CprE 281: Digital Logic
Midterm 2: Friday Oct 28, 2022

Name:

Lab Section:
(circle one)

Tue 2-5 (\#9)
Wed 7-10 (\# 6)
Wed 11-2 (\#13)
Wed 6-9 (\#12)

ID Number: $\qquad$

Thur 8-11 (\#15)
Thur 11-2 (\#11)
Thur 2-5 (\# 8)
Thur 5-8 (\# 7)

1. True/False Questions ( $10 \times 1 \mathrm{p}$ each $=10 \mathrm{p}$ )
(a) I forgot to write down my name, lab section, and student ID number.

TRUE / FALSE
(b) Any Boolean function can be implemented using only 2-to-4 decoders.

TRUE / FALSE
(c) A full-adder can be constructed using two half-adders and one XOR gate.
(d) The phrase "This is the way" refers to a code.

TRUE / FALSE
(e) A JK flip-flop has 2 inputs, 2 outputs, and can store 2 bits of information.
(f) The characteristic tables of the SR latch and the $\overline{\mathrm{S}} \overline{\mathrm{R}}$ latch are the same.
(g) A demultiplexer changes its outputs on the positive edge of the clock.

TRUE / FALSE
TRUE / FALSE
TRUE / FALSE
(h) A 32-bit hierarchical carry-lookahead adder sets the carries in 7 gate delay
(i) In 1's complement, the magnitude of a number with all bits set to 1 is 0 . TRUE / FALSE
(j) The ALU of the i281 CPU has one 8-bit ripple-carry adder.

TRUE / FALSE
2. Decoder Expressions (5p)

Draw the truth table for the 1-to-2 decoder with enable that is shown below. Then, write the Boolean expressions for the outputs $Z_{0}$ and $Z_{1}$ in terms of the inputs $X$ and $Y$.


## 3. Computations with Adders ( $5 \times 3 p$ each $=15$ p)

In all problems below, the binary numbers are stored in 2 's complement representation. For each of the following, assign either a 0 or a 1 to each input and output of the 5-bit adder such that it computes the given expression. The problem in a) is already solved.
a) $(+5)+(+6)=+11$

c) $(+14)+(-6)=$

d) $(-4)+(-5)=$

e) $(-7)-(+9)=$

f) $(-11)-(-2)=$

4. Basic Circuits ( $\mathbf{3} \times 5 \mathrm{p}$ each $=15 \mathrm{p}$ ).
(a) Draw the complete wiring diagram for a full-adder using only 2 -input logic gates. Clearly label all inputs and outputs.
(b) Draw the complete wiring diagram for a gated $D$ latch (with NOR gates for the latch). Clearly label all inputs and outputs.
(c) Draw the complete wiring diagram for a 1-to-2 demultiplexer using only NAND gates. Clearly label all inputs and outputs.
5. Number Conversions $(3 p+4 p+4 p+4 p=15 p)$
(a) Convert $191_{10}$ to hexadecimal.
(b) Convert $\mathbf{- 6 5 1 0}$ to an 8-bit binary number in 2's complement representation.
(c) Convert the following 32-bit float number (in IEEE 754 format) to decimal.

1100000100111000000000000000000
(d) Write down the 32-bit floating point representation (in IEEE 754 format) for -42.0

## 6. Flip-Flops and Timing Diagrams ( $3 \times 5 \mathrm{p}=15 \mathrm{p}$ )

Complete the timing diagram for the specified flip-flop such that the output $\mathbf{Q}$ will be as indicated. Assume that the input signal can change only on the vertical lines. Also, assume that the setup time $t_{s u}$ and the hold time $t_{h}$ are each equal to the width of one square.
a) Complete the timing diagram for the $D$ input to a positive-edge triggered $D$ flip-flop.
D

|  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Clock



Q

b) Complete the timing diagram for the $T$ input to a positive-edge triggered $T$ flip-flop.


Q

c) Complete the timing diagram for the K input to a positive-edge triggered JK flip-flop. If more than one value is possible for $J$ at any time, indicate that with a don't care (d).

7. Mystery Circuit ( $\mathbf{3 \times 5 p = 1 5 p )}$

a) Draw the truth table for the outputs $P$ and $Q$ as functions of $A, B$, and $C$.
b) Use K-maps to find the minimum-cost SOP expressions for $\mathbf{P}$ and $\mathbf{Q}$.
c) What type of familiar circuit is this equivalent to? Explain.
8. Implement a JK flip-flop using a T flip-flop. $(\mathbf{3 p}+4 \mathrm{p}+4 \mathrm{p}=10 \mathrm{p})$

You need to implement a JK flip-flop, but you only have a T flip-flop and some extra logic gates (ANDs, ORs, NOTs, and XORs). You also know the effects of the $J$ and $K$ inputs on the output $Q$. Hopefully, that is all you need to get the job done.
a) Draw the truth table for a positive-edge-triggered JK flip-flop. Hint: the inputs are $J, K$, and $Q(t)$; the output is $Q(t+1)$.
b) Add a column $T$ to the truth table and infer the value of $T$ that will cause the transition from $Q(t)$ to $Q(t+1)$. Use a K-map to derive the minimum-cost SOP expression for T. (4p)
c) Draw the circuit for the JK flip-flop, using the graphical symbol for a T flip-flop and any other necessary logic gates. Clearly label all inputs, outputs, and pins.
9. Alternative Implementation ( $3 \times 5 p$ each $=15 p$ )
a) Draw the truth table for the function $f(\mathbf{a}, \mathbf{b}, \mathbf{c})=\overline{\mathbf{a}+\mathbf{b}}+\overline{\mathbf{c}}(\overline{\mathbf{a}+\overline{\mathbf{b}}})$.
b) Implement this function using a minimal number of 2-to-1 multiplexers. You must use only 2-to-1 multiplexers and no other logic gates. Assume that the signals a, b, and care available only in their non-inverted form. You can also use the constants 0 and 1. Clearly label all inputs, outputs, and pins.
c) Implement this function using one 4-to-1 multiplexer and one XOR gate. Clearly label all inputs, outputs, and pins.

## 10. Error-Correcting Code ( $7 \mathrm{p}+\mathbf{8 p}=\mathbf{1 5 p}$ ) [Use the space on the next page if needed.]

(a) The Hamming $(7,4)$ code is a popular error-correcting code that is used to store or transfer data that can be corrupted by noise. Given four data bits ( $\mathrm{d} 4, \mathrm{~d} 3$, d 2 , and d 1 ) this code computes three parity bits ( $\mathrm{p} 3, \mathrm{p} 2$, and p 1 ) and interleaves them to construct a 7 -bit message $m 7, m 6, m 5, m 4, m 3, m 2, m 1$ from d4, d3, d2, p3, d1, p2, p1. Parity bit p1 is computed from d1, d2, and d4. Bit p2 from d1, d3, and d4. And bit p3 from d2, d3, and d4. In all cases, what is computed is even parity. That is, the parity bit is set to 0 if the number of 1 's in the three corresponding data bits is even. Otherwise, it is set to $\mathbf{1}$. Draw a circuit that encodes a 7 -bit message given 4 -bit input data. Explain your solution.
(b) When the message is received the code has the ability to detect and correct 1-bit errors. Three new parity bits are computed: P3, P2, and P1 (note the capital letter). P1 is 0 if the number of 1 's in $\mathrm{m} 7, \mathrm{~m} 5, \mathrm{~m} 3$, and m 1 is even. Otherwise, it is 1 . Similarly for $P 2$, which depends on $\mathrm{m} 7, \mathrm{~m} 6, \mathrm{~m} 3$, and m 2 . And P 3 , which is computed from $\mathrm{m} 7, \mathrm{mb}$, m 5 , and m 4 . If $\mathbf{P 3}=\mathbf{P 2}=\mathbf{P 1}=\mathbf{0}$, then the message was not corrupted. When $\mathbf{P} 3, \mathbf{P} 2, \mathbf{P} 1$ is interpreted as a binary number it points to the 1-based index of the bit in $\mathrm{m} 7, \mathrm{~m} 6, \mathrm{m5}, \mathrm{m4}, \mathrm{~m} 3, \mathrm{~m} 2, \mathrm{~m} 1$ that is wrong. This bit can be corrected by simply flipping its value (from 1 to 0 or from 0 to 1). Draw a circuit that uses this method to detect and correct 1-bit errors. Explain.

| Question | Max | Score |
| :--- | ---: | :--- |
| 1. True/False | 10 |  |
| 2. Decoder Expressions | 5 |  |
| 3. Computations with Adders | 15 |  |
| 4. Basic Circuits | 15 |  |
| 5. Number Conversions | 15 |  |
| 6. Flip-Flops | 15 |  |
| 7. Mystery Circuit | 15 |  |
| 8. JK flip-flop with T flip-flop | 10 |  |
| 9. Alternative Implementation | 15 |  |
| 10. Error-Correcting Code | 15 |  |
| TOTAL: | 130 |  |

