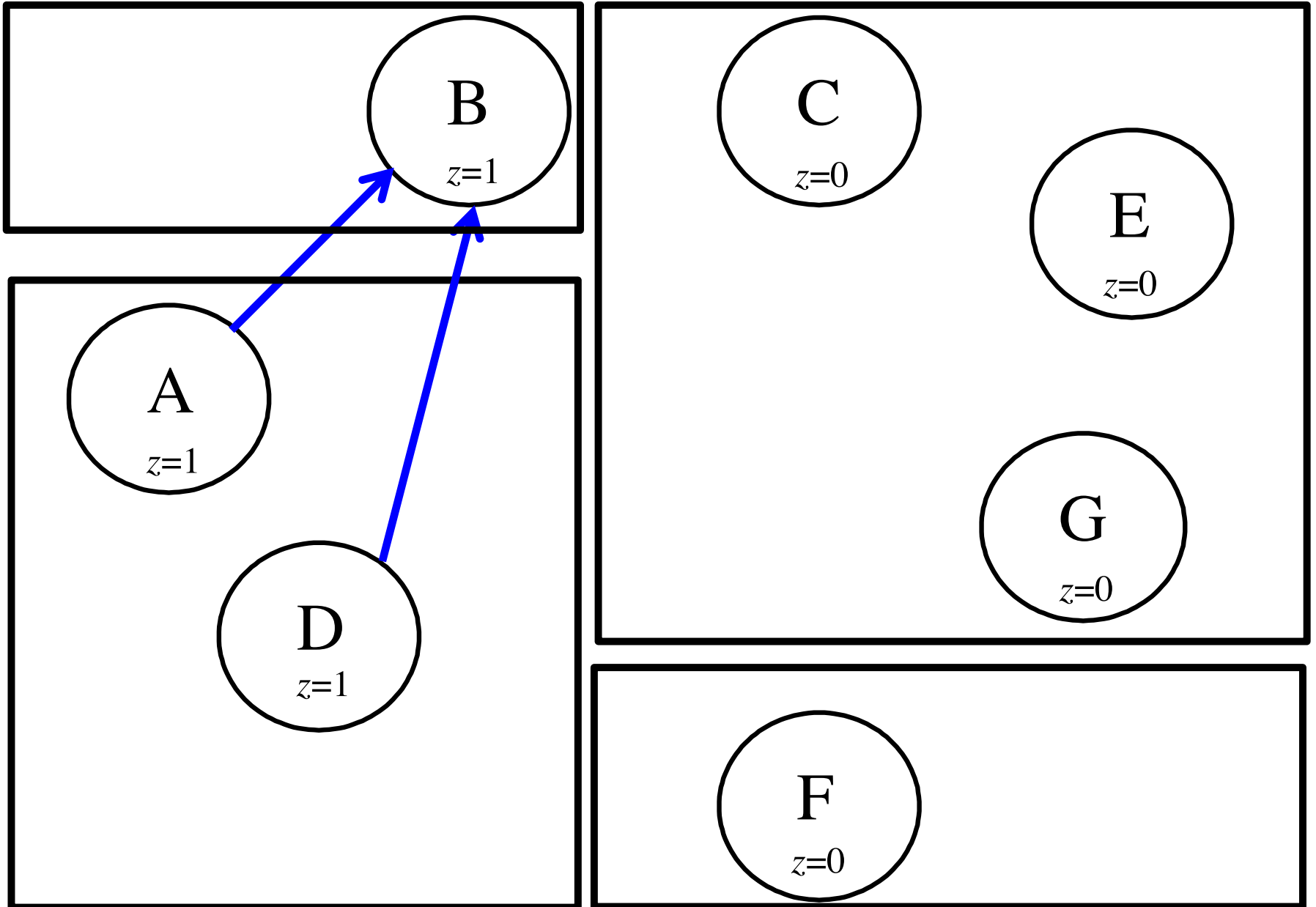
# Partition #4

(AD)(B)(CEG)(F)



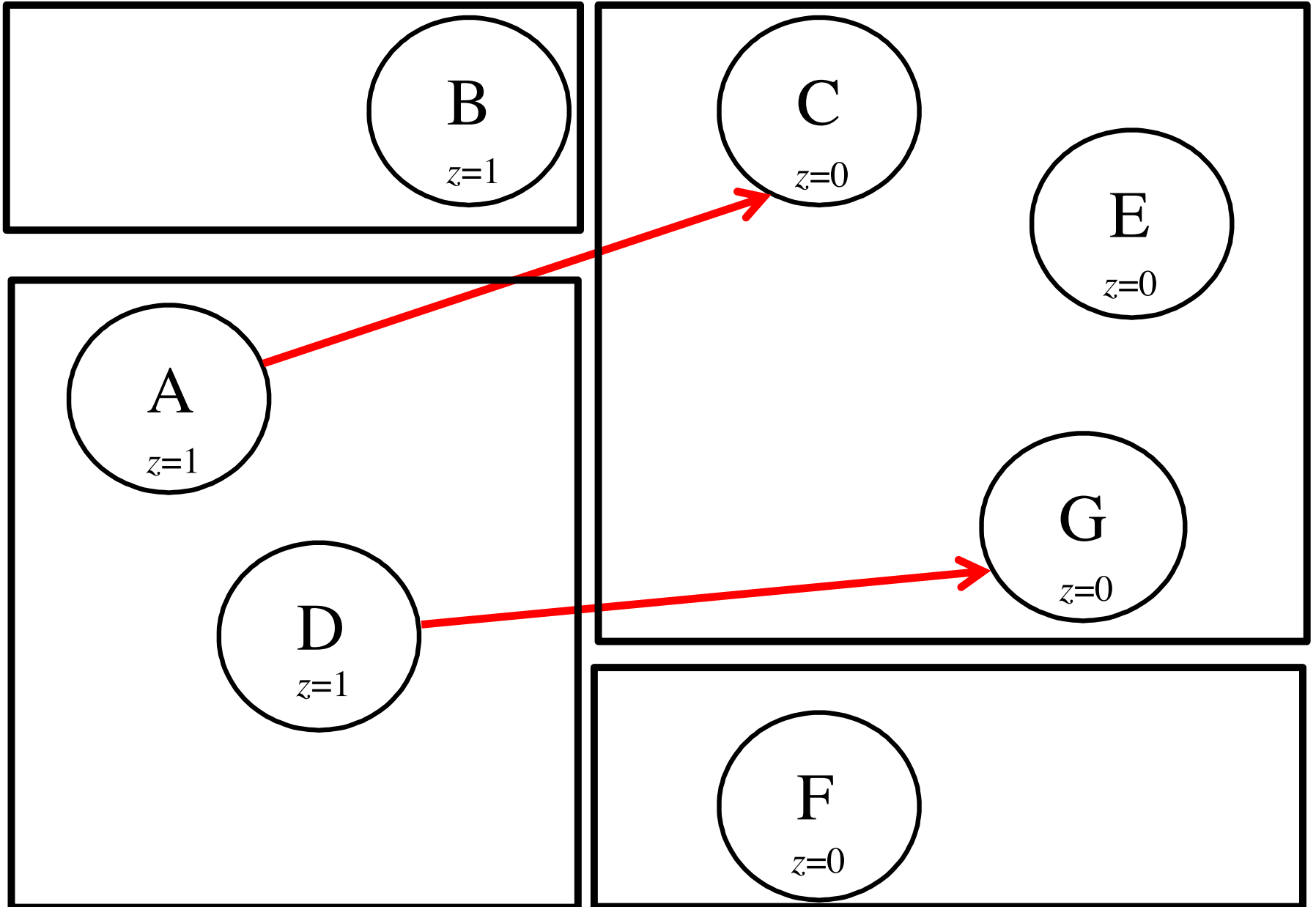
# Partition #5.1

(Examine the 0-successors of AD)



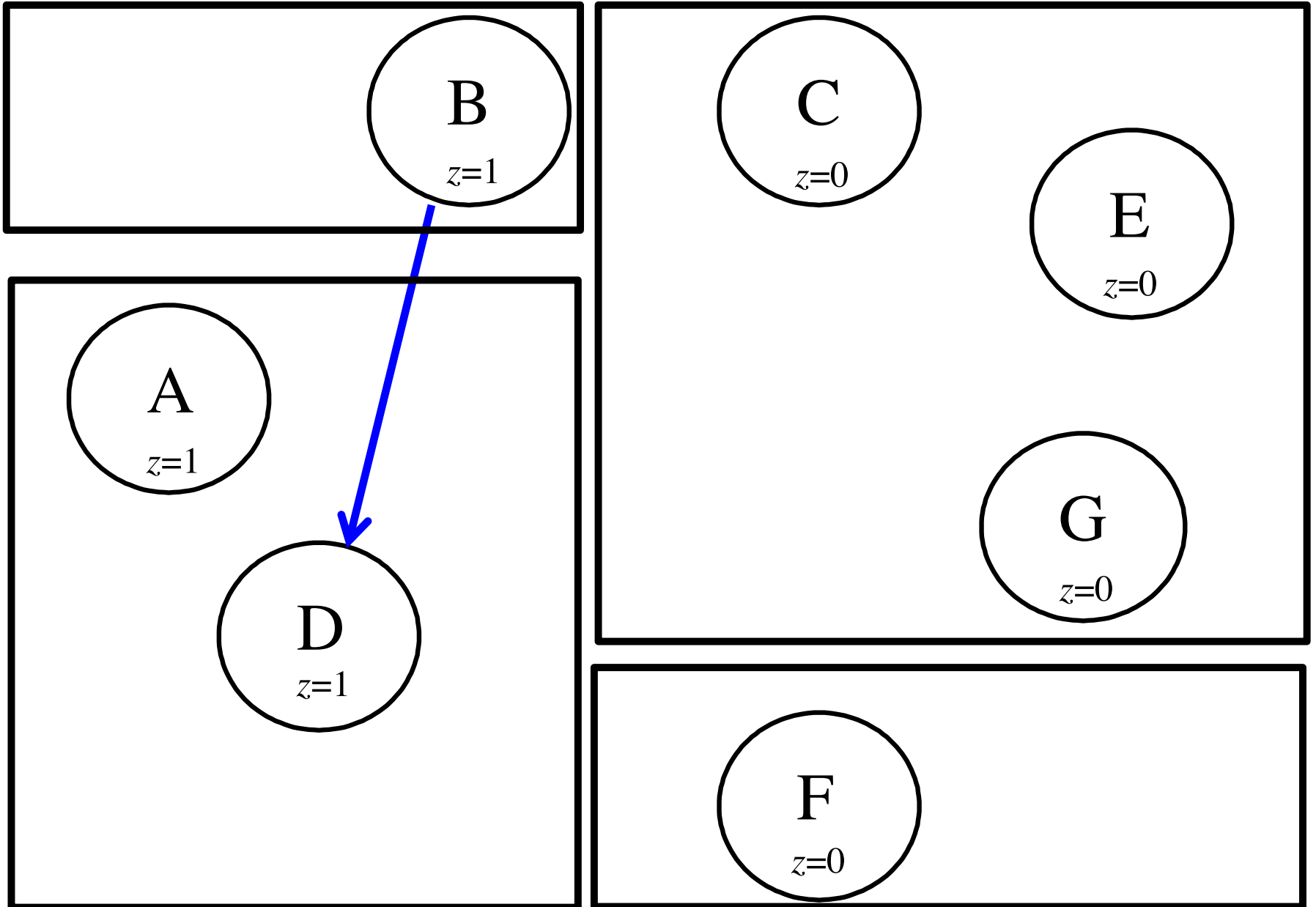
# Partition #5.1

(Examine the 1-successors of AD)



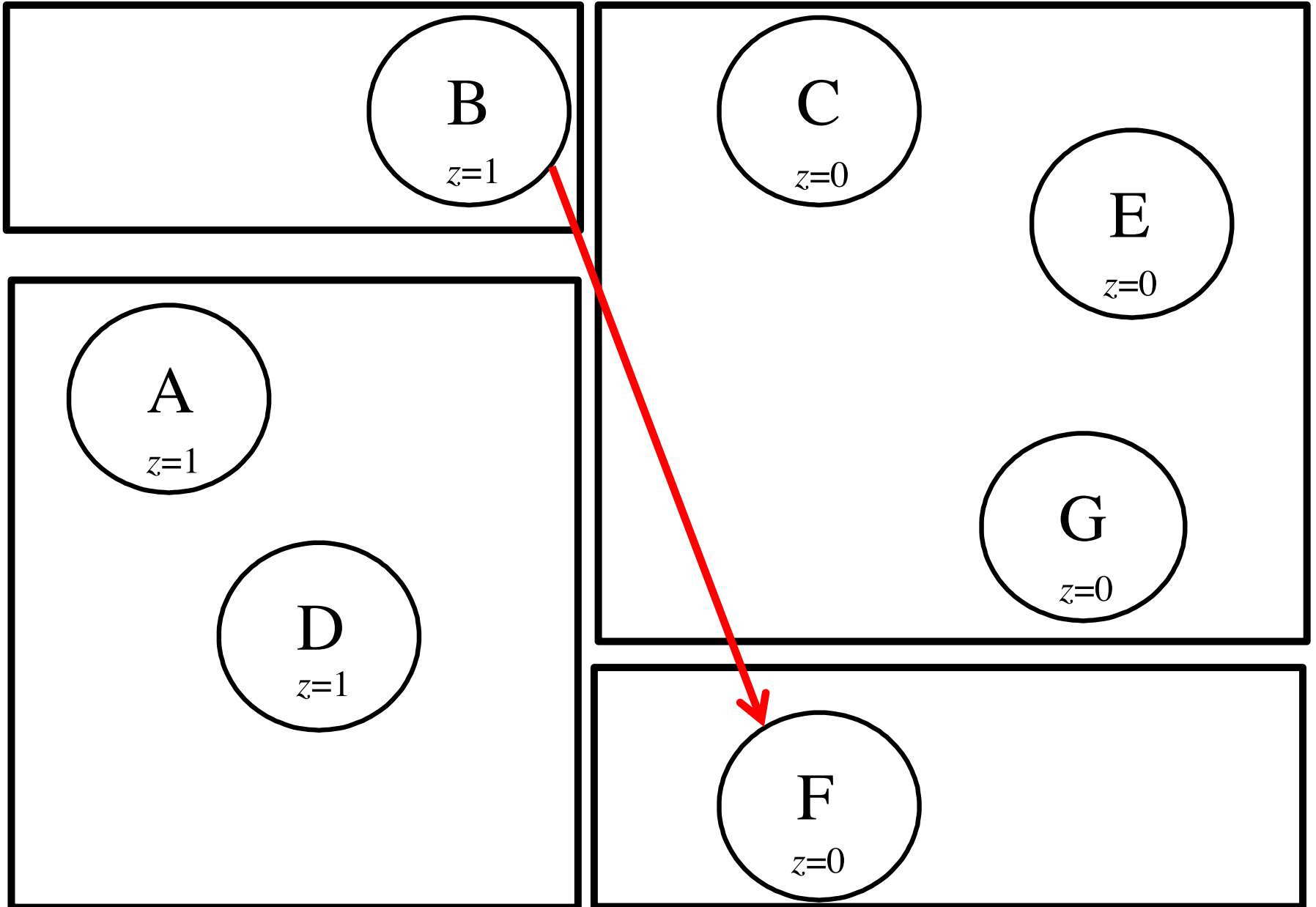
# Partition #5.2

(Examine the 0-successors of B)



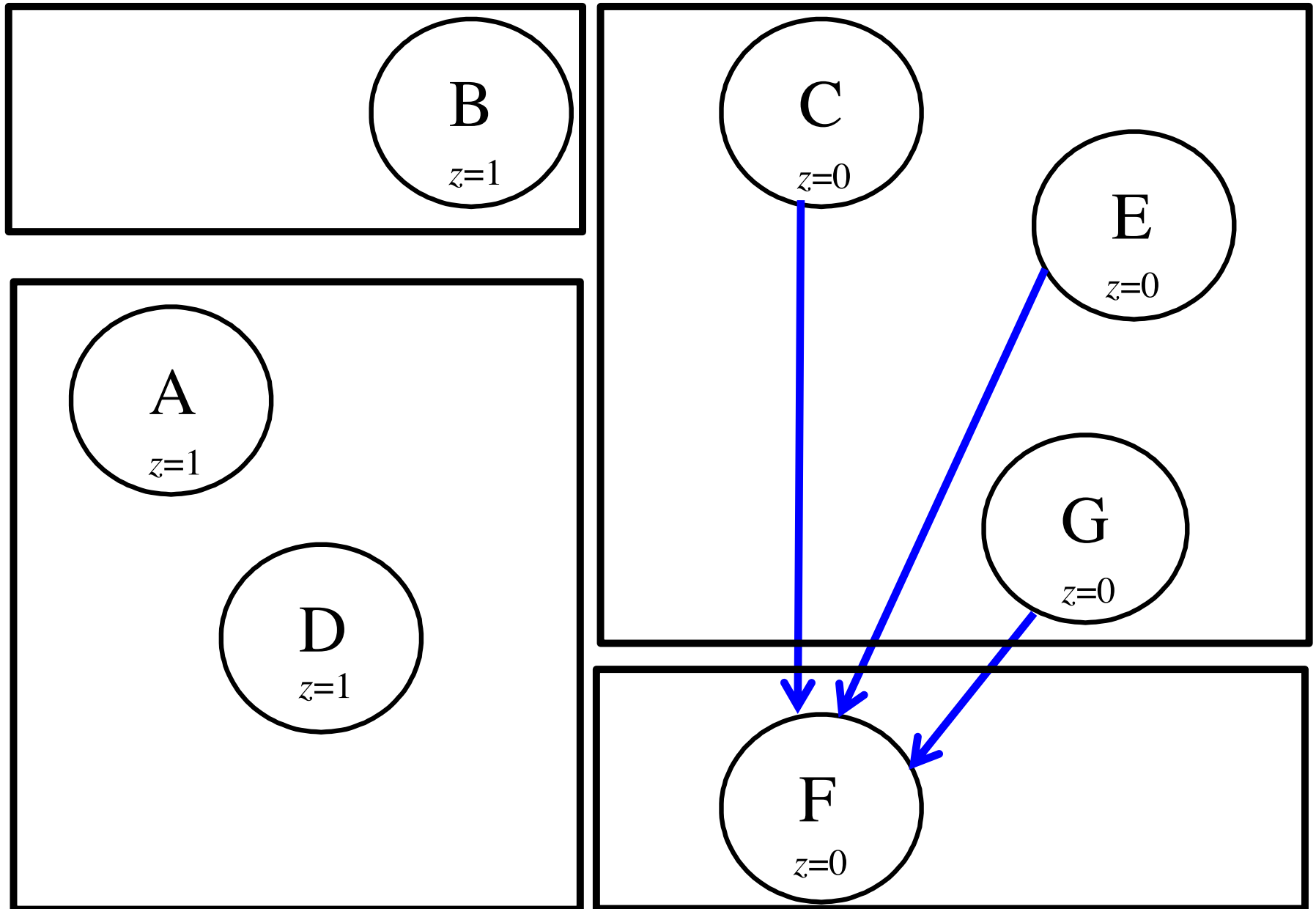
# Partition #5.2

(Examine the 1-successors of B)



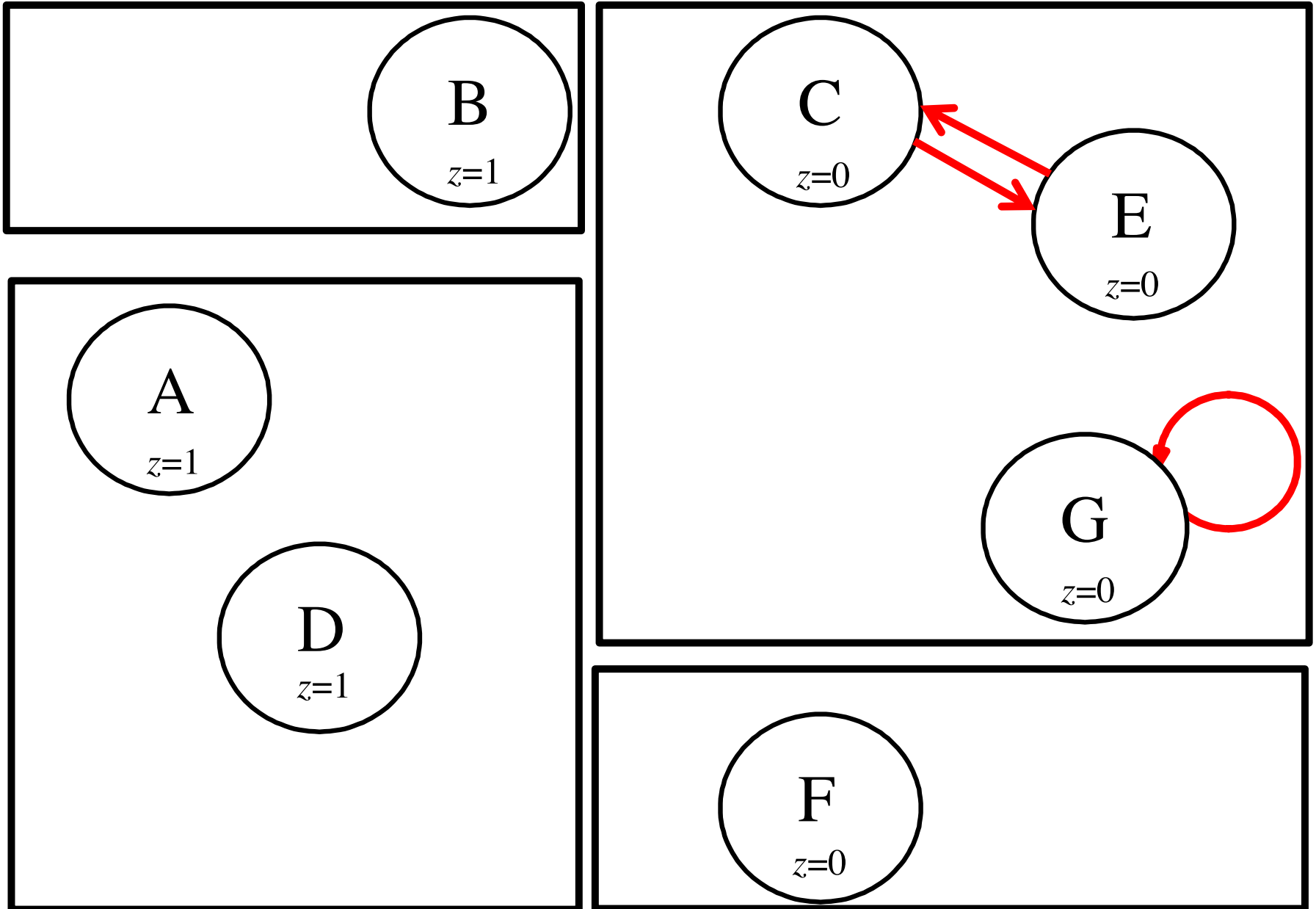
# Partition #5.3

(Examine the 0-successors of CEG)



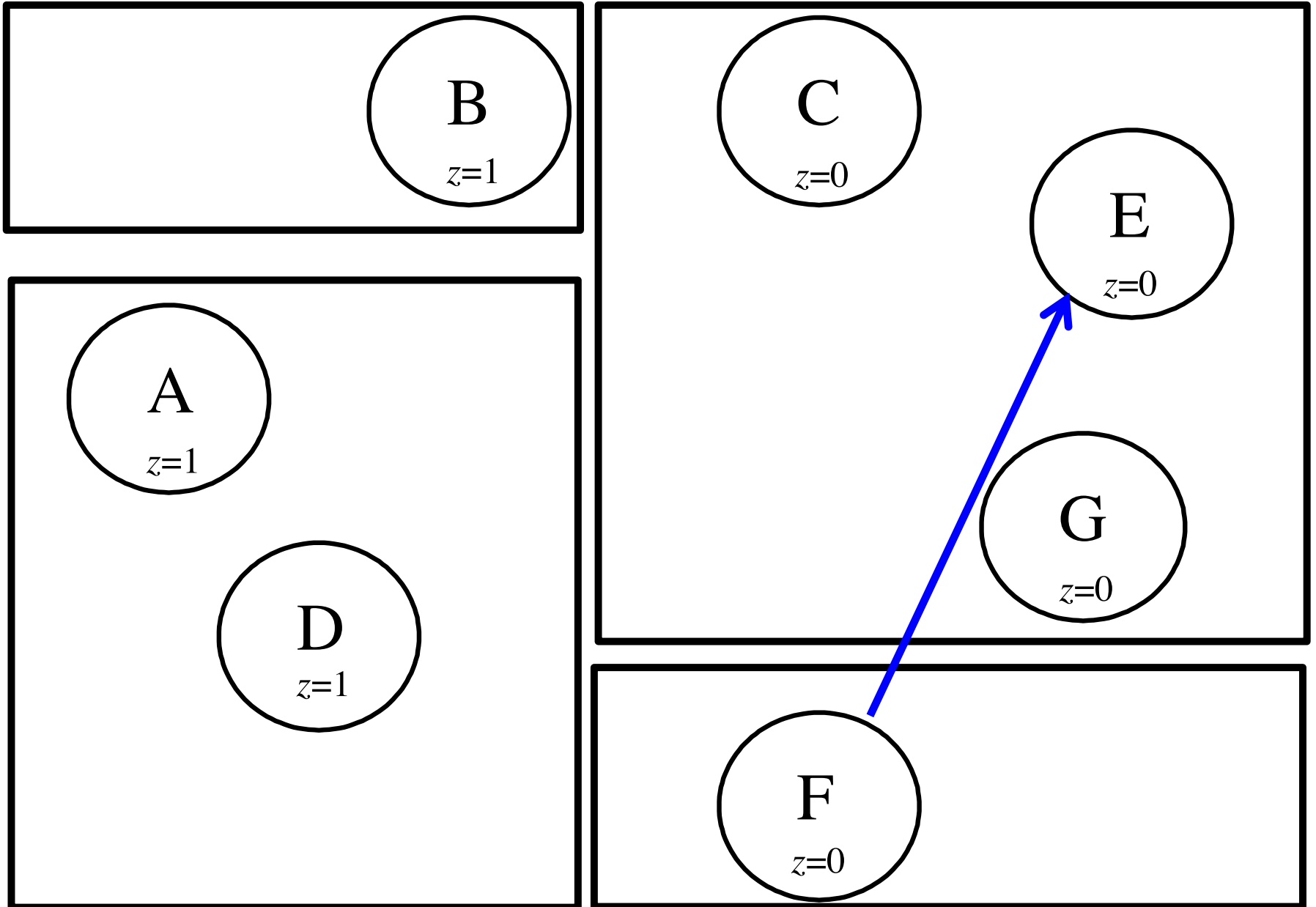
# Partition #5.3

(Examine the 1-successors of CEG)



# Partition #5.4

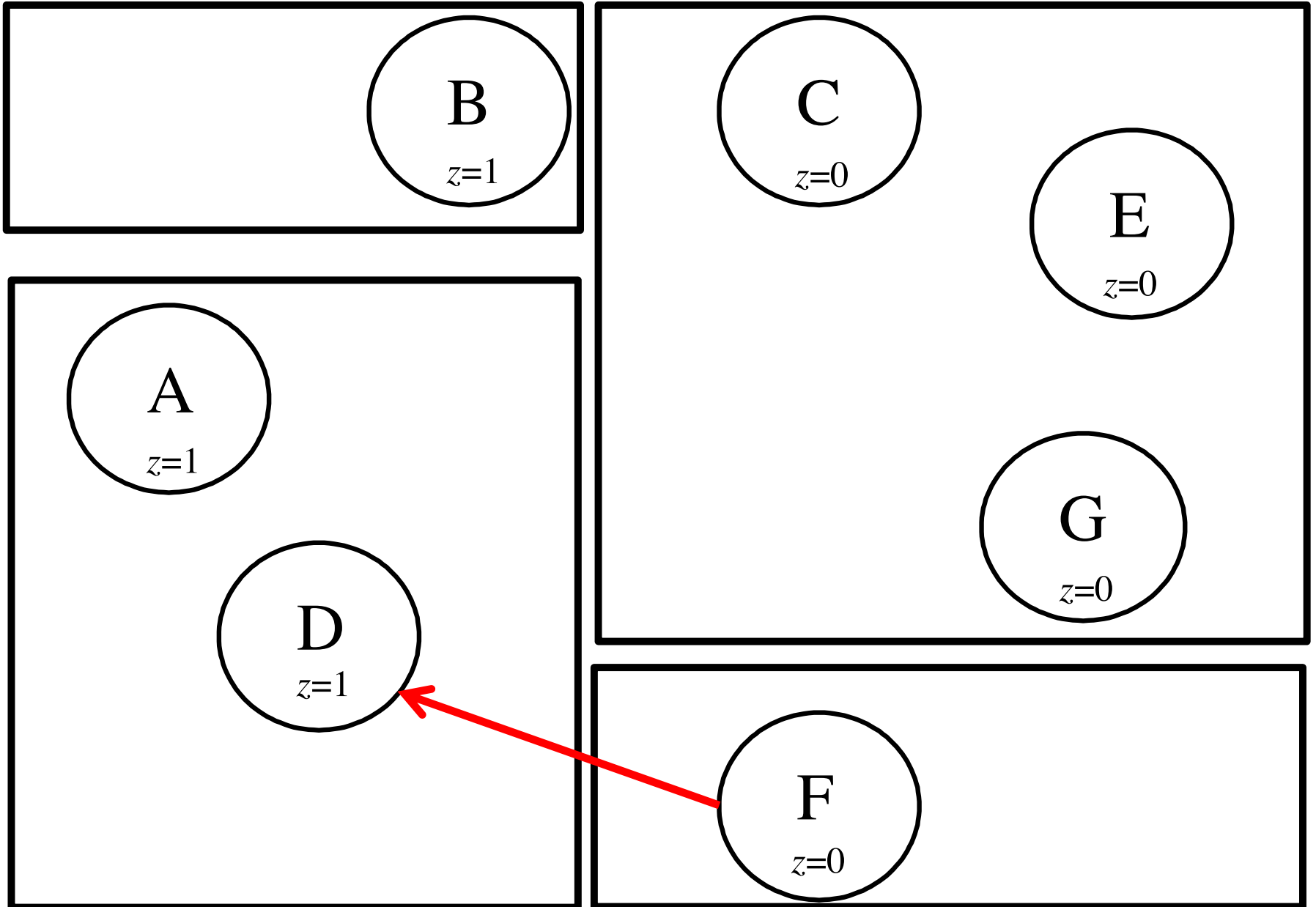
(Examine the 0-successors of F)





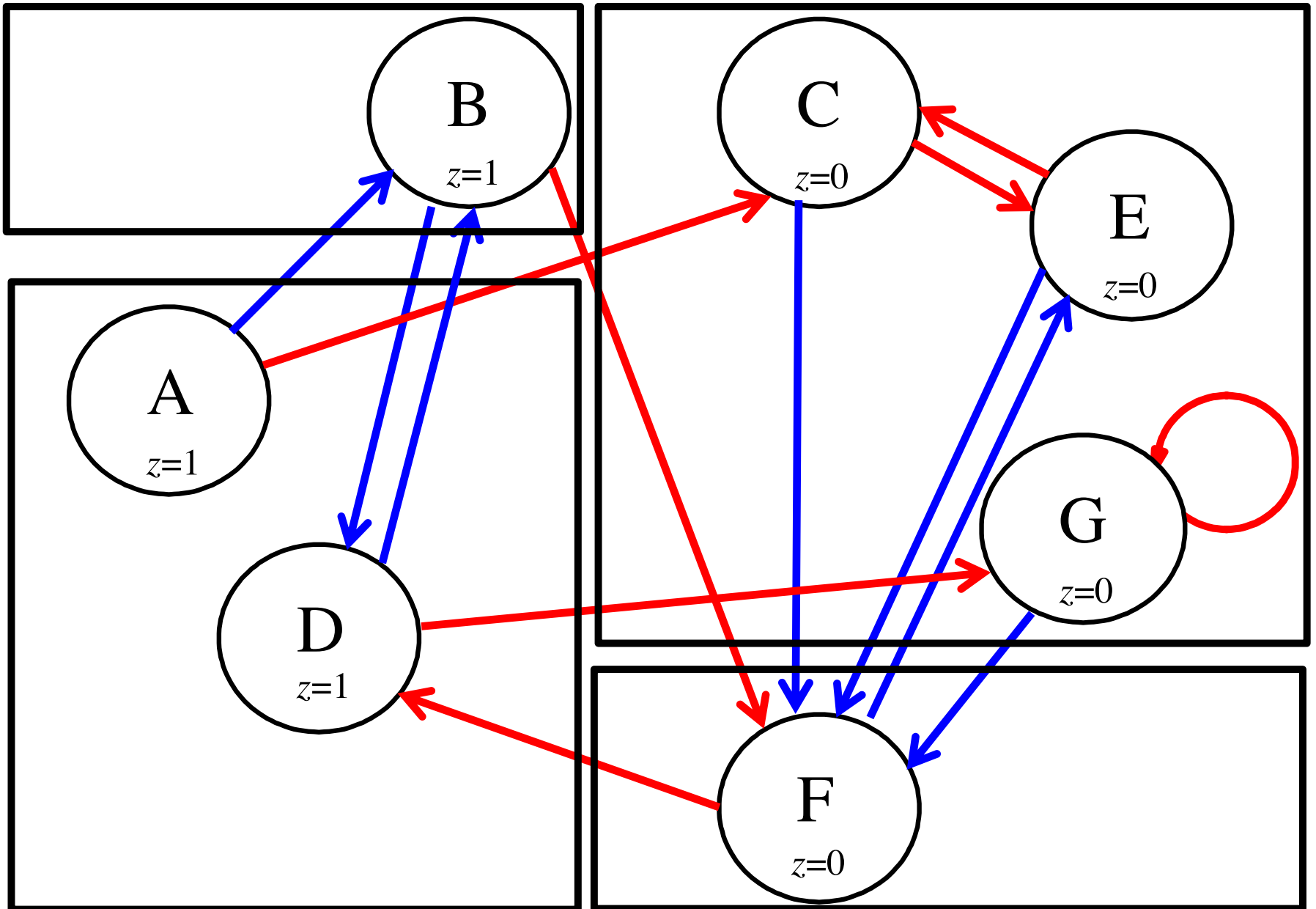
# Partition #5.4

(Examine the 1-successors of F)



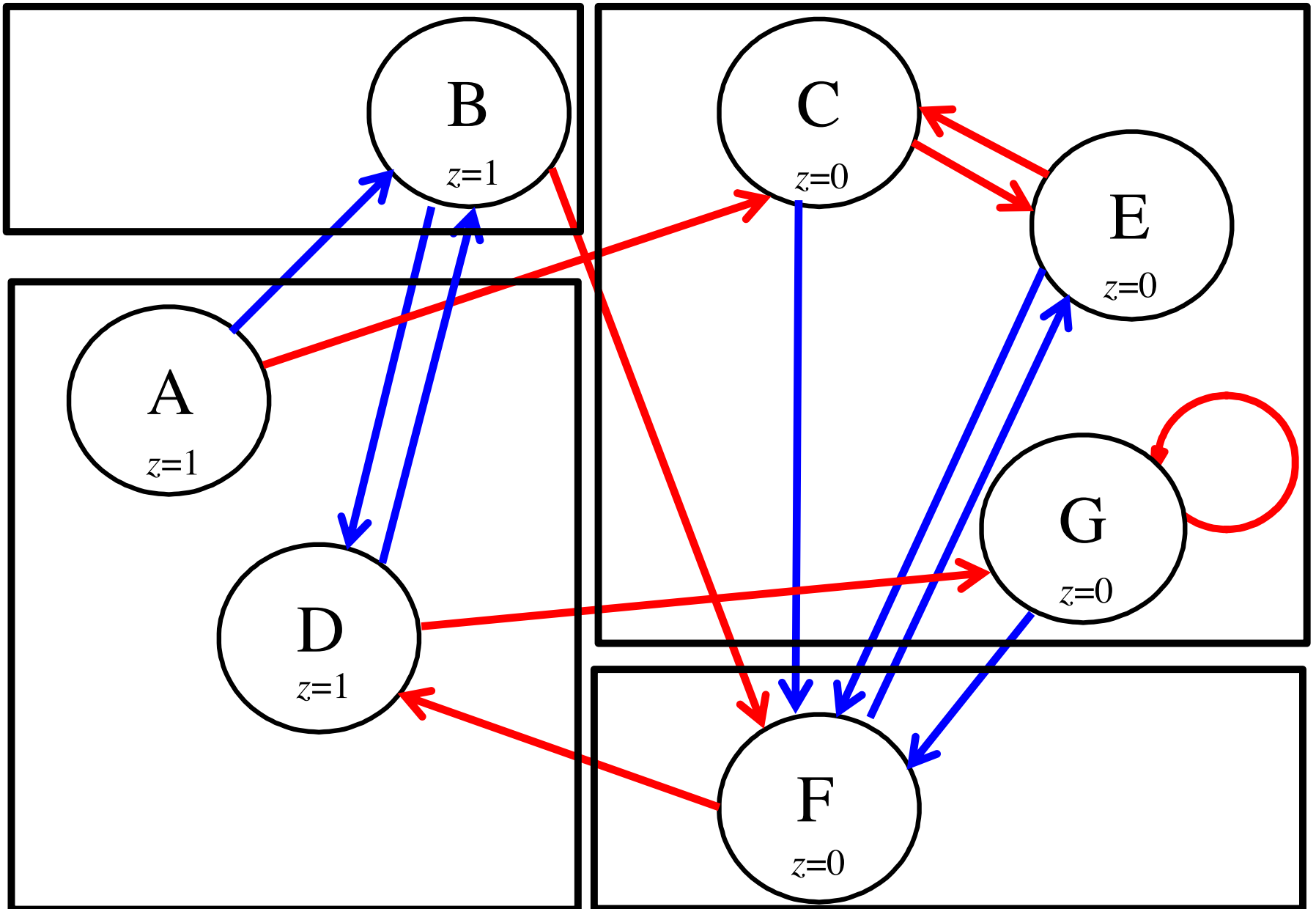
# Partition #5

(AD)(B)(CEG)(F)



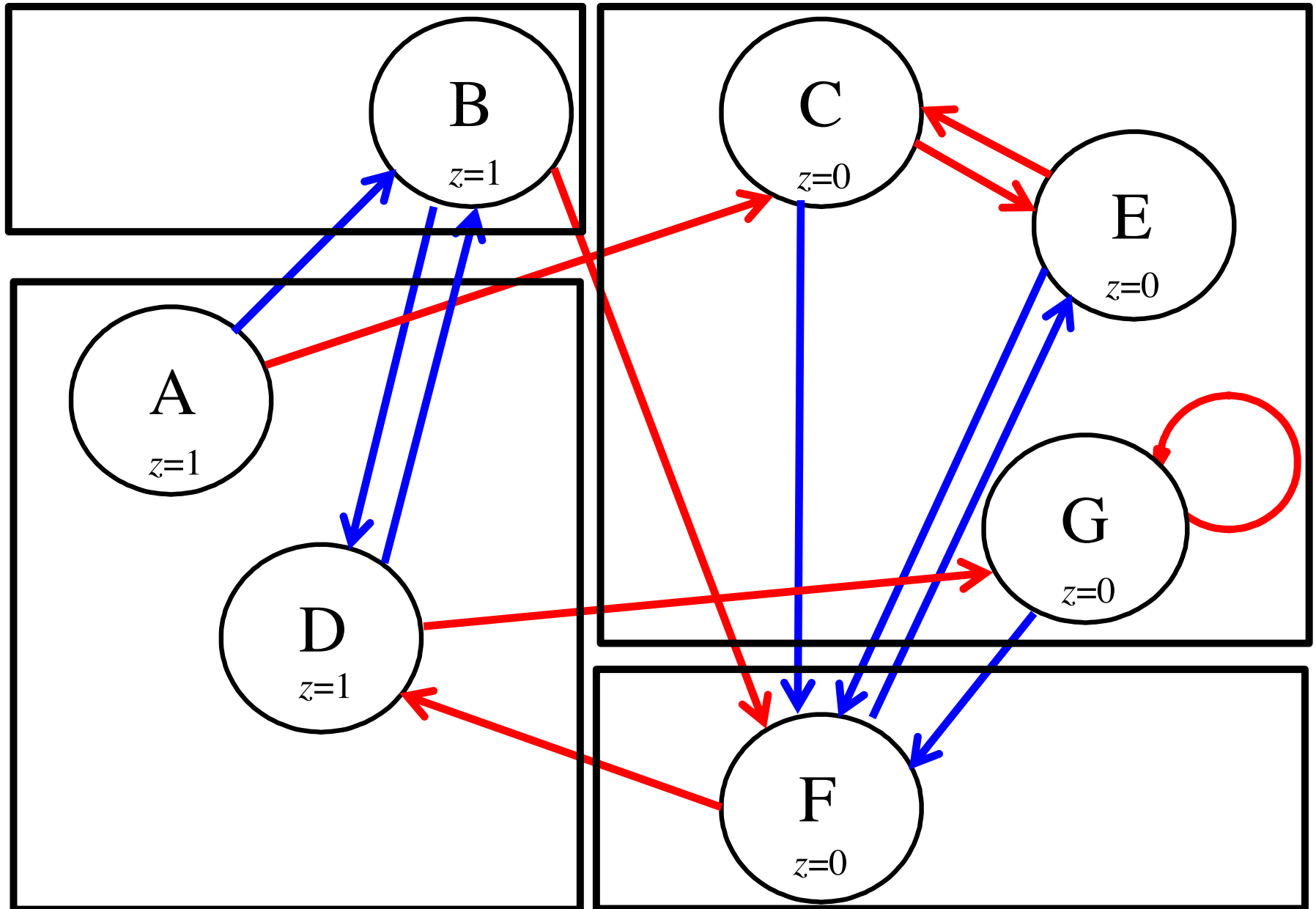
# Partition #4

(AD)(B)(CEG)(F)



# Partition #5

(This is the same as #4 so we can stop here)



# Minimized state table

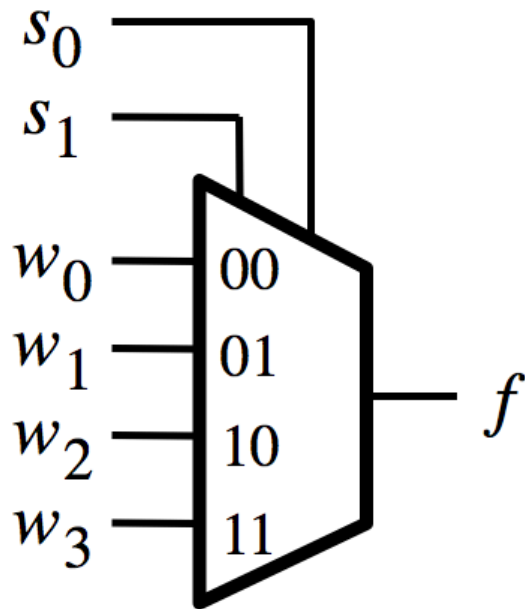
Present state	Nextstate		Output z
	w = 0	w = 1	
A	B	C	1
B	A	F	1
C	F	C	0
F	C	A	0

# Multiplexers

# 4-1 Multiplexer (Definition)

- Has four inputs:  $w_0$ ,  $w_1$ ,  $w_2$ ,  $w_3$
- Also has two select lines:  $s_1$  and  $s_0$
- If  $s_1=0$  and  $s_0=0$ , then the output  $f$  is equal to  $w_0$
- If  $s_1=0$  and  $s_0=1$ , then the output  $f$  is equal to  $w_1$
- If  $s_1=1$  and  $s_0=0$ , then the output  $f$  is equal to  $w_2$
- If  $s_1=1$  and  $s_0=1$ , then the output  $f$  is equal to  $w_3$

# Graphical Symbol and Truth Table



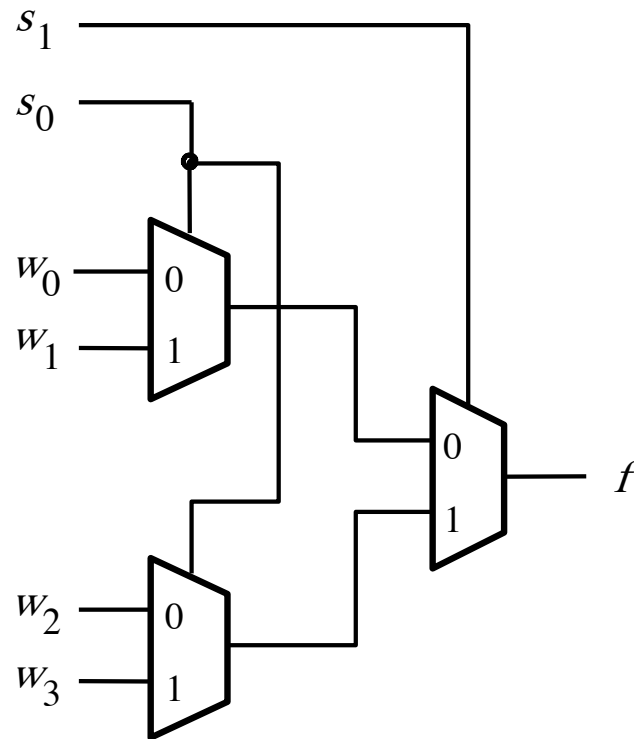
(a) Graphic symbol

$s_1$	$s_0$	$f$
0	0	$w_0$
0	1	$w_1$
1	0	$w_2$
1	1	$w_3$

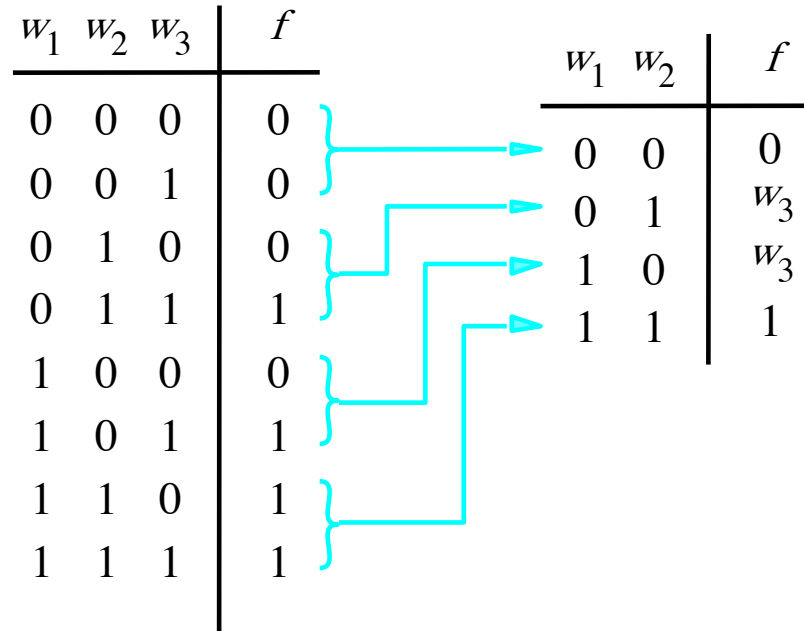
(b) Truth table



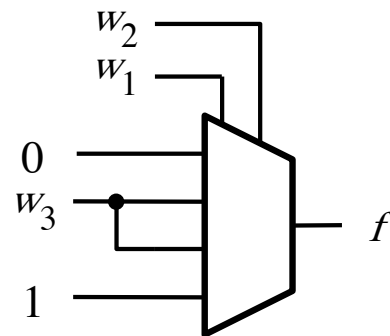
# Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



# Implementation of a logic function



(a) Modified truth table



(b) Circuit

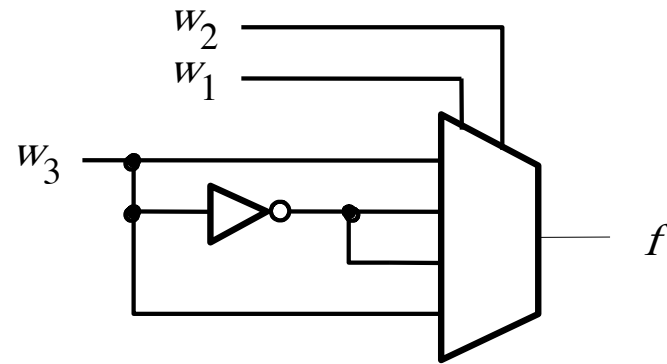
# Implementation of 3-input XOR with a 4-to-1 Multiplexer

$w_1$	$w_2$	$w_3$	$f$	
0	0	0	0	} $w_3$
0	0	1	1	
0	1	0	1	} $\bar{w}_3$
0	1	1	0	
1	0	0	1	} $\bar{w}_3$
1	0	1	0	
1	1	0	0	} $w_3$
1	1	1	1	

# Implementation of 3-input XOR with a 4-to-1 Multiplexer

$w_1$	$w_2$	$w_3$	$f$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

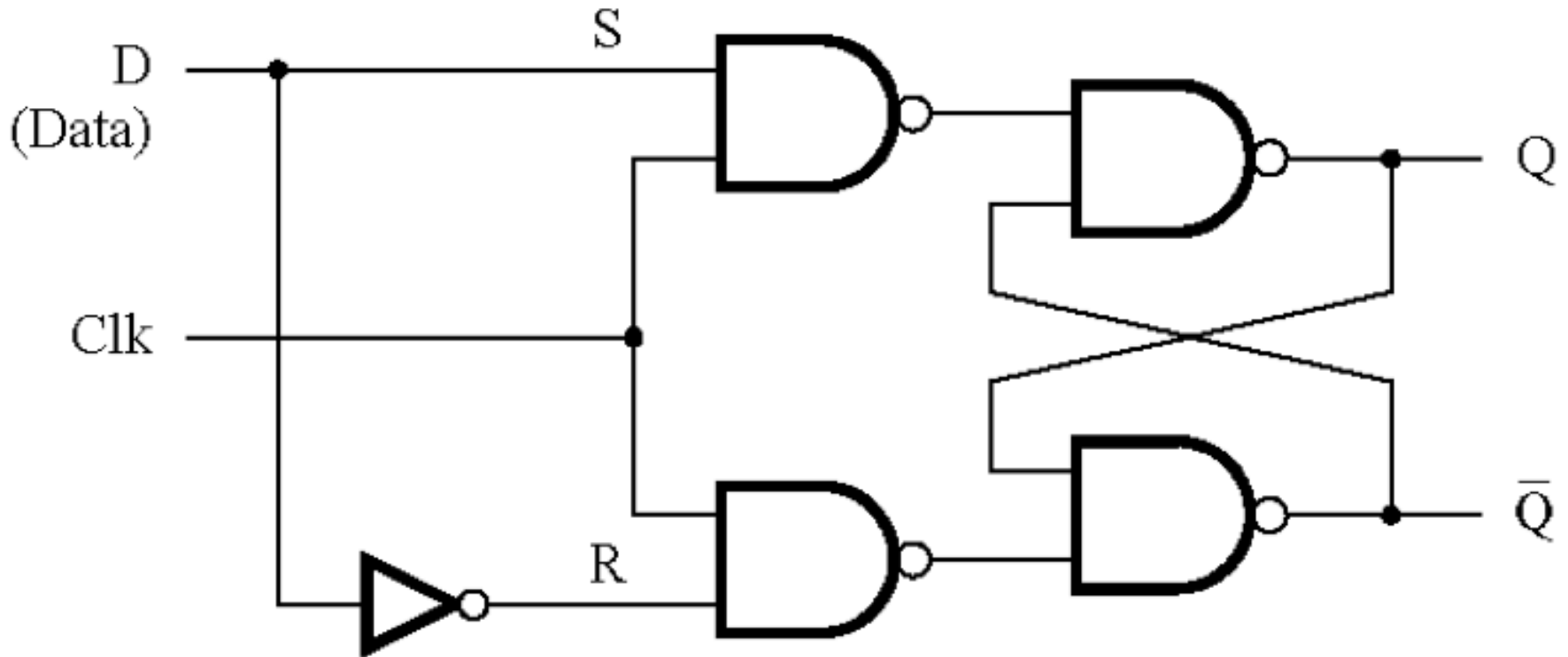
(a) Truth table



(b) Circuit

# **Gated D Latch**

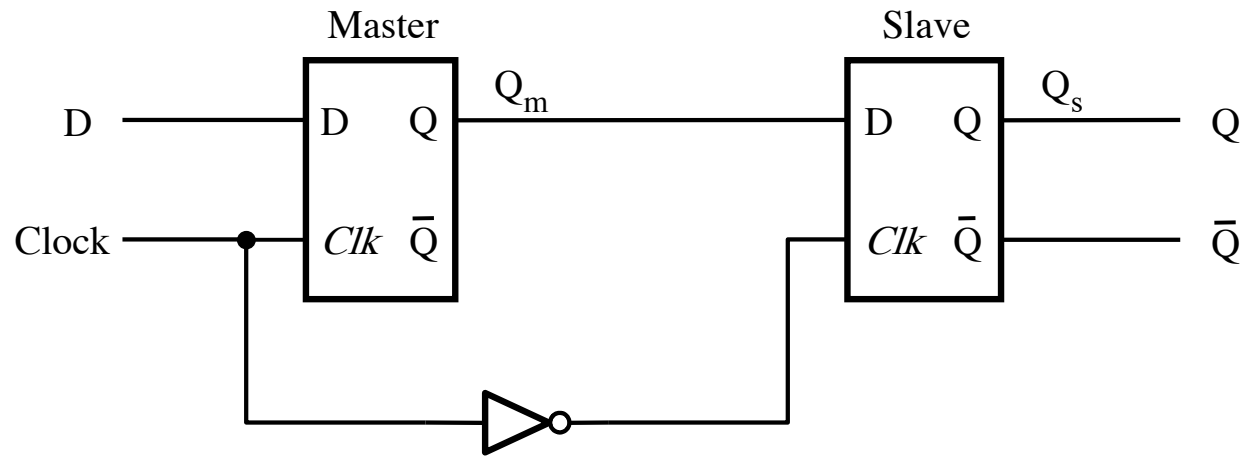
# Circuit Diagram for the Gated D Latch



[ Figure 5.7a from the textbook ]

# Edge-Triggered D Flip-Flops

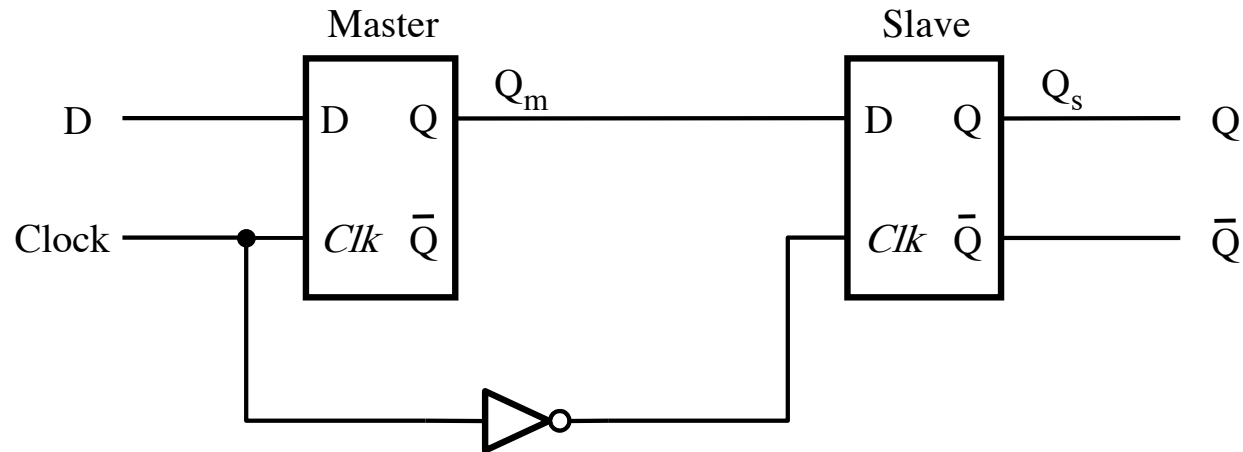
# Master-Slave D Flip-Flop



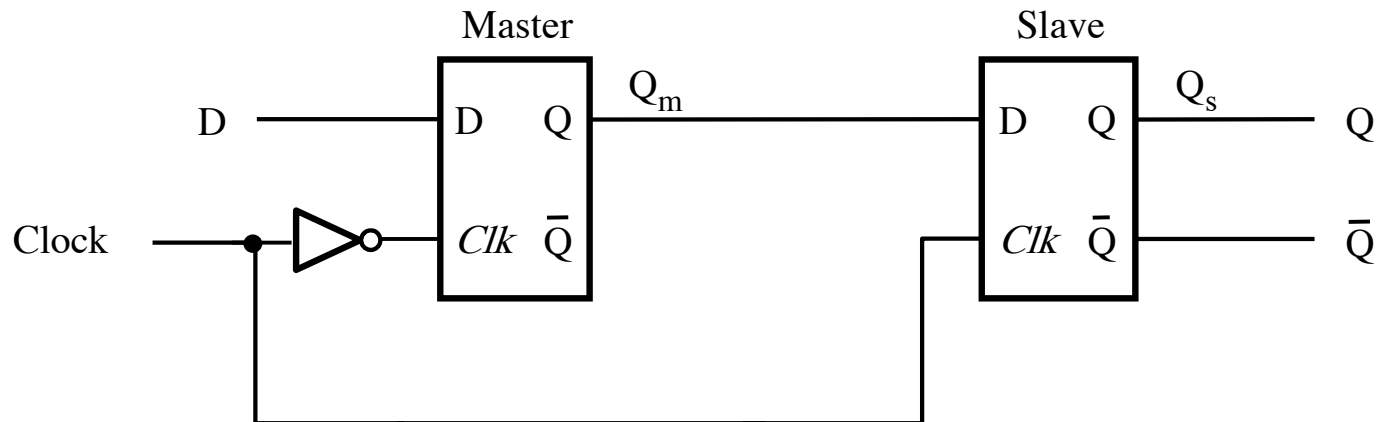
(a) Circuit



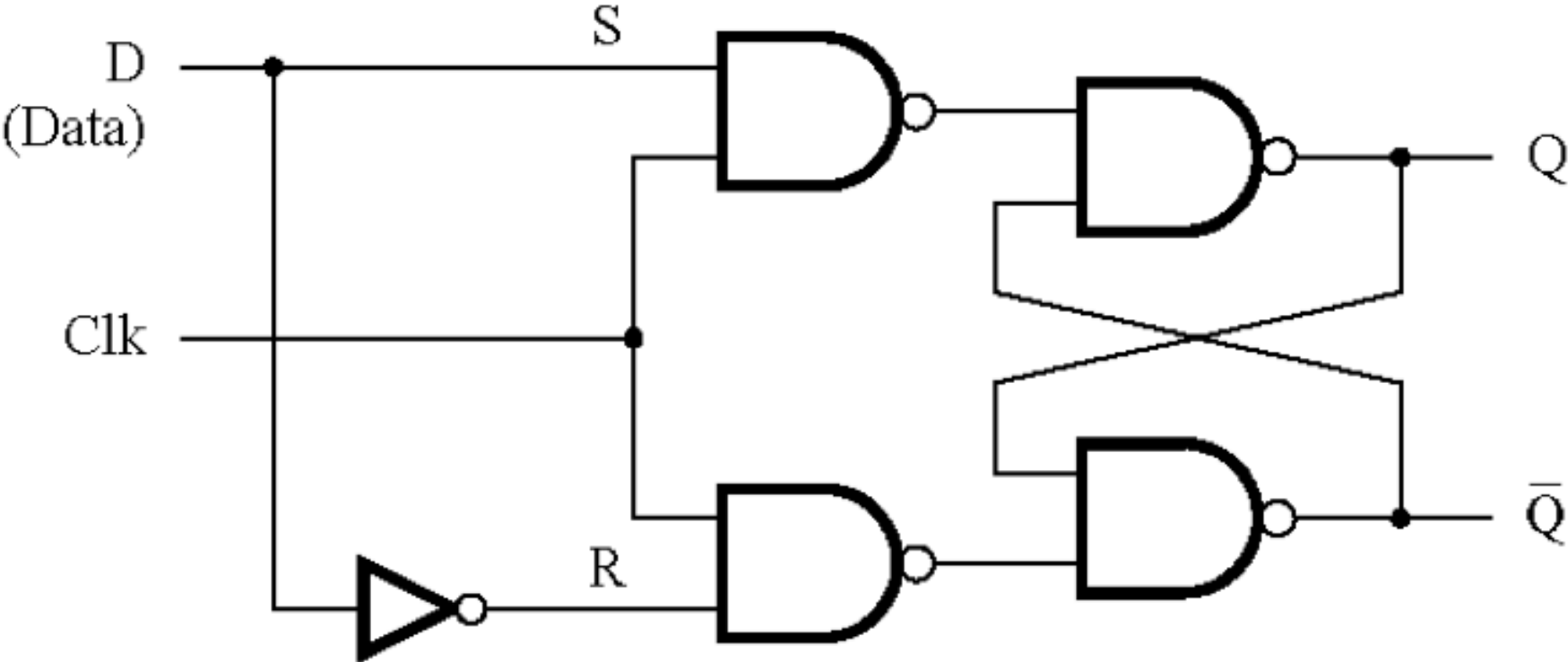
# Negative-Edge-Triggered Master-Slave D Flip-Flop



# Positive-Edge-Triggered Master-Slave D Flip-Flop

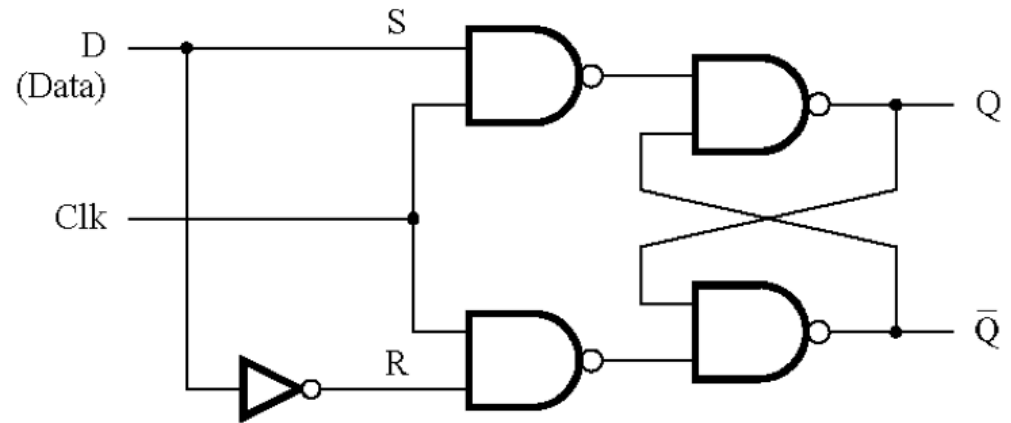
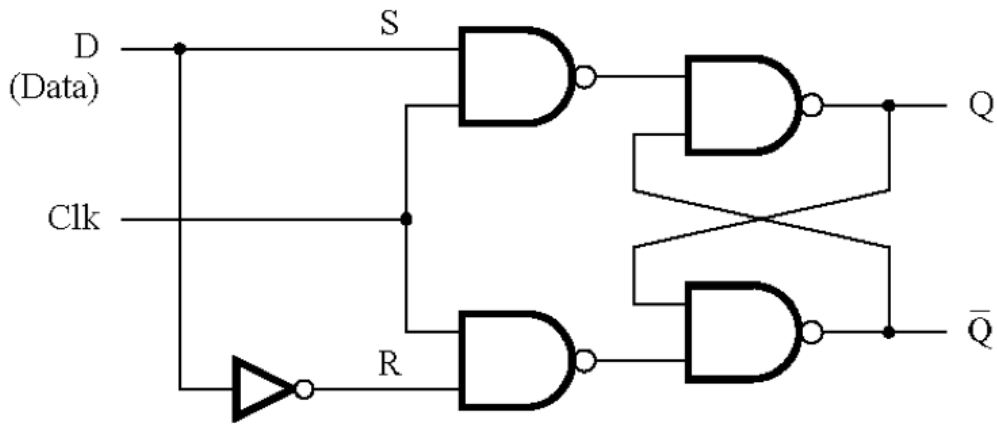


# Circuit Diagram for the Gated D Latch

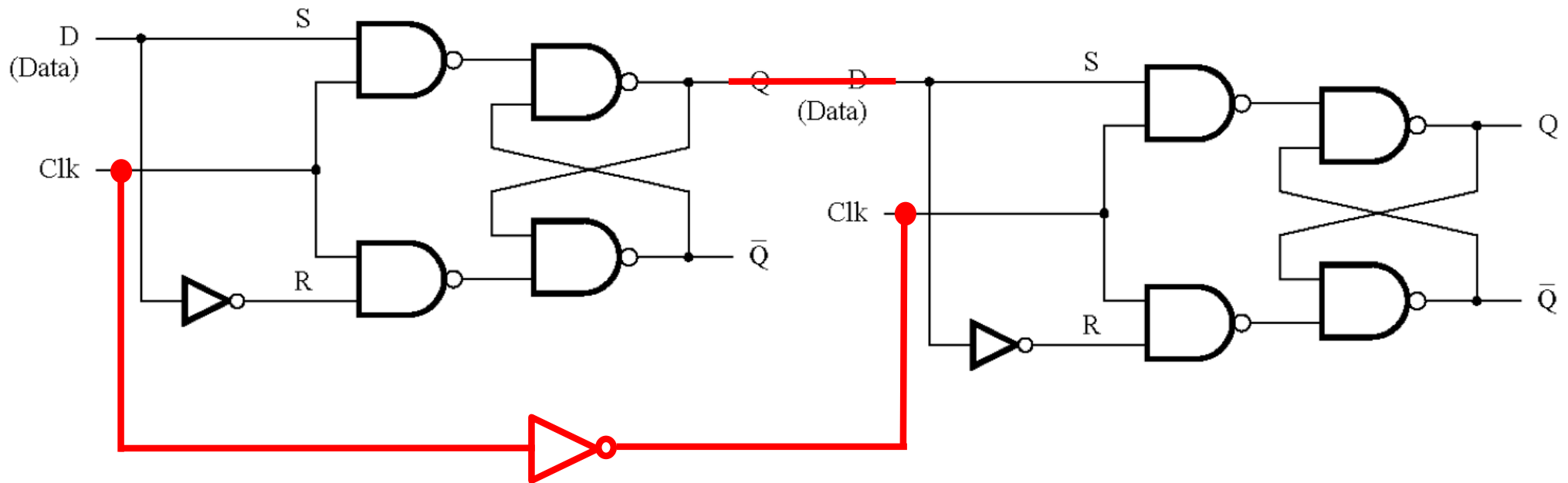


[ Figure 5.7a from the textbook ]

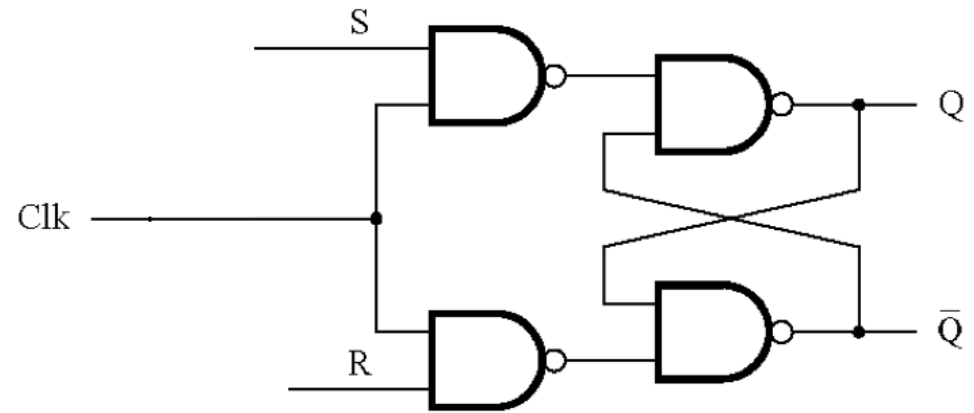
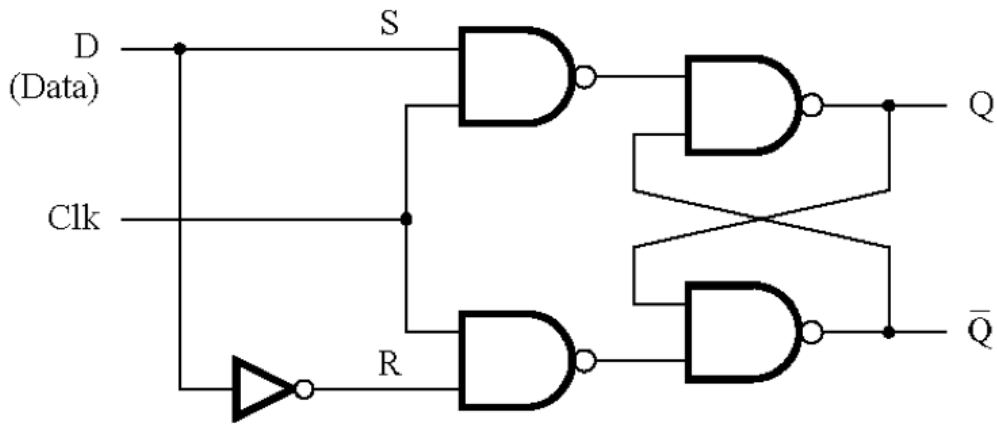
# Constructing a D Flip-Flop



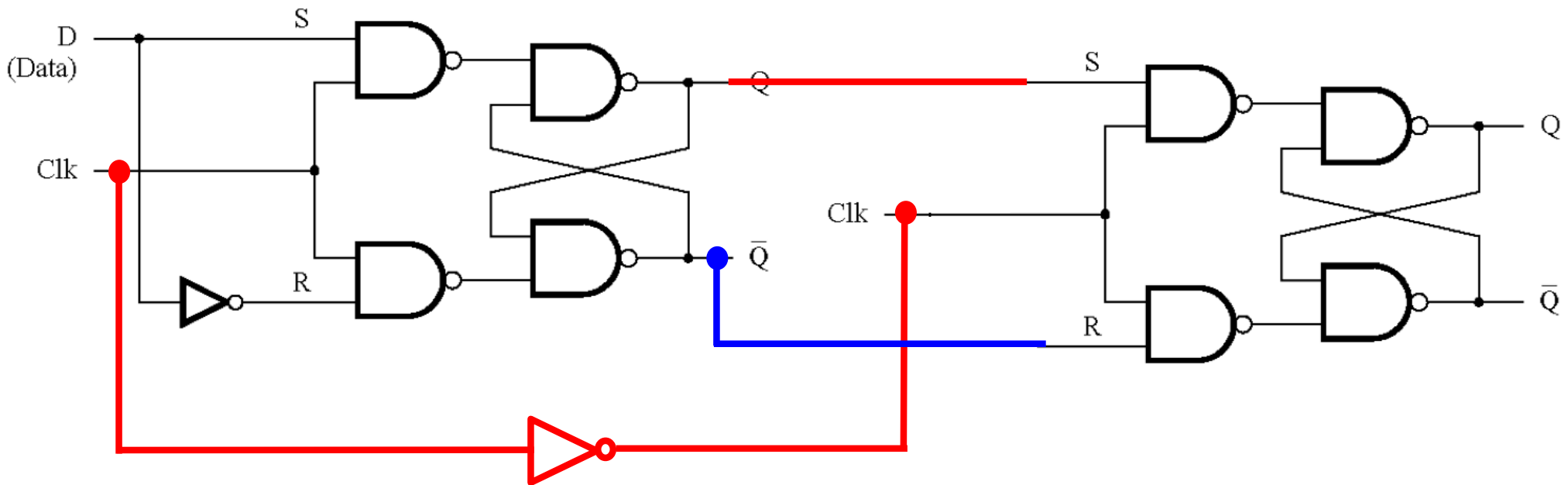
# Constructing a D Flip-Flop



# Constructing a D Flip-Flop (with one less NOT gate)

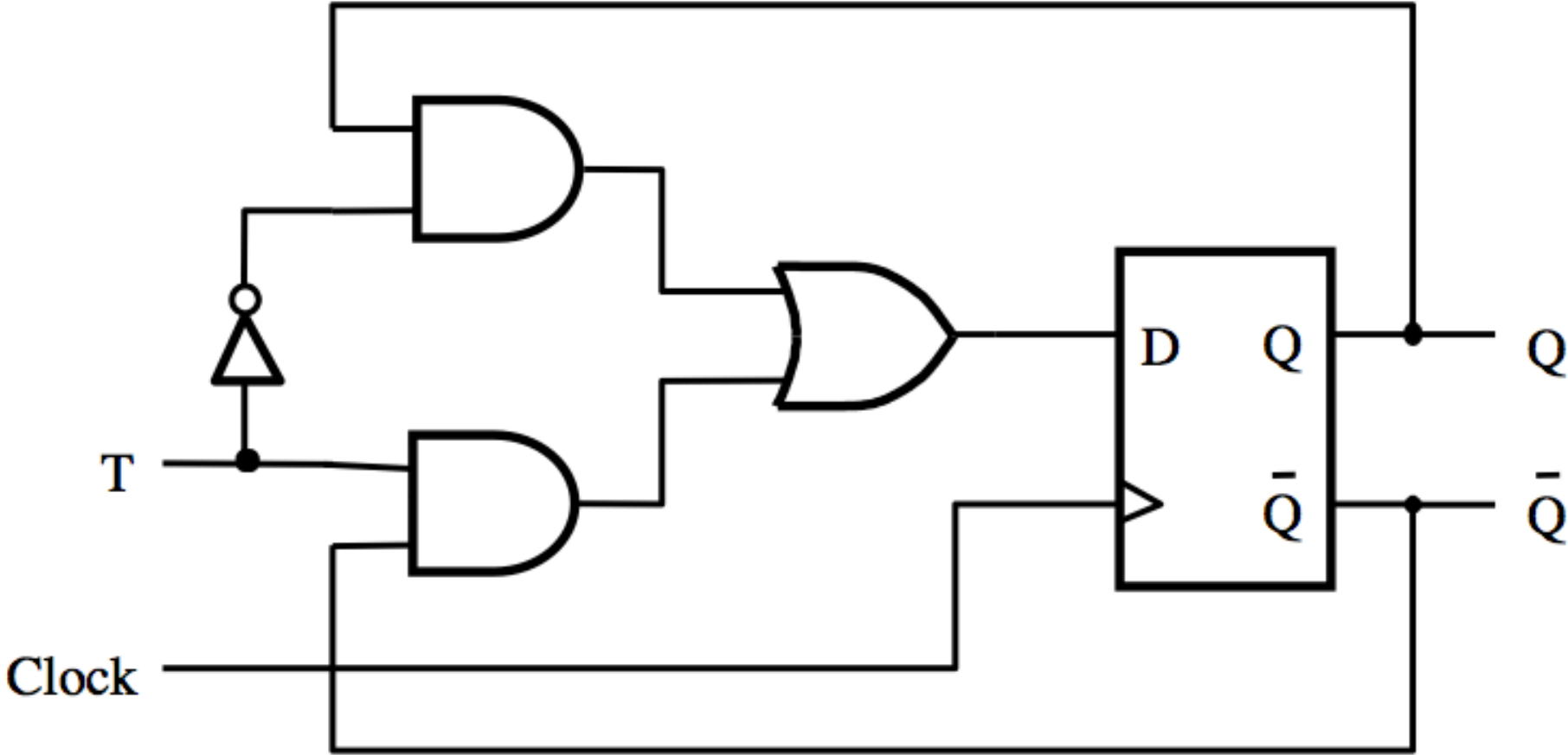


# Constructing a D Flip-Flop (with one less NOT gate)



# T Flip-Flop

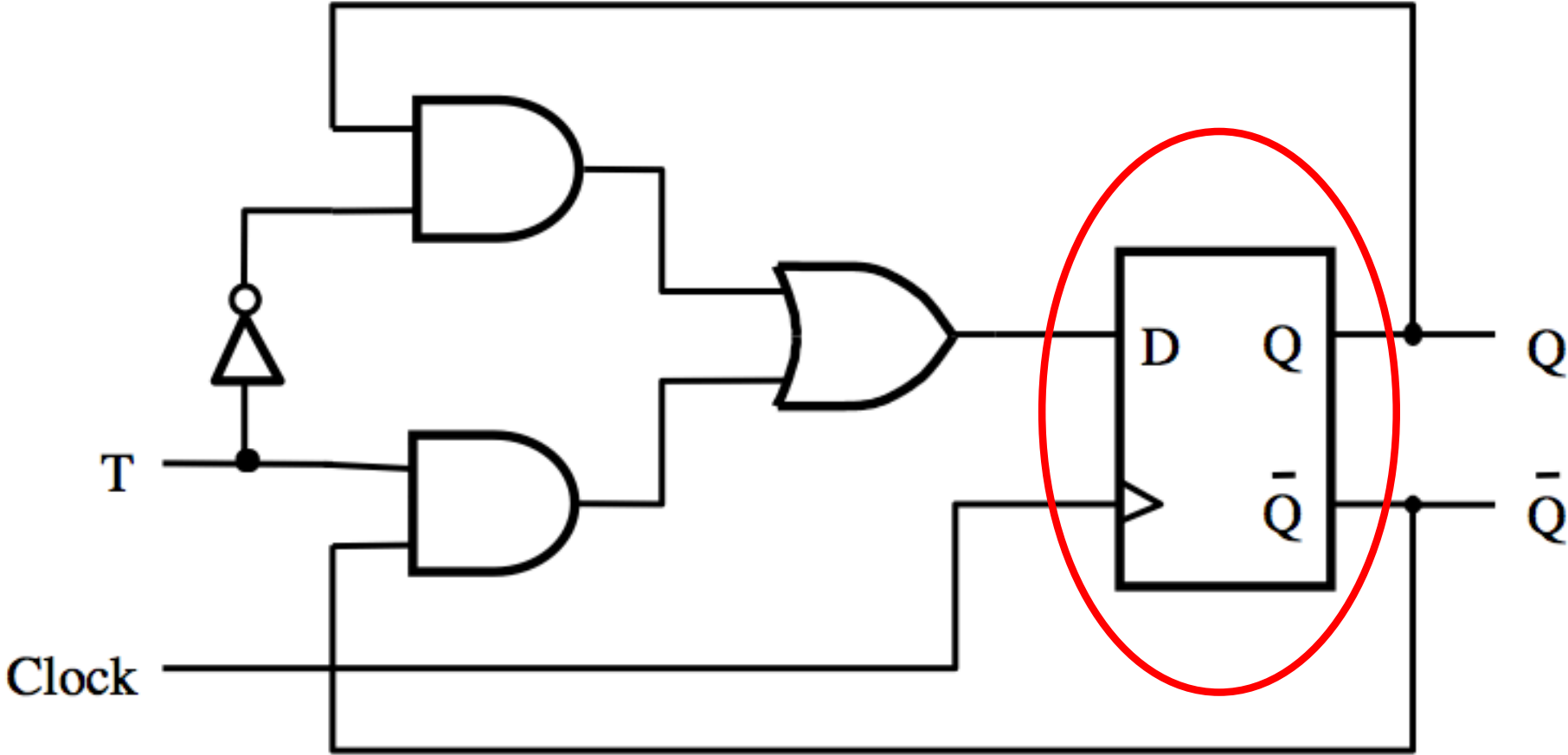
# T Flip-Flop



[ Figure 5.15a from the textbook ]



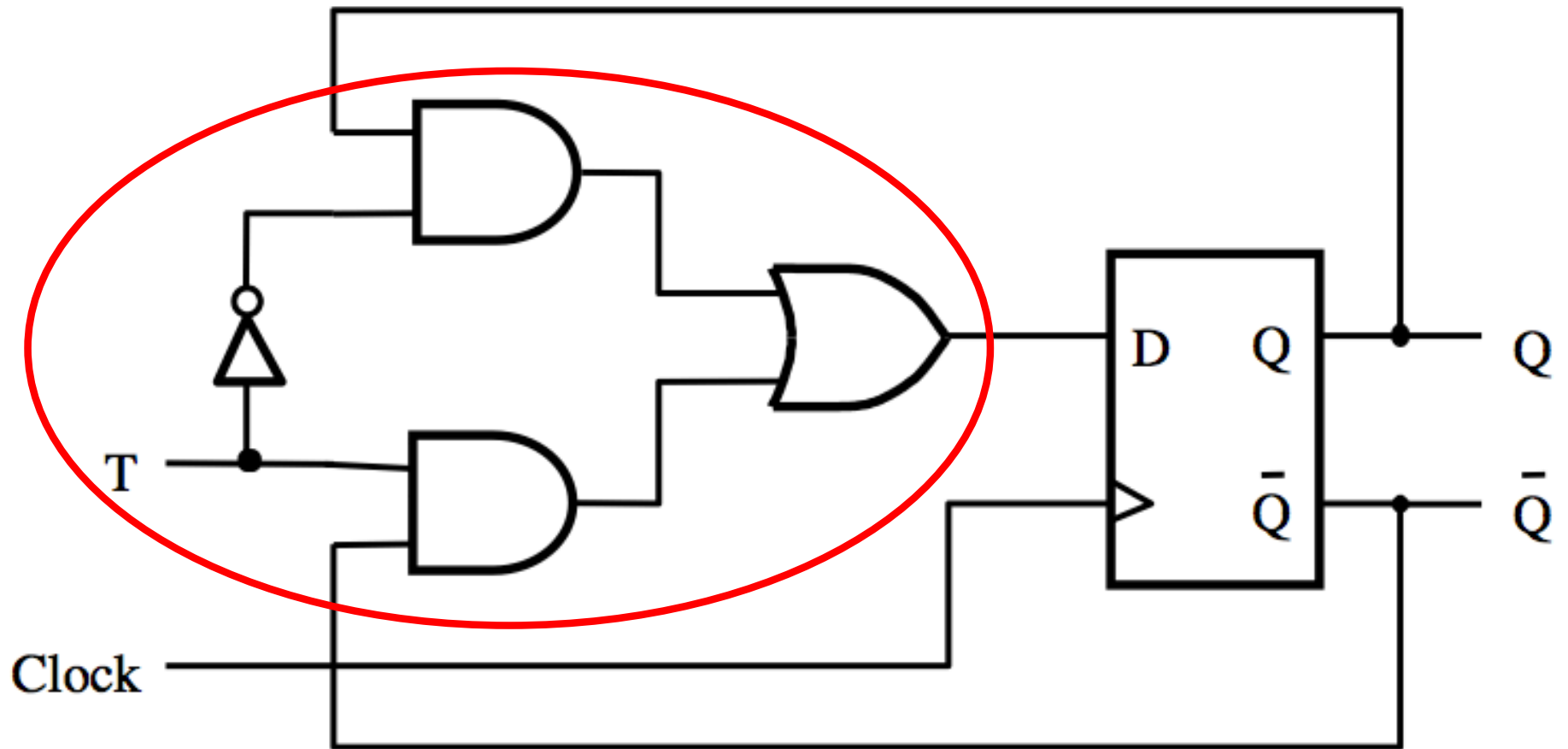
# T Flip-Flop



Positive-edge-triggered  
D Flip-Flop

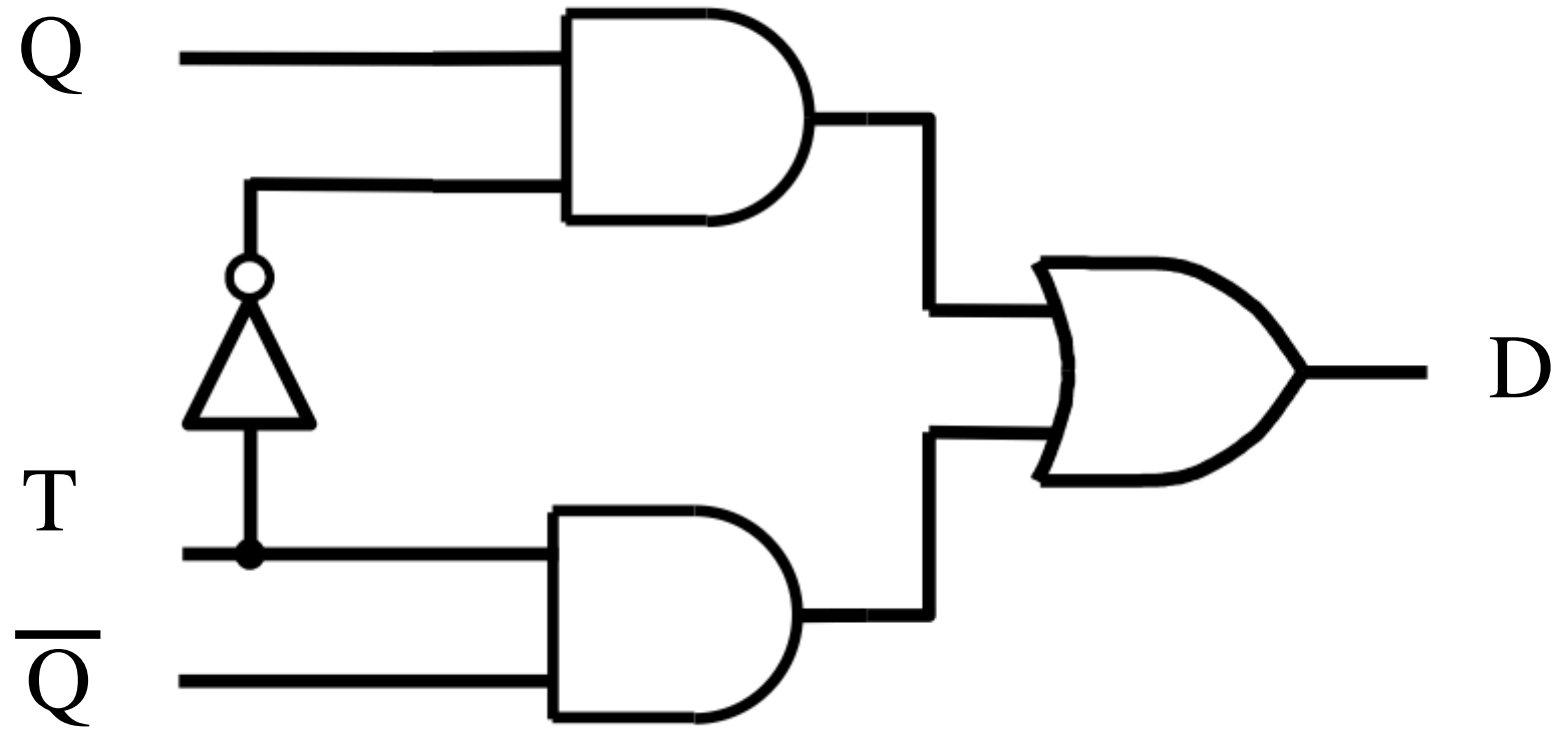
[ Figure 5.15a from the textbook ]

# T Flip-Flop

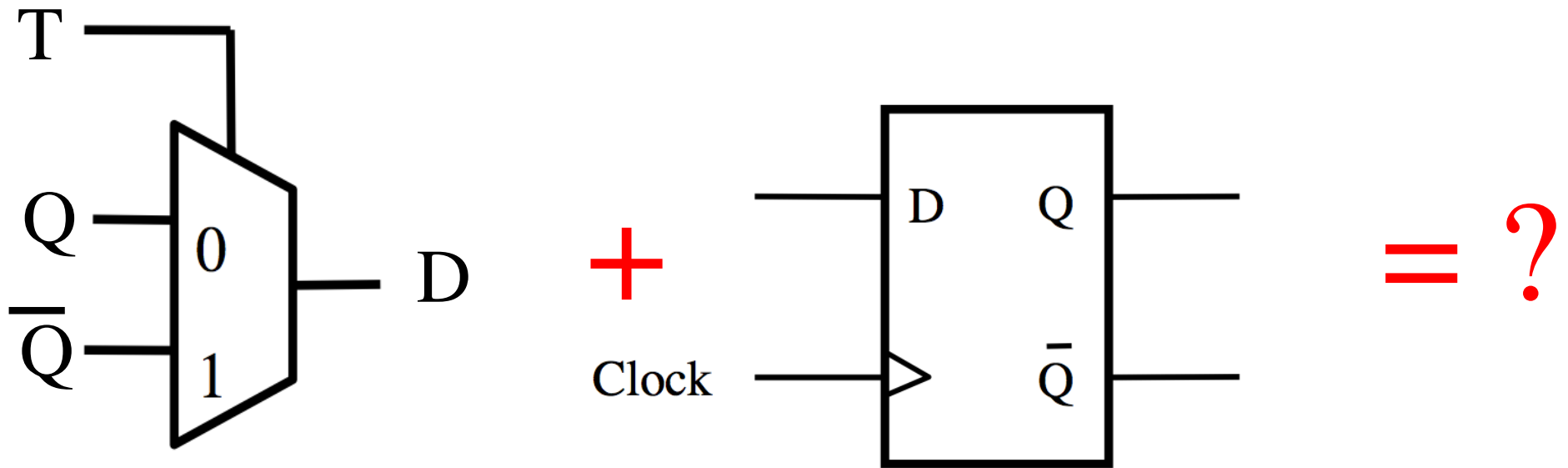


What is this?

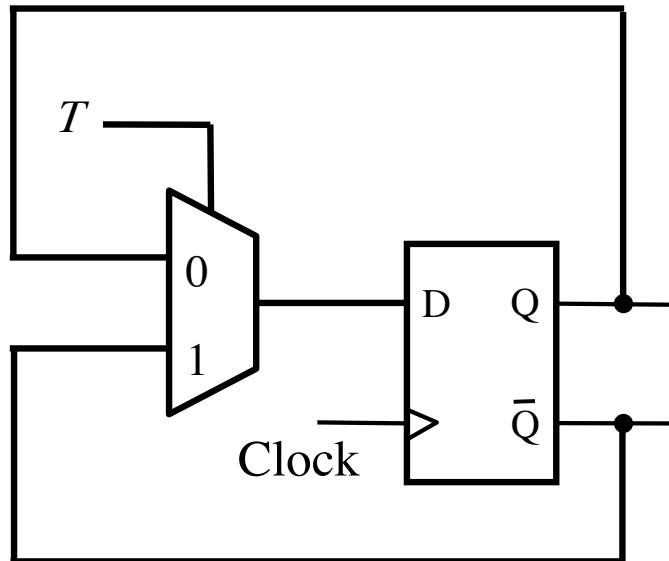
**What is this?**



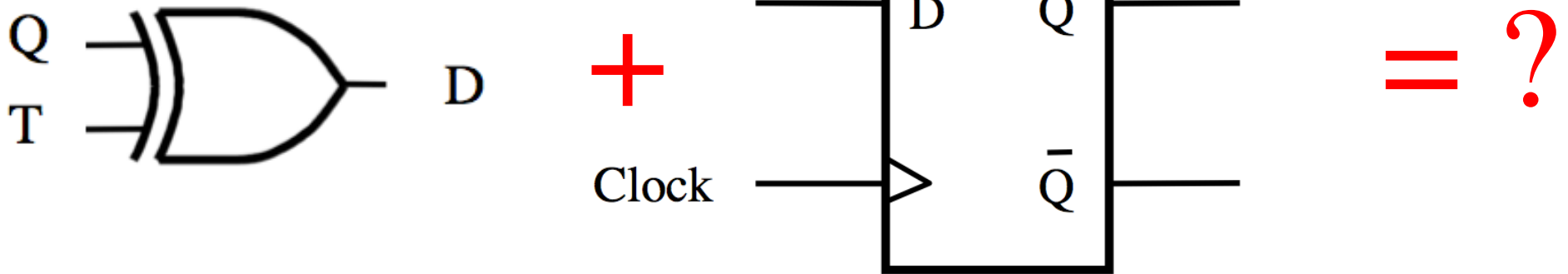
# What is this?



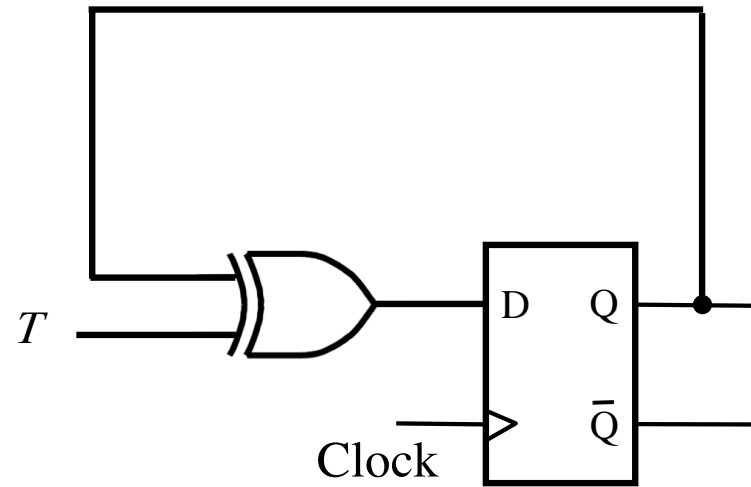
# T Flip-Flop



# What is this?



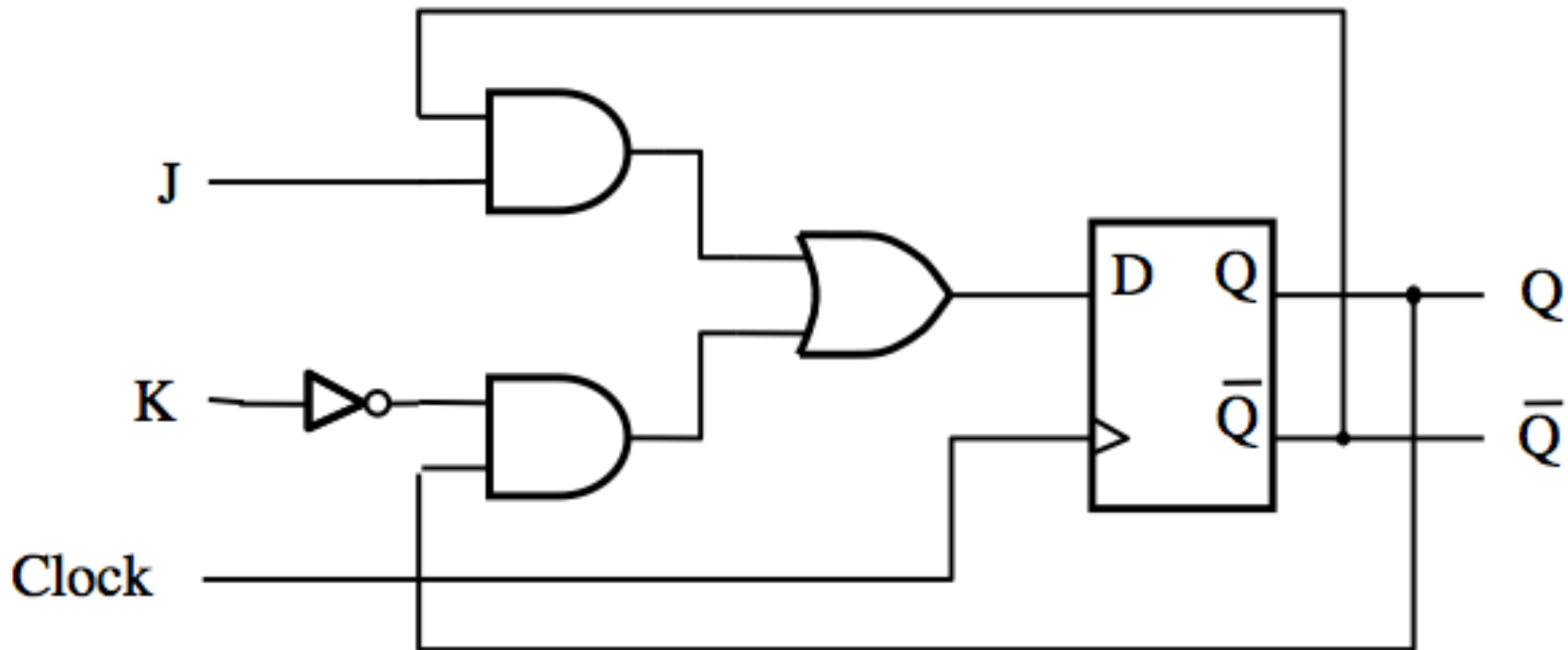
# T Flip-Flop



# **JK Flip-Flop**

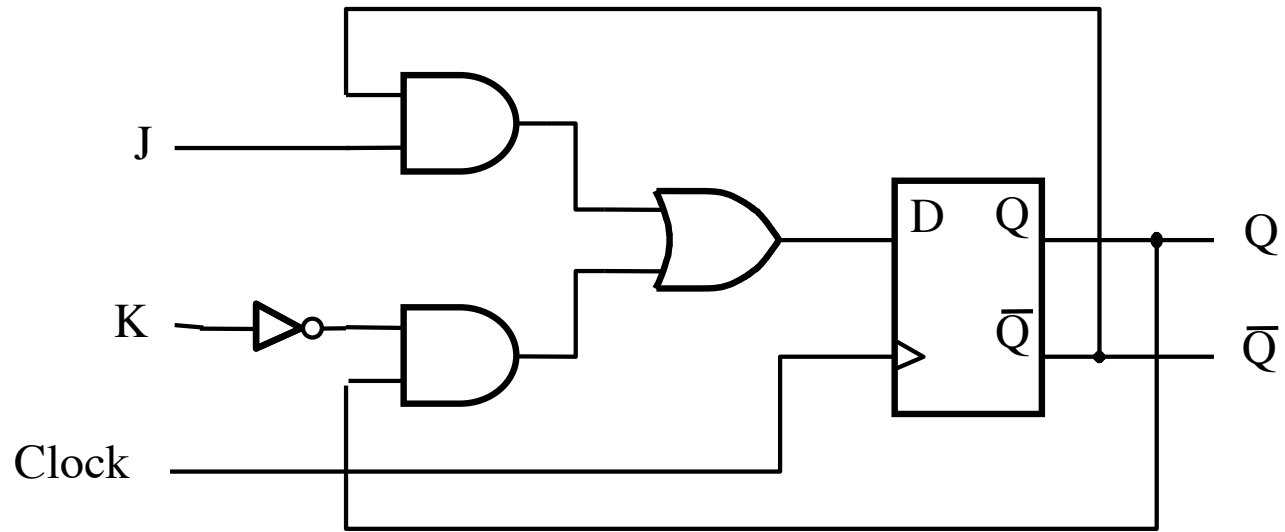


# JK Flip-Flop



$$D = \overline{J}Q + \overline{K}Q$$

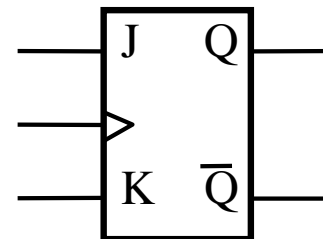
# JK Flip-Flop



(a) Circuit

J	K	$Q(t+1)$
0	0	$Q(t)$
0	1	0
1	0	1
1	1	$\bar{Q}(t)$

(b) Truth table



(c) Graphical symbol

# **JK Flip-Flop (How it Works)**

**A versatile circuit that can be used both as a  
SR flip-flop and as a T flip flop**

**If  $J=0$  and  $S =0$  it stays in the same state**

**Just like SR It can be set and reset**

**$J=S$  and  $K=R$**

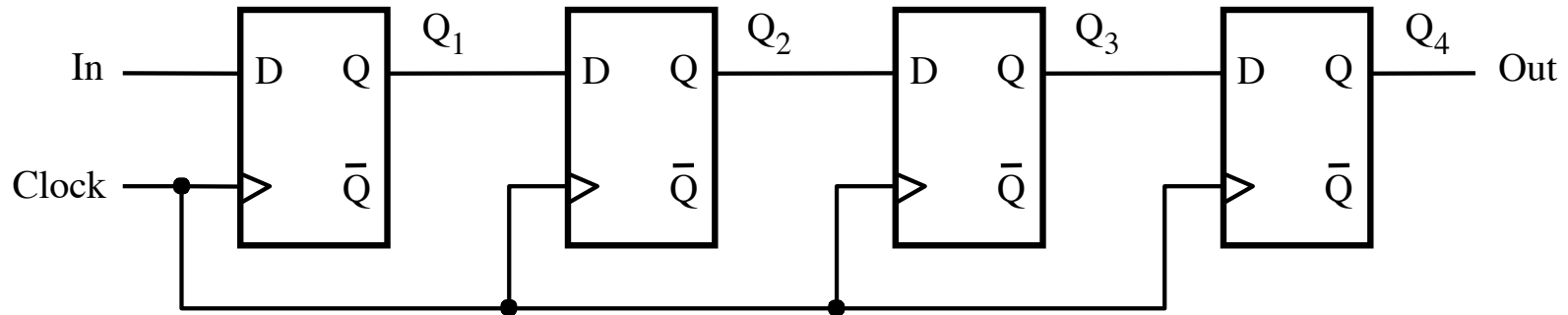
**If  $J=K$  then it behaves as a T flip-flop**

# Registers

# **Register (Definition)**

**An n-bit structure consisting of flip-flops**

# A simple shift register

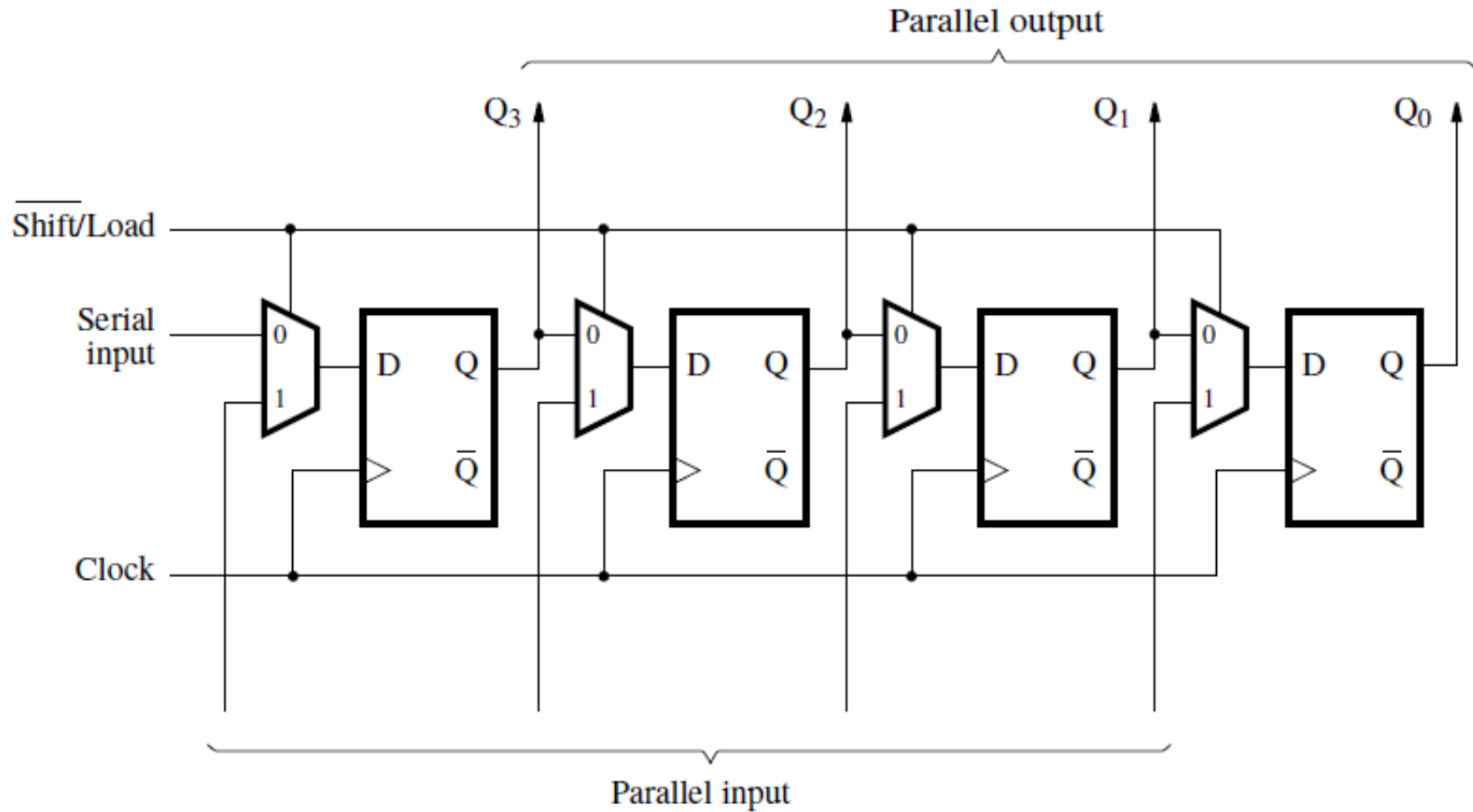


(a) Circuit

	In	$Q_1$	$Q_2$	$Q_3$	$Q_4 = \text{Out}$
$t_0$	1	0	0	0	0
$t_1$	0	1	0	0	0
$t_2$	1	0	1	0	0
$t_3$	1	1	0	1	0
$t_4$	1	1	1	0	1
$t_5$	0	1	1	1	0
$t_6$	0	0	1	1	1
$t_7$	0	0	0	1	1

(b) A sample sequence

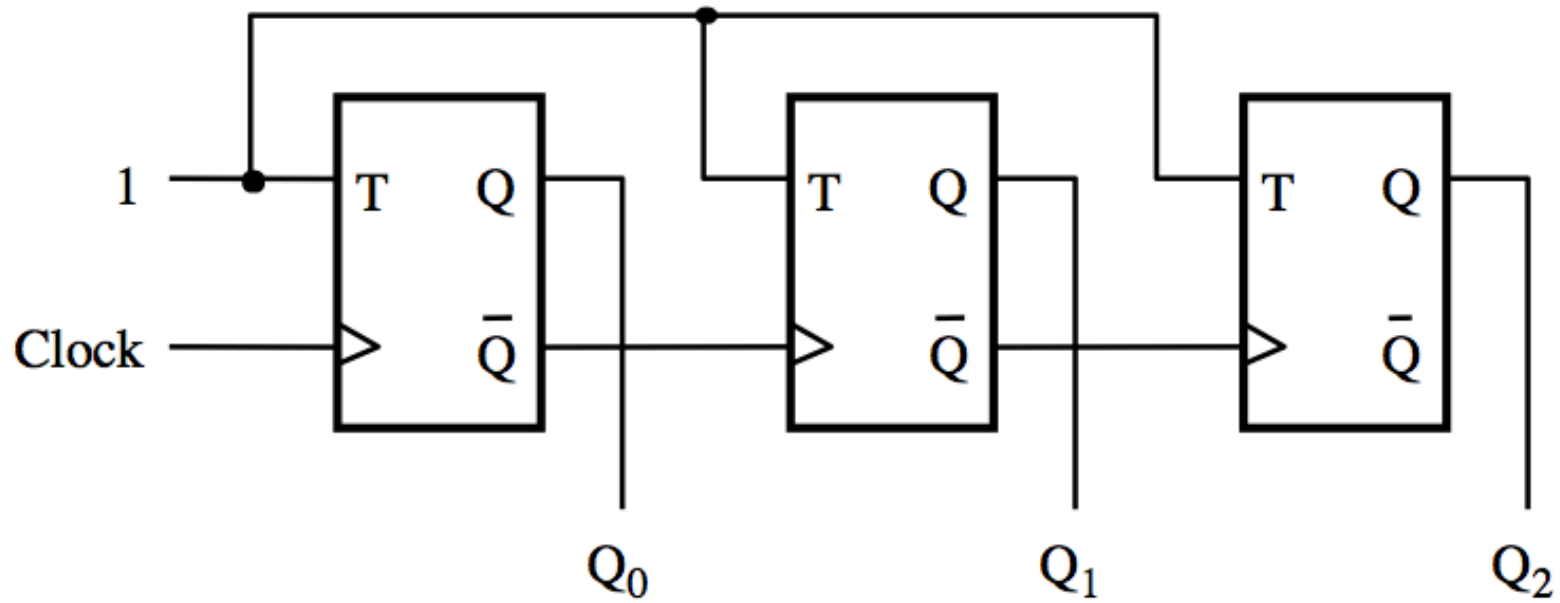
# Parallel-access shift register



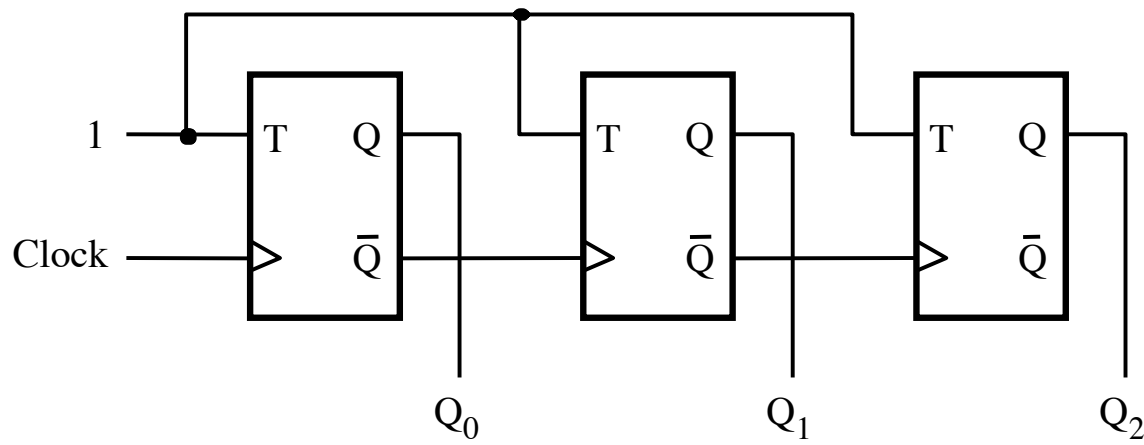
# Counters



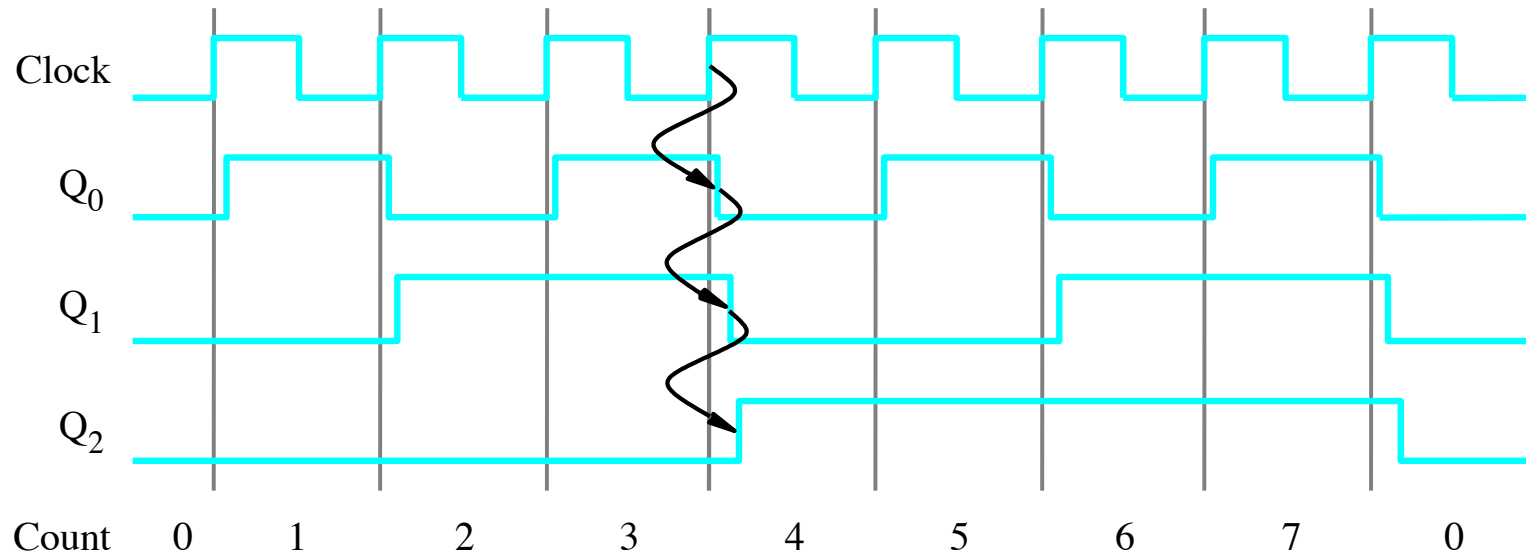
# A three-bit up-counter



# A three-bit up-counter

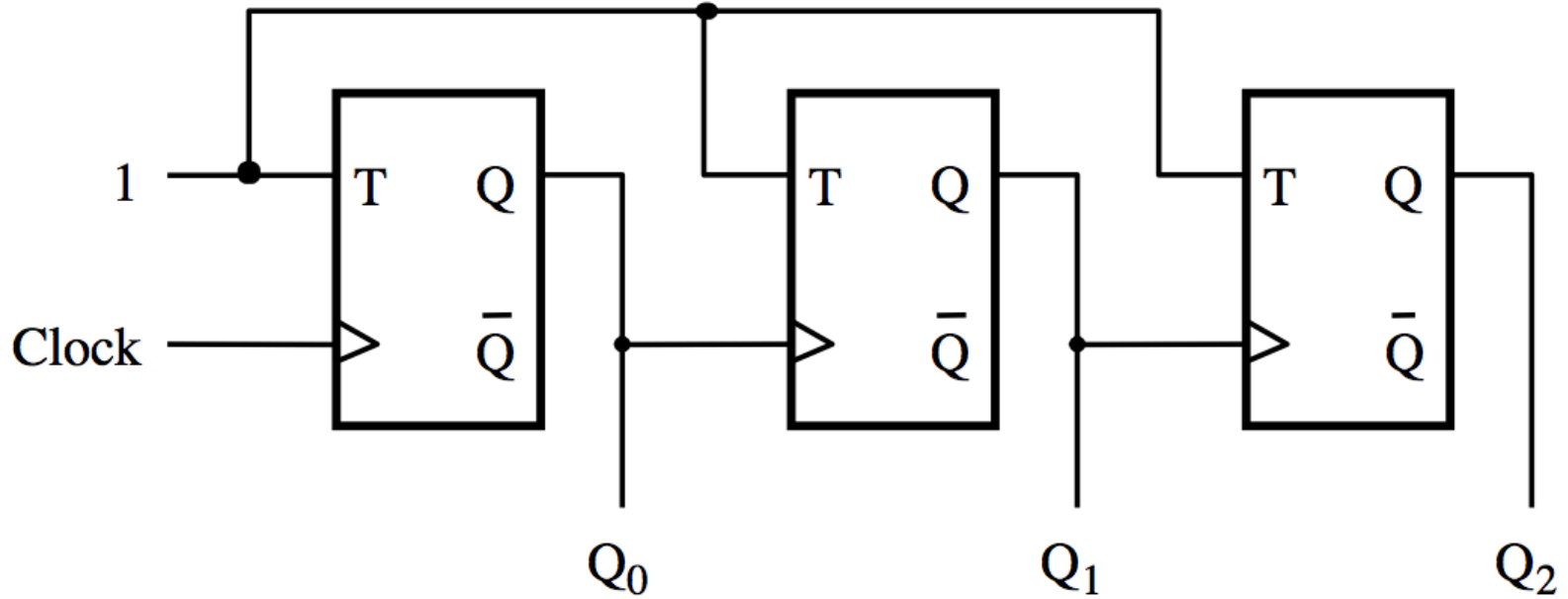


(a) Circuit

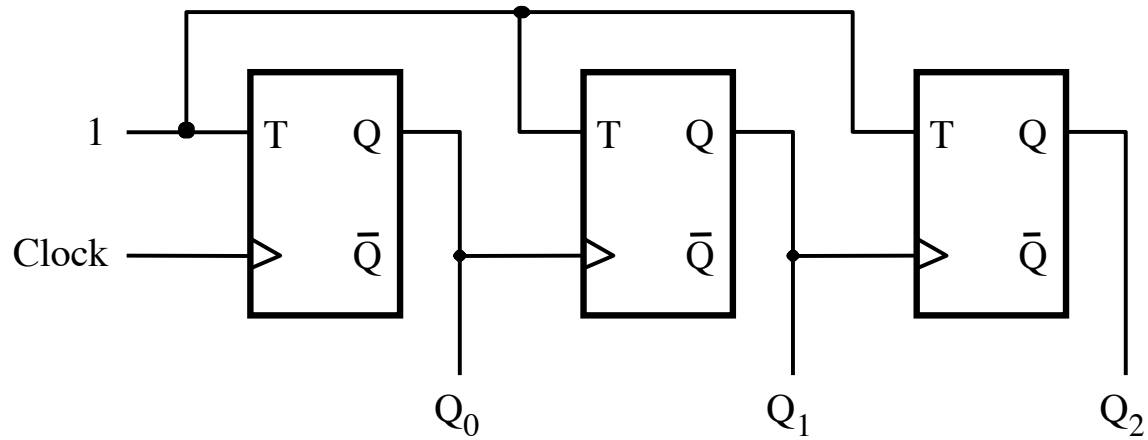


(b) Timing diagram

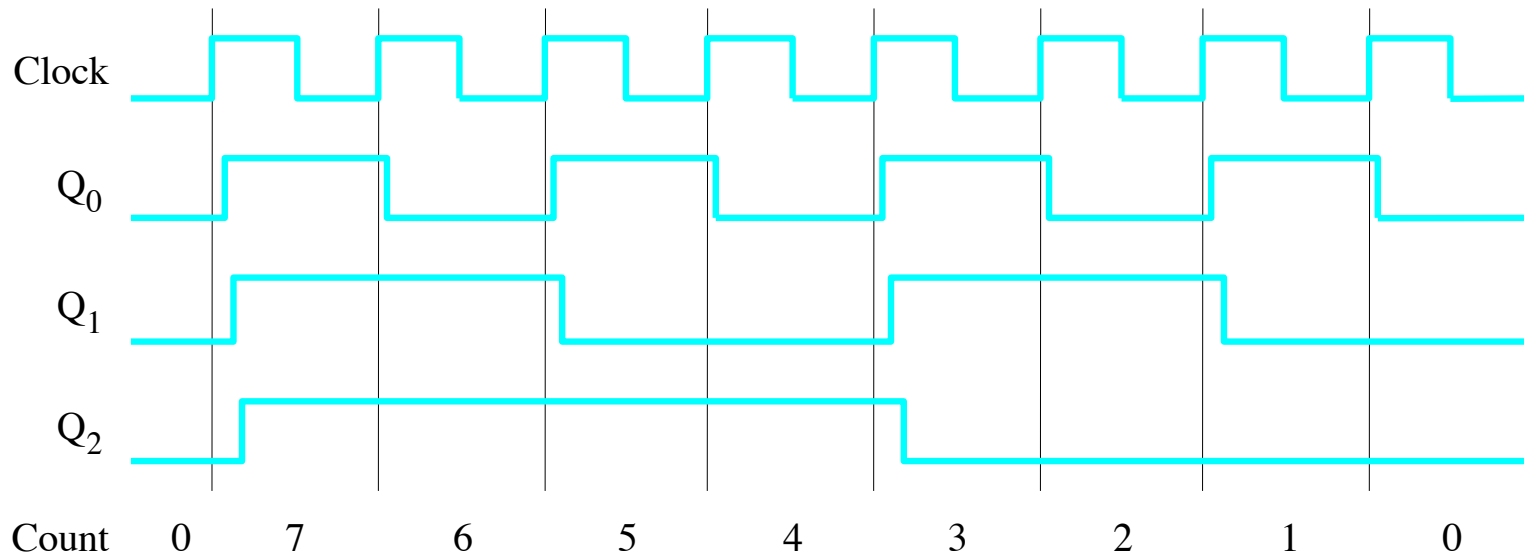
# A three-bit down-counter



# A three-bit down-counter



(a) Circuit

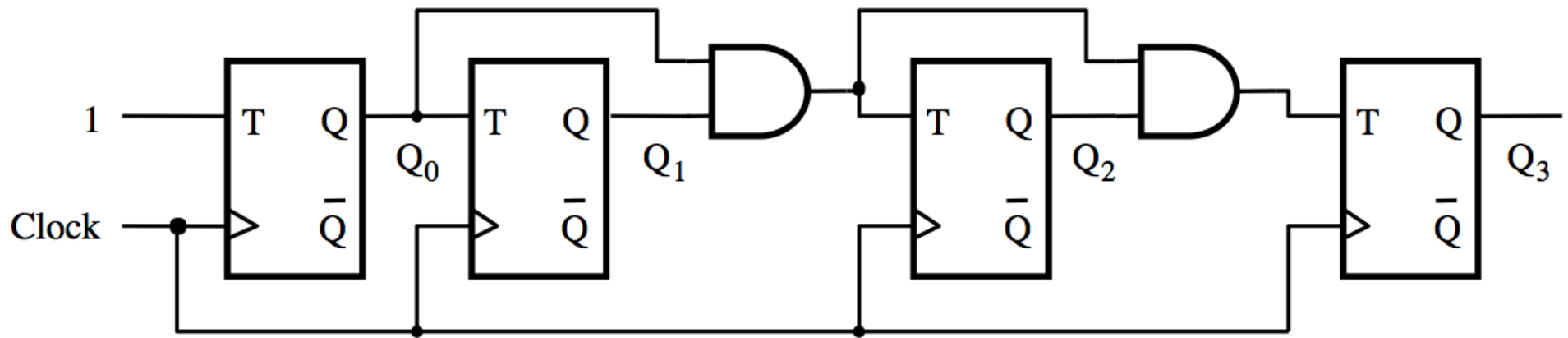


(b) Timing diagram

[ Figure 5.20 from the textbook ]

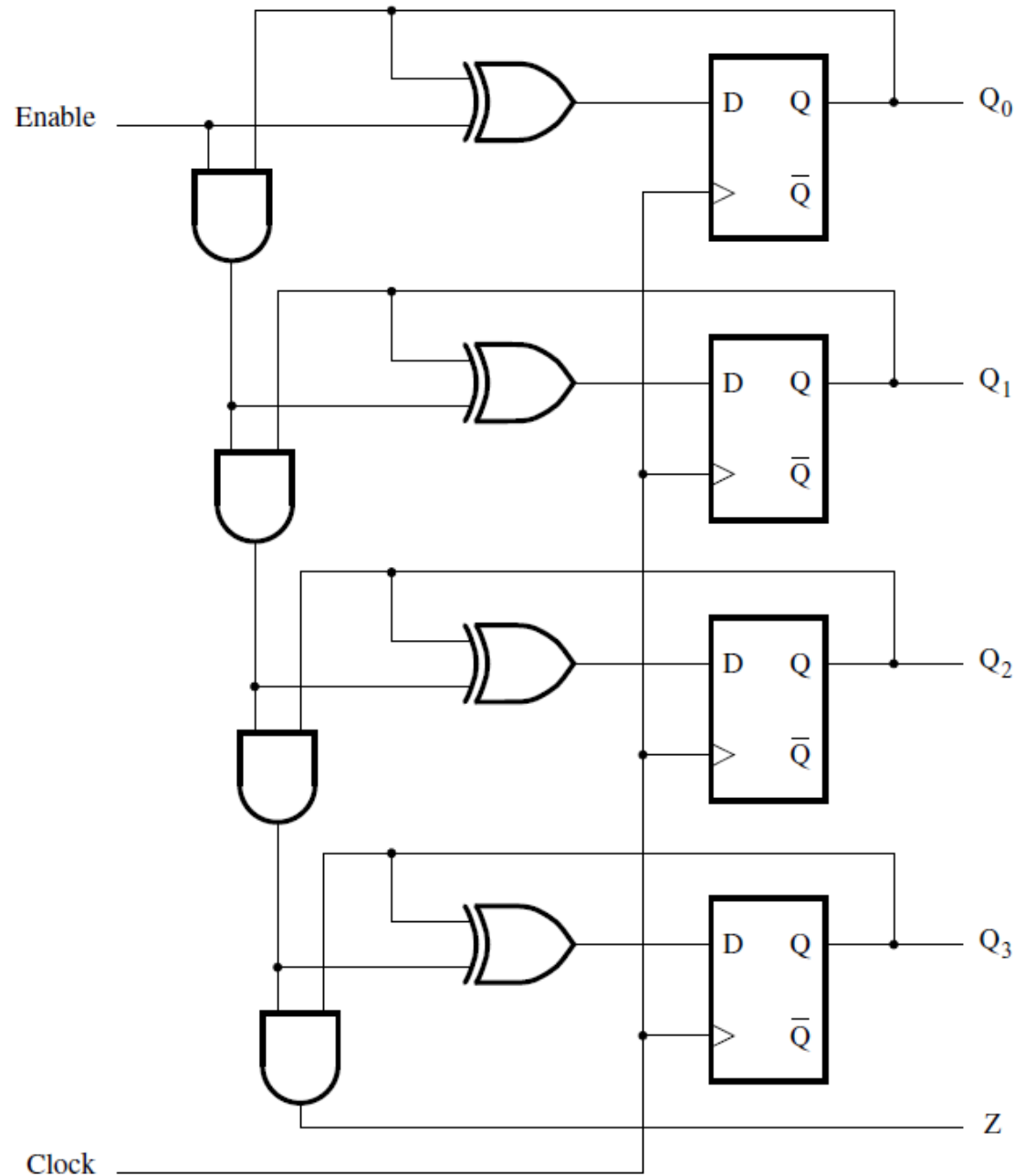
# **Synchronous Counters**

# A four-bit synchronous up-counter



# **Synchronous Counter with D Flip-Flops**

# A four-bit counter with D flip-flops

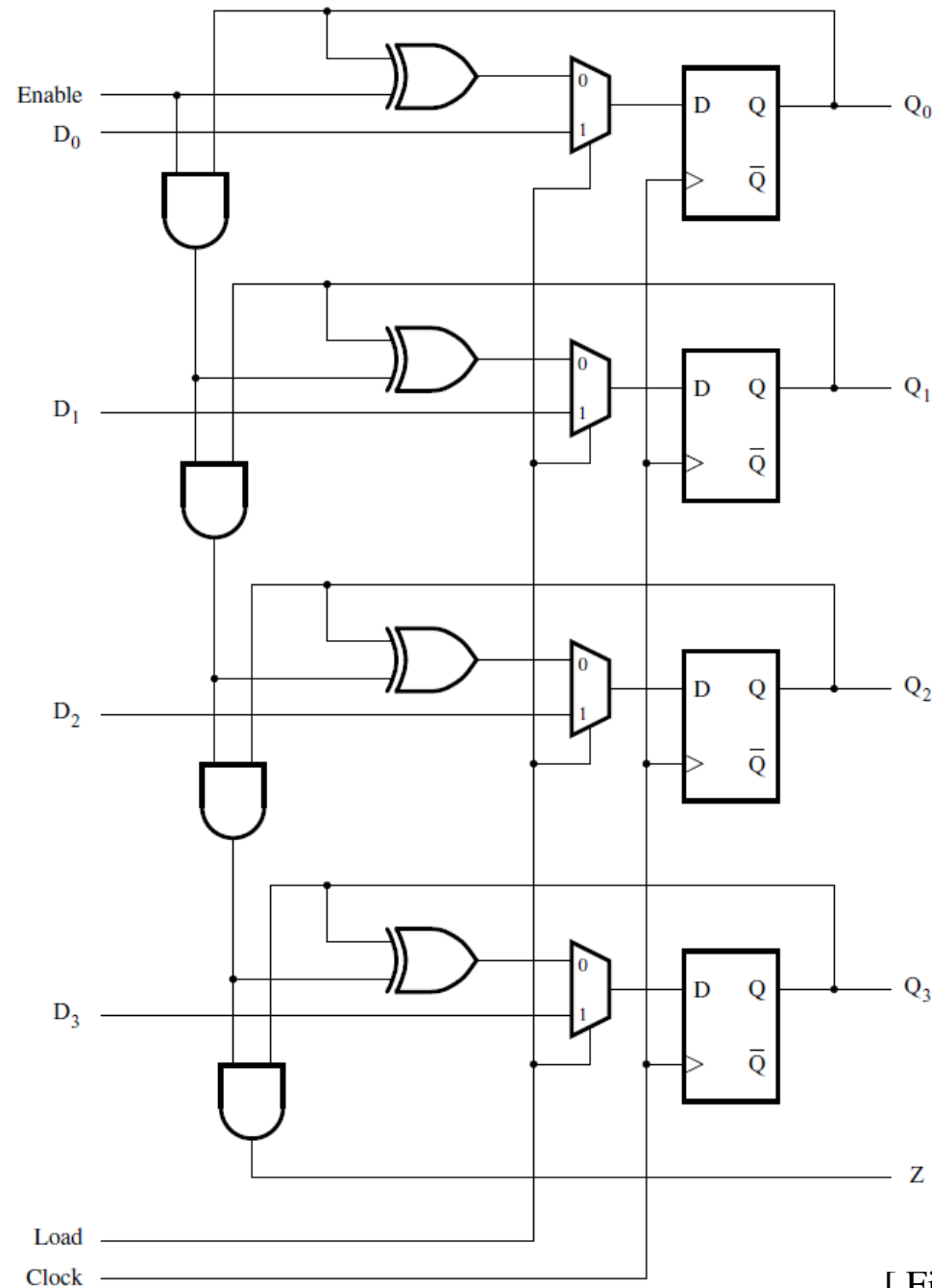


[ Figure 5.23 from the textbook ]



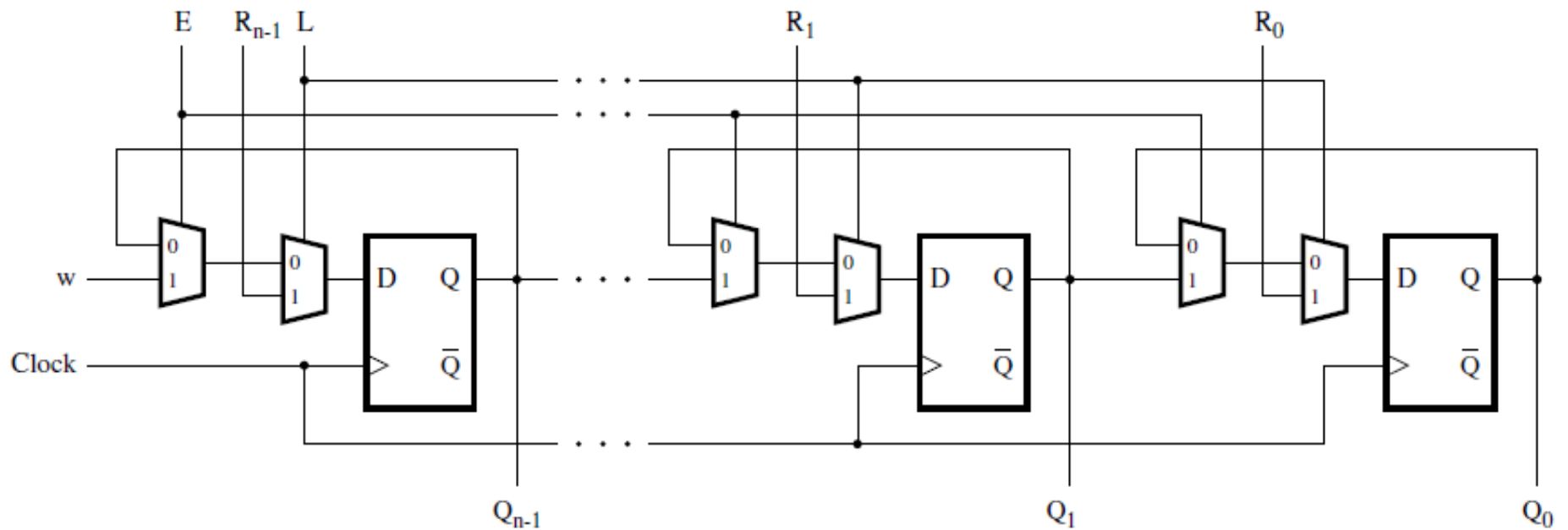
# **Counters with Parallel Load**

# A counter with parallel-load capability



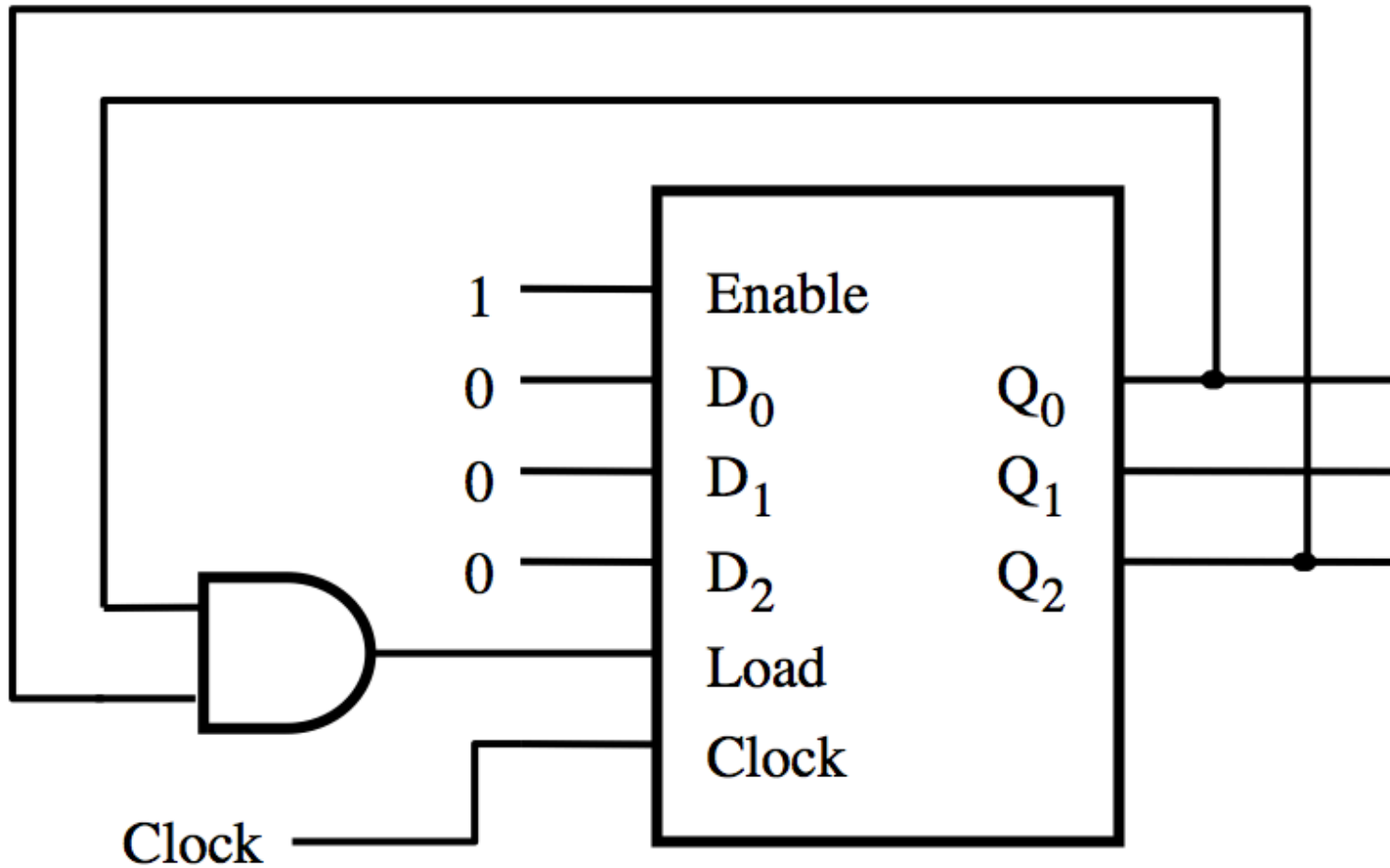
[ Figure 5.24 from the textbook ]

# A shift register with parallel load and enable control inputs

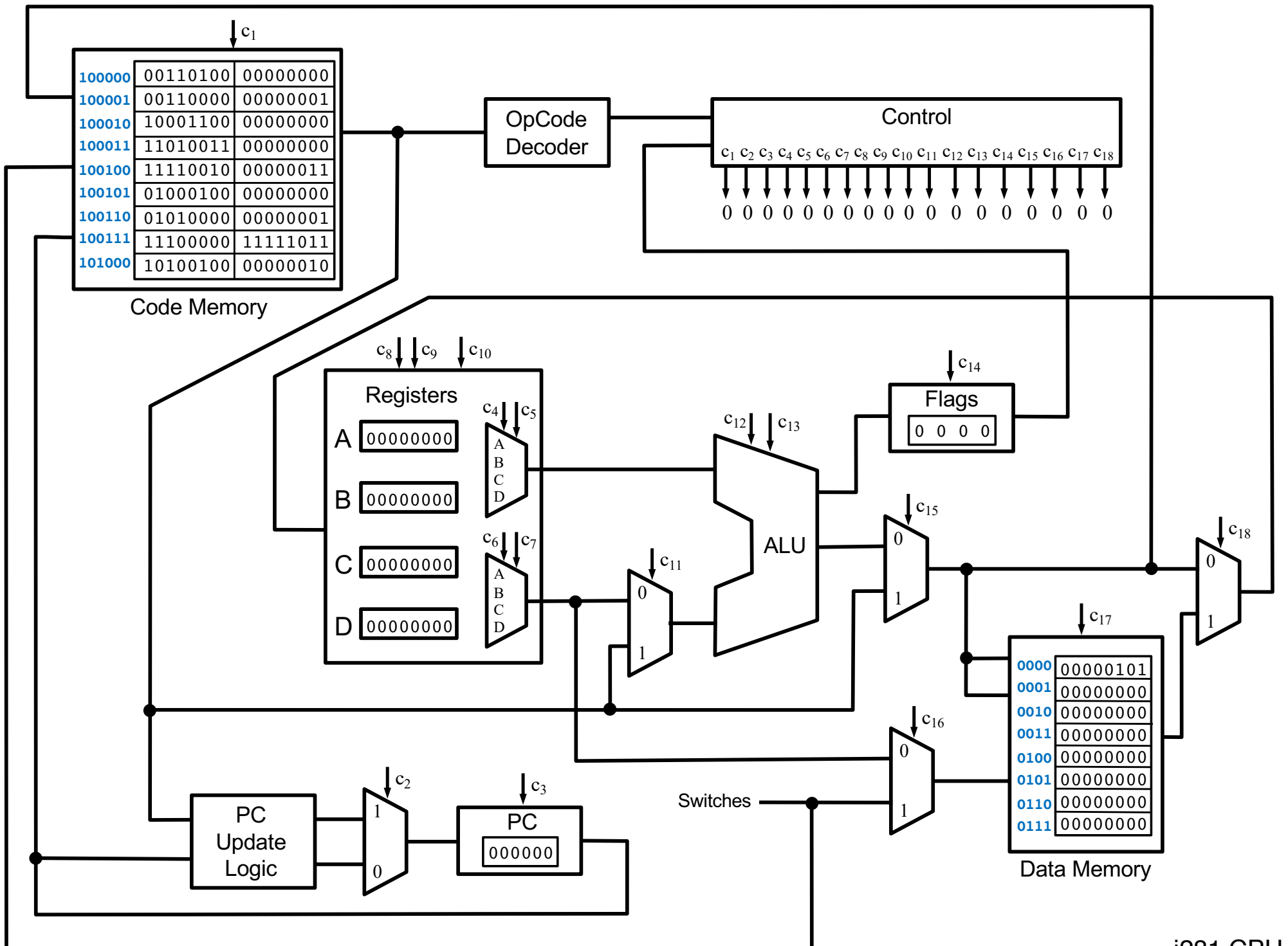


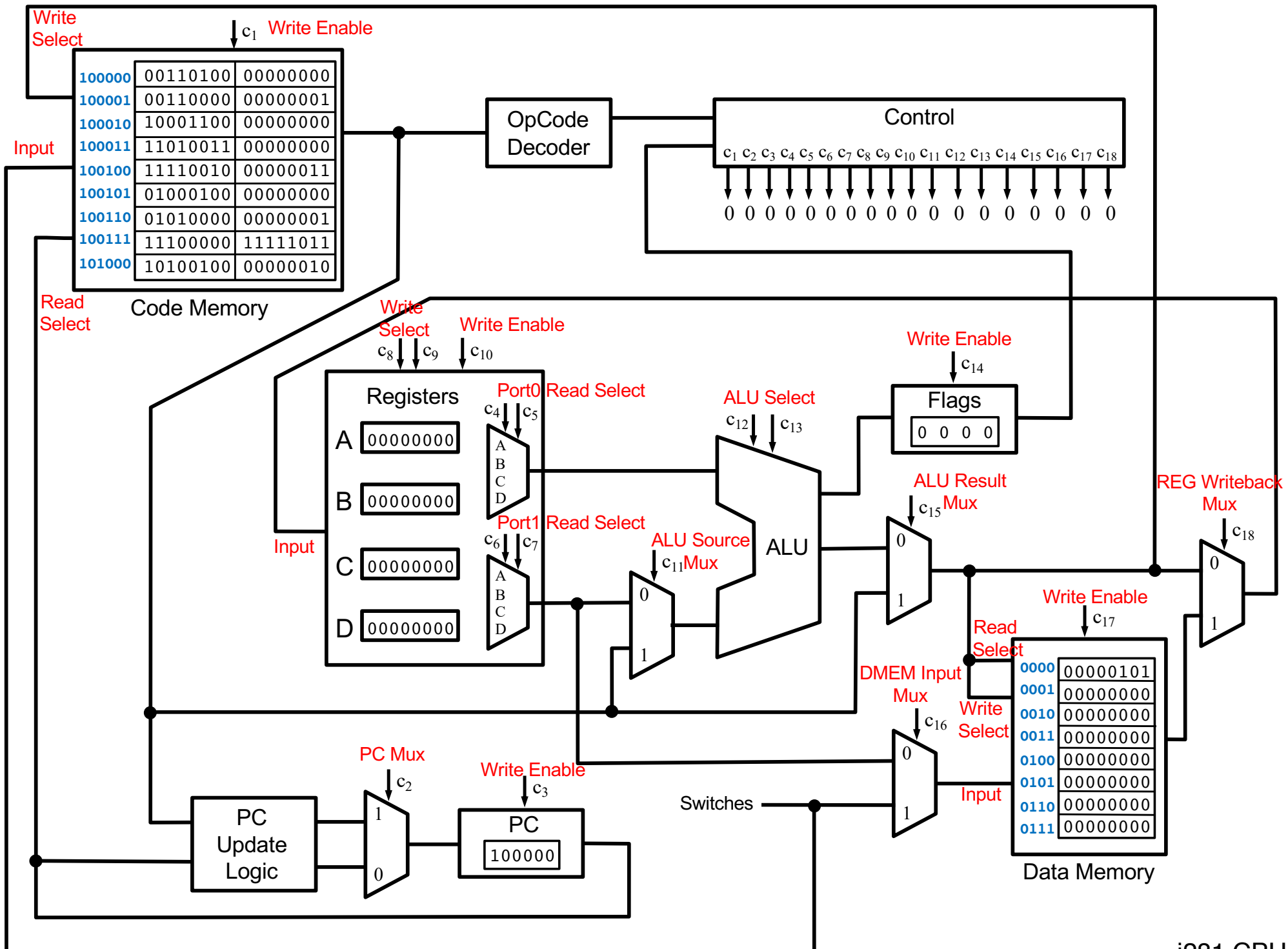
[ Figure 5.59 from the textbook ]

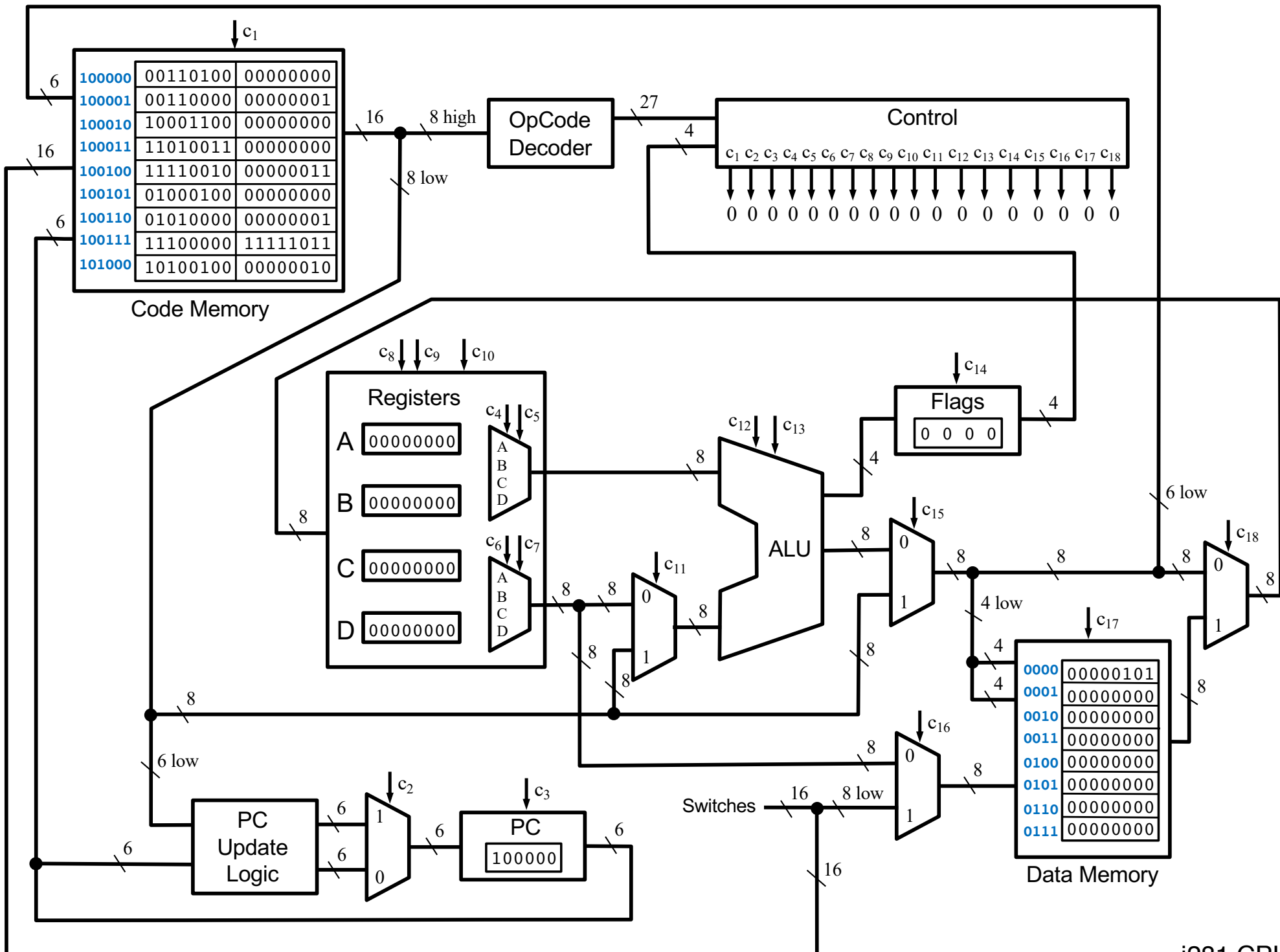
# What does this circuit do?



**i281 CPU**









# Add the numbers from 1 to 5

```
// C Version
// using a for loop

int main()
{
    int N=5;
    int i, sum;

    sum=0;
    for(i=1; i<=N; i++) {
        sum+=i;
    }

    // printf("%d\n", sum);
}
```

```
; Assembly Version

.data
N        BYTE    5
i        BYTE    ?
sum      BYTE    ?

.code

        LOADI   B, 0        ; sum=0
        LOADI   A, 1        ; i=1
        LOAD    D, [N]      ; register_D=N
Loop:   CMP     A, D        ; i<=N ?
        BRG     End        ; exit if i>N
Add:    ADD     B, A        ; sum+=i
        ADDI   A, 1        ; i++
        JUMP   Loop        ; next iteration
End:    STORE  [sum], B    ; write B to sum
```

# Mapping Assembly to Machine Code

**.data**

**N**        **BYTE**    **5**  
**i**        **BYTE**    **?**  
**sum**     **BYTE**    **?**

**Data Memory:**

00000101  
00000000  
00000000

**.code**

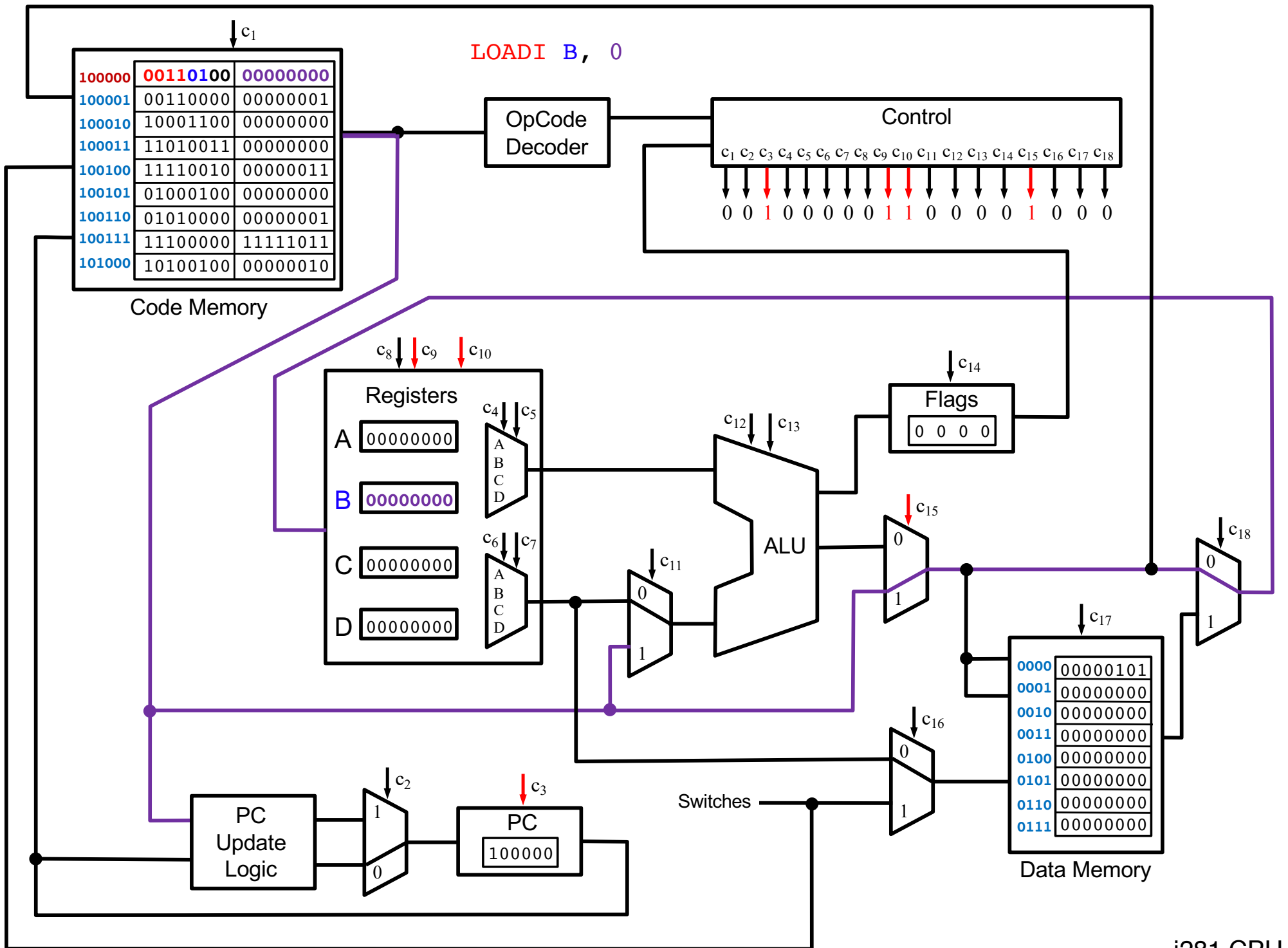
**LOADI**  **B, 0**  
          **LOADI**  **A, 1**  
          **LOAD**   **D, [N]**  
**Loop:**   **CMP**    **A, D**  
          **BRG**    **End**  
**Add:**   **ADD**    **B, A**  
          **ADDI**   **A, 1**  
          **JUMP**   **Loop**  
**End:**    **STORE**  **[sum], B**

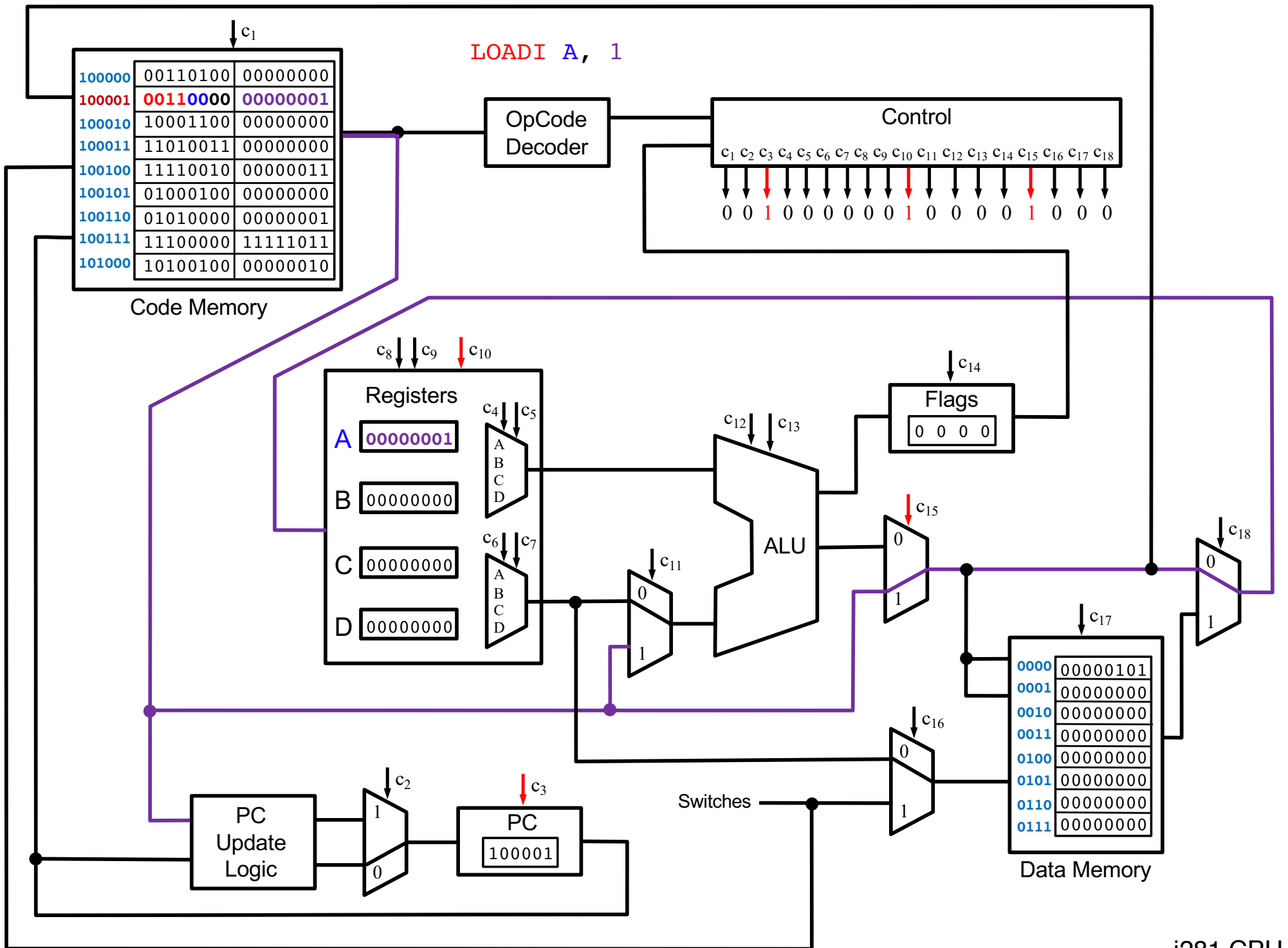
**Code Memory:**

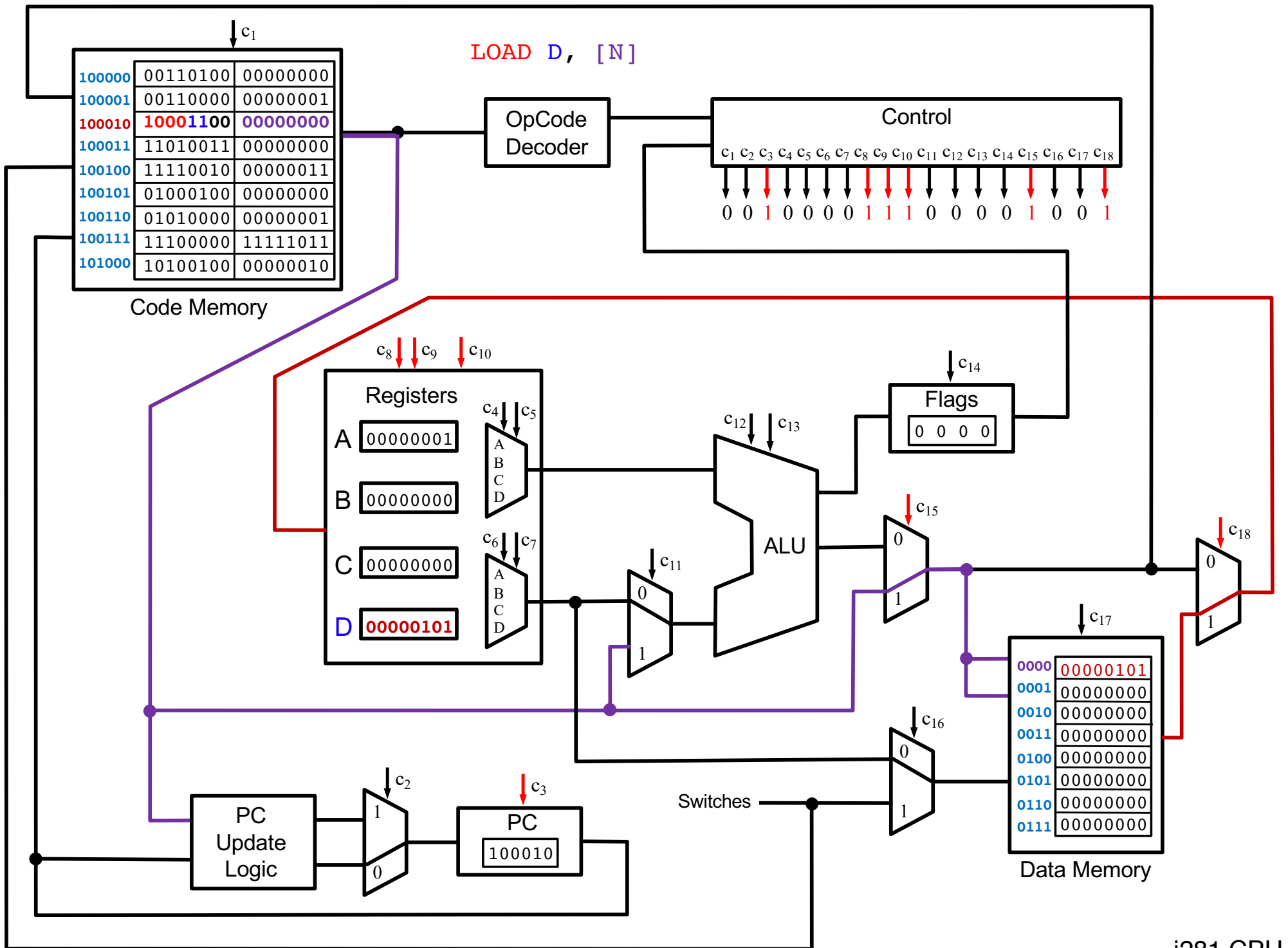
0011010000000000  
0011000000000001  
1000110000000000  
1101001100000000  
1111001000000011  
0100010000000000  
0101000000000001  
1110000011111011  
1010010000000010

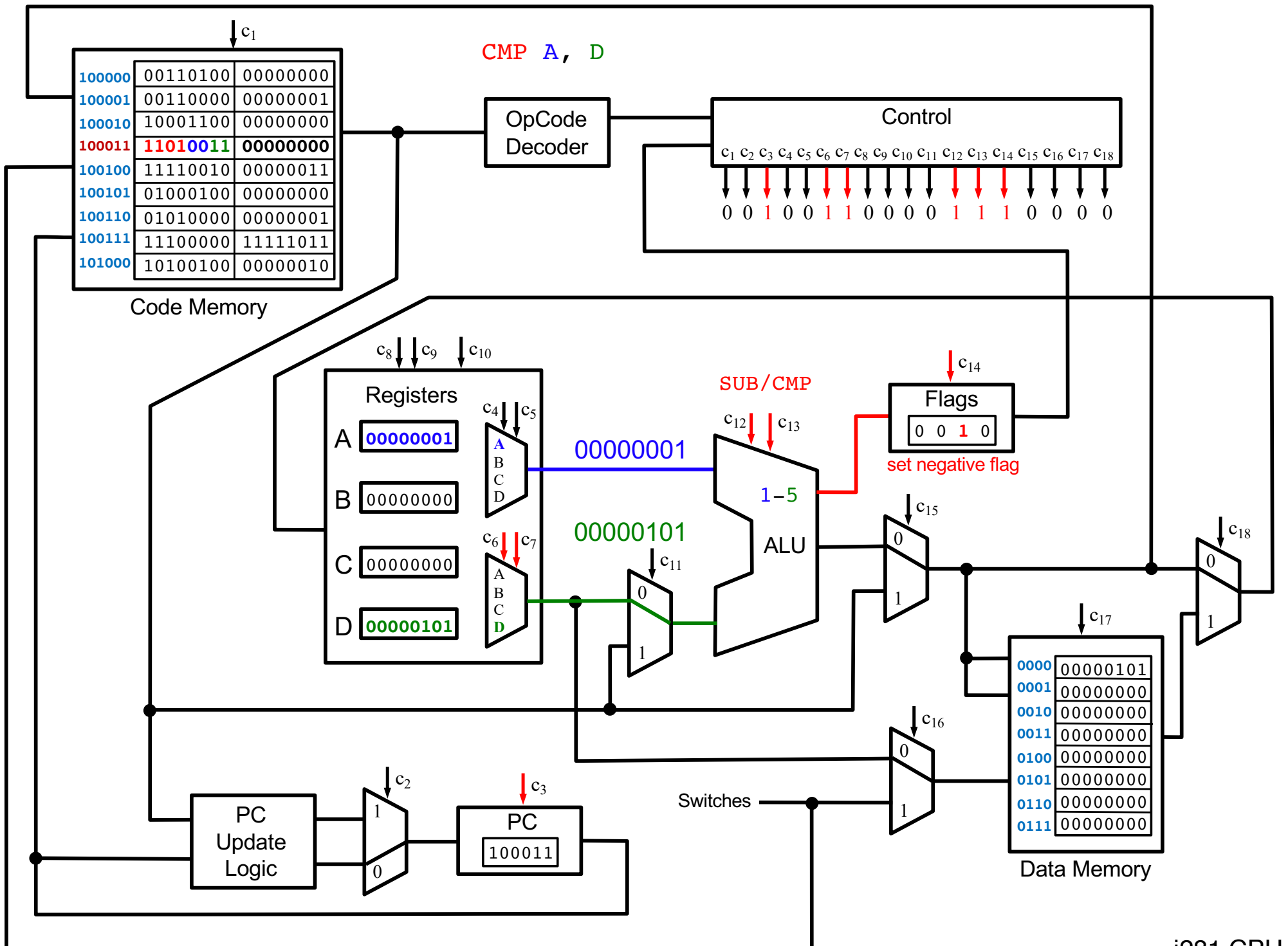
Assembly Language

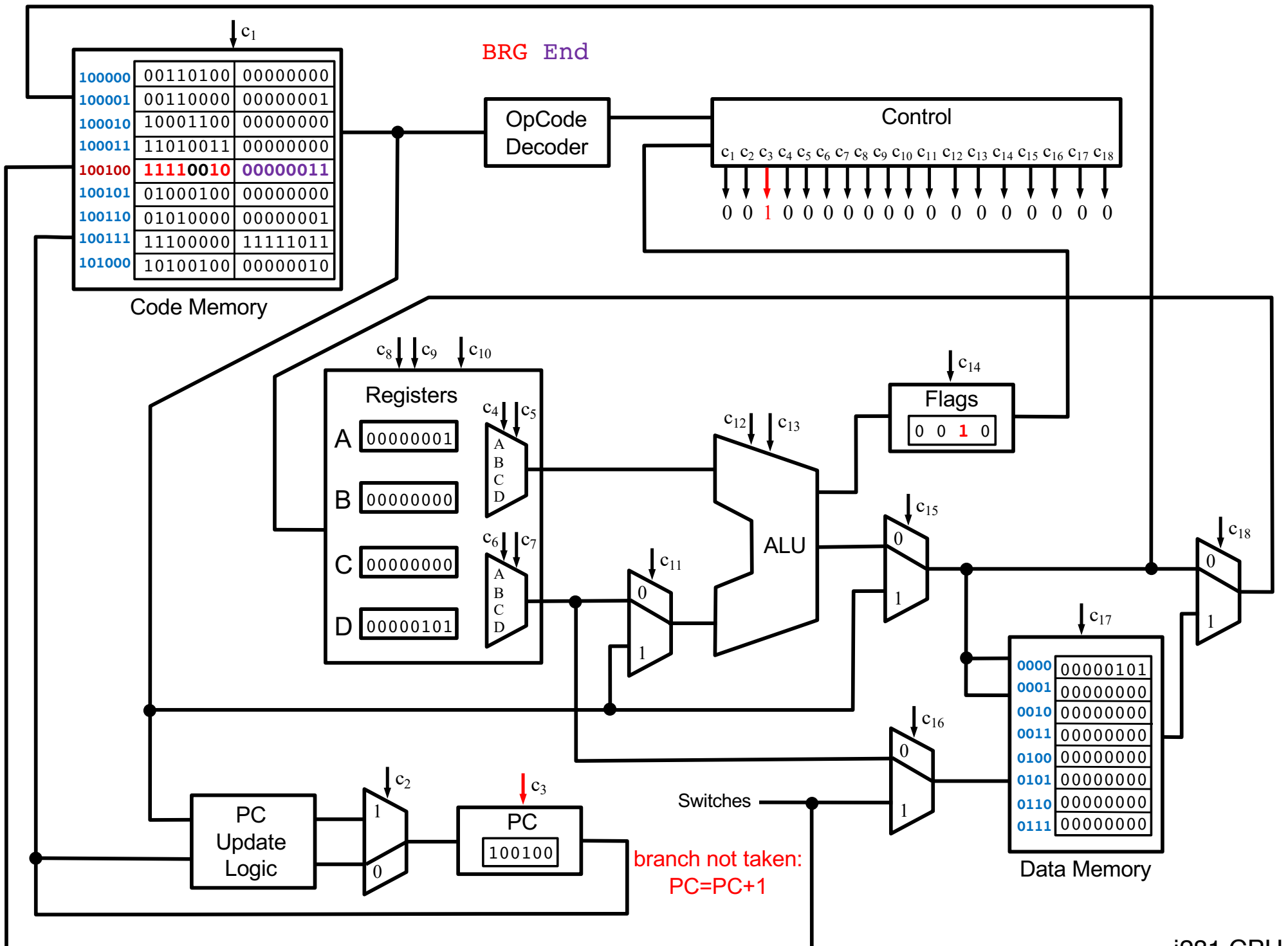
Machine Language

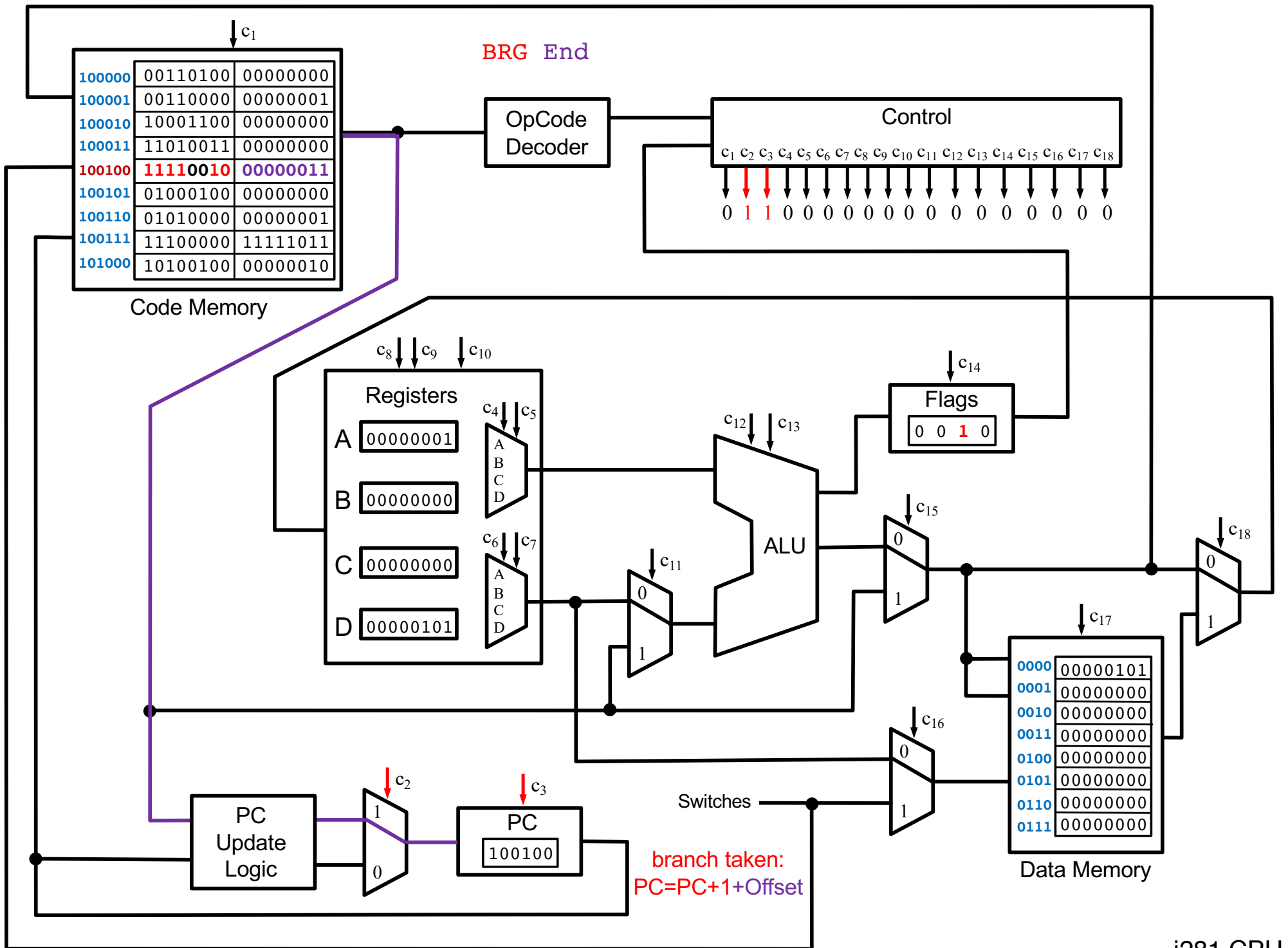














# Bubble Sort

# C Version

```
int array[] = {7, 3, 2, 1, 6, 4, 5, 8};
int last = 7; // last valid index in the array
int temp;
int i, j;

int main()
{
    for (i = 0; i < last; i++)
        for (j = 0; j < last-i; j++)
            if (array[j] > array[j+1]){
                temp = array[j];
                array[j] = array[j+1];
                array[j+1] = temp;
            }

    //for(i = 0; i < N; i++){
    //    printf("%d, ", array[i]);
    //}
}
```

# Assembly Version

```
.data
array  BYTE 7, 3, 2, 1, 6, 4, 5, 8
last   BYTE 7
temp   BYTE ?

.code

Outer:  LOADI  A, 0           ; i = 0;
        LOAD  D, [last]     ; Load last into D
        LOADI B, 0           ; j = 0;
        CMP   A, D           ; i < last
        BRGE  End           ; If i >= last break out of the outer loop
Inner:  LOAD  D, [last]     ; Re-Load last into D (this register is shared)
        SUB   D, A           ; D = D - A (i.e., D = last - i)
        CMP   B, D           ; j < last - i
        BRGE  Iinc          ; If j >= last-i branch to Iinc
If:     LOADF C, [array+B]   ; C = array[j]
        LOADF D, [array+B+1] ; D = array[j+1] (compiler adds 1 to addr. of array)
        CMP   D, C           ; if array[j+1] < array[j] (switched direction)
        BRGE  Jinc          ;
Swap:   STOREF [array+B], D
        STOREF [array+B+1], C
Jinc:   ADDI  B, 1           ; j++
        JUMP  Inner
Iinc:   ADDI  A, 1           ; i++
        JUMP  Outer
End:    NOOP                ; Do nothing

; Register allocation:
; A: i
; B: j
; C: array[j]
; D: last, array[j+1]

; Notes: i and j are optimized away. They exist only in registers, not in the main memory.
```

# Machine Code Version

```
.data
array  BYTE 7, 3, 2, 1, 6, 4, 5, 8
last   BYTE 7
temp   BYTE ?
```

```
.code

        LOADI  A, 0
Outer:  LOAD    D, [last]
        LOADI  B, 0
        CMP    A, D
        BRGE  End
Inner:  LOAD    D, [last]
        SUB    D, A
        CMP    B, D
        BRGE  Iinc
If:     LOADF  C, [array+B]
        LOADF  D, [array+B+1]
        CMP    D, C
        BRGE  Jinc
Swap:   STOREF [array+B], D
        STOREF [array+B+1], C
Jinc:   ADDI   B, 1
        JUMP  Inner
Iinc:   ADDI   A, 1
        JUMP  Outer
End:    NOOP
```

# Machine Code Version

		Data Memory:
.data		
array	BYTE 7, 3, 2, 1, 6, 4, 5, 8	00000111
last	BYTE 7	00000011
temp	BYTE ?	00000010
		00000001
.code		00000110
Outer:	LOADI A, 0	00000100
	LOAD D, [last]	00000101
	LOADI B, 0	00001000
	CMP A, D	00000111
	BRGE End	00000000
Inner:	LOAD D, [last]	
	SUB D, A	
	CMP B, D	
	BRGE Iinc	
If:	LOADF C, [array+B]	
	LOADF D, [array+B+1]	
	CMP D, C	
	BRGE Jinc	
Swap:	STOREF [array+B], D	
	STOREF [array+B+1], C	
Jinc:	ADDI B, 1	
	JUMP Inner	
Iinc:	ADDI A, 1	
	JUMP Outer	
End:	NOOP	

# Machine Code Version

```
.data
array    BYTE 7, 3, 2, 1, 6, 4, 5, 8
last     BYTE 7
temp     BYTE ?

.code

Outer:   LOADI  A, 0
         LOAD   D, [last]
         LOADI  B, 0
         CMP    A, D
         BRGE   End
Inner:   LOAD   D, [last]
         SUB    D, A
         CMP    B, D
         BRGE   Iinc
If:      LOADF  C, [array+B]
         LOADF  D, [array+B+1]
         CMP    D, C
         BRGE   Jinc
Swap:    STOREF [array+B], D
         STOREF [array+B+1], C
Jinc:    ADDI   B, 1
         JUMP   Inner
Iinc:    ADDI   A, 1
         JUMP   Outer
End:     NOOP
```

## Data Memory:

```
00000111 //array[0]
00000011 //array[1]
00000010 //array[2]
00000001 //array[3]
00000110 //array[4]
00000100 //array[5]
00000101 //array[6]
00001000 //array[7]
00000111 //last
00000000 //temp
```

# Machine Code Version

		Address	Data Memory:
<b>.data</b>			
array	BYTE 7, 3, 2, 1, 6, 4, 5, 8	0000	00000111 //array[0]
last	BYTE 7	0001	00000011 //array[1]
temp	BYTE ?	0010	00000010 //array[2]
		0011	00000001 //array[3]
<b>.code</b>		0100	00000110 //array[4]
Outer:	LOADI A, 0	0101	00000100 //array[5]
	LOAD D, [last]	0110	00000101 //array[6]
	LOADI B, 0	0111	00001000 //array[7]
	CMP A, D	1000	00000111 //last
	BRGE End	1001	00000000 //temp
Inner:	LOAD D, [last]		
	SUB D, A		
	CMP B, D		
	BRGE Inc		
If:	LOADF C, [array+B]		
	LOADF D, [array+B+1]		
	CMP D, C		
	BRGE Jinc		
Swap:	STOREF [array+B], D		
	STOREF [array+B+1], C		
Jinc:	ADDI B, 1		
	JUMP Inner		
Inc:	ADDI A, 1		
	JUMP Outer		
End:	NOOP		

# Machine Code Version

		Address	Data Memory:
.data			
array	BYTE 7, 3, 2, 1, 6, 4, 5, 8	0000	00000111 //array[0]
last	BYTE 7	0001	00000011 //array[1]
temp	BYTE ?	0010	00000010 //array[2]
		0011	00000001 //array[3]
.code		0100	00000110 //array[4]
Outer:	LOADI A, 0	0101	00000100 //array[5]
	LOAD D, [last]	0110	00000101 //array[6]
	LOADI B, 0	0111	00001000 //array[7]
	CMP A, D	1000	00000111 //last
	BRGE End	1001	00000000 //temp
Inner:	LOAD D, [last]	1010	00000000
	SUB D, A	1011	00000000
	CMP B, D	1100	00000000
	BRGE Iinc	1101	00000000
If:	LOADF C, [array+B]	1110	00000000
	LOADF D, [array+B+1]	1111	00000000
	CMP D, C		
	BRGE Jinc		
Swap:	STOREF [array+B], D		
	STOREF [array+B+1], C		
Jinc:	ADDI B, 1		
	JUMP Inner		
Iinc:	ADDI A, 1		
	JUMP Outer		
End:	NOOP		



# Machine Code Version

```
.data
array  BYTE 7, 3, 2, 1, 6, 4, 5, 8
last   BYTE 7
temp   BYTE ?
```

```
.code

        LOADI  A, 0
Outer:  LOAD   D, [last]
        LOADI  B, 0
        CMP    A, D
        BRGE  End
Inner:  LOAD   D, [last]
        SUB    D, A
        CMP    B, D
        BRGE  Iinc
If:     LOADF  C, [array+B]
        LOADF  D, [array+B+1]
        CMP    D, C
        BRGE  Jinc
Swap:   STOREF [array+B], D
        STOREF [array+B+1], C
Jinc:   ADDI   B, 1
        JUMP  Inner
Iinc:   ADDI   A, 1
        JUMP  Outer
End:    NOOP
```

# Machine Code Version

```
.data
array  BYTE 7, 3, 2, 1, 6, 4, 5, 8
last   BYTE 7
temp   BYTE ?
```

```
.code

Outer:  LOADI  A, 0
        LOAD  D, [last]
        LOADI B, 0
        CMP   A, D
        BRGE  End
Inner:  LOAD  D, [last]
        SUB   D, A
        CMP   B, D
        BRGE  Iinc
If:     LOADF C, [array+B]
        LOADF D, [array+B+1]
        CMP   D, C
        BRGE  Jinc
Swap:   STOREF [array+B], D
        STOREF [array+B+1], C
Jinc:   ADDI  B, 1
        JUMP  Inner
Iinc:   ADDI  A, 1
        JUMP  Outer
End:    NOOP
```

## Code Memory:

```
0011000000000000
1000110000001000
0011010000000000
1101001100000000
1111001100001110
1000110000001000
0110110000000000
1101011100000000
1111001100001000
1001100100000000
1001110100000001
1101111000000000
1111001100000010
1011110100000000
1011100100000001
0101010000000001
1110000011110100
0101000000000001
1110000011101110
0000000000000000
```

# Machine Code Version

```
.data
array  BYTE 7, 3, 2, 1, 6, 4, 5, 8
last   BYTE 7
temp   BYTE ?
```

```
.code
                                Address  Code Memory:
Outer:  LOADI  A, 0                100000  0011000000000000
        LOAD   D, [last]           100001  1000110000001000
        LOADI  B, 0                100010  0011010000000000
        CMP    A, D                100011  1101001100000000
        BRGE   End                 100100  1111001100001110
Inner:  LOAD   D, [last]           100101  1000110000001000
        SUB    D, A                100110  0110110000000000
        CMP    B, D                100111  1101011100000000
        BRGE   Iinc               101000  1111001100001000
If:     LOADF  C, [array+B]         101001  1001100100000000
        LOADF  D, [array+B+1]     101010  1001110100000001
        CMP    D, C                101011  1101111000000000
        BRGE   Jinc               101100  1111001100000010
Swap:   STOREF [array+B], D        101101  1011110100000000
        STOREF [array+B+1], C     101110  1011100100000001
Jinc:   ADDI   B, 1                101111  0101010000000001
        JUMP   Inner              110000  1110000011110100
Iinc:   ADDI   A, 1                110001  0101000000000001
        JUMP   Outer              110010  1110000011101110
End:    NOOP                       110011  0000000000000000
```



**Questions?**

**THE END**