



# **CprE 281: Digital Logic**

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**<http://www.ece.iastate.edu/~alexs/classes/>**

# Incompletely Specified Functions & Multiple-Output Circuits

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*Iowa State University, Ames, IA*  
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# **Administrative Stuff**

- **HW4 is due on Monday Sep 20 @ 4 pm**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**

# **Administrative Stuff**

- **Midterm Exam #1**
- **When: Friday Sep 24.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **It will be in this room.**

# Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

# Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**
  
- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**
  
- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**
  
- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**
  
- **Something from Star Wars**

# **Quick Review**

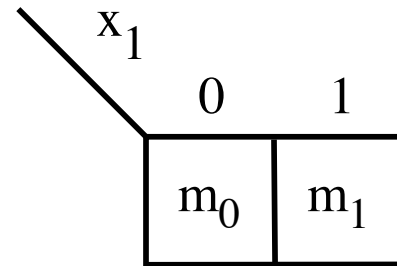
# One-Variable K-Map



# One-Variable K-map

$x_1$	$f$
0	$m_0$
1	$m_1$

(a) Truth table

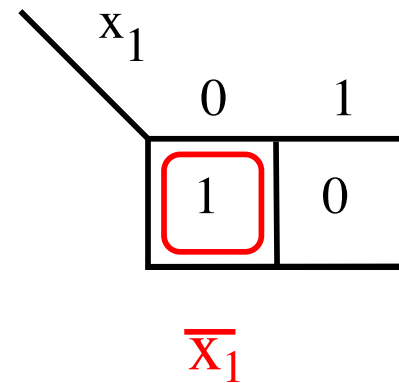


(b) Karnaugh map

# One-Variable K-map

$x_1$	$f$
0	1
1	0

(a) Truth table



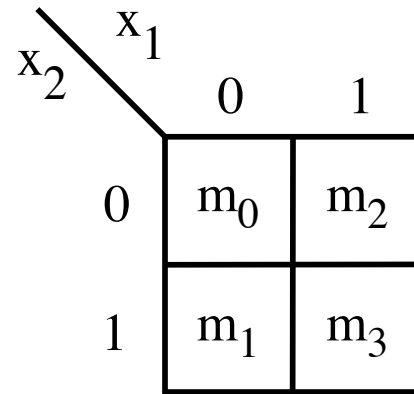
(b) Karnaugh map

# Two-Variable K-Map

# Two-Variable K-map

$x_1$	$x_2$	
0	0	$m_0$
0	1	$m_1$
1	0	$m_2$
1	1	$m_3$

(a) Truth table



(b) Karnaugh map

# These are all valid groupings

	$x_1$	0	1
$x_2$	0	1	0
	1	0	0

$$\overline{x_1} \overline{x_2}$$

	$x_1$	0	1
$x_2$	0	0	0
	1	1	0

$$\overline{x_1} x_2$$

	$x_1$	0	1
$x_2$	0	0	1
	1	0	0

$$x_1 \overline{x_2}$$

	$x_1$	0	1
$x_2$	0	0	0
	1	0	1

$$x_1 x_2$$

# These are all valid groupings

	$x_1$	0	1
$x_2$	0	1	0
	1	1	0

$\bar{x}_1$

	$x_1$	0	1
$x_2$	0	0	1
	1	0	1

$x_1$

	$x_1$	0	1
$x_2$	0	1	1
	1	0	0

$\bar{x}_2$

	$x_1$	0	1
$x_2$	0	0	0
	1	1	1

$x_2$

# This one is valid too

$x_2 \backslash x_1$	0	1
0	1	1
1	1	1

In this case the result is the constant function 1.

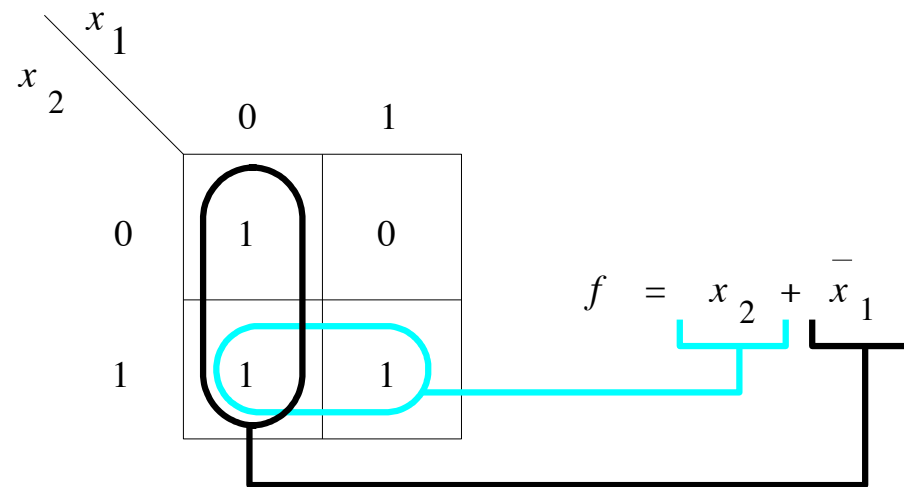
# Why are these two not valid?

	$x_1$		
$x_2$		0	1
0		1	0
1		0	1

	$x_1$		
$x_2$		0	1
0		0	1
1		1	0



# Minimization Example with a two-variable K-map



# Three-Variable K-Map

# Three-Variable K-map

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

# Location of three-variable minterms

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

$x_3$	$x_1 x_2$	00	01	11	10
0		$m_0$	$m_2$	$m_6$	$m_4$
1		$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

**Notice the placement of**

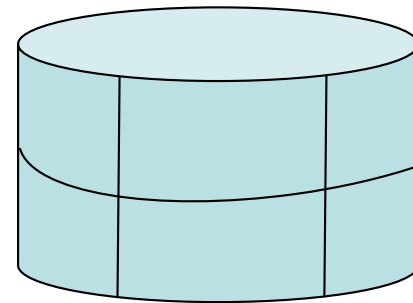
- **Variables**
- **Binary pair values**
- **Minterms**

# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$



adjacent  
columns



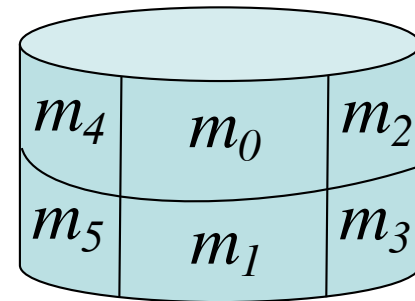
As if the K-map were  
drawn on a cylinder

# Adjacency Rules

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$



adjacent  
columns



As if the K-map were  
drawn on a cylinder

# Some **Invalid** Groupings

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	1	0	0
	1	0	0	1	0

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	1	0
	1	0	1	0	0

Can't group diagonally.

# Some **Invalid** Groupings

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	1	1	0
	1	0	0	0	0

		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	0	0	0
	1	0	1	1	1

Can't group three in a row.  
Each side must be a power of 2.



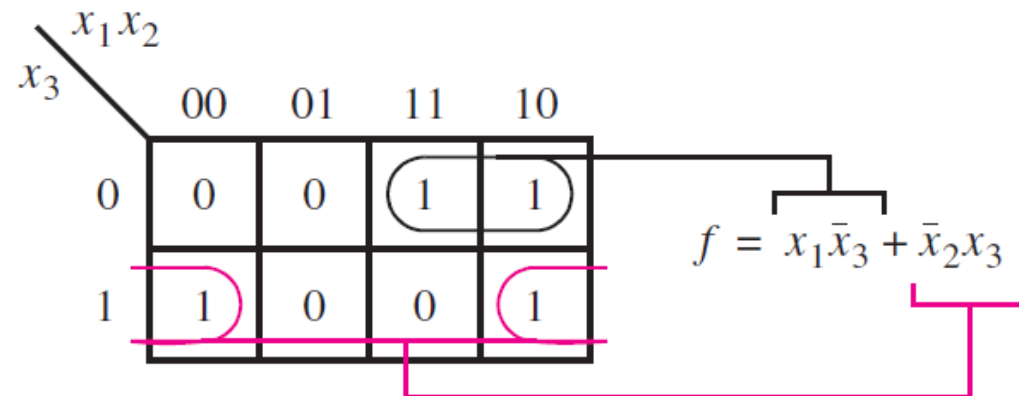
# Some **Invalid** Groupings

		$x_1x_2$			
		00	01	11	10
$x_3$	0	1	<b>0</b>	1	1
	1	0	0	0	0

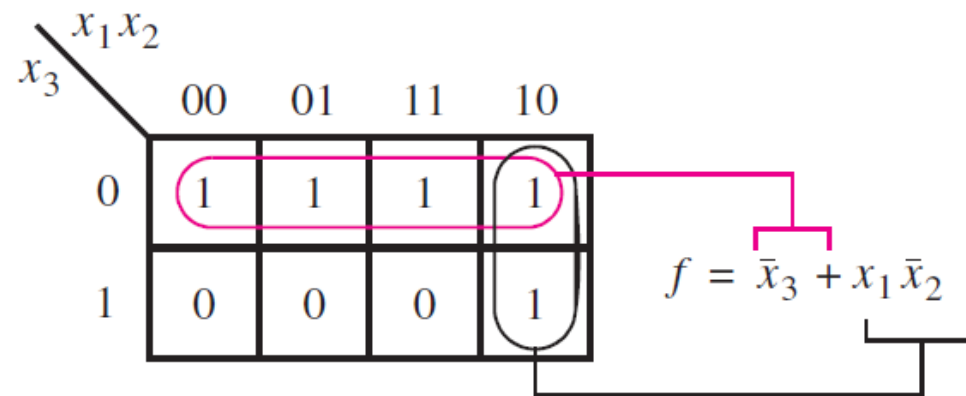
		$x_1x_2$			
		00	01	11	10
$x_3$	0	0	<b>0</b>	1	0
	1	0	1	1	0

Can't group zeros and ones together.

# Three-Variable K-map



(a) The function of Figure 2.23



(b) The function of Figure 2.48

# Two Different Ways to Draw the K-map

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

		$x_2 x_3$			
		00	01	11	10
$x_1$	0	$m_0$	$m_1$	$m_3$	$m_2$
	1	$m_4$	$m_5$	$m_7$	$m_6$

# Another Way to Draw 3-variable K-map

$x_1$	$x_2$	$x_3$	
0	0	0	$m_0$
0	0	1	$m_1$
0	1	0	$m_2$
0	1	1	$m_3$
1	0	0	$m_4$
1	0	1	$m_5$
1	1	0	$m_6$
1	1	1	$m_7$

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

(b) Karnaugh map

		$x_1$	
		0	1
$x_2 x_3$	00	$m_0$	$m_4$
	01	$m_1$	$m_5$
	11	$m_3$	$m_7$
	10	$m_2$	$m_6$

# There are 4 different versions!

		$x_1x_2$			
		00	01	11	10
$x_3$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

		$x_2x_3$			
		00	01	11	10
$x_1$	0	$m_0$	$m_1$	$m_3$	$m_2$
	1	$m_4$	$m_5$	$m_7$	$m_6$

		$x_3$	
		0	1
$x_1x_2$	00	$m_0$	$m_1$
	01	$m_2$	$m_3$
	11	$m_6$	$m_7$
	10	$m_4$	$m_5$

		$x_1$	
		0	1
$x_2x_3$	00	$m_0$	$m_4$
	01	$m_1$	$m_5$
	11	$m_3$	$m_7$
	10	$m_2$	$m_6$

# Gray Code

- **Sequence of binary codes**
- **Consecutive lines vary by only 1 bit**

	000
	001
00	011
01	010
11	110
10	111
	101
	100

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	$m_0$	$m_2$	$m_6$	$m_4$
	1	$m_1$	$m_3$	$m_7$	$m_5$

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101



# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These two neighbors  
differ only in the LAST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These two neighbors  
differ only in the FIRST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

$x_2$	$s$	$x_1$				
			00	01	11	10
0			000	010	110	100
1			001	011	111	101

These two neighbors  
differ only in the FIRST bit

# Gray Code & K-map

	$s$	$x_1$	$x_2$
$m_0$	0	0	0
$m_1$	0	0	1
$m_2$	0	1	0
$m_3$	0	1	1
$m_4$	1	0	0
$m_5$	1	0	1
$m_6$	1	1	0
$m_7$	1	1	1

		$s \ x_1$			
		00	01	11	10
$x_2$	0	000	010	110	100
	1	001	011	111	101

These four neighbors  
differ in the FIRST and LAST bit

They are similar in their MIDDLE bit

**Why is it OK to combine  
a group of four ones?**

**The K-Map theory uses the combining theorems of Boolean algebra**

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

# The K-Map theory uses the combining theorems of Boolean algebra

optimization by 1's

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$



**The K-Map theory uses the  
combining theorems of Boolean algebra**

$$14a. \quad \mathbf{x \cdot y + x \cdot \bar{y} = x}$$

$$14b. \quad \mathbf{(x + y) \cdot (x + \bar{y}) = x}$$

*optimization by 0's*

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Theorem 14a is behind the K-Map theory.  
But that theorem is just for two variables.  
Why is this grouping of four ones possible?

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + x y \bar{z} + \bar{x} y z + x y z$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{\bar{x} y \bar{z} + x y \bar{z}}_{(\bar{x} y + x y) \bar{z}} + \underbrace{\bar{x} y z + x y z}_{(\bar{x} y + x y) z}$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = (\bar{x}y + xy)\bar{z} + (\bar{x}y + xy)z$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{(\bar{x}y + xy)}_{y\bar{z} \text{ (by 14a)}} \bar{z} + \underbrace{(\bar{x}y + xy)}_{yz \text{ (by 14a)}} z$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y \bar{z} + y z$$



# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{y \bar{z} + y z}_{y \text{ (by 14a)}}$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y$$

# Why can we group these four ones?

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + x y \bar{z} + \bar{x} y z + x y z$$

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} y \bar{z} + \bar{x} y z + x y \bar{z} + x y z$$

# Alternative Derivation

		x y			
	z	00	01	11	10
0		0	1	1	0
1		0	1	1	0

$$f = \underbrace{\bar{x} y \bar{z} + \bar{x} y z}_{\bar{x} (y \bar{z} + y z)} + \underbrace{x y \bar{z} + x y z}_{x (y \bar{z} + y z)}$$

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x} (y \bar{z} + y z) + x (y \bar{z} + y z)$$



# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \underbrace{\bar{x} (y \bar{z} + y z)}_{\bar{x} y \text{ (by 14a)}} + \underbrace{x (y \bar{z} + y z)}_{x y \text{ (by 14a)}}$$

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = \bar{x}y + xy$$

# Alternative Derivation

		x y			
	z	00	01	11	10
0		0	1	1	0
1		0	1	1	0

$$f = \underbrace{\bar{x}y + xy}_{y \text{ (by 14a)}}$$

# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

$$f = y$$

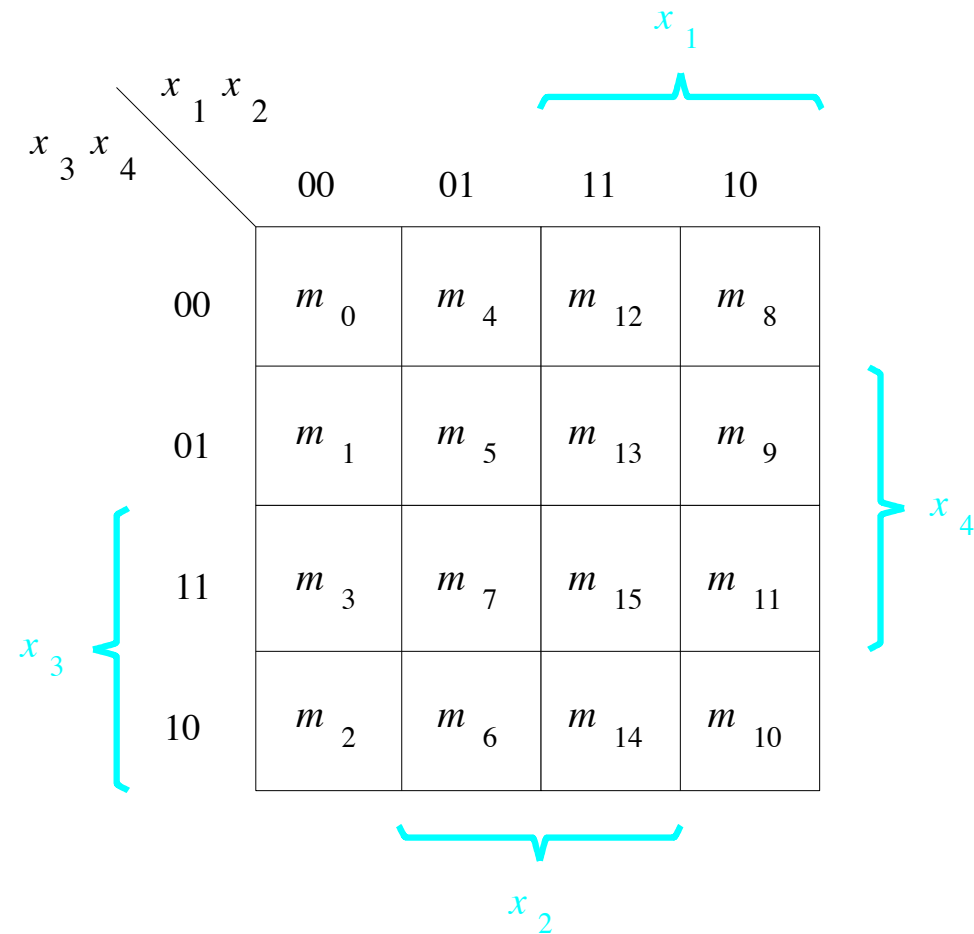
# Alternative Derivation

		x y			
		00	01	11	10
z	0	0	1	1	0
	1	0	1	1	0

Answer: We can combine them because Theorem 14a is applied three times, not just once.

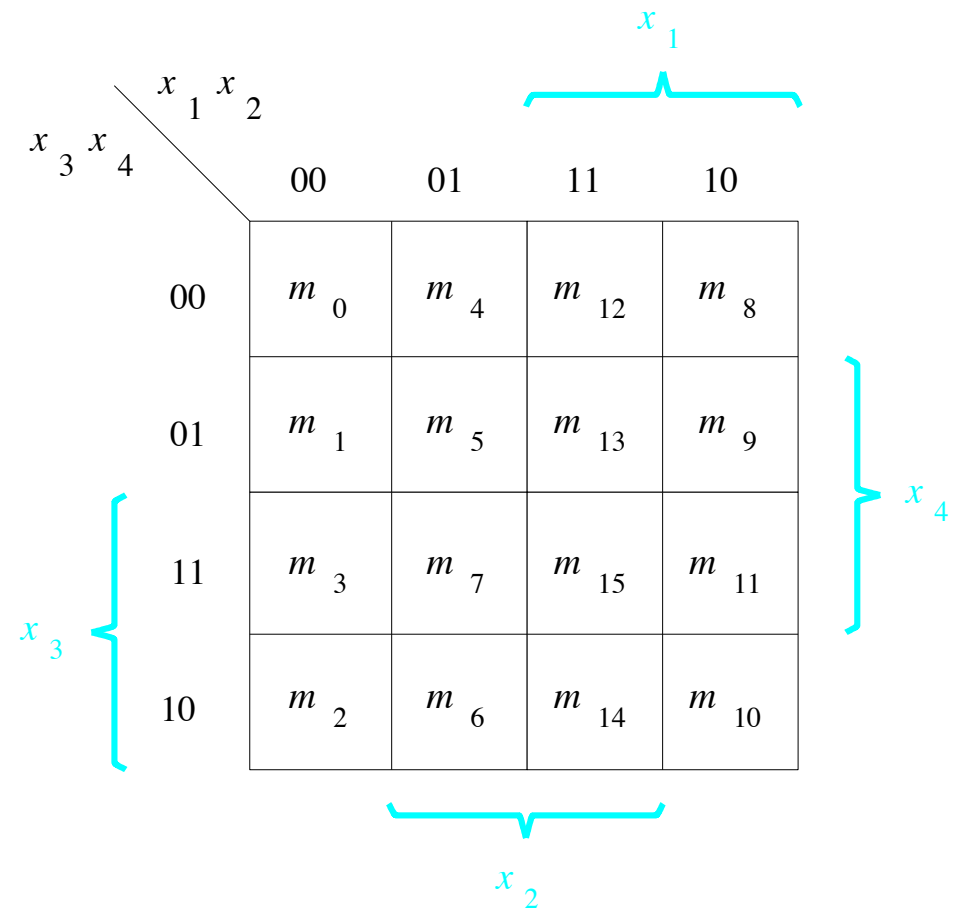
# Four-Variable K-Map

# A four-variable Karnaugh map



# A four-variable Karnaugh map

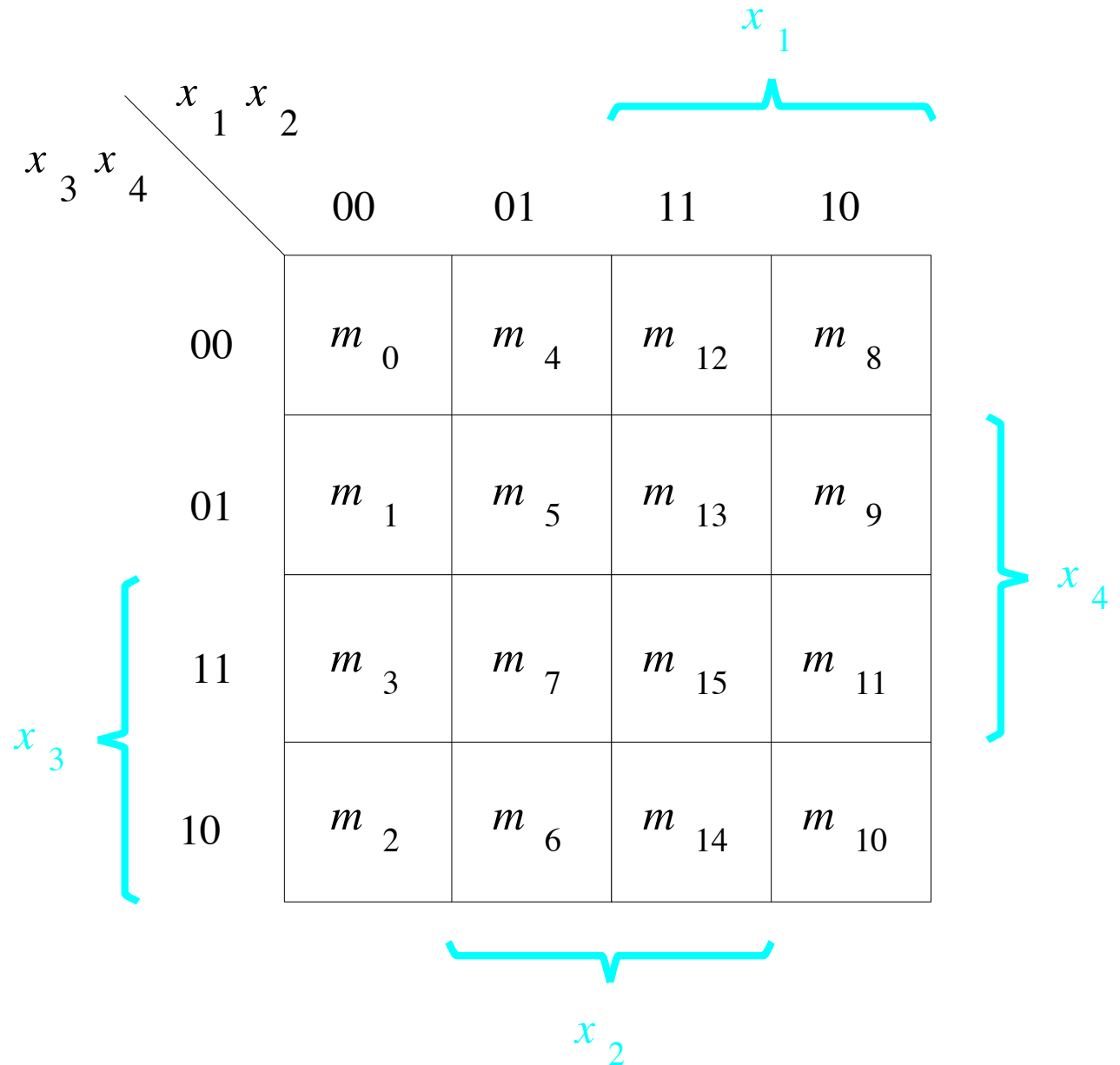
x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15





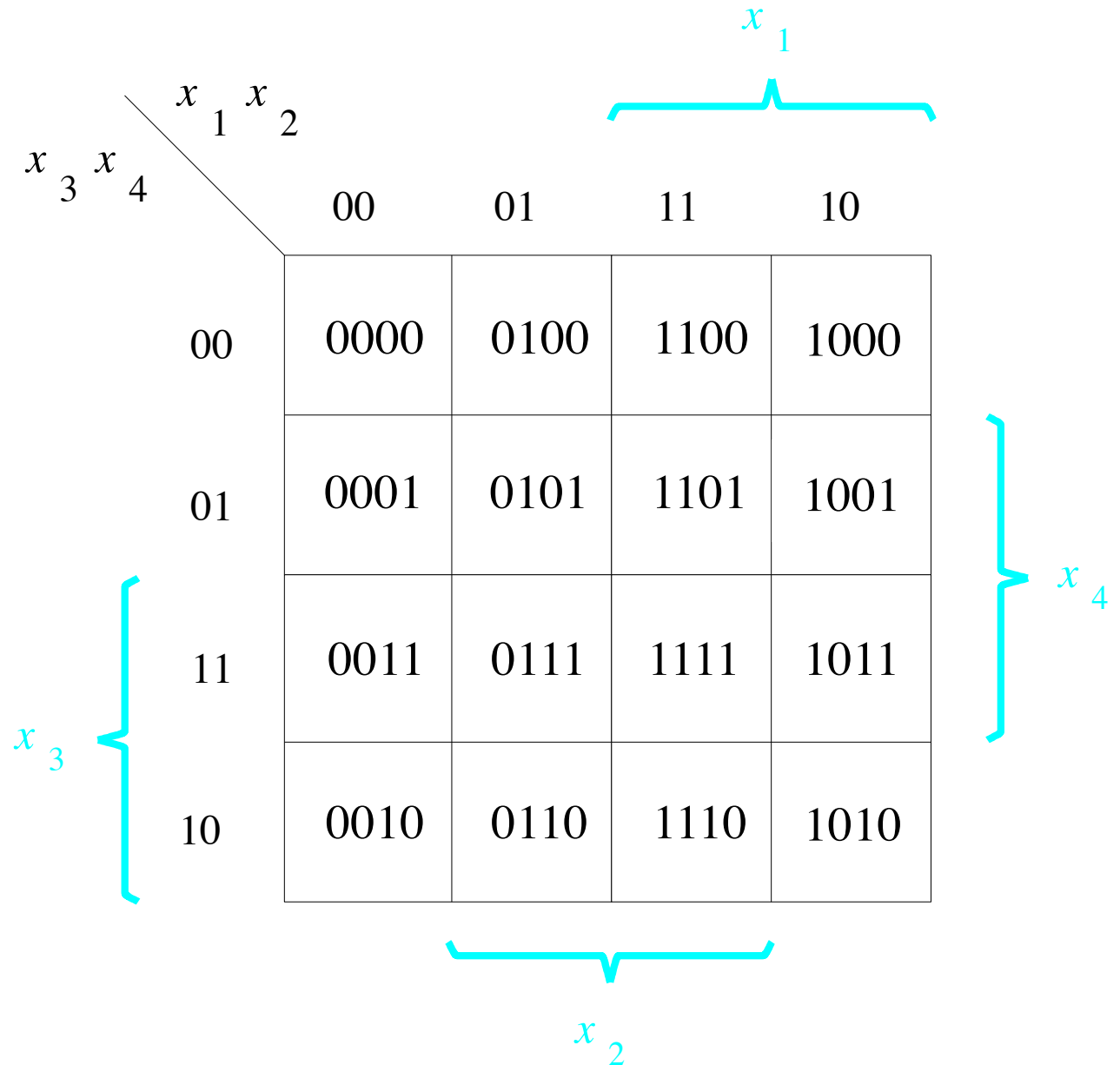
# Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



# Gray Code & K-map

x1	x2	x3	x4	
0	0	0	0	m0
0	0	0	1	m1
0	0	1	0	m2
0	0	1	1	m3
<hr/>				
0	1	0	0	m4
0	1	0	1	m5
0	1	1	0	m6
0	1	1	1	m7
<hr/>				
1	0	0	0	m8
1	0	0	1	m9
1	0	1	0	m10
1	0	1	1	m11
<hr/>				
1	1	0	0	m12
1	1	0	1	m13
1	1	1	0	m14
1	1	1	1	m15



# Adjacency Rules

$x_3$	$x_1x_2$	00	01	11	10
0		$m_0$	$m_2$	$m_6$	$m_4$
1		$m_1$	$m_3$	$m_7$	$m_5$

adjacent  
columns

$x_3x_4$	$x_1x_2$	00	01	11	10
00		$m_0$	$m_4$	$m_{12}$	$m_8$
01		$m_1$	$m_5$	$m_{13}$	$m_9$
11		$m_3$	$m_7$	$m_{15}$	$m_{11}$
10		$m_2$	$m_6$	$m_{14}$	$m_{10}$

adjacent  
columns

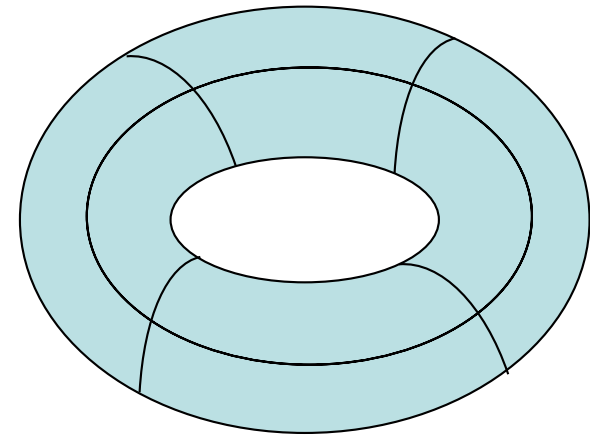
adjacent  
rows

# Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

adjacent  
rows

adjacent  
columns



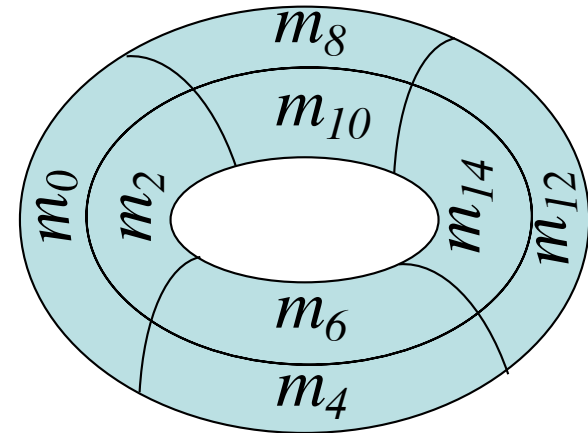
As if the K-map were  
drawn on a torus

# Adjacency Rules

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	$m_0$	$m_4$	$m_{12}$	$m_8$
01	$m_1$	$m_5$	$m_{13}$	$m_9$
11	$m_3$	$m_7$	$m_{15}$	$m_{11}$
10	$m_2$	$m_6$	$m_{14}$	$m_{10}$

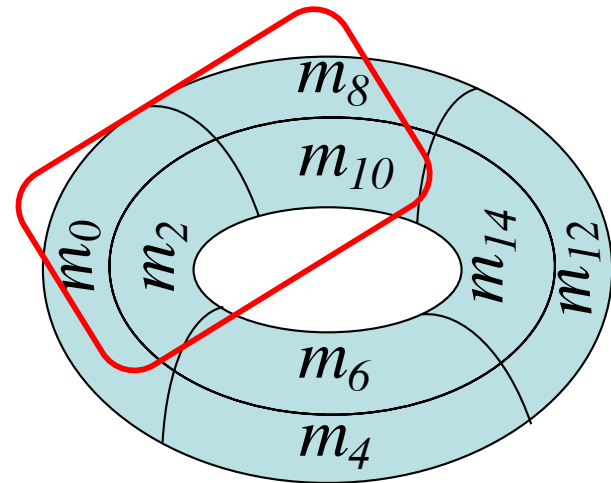
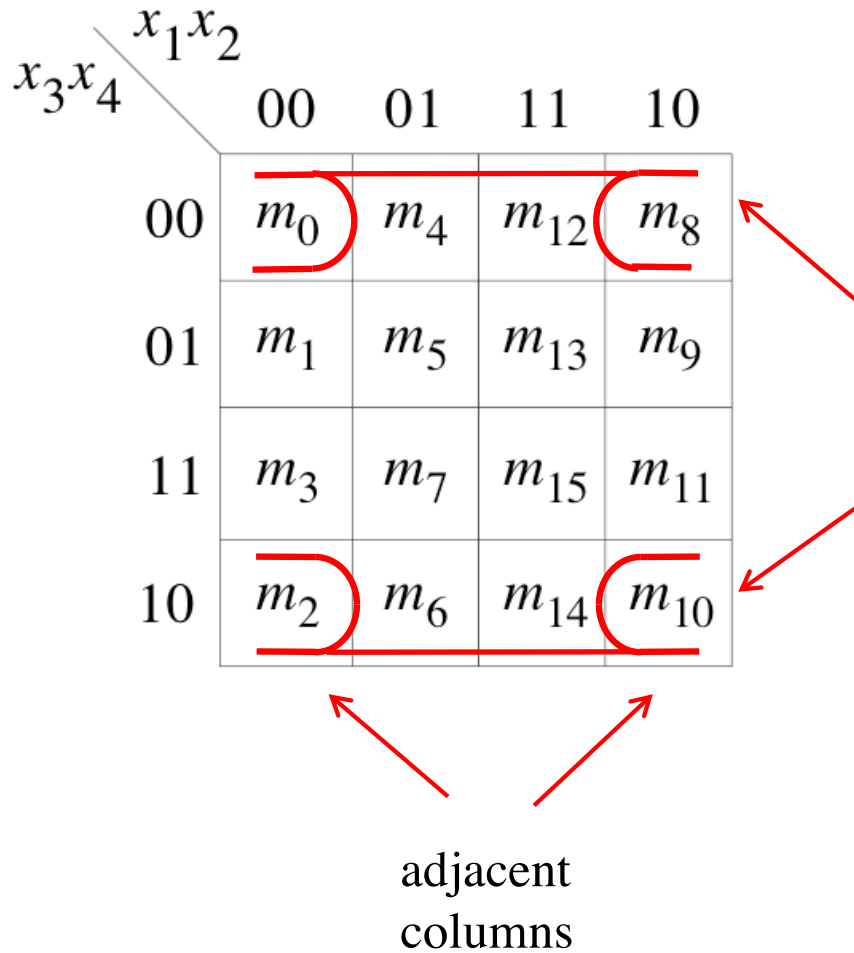
adjacent  
rows

adjacent  
columns



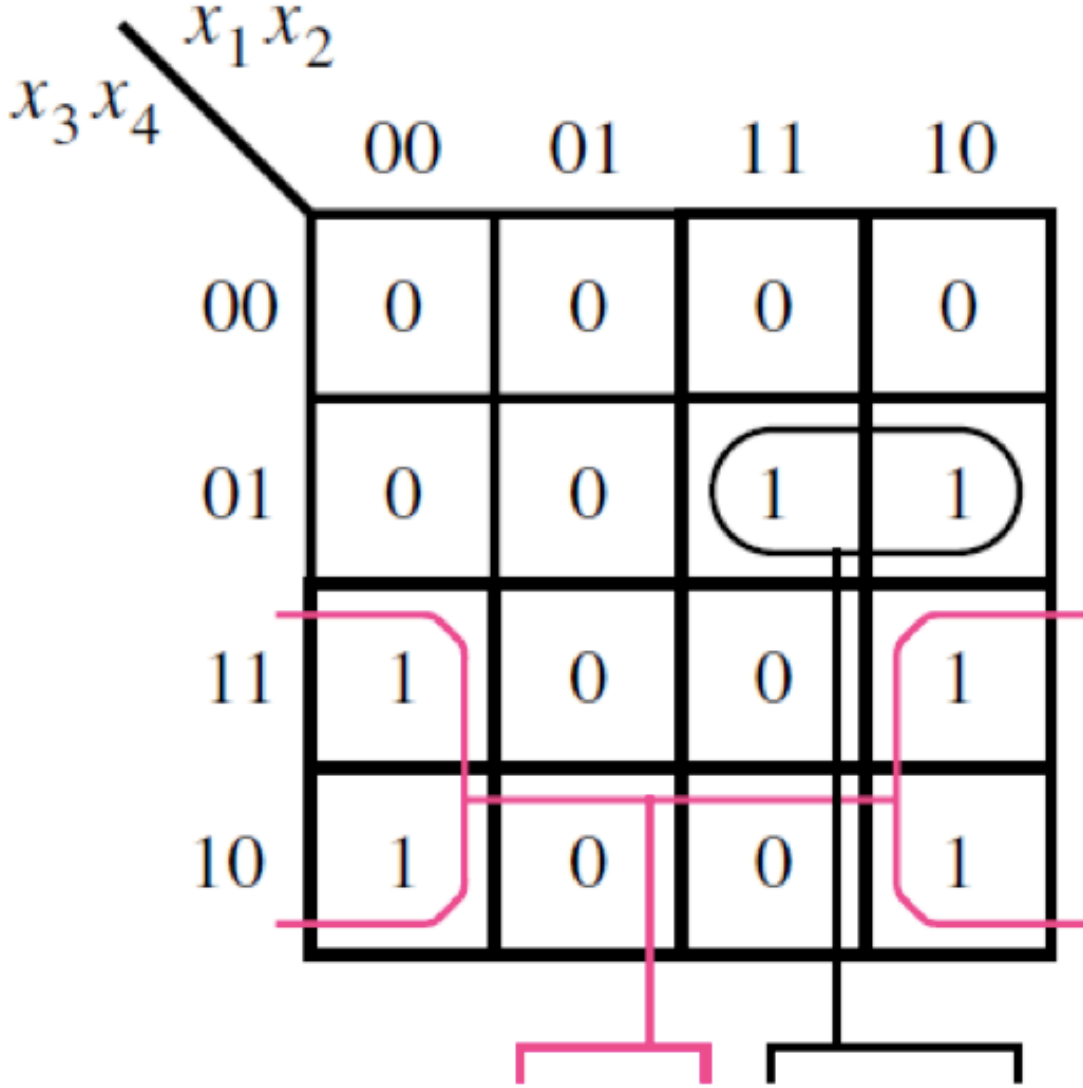
As if the K-map were  
drawn on a torus

# Adjacency Rules



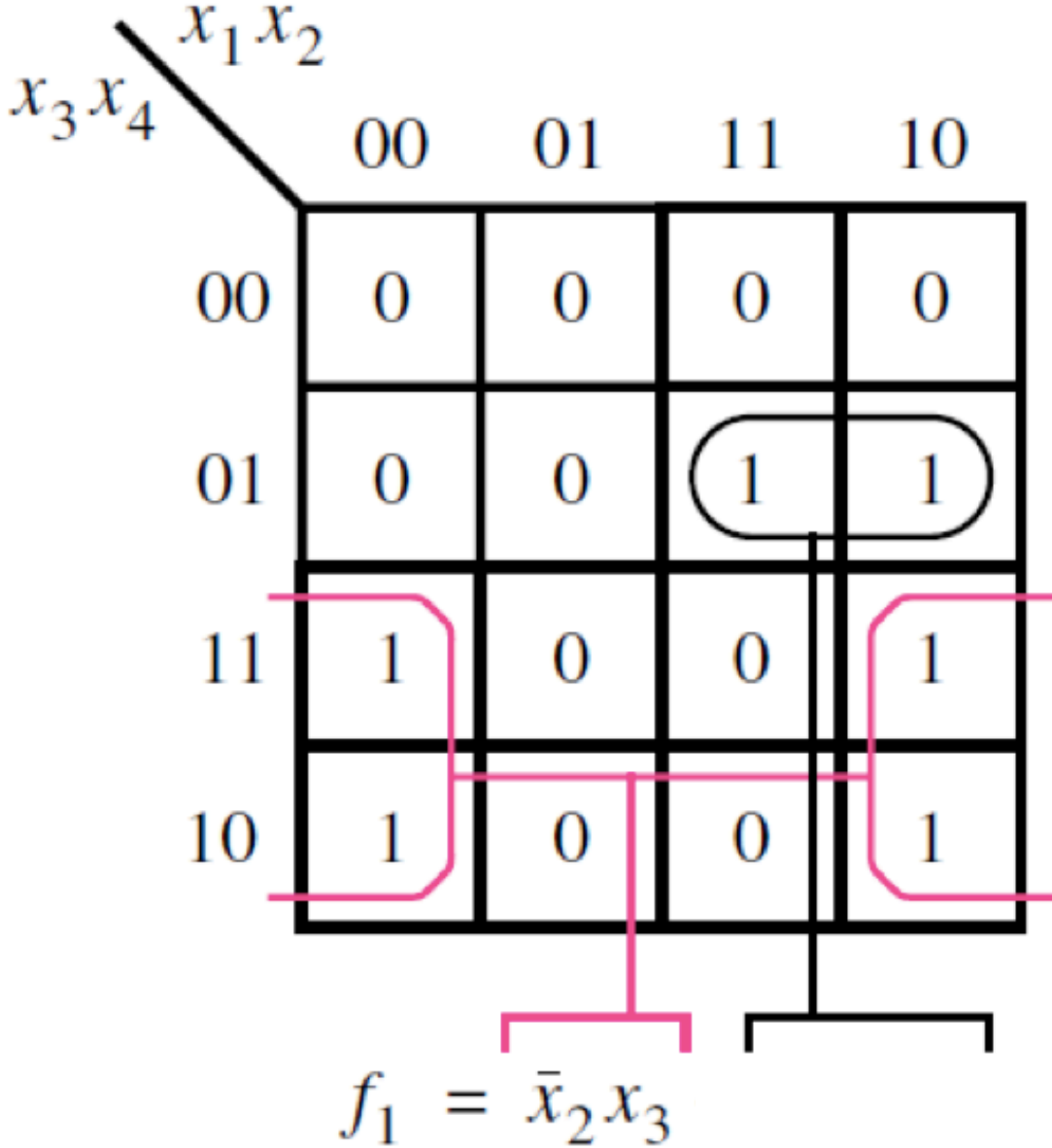
As if the K-map were drawn on a torus

# Example of a four-variable Karnaugh map



[ Figure 2.54 from the textbook ]

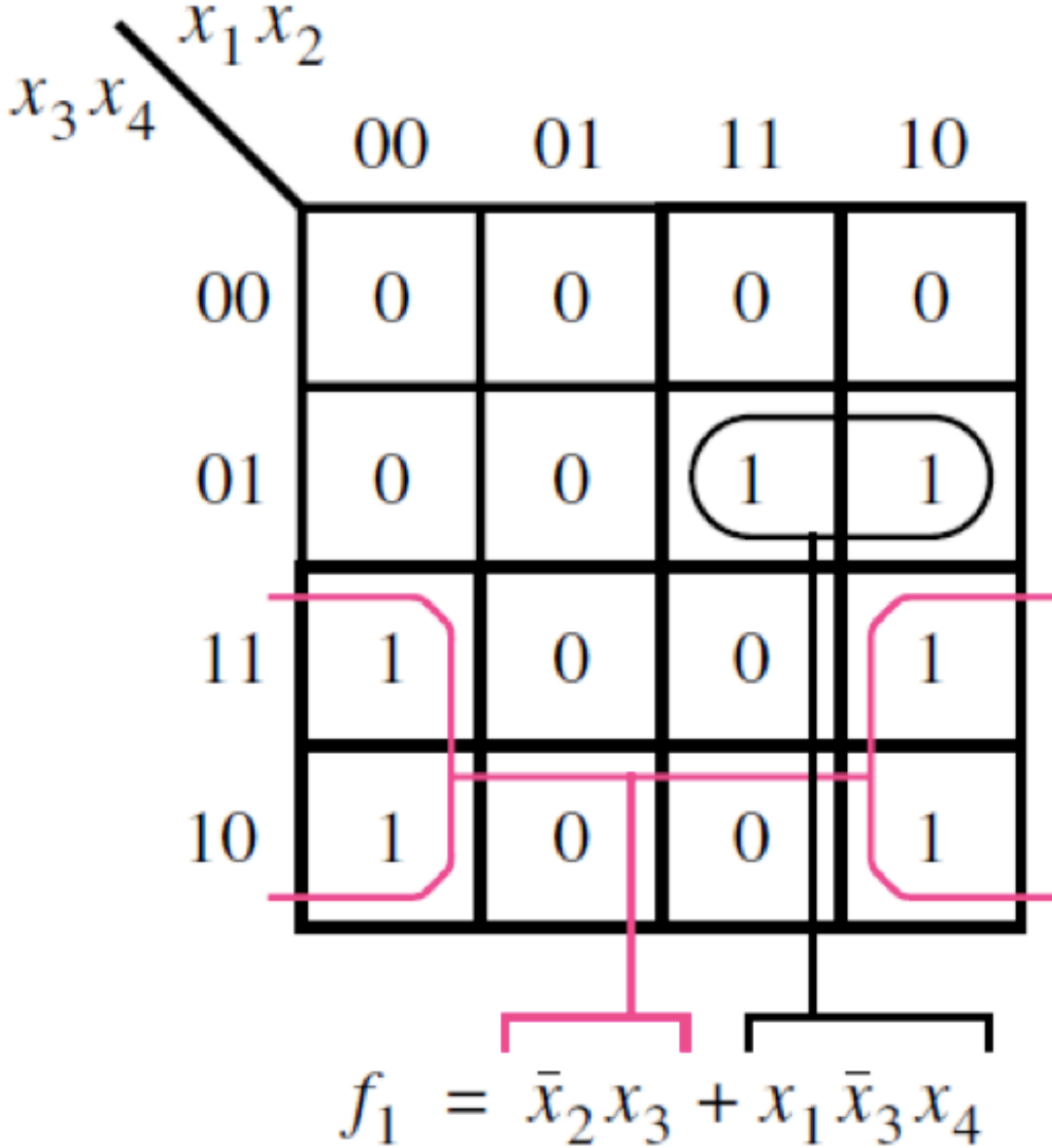
# Example of a four-variable Karnaugh map



[ Figure 2.54 from the textbook ]

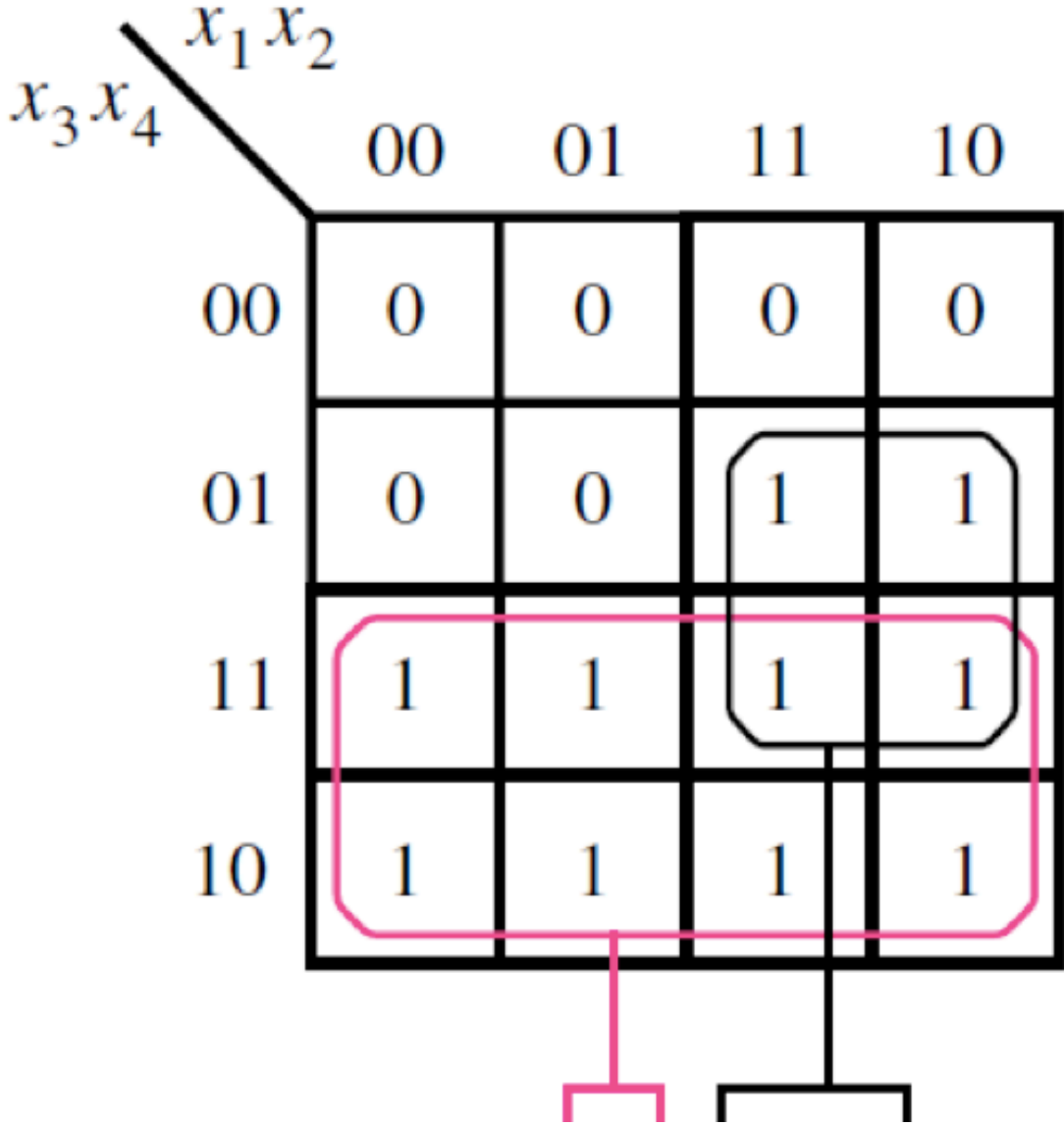


# Example of a four-variable Karnaugh map



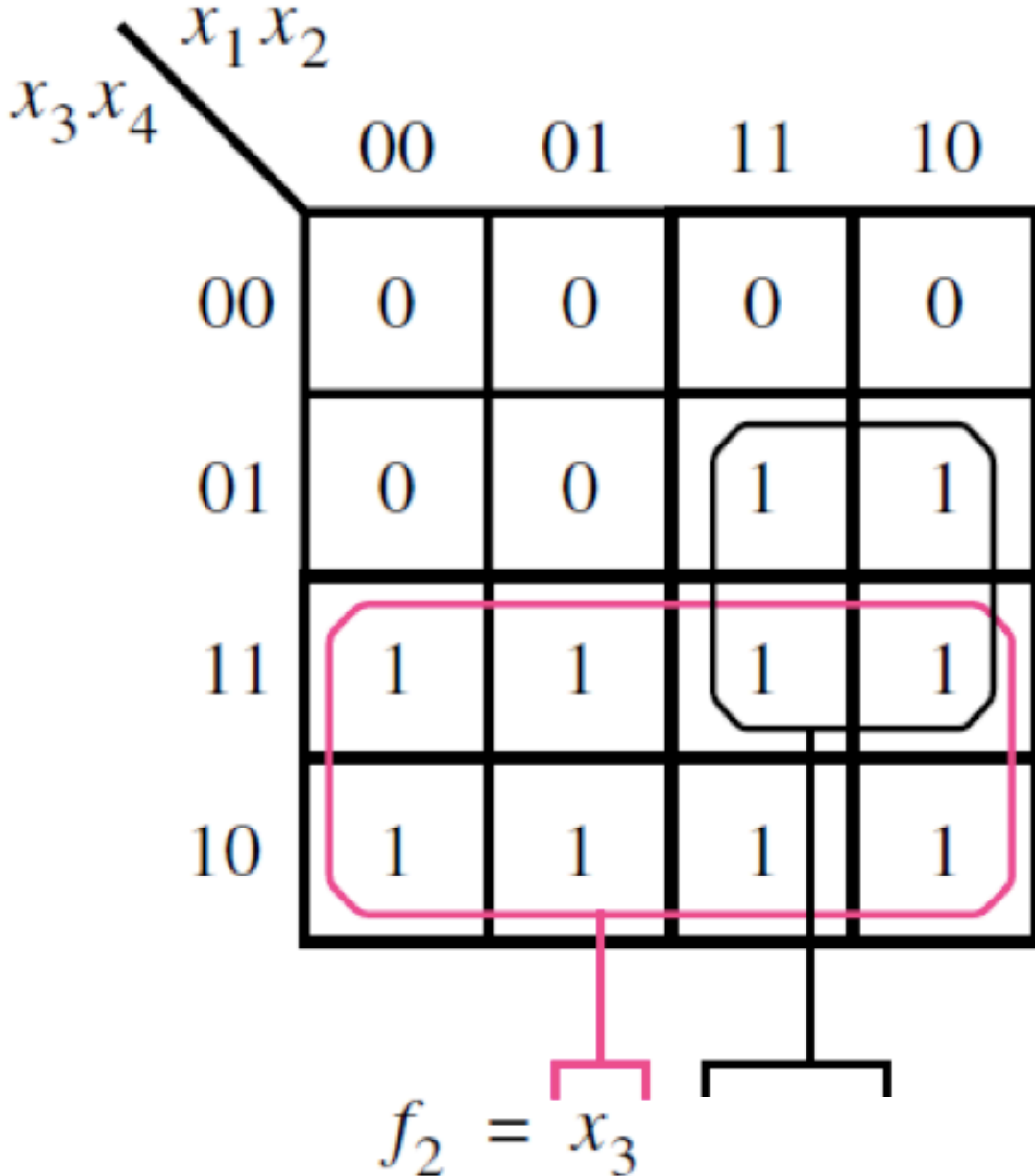
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map



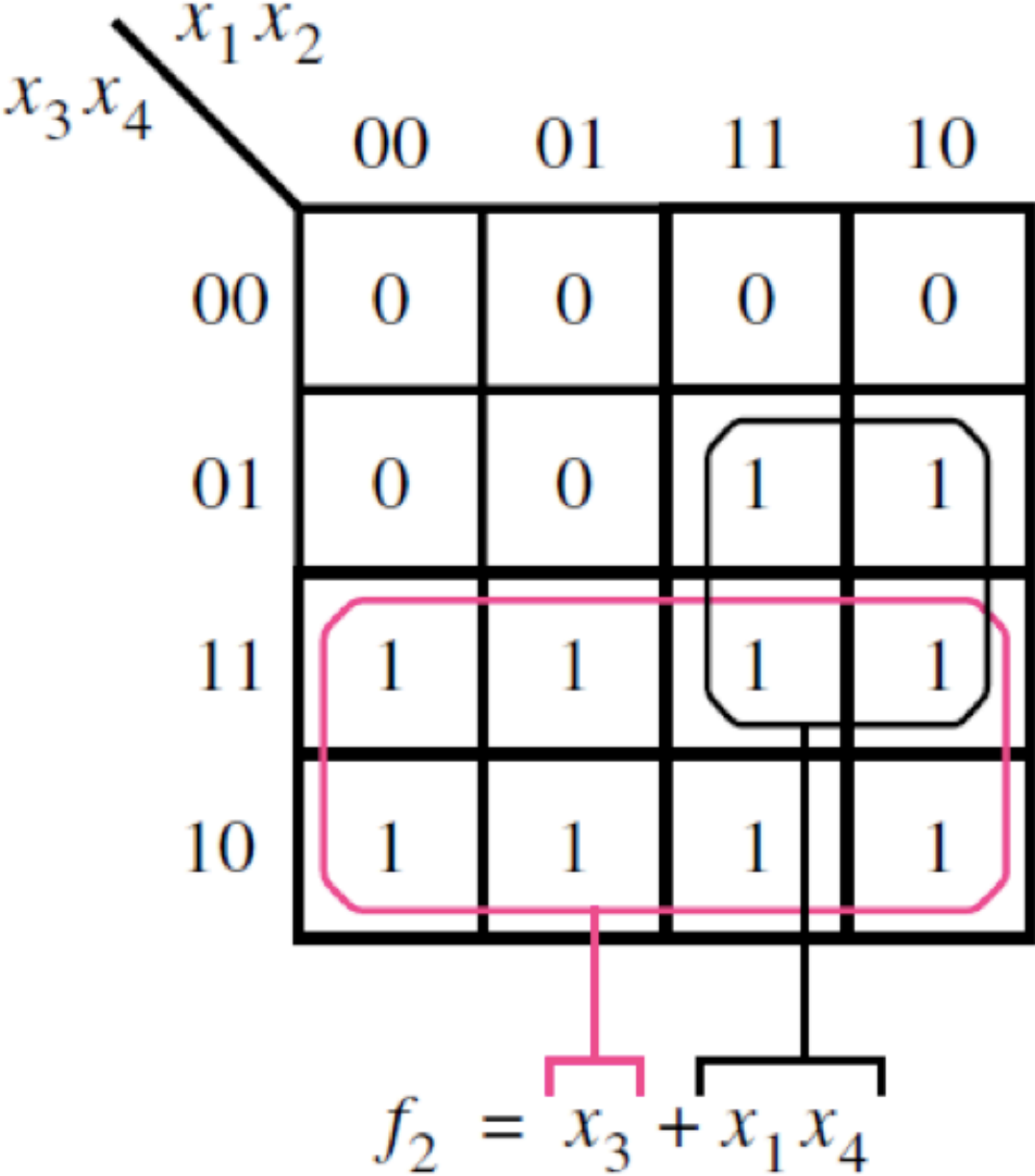
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map



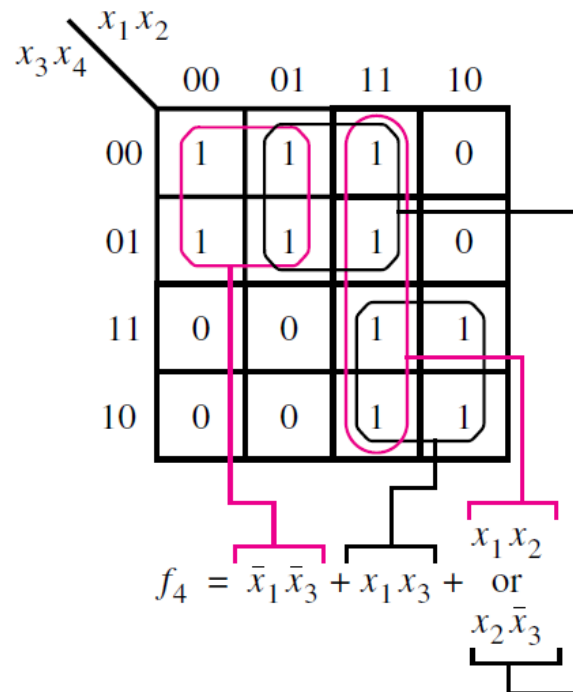
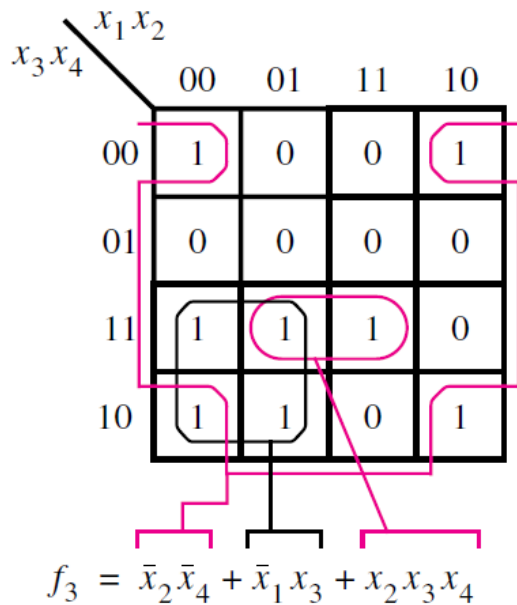
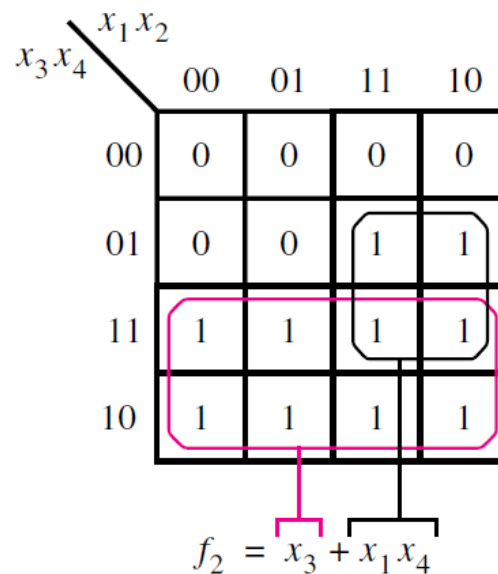
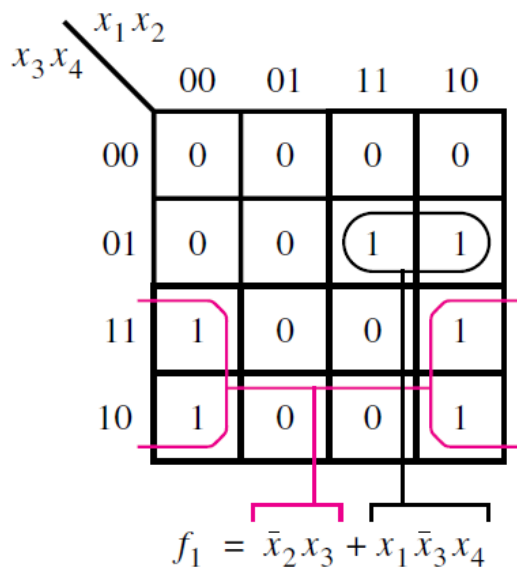
[ Figure 2.54 from the textbook ]

# Example of a four-variable Karnaugh map



[ Figure 2.54 from the textbook ]

# Other Four-Variable K-map Examples



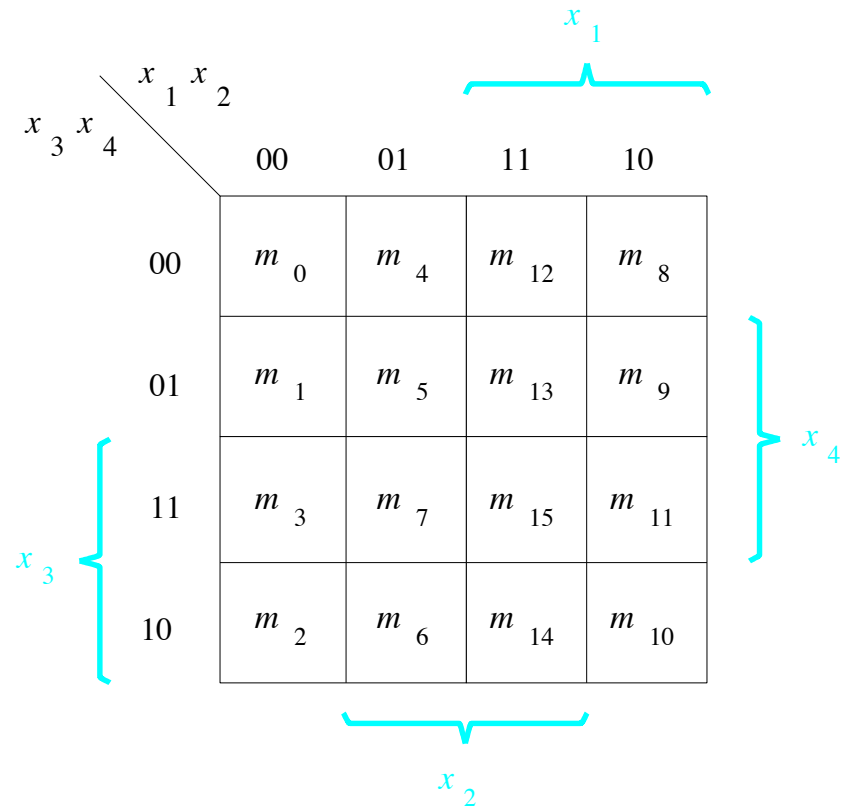
[ Figure 2.54 from the textbook ]

**Example:**  
**Incompletely Specified Function**

# Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \Sigma m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1$	$x_2$	$x_3$	$x_4$	$f$
0	0	0	0	$m_0$
0	0	0	1	$m_1$
0	0	1	0	$m_2$
0	0	1	1	$m_3$
0	1	0	0	$m_4$
0	1	0	1	$m_5$
0	1	1	0	$m_6$
0	1	1	1	$m_7$
1	0	0	0	$m_8$
1	0	0	1	$m_9$
1	0	1	0	$m_{10}$
1	0	1	1	$m_{11}$
1	1	0	0	$m_{12}$
1	1	0	1	$m_{13}$
1	1	1	0	$m_{14}$
1	1	1	1	$m_{15}$



# Three Ways to Specify the Function

$$f(x_1, x_2, x_3, x_4) = \sum m(2, 4, 5, 6, 10) + D(12, 13, 14, 15)$$

$x_1$	$x_2$	$x_3$	$x_4$	$f$
0	0	0	0	$m_0$ 0
0	0	0	1	$m_1$ 0
0	0	1	0	$m_2$ 1
0	0	1	1	$m_3$ 0
0	1	0	0	$m_4$ 1
0	1	0	1	$m_5$ 1
0	1	1	0	$m_6$ 1
0	1	1	1	$m_7$ 1
1	0	0	0	$m_8$ 0
1	0	0	1	$m_9$ 0
1	0	1	0	$m_{10}$ 1
1	0	1	1	$m_{11}$ 0
1	1	0	0	$m_{12}$ d
1	1	0	1	$m_{13}$ d
1	1	1	0	$m_{14}$ d
1	1	1	1	$m_{15}$ d

		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	0	1	d	0
	01	0	1	d	0
	11	0	0	d	0
	10	1	1	d	1



# SOP implementation

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

(a) SOP implementation

# POS implementation

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	d	0
01	0	1	d	0
11	0	0	d	0
10	1	1	d	1

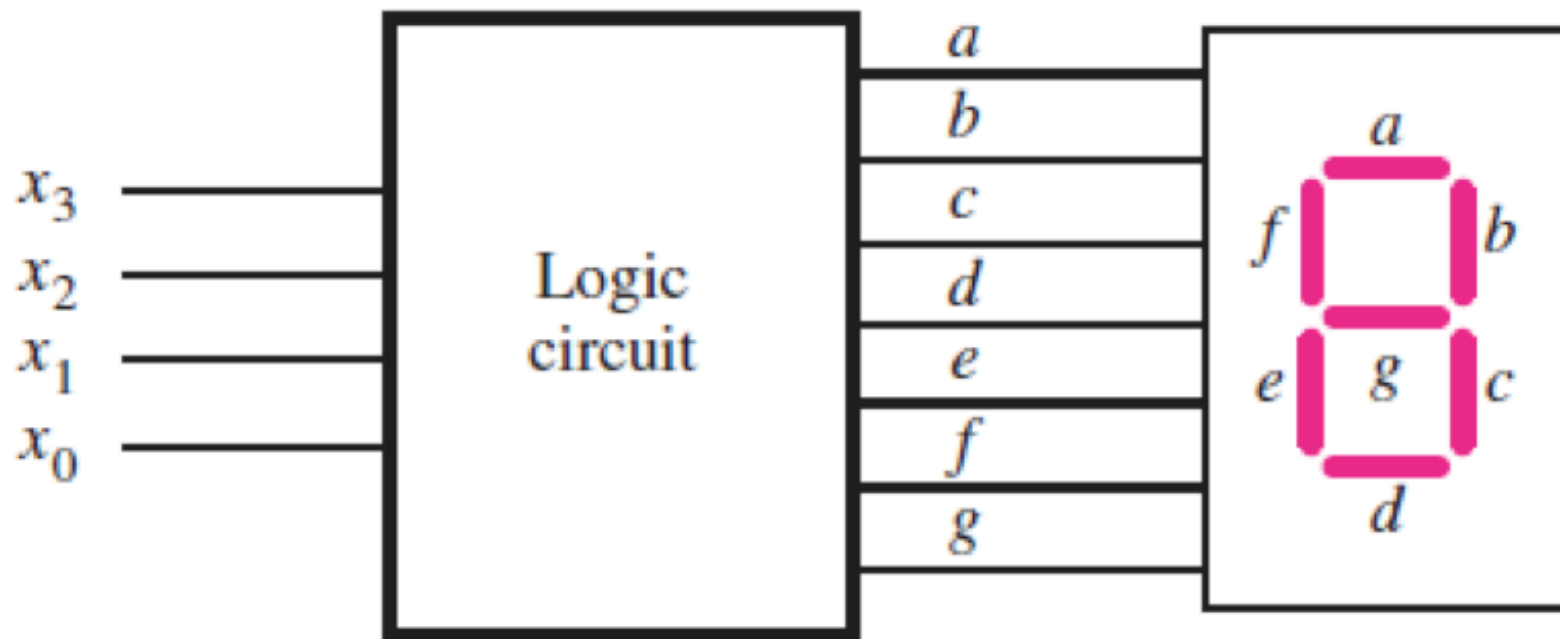
$(x_2 + x_3)$

$(\bar{x}_3 + \bar{x}_4)$

(b) POS implementation

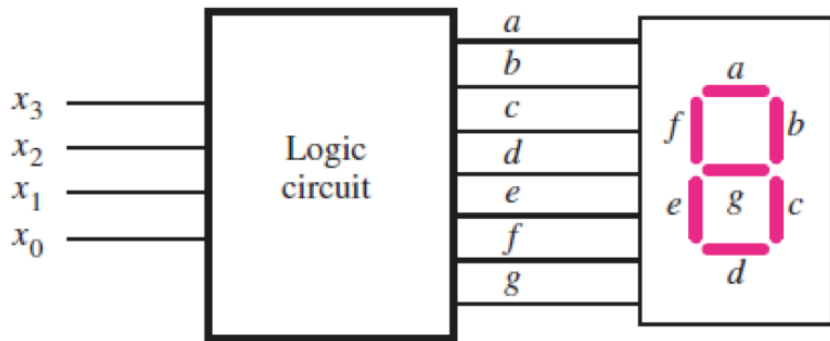
**Example:**  
**A circuit with multiple outputs**

# Seven-Segment Indicator



(a) Logic circuit and 7-segment display

# Seven-Segment Indicator



(a) Logic circuit and 7-segment display

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table

# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0							
1011	1	0	1	1							
1100	1	1	0	0							
1101	1	1	0	1							
1110	1	1	1	0							
1111	1	1	1	1							

# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
d	1	0	1	0	d	d	d	d	d	d	d
d	1	0	1	1	d	d	d	d	d	d	d
d	1	1	0	0	d	d	d	d	d	d	d
d	1	1	0	1	d	d	d	d	d	d	d
d	1	1	1	0	d	d	d	d	d	d	d
d	1	1	1	1	d	d	d	d	d	d	d

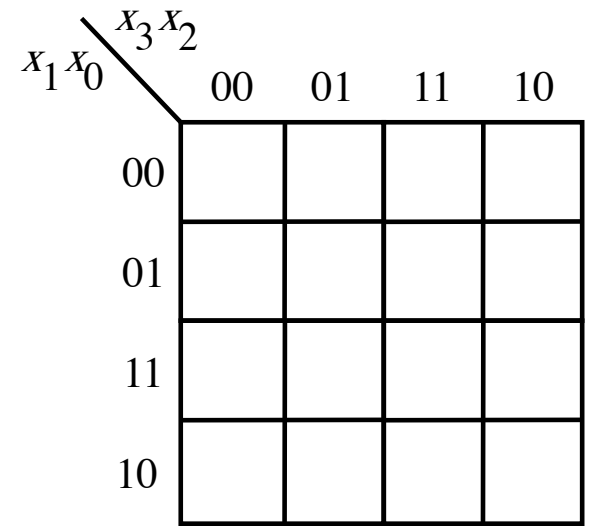
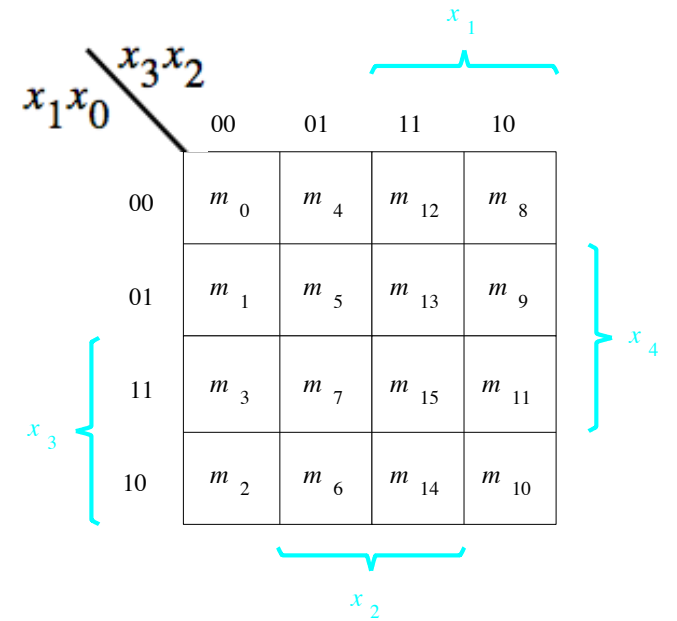
# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d



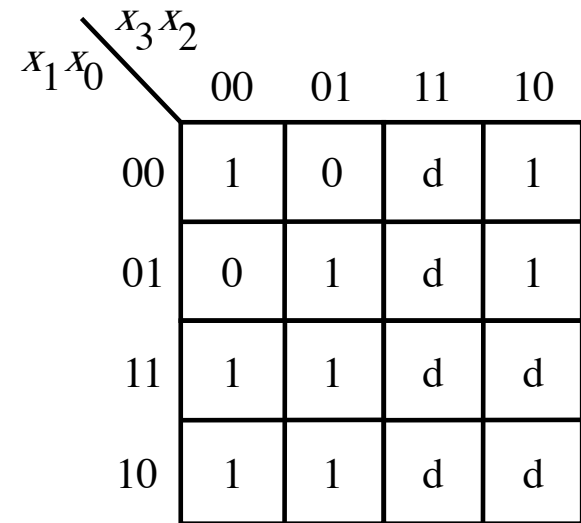
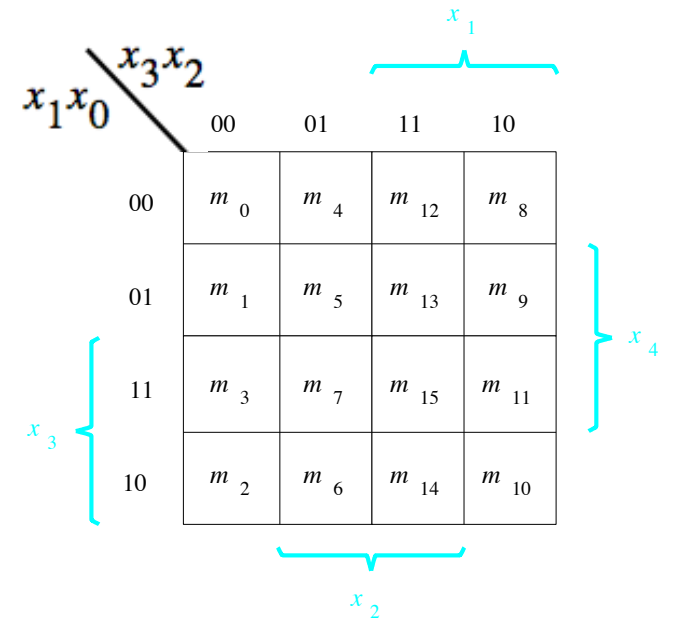
# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d



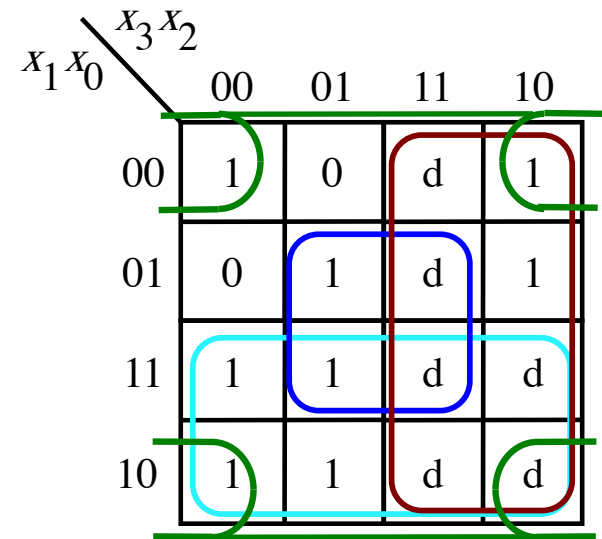
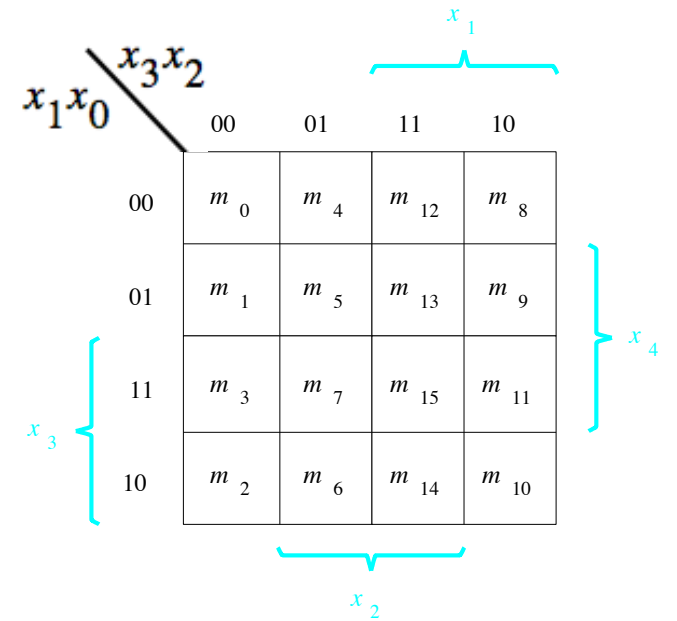
# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	d	d	d	d	d	d	d
1011	1	0	1	1	d	d	d	d	d	d	d
1100	1	1	0	0	d	d	d	d	d	d	d
1101	1	1	0	1	d	d	d	d	d	d	d
1110	1	1	1	0	d	d	d	d	d	d	d
1111	1	1	1	1	d	d	d	d	d	d	d



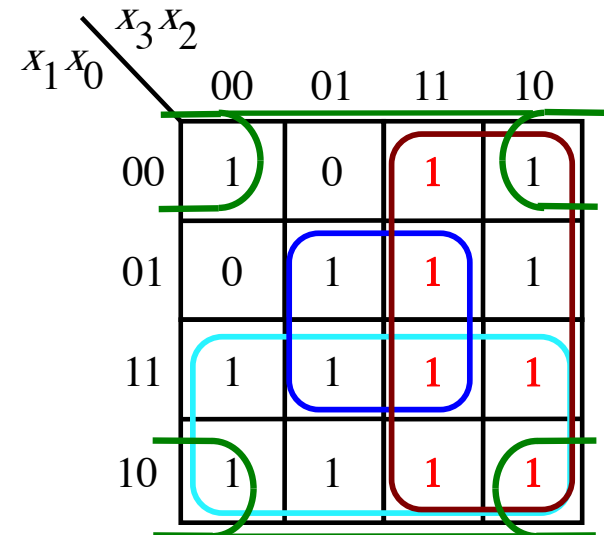
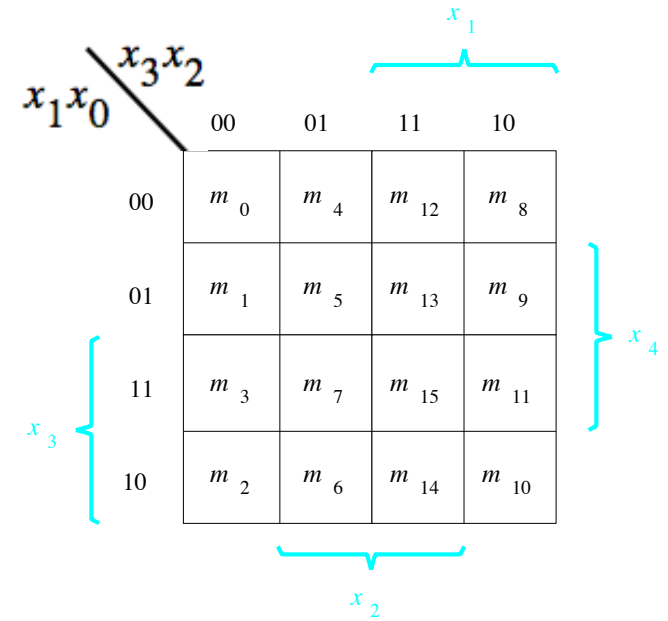
# Seven-Segment Indicator

$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	d	d	d	d	d	d	d
1	0	1	1	d	d	d	d	d	d	d
1	1	0	0	d	d	d	d	d	d	d
1	1	0	1	d	d	d	d	d	d	d
1	1	1	0	d	d	d	d	d	d	d
1	1	1	1	d	d	d	d	d	d	d



# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d



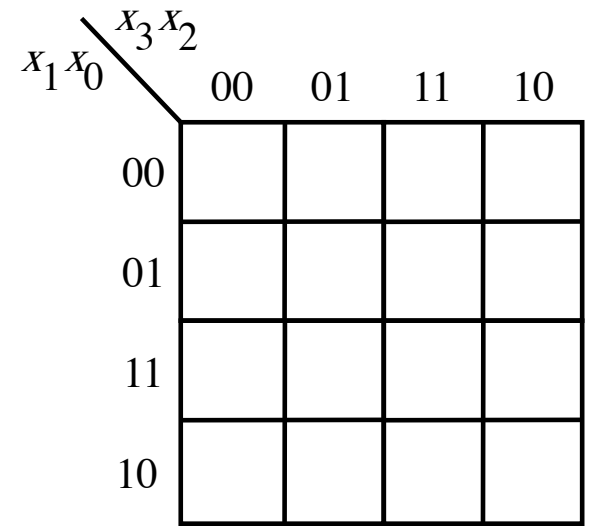
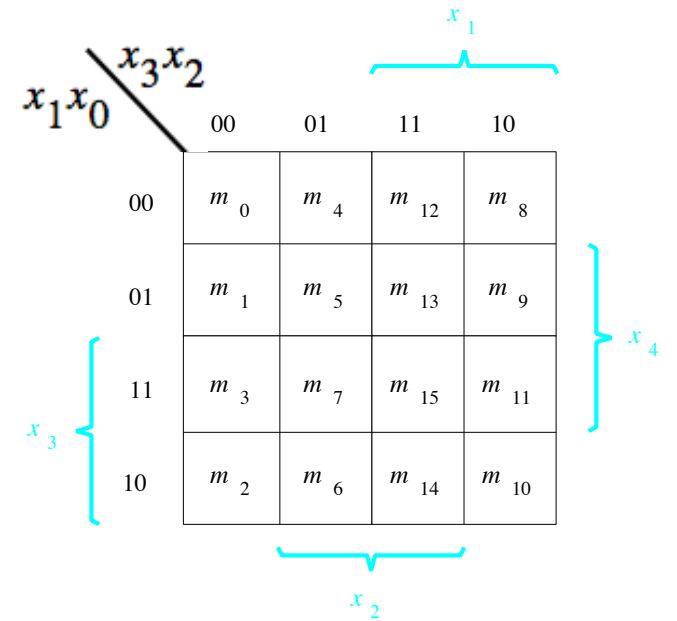
In this case all d's were treated as 1's.

# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1
d	1	0	1	0	1	d	d	d	d	d	d
e	1	0	1	1	1	d	d	d	d	d	d
f	1	1	0	0	1	d	d	d	d	d	d
g	1	1	0	1	1	d	d	d	d	d	d
h	1	1	1	0	1	d	d	d	d	d	d
i	1	1	1	1	1	d	d	d	d	d	d

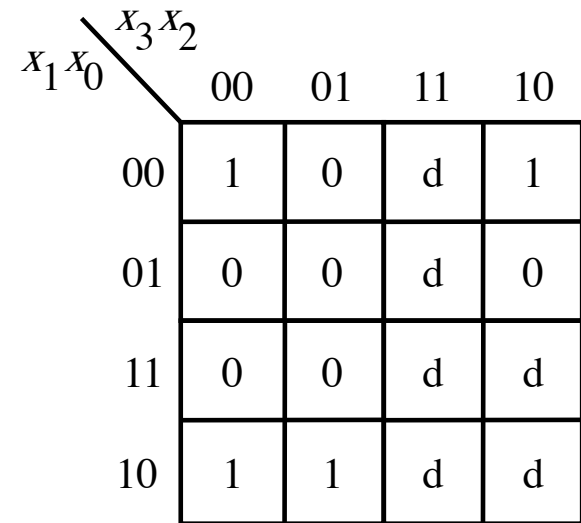
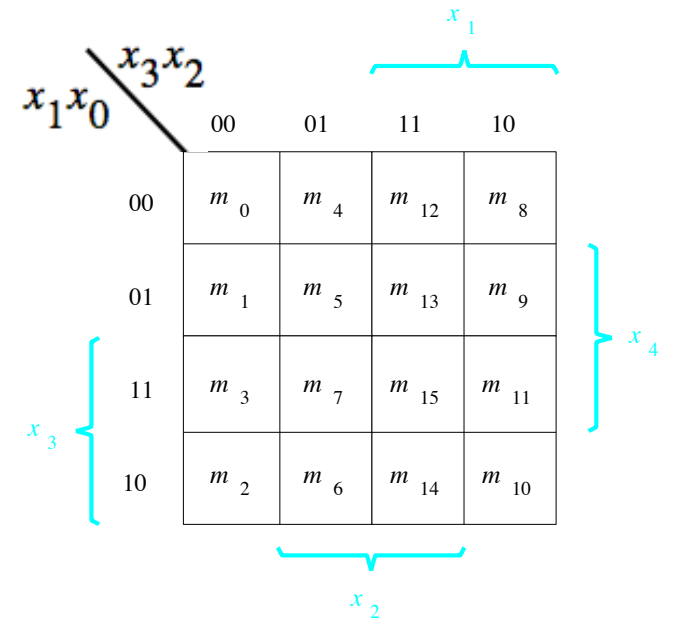
# Seven-Segment Indicator

$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d



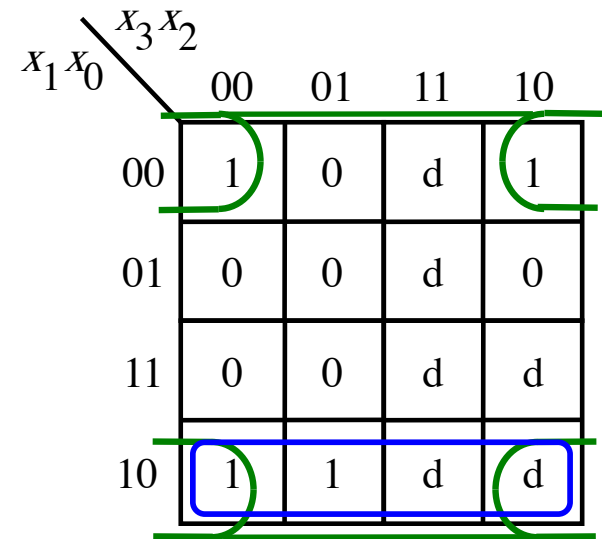
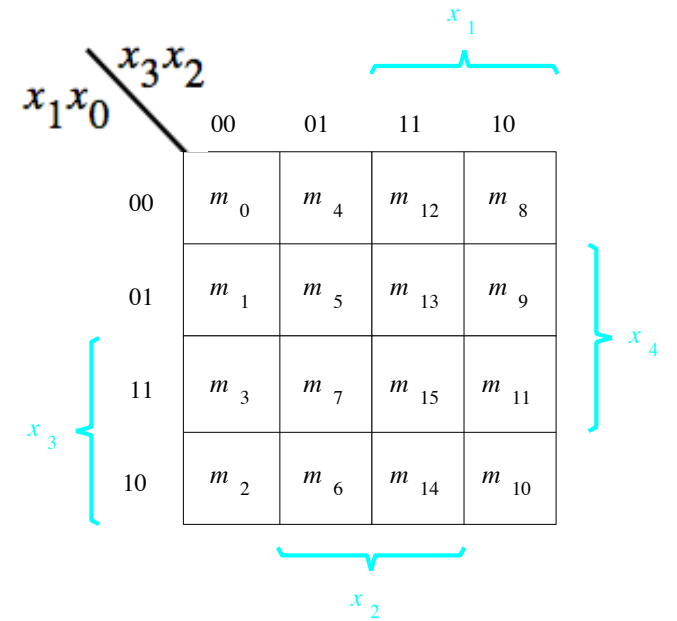
# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0000	0	0	0	0	1	1	1	1	1	1	0
0001	0	0	0	1	0	1	1	0	0	0	0
0010	0	0	1	0	1	1	0	1	1	0	1
0011	0	0	1	1	1	1	1	1	0	0	1
0100	0	1	0	0	0	1	1	0	0	1	1
0101	0	1	0	1	1	0	1	1	0	1	1
0110	0	1	1	0	1	0	1	1	1	1	1
0111	0	1	1	1	1	1	1	0	0	0	0
1000	1	0	0	0	1	1	1	1	1	1	1
1001	1	0	0	1	1	1	1	1	0	1	1
1010	1	0	1	0	1	d	d	d	d	d	d
1011	1	0	1	1	1	d	d	d	d	d	d
1100	1	1	0	0	1	d	d	d	d	d	d
1101	1	1	0	1	1	d	d	d	d	d	d
1110	1	1	1	0	1	d	d	d	d	d	d
1111	1	1	1	1	1	d	d	d	d	d	d



# Seven-Segment Indicator

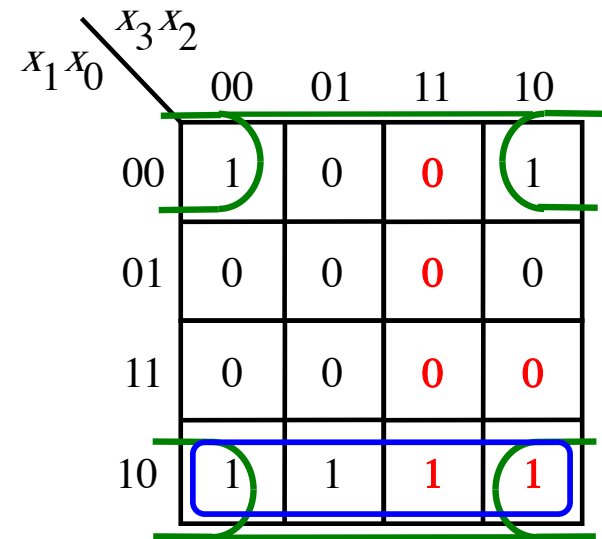
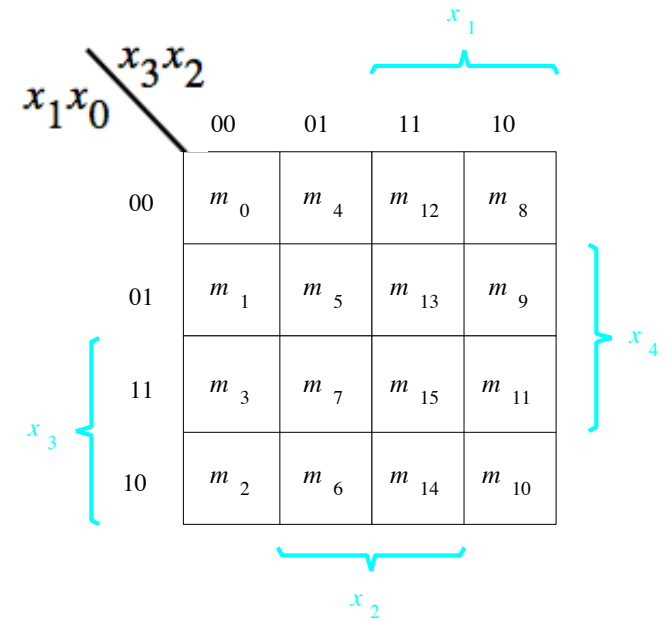
$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	d	d	d
1	0	1	1	1	d	d	d	d	d	d
1	1	0	0	1	d	d	d	d	d	d
1	1	0	1	1	d	d	d	d	d	d
1	1	1	0	1	d	d	d	d	d	d
1	1	1	1	1	d	d	d	d	d	d





# Seven-Segment Indicator

$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	1	1	1	1	1	1	0
0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	1	1	0	1
0	0	1	1	1	1	1	1	0	0	1
0	1	0	0	0	1	1	0	0	1	1
0	1	0	1	1	0	1	1	0	1	1
0	1	1	0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	1	1	1	1
1	0	0	1	1	1	1	1	0	1	1
1	0	1	0	1	d	d	d	1	d	d
1	0	1	1	1	d	d	d	0	d	d
1	1	0	0	1	d	d	d	0	d	d
1	1	0	1	1	d	d	d	0	d	d
1	1	1	0	1	d	d	d	1	d	d
1	1	1	1	1	d	d	d	0	d	d

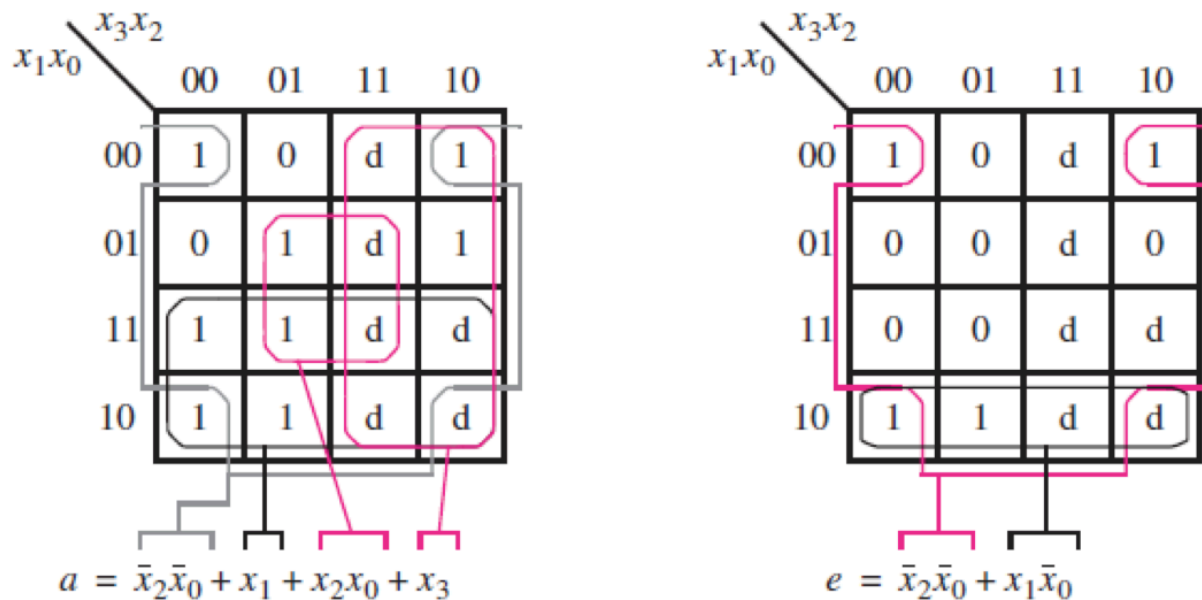


In this case some d's were treated as 1's, others as 0's.

# Seven-Segment Indicator

	$x_3$	$x_2$	$x_1$	$x_0$	$a$	$b$	$c$	$d$	$e$	$f$	$g$
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1	0	1	1	0	0	0	0
2	0	0	1	0	1	1	0	1	1	0	1
3	0	0	1	1	1	1	1	1	0	0	1
4	0	1	0	0	0	1	1	0	0	1	1
5	0	1	0	1	1	0	1	1	0	1	1
6	0	1	1	0	1	0	1	1	1	1	1
7	0	1	1	1	1	1	1	0	0	0	0
8	1	0	0	0	1	1	1	1	1	1	1
9	1	0	0	1	1	1	1	1	0	1	1

(b) Truth table



(c) The Karnaugh maps for outputs  $a$  and  $e$ .

# **Another Example**

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function  $f_2$

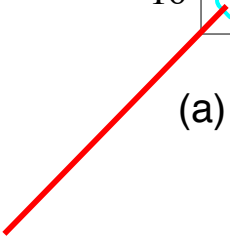
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

(b) Function  $f_2$

$\bar{x}_1 x_3$



$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$\bar{x}_1 x_3$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$$\overline{x_1} x_3$$



$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$$\overline{x_1} x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$\bar{X}_1 \bar{X}_3$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$\bar{X}_1 X_3$

$\bar{X}_1 X_3$

	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$$\bar{x}_1 x_3$$

$$x_1 \bar{x}_3$$

$$\bar{x}_2 \bar{x}_3 x_4$$

	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$$x_2 x_3 x_4$$

$$\bar{x}_1 x_3$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$\bar{x}_1 \bar{x}_3$

$x_2 \bar{x}_3 x_4$

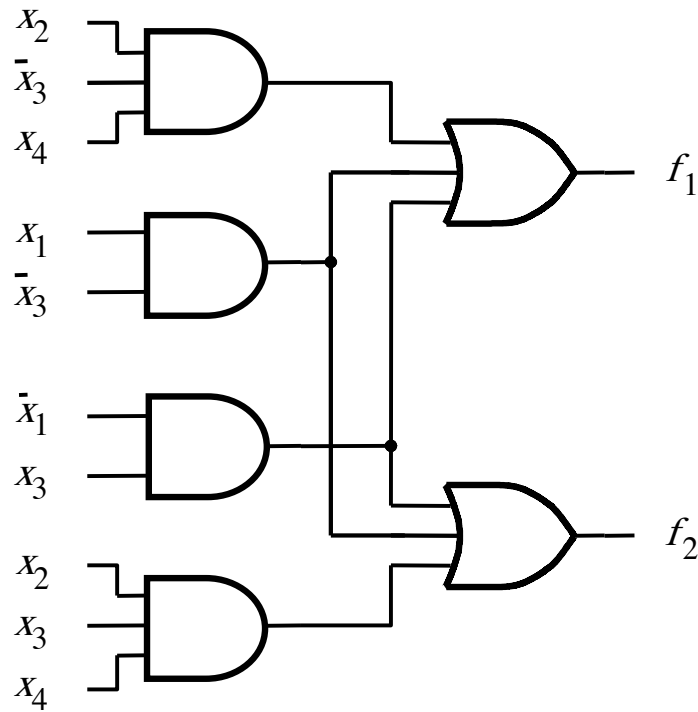
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$x_2 x_3 x_4$

$\bar{x}_1 x_3$

$\bar{x}_1 x_3$



(c) Combined circuit for  $f_1$  and  $f_2$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$\bar{X}_1 \bar{X}_3$

$\bar{X}_2 \bar{X}_3 X_4$

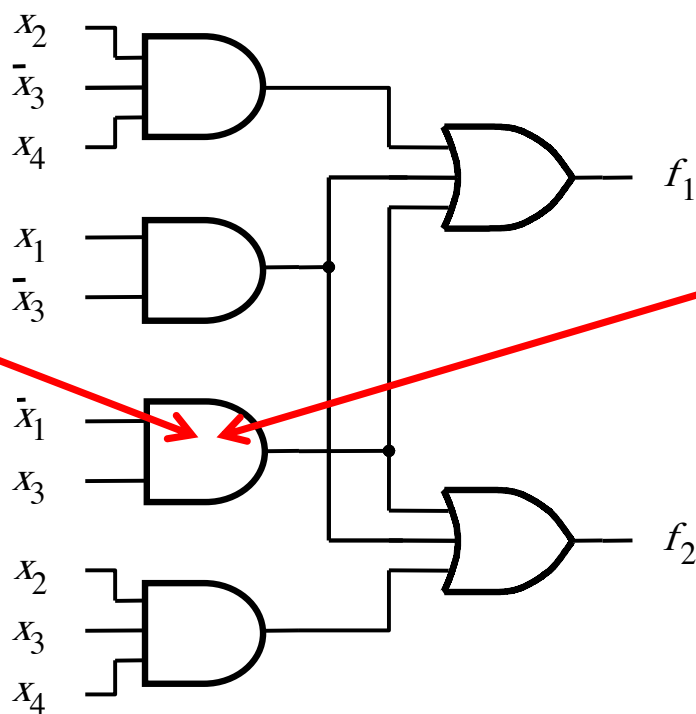
$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$

$X_2 X_3 X_4$

$\bar{X}_1 \bar{X}_3$

$\bar{X}_1 \bar{X}_3$



(c) Combined circuit for  $f_1$  and  $f_2$

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

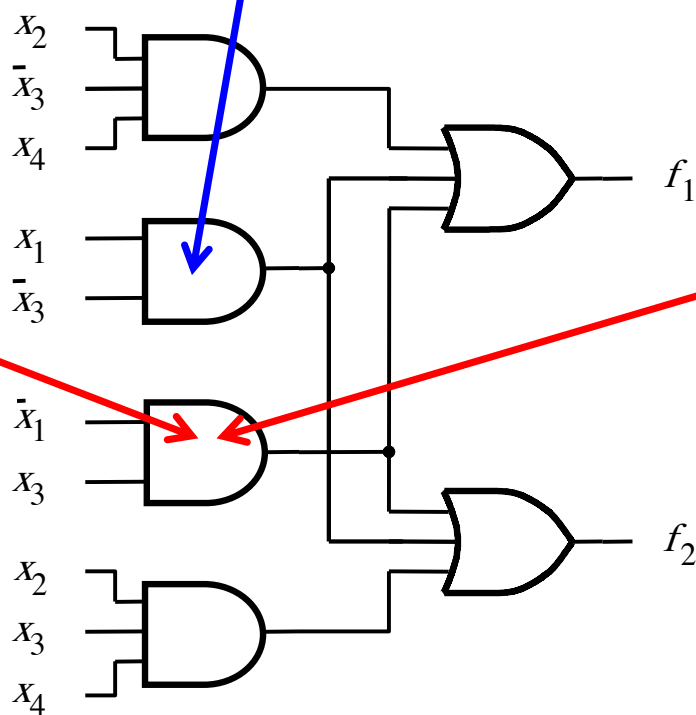
(b) Function  $f_2$

$\bar{x}_2 \bar{x}_3 x_4$

$x_2 x_3 x_4$

$\bar{x}_1 x_3$

$\bar{x}_1 x_3$



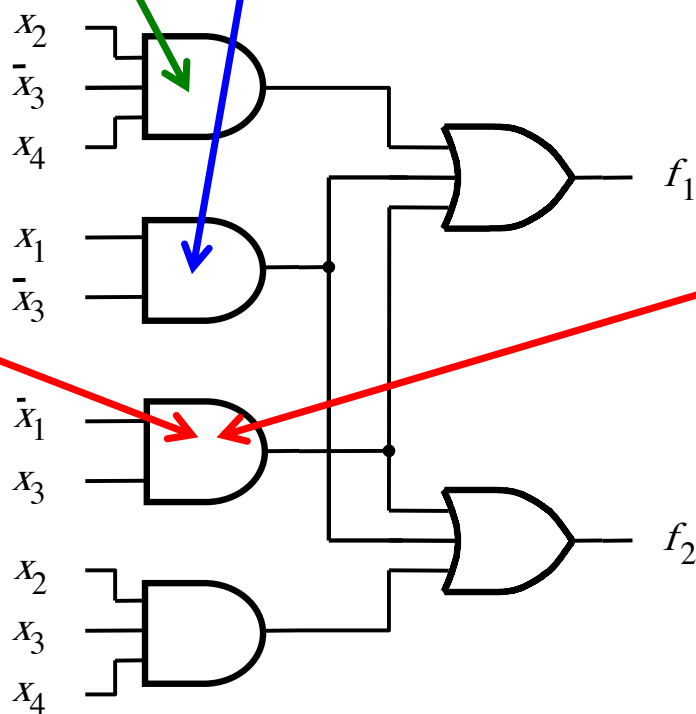
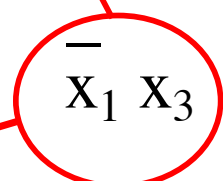
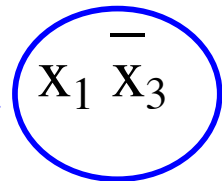
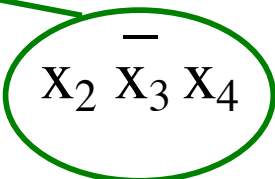
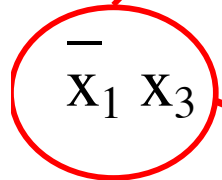
(c) Combined circuit for  $f_1$  and  $f_2$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$



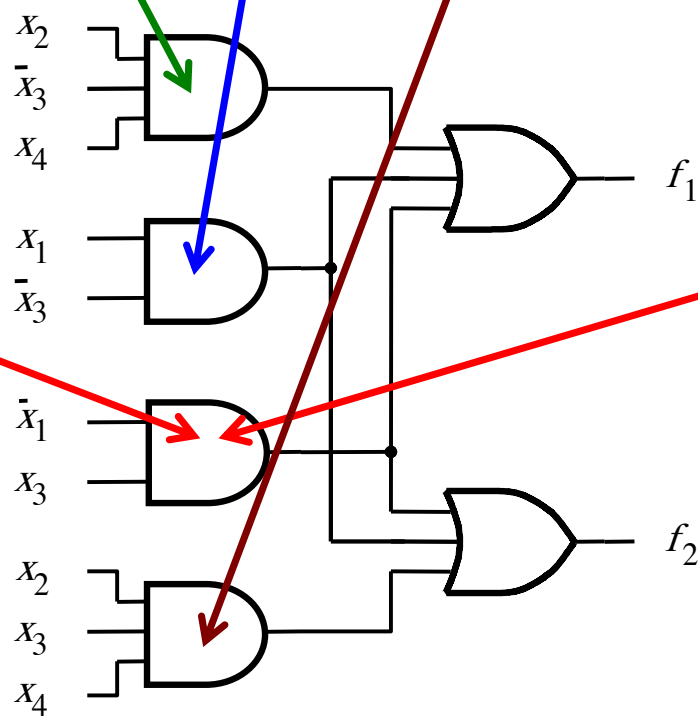
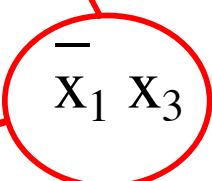
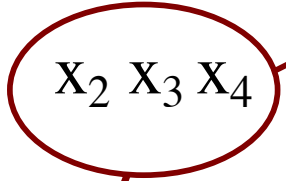
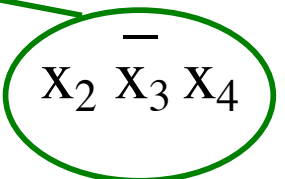
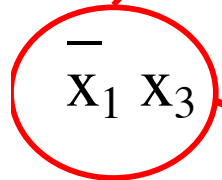
(c) Combined circuit for  $f_1$  and  $f_2$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00			1	1
01			1	1
11	1	1	1	
10	1	1		

(b) Function  $f_2$



(c) Combined circuit for  $f_1$  and  $f_2$

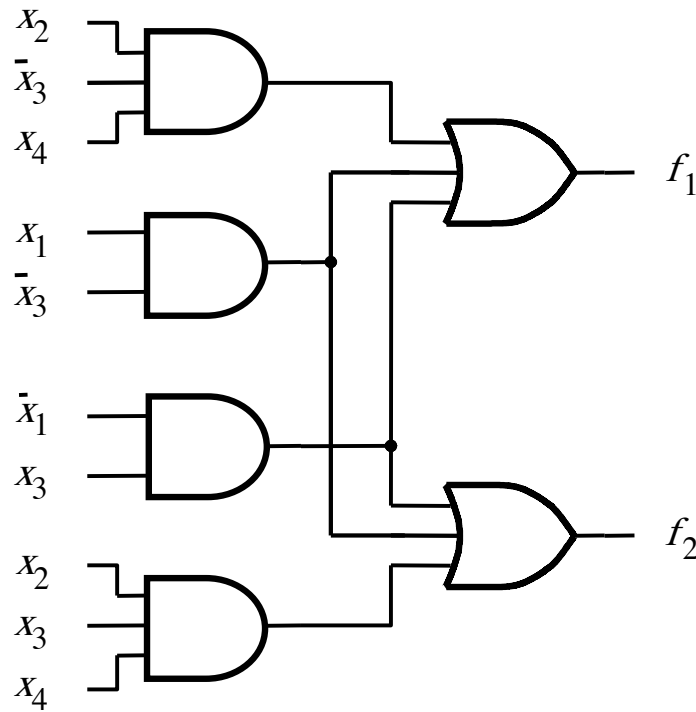


	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01		1	1	1
11	1	1		
10	1	1		

(a) Function  $f_1$

	$x_1 x_2$			
$x_3 x_4$	00	01	11	10
00			1	1
01			1	1
11	1	1		
10	1	1		

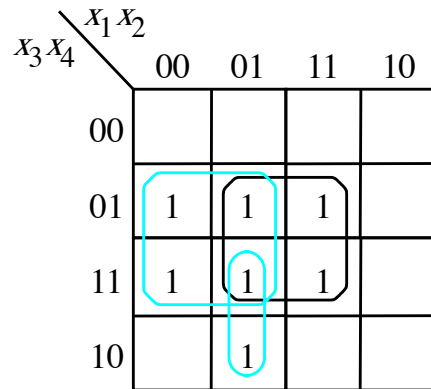
(b) Function  $f_2$



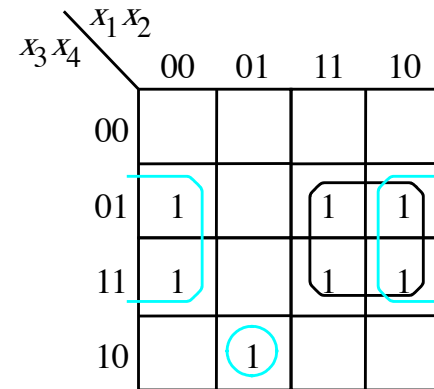
(c) Combined circuit for  $f_1$  and  $f_2$

**Yet Another Example**

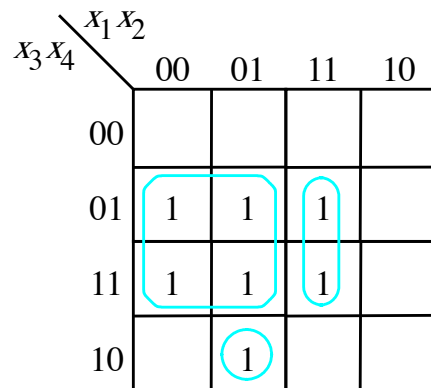
# Individual vs Joint Optimization



(a) Optimal realization of  $f_3$

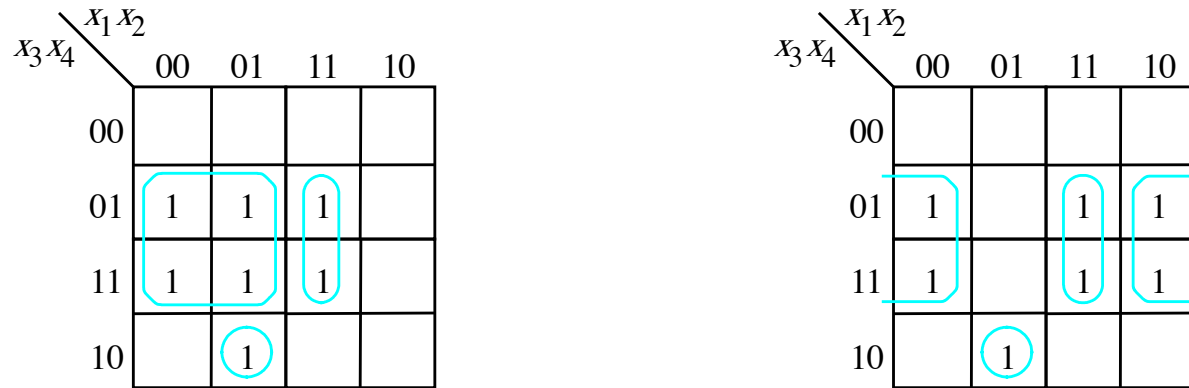


(b) Optimal realization of  $f_4$

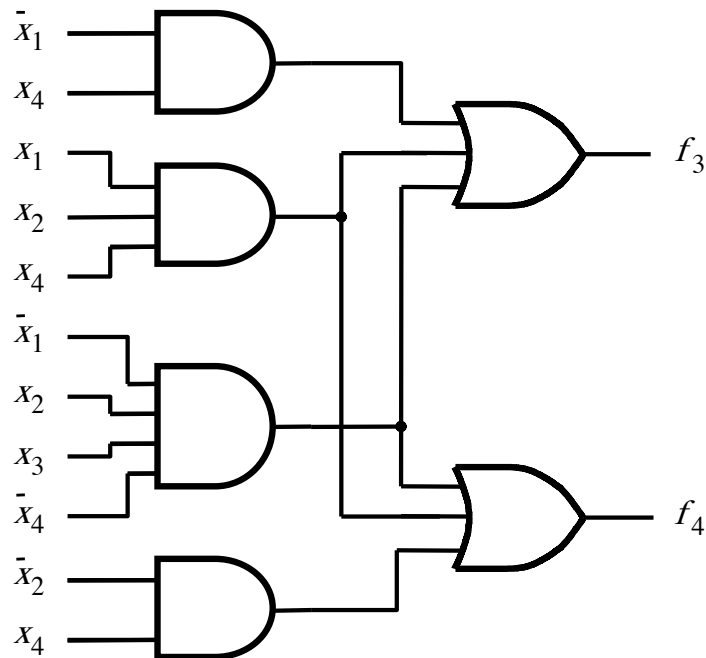


(c) Optimal realization of  $f_3$  and  $f_4$  together

# Individual vs Joint Optimization

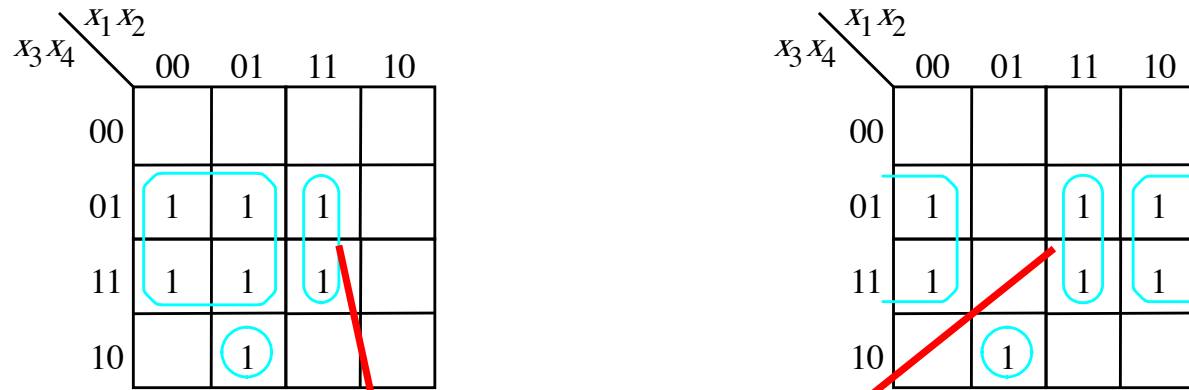


(c) Optimal realization of  $f_3$  and  $f_4$  together

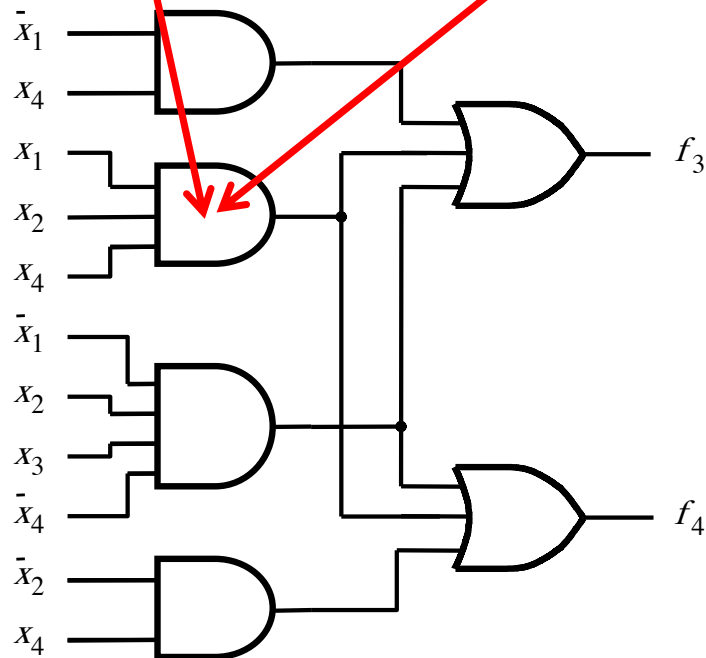


(d) Combined circuit for  $f_3$  and  $f_4$

# Individual vs Joint Optimization



(c) Optimal realization of  $f_3$  and  $f_4$  together



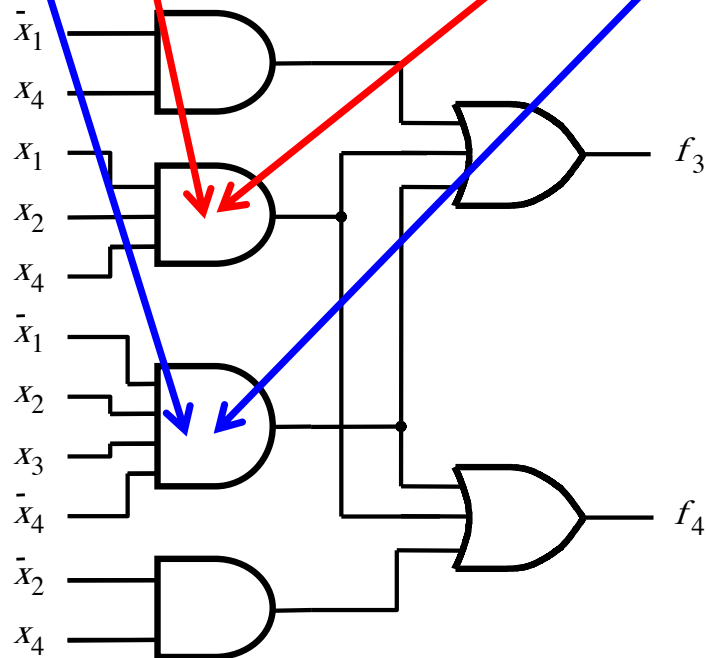
(d) Combined circuit for  $f_3$  and  $f_4$

# Individual vs Joint Optimization

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00				
01	1	1	1	
11	1	1	1	
10		1		

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00				
01	1		1	1
11	1		1	1
10		1		

(c) Optimal realization of  $f_3$  and  $f_4$  together



(d) Combined circuit for  $f_3$  and  $f_4$

**Questions?**

**THE END**