



# **CprE 281: Digital Logic**

**Instructor: Alexander Stoytchev**

**<http://www.ece.iastate.edu/~alexs/classes/>**

# **Synchronous Sequential Circuits**

## **Basic Design Steps**

*CprE 281: Digital Logic*  
*Iowa State University, Ames, IA*  
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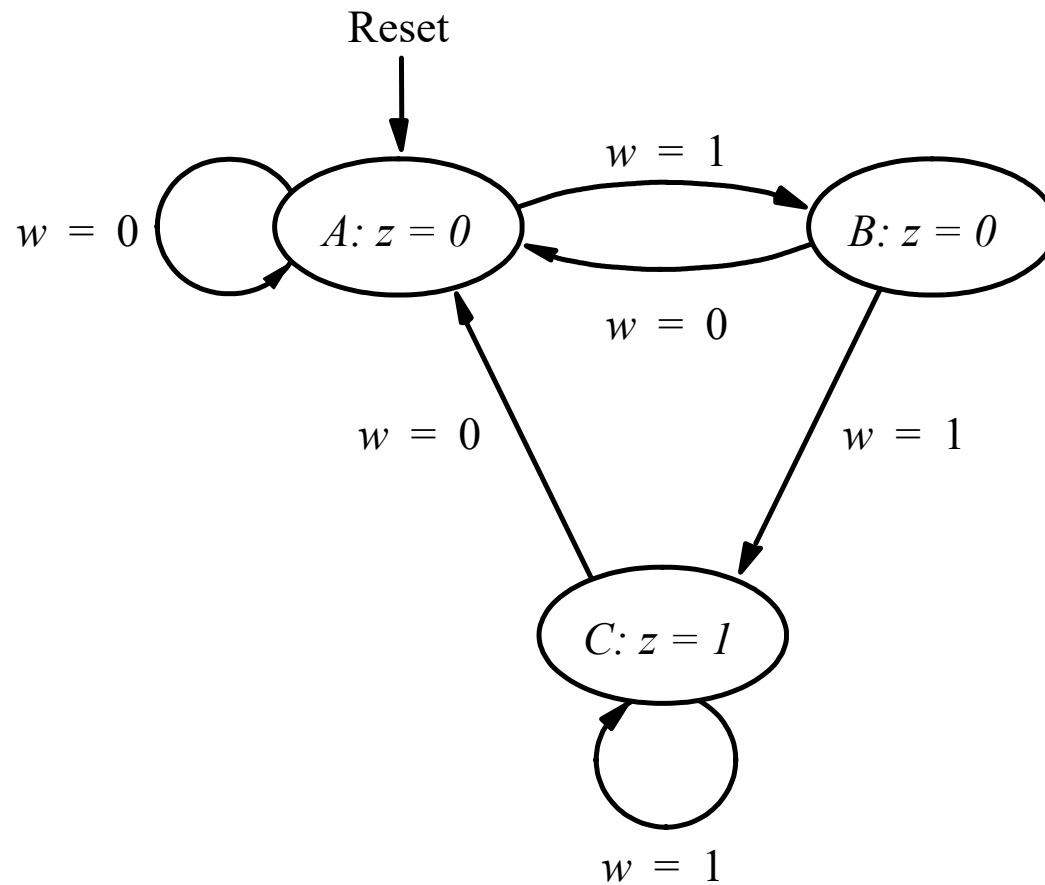
# **Administrative Stuff**

- **Homework 10 is due next Monday.**
- **Start thinking about your final project ideas.**

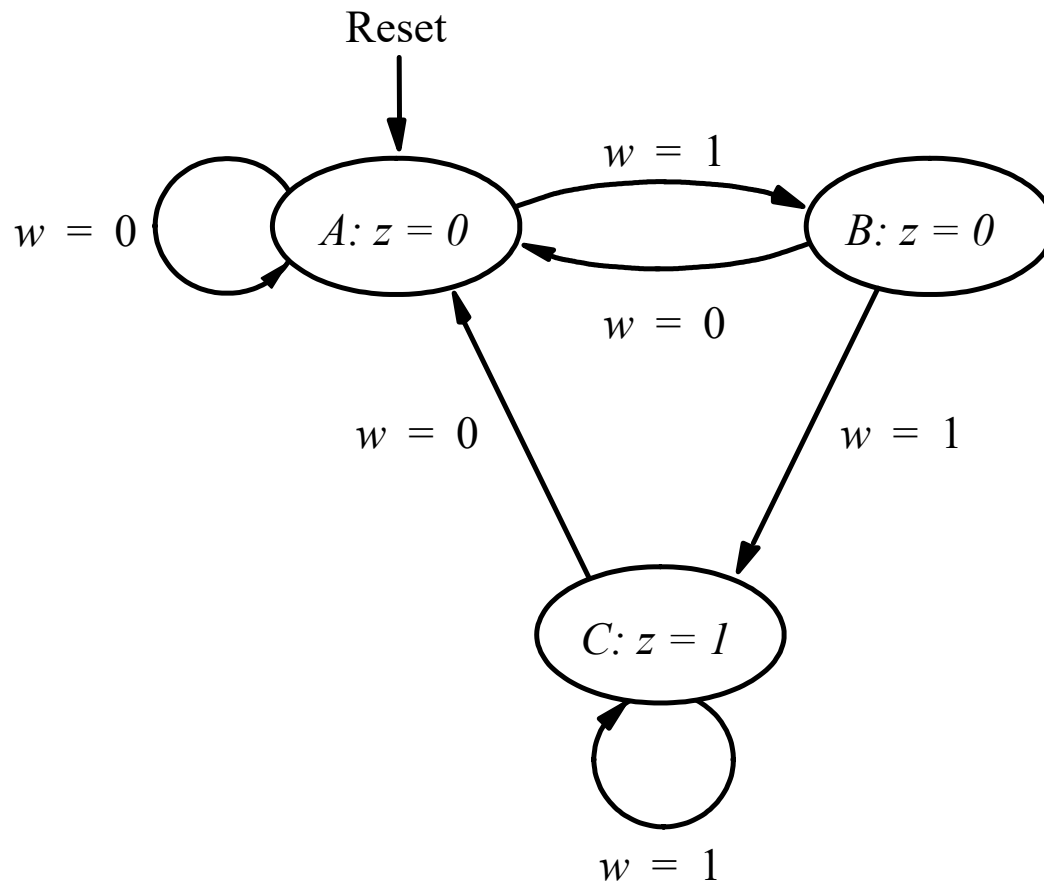
# **First Design Pattern: Moore Machines**

# Moore Machine:

A Type of Finite State Machine (FSM)

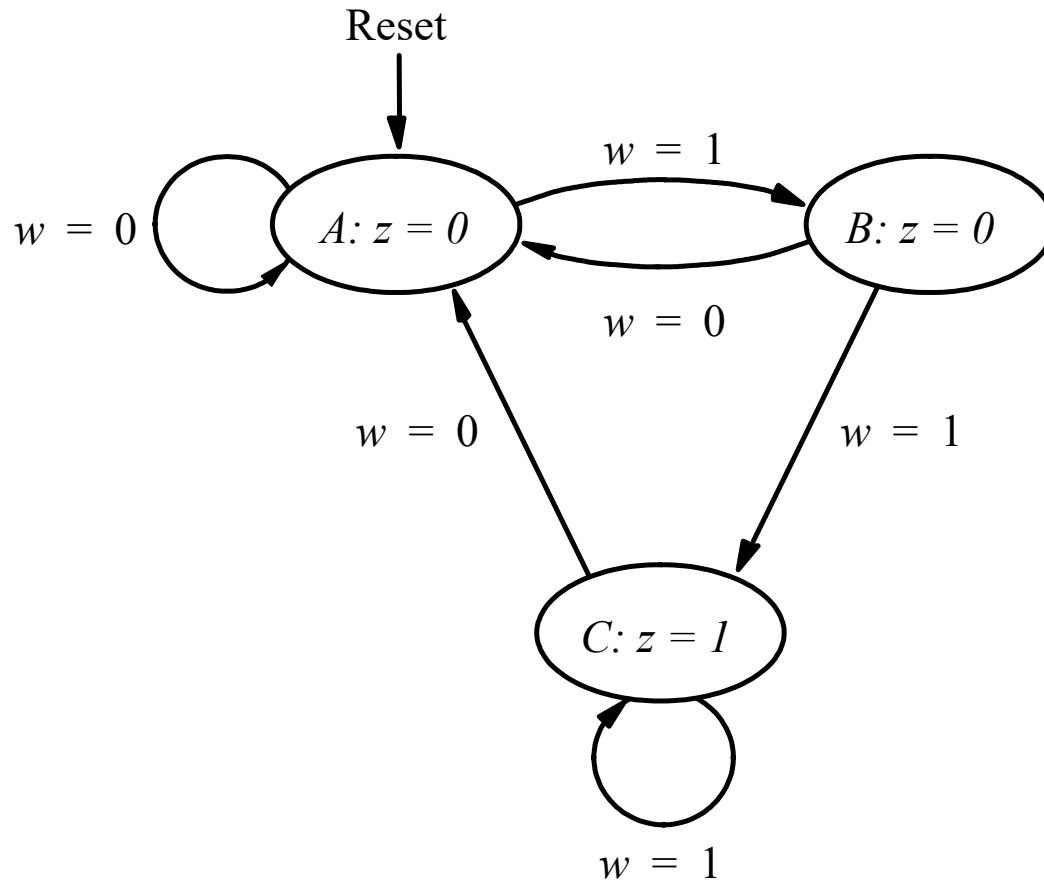


[ Figure 6.3 from the textbook ]

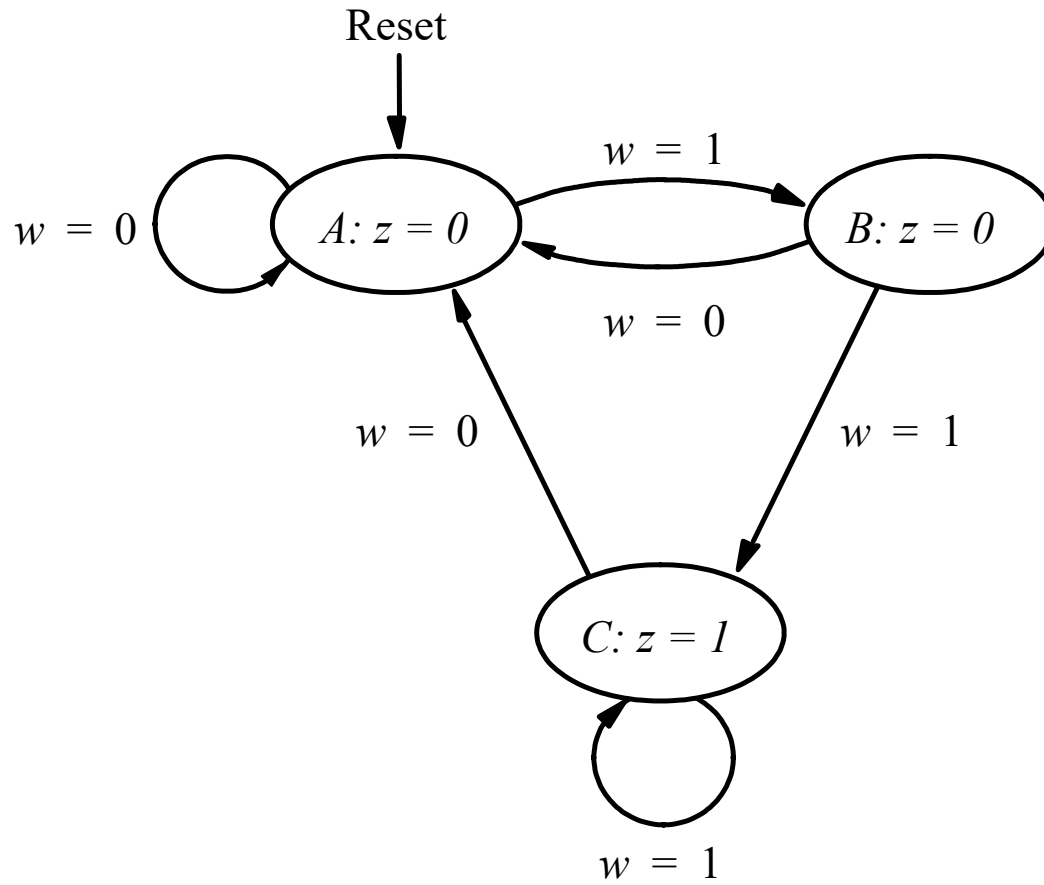


- Finite number of states (nodes).
- Discrete state transitions (edges).
- Only “in” one state at a time.
- One reset state
- Every state has an outgoing state transition for each possible input.

[ Figure 6.3 from the textbook ]

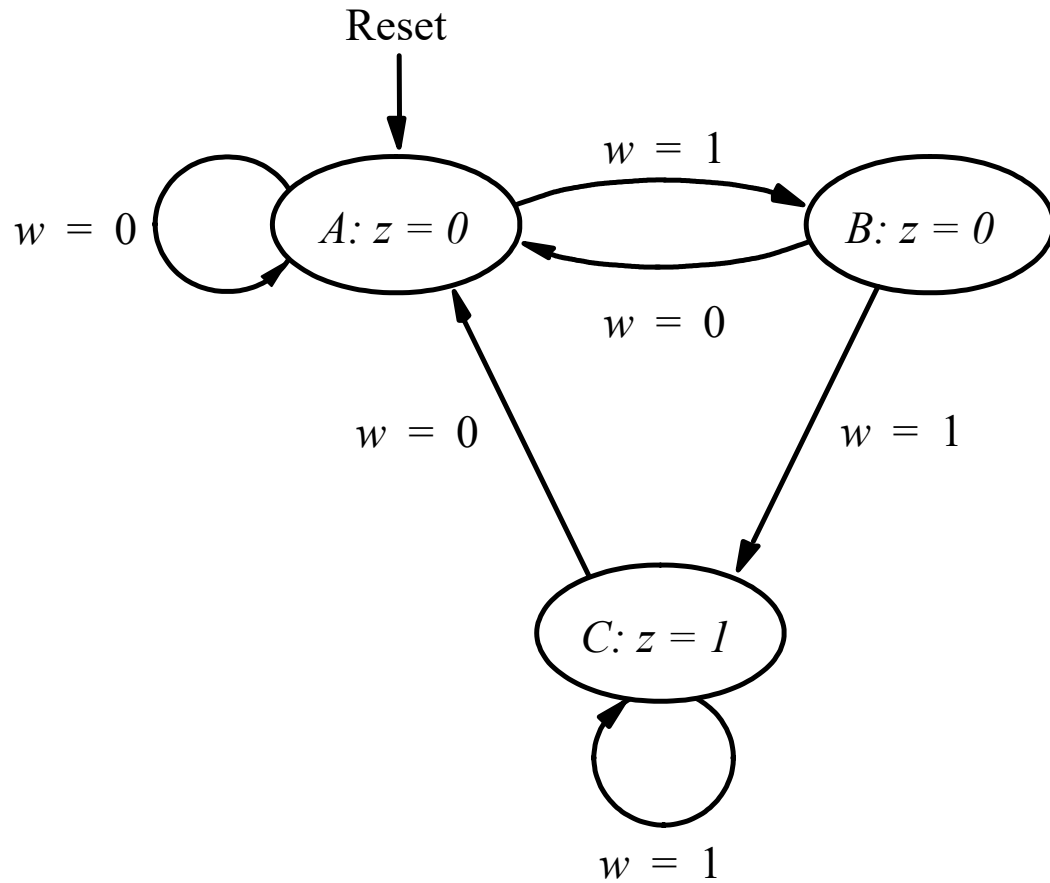


**Key:** The next state depends on both the current state and the current input.



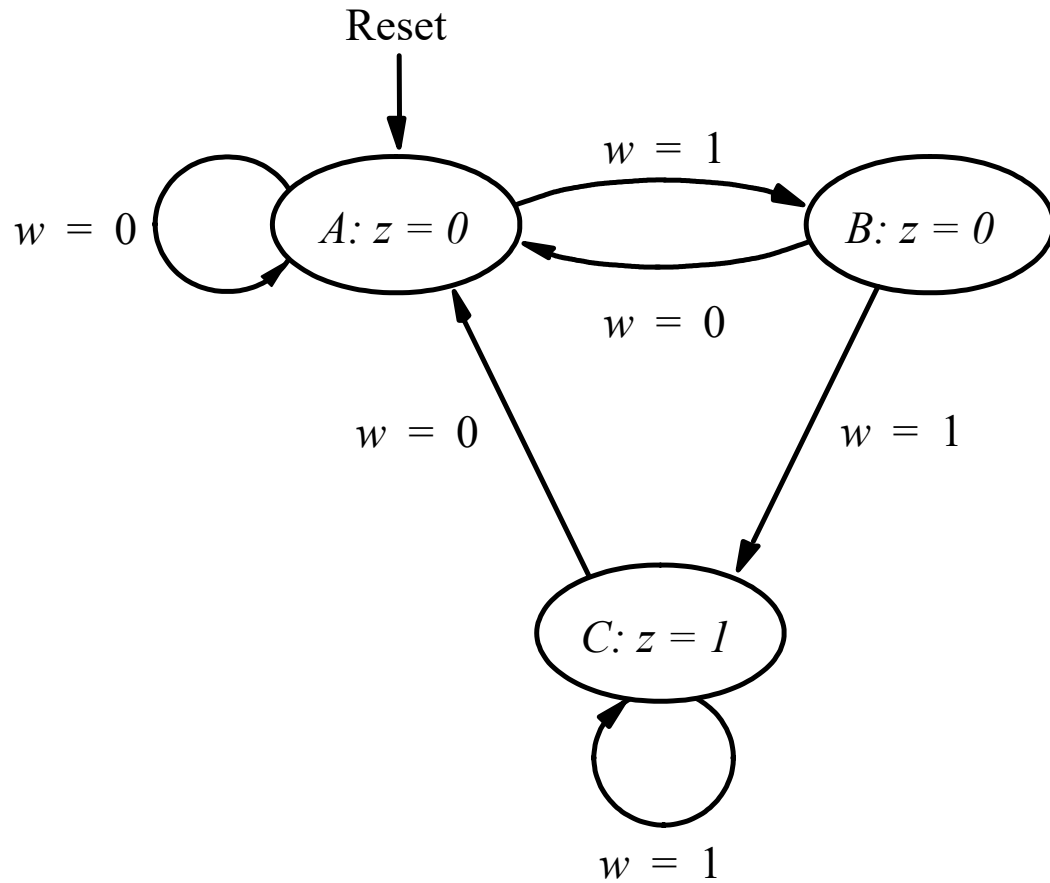
**Key:** The output depends only on the current state.



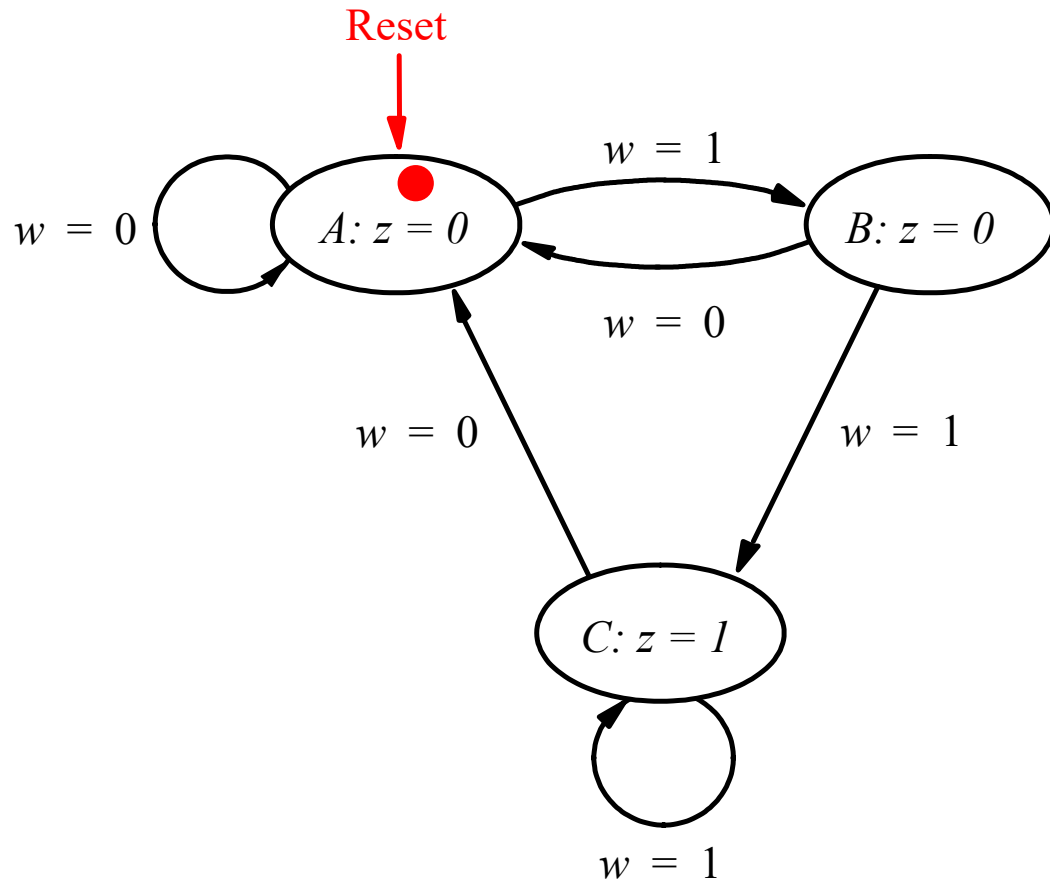


Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

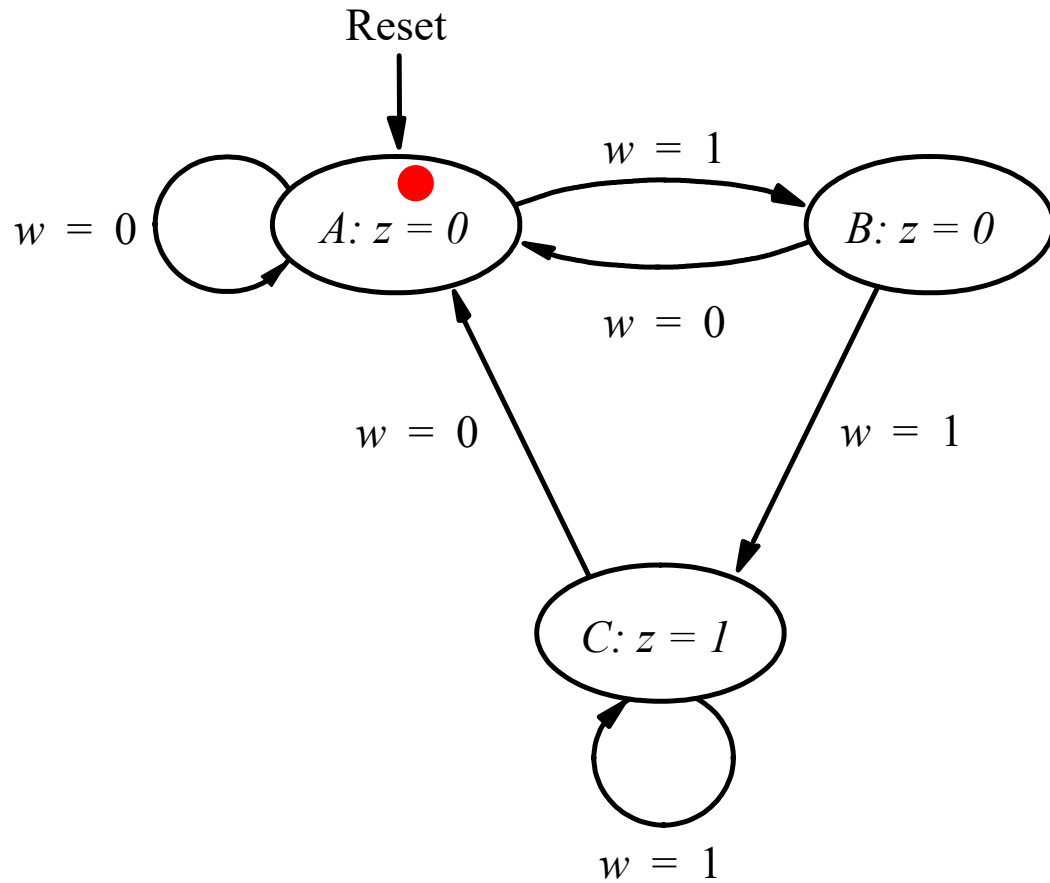
**Let's do a simulation**



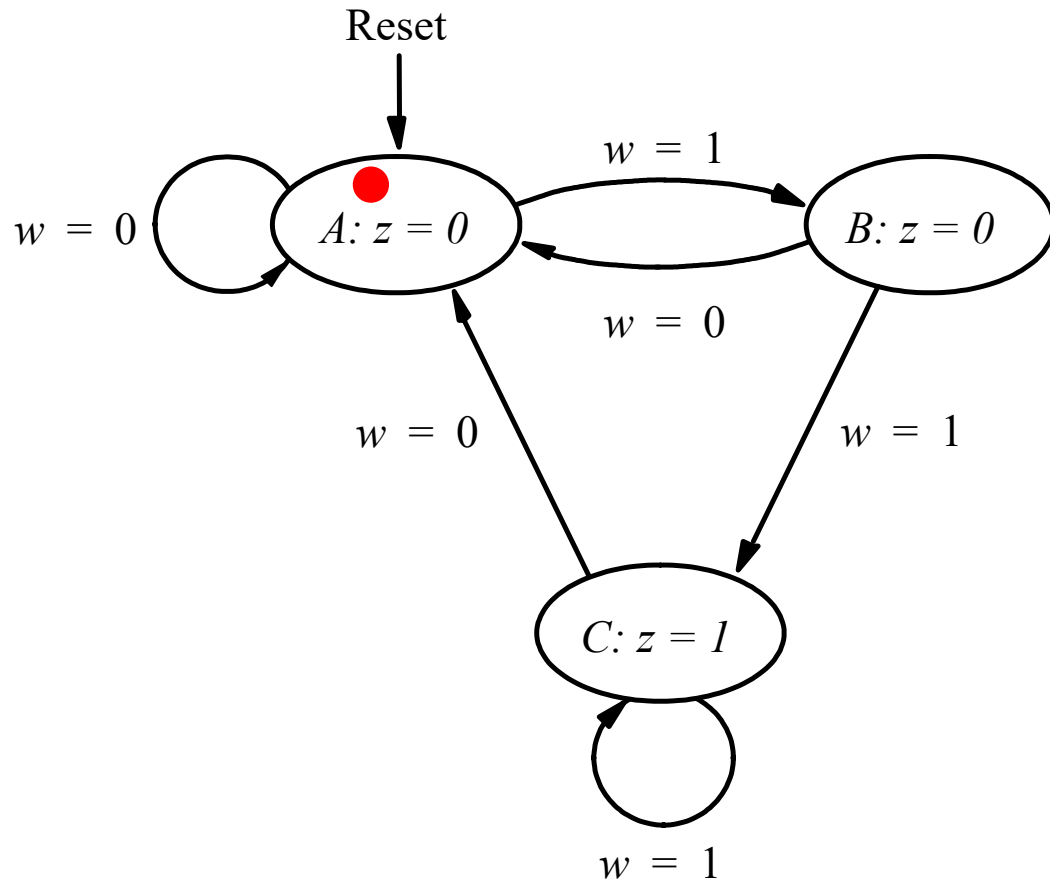
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



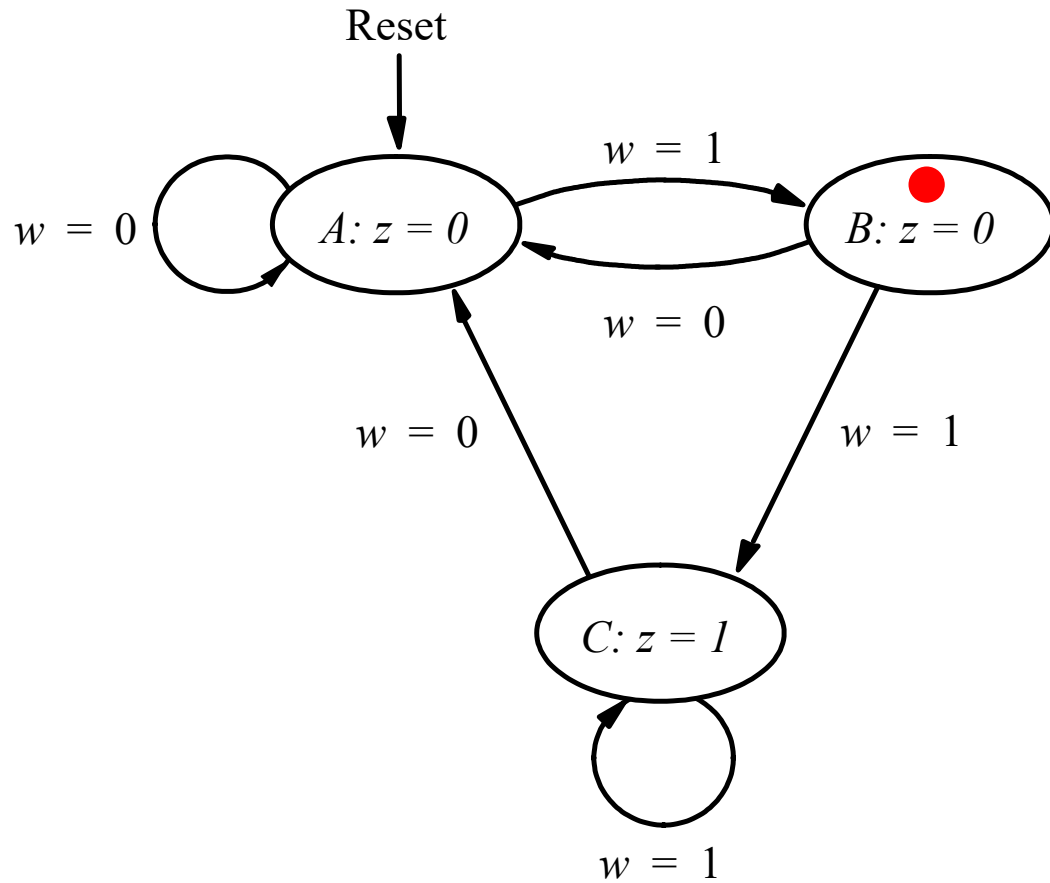
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



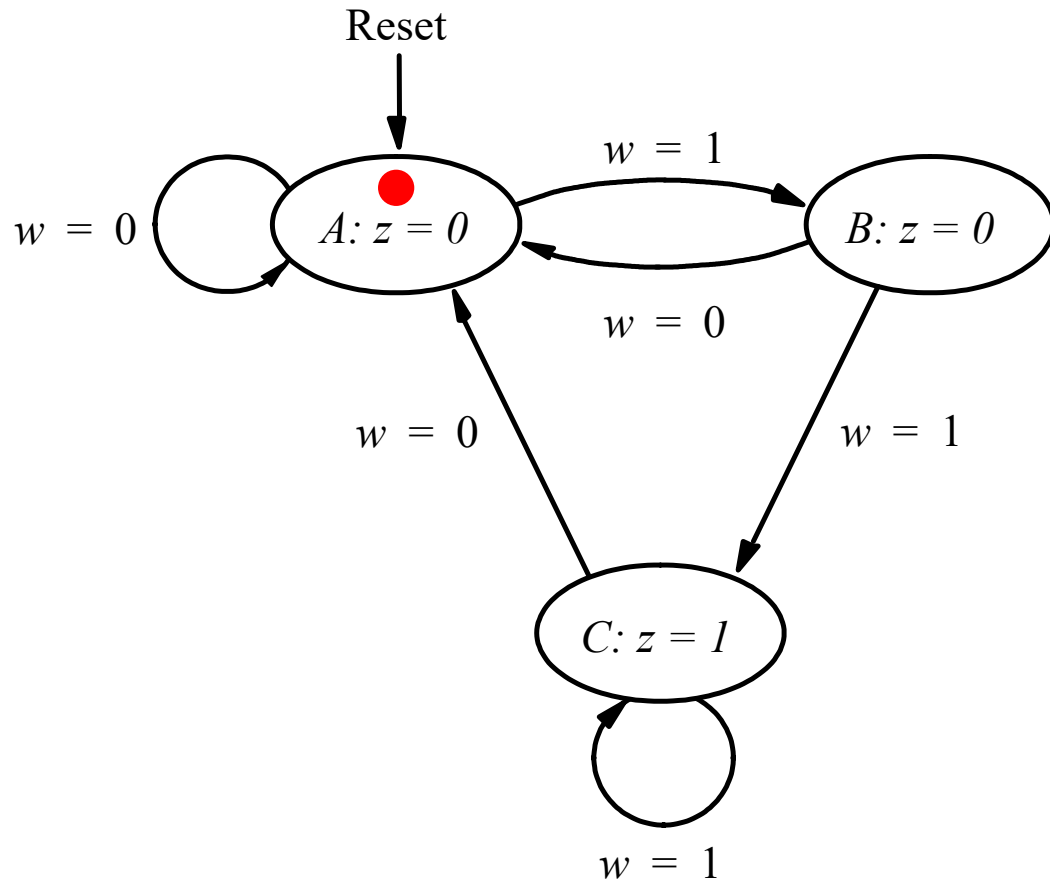
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

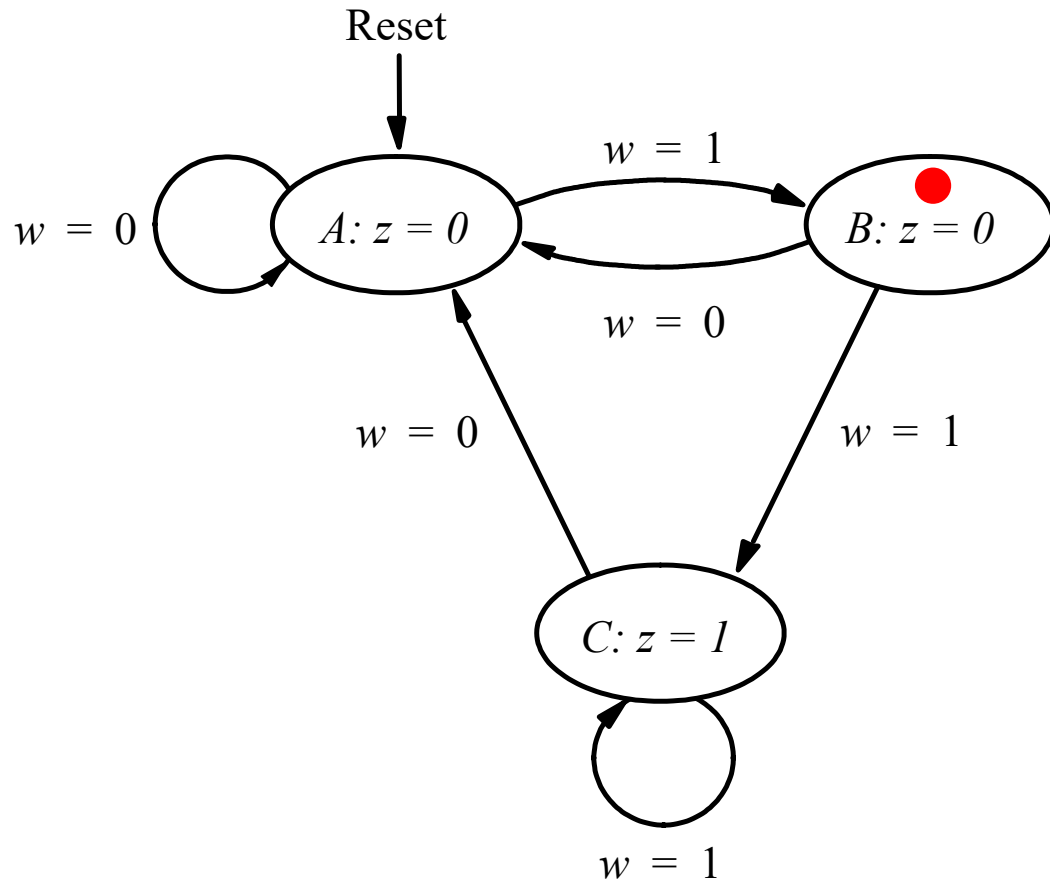


Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

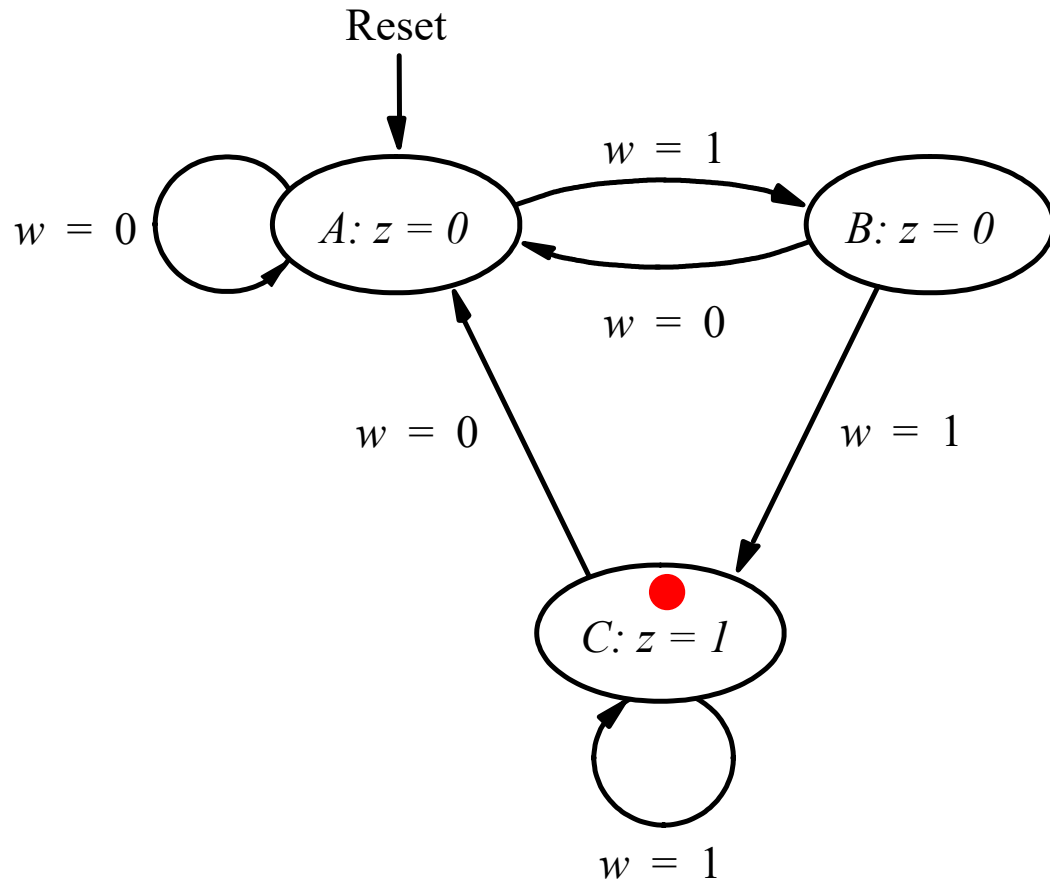


Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

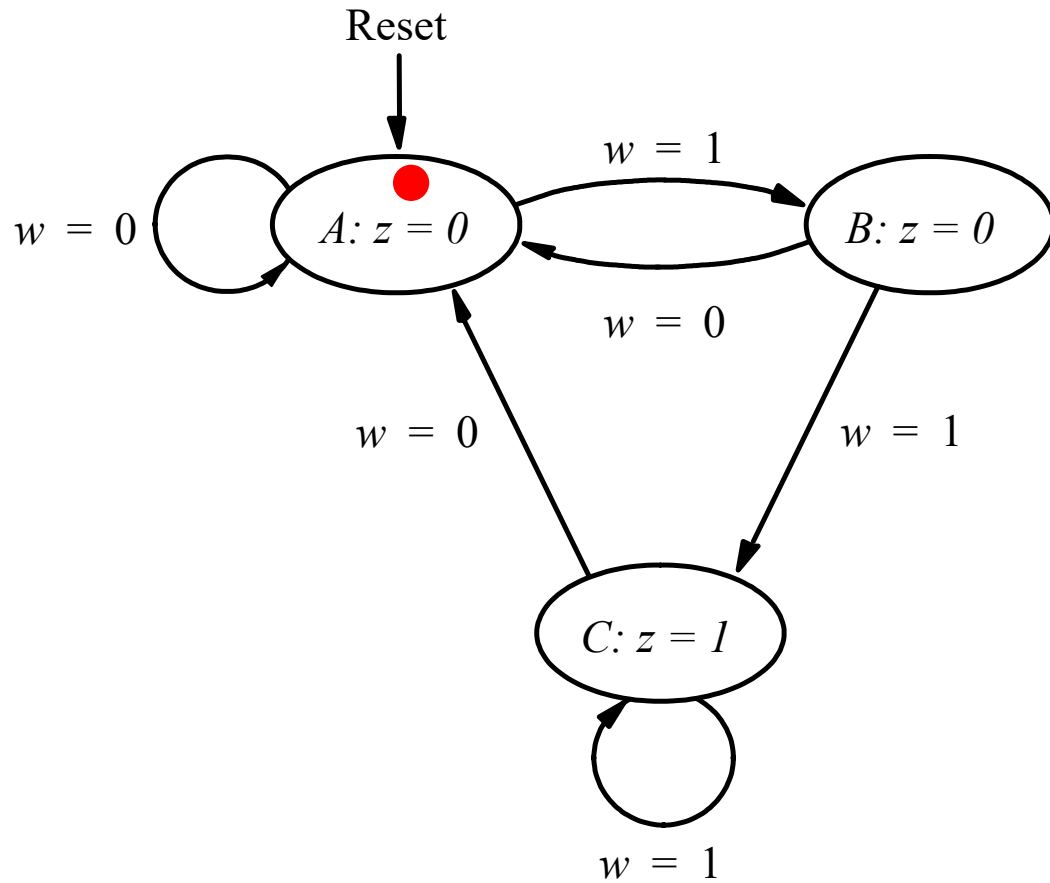




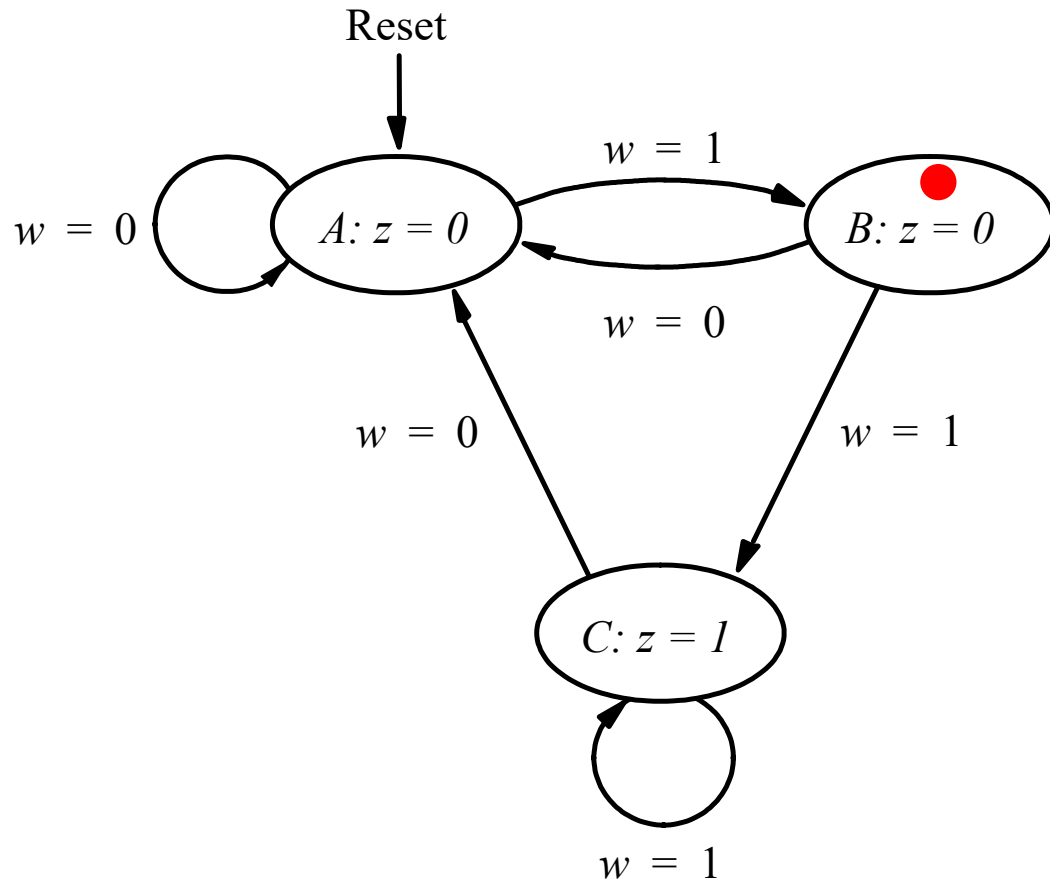
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



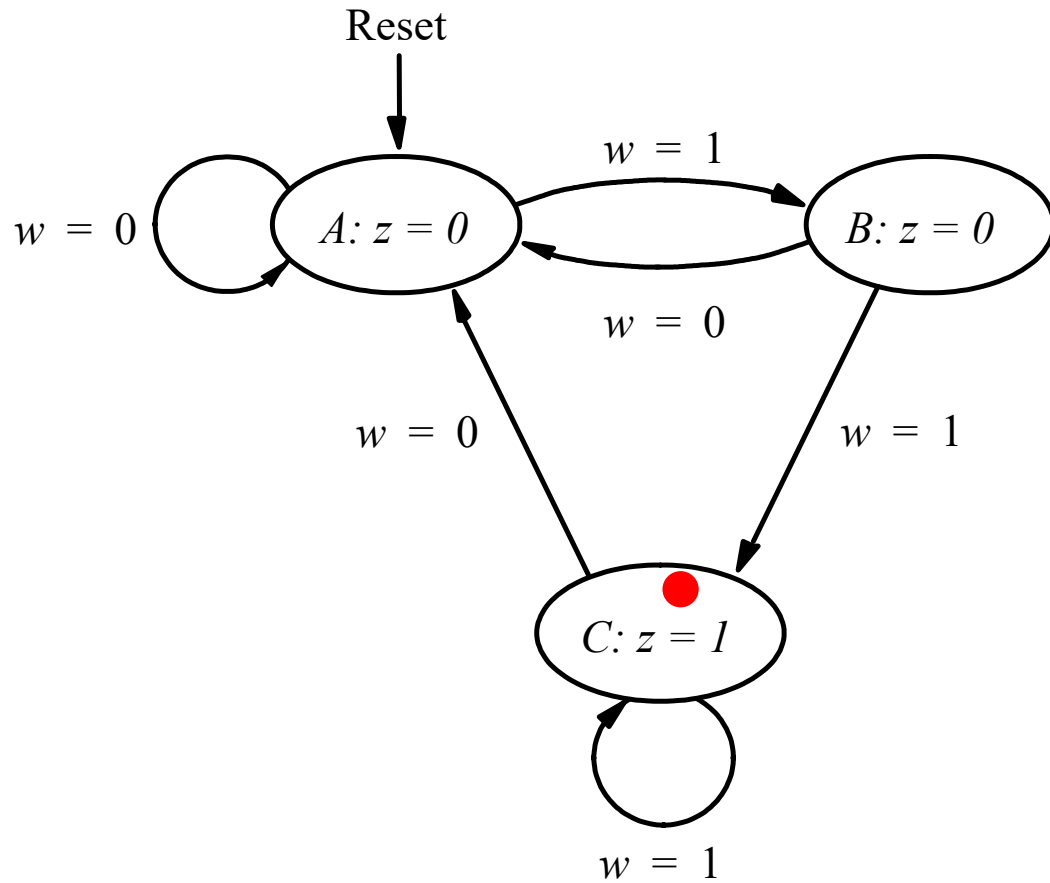
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



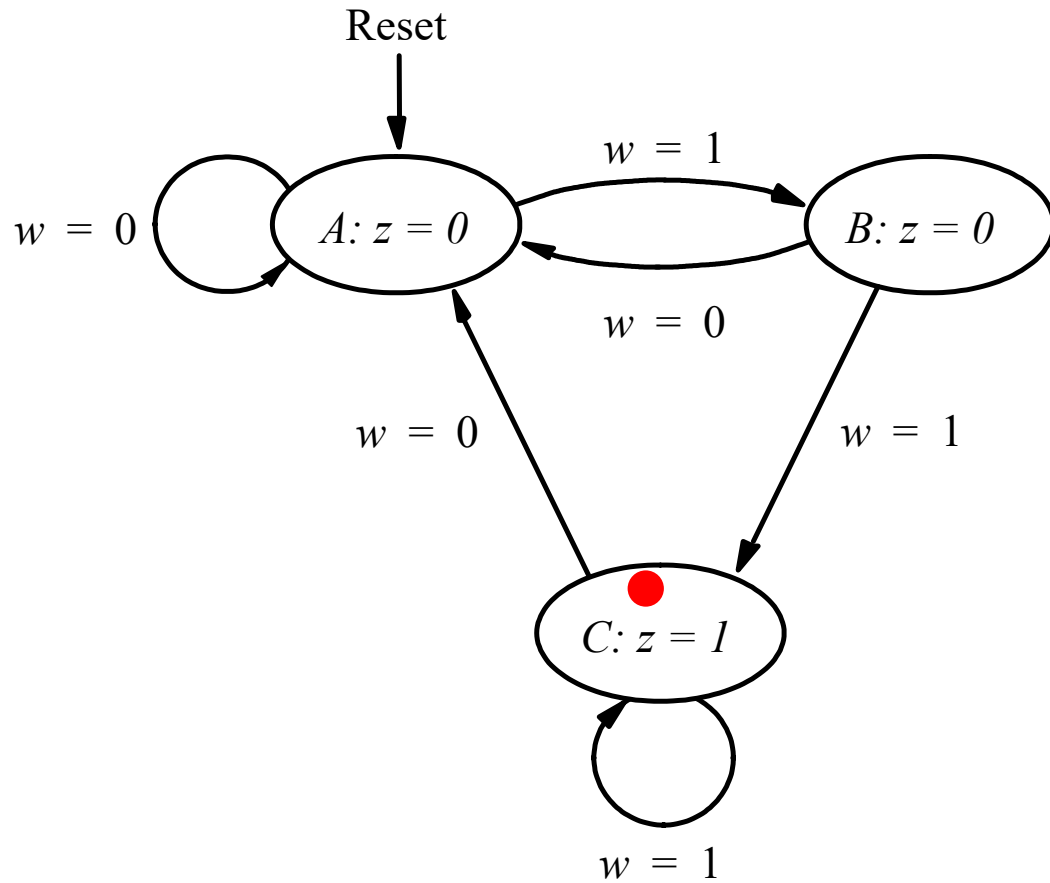
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



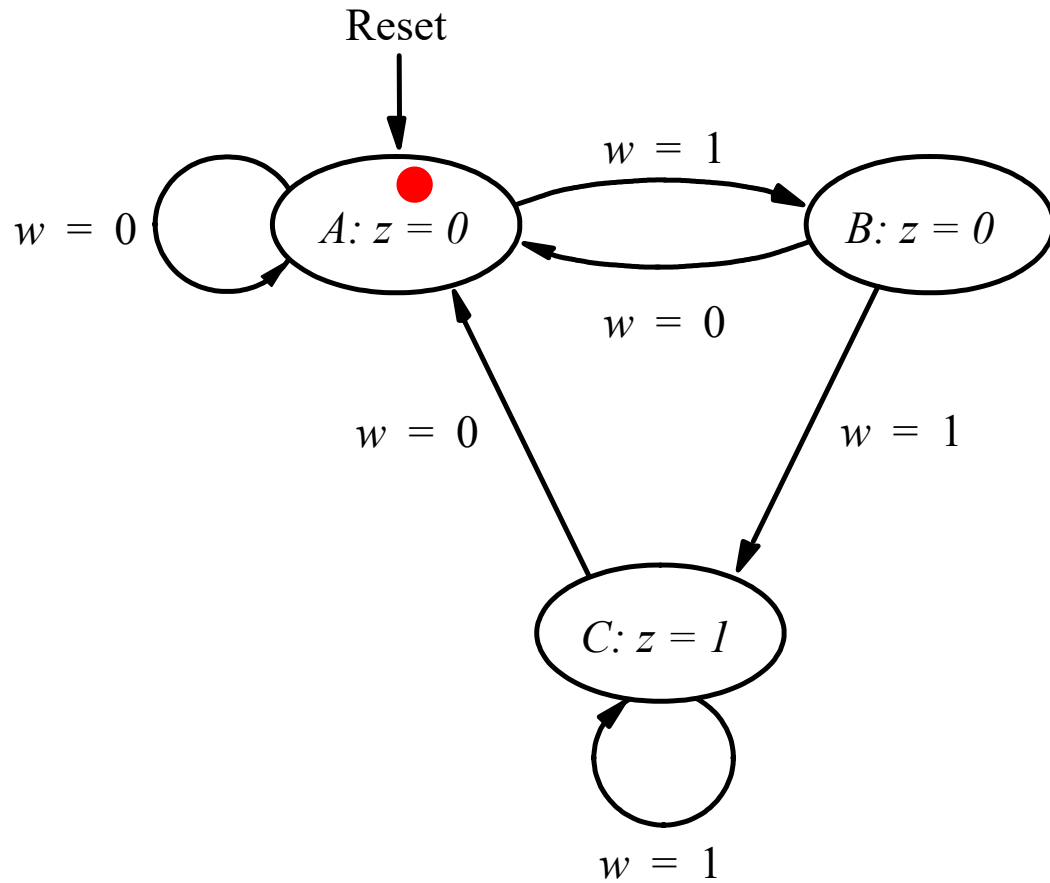
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



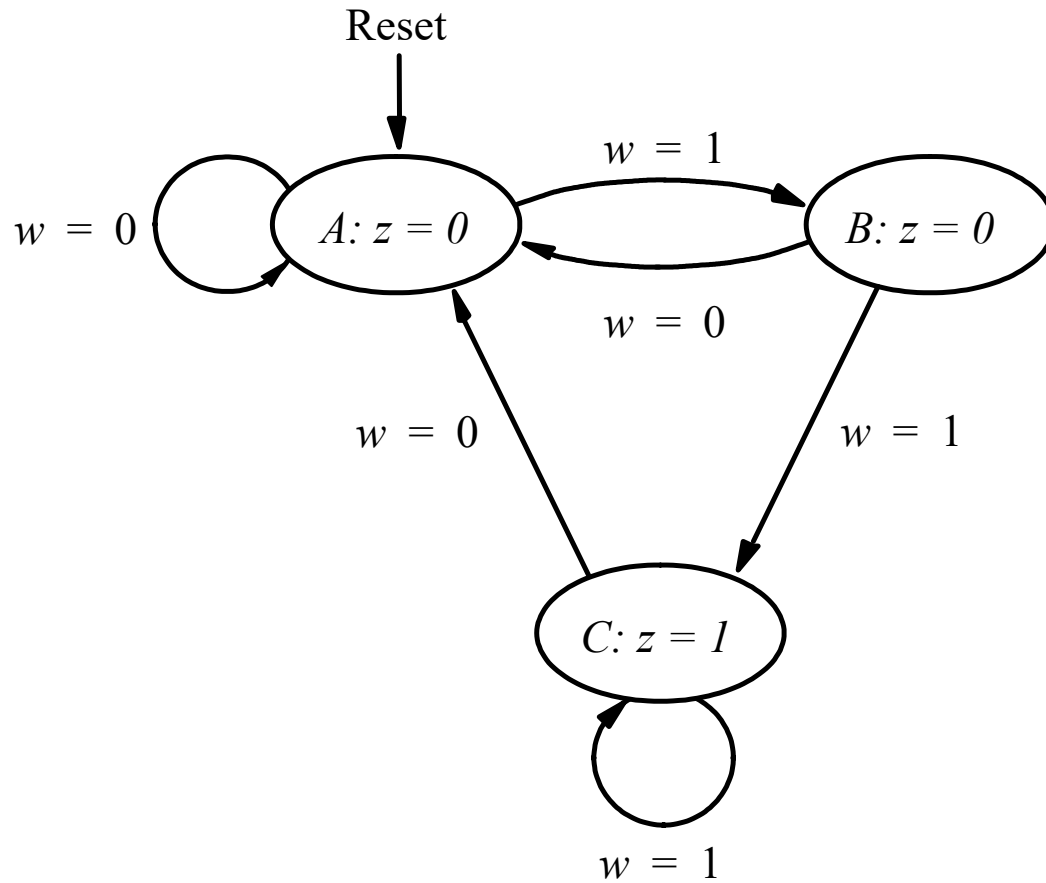
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



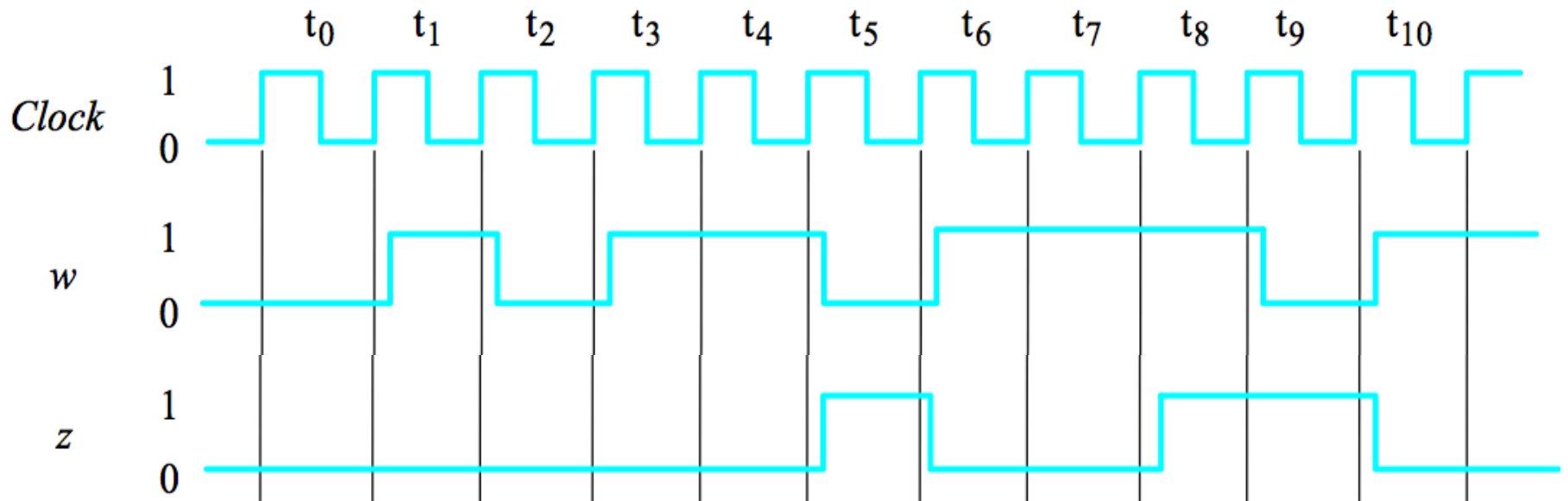




In general, we need to start tracing from the beginning to know which state the FSM is in. It may not be clear from a short sequence of outputs.

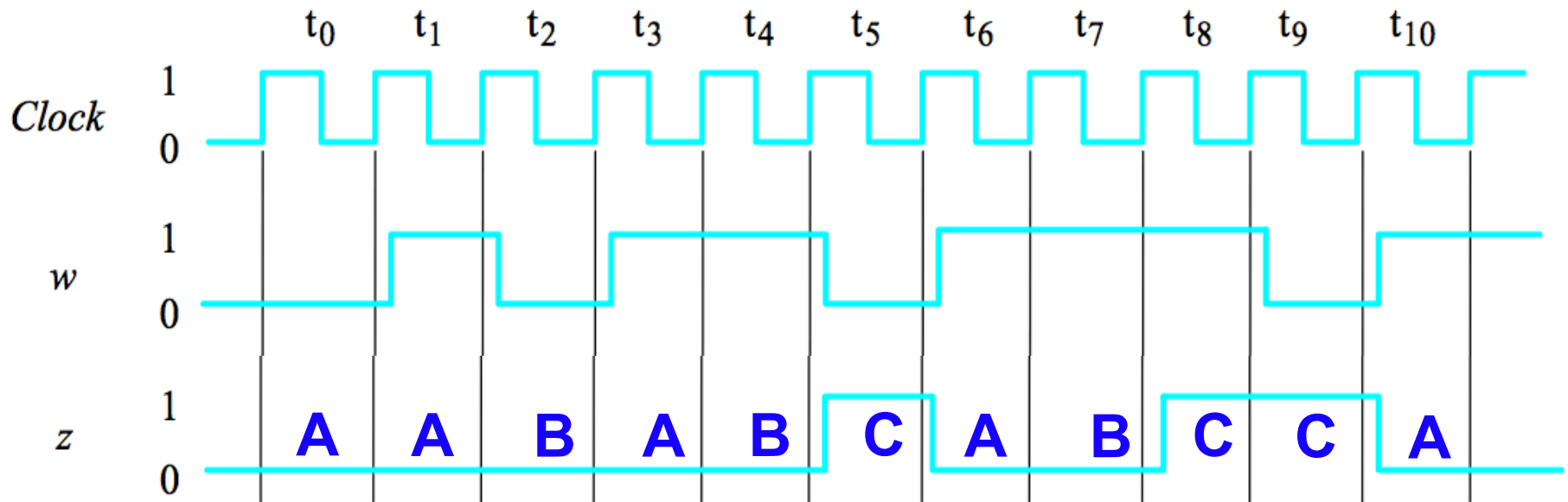
Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

# Inferring the States

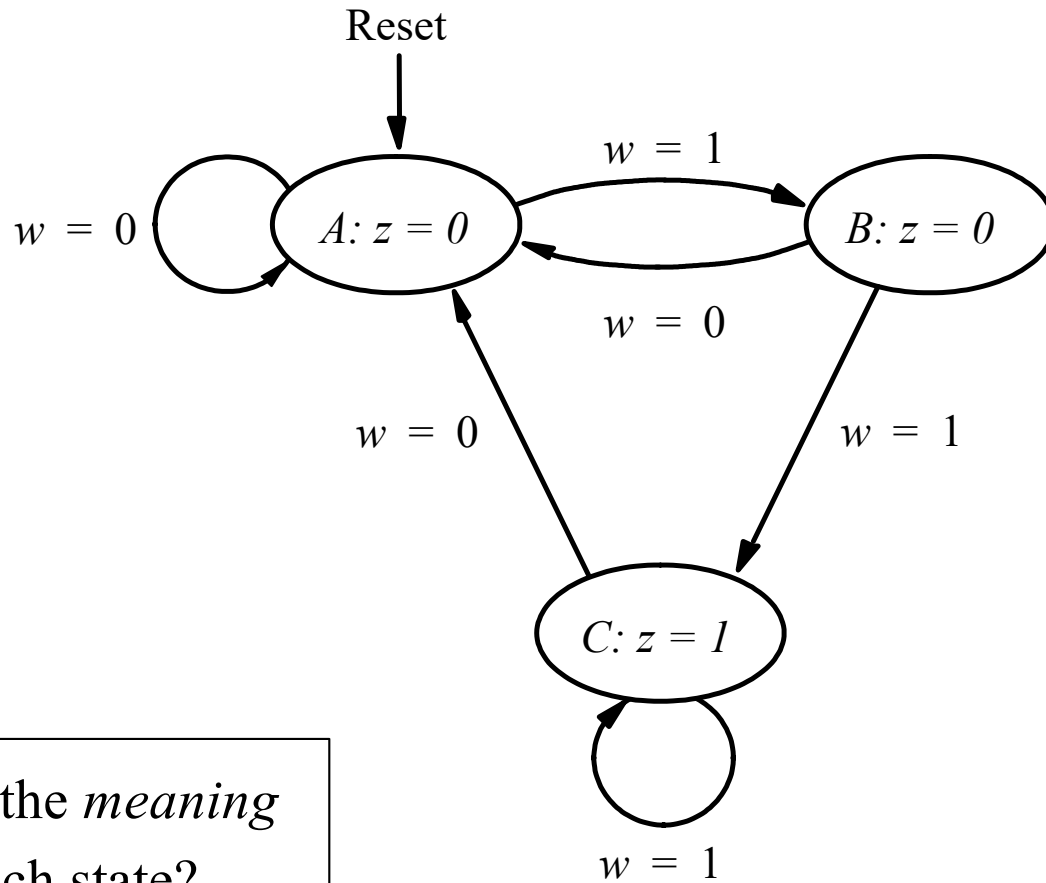


Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
<i>w</i> :	0	1	0	1	1	0	1	1	1	0	1
<i>z</i> :	0	0	0	0	0	1	0	0	1	1	0

# Inferring the States



Clockcycle:	t <sub>0</sub>	t <sub>1</sub>	t <sub>2</sub>	t <sub>3</sub>	t <sub>4</sub>	t <sub>5</sub>	t <sub>6</sub>	t <sub>7</sub>	t <sub>8</sub>	t <sub>9</sub>	t <sub>10</sub>
w:	0	1	0	1	1	0	1	1	1	0	1
z:	0	0	0	0	0	1	0	0	1	1	0



What is the *meaning*  
of each state?

Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

# What is a State?

It is not really a memory of every past input  
(We might run out of space to remember it all!)

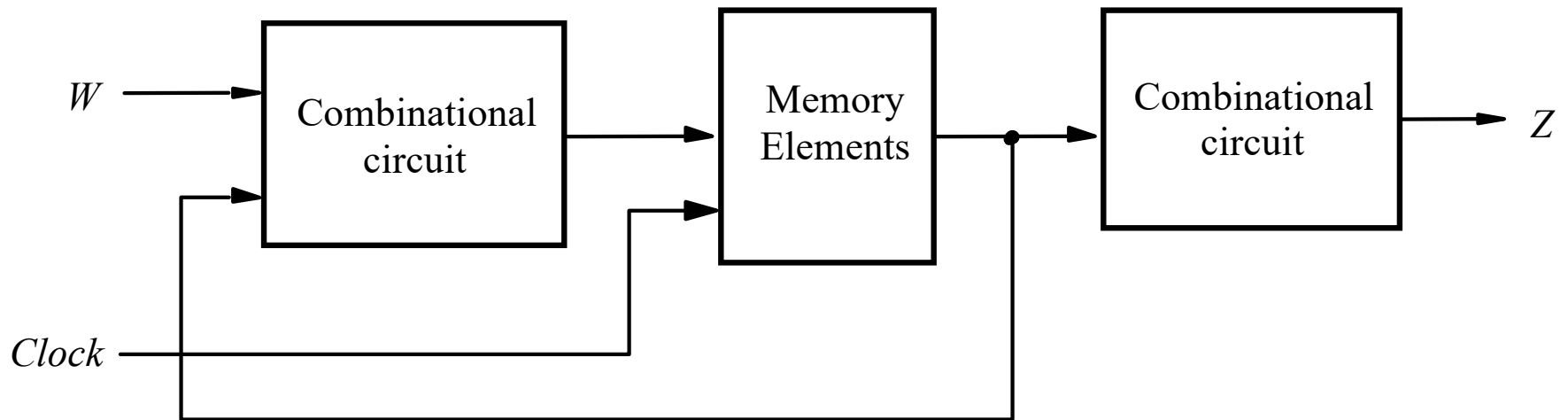
Rather, it is a characterization or snapshot of  
the pattern of inputs that have come before.

# Moore Machine Implementation

The state diagram is just an illustration to help us describe and reason about how the FSM will behave in each of its states.

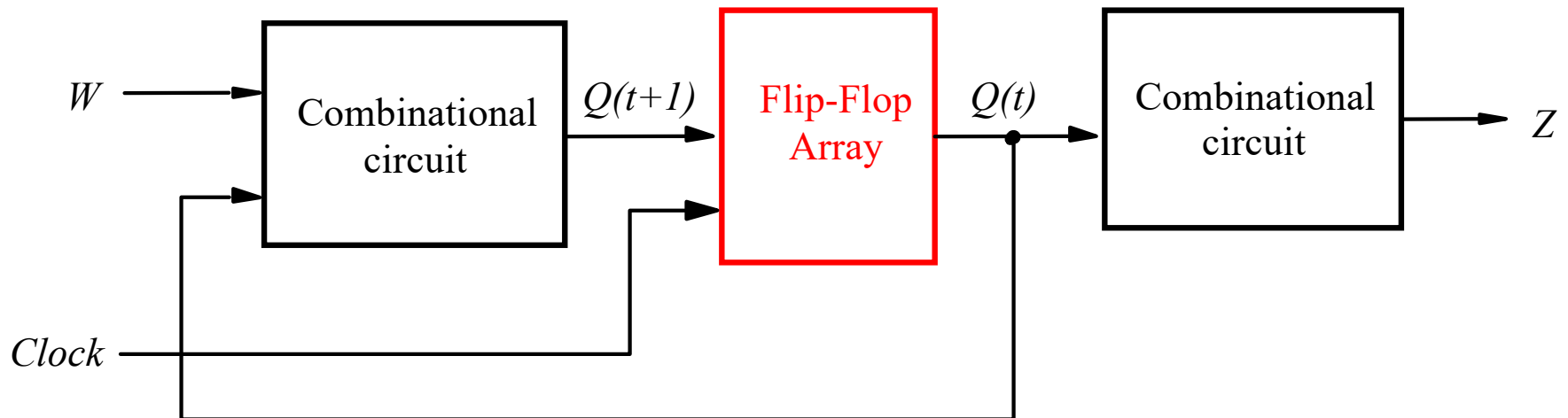
So, how do we turn it into a circuit?

# Moore Machine Implementation



Note: The  $W$  and  $Z$  lines need not be wires. They can be buses.

# State Storage

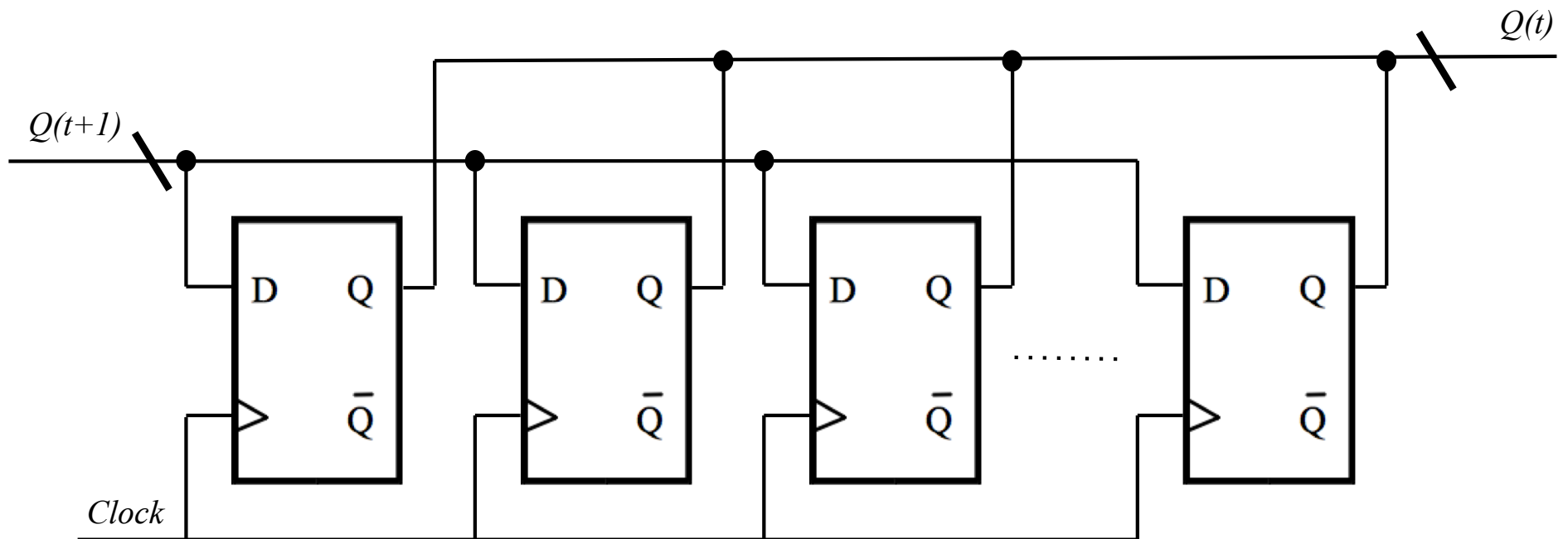


Any usable “memory” of the preceding input sequence is encoded in the flip-flop array.



# FSM States

The Flip-Flop array stores an encoding of the current state.



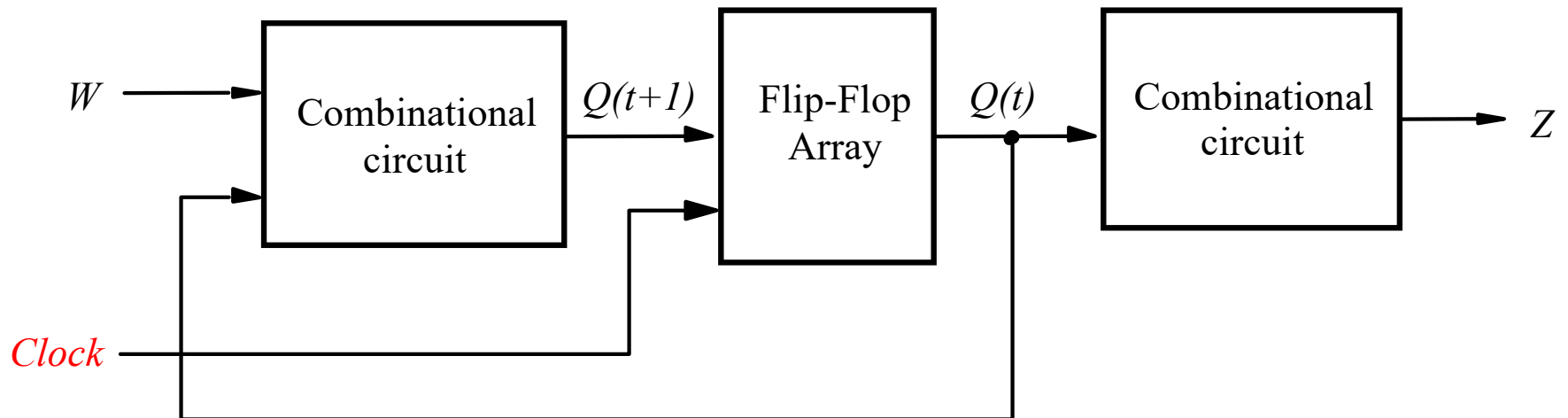
# State Encoding

Each of the states in our design is identified by a distinct code.

If we use 3 flip-flops, then the FSM can have up to  $2^3 = 8$  distinct states.

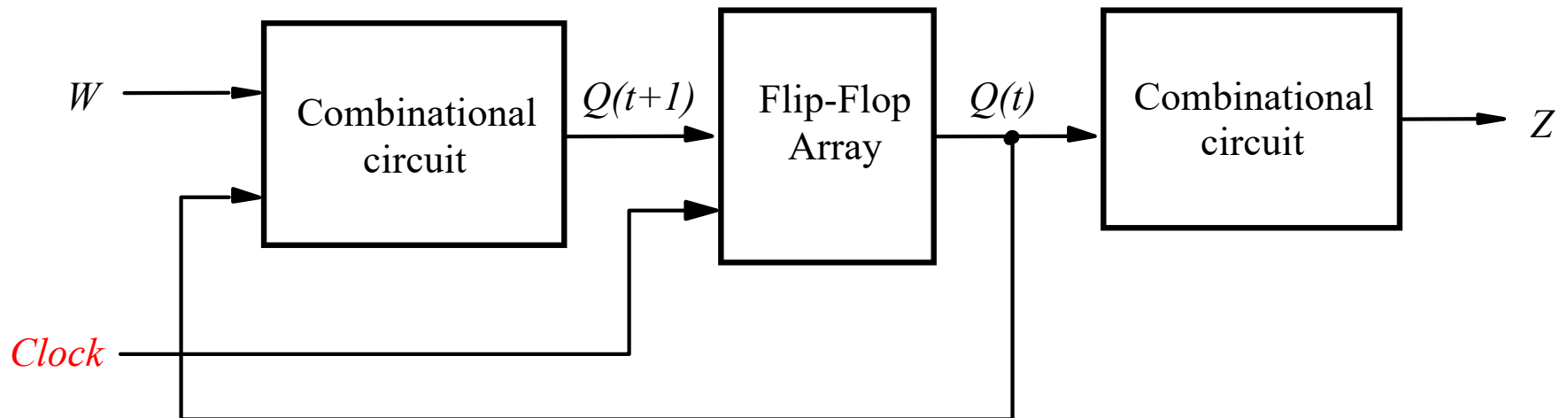
So, when the flip-flop array contains the code *011*, we say that the machine is in state *011*.

# Synchronous Design



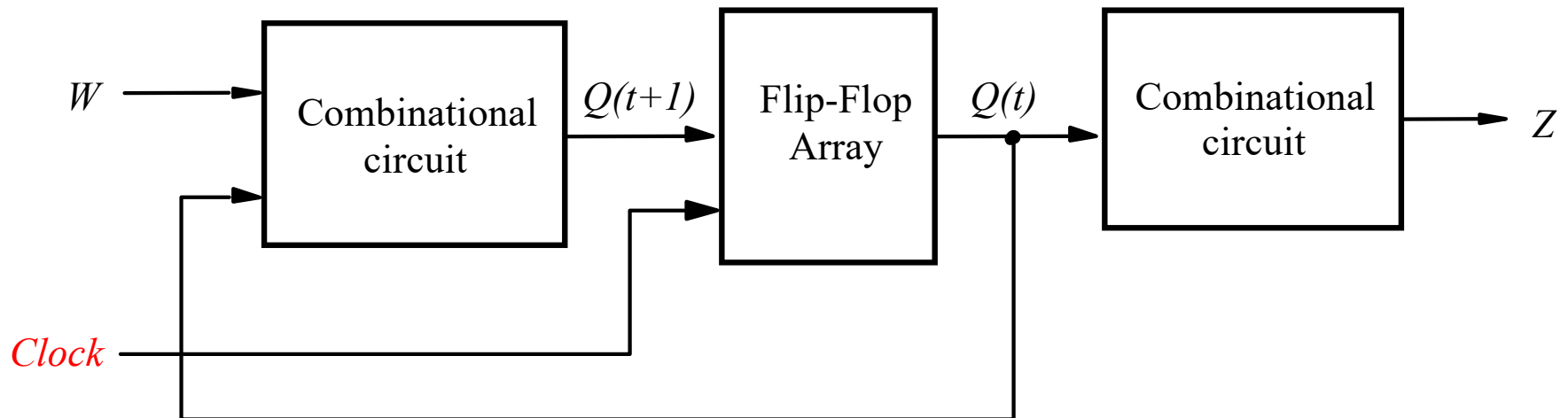
*Every active clock edge causes a state transition.*

# Synchronous Design



We expect the input signals to be stable before the *active clock edge* occurs.

# Synchronous Design



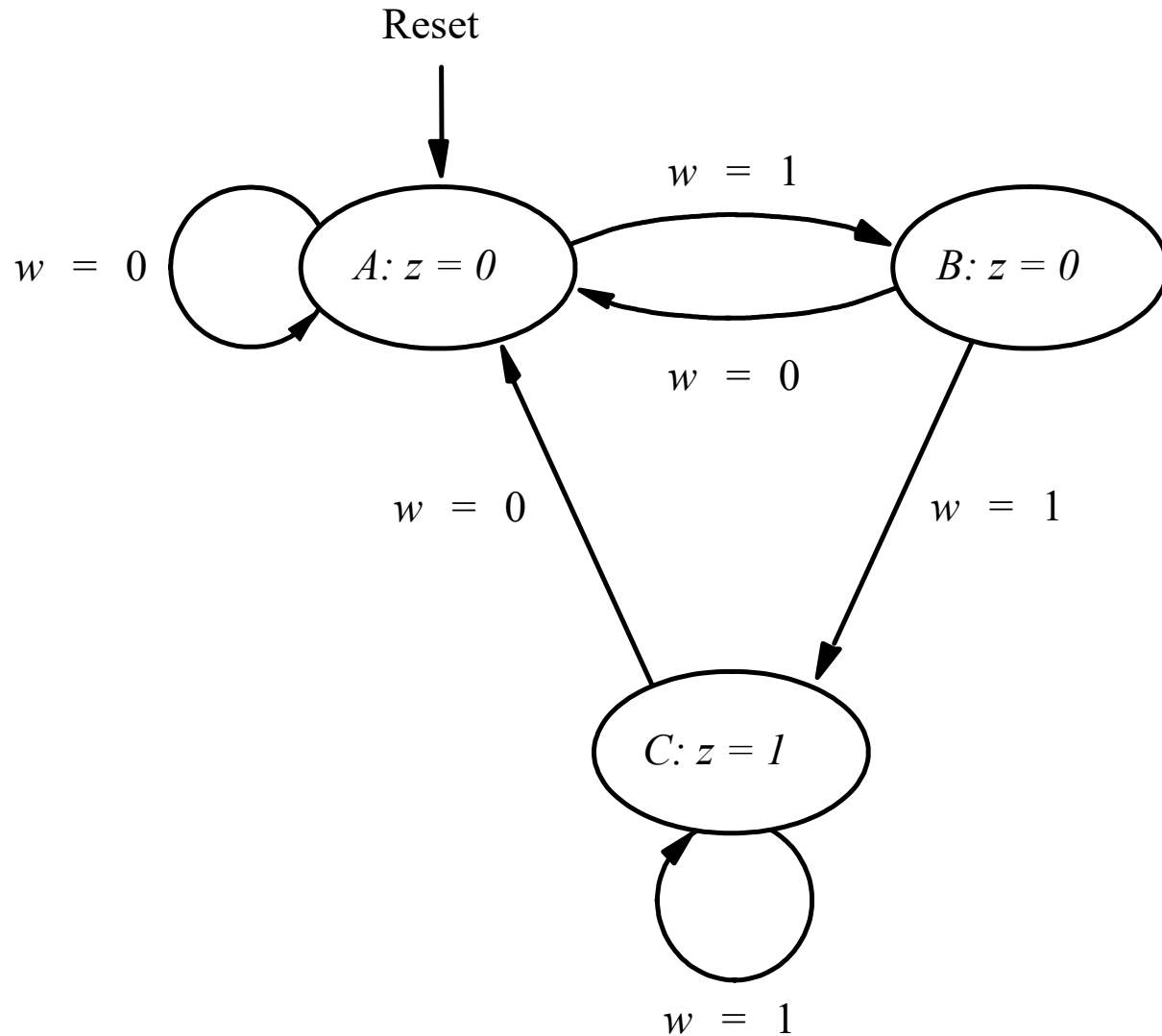
There is a whole other class of sequential circuits that are asynchronous, but we will not study them in this course.

# Sequential Circuits: Key Ideas

The current output depends on something about the preceding sequence of inputs (and maybe the current output).

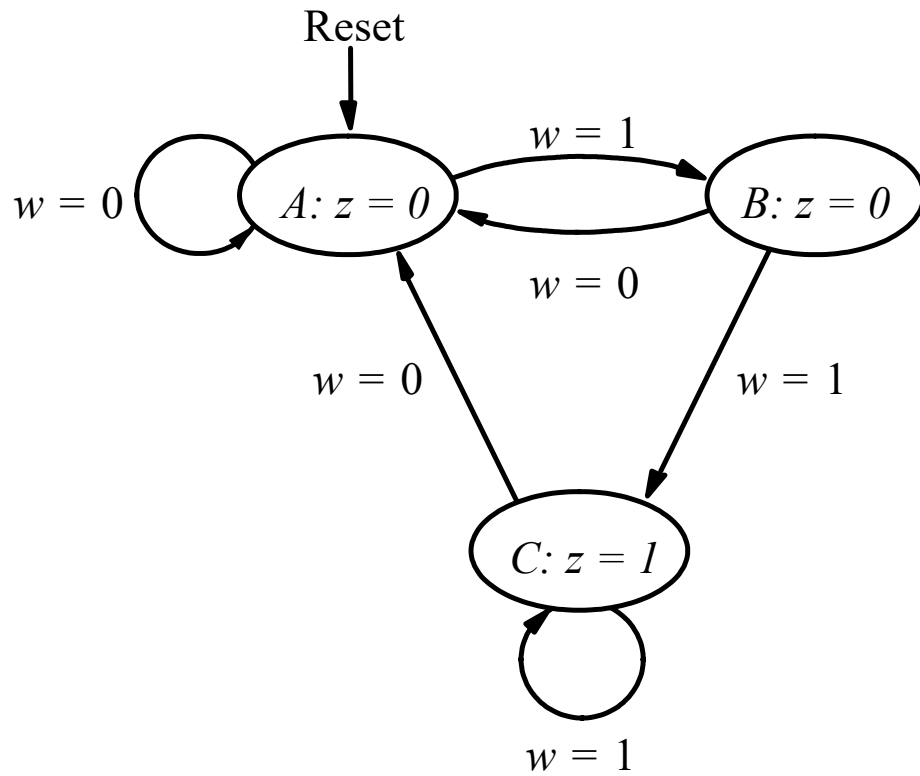
Using *memory elements* (i.e., flip-flops), we design the circuit to remember some *relevant* information about the prior inputs.

**Example**

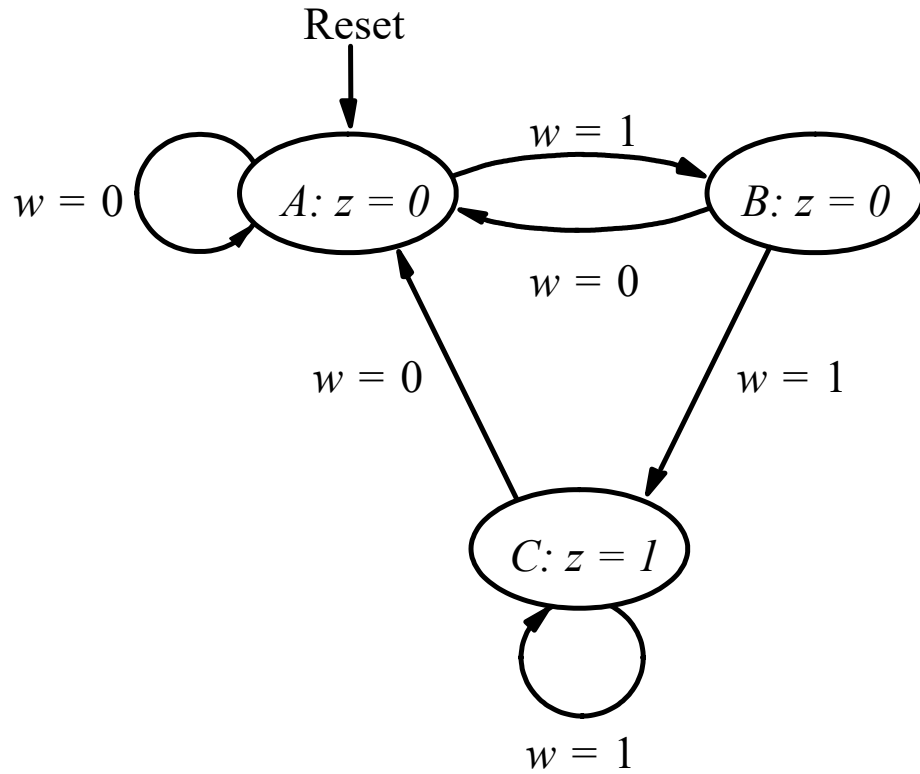


We need to find both the *next state logic* and the *output logic* implied by this machine.





Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A			
B			
C			



Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

[ Figure 6.4 from the textbook ]

# How to represent the States?

One way is to encode each state with a 2-bit binary number

A ~ 00

B ~ 01

C ~ 10

# How to represent the states?

One way is to encode each state with a 2-bit binary number

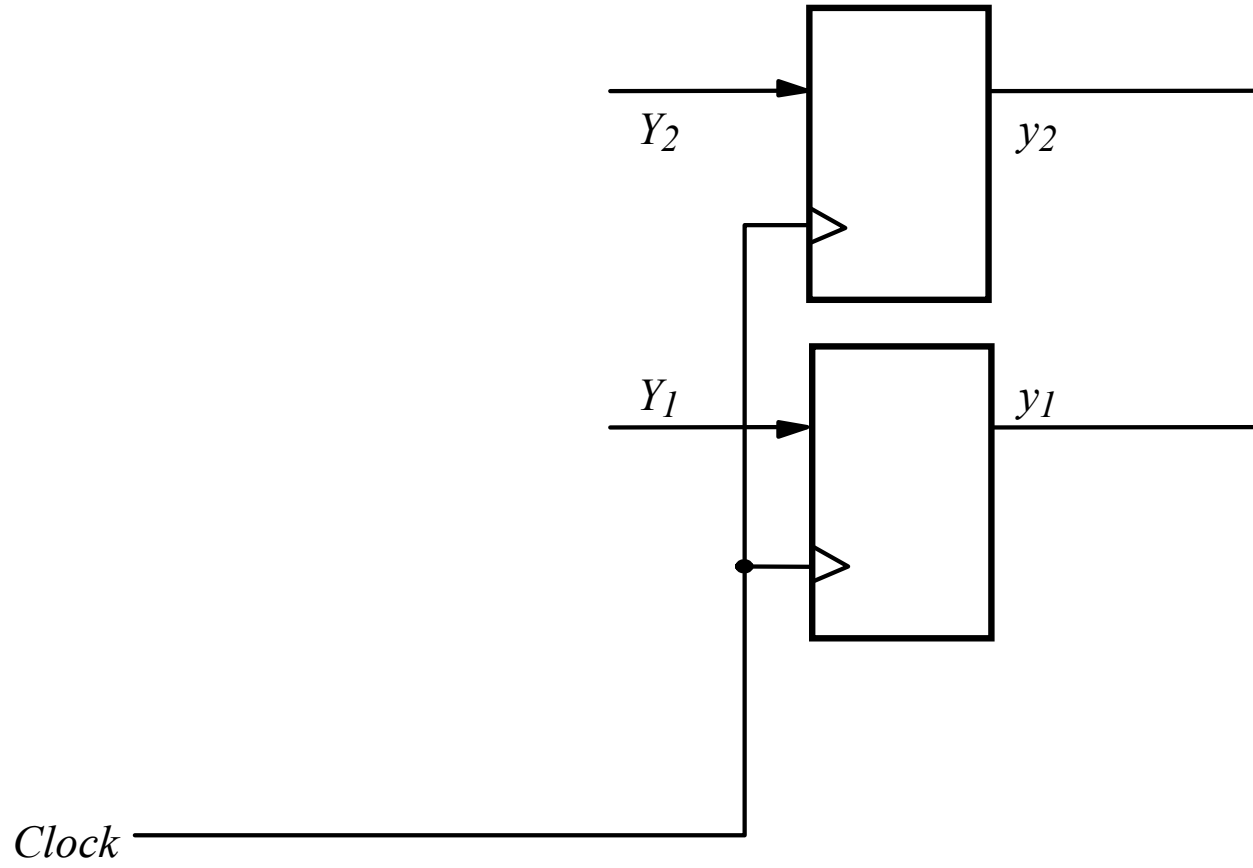
A ~ 00

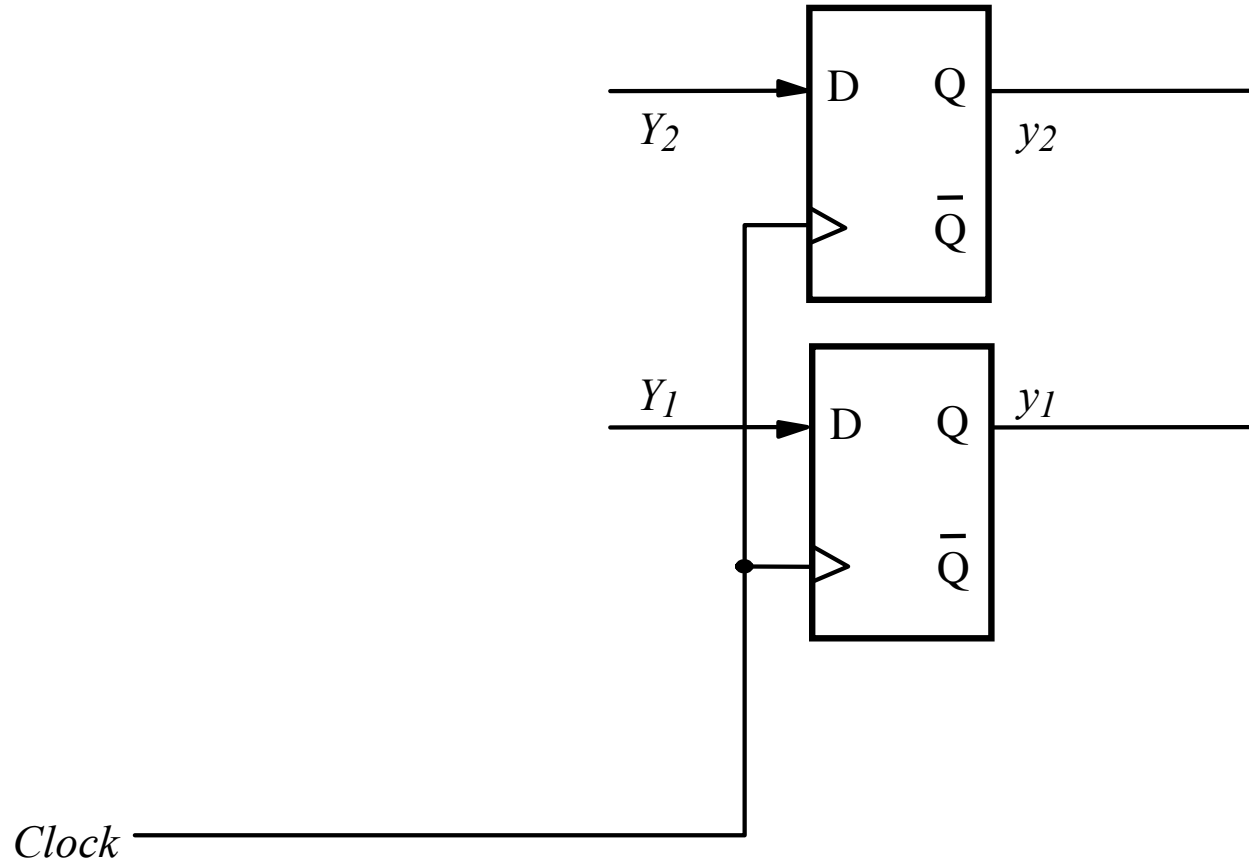
B ~ 01

C ~ 10

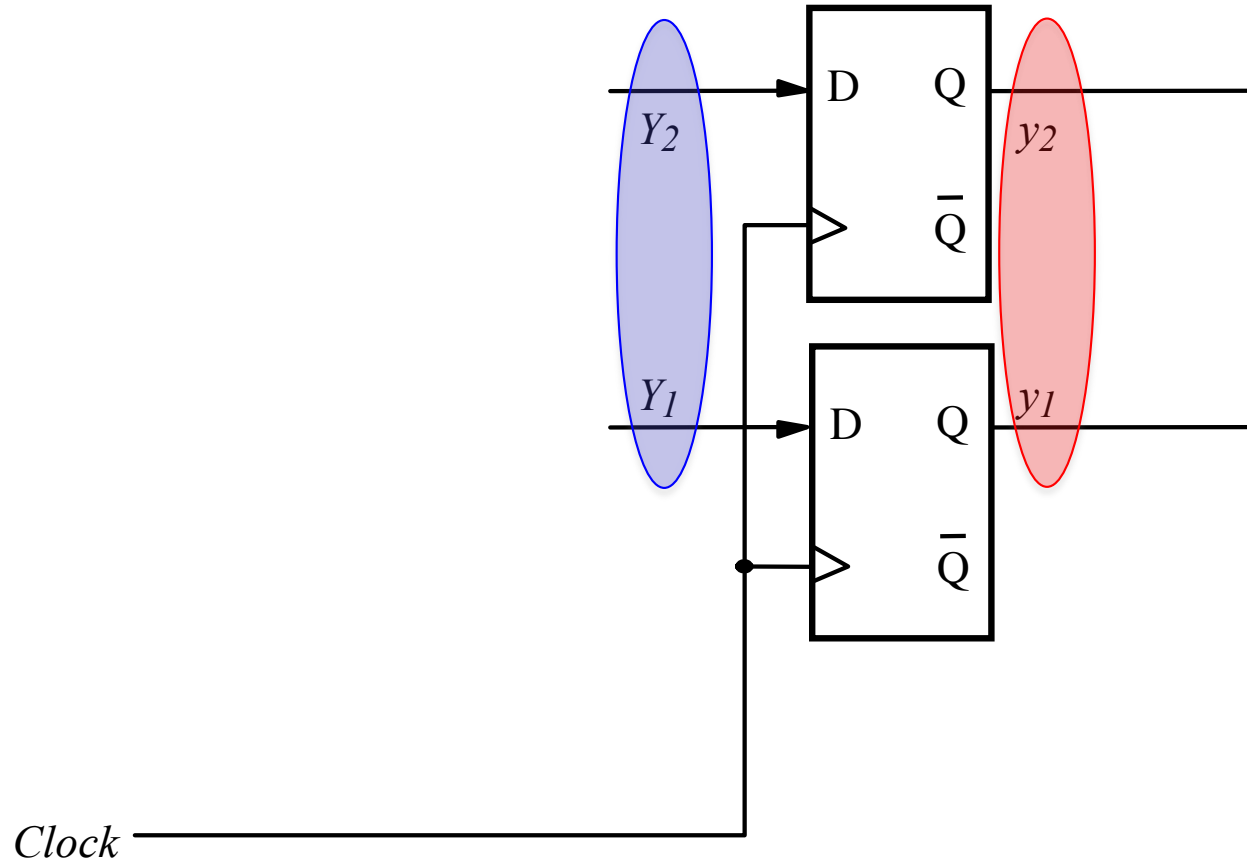
How many flip-flops do we need?

**Let's use **two** flip-flops  
to hold the machine's state**





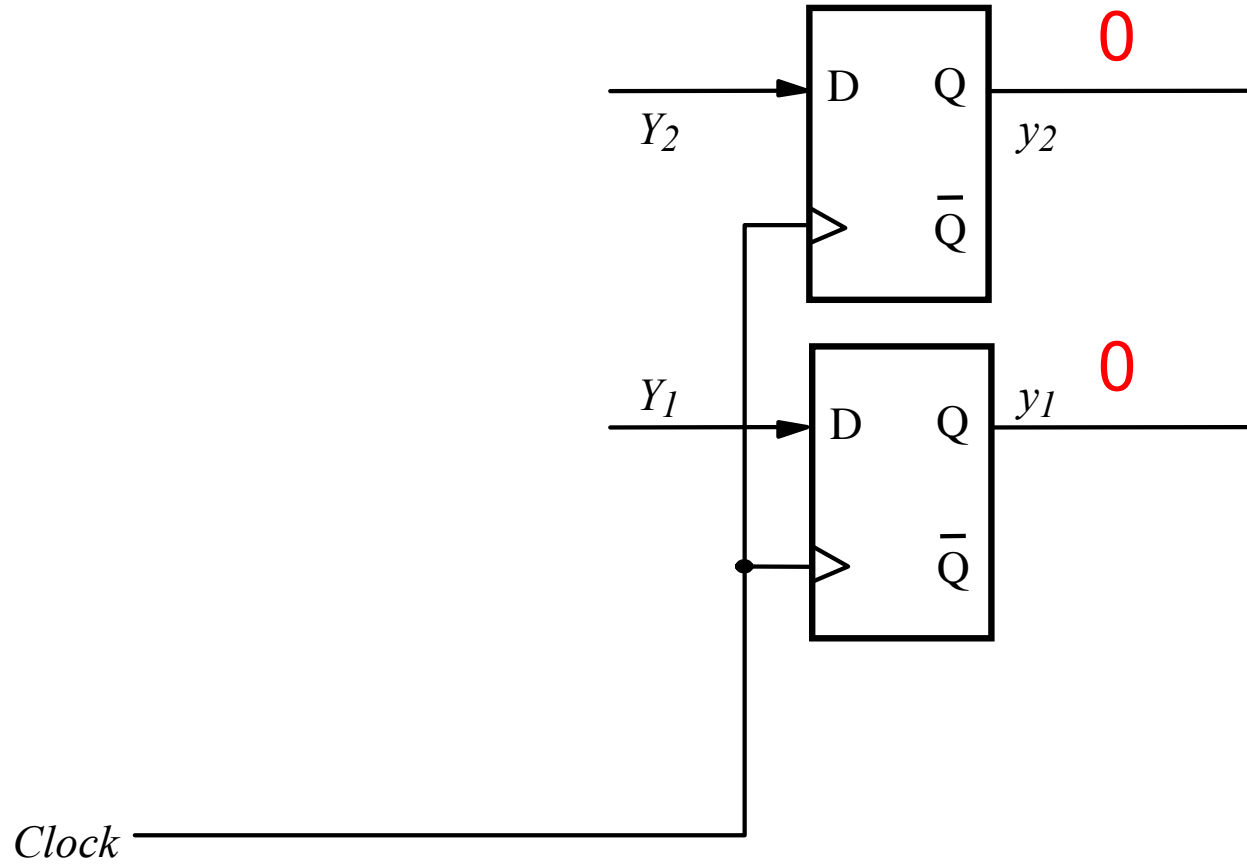
Let's pick D Flip-Flops.



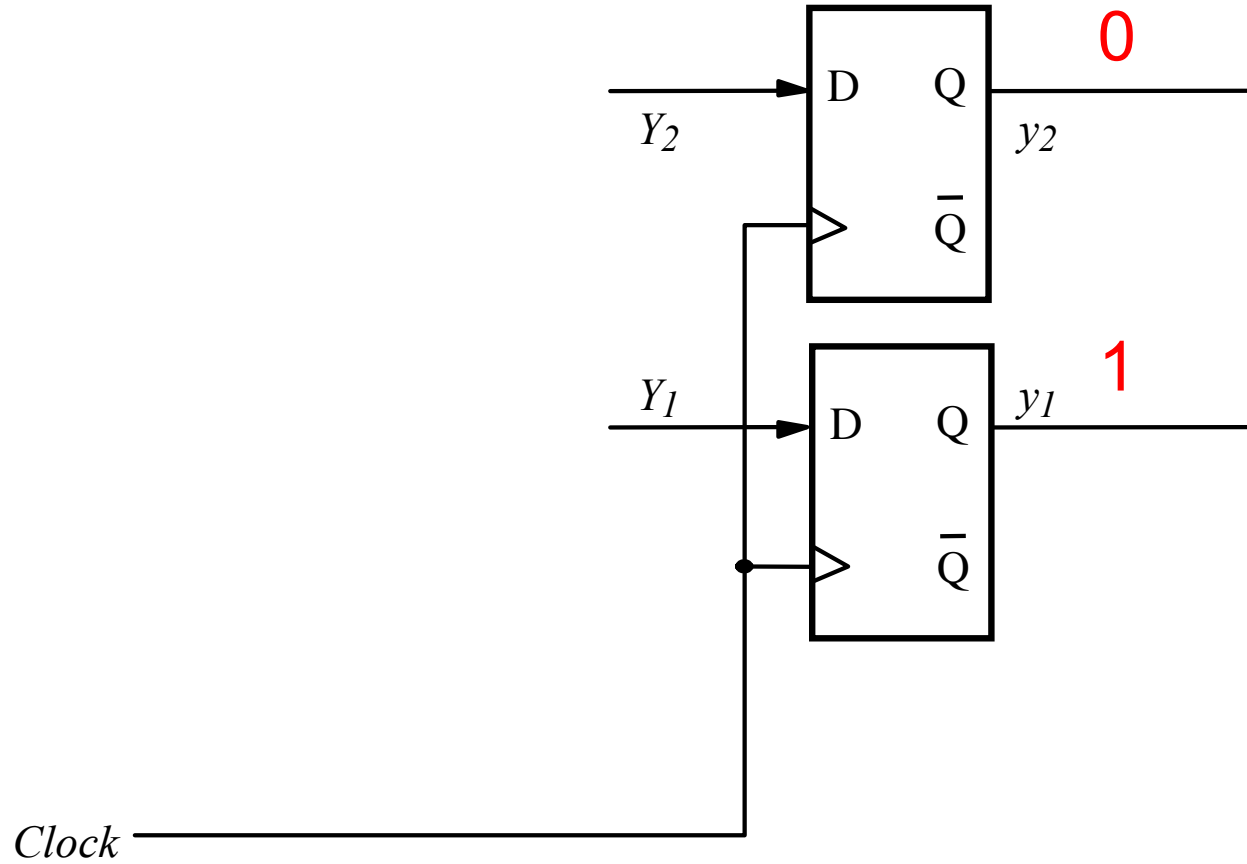
We will call  $y_1$  and  $y_2$  the *present state variables*.

We will call  $Y_1$  and  $Y_2$  the *next state variables*.

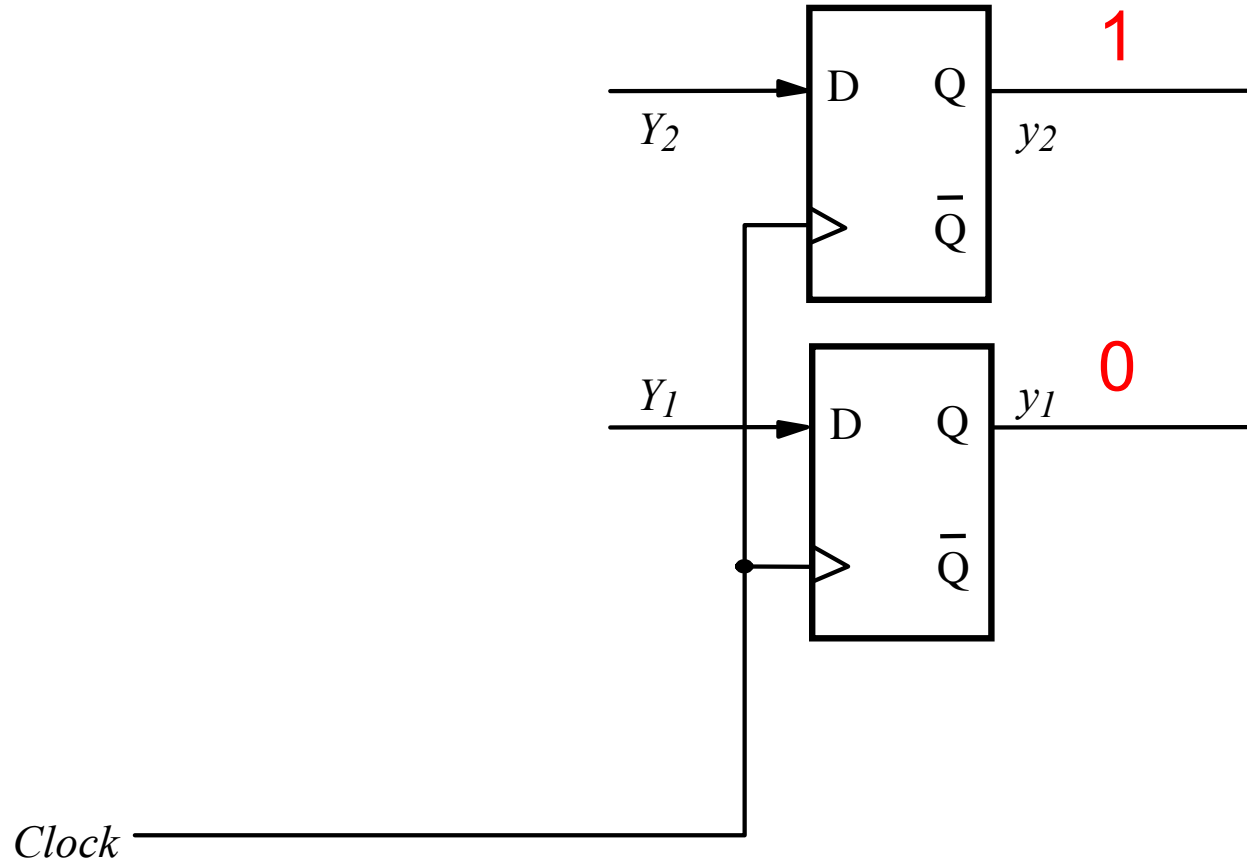




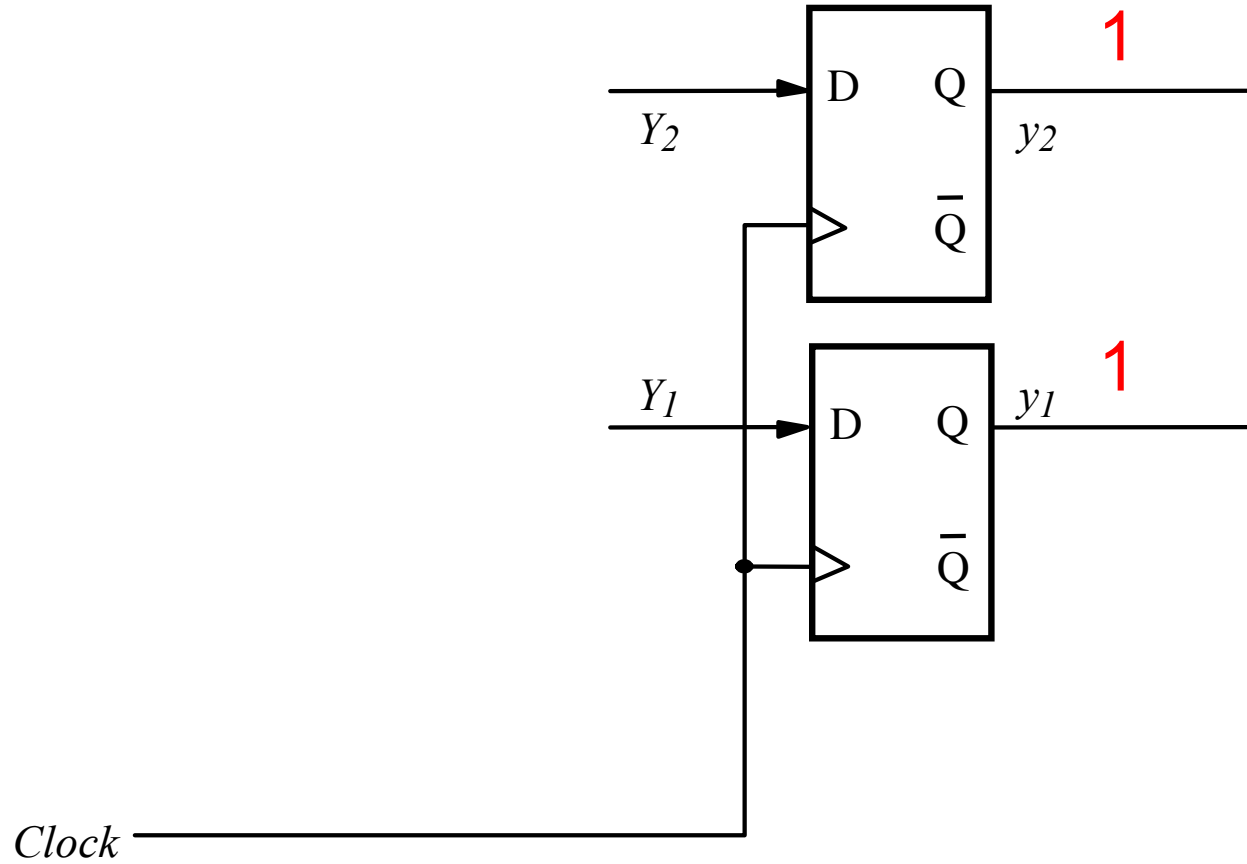
Two zeros on the output JOINTLY represent state A.



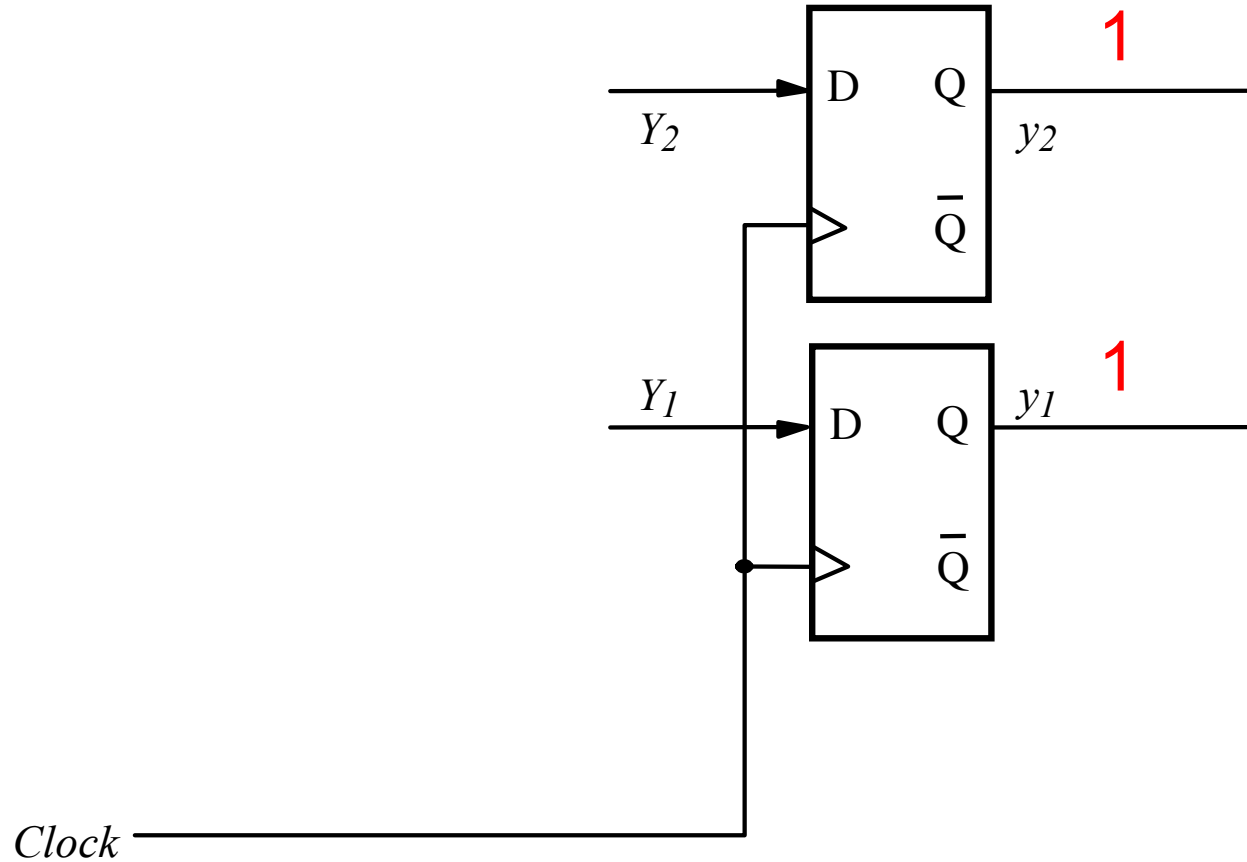
This flip-flop output pattern represents state B.



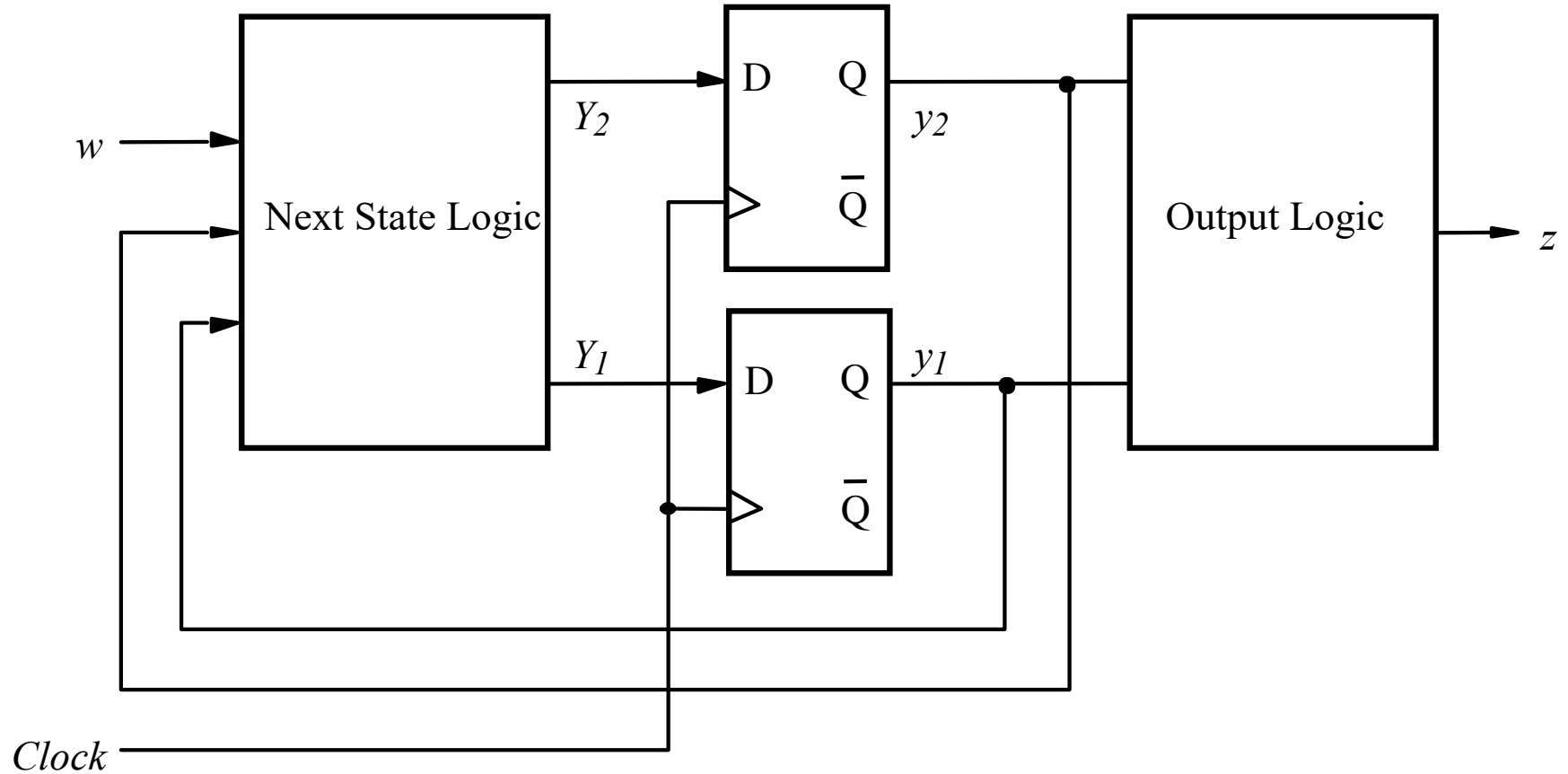
This flip-flop output pattern represents state C.



What does this flip-flop output pattern represent?

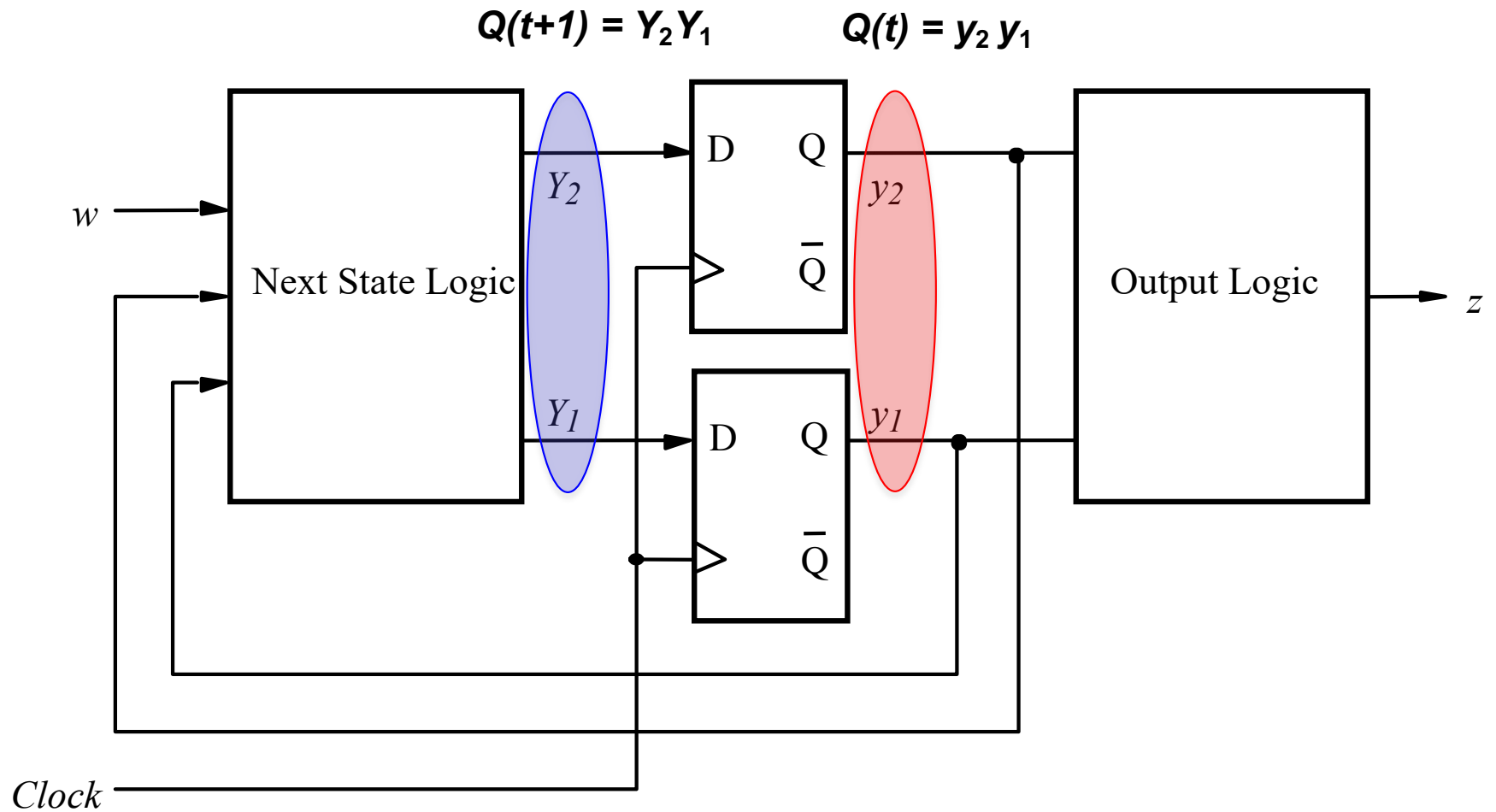


This would be state D, but we don't have one in this example. So this is an impossible state.



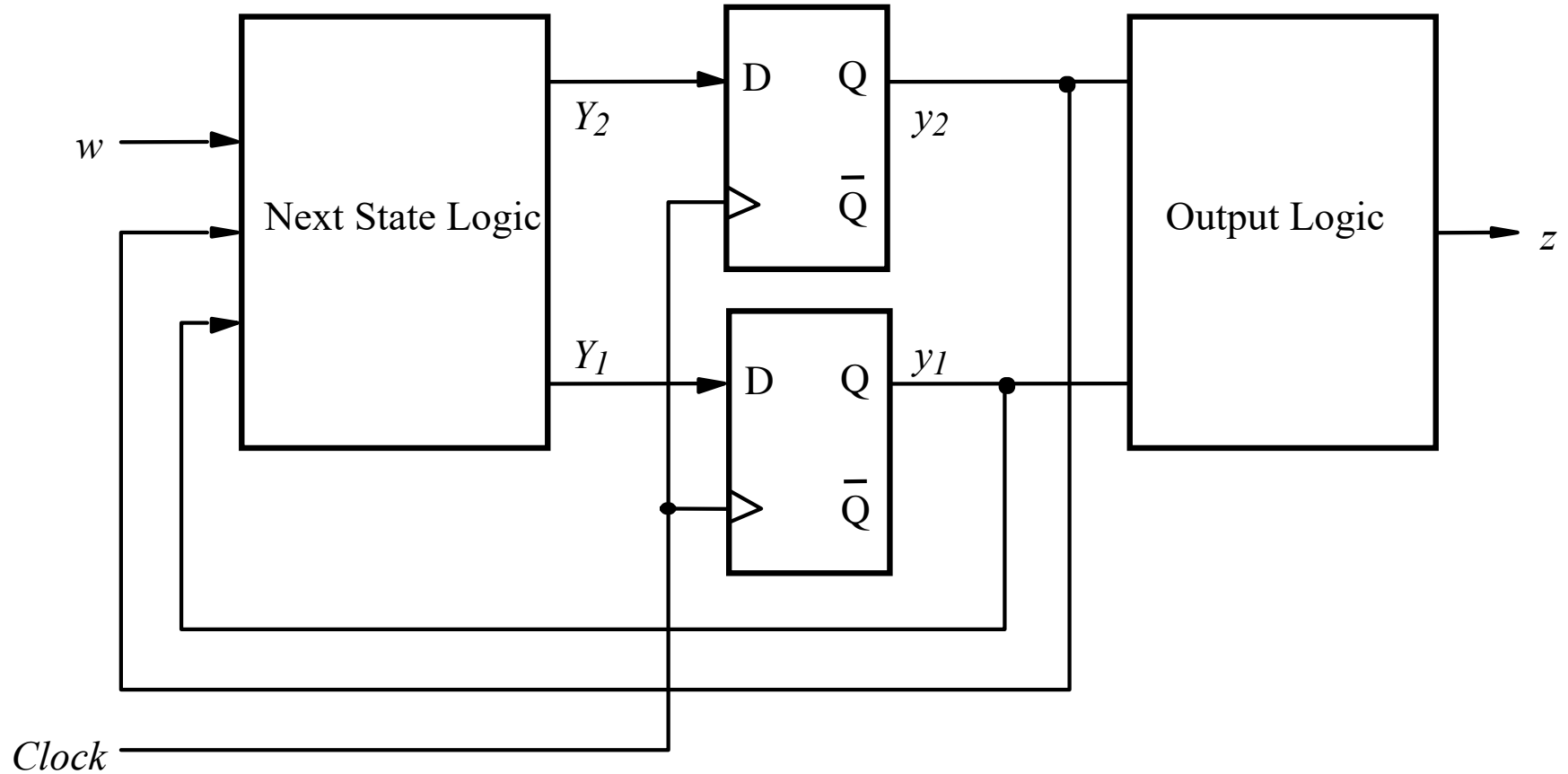
We will call  $y_1$  and  $y_2$  the *present state variables*.

We will call  $Y_1$  and  $Y_2$  the *next state variables*.



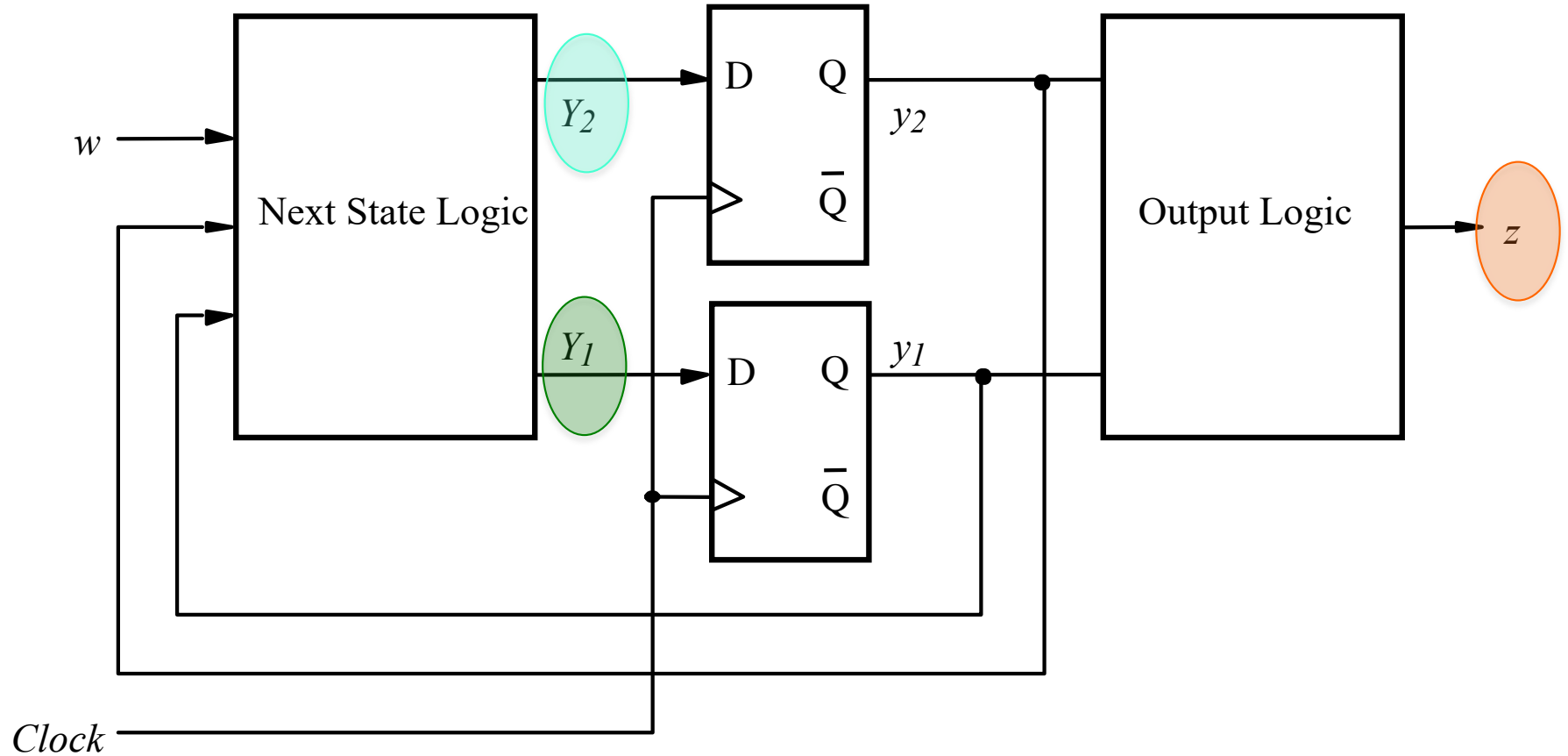
We will call  $y_1$  and  $y_2$  the *present state variables*.

We will call  $Y_1$  and  $Y_2$  the *next state variables*.



We need to find logic expressions for  $Y_1(w, y_1, y_2)$ ,  $Y_2(w, y_1, y_2)$ , and  $z(y_1, y_2)$ .





We need to find logic expressions for  $Y_1(w, y_1, y_2)$ ,  $Y_2(w, y_1, y_2)$ , and  $z(y_1, y_2)$ .

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

Suppose we encoded our states in the same order in which they were labeled:

A ~ 00

B ~ 01

C ~ 10

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	00		
B	01		
C	10		
	11		

The finite state machine will never reach a state encoded as 11.

[ Figure 6.6 from the textbook ]

Present state	Next state		Output $z$
	$w = 0$	$w = 1$	
A	A	B	0
B	A	C	0
C	A	C	1

	Present state	Next state		Output $z$
		$w = 0$	$w = 1$	
	$y_2 y_1$	$Y_2 Y_1$	$Y_2 Y_1$	
A	00	00	01	0
B	01	00	10	0
C	10	00	10	1
	11	<i>dd</i>	<i>dd</i>	<i>d</i>

We arbitrarily chose these as our state encodings. We could have used others.

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	
0	1	
1	0	
1	1	

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	<i>d</i>

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	<i>d</i>

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	dd	dd	d

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	d	
1	0	0		
1	0	1		
1	1	0		
1	1	1		

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]



$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	$d$	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	$d$	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	$d$

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	
0	0	1	0	
0	1	0	0	
0	1	1	d	
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	d	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	
1	0	1	1	
1	1	0	1	
1	1	1	d	

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	$dd$	$dd$	$d$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

$$Q(t) = y_2y_1 \text{ and } Q(t+1) = Y_2Y_1$$

Present state $y_2y_1$	Next state		Output $z$
	$w = 0$	$w = 1$	
	$Y_2Y_1$	$Y_2Y_1$	
00	00	01	0
01	00	10	0
10	00	10	1
11	<i>dd</i>	<i>dd</i>	<i>d</i>

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

[ Figure 6.6 from the textbook ]

Note that the textbook draws these K-Maps differently from all previous K-maps (the most significant bit indexes the rows).

$Y_1$

$y_2 y_1$

$w$		00	01	11	10
0		0	0	d	0
1		1	0	d	0

$Y_2$

$y_2 y_1$

$w$		00	01	11	10
0		0	0	d	0
1		0	1	d	1

$z$

$y_1$

$y_2$

		0	1
0		0	0
1		1	d

$$Q(t) = y_2 y_1 \text{ and } Q(t+1) = Y_2 Y_1$$

$w$	$y_2$	$y_1$	$Y_2$	$Y_1$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	d	d
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	d	d

$y_2$	$y_1$	$z$
0	0	0
0	1	0
1	0	1
1	1	d

# Don't care conditions simplify the combinatorial logic

$Y_1$

$w$	$y_2 y_1$	00	01	11	10
0		0	0	d	0
1		1	0	d	0

$Y_2$

$w$	$y_2 y_1$	00	01	11	10
0		0	0	d	0
1		0	1	d	1

$z$

$y_2$	$y_1$	0	1
0		0	0
1		1	d

Ignoring don't cares

$$Y_1 = w\bar{y}_1\bar{y}_2$$

$$Y_2 = wy_1\bar{y}_2 + \bar{w}y_1y_2$$

$$z = \bar{y}_1y_2$$

Using don't cares

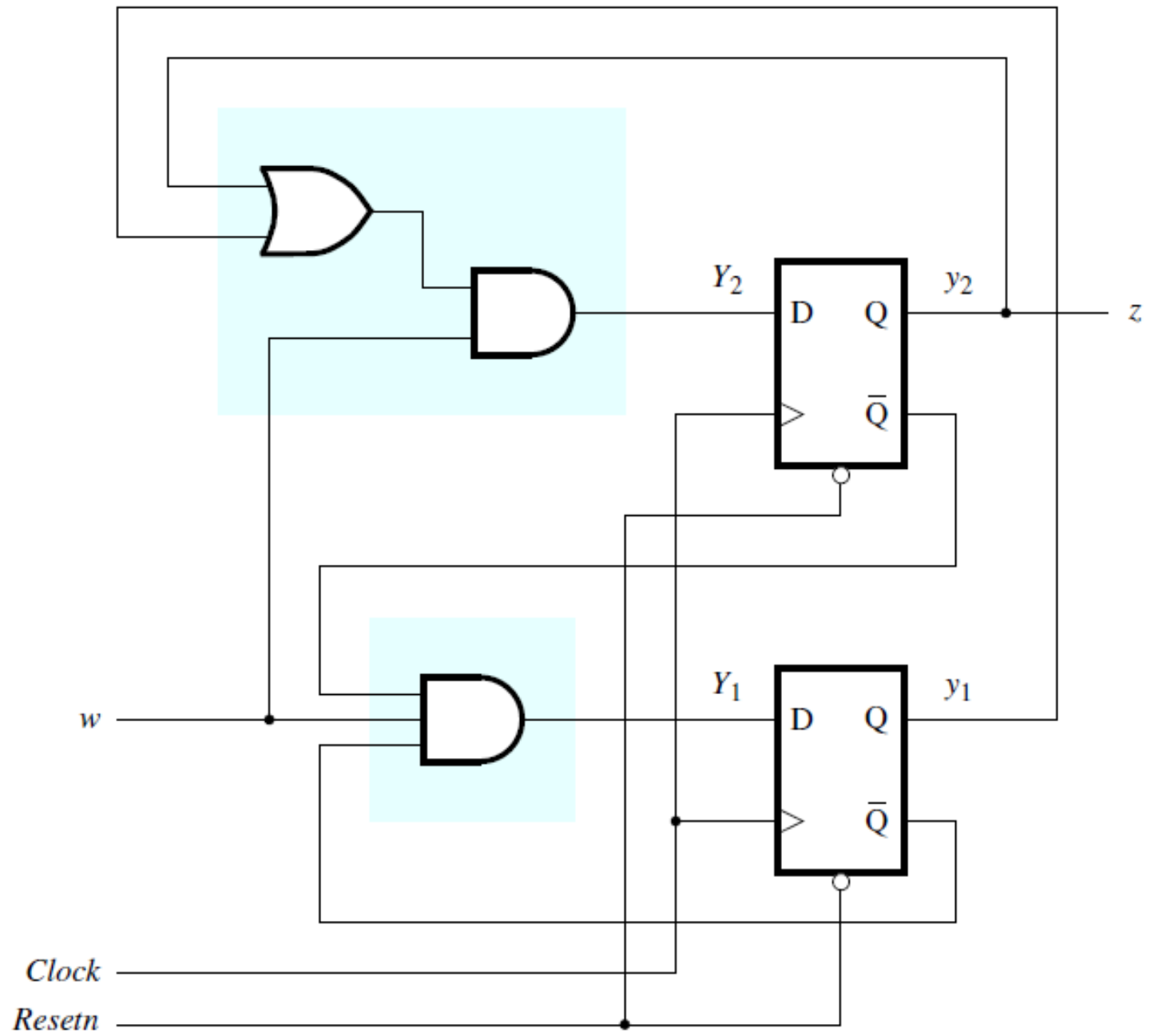
$$Y_1 = w\bar{y}_1\bar{y}_2$$

$$Y_2 = wy_1 + wy_2$$

$$= w(y_1 + y_2)$$

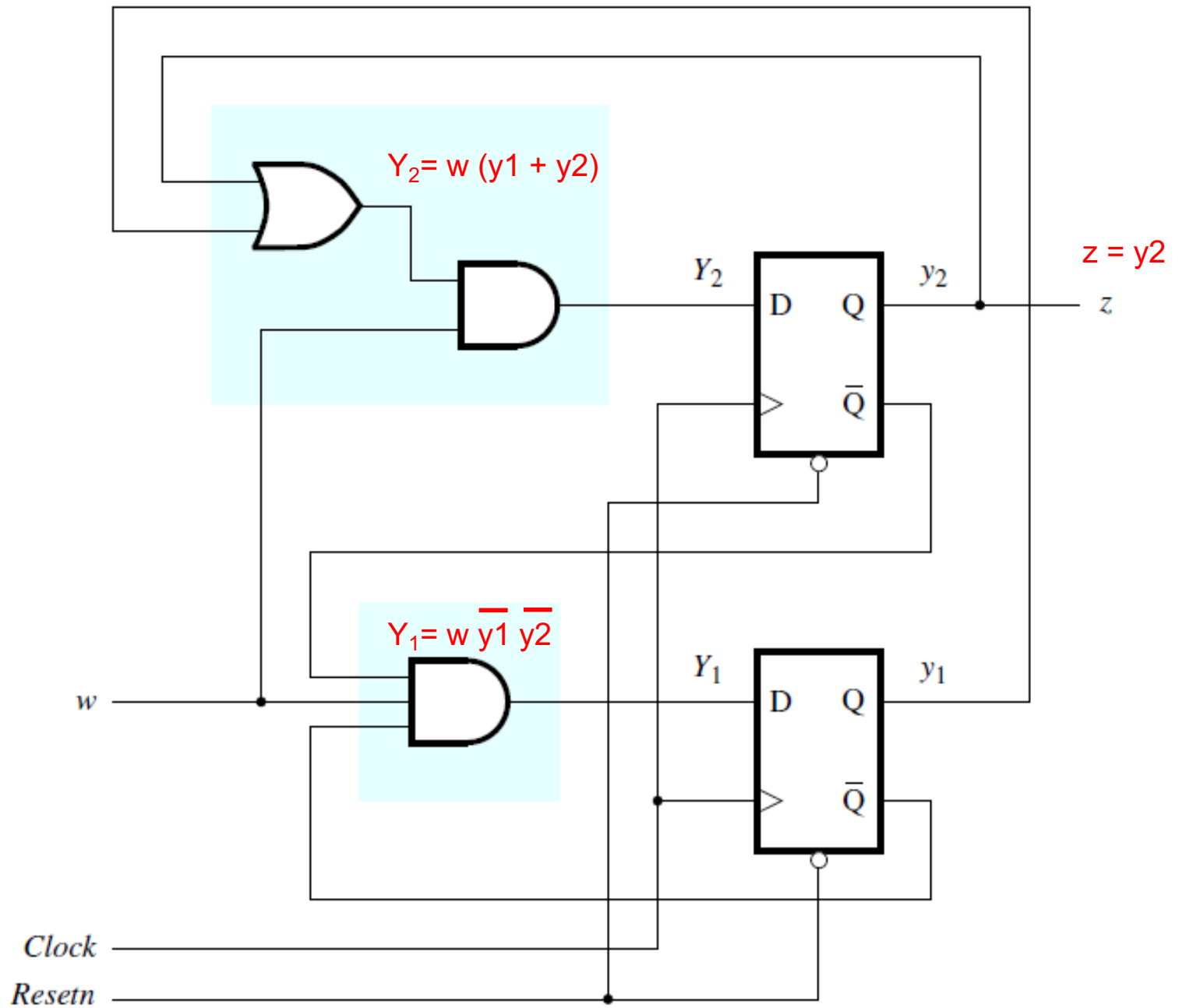
$$z = y_2$$

[ Figure 6.7 from the textbook ]



[ Figure 6.8 from the textbook ]

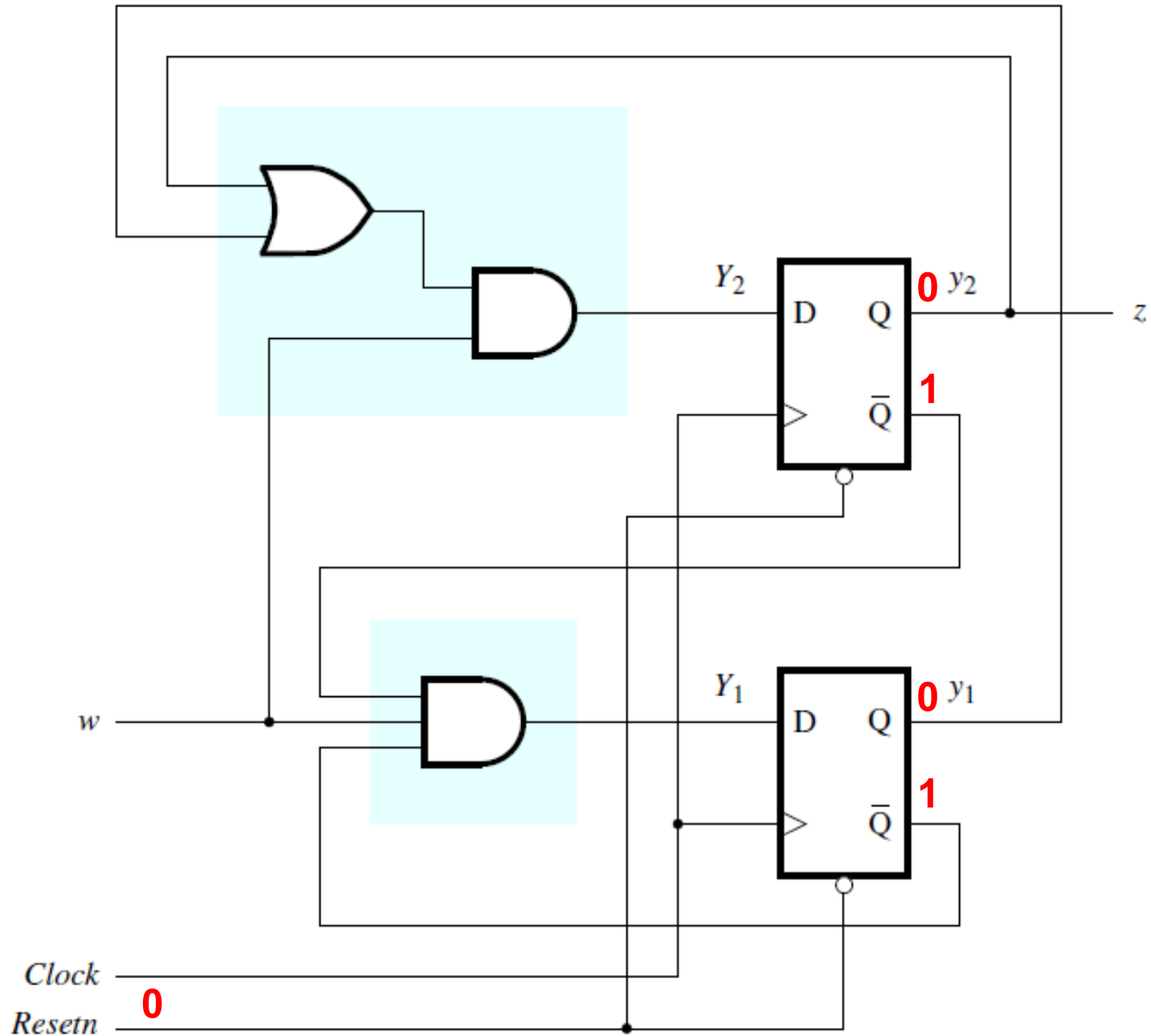




[ Figure 6.8 from the textbook ]



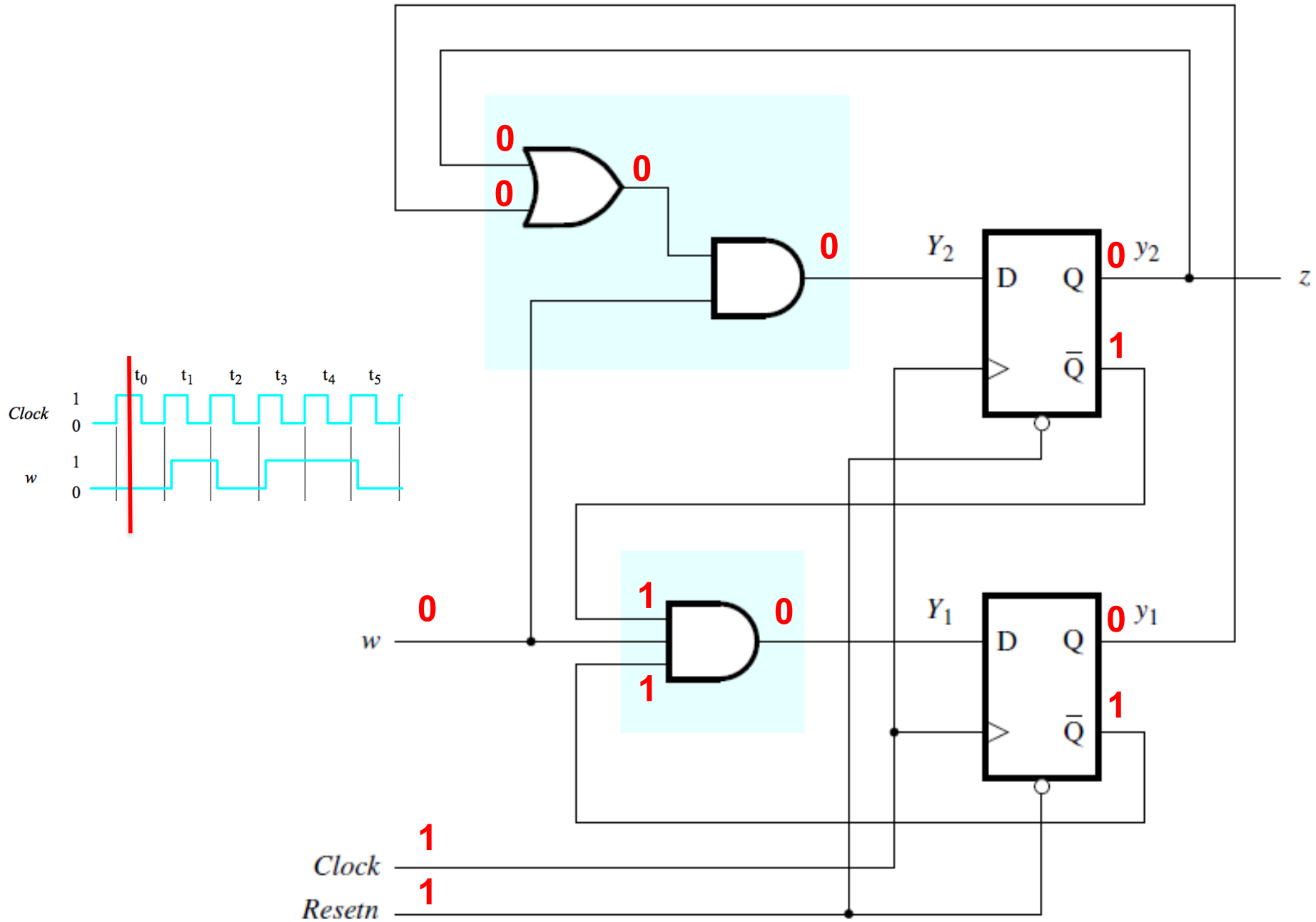
State A=00



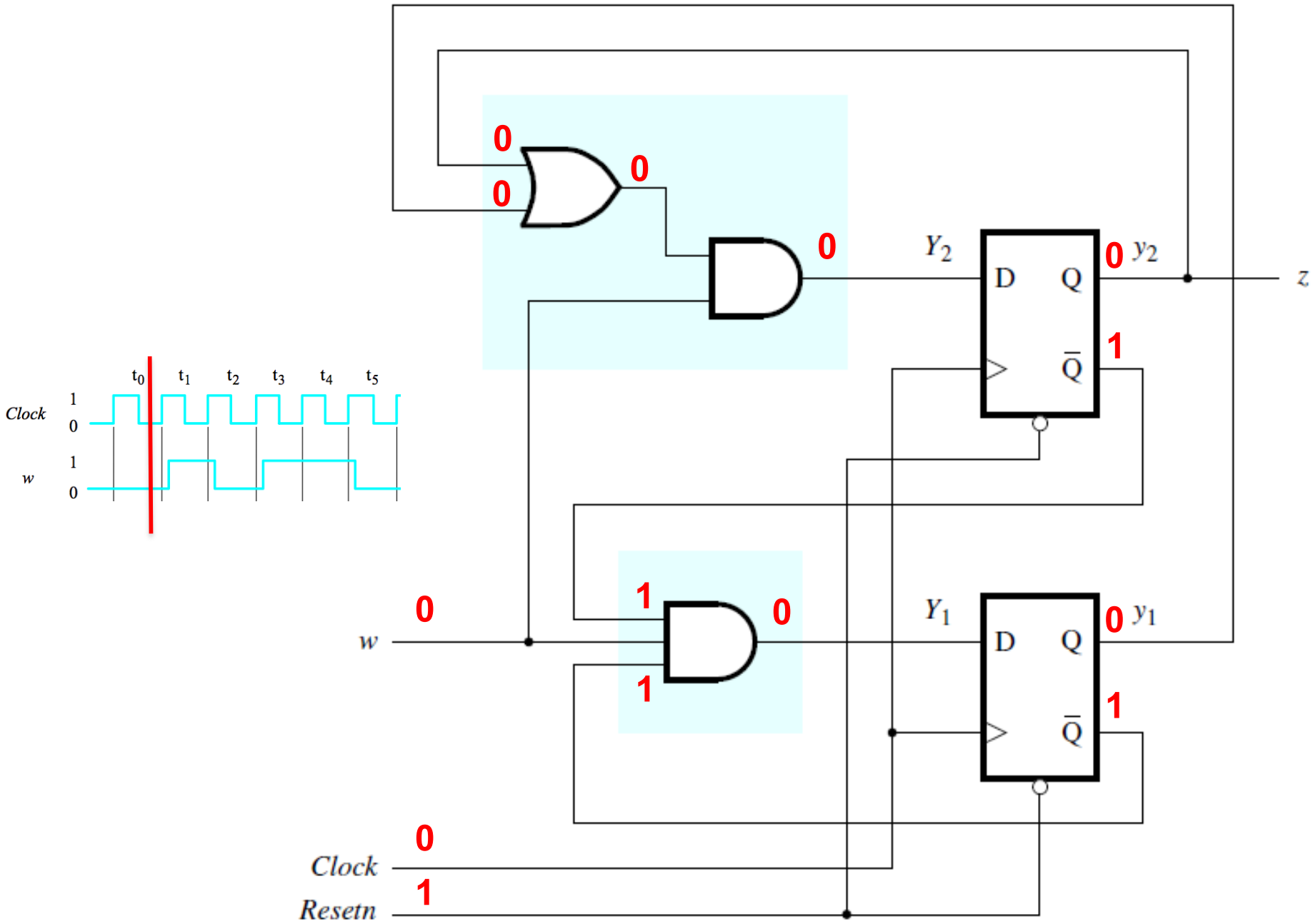
Finally, we add a reset signal. When it is equal to zero it puts the machine back to its start state, which is state 00 in this case.

[ Figure 6.8 from the textbook ]

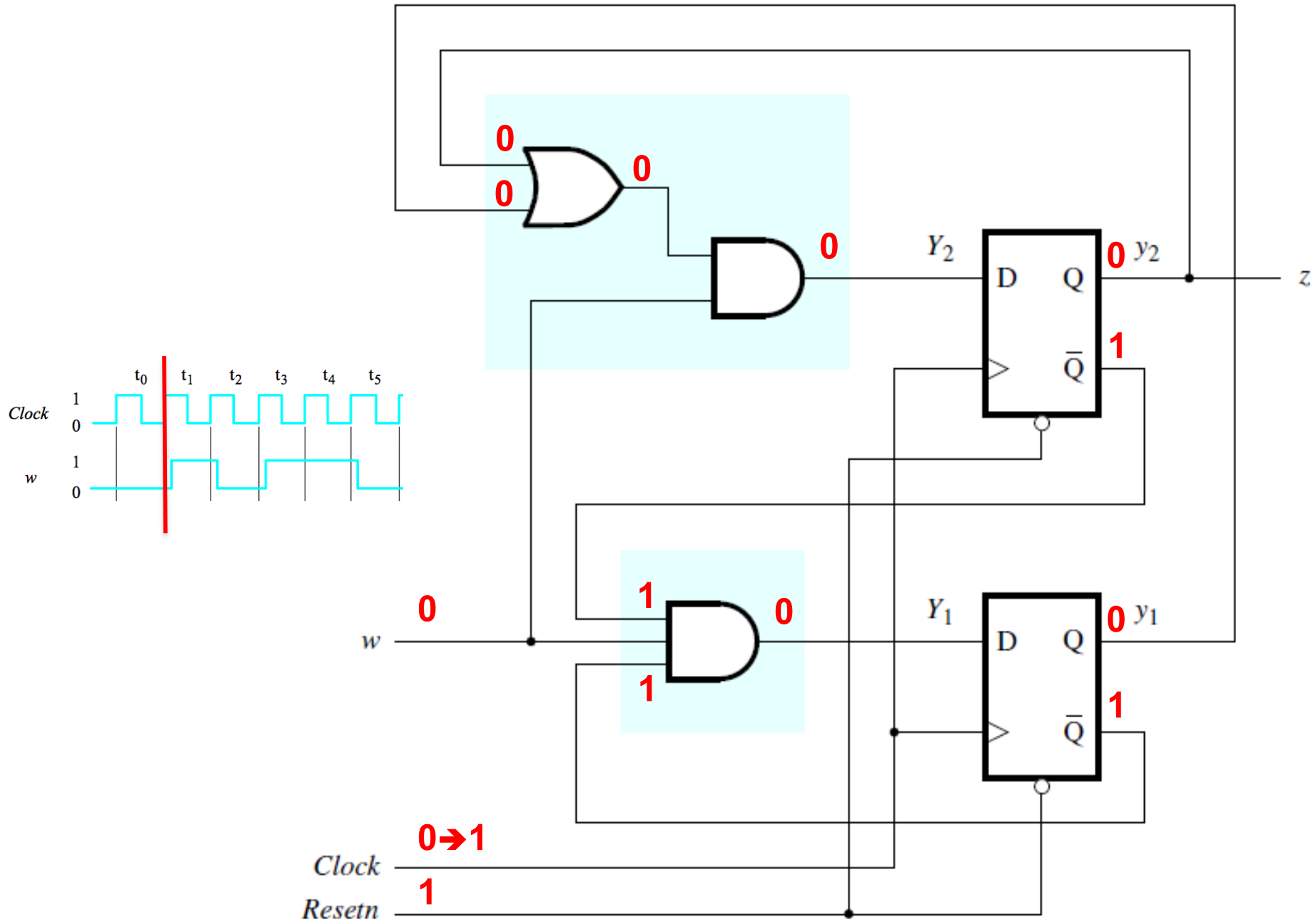
State A=00



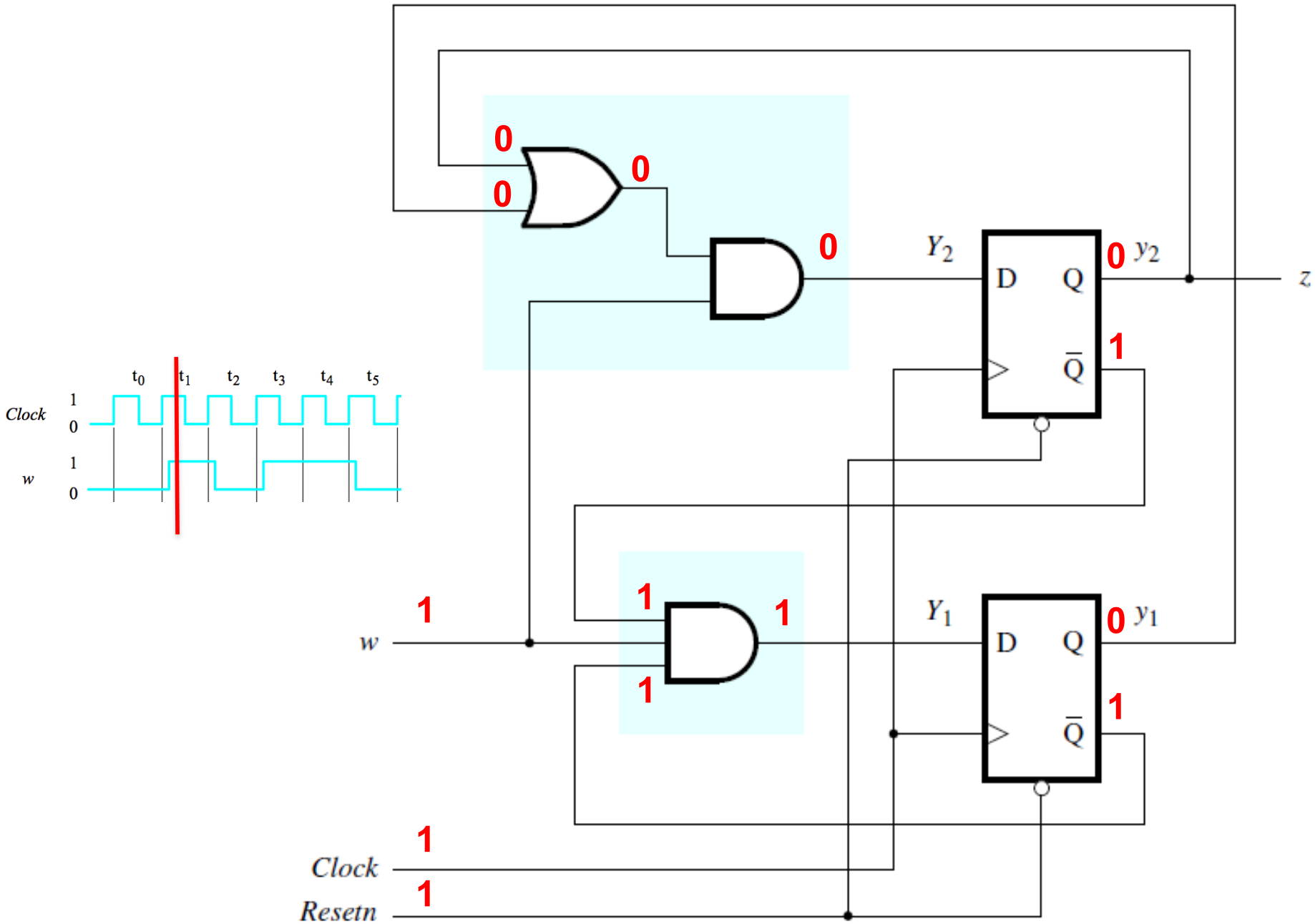
State A=00



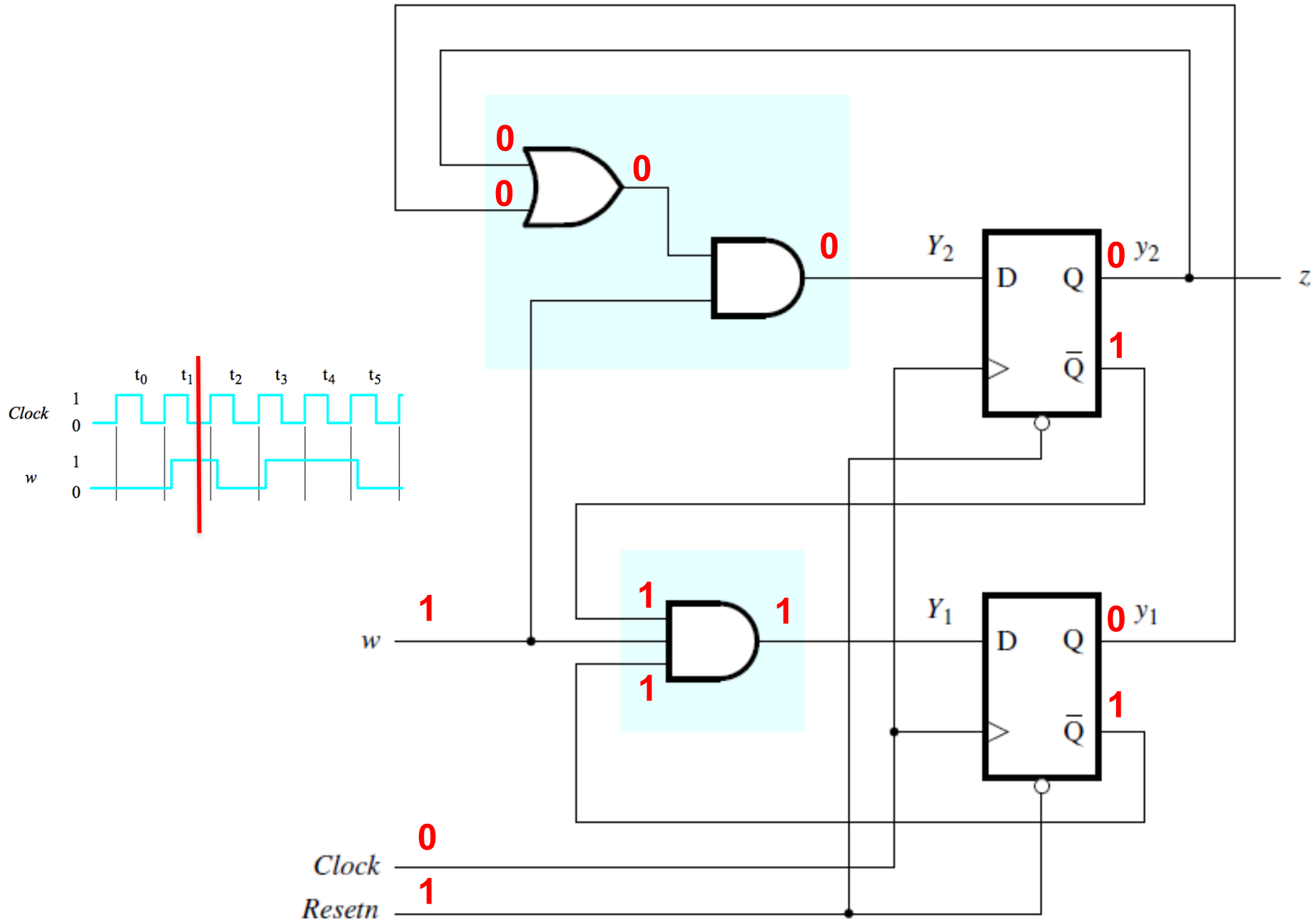
State A=00



State A=00

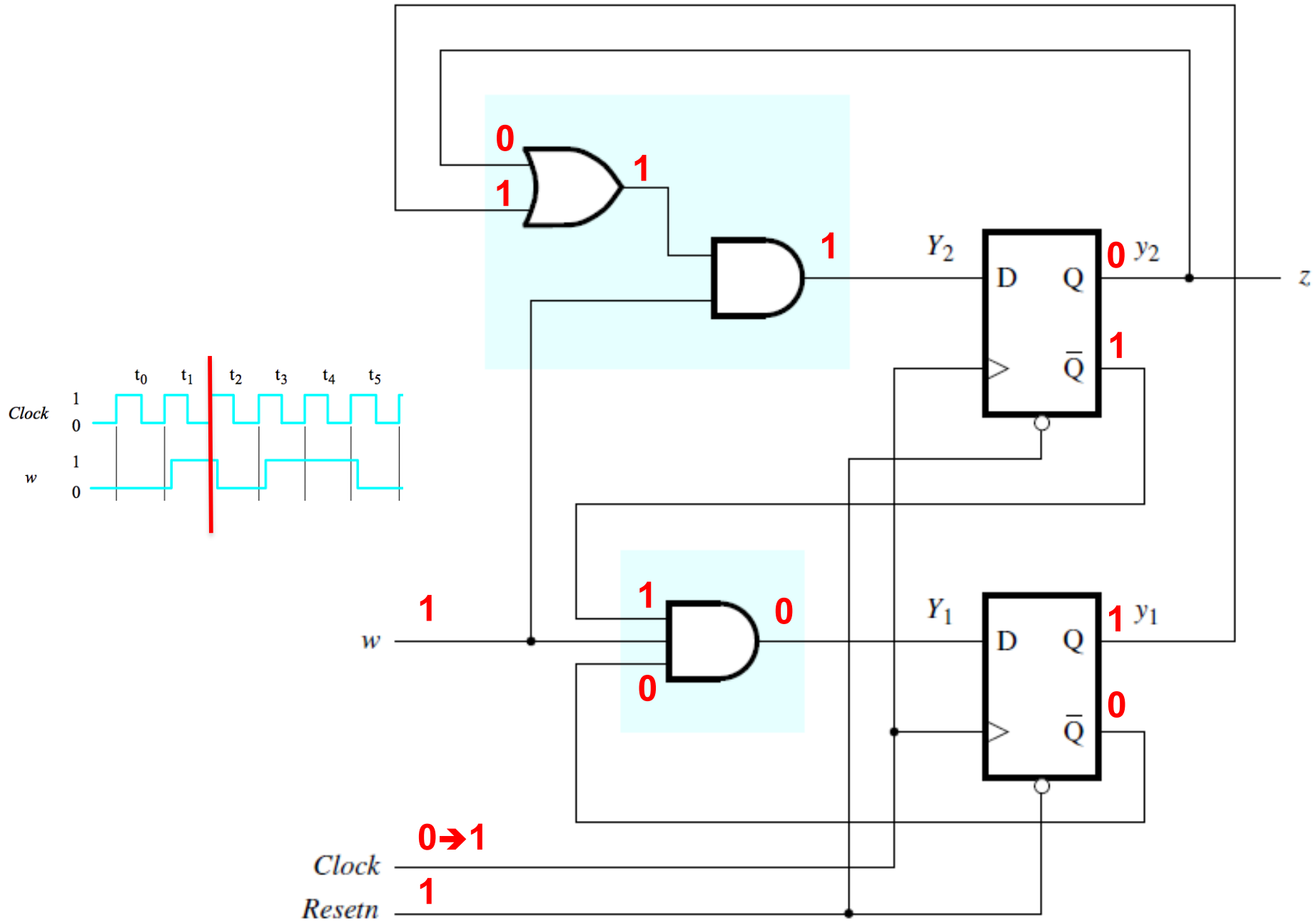


State A=00

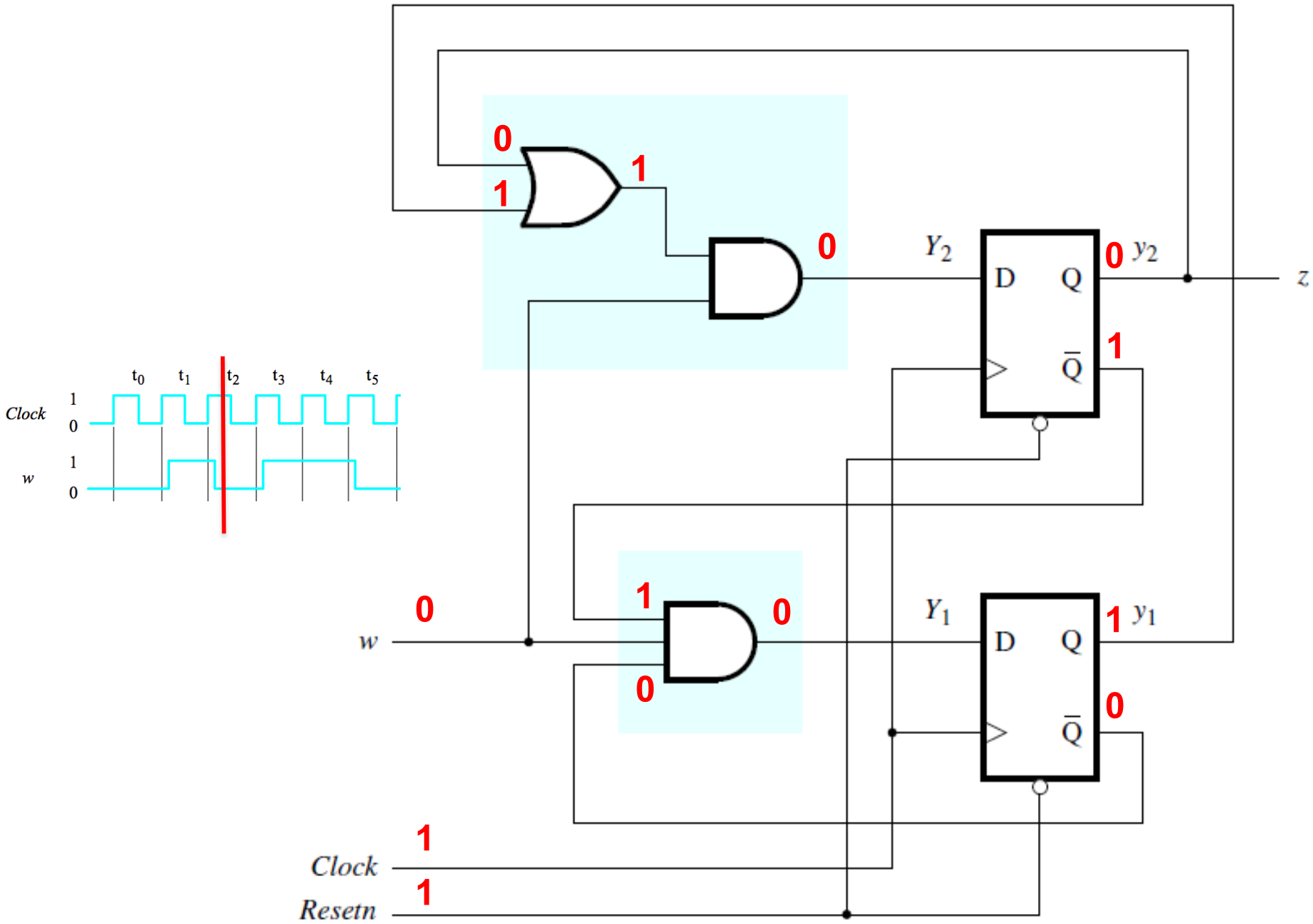




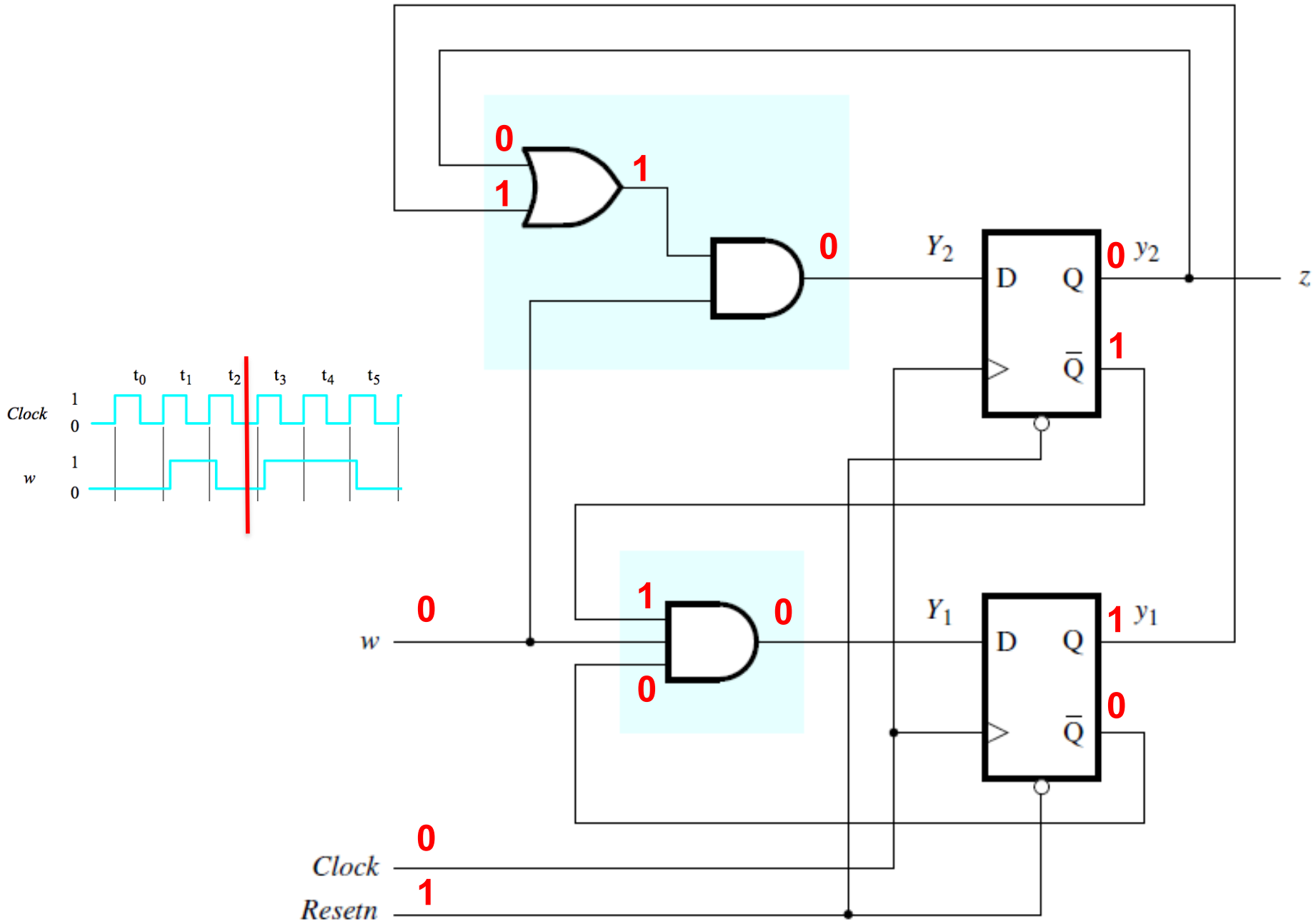
State B=01



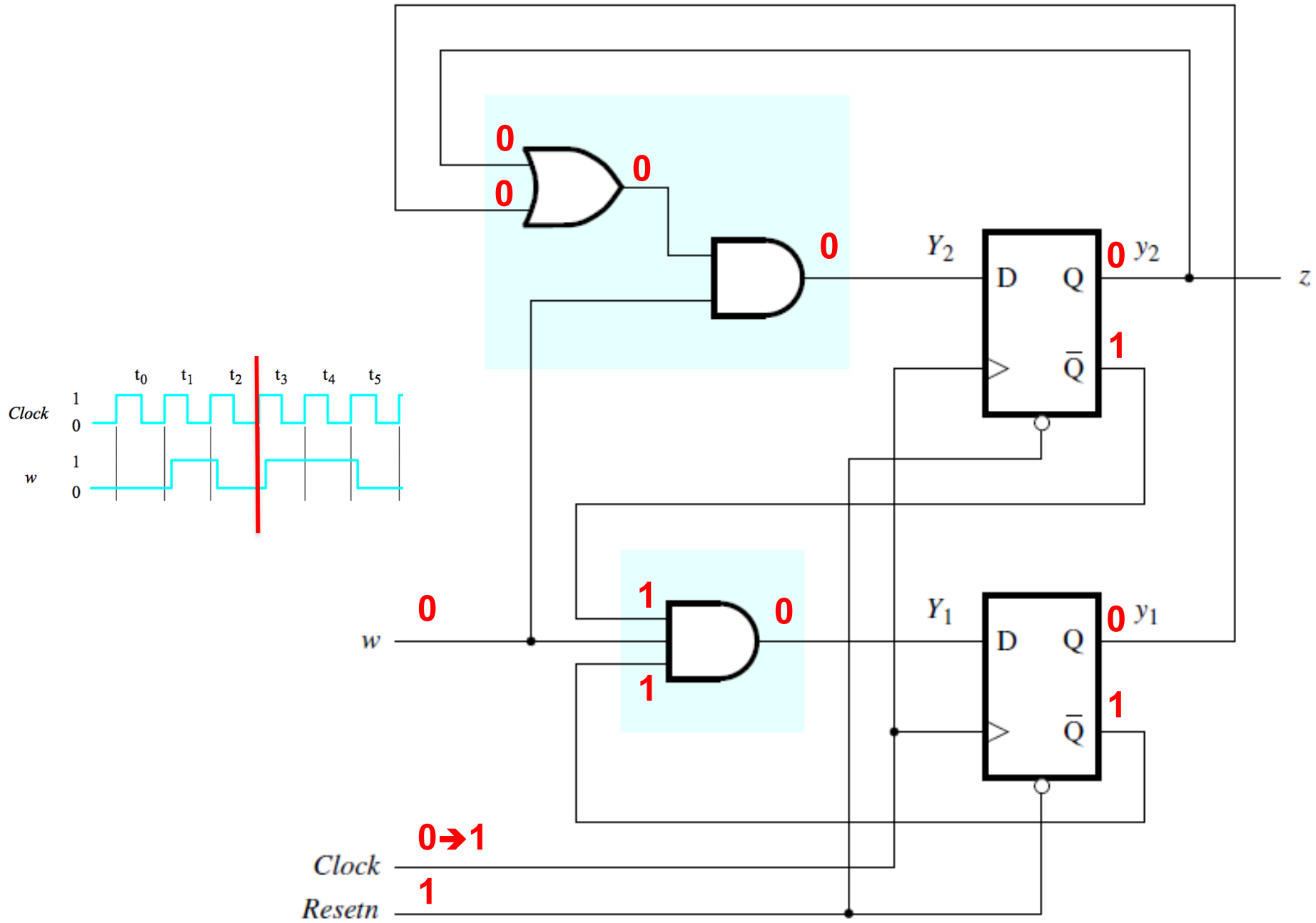
State B=01



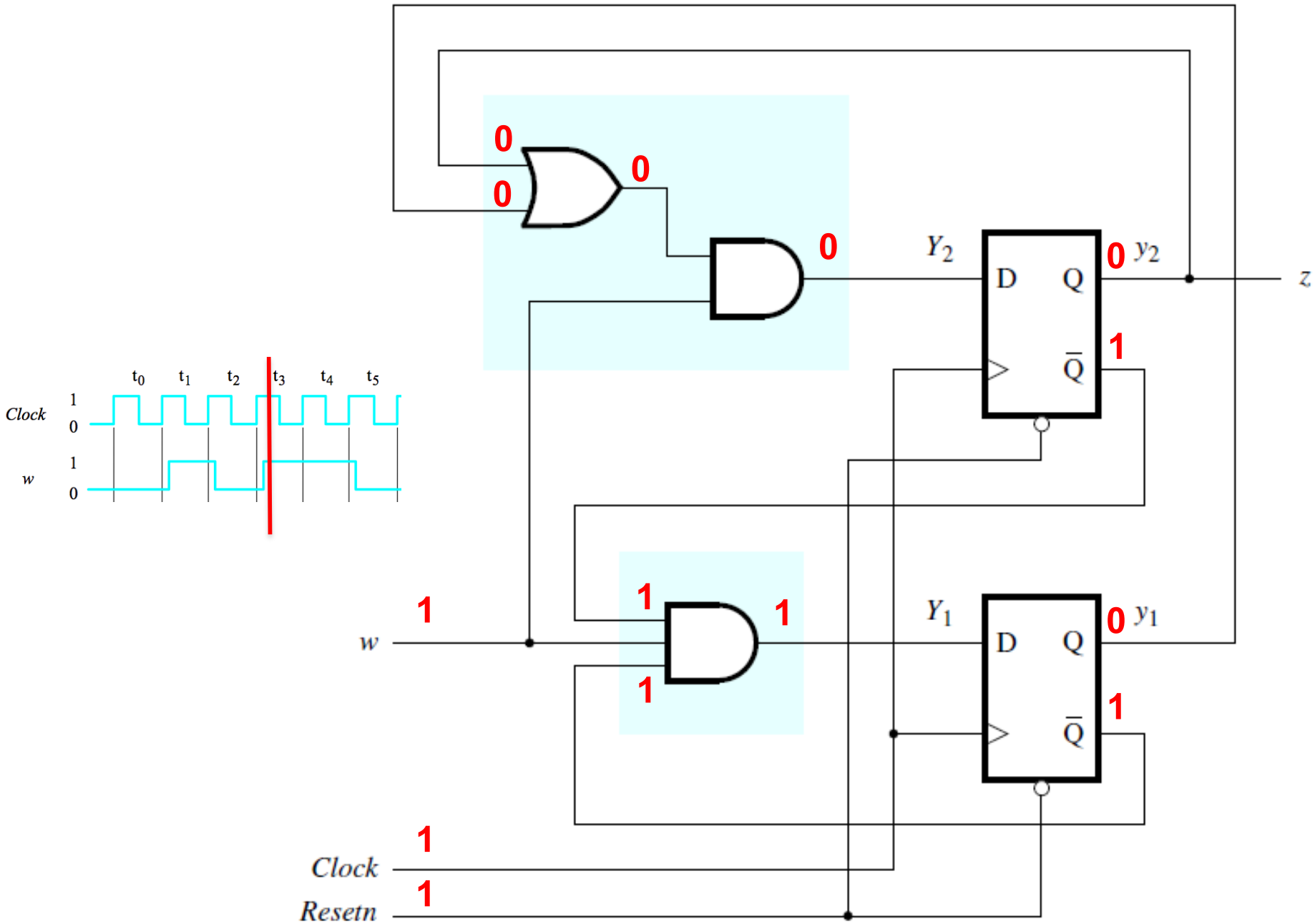
State B=01



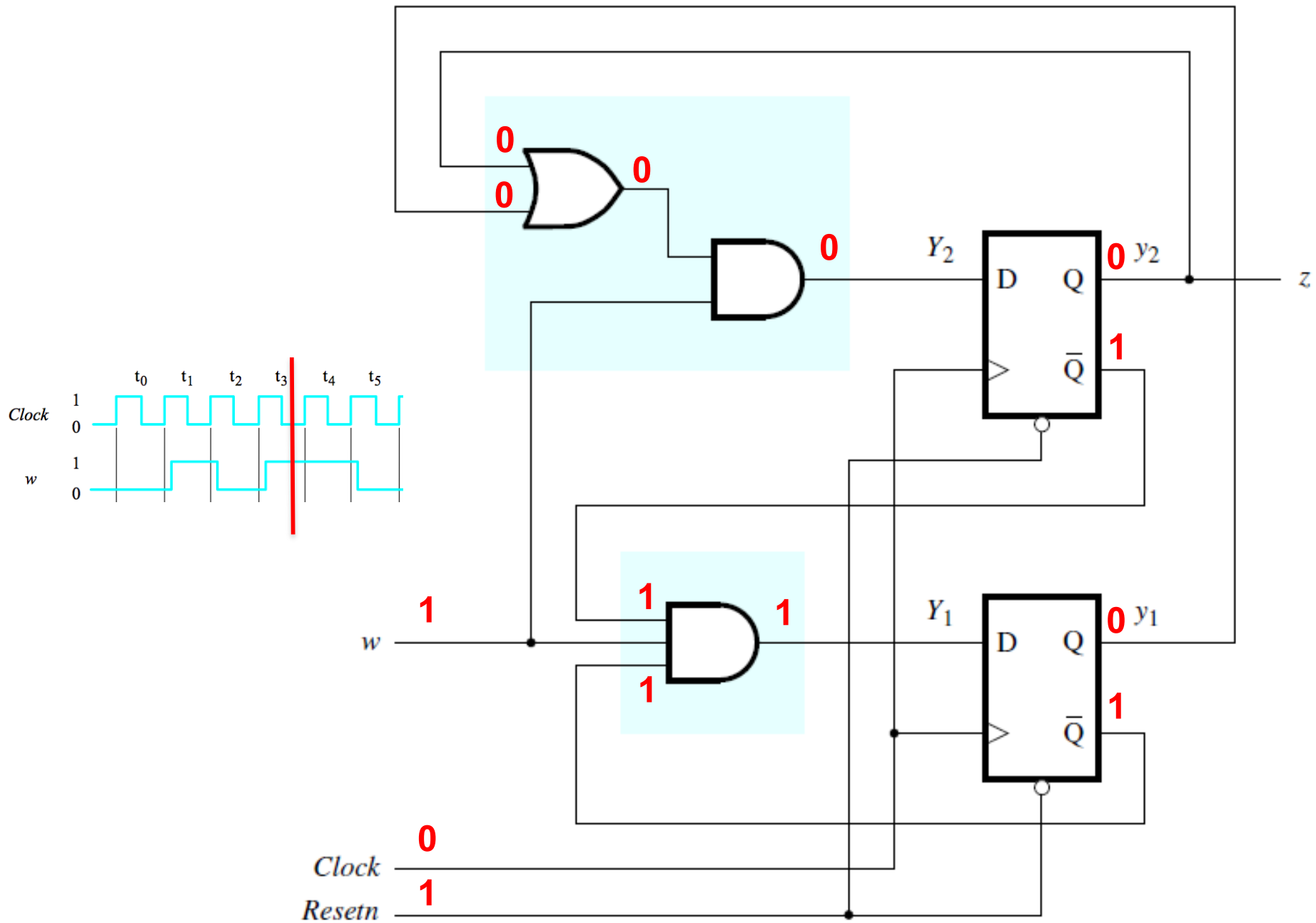
State A=00



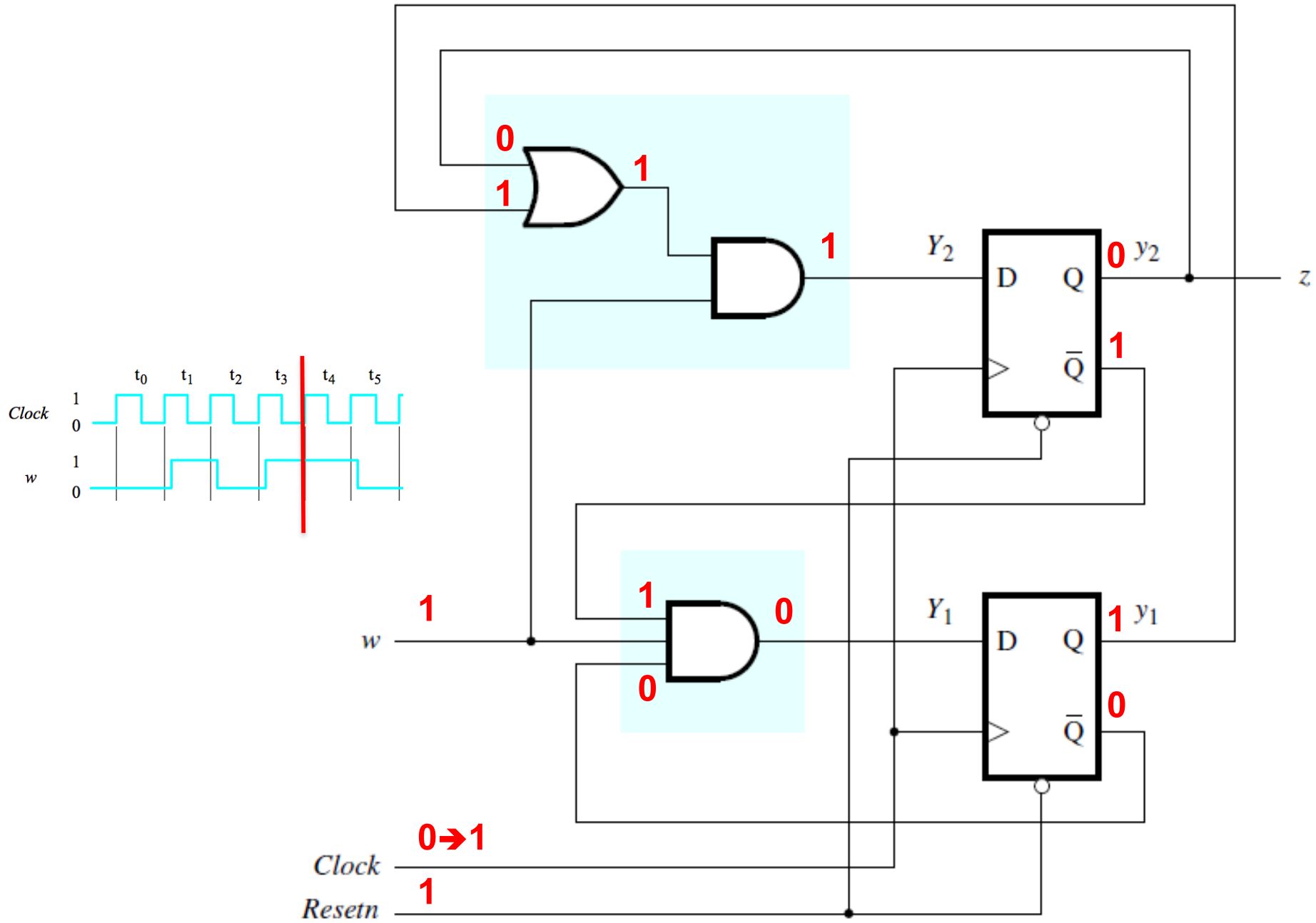
State A=00



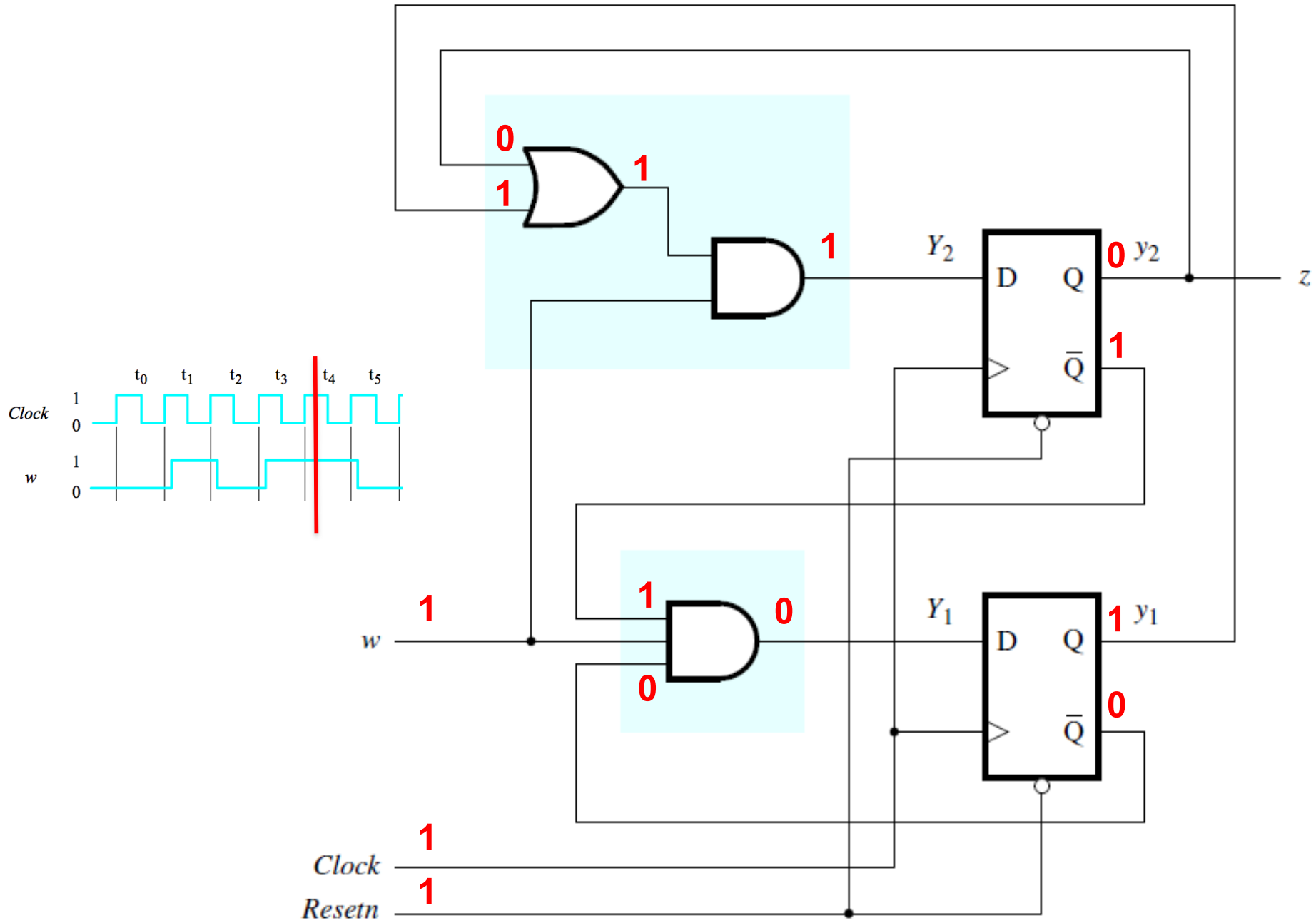
State A=00



State B=01

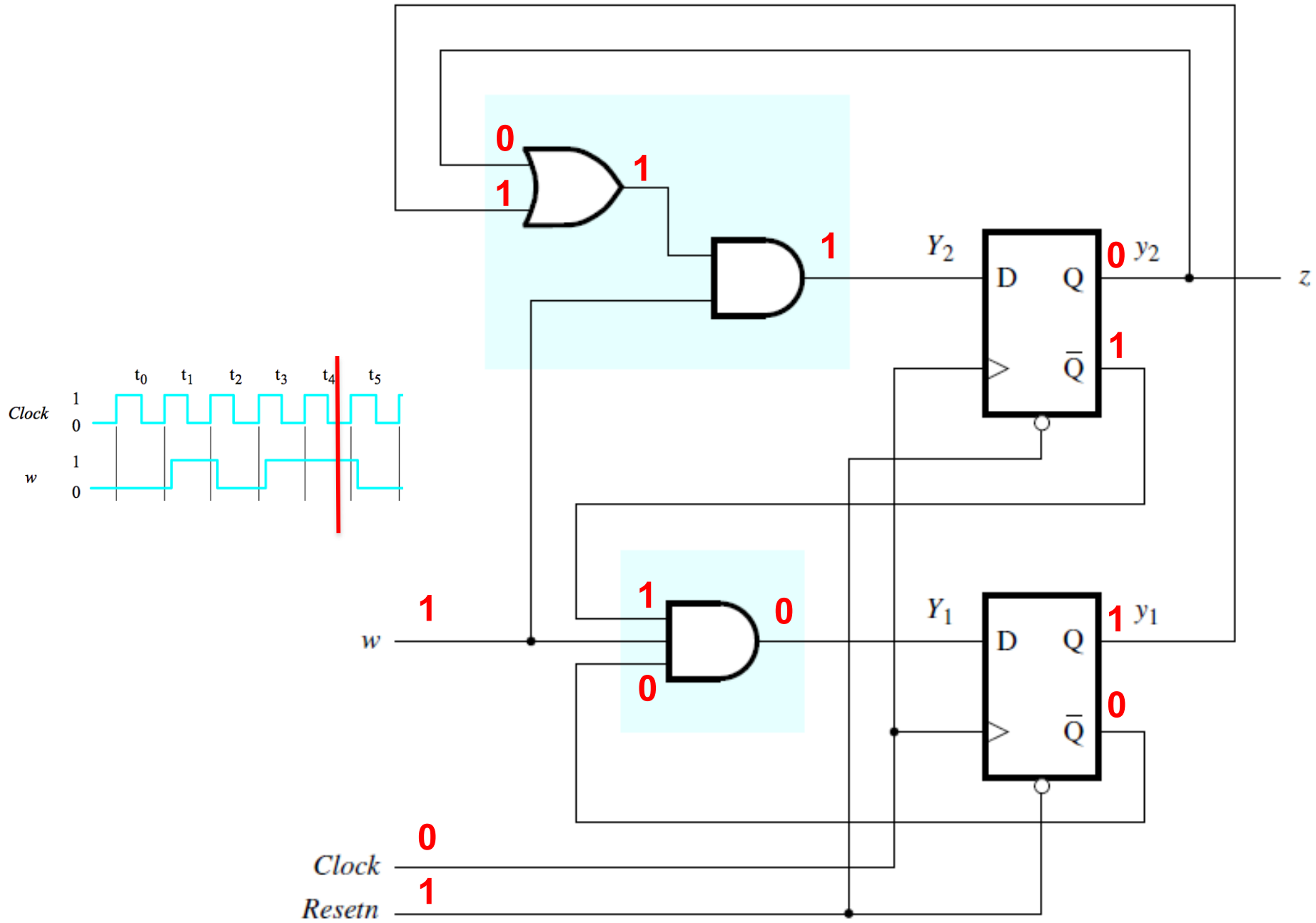


State B=01

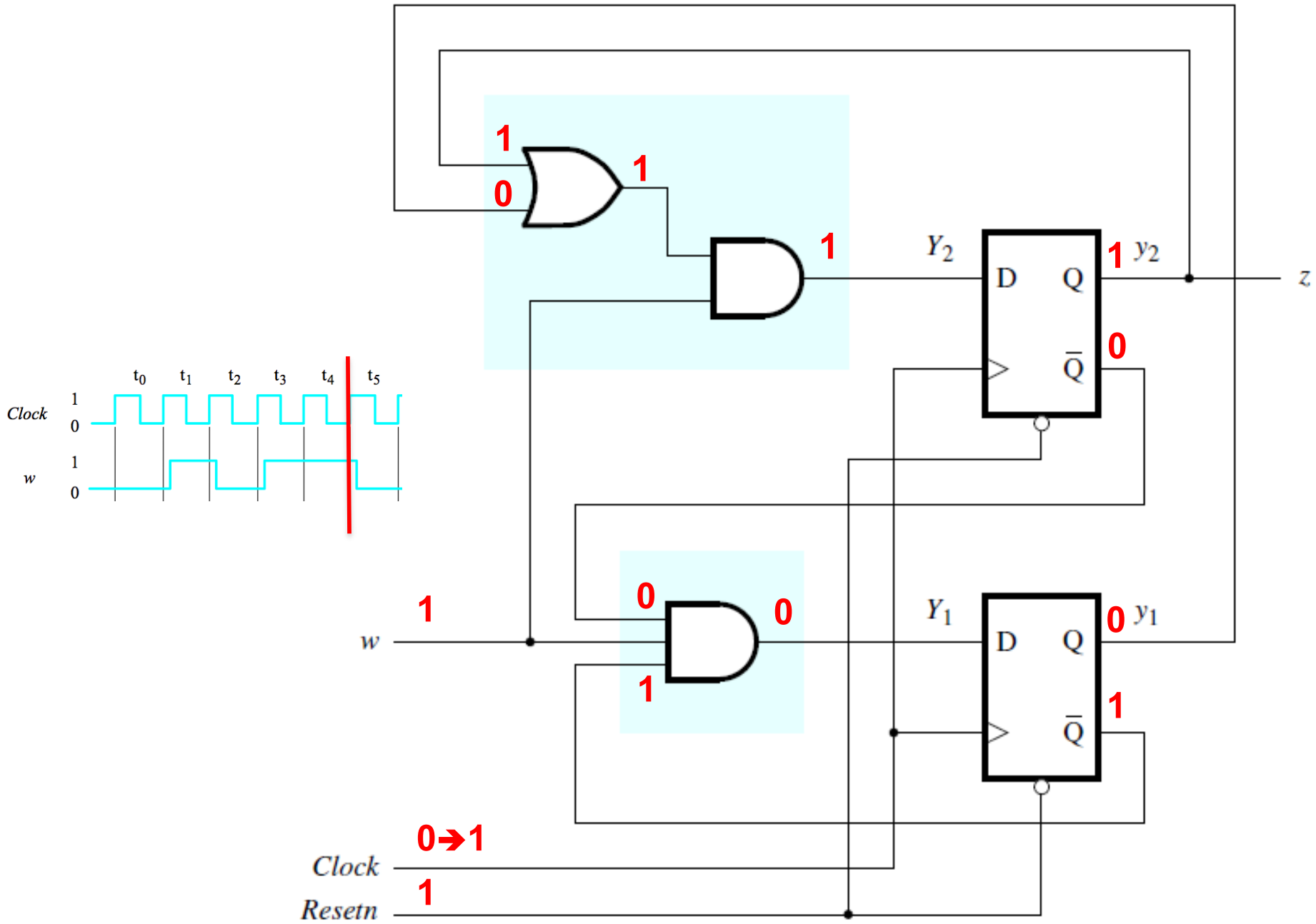




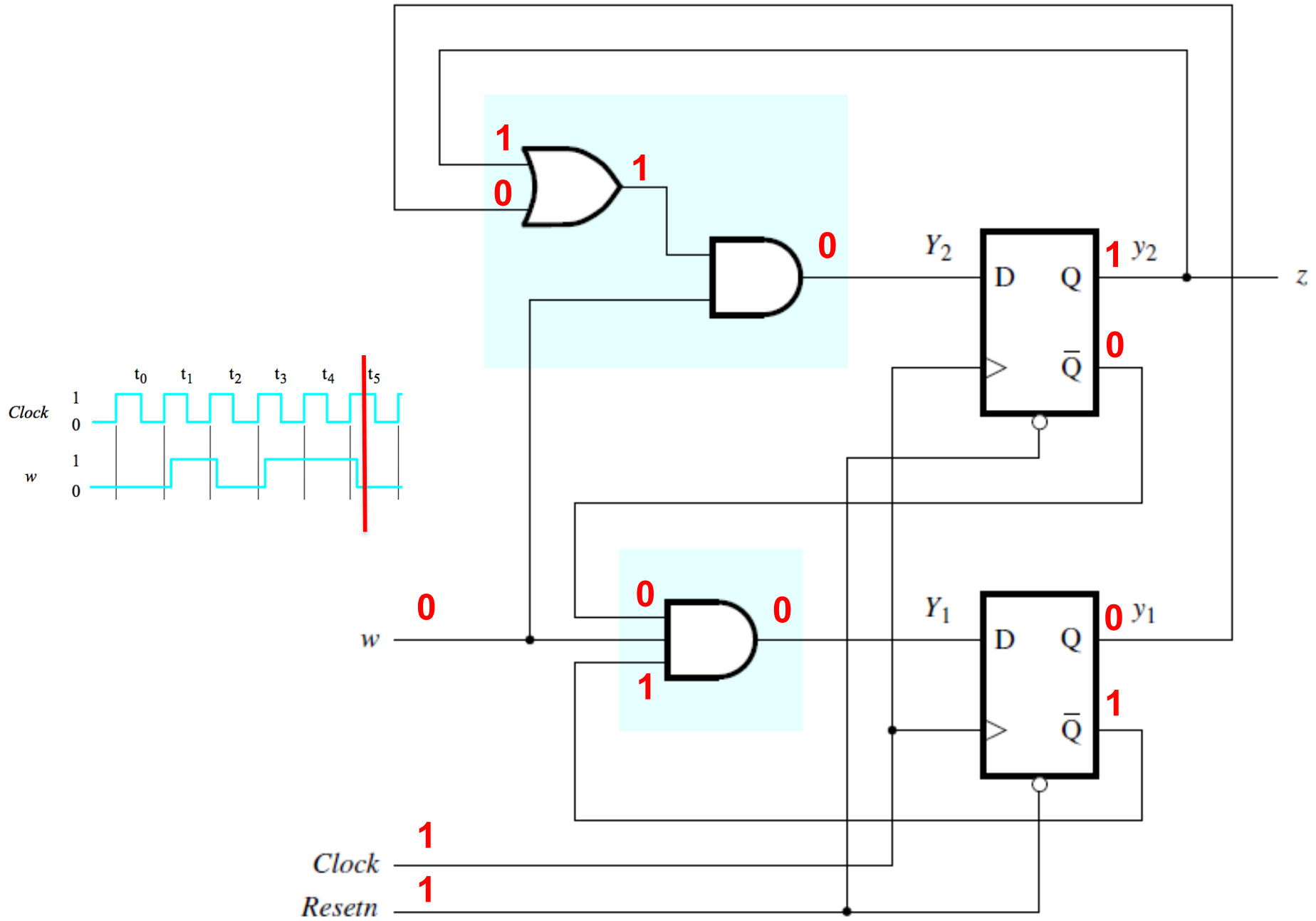
State B=01



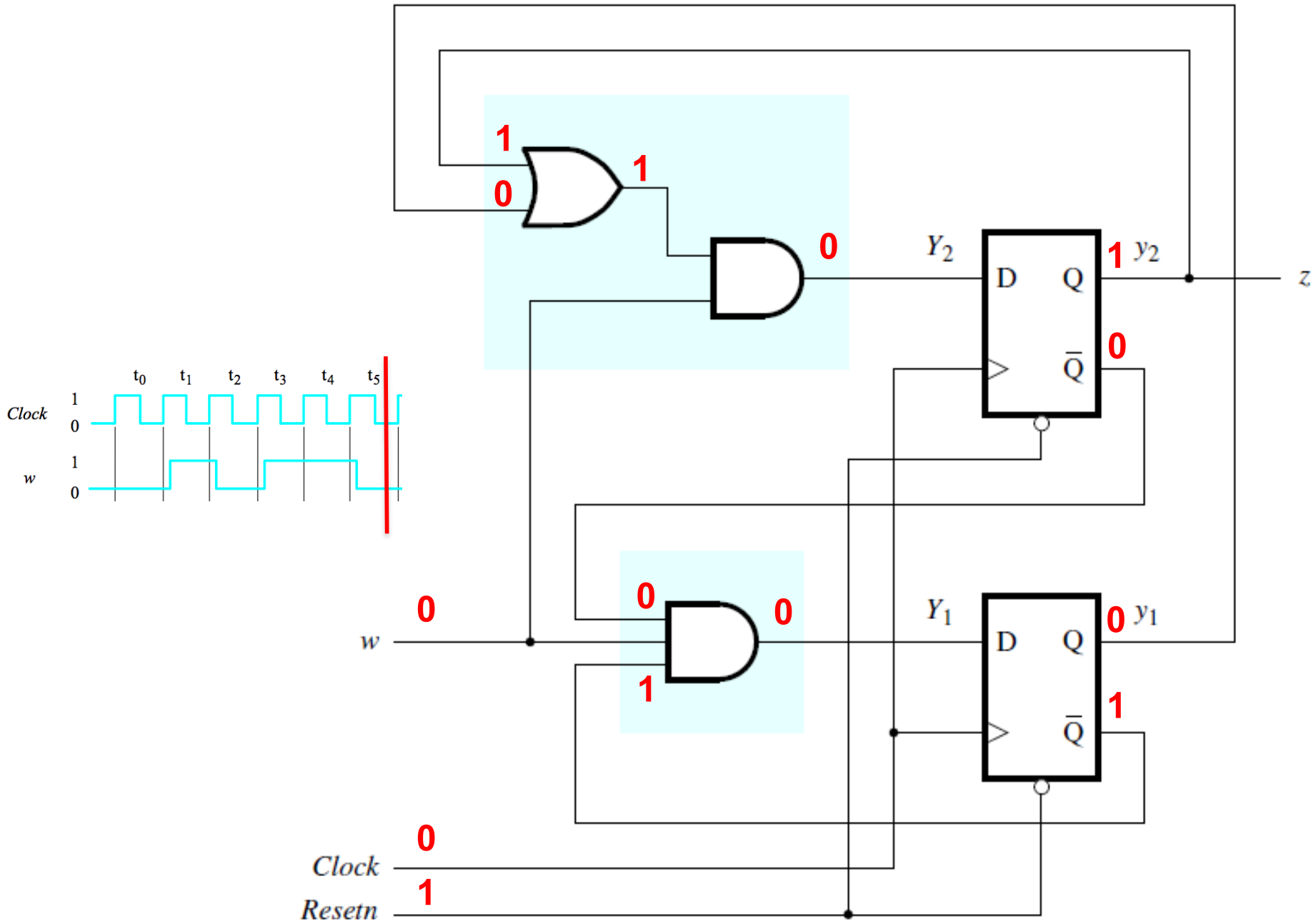
State C=10



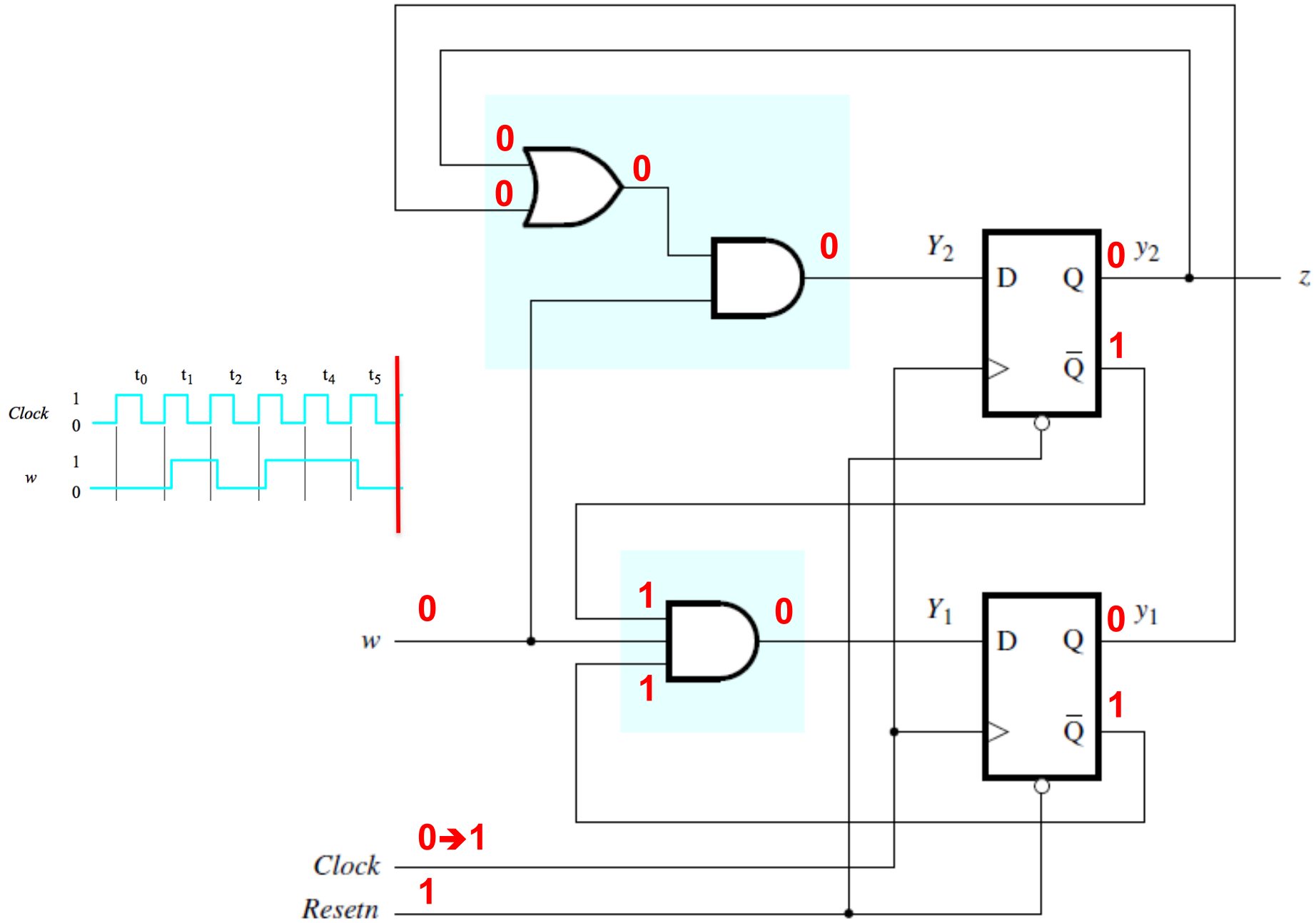
State C=10



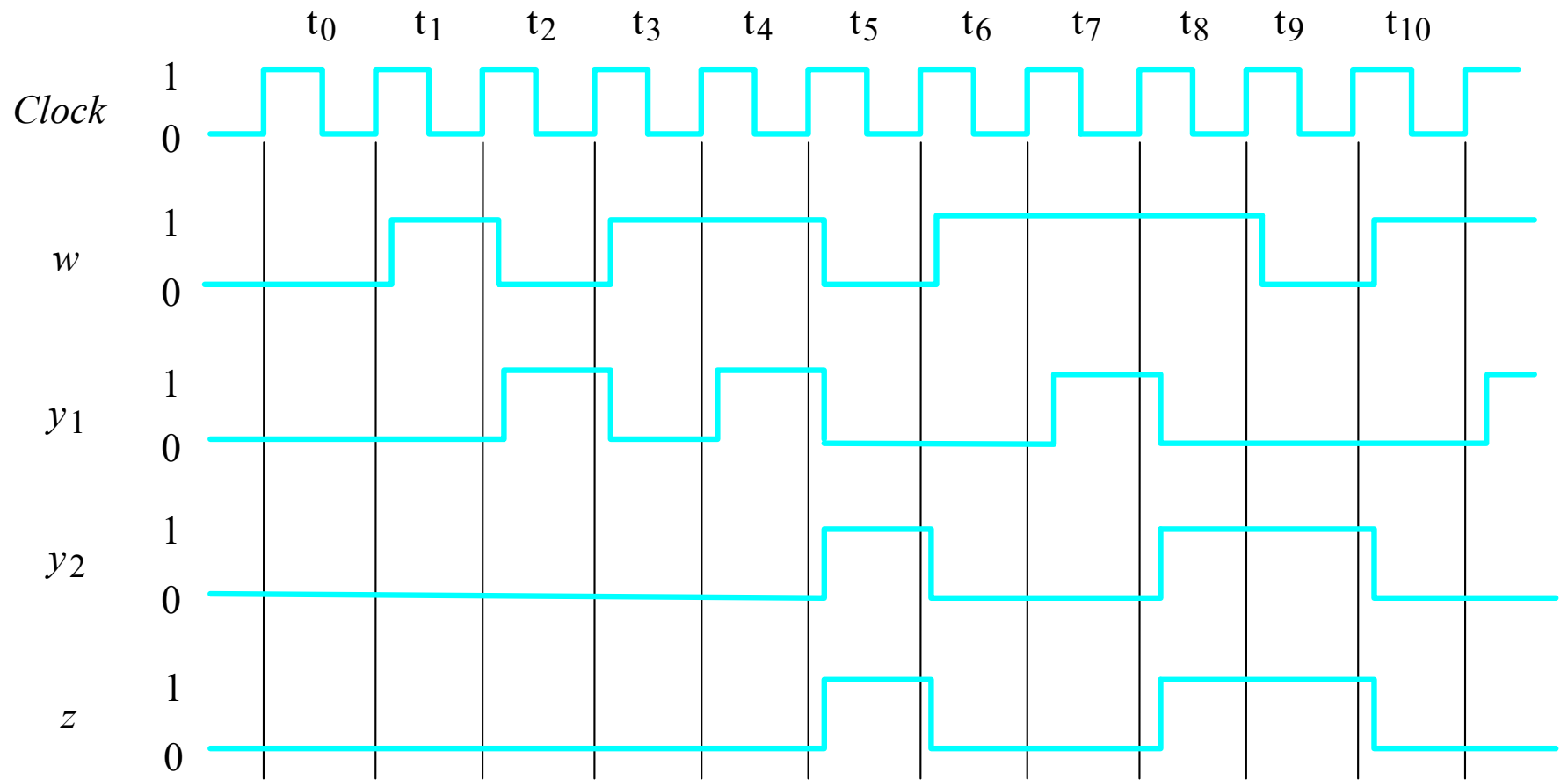
State C=10



State A=00

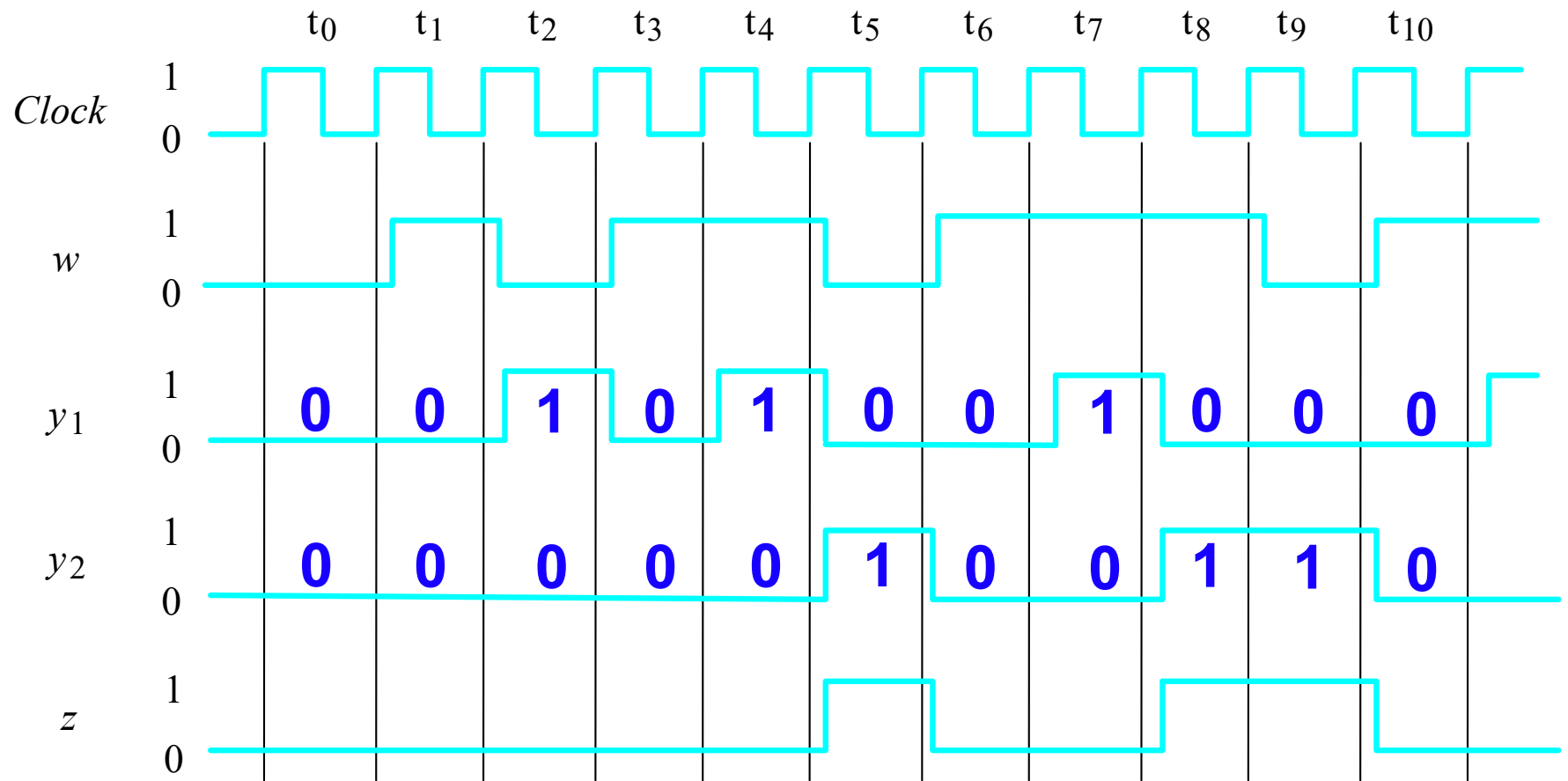


<b>Clockcycle:</b>	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

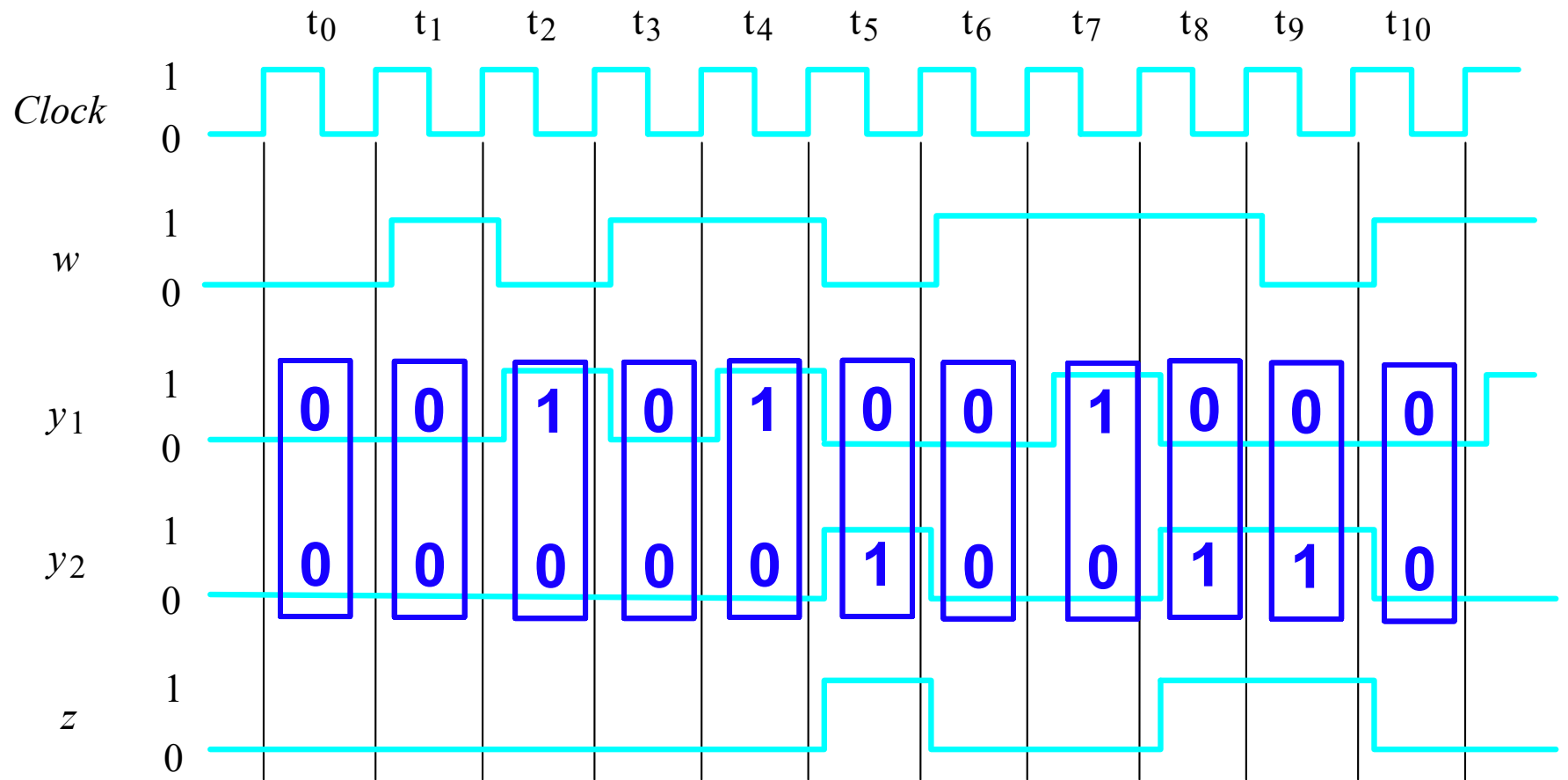


[ Figure 6.9 from the textbook ]

Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

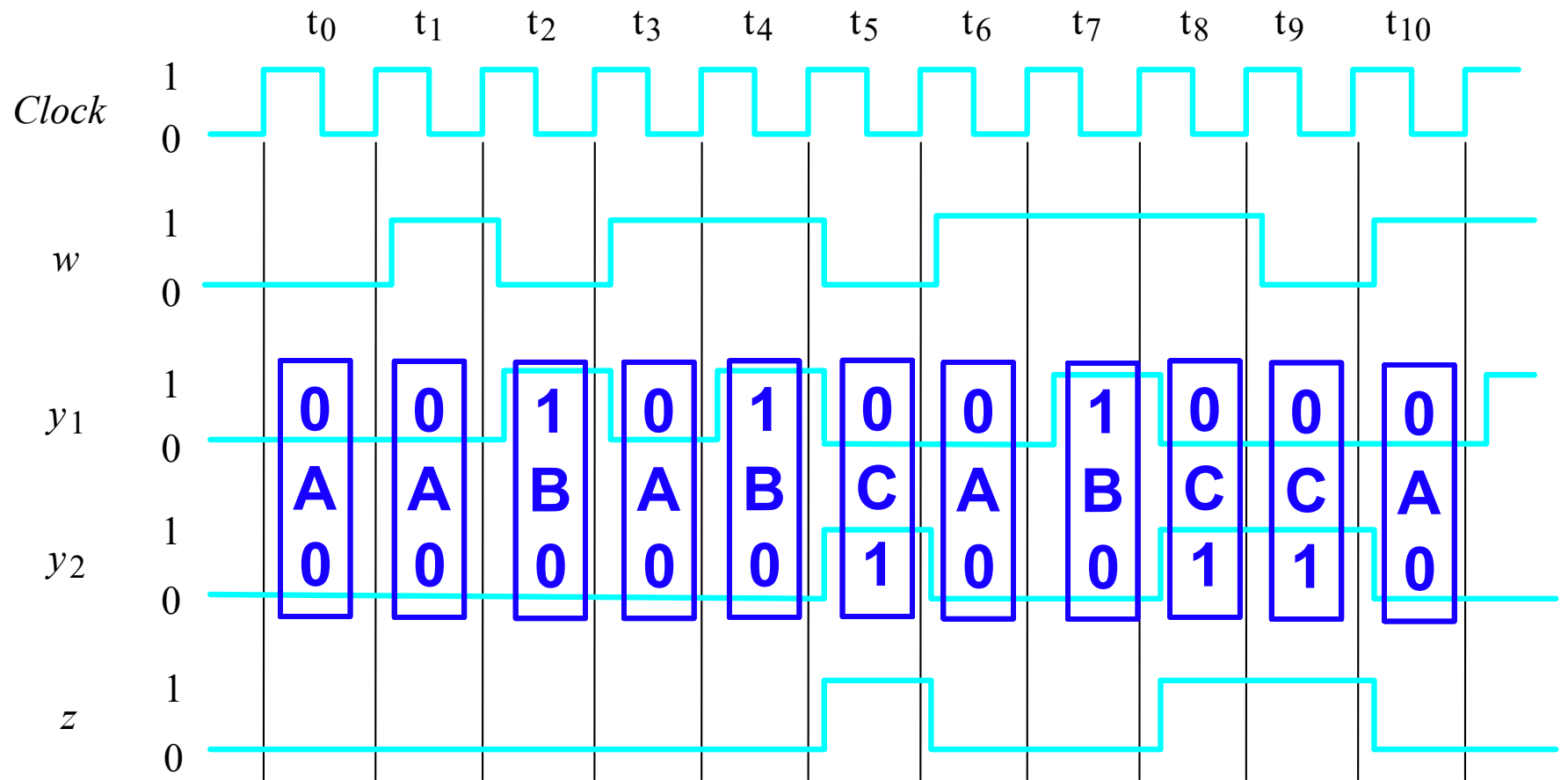


Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0

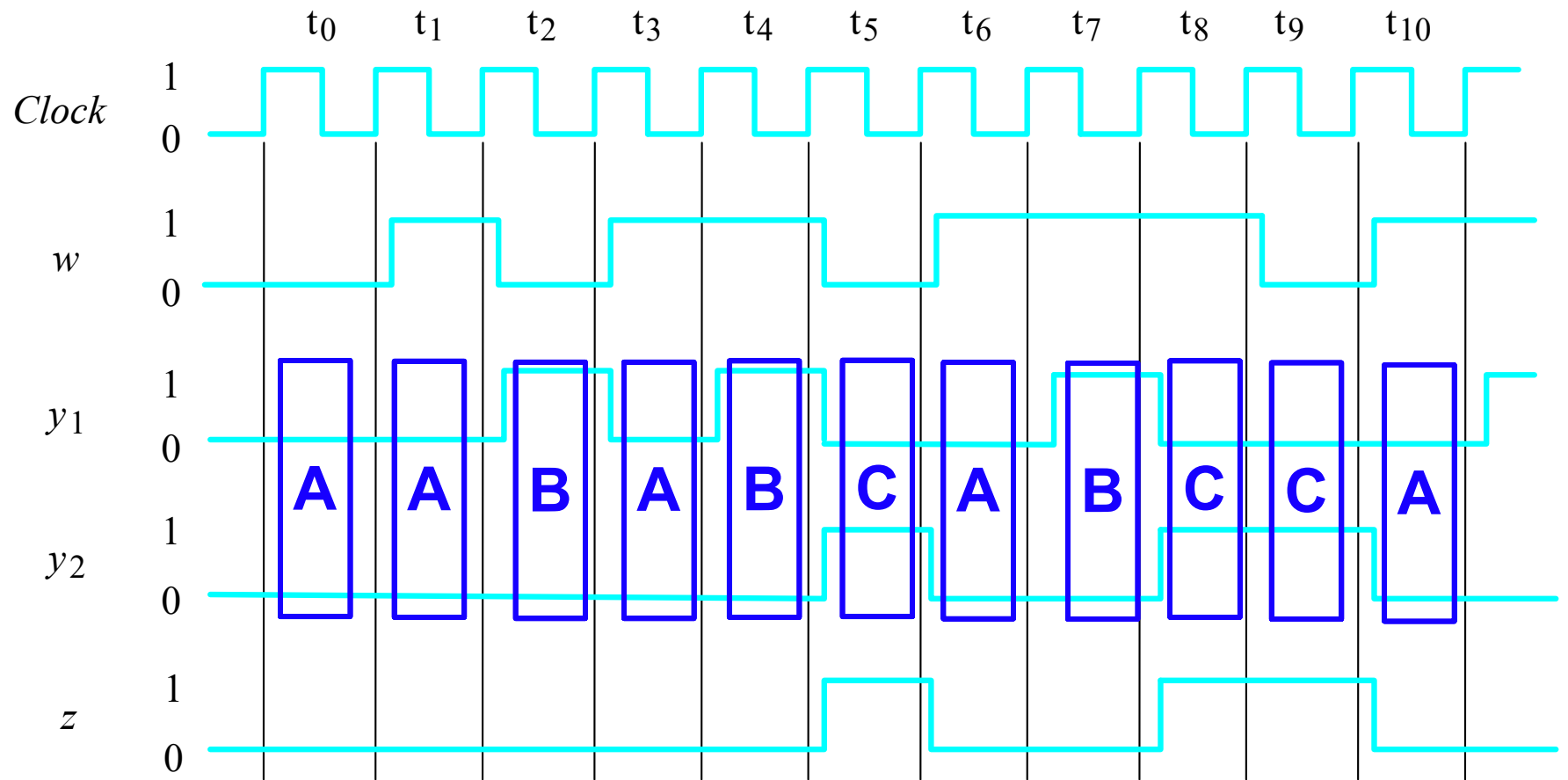




Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



Clockcycle:	$t_0$	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$	$t_8$	$t_9$	$t_{10}$
$w$ :	0	1	0	1	1	0	1	1	1	0	1
$z$ :	0	0	0	0	0	1	0	0	1	1	0



# Summary: Designing a Moore Machine

- Obtain the circuit specification.
- Derive a state diagram.
- Derive the state table.
- Decide on a state encoding.
- Encode the state table.
- Derive the output logic and next-state logic.
- Add a reset signal.

**Questions?**

**THE END**