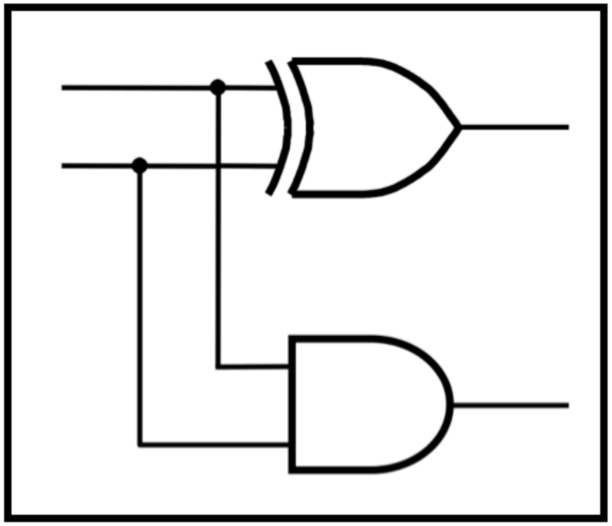


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Design Examples

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- HW3 is due on Monday Sep 13 @ 4pm

- Please write clearly on the first page the following three things:

    - Your First and Last Name
    - Your Student ID Number
    - Your Lab Section Letter

- Submit on Canvas as *one* PDF file.

- Please orient your pages such that the text can be read without the need to rotate the page.

Quick Review

Axioms of Boolean Algebra

1a. $0 \bullet 0 = 0$

1b. $1 + 1 = 1$

2a. $1 \bullet 1 = 1$

2b. $0 + 0 = 0$

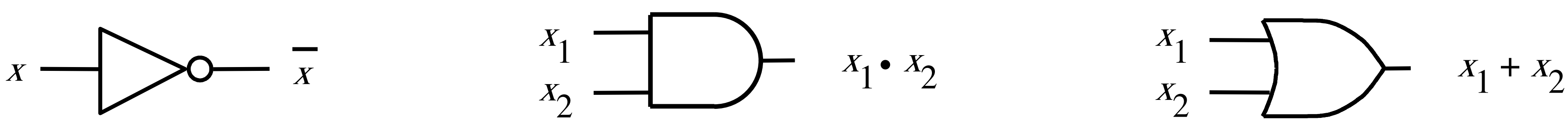
3a. $0 \bullet 1 = 1 \bullet 0 = 0$

3b. $1 + 0 = 0 + 1 = 1$

4a. If $x=0$, then $\overline{x} = 1$

4b. If $x=1$, then $\overline{x} = 0$

The Three Basic Logic Gates



NOT gate AND gate OR gate

[Figure 2.8 from the textbook]

Single-Variable Theorems

5a. $x \bullet 0 = 0$

5b. $x + 1 = 1$

6a. $x \bullet 1 = x$

6b. $x + 0 = x$

7a. $x \bullet x = x$

7b. $x + x = x$

8a. $x \bullet \overline{x} = 0$

8b. $x + \overline{x} = 1$

9. $\overline{\overline{x}} = x$

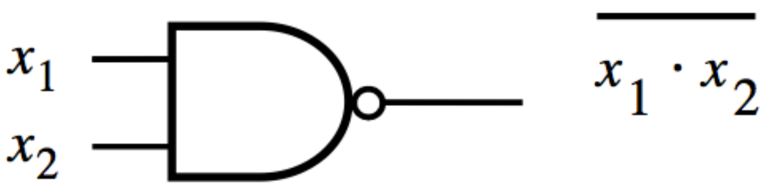
Two- and Three-Variable Properties

10a.	$x \bullet y = y \bullet x$	Commutative
10b.	$x + y = y + x$	
11a.	$x \bullet (y \bullet z) = (x \bullet y) \bullet z$	Associative
11b.	$x + (y + z) = (x + y) + z$	
12a.	$x \bullet (y + z) = x \bullet y + x \bullet z$	Distributive
12b.	$x + y \bullet z = (x + y) \bullet (x + z)$	
13a.	$x + x \bullet y = x$	Absorption
13b.	$x \bullet (x + y) = x$	

Two- and Three-Variable Properties

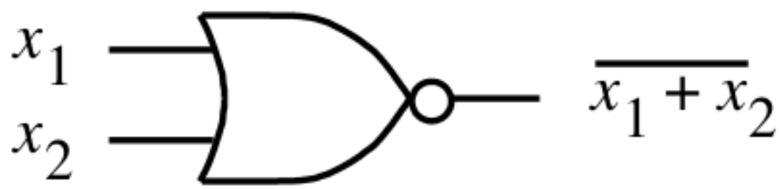
14a.	$x \bullet y + x \bullet \overline{y} = x$	Combining
14b.	$(x + y) \bullet (x + \overline{y}) = x$	
15a.	$\overline{x \bullet y} = \overline{x} + \overline{y}$	DeMorgan's theorem
15b.	$\overline{x + y} = \overline{x} \bullet \overline{y}$	
16a.	$x + \overline{x} \bullet y = x + y$	
16b.	$x \bullet (\overline{x} + y) = x \bullet y$	
17a.	$x \bullet y + y \bullet z + \overline{x} \bullet z = x \bullet y + \overline{x} \bullet z$	Consensus
17b.	$(x+y) \bullet (y+z) \bullet (\overline{x}+z) = (x+y) \bullet (\overline{x}+z)$	

NAND Gate



x_1	x_2	f
0	0	1
0	1	1
1	0	1
1	1	0

NOR Gate



x_1	x_2	f
0	0	1
0	1	0
1	0	0
1	1	0

Why do we need two more gates?

They can be implemented with fewer transistors.

Each of the new gates can be used to implement the three basic logic gates: NOT, AND, OR.

Implications

Any Boolean function can be implemented with only NAND gates!

Implications

Any Boolean function can be implemented with only NAND gates!

The same is also true for NOR gates!

Minterms
(for two variables)

The Four Minterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

$m_0(x, y)$

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

$m_1(x, y)$

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

$m_2(x, y)$

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

$m_3(x, y)$

The Four Minterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

$m_0(x, y)$ $m_1(x, y)$ $m_2(x, y)$ $m_3(x, y)$

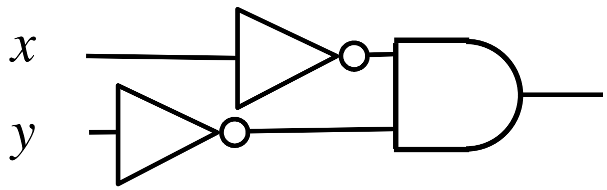
The Four Minterms

x	y		$m_0(x, y)$	$m_1(x, y)$	$m_2(x, y)$	$m_3(x, y)$
0	0		1	0	0	0
0	1		0	1	0	0
1	0		0	0	1	0
1	1		0	0	0	1

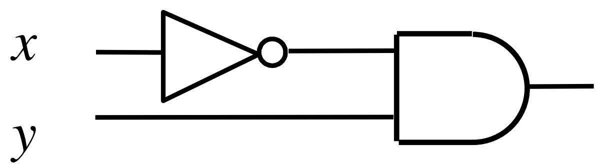
The Four Minterms

x	y		$\bar{x}\,\bar{y}$	$\bar{x}\,y$	$x\,\bar{y}$	$x\,y$
0	0		1	0	0	0
0	1		0	1	0	0
1	0		0	0	1	0
1	1		0	0	0	1

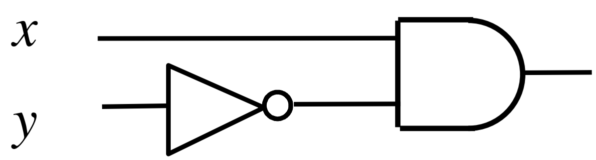
Circuits for the four minterms



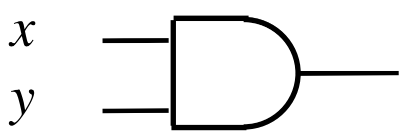
$m_0(x, y) = \overline{x}\ \overline{y}$



$m_1(x, y) = \overline{x}\ y$



$m_2(x, y) = x\ \overline{y}$



$m_3(x, y) = x\ y$

Maxterms
(for two variables)

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

$M_0(x, y)$

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

$M_1(x, y)$

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

$M_2(x, y)$

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

$M_3(x, y)$

The Four Maxterms

x	y		$M_0(x, y)$	$M_1(x, y)$	$M_2(x, y)$	$M_3(x, y)$
0	0		0	1	1	1
0	1		1	0	1	1
1	0		1	1	0	1
1	1		1	1	1	0

The Four Maxterms

x	y		x + y	$x + \overline{y}$	$\overline{x} + y$	$\overline{x} + \overline{y}$
0	0		0	1	1	1
0	1		1	0	1	1
1	0		1	1	0	1
1	1		1	1	1	0

Minterms and Maxterms
(for two variables)

Minterms and Maxterms

x	y	m_0
0	0	1
0	1	0
1	0	0
1	1	0

x	y	m_1
0	0	0
0	1	1
1	0	0
1	1	0

x	y	m_2
0	0	0
0	1	0
1	0	1
1	1	0

x	y	m_3
0	0	0
0	1	0
1	0	0
1	1	1

x	y	M_0
0	0	0
0	1	1
1	0	1
1	1	1

x	y	M_1
0	0	1
0	1	0
1	0	1
1	1	1

x	y	M_2
0	0	1
0	1	1
1	0	0
1	1	1

x	y	M_3
0	0	1
0	1	1
1	0	1
1	1	0

Minterms and Maxterms

$m_0(x, y) = \overline{x}\,\overline{y}$

$M_0(x, y) = x + y$

$m_1(x, y) = \overline{x}\, y$

$M_1(x, y) = x + \overline{y}$

$m_2(x, y) = x\,\overline{y}$

$M_2(x, y) = \overline{x} + y$

$m_3(x, y) = x\, y$

$M_3(x, y) = \overline{x} + \overline{y}$

Minterms
(for three variables)

The Eight Minterms

x	y	z	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

The Eight Minterms

x	y	z	m_0	m_1	m_2	m_3	m_4	m_5	m_6	m_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Expressions for the Minterms

$$m_0 = \overline{x}\ \overline{y}\ \overline{z}$$

$$m_1 = \overline{x}\ \overline{y}\ z$$

$$m_2 = \overline{x}\ y\ \overline{z}$$

$$m_3 = \overline{x}\ y\ z$$

$$m_4 = x\ \overline{y}\ \overline{z}$$

$$m_5 = x\ \overline{y}\ z$$

$$m_6 = x\ y\ \overline{z}$$

$$m_7 = x\ y\ z$$

Expressions for the Minterms

0	0	0	$m_0 = \bar{x}\,\bar{y}\,\bar{z}$
0	0	1	$m_1 = \bar{x}\,\bar{y}\,z$
0	1	0	$m_2 = \bar{x}\,y\,\bar{z}$
0	1	1	$m_3 = \bar{x}\,y\,z$
1	0	0	$m_4 = x\,\bar{y}\,\bar{z}$
1	0	1	$m_5 = x\,\bar{y}\,z$
1	1	0	$m_6 = x\,y\,\bar{z}$
1	1	1	$m_7 = x\,y\,z$

The bars coincide
with the 0's
in the binary expansion
of the minterm sub-index

Maxterms
(for three variables)

The Eight Maxterms

x	y	z	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

The Eight Maxterms

x	y	z	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7
0	0	0	0	1	1	1	1	1	1	1
0	0	1	1	0	1	1	1	1	1	1
0	1	0	1	1	0	1	1	1	1	1
0	1	1	1	1	1	0	1	1	1	1
1	0	0	1	1	1	1	0	1	1	1
1	0	1	1	1	1	1	1	0	1	1
1	1	0	1	1	1	1	1	1	0	1
1	1	1	1	1	1	1	1	1	1	0

Expressions for the Maxterms

$$M_0 = x + y + z$$

$$M_1 = x + y + \overline{z}$$

$$M_2 = x + \overline{y} + z$$

$$M_3 = x + \overline{y} + \overline{z}$$

$$M_4 = \overline{x} + y + z$$

$$M_5 = \overline{x} + y + \overline{z}$$

$$M_6 = \overline{x} + \overline{y} + z$$

$$M_7 = \overline{x} + \overline{y} + \overline{z}$$

Expressions for the Maxterms

0 0 0	$M_0 = x + y + z$	
0 0 1	$M_1 = x + y + \overline{z}$	
0 1 0	$M_2 = x + \overline{y} + z$	
0 1 1	$M_3 = x + \overline{y} + \overline{z}$	The bars coincide with the 1's in the binary expansion of the maxterm sub-index
1 0 0	$M_4 = \overline{x} + y + z$	
1 0 1	$M_5 = \overline{x} + y + \overline{z}$	
1 1 0	$M_6 = \overline{x} + \overline{y} + z$	
1 1 1	$M_7 = \overline{x} + \overline{y} + \overline{z}$	

Minterms and Maxterms (for three variables)

Minterms and Maxterms

$m_0 = \bar{x}\,\bar{y}\,\bar{z}$	$M_0 = x + y + z$
$m_1 = \bar{x}\,\bar{y}\,z$	$M_1 = x + y + \bar{z}$
$m_2 = \bar{x}\,y\,\bar{z}$	$M_2 = x + \bar{y} + z$
$m_3 = \bar{x}\,y\,z$	$M_3 = x + \bar{y} + \bar{z}$
$m_4 = x\,\bar{y}\,\bar{z}$	$M_4 = \bar{x} + y + z$
$m_5 = x\,\bar{y}\,z$	$M_5 = \bar{x} + y + \bar{z}$
$m_6 = x\,y\,\bar{z}$	$M_6 = \bar{x} + \bar{y} + z$
$m_7 = x\,y\,z$	$M_7 = \bar{x} + \bar{y} + \bar{z}$

Synthesis Example

Truth table for a three-way light control

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[Figure 2.31 from the textbook]

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

Let's Derive the SOP form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let’s Derive the SOP form

x_1	x_2	x_3	f	
0	0	0	0	
0	0	1	1	$x_1\ x_2\ x_3$
0	1	0	1	$x_1\ x_2\ x_3$
0	1	1	0	
1	0	0	1	$x_1\ x_2\ x_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1\ x_2\ x_3$

Let's Derive the SOP form

x_1	x_2	x_3	f	
0	0	0	0	
0	0	1	1	$\overline{x}_1 \overline{x}_2 x_3$
0	1	0	1	$\overline{x}_1 x_2 \overline{x}_3$
0	1	1	0	
1	0	0	1	$x_1 \overline{x}_2 \overline{x}_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1 x_2 x_3$

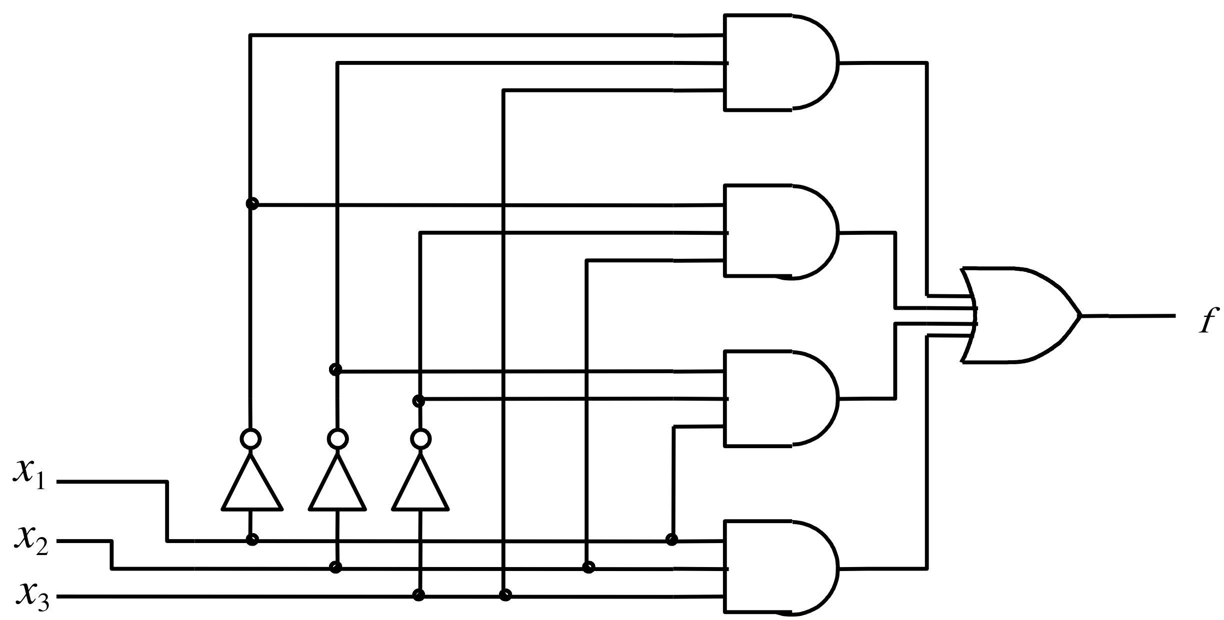
Let's Derive the SOP form

x_1	x_2	x_3	f	
0	0	0	0	
0	0	1	1	$\overline{x}_1 \overline{x}_2 x_3$
0	1	0	1	$\overline{x}_1 x_2 \overline{x}_3$
0	1	1	0	
1	0	0	1	$x_1 \overline{x}_2 \overline{x}_3$
1	0	1	0	
1	1	0	0	
1	1	1	1	$x_1 x_2 x_3$

$$f = m_1 + m_2 + m_4 + m_7$$

$$= \bar{x}_1\bar{x}_2x_3 + \bar{x}_1x_2\bar{x}_3 + x_1\bar{x}_2\bar{x}_3 + x_1x_2x_3$$

Sum-of-products realization



[Figure 2.32a from the textbook]

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

[Figure 2.31 from the textbook]

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1x_2x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

Let's Derive the POS form

x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Let’s Derive the POS form

x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + x_2 + x_3)$
1	0	0	1	
1	0	1	0	$(x_1 + x_2 + x_3)$
1	1	0	0	$(x_1 + x_2 + x_3)$
1	1	1	1	

Let's Derive the POS form

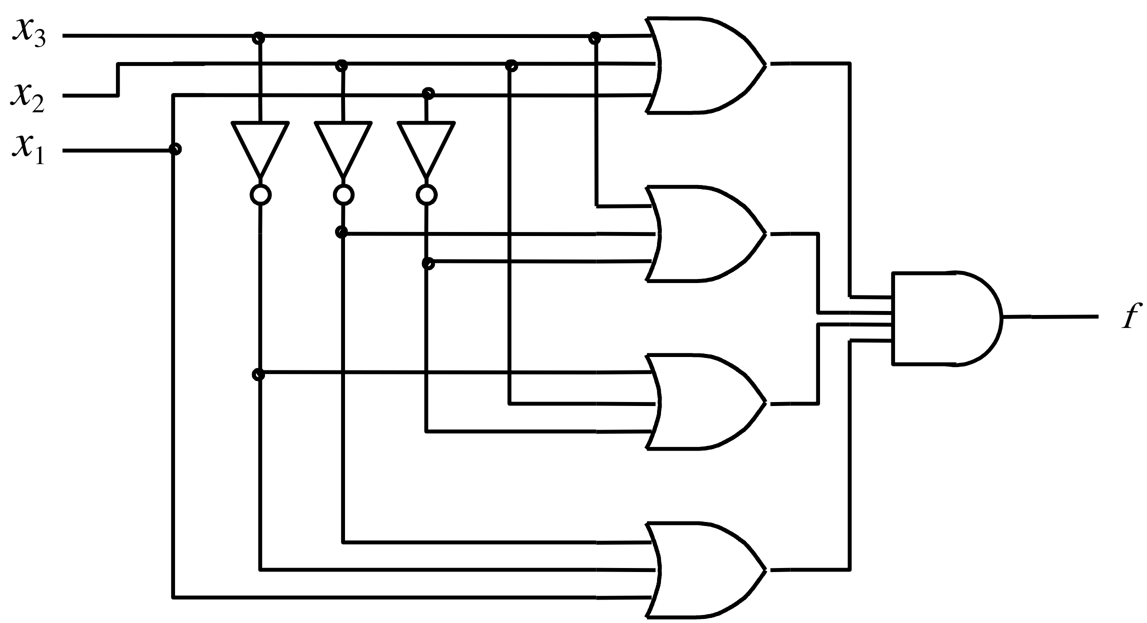
x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \overline{x}_2 + \overline{x}_3)$
1	0	0	1	
1	0	1	0	$(\overline{x}_1 + x_2 + \overline{x}_3)$
1	1	0	0	$(\overline{x}_1 + \overline{x}_2 + x_3)$
1	1	1	1	

Let's Derive the POS form

x_1	x_2	x_3	f	
0	0	0	0	$(x_1 + x_2 + x_3)$
0	0	1	1	
0	1	0	1	
0	1	1	0	$(x_1 + \overline{x}_2 + \overline{x}_3)$
1	0	0	1	
1	0	1	0	$(\overline{x}_1 + x_2 + \overline{x}_3)$
1	1	0	0	$(\overline{x}_1 + \overline{x}_2 + x_3)$
1	1	1	1	

$$\begin{aligned} f &= M_0 \cdot M_3 \cdot M_5 \cdot M_6 \\ &= (x_1 + x_2 + x_3)(x_1 + \bar{x}_2 + \bar{x}_3)(\bar{x}_1 + x_2 + \bar{x}_3)(\bar{x}_1 + \bar{x}_2 + x_3) \end{aligned}$$

Product-of-sums realization



[Figure 2.32b from the textbook]

Function Synthesis

Example 2.10

Implement the function $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1x_2x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1x_2x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

$$f(x_1, x_2, x_3) = \sum m(2, 3, 4, 6, 7)$$

- **The SOP expression is:**

$$\begin{aligned} f &= m_2 + m_3 + m_4 + m_6 + m_7 \\ &= \bar{x}_1 x_2 \bar{x}_3 + \bar{x}_1 x_2 x_3 + x_1 \bar{x}_2 \bar{x}_3 + x_1 x_2 \bar{x}_3 + x_1 x_2 x_3 \end{aligned}$$

- **This could be simplified as follows:**

$$\begin{aligned} f &= \bar{x}_1 x_2 (\bar{x}_3 + x_3) + x_1 (\bar{x}_2 + x_2) \bar{x}_3 + x_1 x_2 (\bar{x}_3 + x_3) \\ &= \bar{x}_1 x_2 + x_1 \bar{x}_3 + x_1 x_2 \\ &= (\bar{x}_1 + x_1) x_2 + x_1 \bar{x}_3 \\ &= x_2 + x_1 \bar{x}_3 \end{aligned}$$

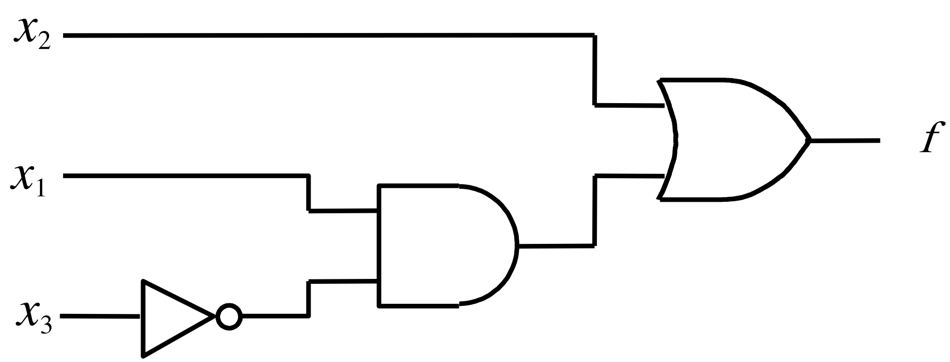
Recall Property 14a

14a. $x \bullet y + x \bullet \overline{y} = x$ Combining

14b. $(x + y)\bullet(x + \overline{y}) = x$

SOP realization of the function

The SOP expression is: $f = x_2 + x_1\overline{x}_3$



[Figure 2.30a from the textbook]

Example 2.12

Implement the function $f(x_1, x_2, x_3) = \Pi\, M(0, 1, 5)$,

which is equivalent to $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

$$f(x_1, x_2, x_3) = \Pi M(0, 1, 5)$$

- **The POS expression is:**

$$\begin{aligned} f &= M_0 \cdot M_1 \cdot M_5 \\ &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3) \end{aligned}$$

- **This could be simplified as follows:**

$$\begin{aligned} f &= (x_1 + x_2 + x_3)(x_1 + x_2 + \overline{x}_3)(x_1 + x_2 + \overline{x}_3)(\overline{x}_1 + x_2 + \overline{x}_3) \\ &= ((x_1 + x_2) + x_3)((x_1 + x_2) + \overline{x}_3)(x_1 + (x_2 + \overline{x}_3))(\overline{x}_1 + (x_2 + \overline{x}_3)) \\ &= ((x_1 + x_2) + x_3\overline{x}_3)(x_1\overline{x}_1 + (x_2 + \overline{x}_3)) \\ &= (x_1 + x_2)(x_2 + \overline{x}_3) \end{aligned}$$

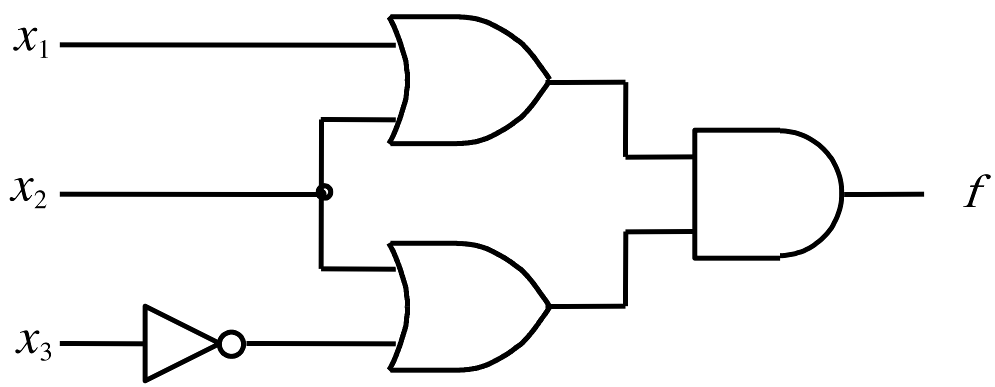
Recall Property 14b

14a. $x \bullet y + x \bullet \overline{y} = x$ Combining

14b. $(x + y)\bullet(x + \overline{y}) = x$

POS realization of the function

The POS expression is: $f = (x_1 + x_2)(x_2 + \overline{x}_3)$



[Figure 2.29a from the textbook]

More Examples

Example 2.14

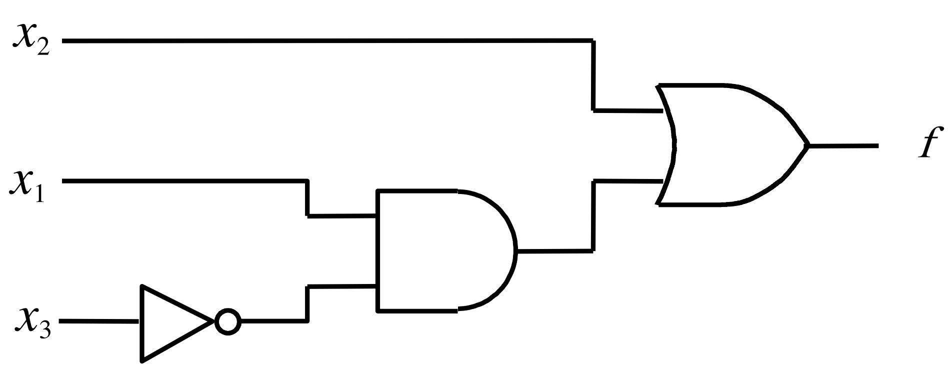
Implement the function $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$ using only NAND gates.

Example 2.14

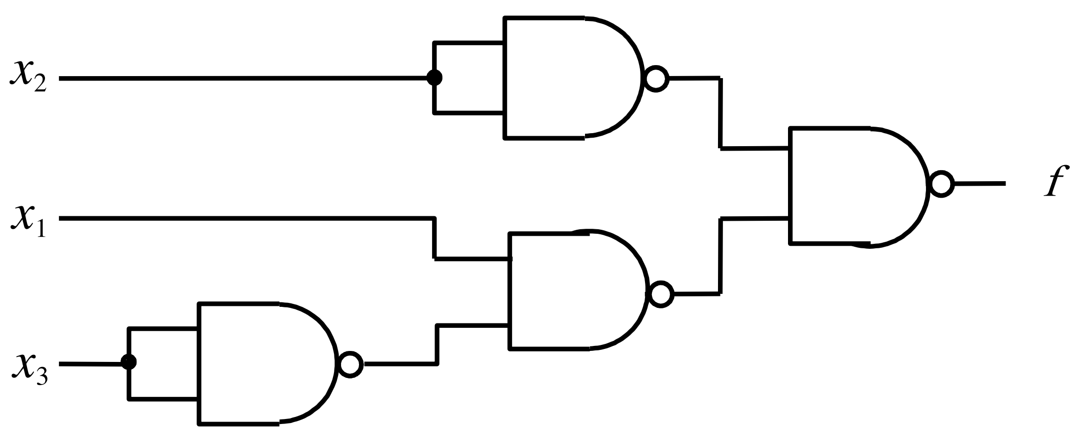
Implement the function $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$ using only NAND gates.

The SOP expression is: $f = x_2 + x_1\overline{x}_3$

NAND-gate realization of the function



(a) SOP implementation



(b) NAND implementation

[Figure 2.30 from the textbook]

Example 2.13

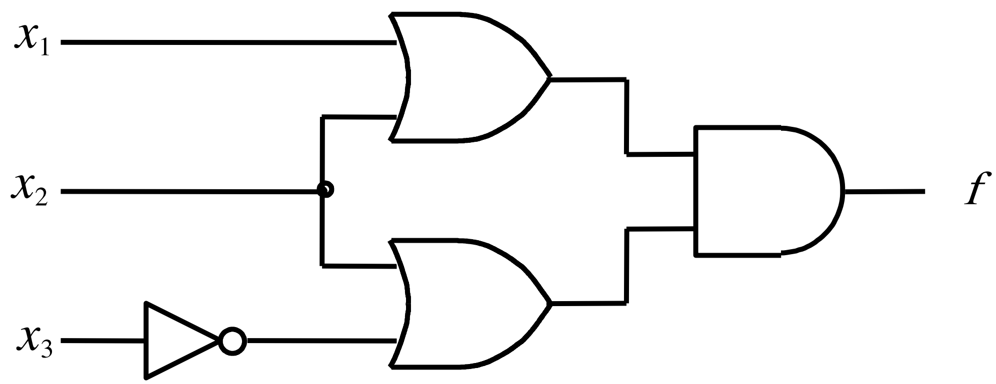
Implement the function $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$ using only NOR gates.

Example 2.13

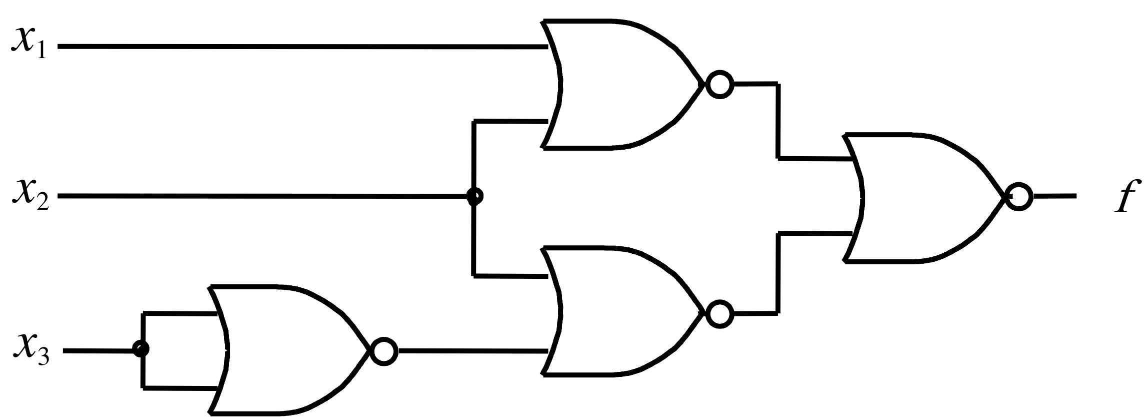
Implement the function $f(x_1, x_2, x_3) = \Sigma\, m(2, 3, 4, 6, 7)$ using only NOR gates.

The POS expression is: $f = (x_1 + x_2)(x_2 + \overline{x}_3)$

NOR-gate realization of the function



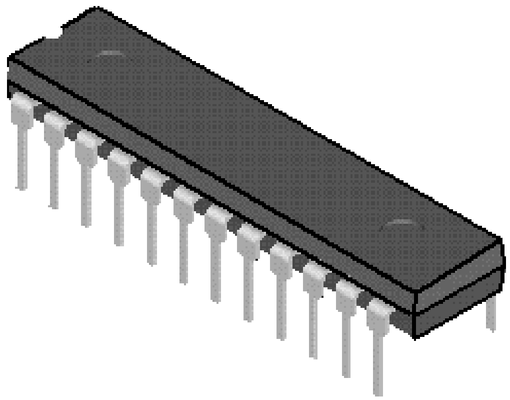
(a) POS implementation



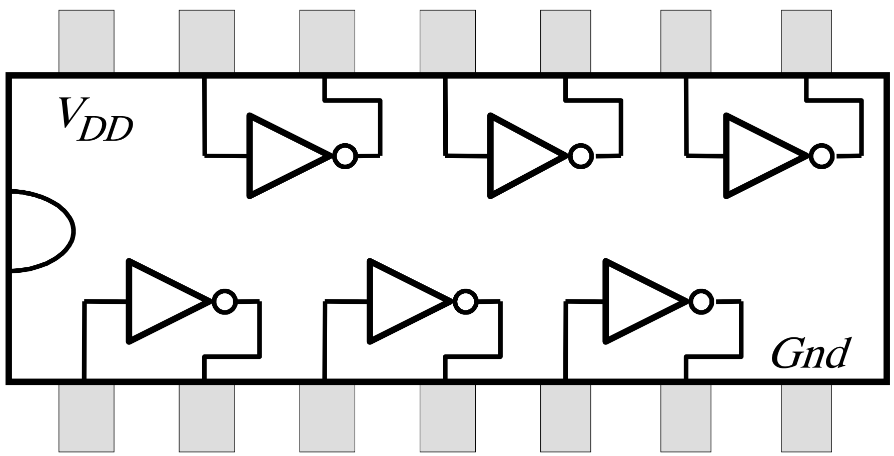
(b) NOR implementation

[Figure 2.29 from the textbook]

Implementation with Chips



(a) Dual-inline package



(b) Structure of 7404 chip

Figure B.21. A 7400-series chip.

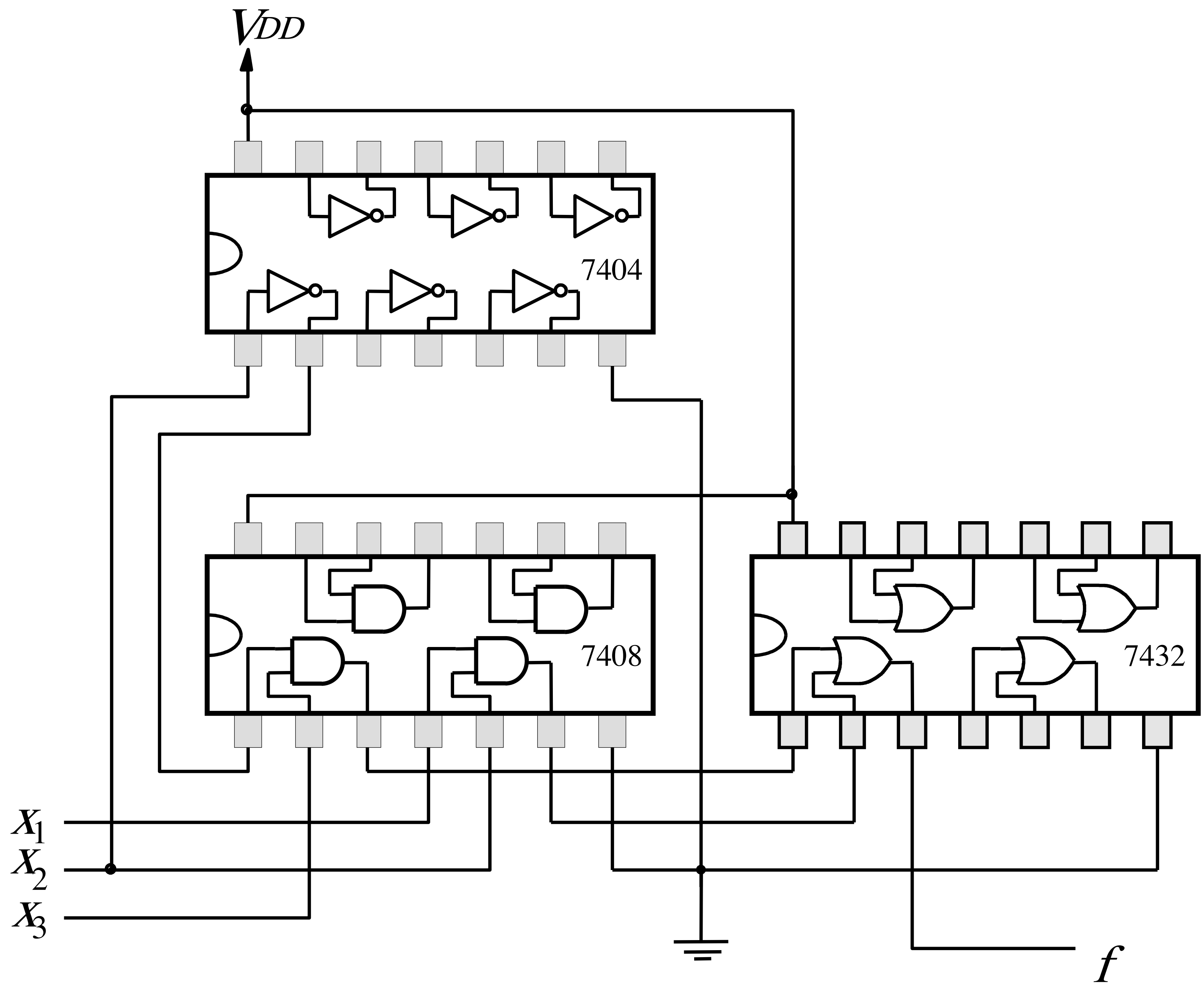


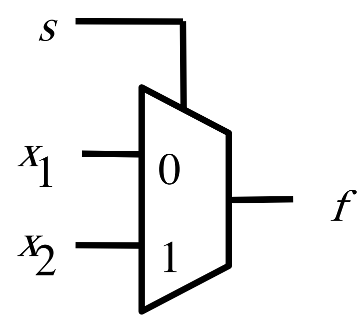
Figure B.22. An implementation of $f = x_1x_2 + \overline{x_2}x_3$.

Multiplexers

2-to-1 Multiplexer (Definition)

- Has two inputs: x_1 and x_2
- Also has another input line s
- If s=0, then the output is equal to x_1
- If s=1, then the output is equal to x_2

Graphical Symbol for a 2-to-1 Multiplexer

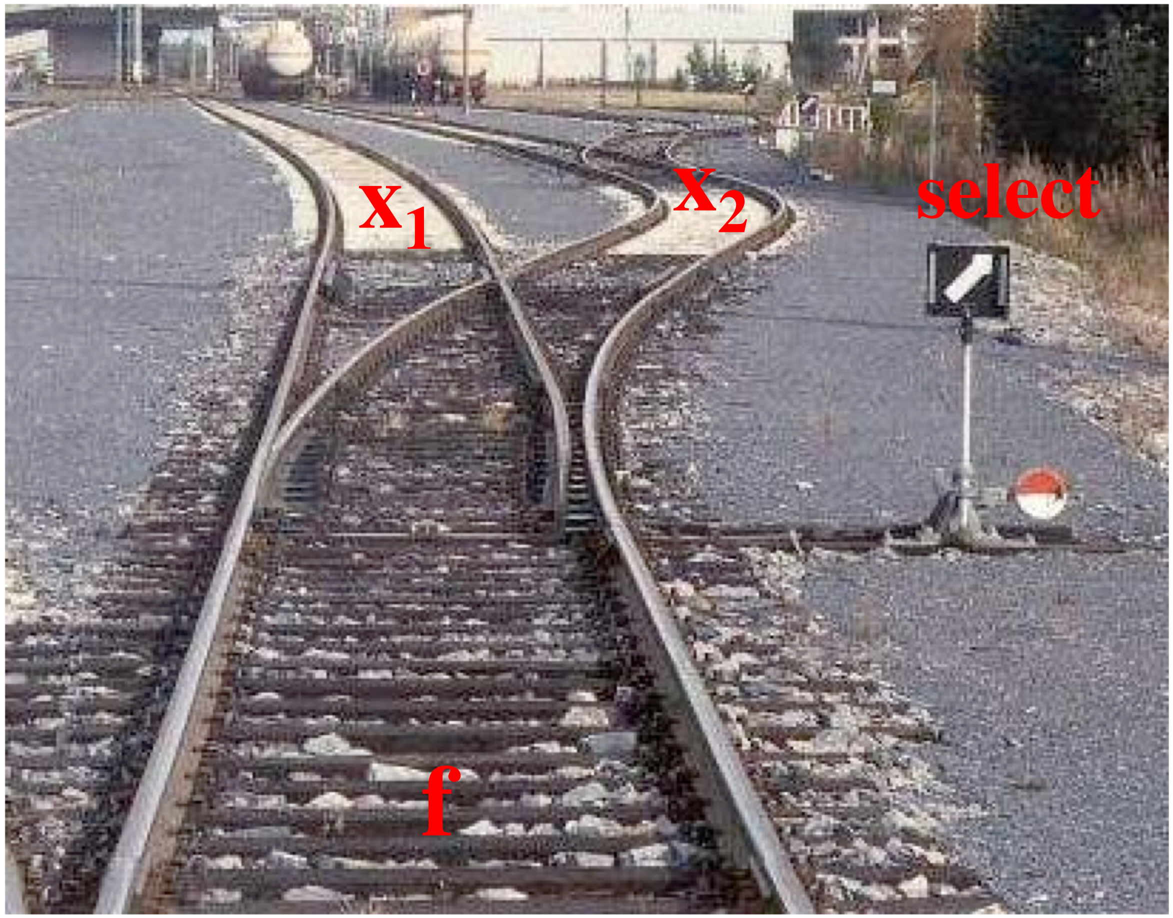


[Figure 2.33c from the textbook]

Analogy: Railroad Switch

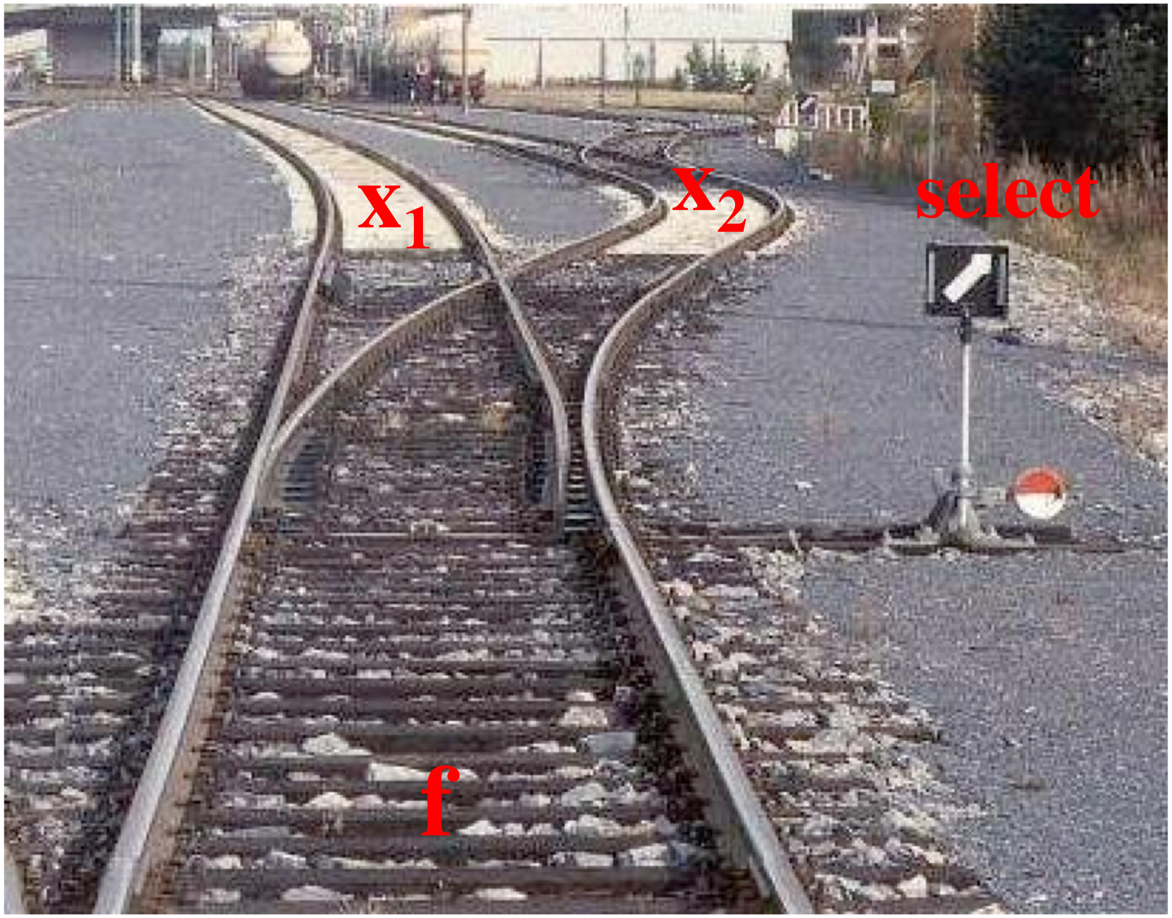
http://en.wikipedia.org/wiki/Railroad_switch]

Analogy: Railroad Switch



http://en.wikipedia.org/wiki/Railroad_switch]

Analogy: Railroad Switch



This is not a perfect analogy because the trains can go in either direction, while the multiplexer would only allow them to go from top to bottom.

http://en.wikipedia.org/wiki/Railroad_switch]

Truth Table for a 2-to-1 Multiplexer

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

[Figure 2.33a from the textbook]

Let’s Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	$\boxed{1}$
0 1 1	$\boxed{1}$
1 0 0	0
1 0 1	$\boxed{1}$
1 1 0	0
1 1 1	$\boxed{1}$

Where should we put the negation signs?

$s\ x_1\ x_2$

$s\ x_1\ x_2$

$s\ x_1\ x_2$

$s\ x_1\ x_2$

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	$\boxed{1}$	$\overline{s}\, x_1\, \overline{x}_2$
0 1 1	$\boxed{1}$	$\overline{s}\, x_1\, x_2$
1 0 0	0	
1 0 1	$\boxed{1}$	$s\, \overline{x}_1\, x_2$
1 1 0	0	
1 1 1	$\boxed{1}$	$s\, x_1\, x_2$

Let's Derive the SOP form

$s\ x_1\ x_2$	$f(s, x_1, x_2)$	
0 0 0	0	
0 0 1	0	
0 1 0	$\boxed{1}$	$\overline{s}\, x_1\, \overline{x_2}$
0 1 1	$\boxed{1}$	$\overline{s}\, x_1\, x_2$
1 0 0	0	
1 0 1	$\boxed{1}$	$s\, \overline{x_1}\, x_2$
1 1 0	0	
1 1 1	$\boxed{1}$	$s\, x_1\, x_2$

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

$$f(s, x_1, x_2) = \overline{s}\, x_1\, (\overline{x_2} + x_2) + s\, (\overline{x_1} + x_1)\, x_2$$

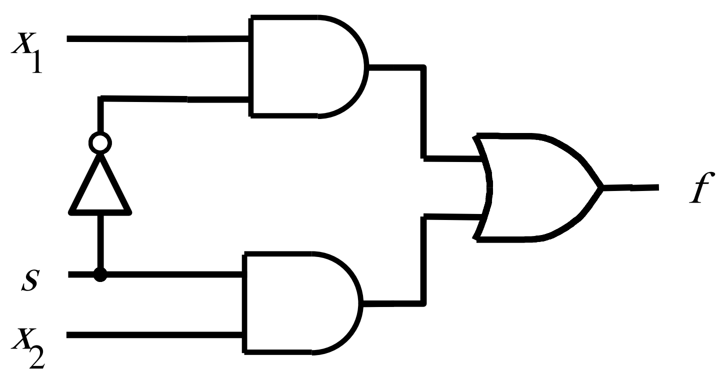
Let's simplify this expression

$$f(s, x_1, x_2) = \overline{s}\, x_1\, \overline{x_2} + \overline{s}\, x_1\, x_2 + s\, \overline{x_1}\, x_2 + s\, x_1\, x_2$$

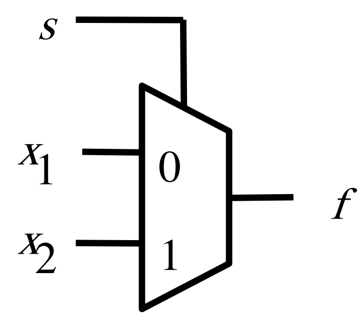
$$f(s, x_1, x_2) = \overline{s}\, x_1\, (\overline{x_2} + x_2) + s\, (\overline{x_1} + x_1)\, x_2$$

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

Circuit for 2-to-1 Multiplexer



(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

[Figure 2.33b-c from the textbook]

More Compact Truth-Table Representation

$s\ x_1\ x_2$	$f(s, x_1, x_2)$
0 0 0	0
0 0 1	0
0 1 0	1
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

(a)Truth table

s	$f(s, x_1, x_2)$
0	x_1
1	x_2

[Figure 2.33 from the textbook]

4-to-1 Multiplexer (Definition)

- Has four inputs: w_0 , w_1, w_2, w_3
- Also has two select lines: s_1 and s_0
- If s_1=0 and s_0=0, then the output f is equal to w_0
- If s_1=0 and s_0=1, then the output f is equal to w_1
- If s_1=1 and s_0=0, then the output f is equal to w_2
- If s_1=1 and s_0=1, then the output f is equal to w_3

4-to-1 Multiplexer (Definition)

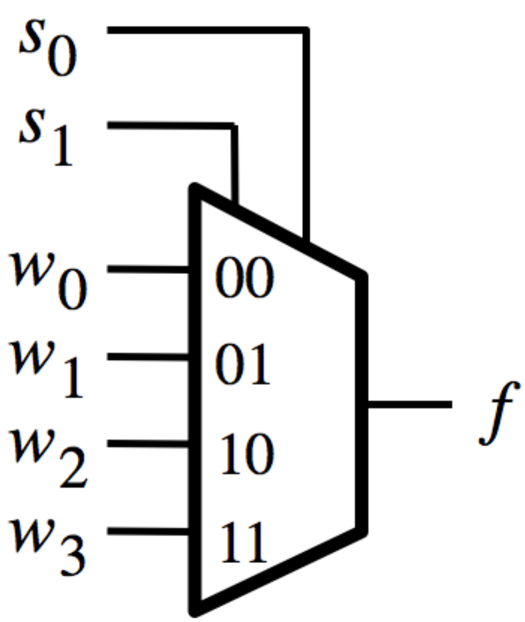
- Has four inputs: w_0 , w_1, w_2, w_3

- Also has two select lines: s_1 and s_0

- If s_1=0 and s_0=0, then the output f is equal to w_0
- If s_1=0 and s_0=1, then the output f is equal to w_1
- If s_1=1 and s_0=0, then the output f is equal to w_2
- If s_1=1 and s_0=1, then the output f is equal to w_3

We'll talk more about this when we get
to chapter 4, but here is a quick preview.

Graphical Symbol and Truth Table



(a) Graphic symbol

s_1	s_0	f
0	0	w_0
0	1	w_1
1	0	w_2
1	1	w_3

(b) Truth table

[Figure 4.2a-b from the textbook]

The long-form truth table

The long-form truth table

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	0	0
		0	0	0	1	1
		0	0	1	0	0
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	1
		0	1	1	0	0
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	1
		1	0	1	0	0
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	1
		1	1	1	0	0
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
0	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	1
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	0
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	0	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	1
		0	1	0	1	1
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	0
		1	0	1	1	0
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	0
		0	1	1	1	0
		1	0	0	0	1
		1	0	0	1	1
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

[http://www.absoluteastronomy.com/topics/Multiplexer]

The long-form truth table

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	0	0
		0	0	0	1	1
		0	0	1	0	0
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	1
		0	1	1	0	0
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	1
		1	0	1	0	0
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	1
		1	1	1	0	0
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
0	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	1
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	0
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	0	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	1
		0	1	0	1	1
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	0
		1	0	1	1	0
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	0
		0	1	1	1	0
		1	0	0	0	1
		1	0	0	1	1
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

[http://www.absoluteastronomy.com/topics/Multiplexer]

The long-form truth table

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	0	0
		0	0	0	1	1
		0	0	1	0	0
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	1
		0	1	1	0	0
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	1
		1	0	1	0	0
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	1
		1	1	1	0	0
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
0	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	1
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	0
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	0	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	1
		0	1	0	1	1
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	0
		1	0	1	1	0
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	0
		0	1	1	1	0
		1	0	0	0	1
		1	0	0	1	1
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

[http://www.absoluteastronomy.com/topics/Multiplexer]

The long-form truth table

S_1	S_0	I_3	I_2	I_1	I_0	F
0	0	0	0	0	0	0
		0	0	0	1	1
		0	0	1	0	0
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	1
		0	1	1	0	0
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	1
		1	0	1	0	0
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	1
		1	1	1	0	0
		1	1	1	1	1

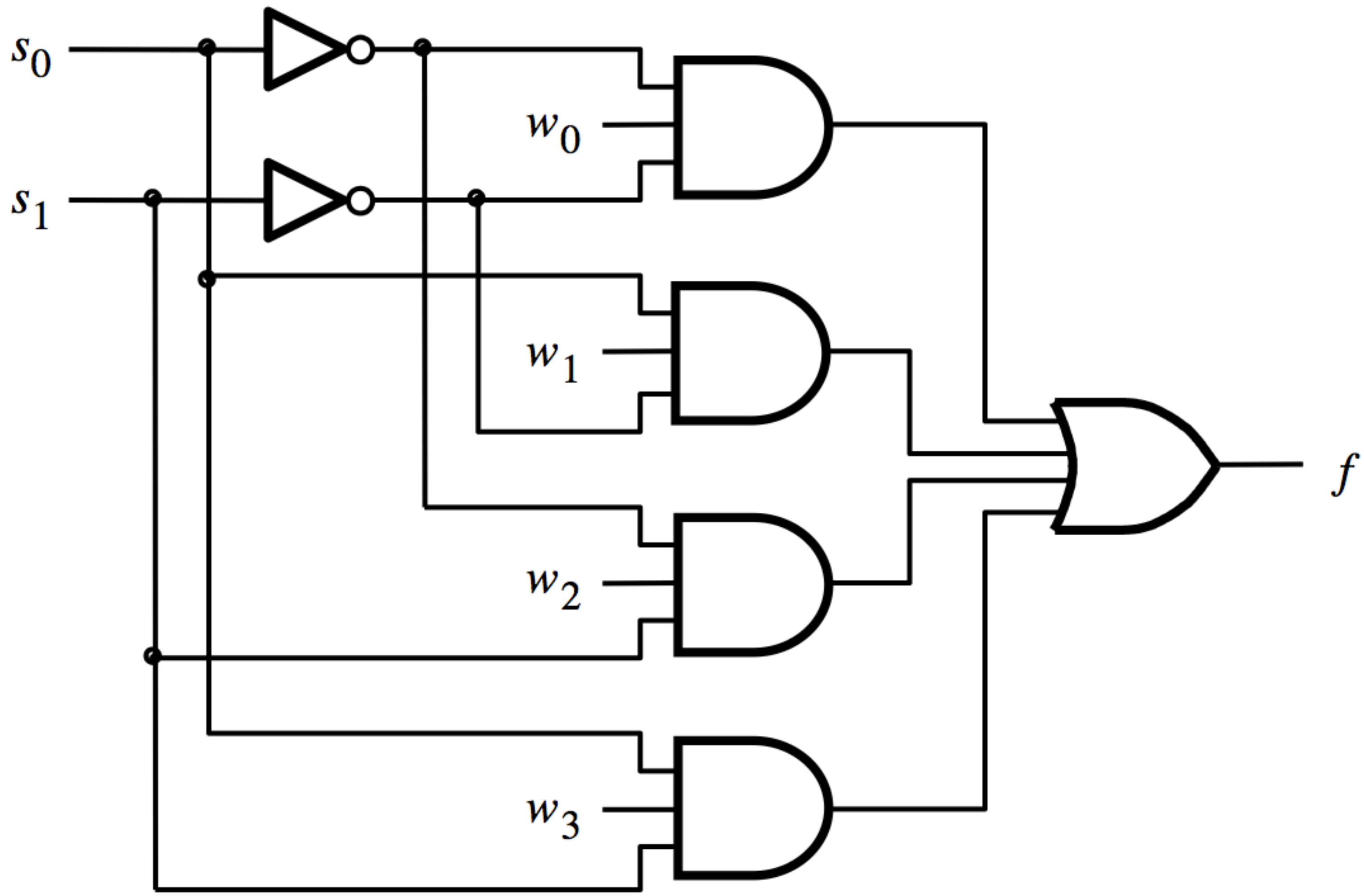
S_1	S_0	I_3	I_2	I_1	I_0	F
0	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	1
		0	0	1	1	1
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	0
		1	1	0	1	0
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	0	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	1
		0	1	0	1	1
		0	1	1	0	1
		0	1	1	1	1
		1	0	0	0	0
		1	0	0	1	0
		1	0	1	0	0
		1	0	1	1	0
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

S_1	S_0	I_3	I_2	I_1	I_0	F
1	1	0	0	0	0	0
		0	0	0	1	0
		0	0	1	0	0
		0	0	1	1	0
		0	1	0	0	0
		0	1	0	1	0
		0	1	1	0	0
		0	1	1	1	0
		1	0	0	0	1
		1	0	0	1	1
		1	0	1	0	1
		1	0	1	1	1
		1	1	0	0	1
		1	1	0	1	1
		1	1	1	0	1
		1	1	1	1	1

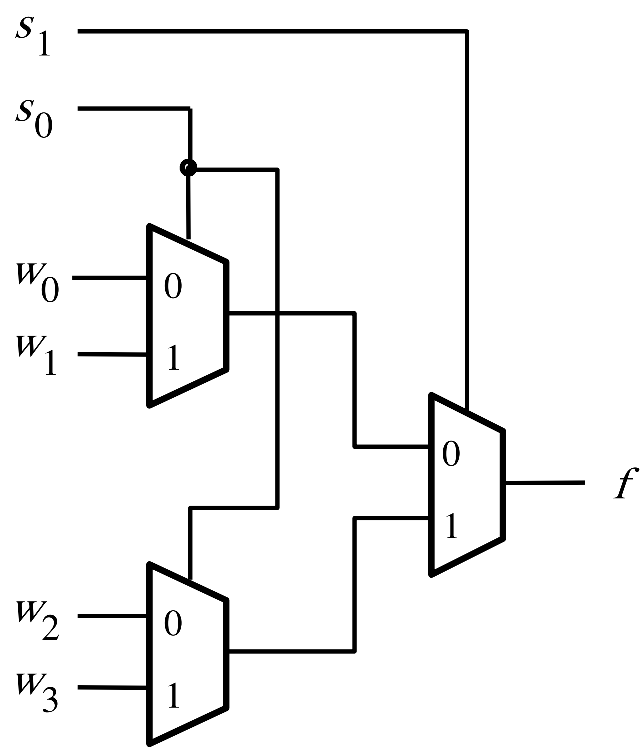
[http://www.absoluteastronomy.com/topics/Multiplexer]

4-to-1 Multiplexer (SOP circuit)



[Figure 4.2c from the textbook]

Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer

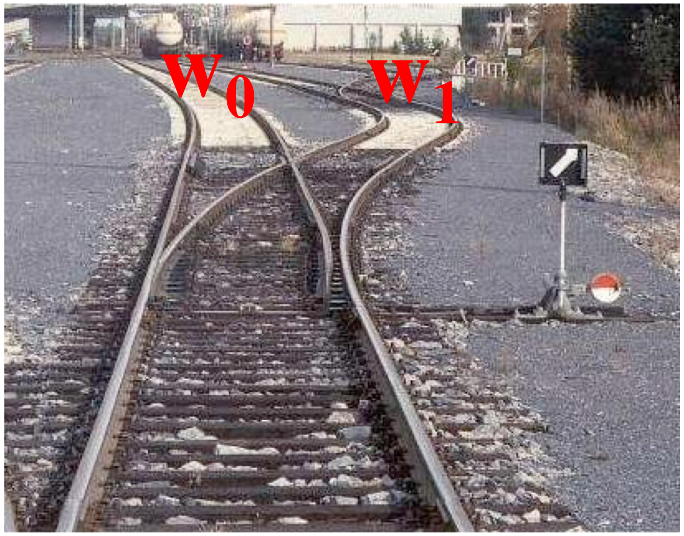


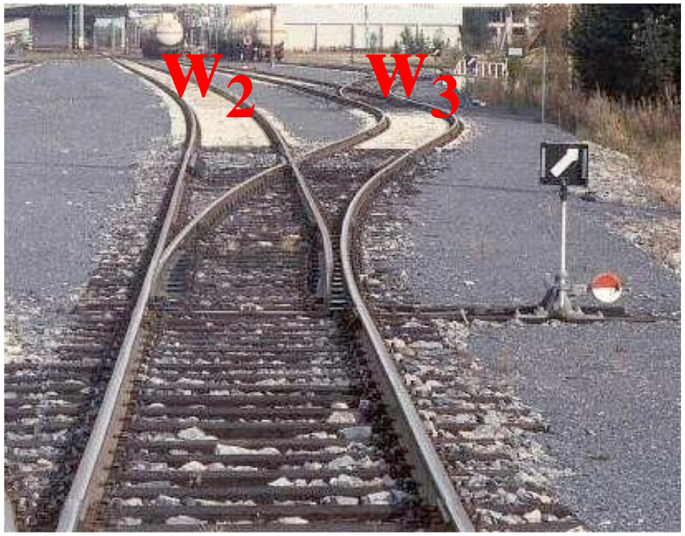
[Figure 4.3 from the textbook]

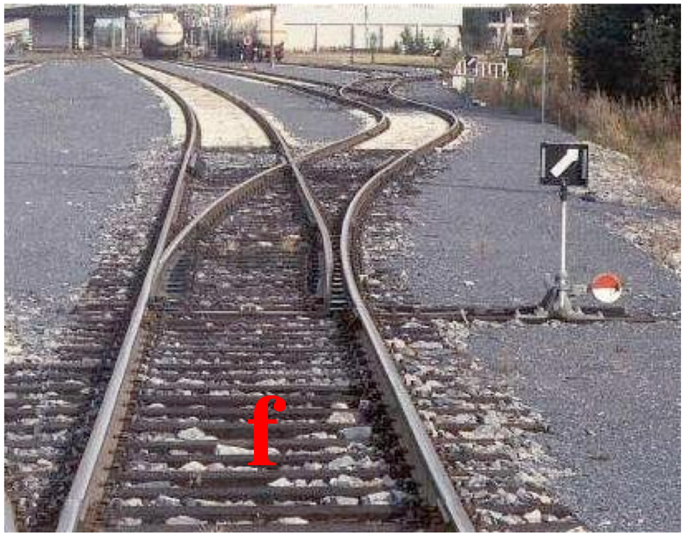
Analogy: Railroad Switches

http://en.wikipedia.org/wiki/Railroad_switch]

Analogy: Railroad Switches



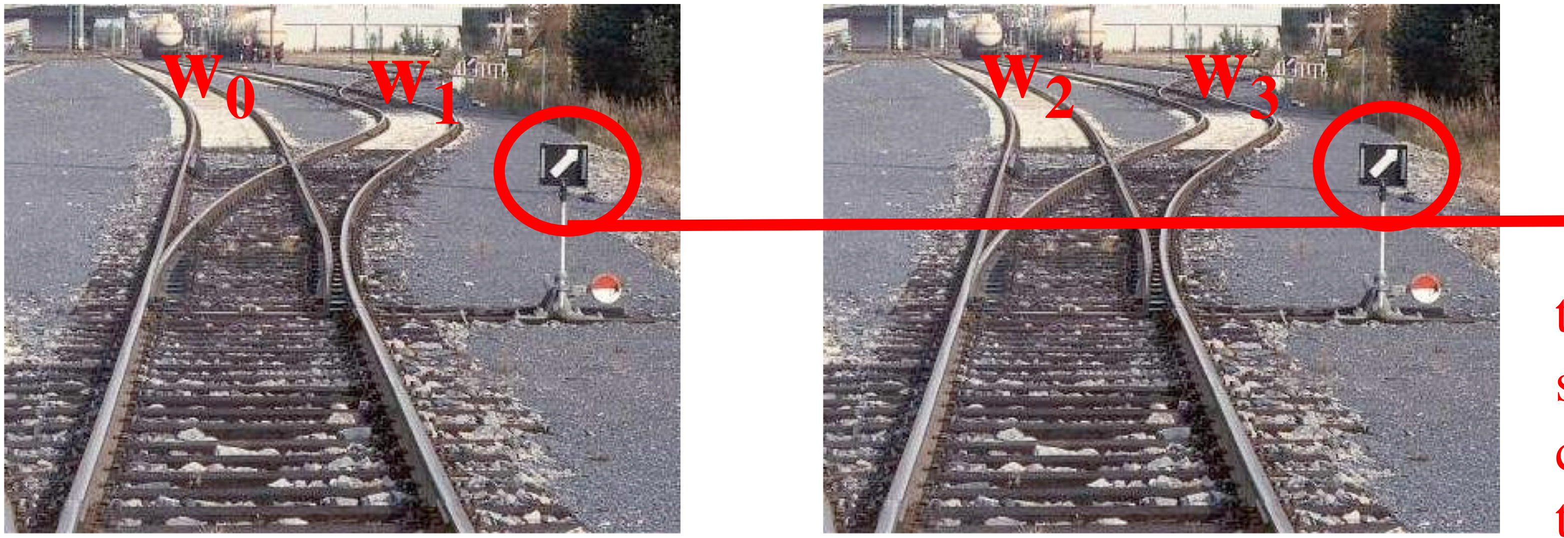




s_1

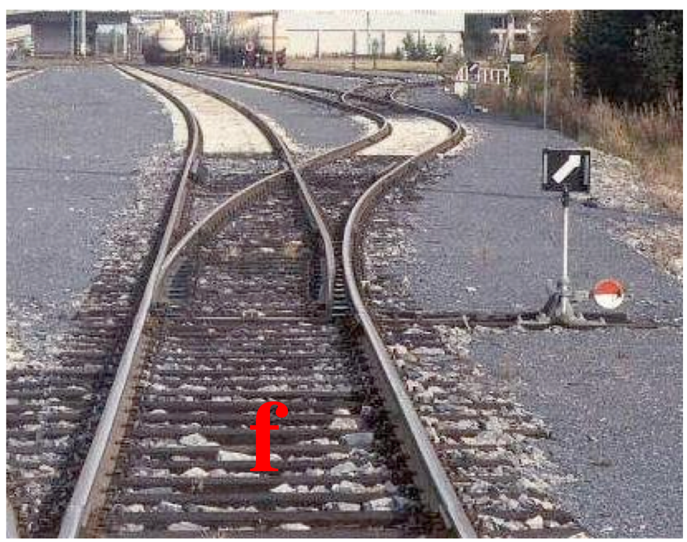
http://en.wikipedia.org/wiki/Railroad_switch]

Analogy: Railroad Switches



s_0

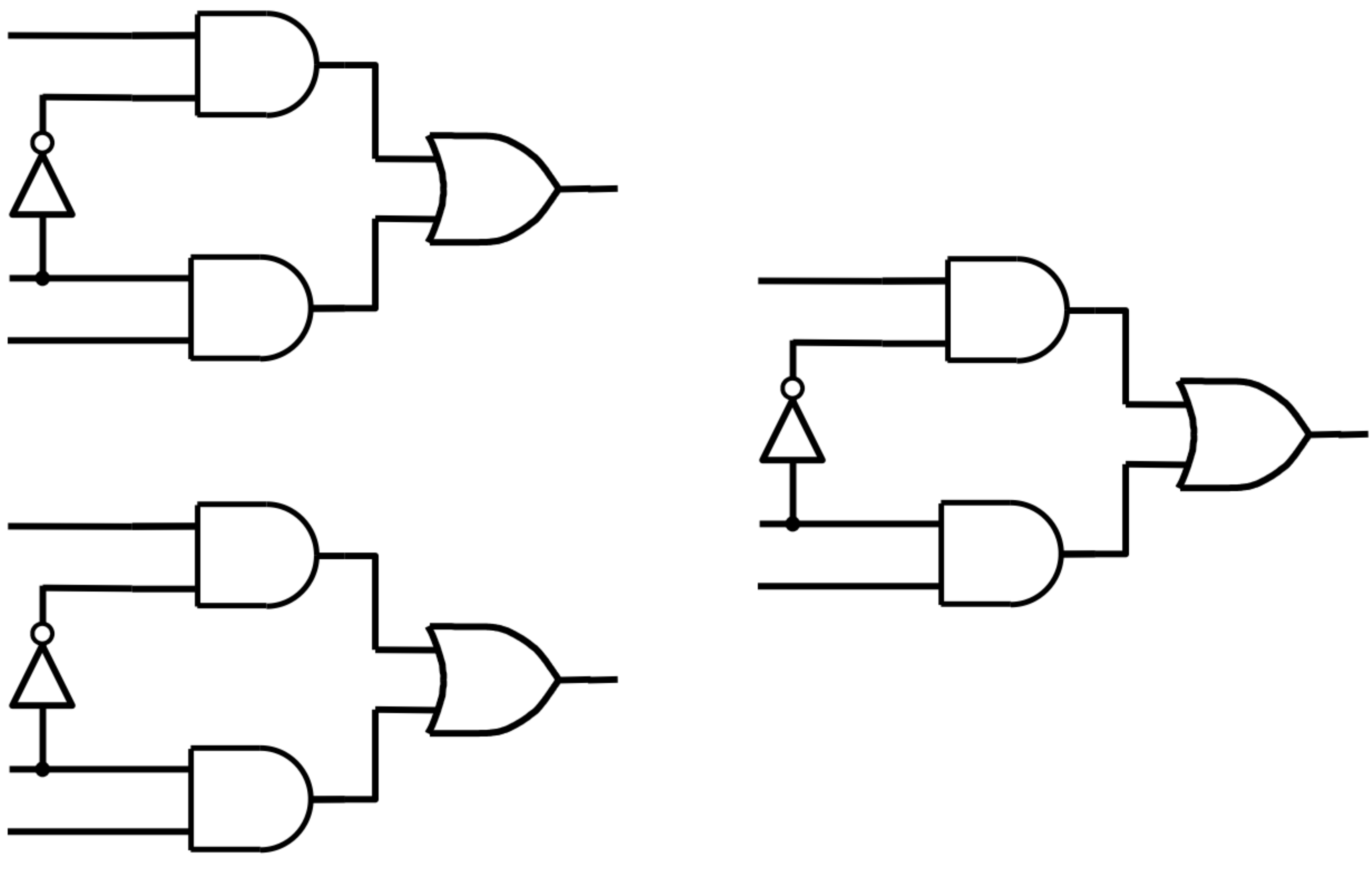
these two switches are controlled together



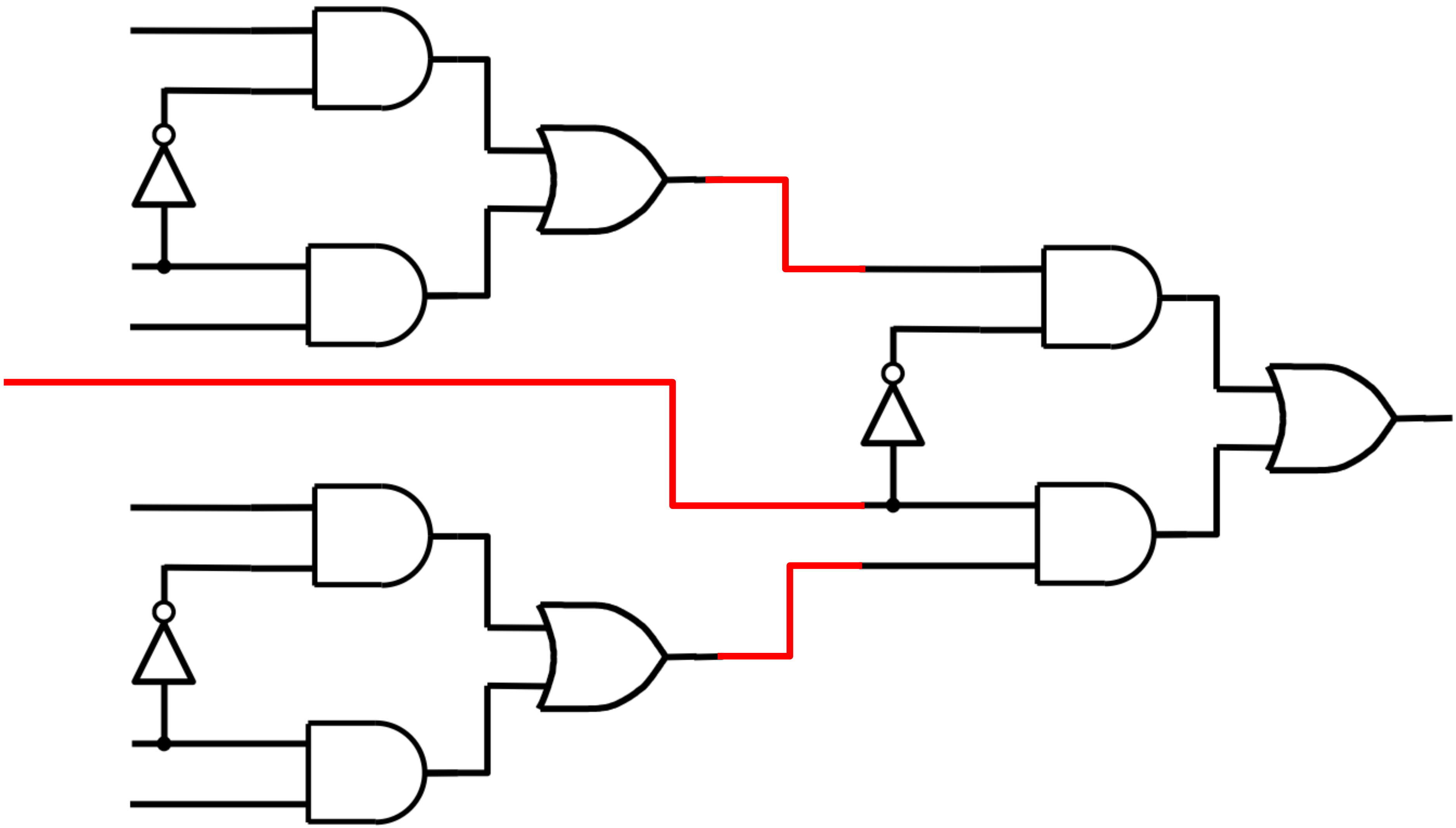
s_1

http://en.wikipedia.org/wiki/Railroad_switch]

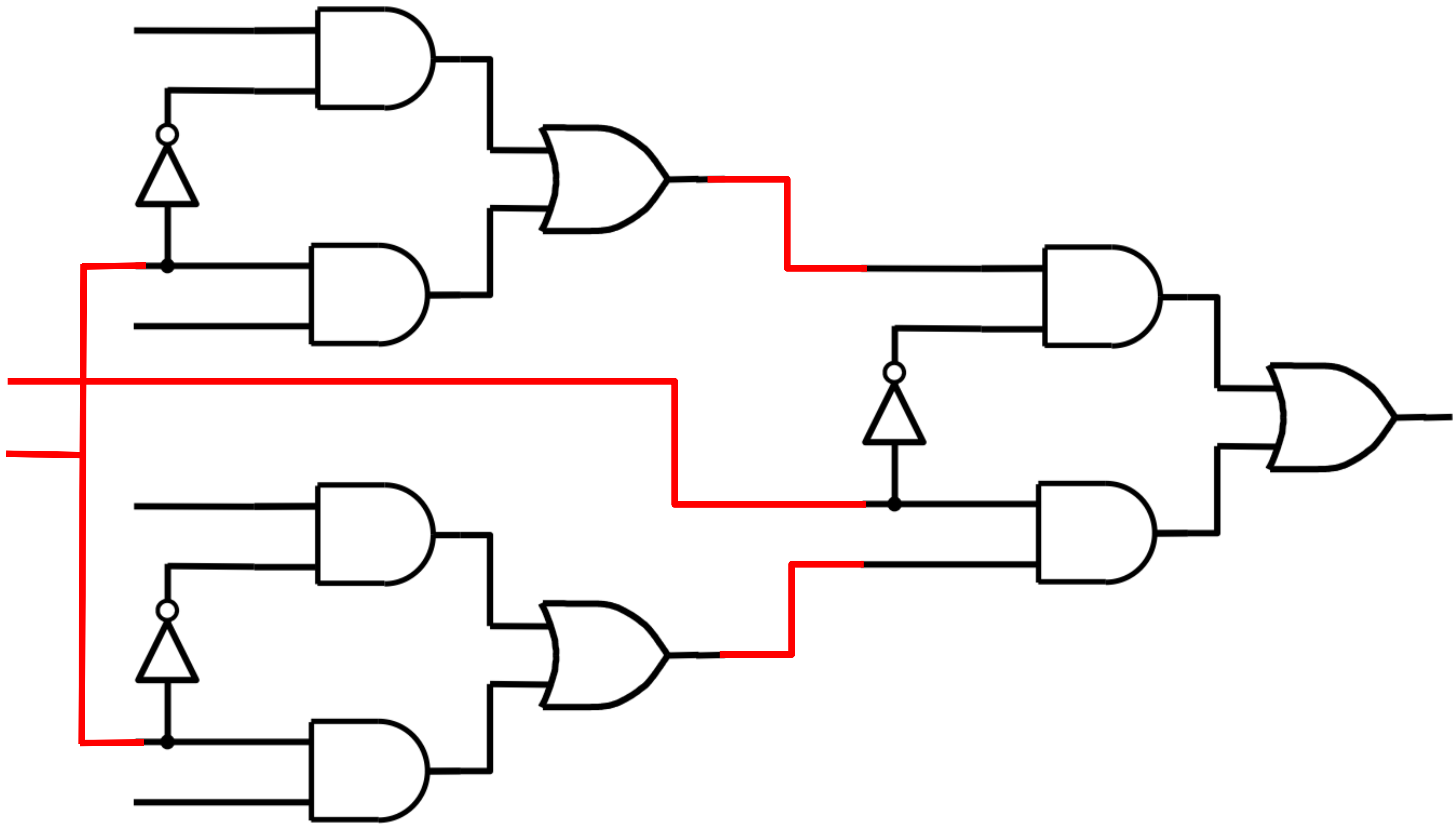
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



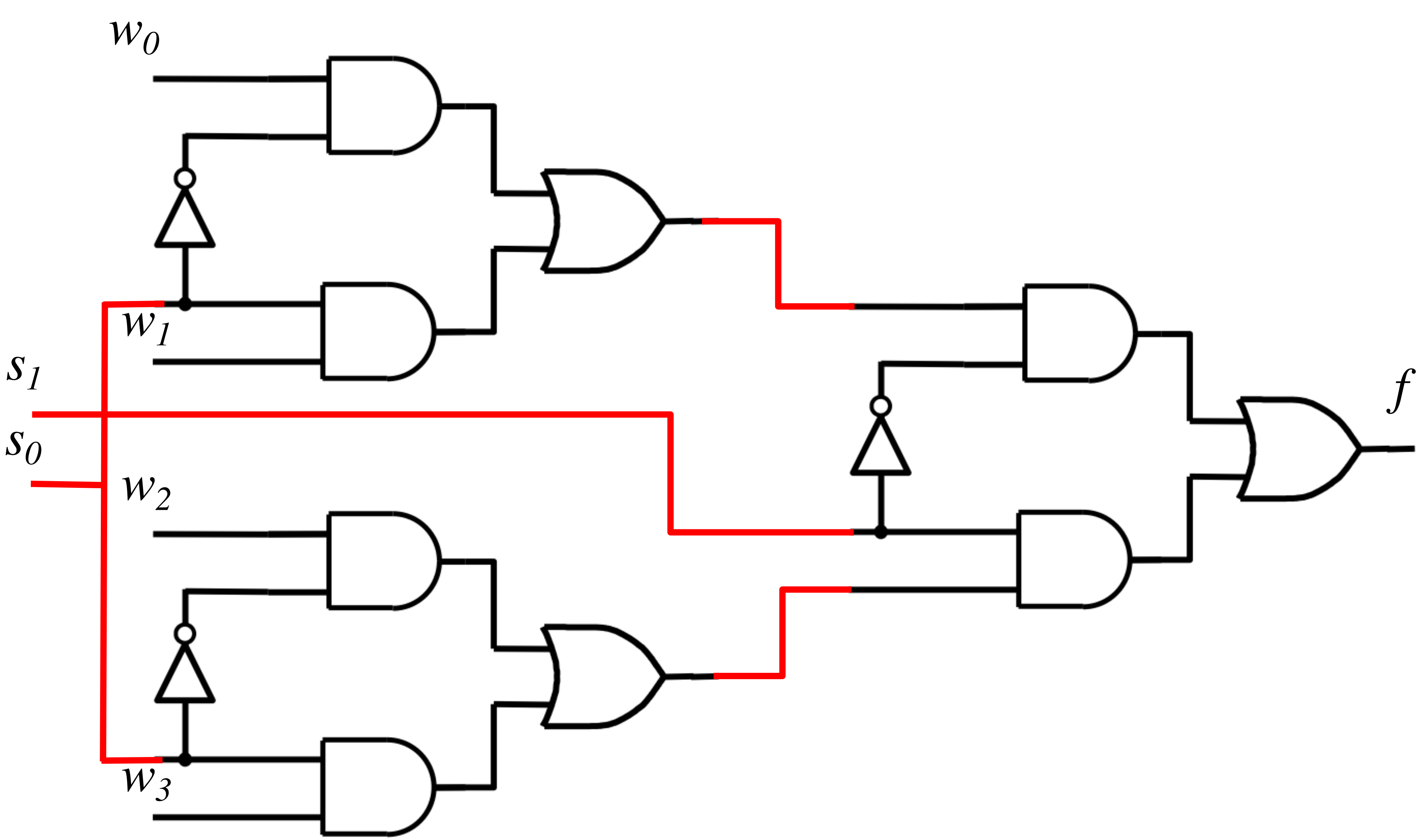
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



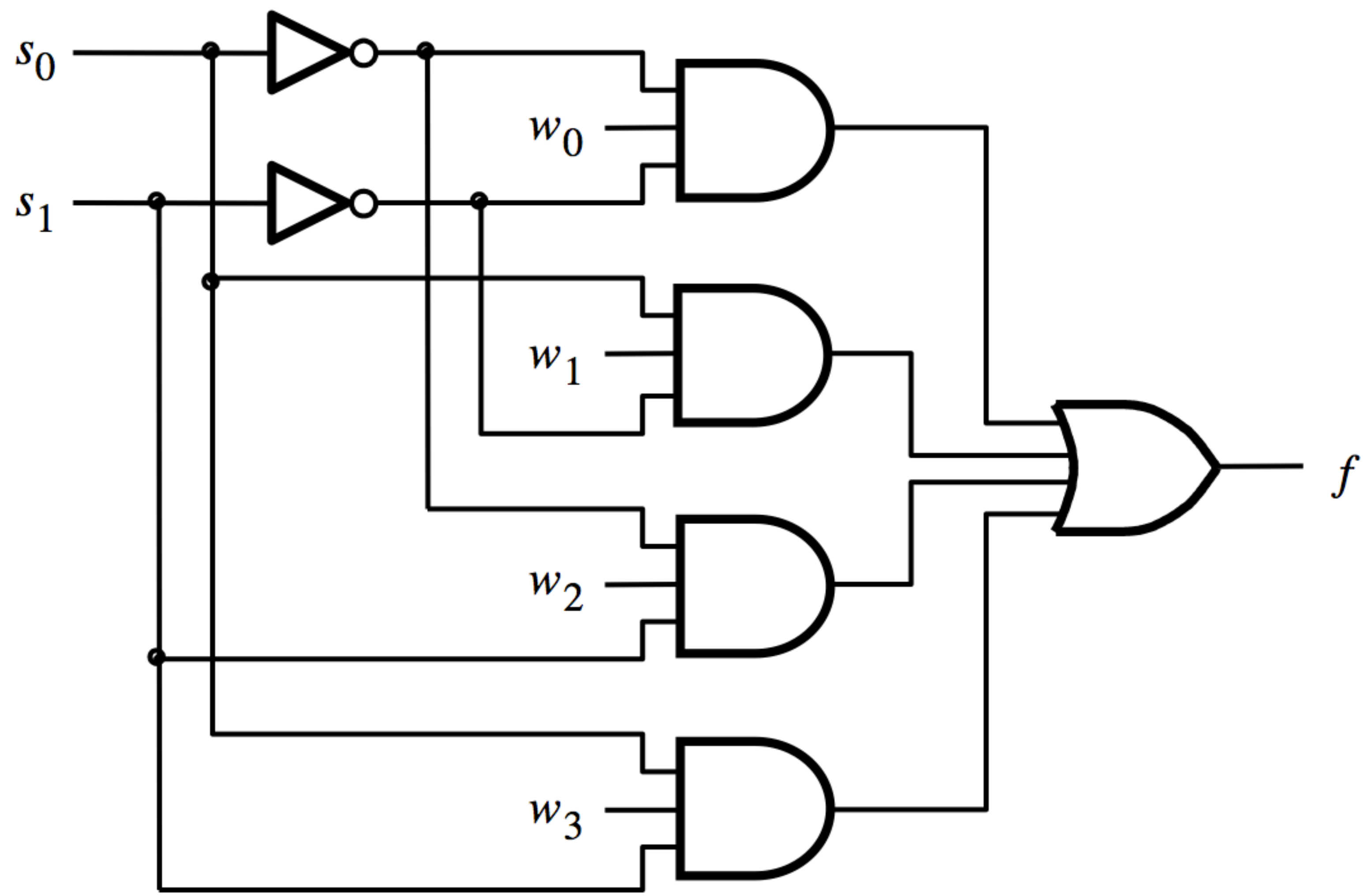
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



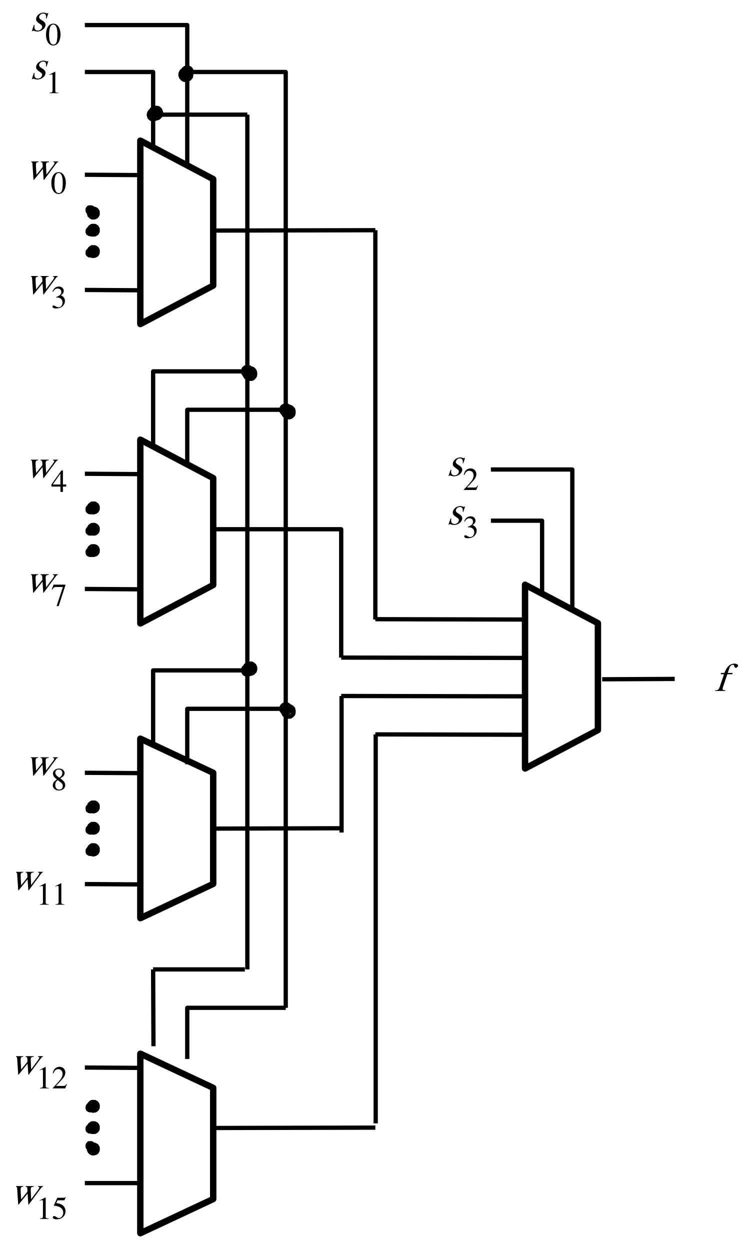
Using three 2-to-1 multiplexers to build one 4-to-1 multiplexer



That is different from the SOP form of the 4-to-1 multiplexer shown below, which uses fewer gates



16-to-1 Multiplexer



[Figure 4.4 from the textbook]

[http://upload.wikimedia.org/wikipedia/commons/2/26/SunsetTracksCrop.JPG]

Questions?

THE END