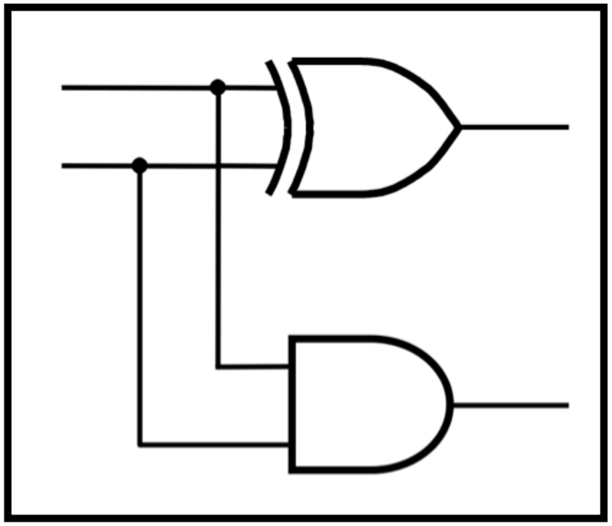


CprE 281: Digital Logic

Instructor: Alexander Stoytchev

http://www.ece.iastate.edu/~alexs/classes/

Examples of Solved Problems

CprE 281: Digital Logic
Iowa State University, Ames, IA

Administrative Stuff

- **HW4 is due today**

Administrative Stuff

- **HW5 is out**
- **It is due on Monday Sep 26 @ 10pm.**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**
- **You can use this as a preparation for the exam.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 23.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **It will be in this room.**
- **Review session: This Wednesday during lecture**

Topics for the Midterm Exam

- Binary Numbers
- Octal Numbers
- Hexadecimal Numbers
- Conversion between the different number systems
- Truth Tables
- Boolean Algebra
- Logic Gates
- Circuit Synthesis with AND, OR, NOT
- Circuit Synthesis with NAND, NOR
- Converting an AND/OR/NOT circuit to NAND circuit
- Converting an AND/OR/NOT circuit to NOR circuit
- SOP and POS expressions

Topics for the Midterm Exam

- Mapping a Circuit to Verilog code
- Mapping Verilog code to a circuit

- Multiplexers
- Venn Diagrams
- K-maps for 2, 3, and 4 variables

- Minimization of Boolean expressions using theorems
- Minimization of Boolean expressions with K-maps

- Incompletely specified functions (with don't cares)
- Functions with multiple outputs

- Something from Star Wars

All possible Boolean functions with two input variables

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

constant 0

constant 1

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

NOR(x, y)

OR(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

$m_1 = \bar{x}\, y$

$M_1 = x + \bar{y}$

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

NOT(x)

x

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

$m_2 = x\,\overline{y}$

$M_2 = \overline{x} + y$

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

NOT(y)

y

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

XOR(x, y)

XNOR(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

NAND(x, y)   AND(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
		0	NOR	m_1	$\bar{x}$	m_2	$\bar{y}$	XOR	NAND	AND	XNOR	y	M_2	x	M_1	OR	1

There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0 m_0 m_1 $\overline{x}$ m_2 $\overline{y}$ XOR M_3 m_3 XNOR y M_2 x M_1 M_0 1

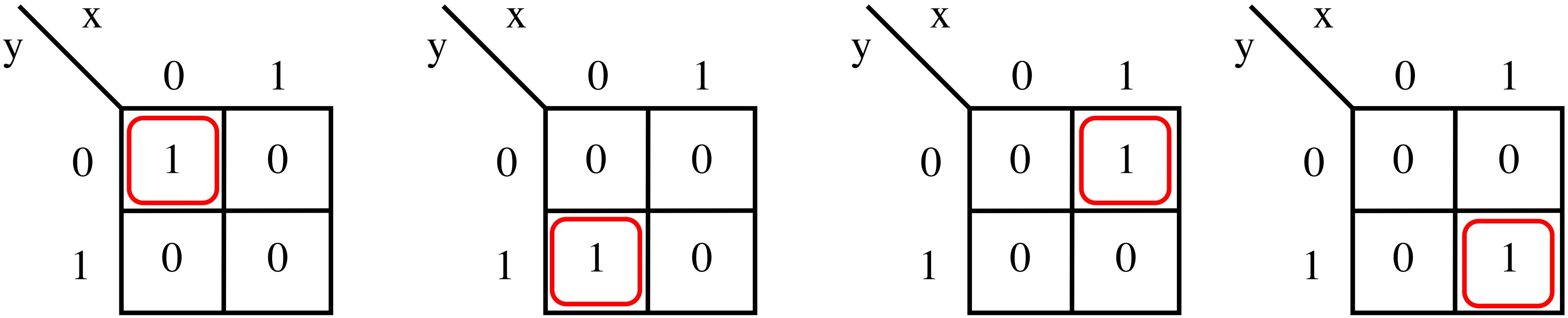
There are 16 possible Boolean functions with two input variables

x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
		0	m_0	m_1	$\overline{x}$	m_2	$\overline{y}$	$m_1 + m_2$	M_3	m_3	$M_1 \bullet M_2$	y	M_2	x	M_1	M_0	1

There are 16 possible Boolean functions with two input variables

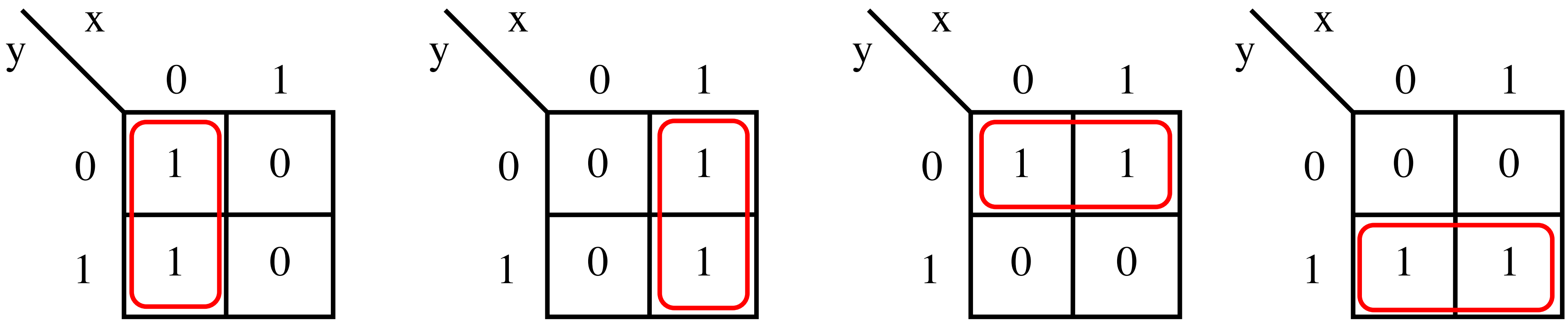
x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
		0	m_0	m_1	$m_0 + m_1$	m_2	$m_0 + m_2$	$m_1 + m_2$	M_3	m_3	$M_1 \bullet M_2$	$M_0 \bullet M_2$	M_2	$M_0 \bullet M_1$	M_1	M_0	1

These are all valid groupings



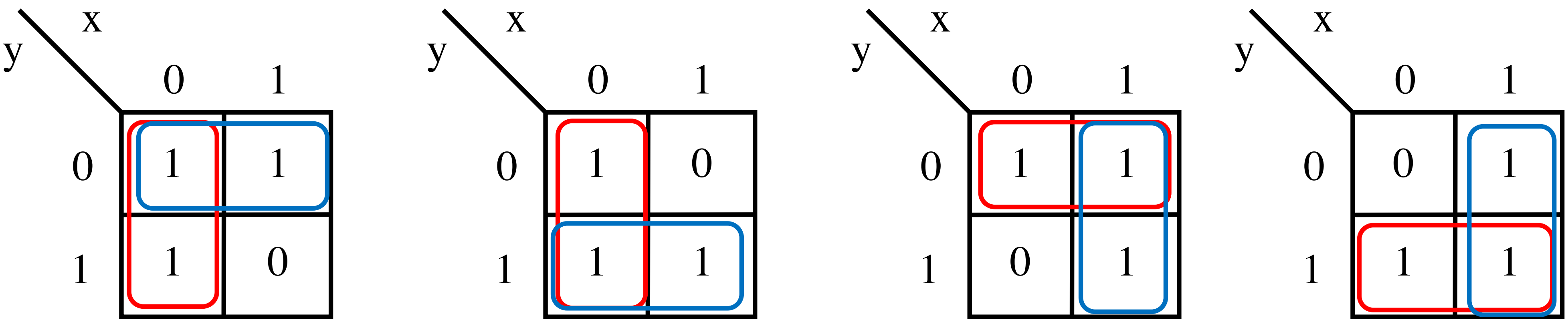
$\bar{x}\,\bar{y}$ $\bar{x}\,y$ $x\,\bar{y}$ $x\,y$

These are all valid groupings



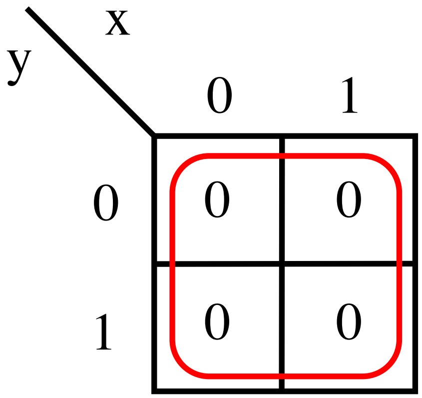
$\bar{x}$ $\quad x \quad$ $\bar{y}$ $\quad y$

These are all valid groupings

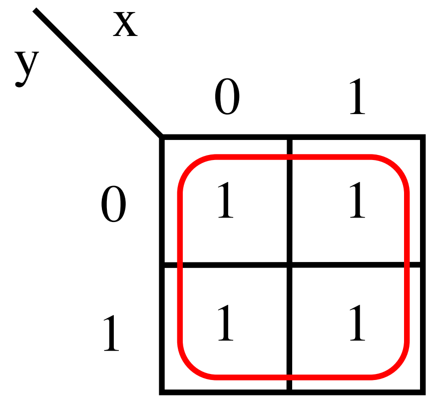


$\overline{x} + \overline{y}$ $\overline{x} + y$ $x + \overline{y}$ $x + y$

These are valid too

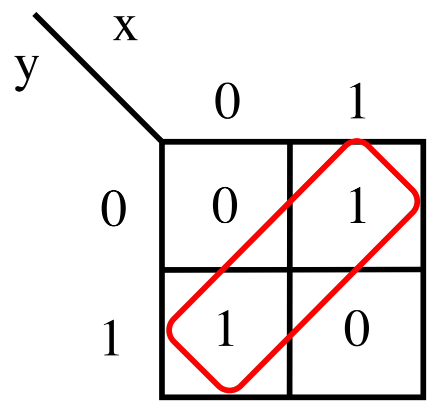


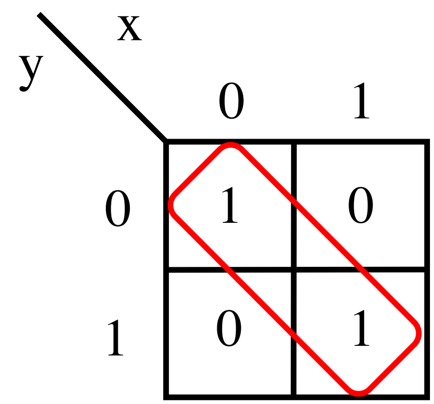
constant 0



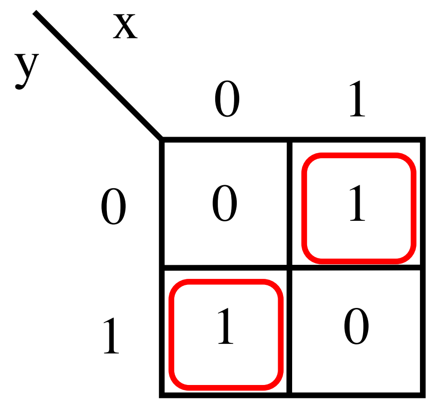
constant 1

These are not valid groupings, but they correspond to XOR and XNOR

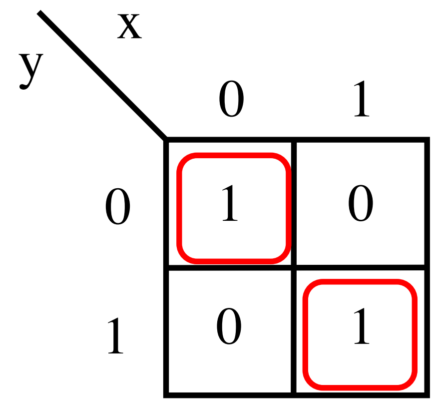




These are not valid groupings, but they correspond to XOR and XNOR



XOR(x, y)

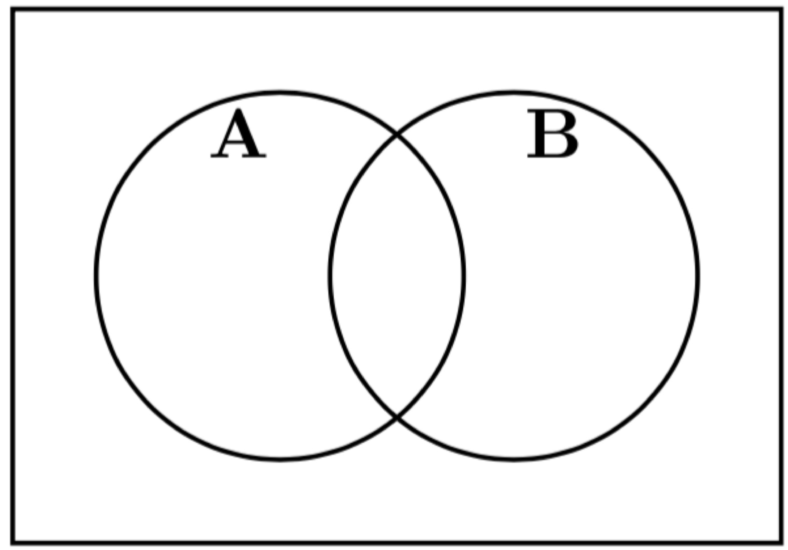


XNOR(x, y)

The Link Between Truth Tables and Venn Diagrams

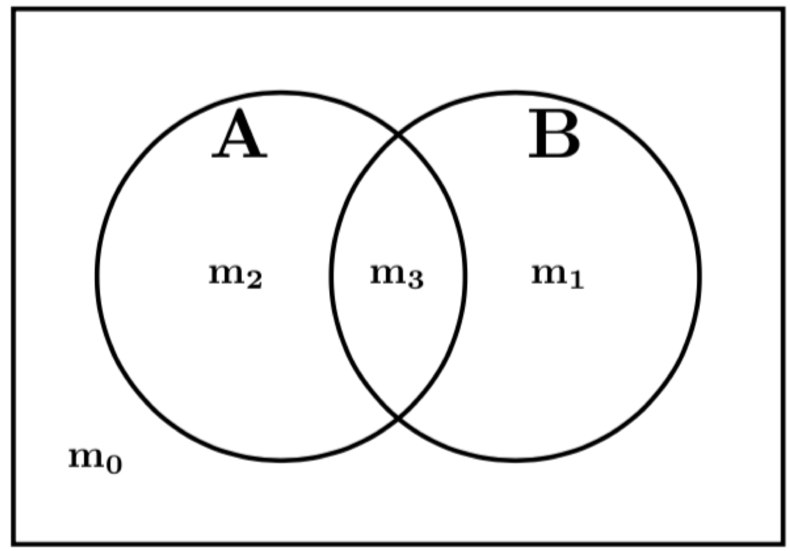
Place the minterms on the Venn diagram

A	B	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3



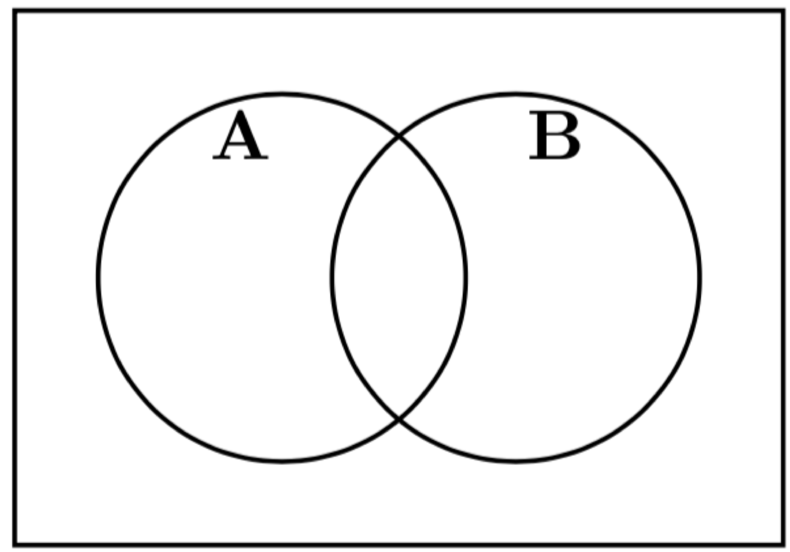
Place the minterms on the Venn diagram

A	B	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3



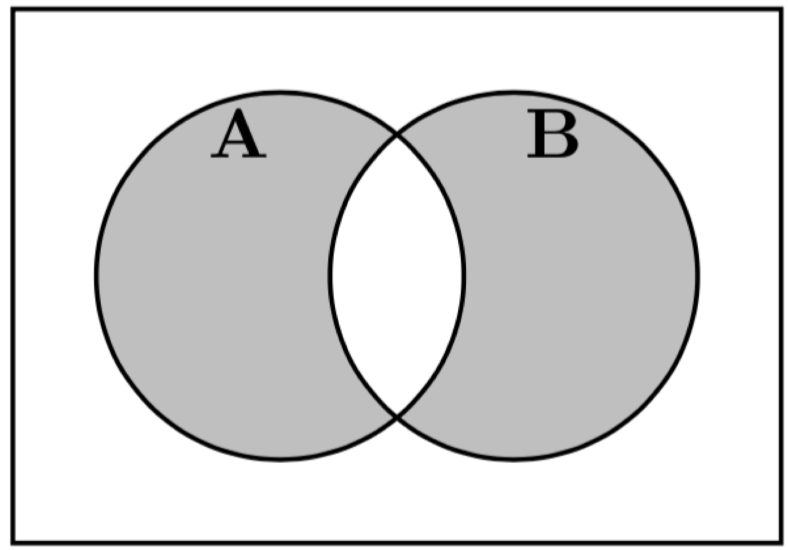
Color the Venn diagram for XOR

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



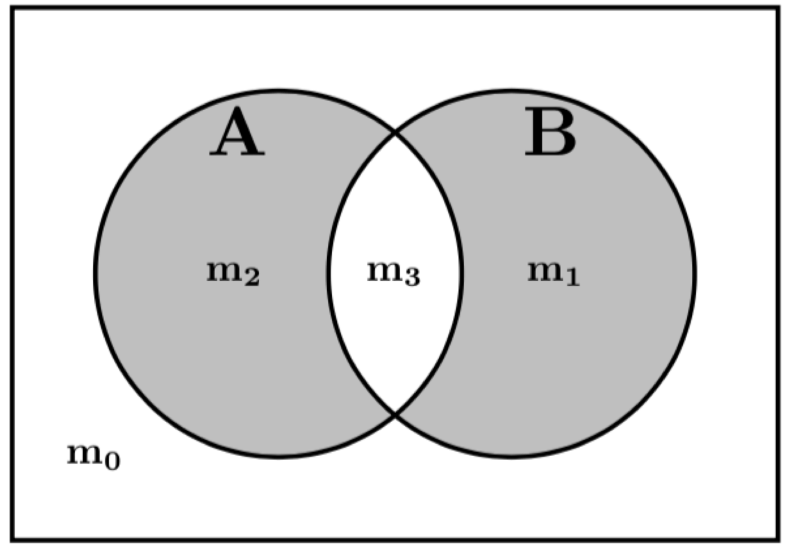
Color the Venn diagram for XOR

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



Color the Venn diagram for XOR

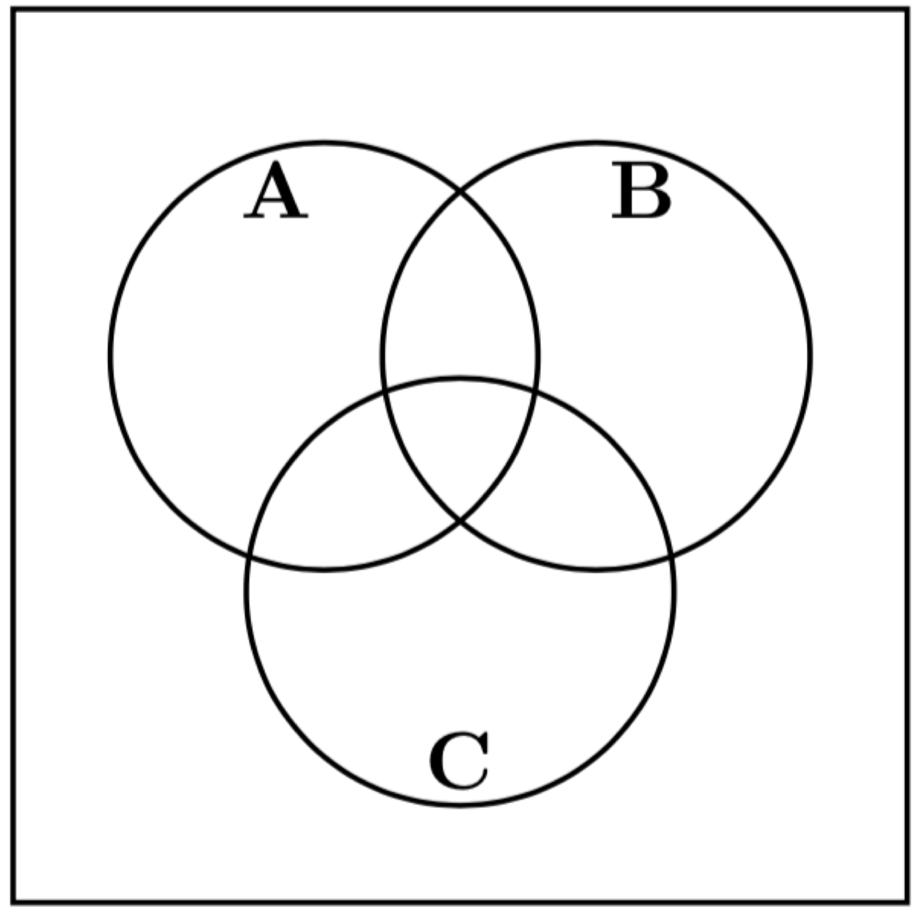
	A	B	F
m_0	0	0	0
m_1	0	1	1
m_2	1	0	1
m_3	1	1	0



$$F = \overline{A}\,B + A\,\overline{B}$$

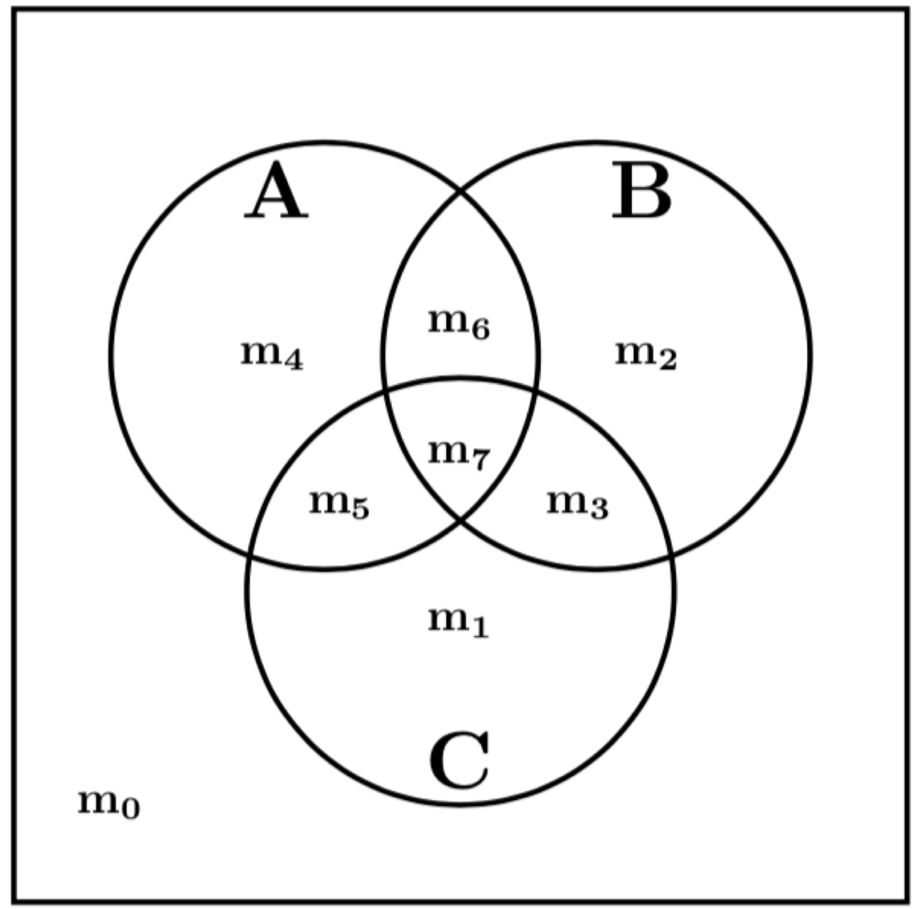
Place the minterms on the Venn diagram

A	B	C	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



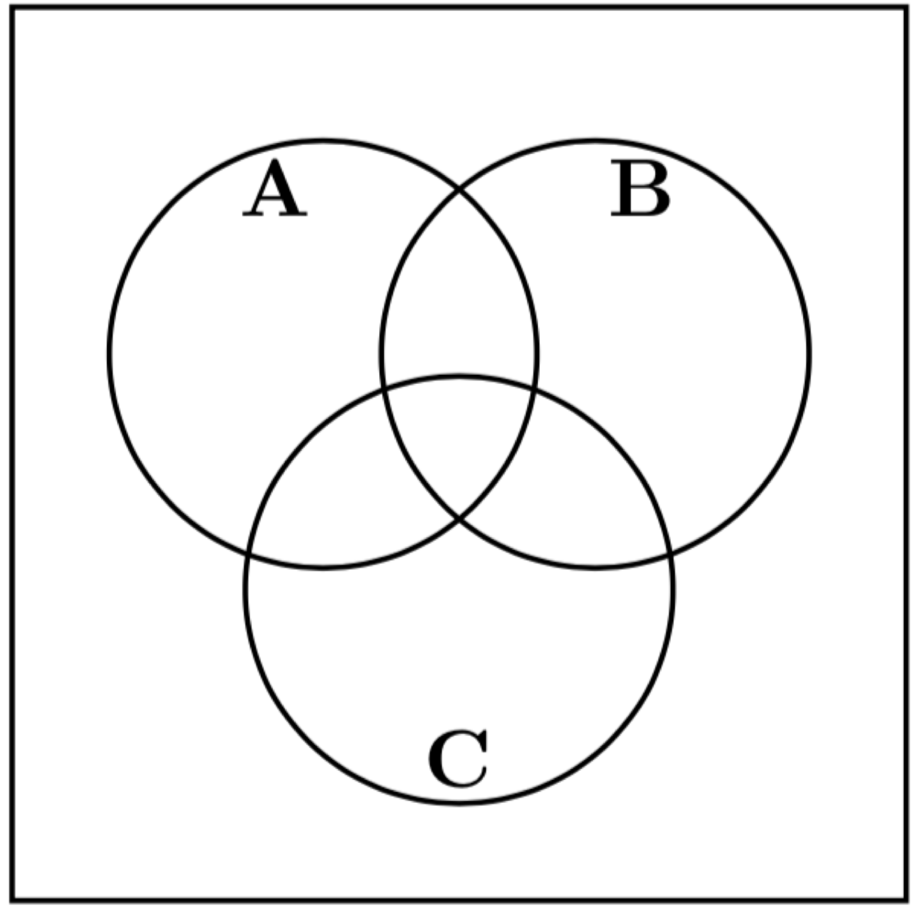
Place the minterms on the Venn diagram

A	B	C	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



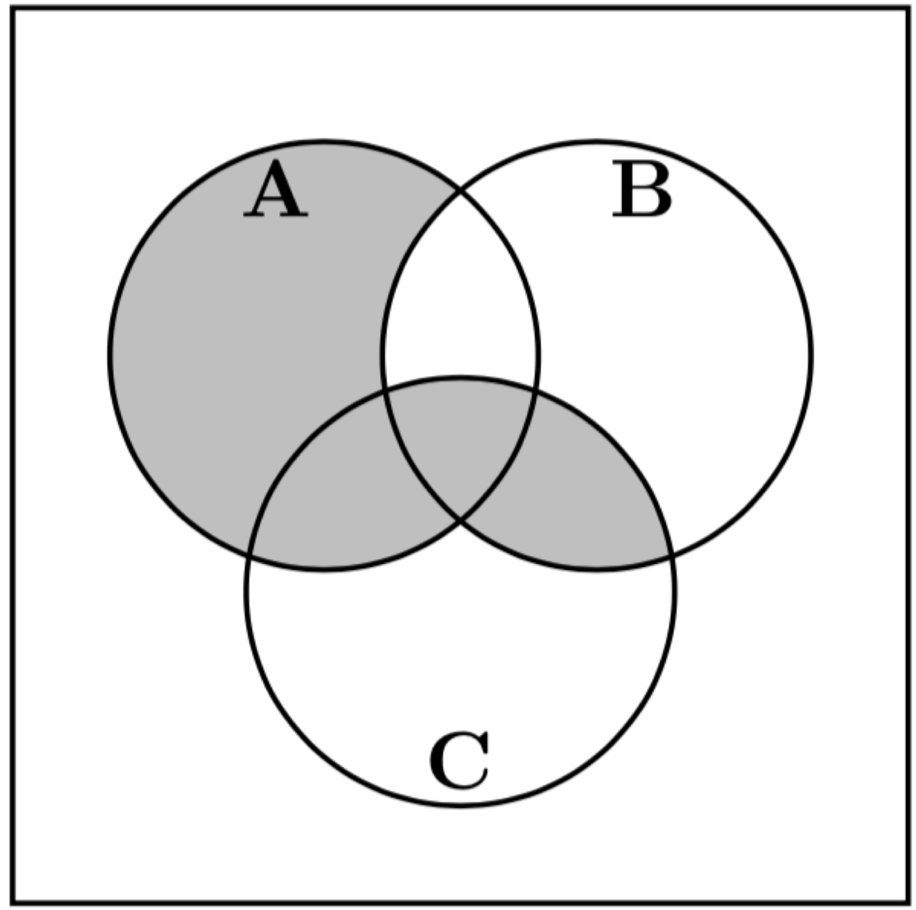
Color the Venn diagram for this function

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



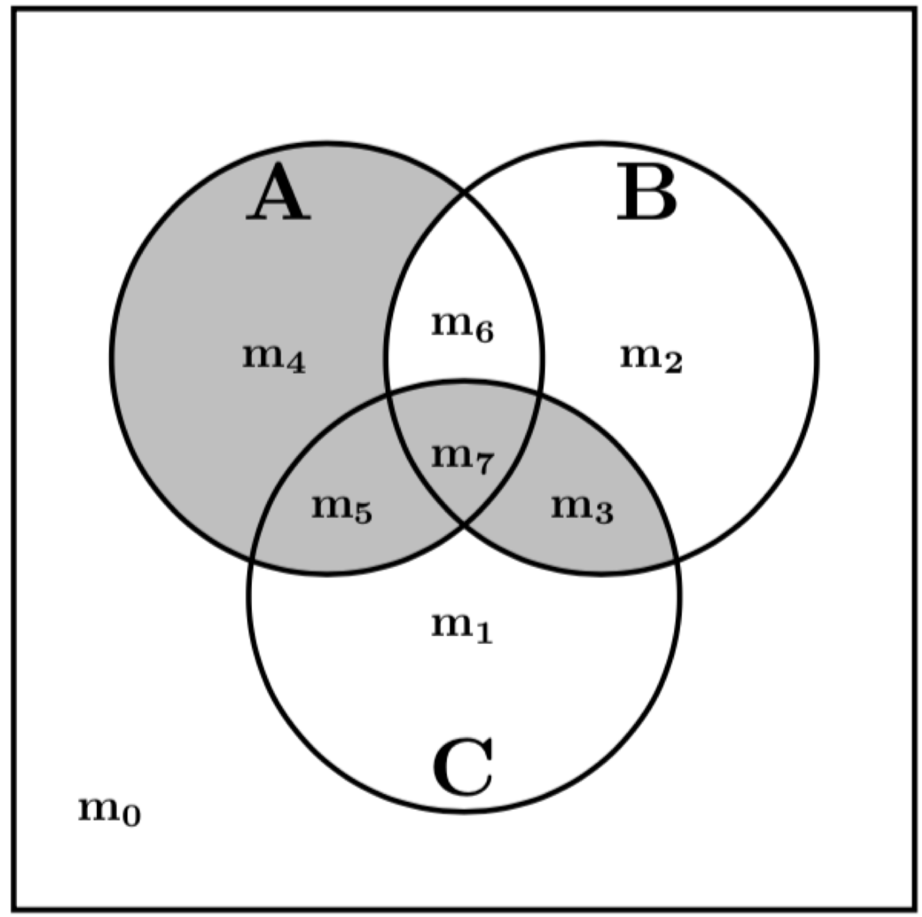
Color the Venn diagram for this function

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



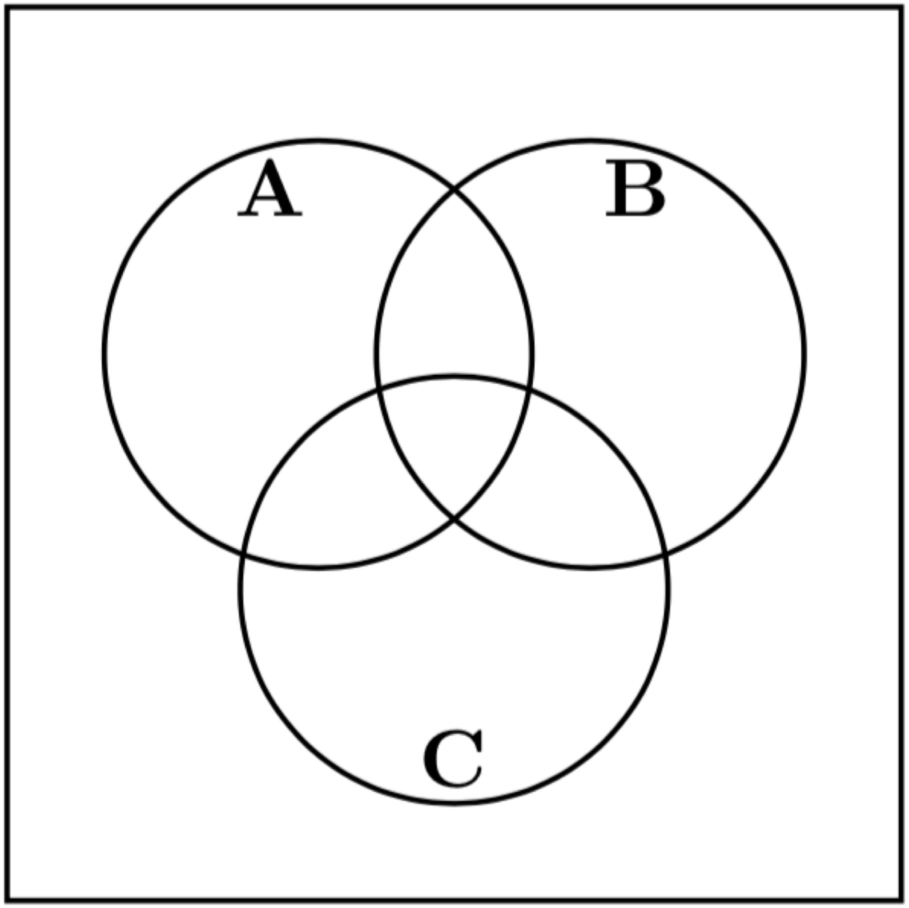
Place the minterms on the Venn diagram

	A	B	C	F
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1



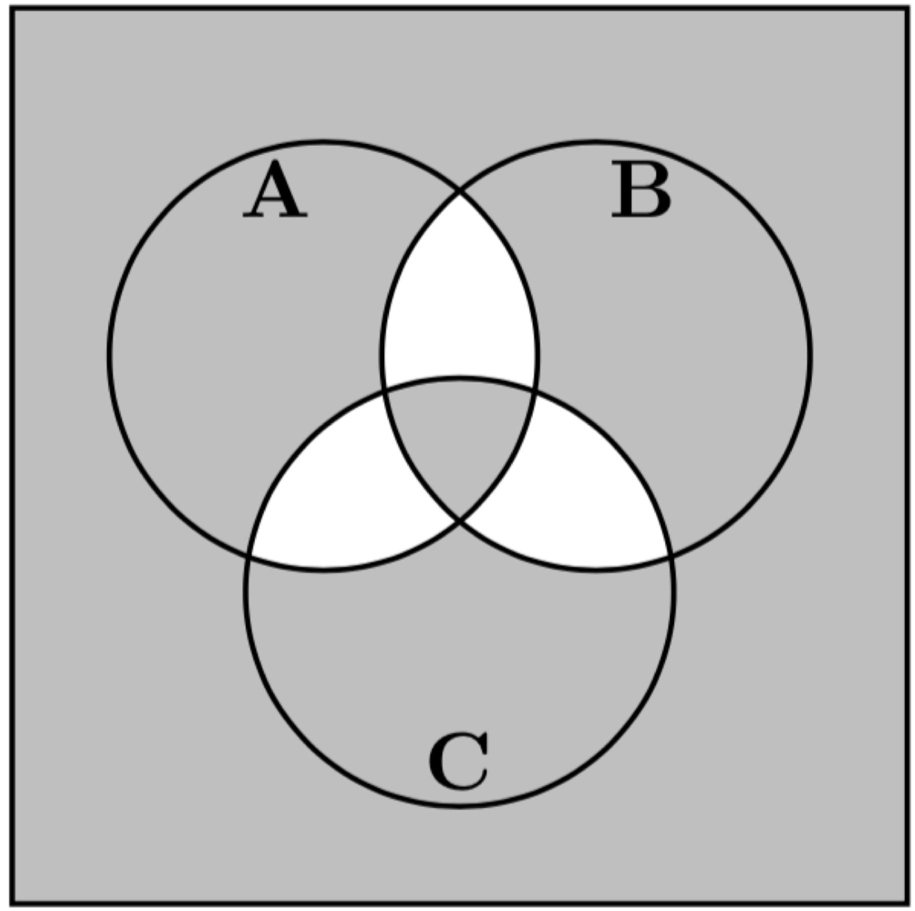
Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



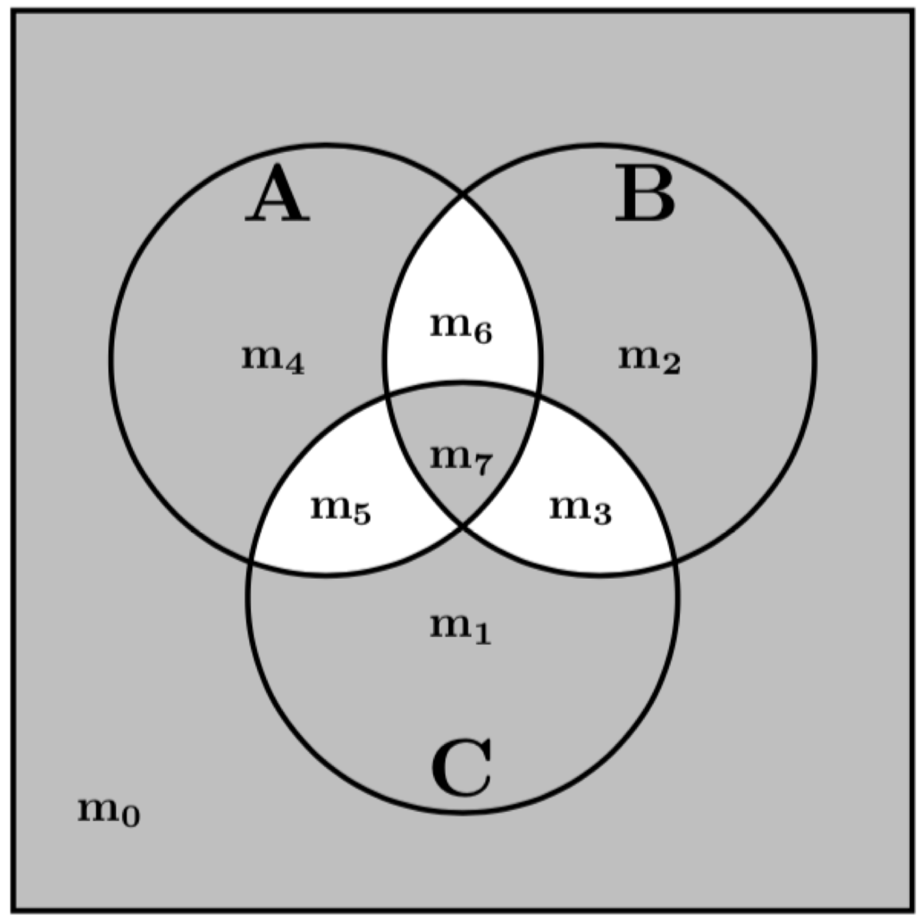
Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

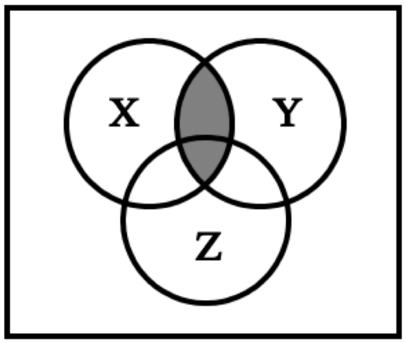


Place the minterms on the Venn diagram

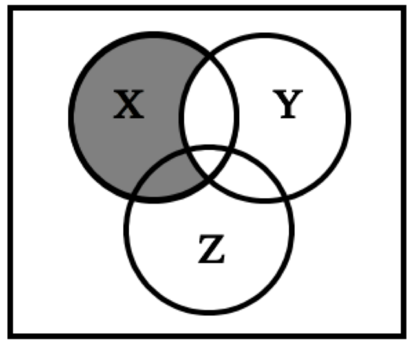
	A	B	C	F
m_0	0	0	0	1
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	0
m_4	1	0	0	1
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1



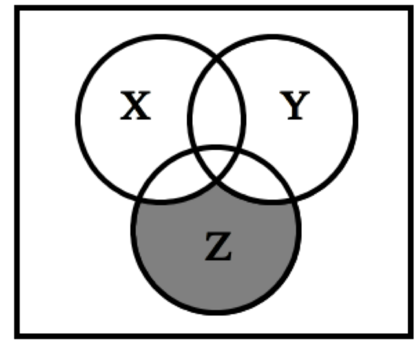
Write the expression that is represented by each of the three Venn diagrams:



(A)

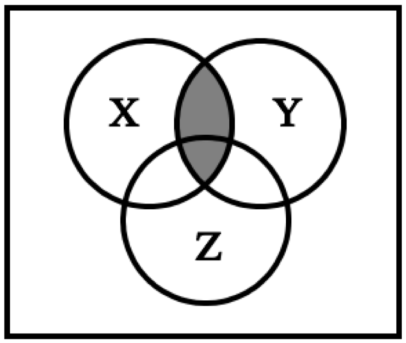


(B)

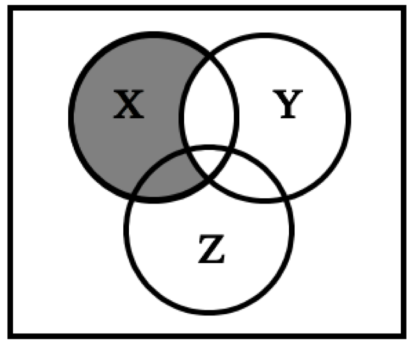


(C)

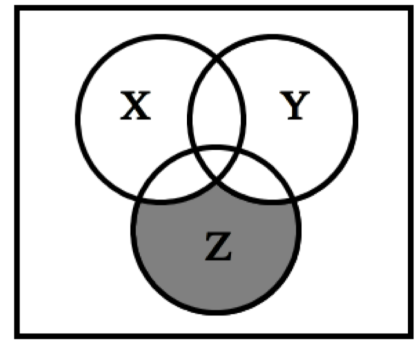
Write the expression that is represented by each of the three Venn diagrams:



(A)



(B)



(C)

$X\,Y$

$X\,\overline{Y}$

$\overline{X}\,\overline{Y}\,Z$

Example 1

Determine if the following equation is valid

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \overset{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x_1}\,\overline{x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	f
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x_1}\,\overline{x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	f
0	0	0	0	1	0	0	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	0	1	0	
4	1	0	0	0	0	1	
5	1	0	1	0	0	1	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x}_1 \overline{x}_3$	$x_2 x_3$	$x_1 \overline{x}_2$	f
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1} x_2$	$x_1 x_3$	$\overline{x_2}\ \overline{x_3}$	f
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1} x_2$	$x_1 x_3$	$\overline{x_2}\ \overline{x_3}$	f
0	0	0	0	0	0	1	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	1	0	0	
4	1	0	0	0	0	1	
5	1	0	1	0	1	0	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1}x_2$	x_1x_3	$\overline{x_2}\,\overline{x_3}$	f
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

f	f
1	1
0	0
1	1
1	1
1	1
1	1
0	0
1	1

They are equal.

Example 2

Design the minimum-cost product-of-sums expression for the function

$$f(x_1, x_2, x_3) = \Sigma\, m(0, 2, 4, 5, 6, 7)$$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

[Figure 2.22 from the textbook]

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

The function is 1 for these rows

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \overline{x}_1\overline{x}_2\overline{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \overline{x}_1\overline{x}_2 x_3$	$M_1 = x_1 + x_2 + \overline{x}_3$
2	0	1	0	$m_2 = \overline{x}_1 x_2\overline{x}_3$	$M_2 = x_1 + \overline{x}_2 + x_3$
3	0	1	1	$m_3 = \overline{x}_1 x_2 x_3$	$M_3 = x_1 + \overline{x}_2 + \overline{x}_3$
4	1	0	0	$m_4 = x_1\overline{x}_2\overline{x}_3$	$M_4 = \overline{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\overline{x}_2 x_3$	$M_5 = \overline{x}_1 + x_2 + \overline{x}_3$
6	1	1	0	$m_6 = x_1 x_2\overline{x}_3$	$M_6 = \overline{x}_1 + \overline{x}_2 + x_3$
7	1	1	1	$m_7 = x_1 x_2 x_3$	$M_7 = \overline{x}_1 + \overline{x}_2 + \overline{x}_3$

The function is 1 for these rows

The function is 0 for these rows

Two different ways to specify the same function f of three variables

$$f(x_1, x_2, x_3) = \Sigma\, m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \Pi\, M(1, 3)$$

Two different ways to specify the same function f of three variables

$$f(x_1, x_2, x_3) = \Sigma\, m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \Pi\, M(1, 3)$$

$$= M_1 \bullet M_3$$

$$= (\, x_1 + x_2 + \overline{x_3}) \bullet (\, x_1 + \overline{x_2} + \overline{x_3})$$

The Minimum POS Expression

$$f(x_1, x_2, x_3) = (x_1 + x_2 + \overline{x_3}) \bullet (x_1 + \overline{x_2} + \overline{x_3})$$

$$= (x_1 + \overline{x_3} + x_2) \bullet (x_1 + \overline{x_3} + \overline{x_2})$$

$$= (x_1 + \overline{x_3})$$

Hint: Use the following Boolean Algebra theorem

14b. $(x + y) \bullet (x + \overline{y}) = x$

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Alternative Solution Using K-Maps

$x_3 \backslash x_1x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

Alternative Solution Using K-Maps

$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	1	1
1	0	0	1	1

Alternative Solution Using K-Maps

$x_3 \backslash x_1x_2$	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$(x_1 + \overline{x}_3)$

Example 3

Problem: A circuit that controls a given digital system has three inputs: x_1, x_2, and x_3. It has to recognize three different conditions:

- Condition A is true if x_3 is true and either x_1 is true or x_2 is false
- Condition B is true if x_1 is true and either x_2 or x_3 is false
- Condition C is true if x_2 is true and either x_1 is true or x_3 is false

The control circuit must produce an output of 1 if at least two of the conditions A, B, and C are true. Design the simplest circuit that can be used for this purpose.

Condition A

Condition A is true if x_3 is true and either x_1 is true or x_2 is false

Condition A

Condition A is true if x_3 is true and either x_1 is true or x_2 is false

$$A = x_3(x_1 + \bar{x}_2) = x_3x_1 + x_3\bar{x}_2$$

Condition B

Condition B is true if x_1 is true and either x_2 or x_3 is false

Condition B

Condition B is true if x_1 is true and either x_2 or x_3 is false

$$B = x_1(\overline{x}_2 + \overline{x}_3) \;=\; x_1\overline{x}_2 + x_1\overline{x}_3$$

Condition C

Condition C is true if x_2 is true and either x_1 is true or x_3 is false

Condition C

Condition C is true if x_2 is true and either x_1 is true or x_3 is false

$$C = x_2(x_1 + \bar{x}_3) \;=\; x_2 x_1 + x_2 \bar{x}_3$$

The output of the circuit can be expressed as

f = AB + AC + BC

$$\begin{aligned} AB &= (x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3) \\ &= x_3x_1x_1\bar{x}_2 + x_3x_1x_1\bar{x}_3 + x_3\bar{x}_2x_1\bar{x}_2 + x_3\bar{x}_2x_1\bar{x}_3 \\ &= x_3x_1\bar{x}_2 + 0 + x_3\bar{x}_2x_1 + 0 \\ &= x_1\bar{x}_2x_3 \end{aligned}$$

The output of the circuit can be expressed as

f = AB + AC + BC

$$
\begin{aligned}
AC &= (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3) \\
&= x_3x_1x_2x_1 + x_3x_1x_2\bar{x}_3 + x_3\bar{x}_2x_2x_1 + x_3\bar{x}_2x_2\bar{x}_3 \\
&= x_3x_1x_2 + 0 + 0 + 0 \\
&= x_1x_2x_3
\end{aligned}
$$

The output of the circuit can be expressed as

$$f = AB + AC + \boxed{BC}$$

$$\begin{aligned} BC &= (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3) \\ &= x_1\bar{x}_2x_2x_1 + x_1\bar{x}_2x_2\bar{x}_3 + x_1\bar{x}_3x_2x_1 + x_1\bar{x}_3x_2\bar{x}_3 \\ &= 0 + 0 + x_1\bar{x}_3x_2 + x_1\bar{x}_3x_2 \\ &= x_1x_2\bar{x}_3 \end{aligned}$$

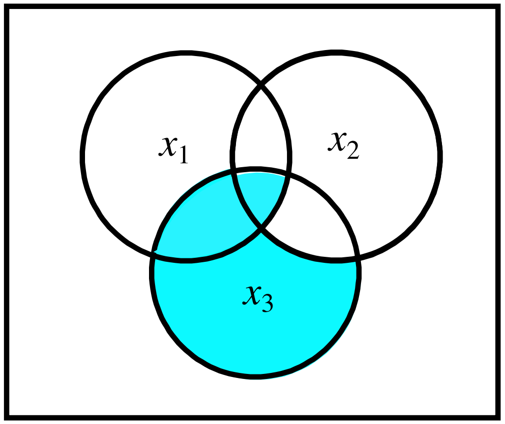
Finally, we get

$$
\begin{aligned}
f &= x_1\bar{x}_2x_3 + x_1x_2x_3 + x_1x_2\bar{x}_3 \\
&= x_1(\bar{x}_2 + x_2)x_3 + x_1x_2(x_3 + \bar{x}_3) \\
&= x_1x_3 + x_1x_2 \\
&= x_1(x_3 + x_2)
\end{aligned}
$$

Example 4

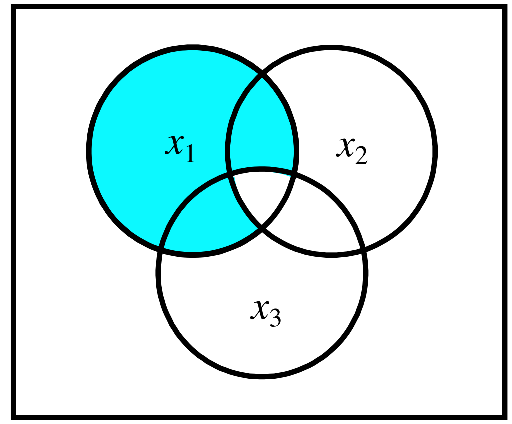
Solve the previous problem using Venn diagrams.

Venn Diagrams
(find the areas that are shaded at least two times)



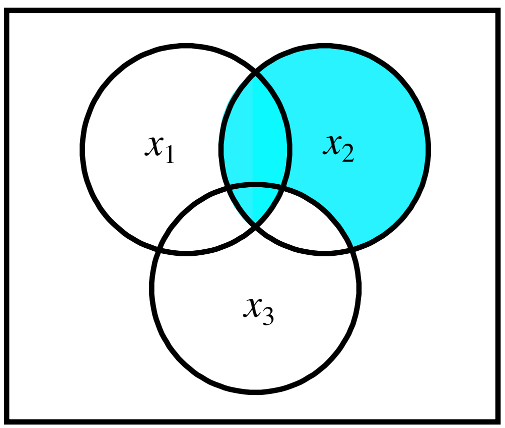
(a) Function *A:*

$$x_3x_1 + x_3\bar{x}_2$$



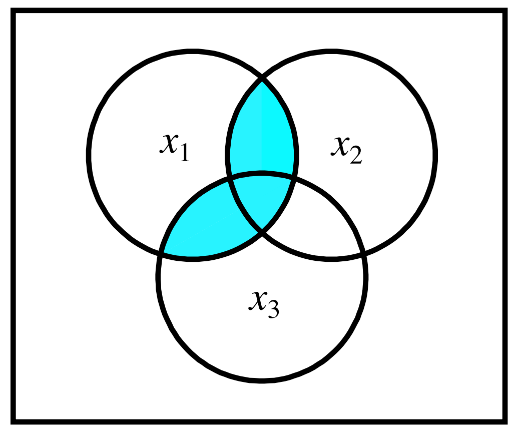
(b) Function *B*

$$x_1\bar{x}_2 + x_1\bar{x}_3$$



(c) Function *C*

$$x_2x_1 + x_2\bar{x}_3$$



(d) Function *f*

$$x_1(x_3 + x_2)$$

[Figure 2.66 from the textbook]

Example 5

Design the minimum-cost SOP and POS expression for the function

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

Let’s Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

Let’s Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

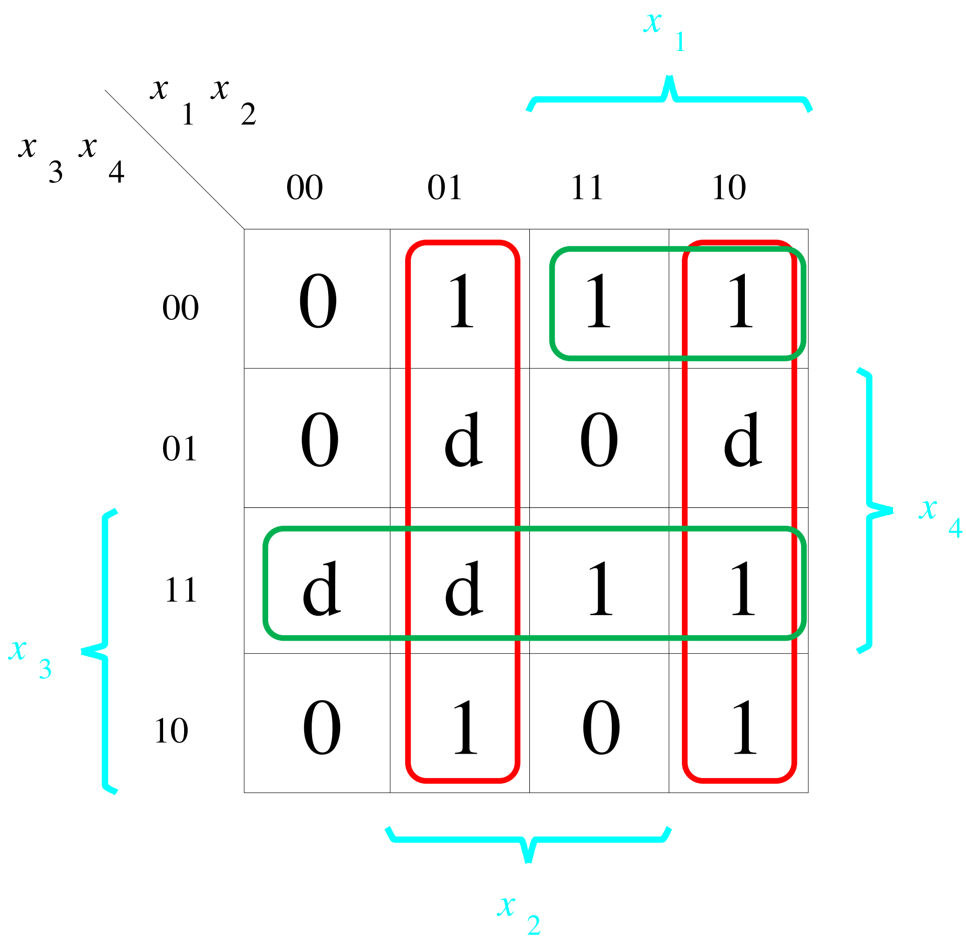
Let’s Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



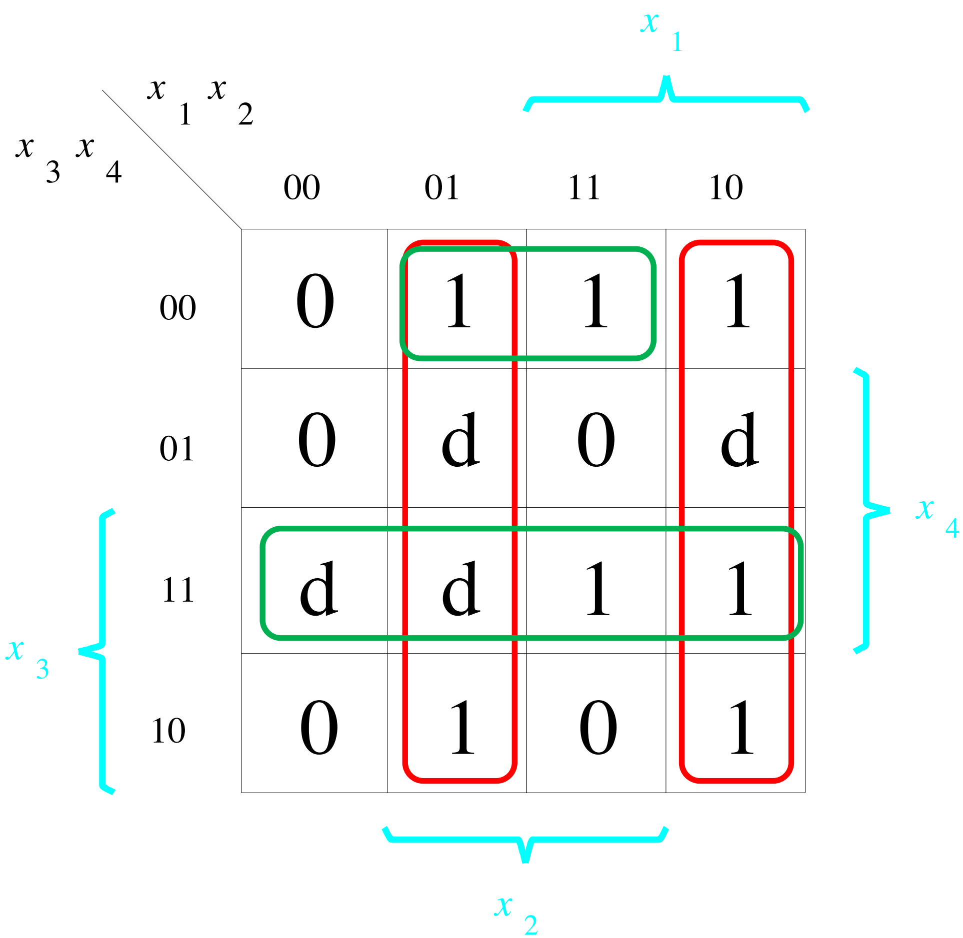
Let's Use a K-Map

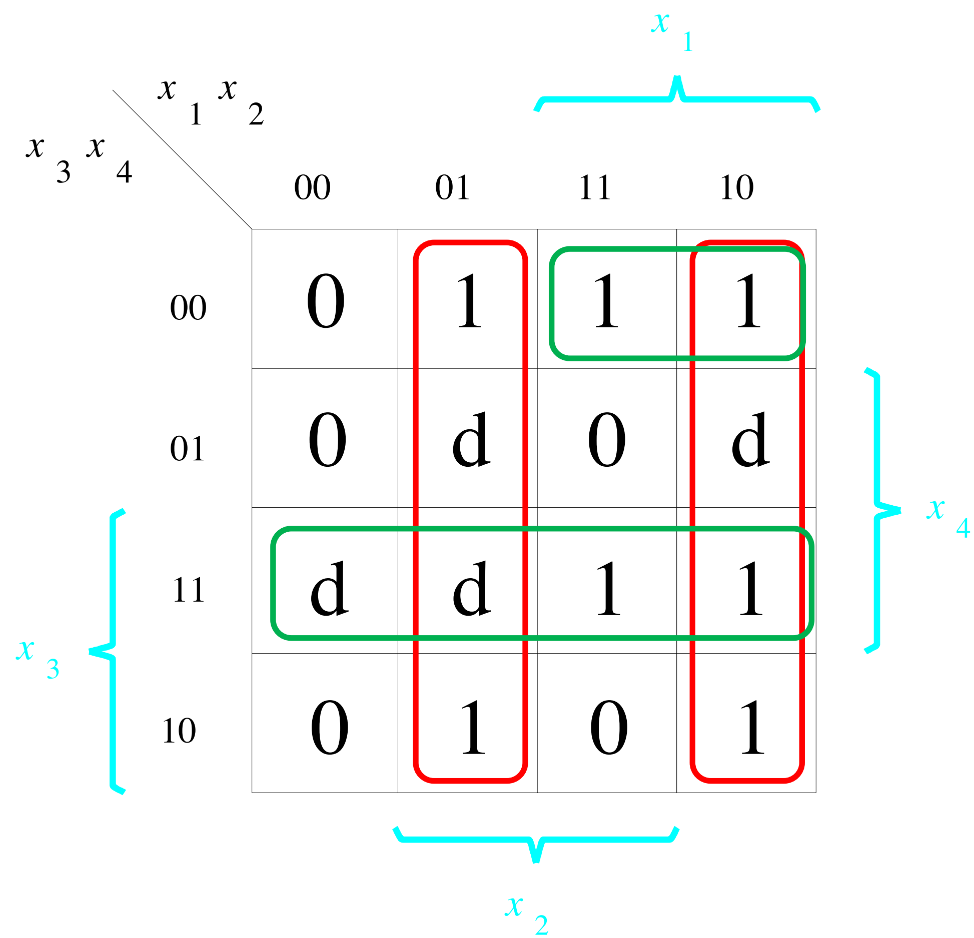
$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

x_3x_4 \ x_1x_2	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

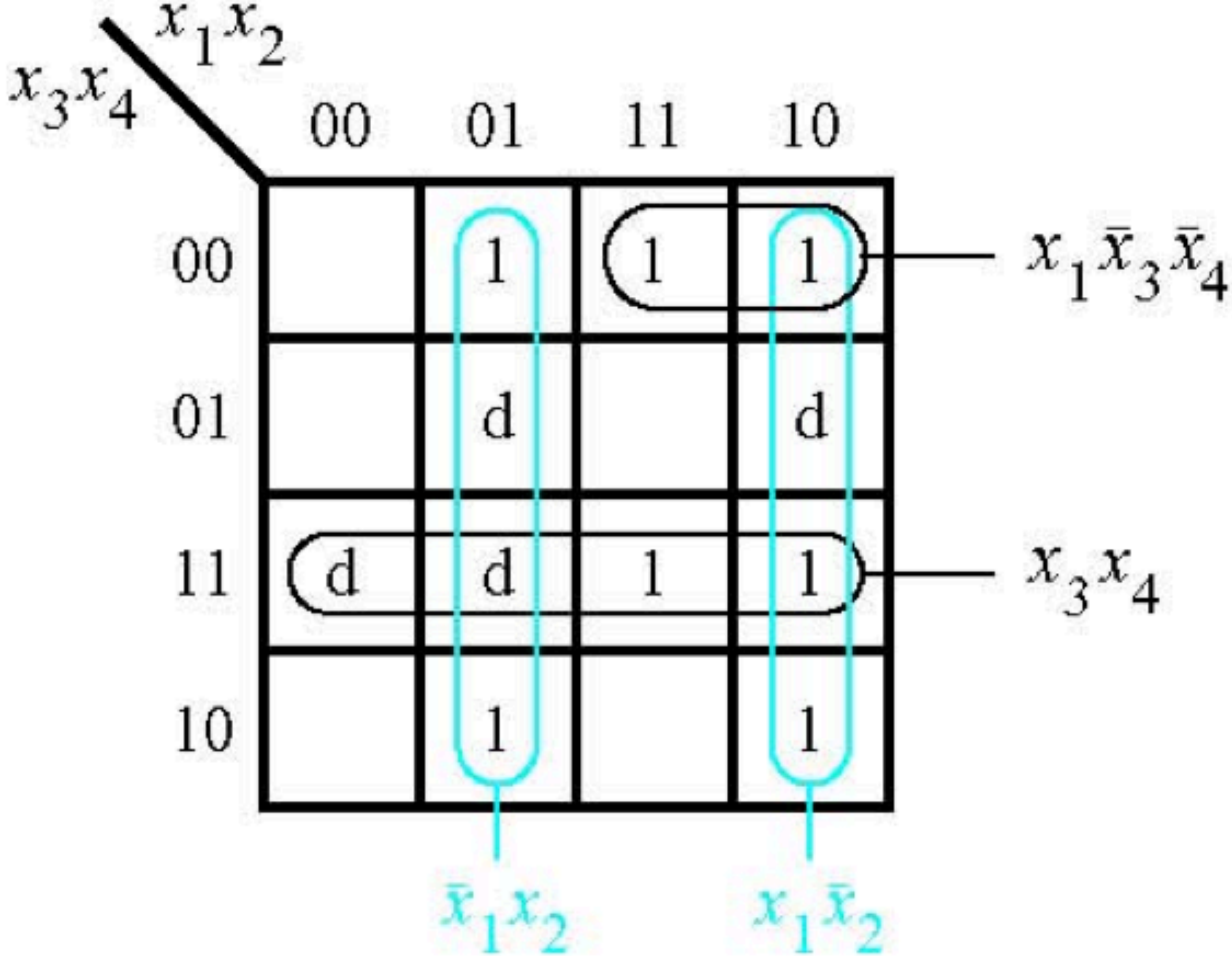
Two Alternative Solutions

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$





The SOP Expression



$$f = \overline{x}_1 x_2 + x_1 \overline{x}_2 + x_1 \overline{x}_3 \overline{x}_4 + x_3 x_4$$

[Figure 2.67a from the textbook]

What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

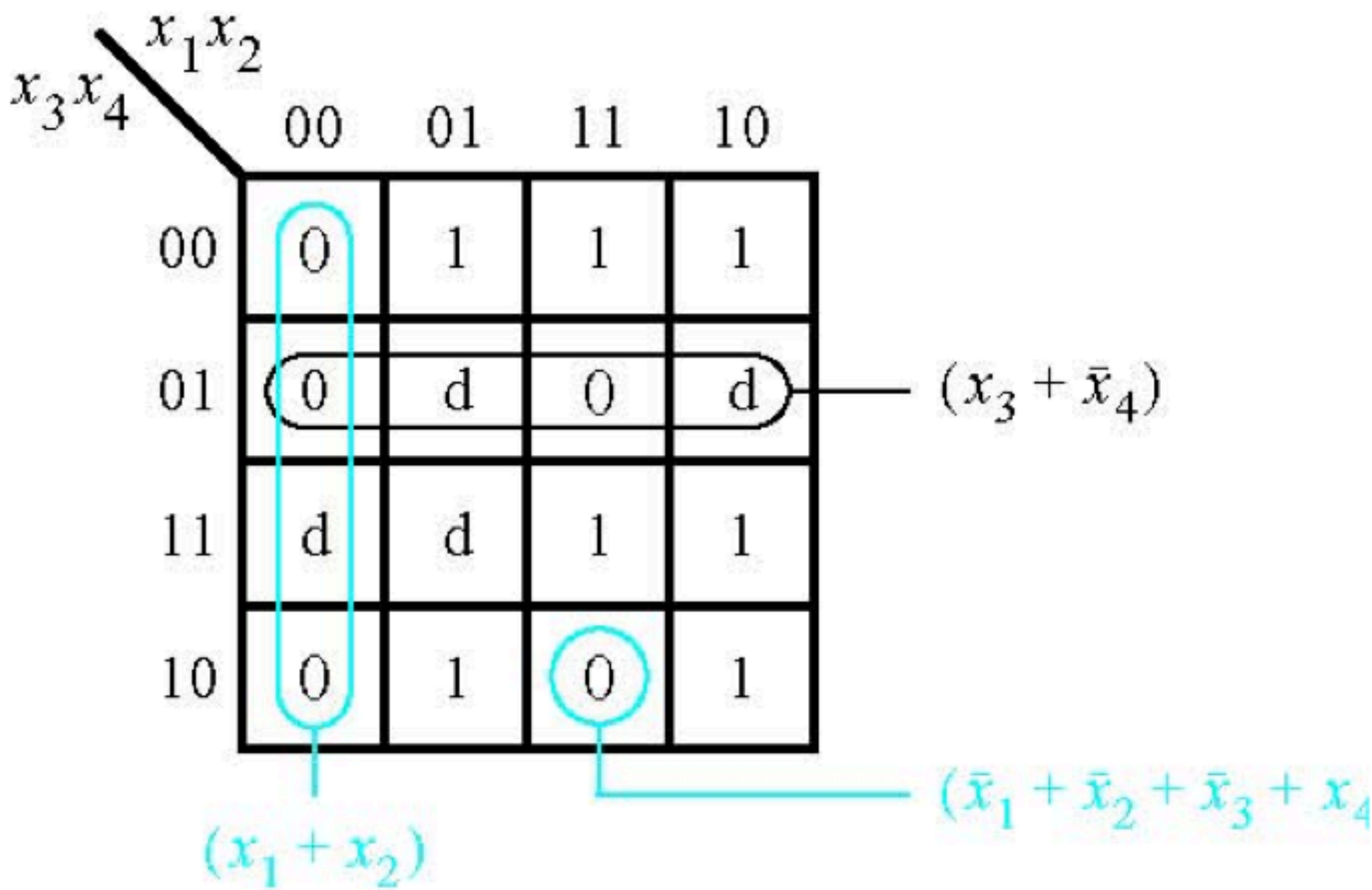
$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \Sigma\, m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

The POS Expression



$$f = (x_1 + x_2) \bullet (x_3 + \overline{x}_4) \bullet (\overline{x}_1 + \overline{x}_2 + \overline{x}_3 + x_4)$$

[Figure 2.67b from the textbook]

Example 6

Use K-maps to find the minimum-cost SOP and POS expression for the function

$$f(x_1, \ldots, x_4) = \overline{x}_1\overline{x}_3\overline{x}_4 + x_3x_4 + \overline{x}_1\overline{x}_2x_4 + x_1x_2\overline{x}_3x_4$$

assuming that there are also don't-cares defined as $D = \sum(9, 12, 14)$.

Let's map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

Let's map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \boxed{\bar{x}_1\bar{x}_3\bar{x}_4} + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

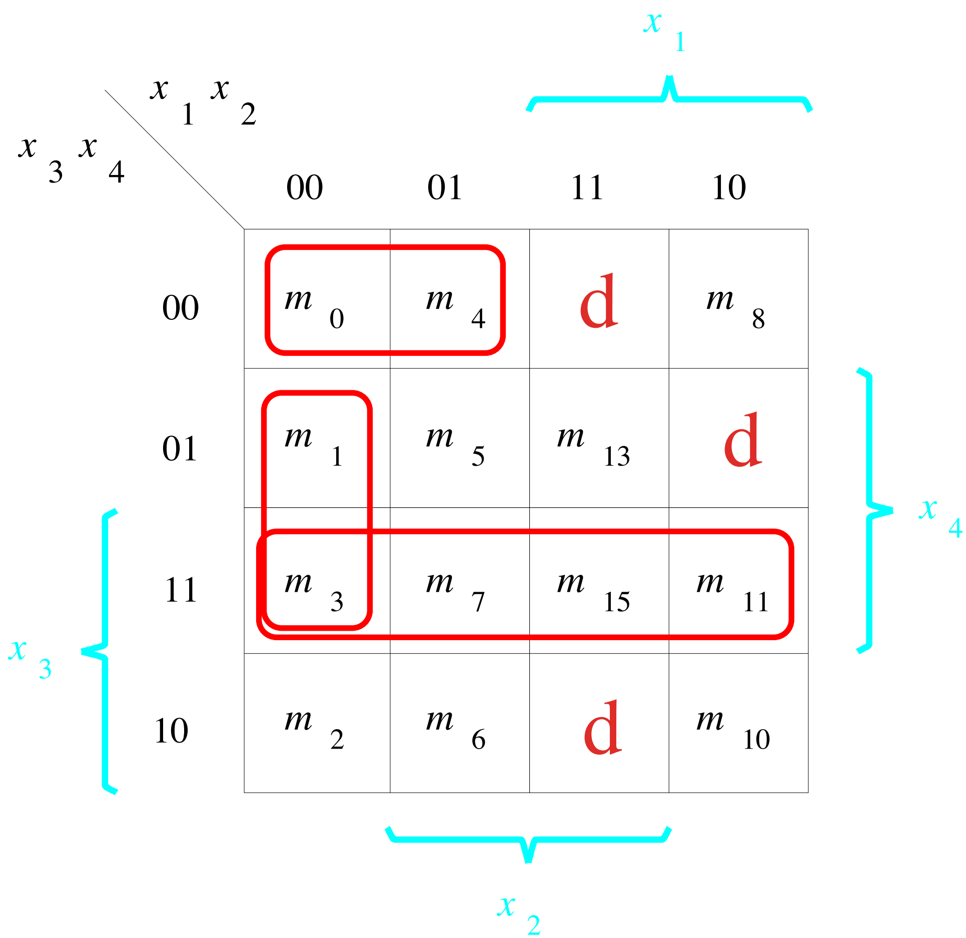
Let's map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \boxed{x_3 x_4} + \bar{x}_1\bar{x}_2 x_4 + x_1 x_2 \bar{x}_3 x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

Let's map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \boxed{\bar{x}_1\bar{x}_2x_4} + x_1x_2\bar{x}_3x_4$$



Let's map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + \boxed{x_1x_2\bar{x}_3x_4}$$

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

x_1 x_2 x_3 x_4

Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

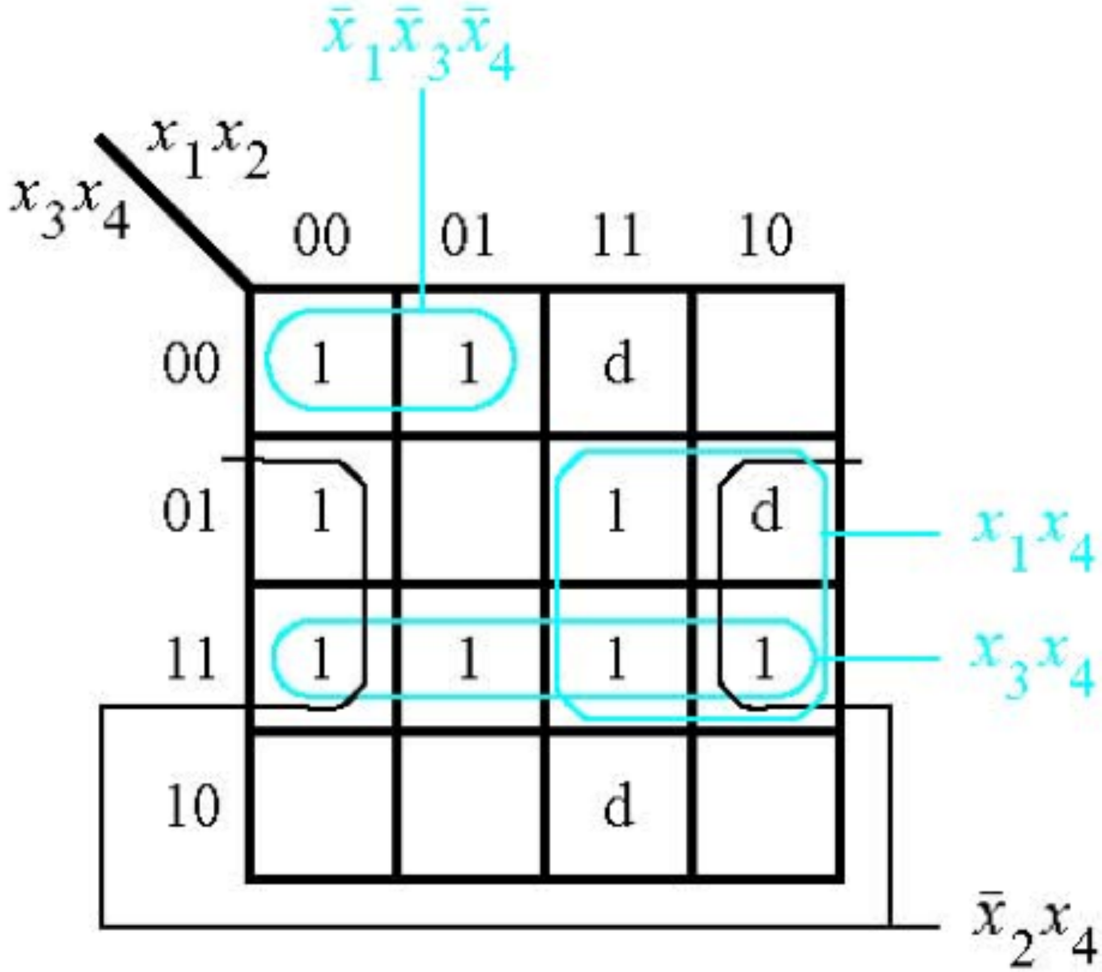
Let’s map the expression to the K-Map

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

The SOP Expression

$$f(x_1, \ldots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



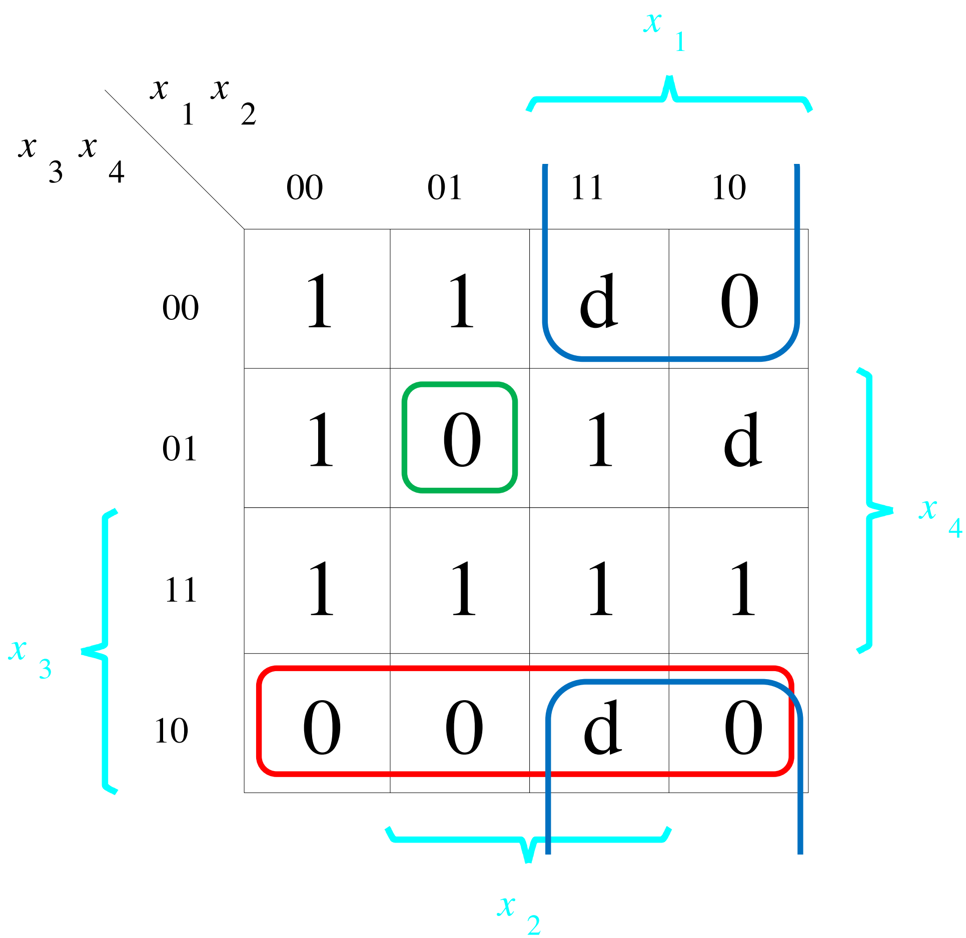
$$f = x_3x_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_4 + x_1x_4$$

[Figure 2.68a from the textbook]

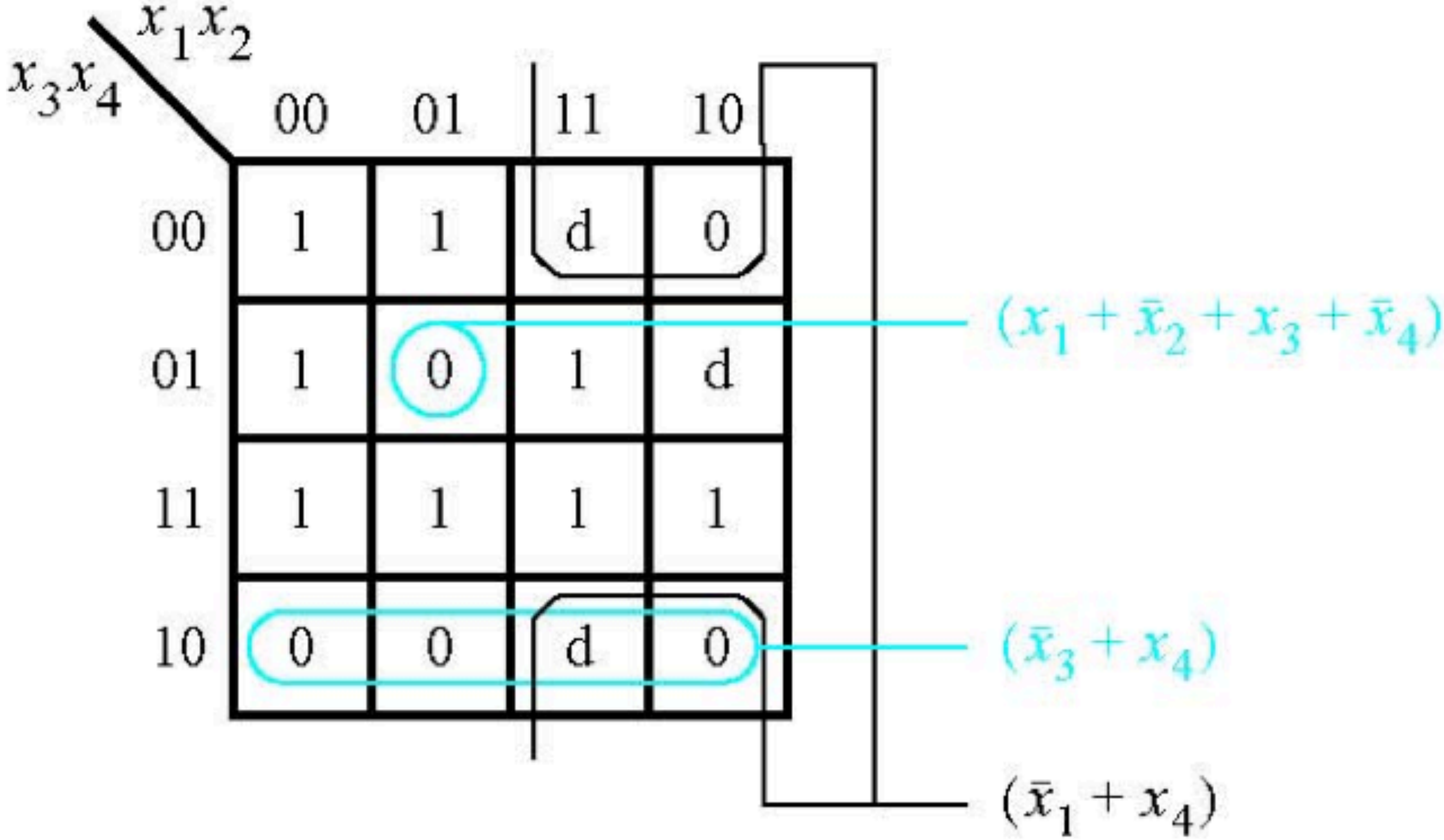
What about the POS Expression?

x_3x_4 \ x_1x_2	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

What about the POS Expression?



The POS Expression



$$f = (\bar{x}_3 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$

[Figure 2.68b from the textbook]

Example 7

Derive the minimum-cost SOP expression for

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1 s_2$$

First, expand the expression using property 12a

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1 s_2$$

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

s_3 \ $s_1 s_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \boxed{\bar{s}_1 s_3} + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

s_3 \ $s_1 s_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \boxed{\bar{s}_2 s_3} + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

s_3 \ $s_1 s_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + \boxed{s_1 s_2}$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

s_3 \ $s_1 s_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

s_3 \ $s_1 s_2$	00	01	11	10
0	0	0	1	0
1	1	1	1	1

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

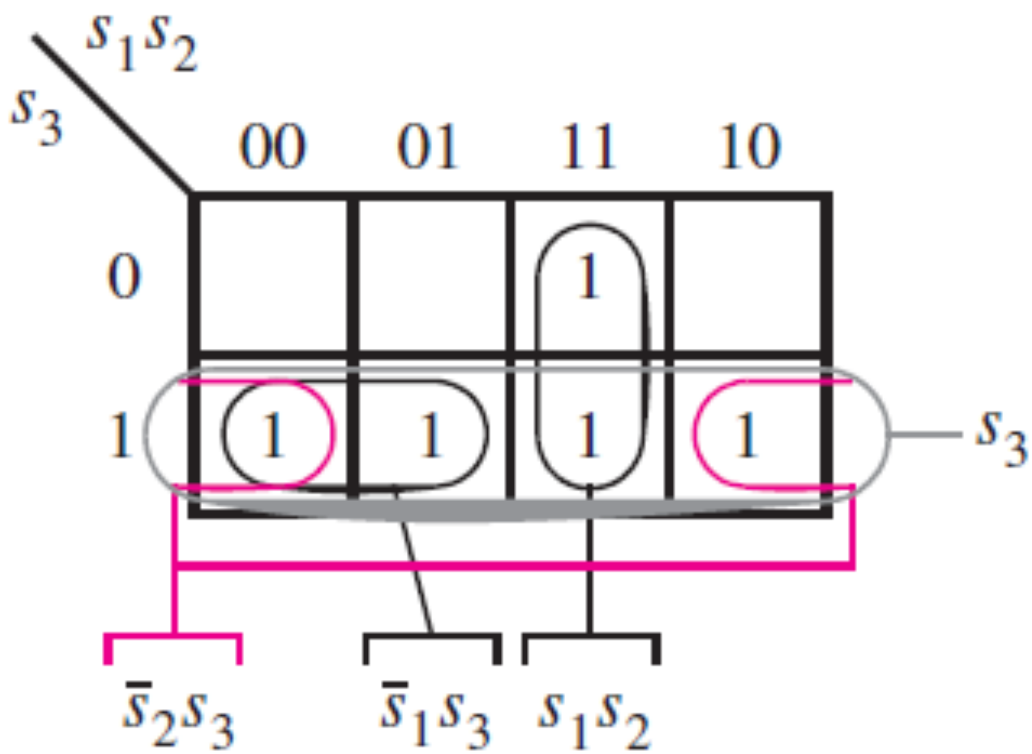
s_3 \ $s_1 s_2$	00	01	11	10
0	0	0	1	0
1	1	1	1	1

(b) Karnaugh map

Simplified Expression: $f = s_3 + s_1 s_2$

Construct the K-Map for this expression

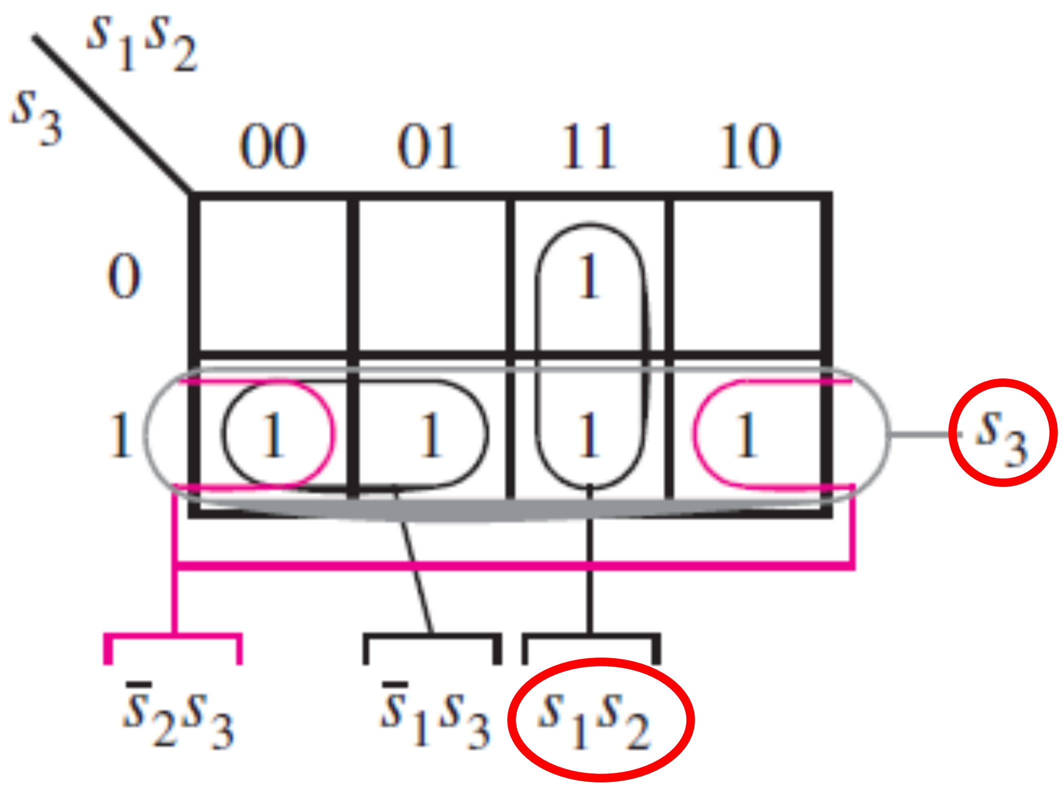
$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



[Figure 2.69 from the textbook]

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



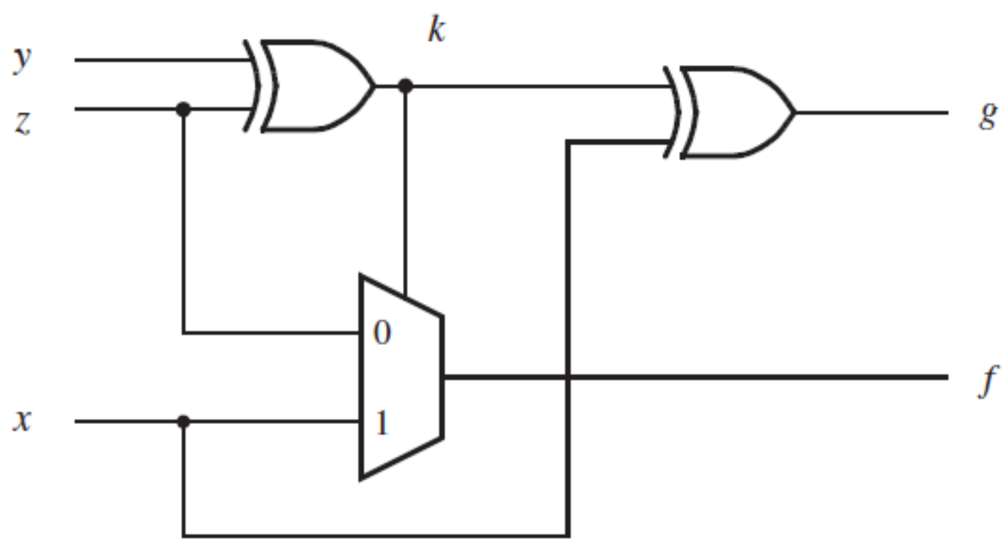
Simplified Expression: $f = s_3 + s_1\ s_2$

[Figure 2.69 from the textbook]

Example 8

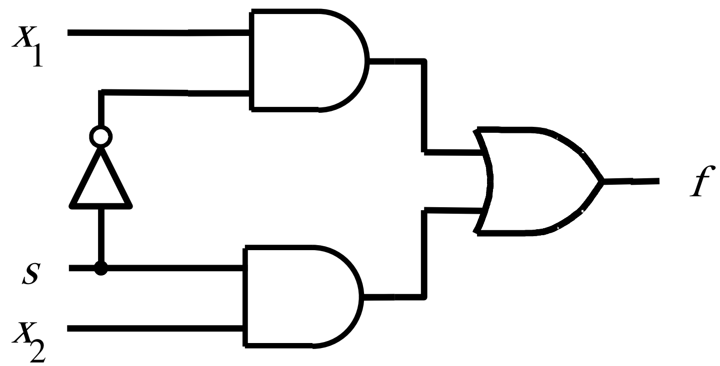
Write the Verilog code for the following circuit …

Logic Circuit

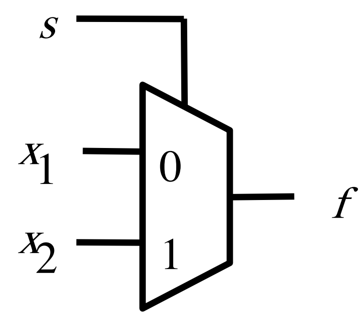


[Figure 2.70 from the textbook]

Circuit for 2-1 Multiplexer



(b) Circuit

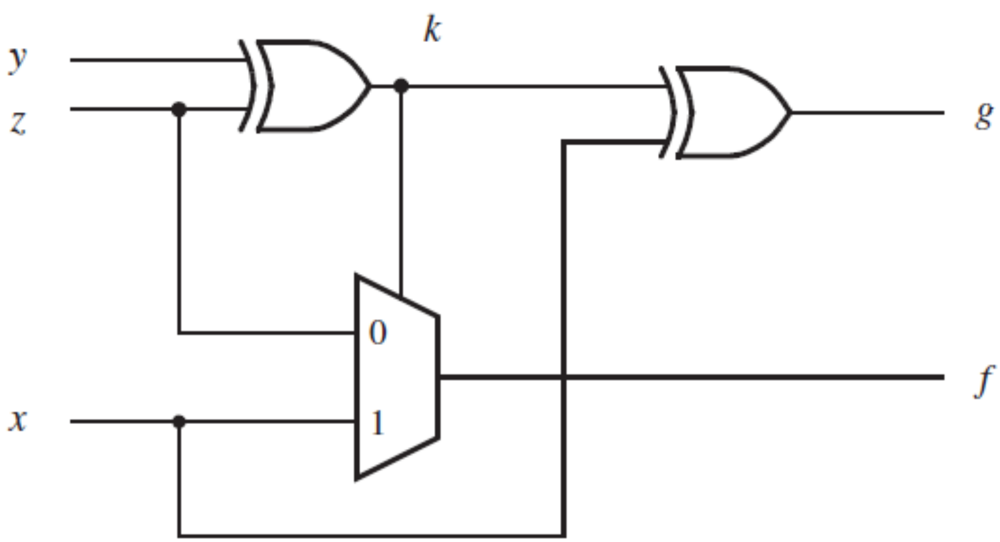


(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s}\, x_1 + s\, x_2$$

[Figure 2.33b-c from the textbook]

Logic Circuit vs Verilog Code




```
module f_g (x, y, z, f, g);
    input x, y, z;
    output f, g;
    wire k;

    assign k = y ^ z;
    assign g = k ^ x;
    assign f = (~k & z) | (k & x);

endmodule
```

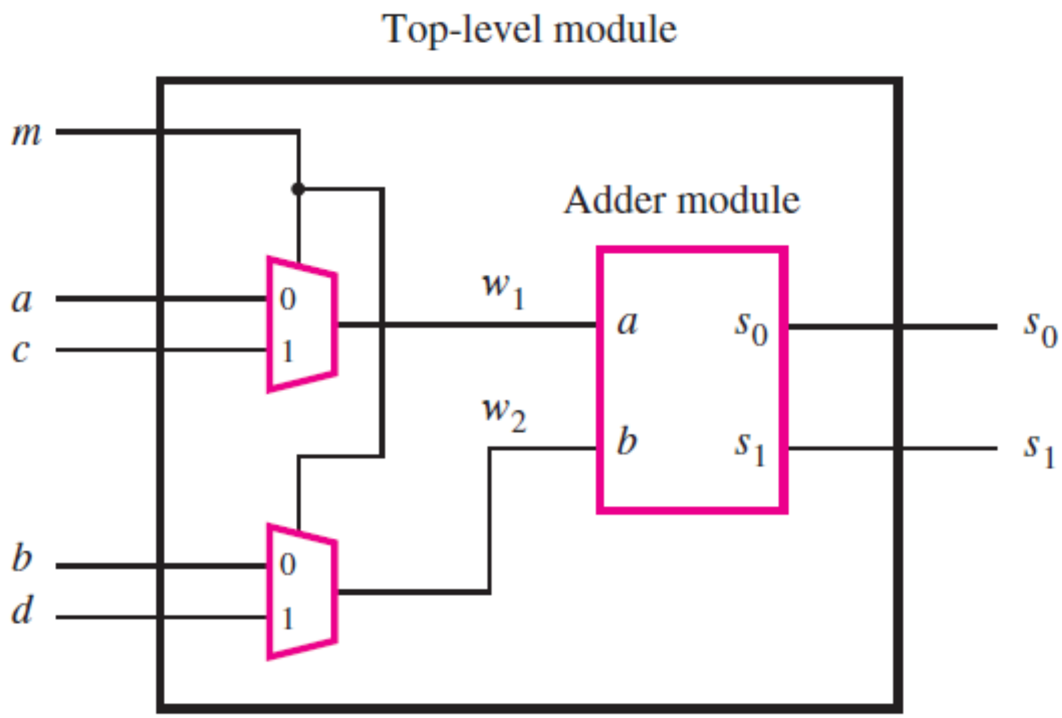
[Figure 2.70 from the textbook]

[Figure 2.71 from the textbook]

Example 9

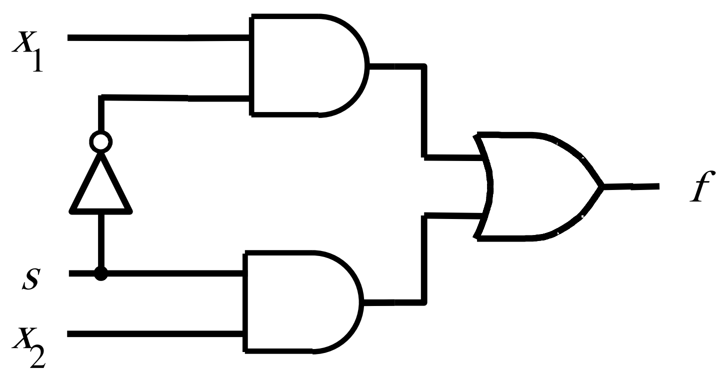
Write the Verilog code for the following circuit …

The Logic Circuit for this Example

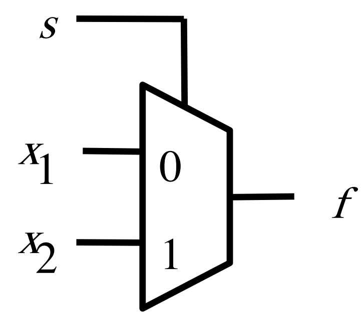


[Figure 2.72 from the textbook]

Circuit for 2-1 Multiplexer



(b) Circuit

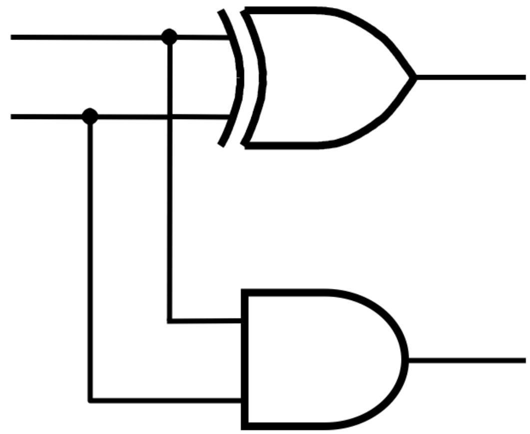


(c) Graphical symbol

$$f(s, x_1, x_2) = \overline{s}\, x_1 + s\, x_2$$

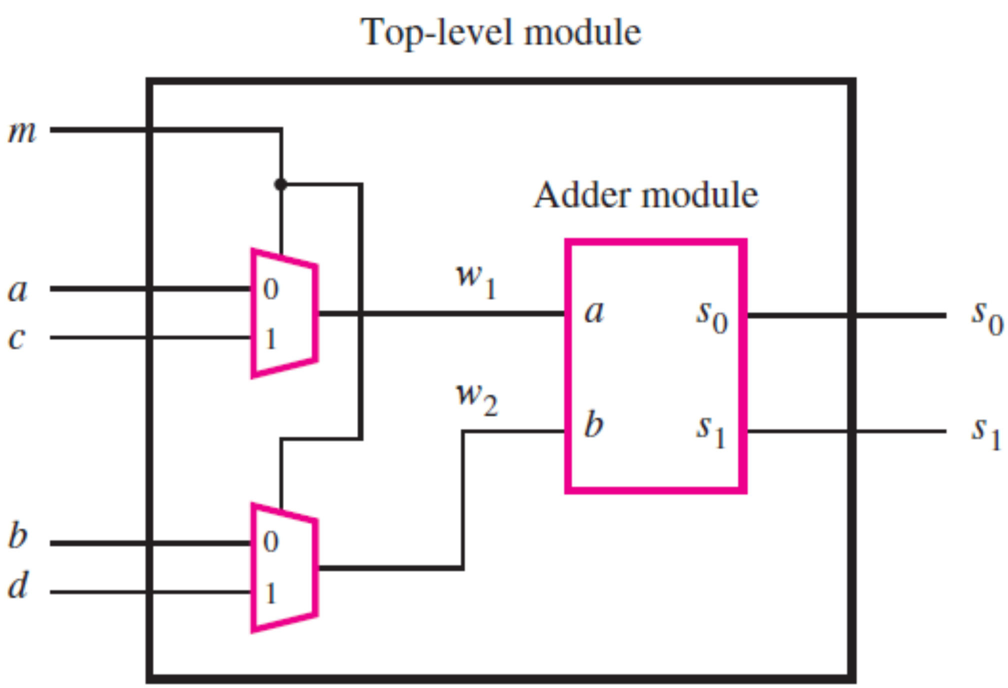
[Figure 2.33b-c from the textbook]

Addition of Binary Numbers



a	b	s_1	s_0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Logic Circuit vs Verilog Code

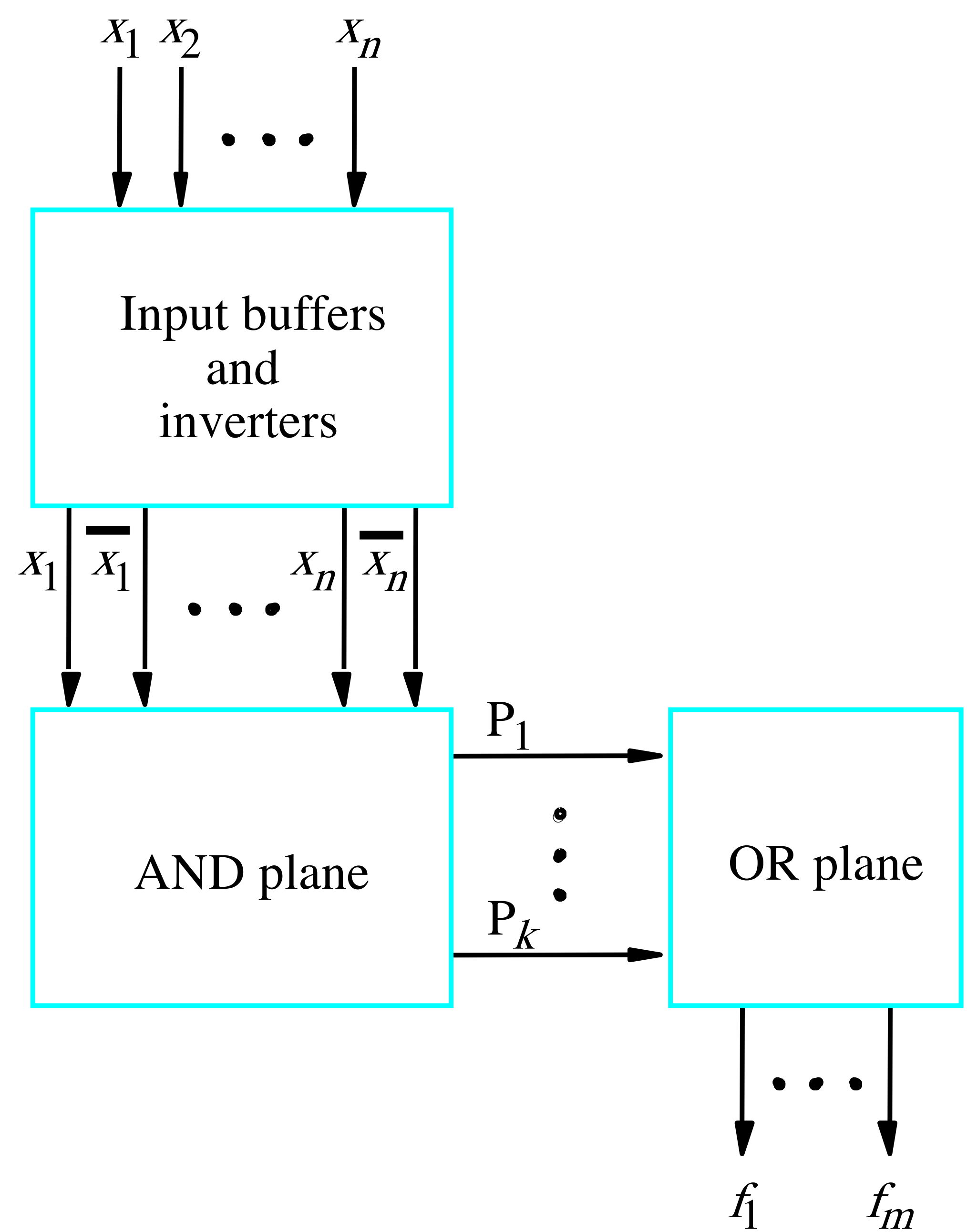



```
module shared (a, b, c, d, m, s1, s0);
    input a, b, c, d, m;
    output s1, s0;
    wire w1, w2;
    mux2to1 U1 (a, c, m, w1);
    mux2to1 U2 (b, d, m, w2);
    adder U3 (w1, w2, s1, s0);
endmodule

module mux2to1 (x1, x2, s, f);
    input x1, x2, s;
    output f;
    assign f = (~s & x1) | (s & x2);
endmodule

module adder (a, b, s1, s0);
    input a, b;
    output s1, s0;
    assign s1 = a & b;
    assign s0 = a ^ b;
endmodule
```

[Figure 2.73 from the textbook]

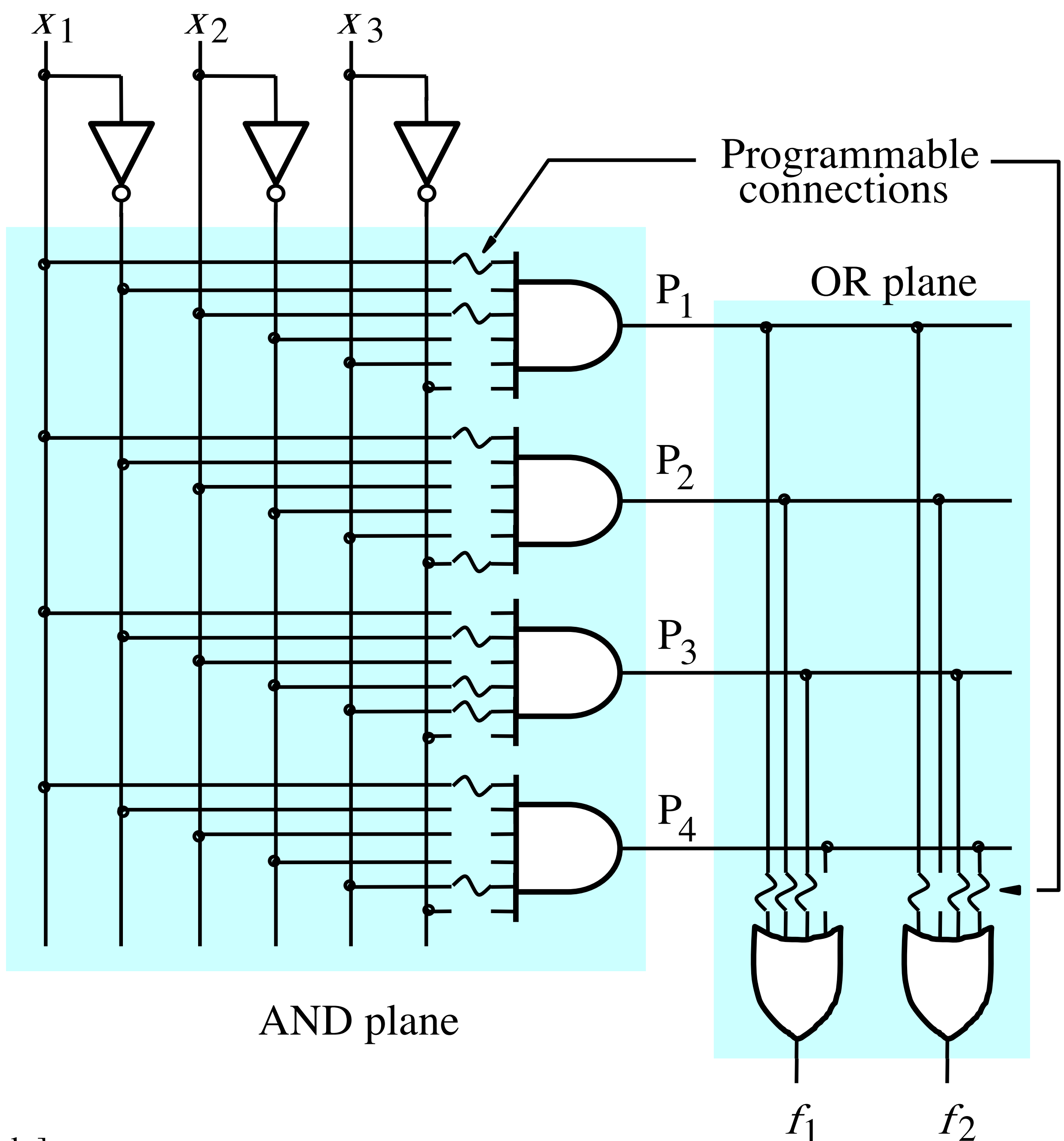
Some material form Appendix B

Programmable Logic Array (PLA)



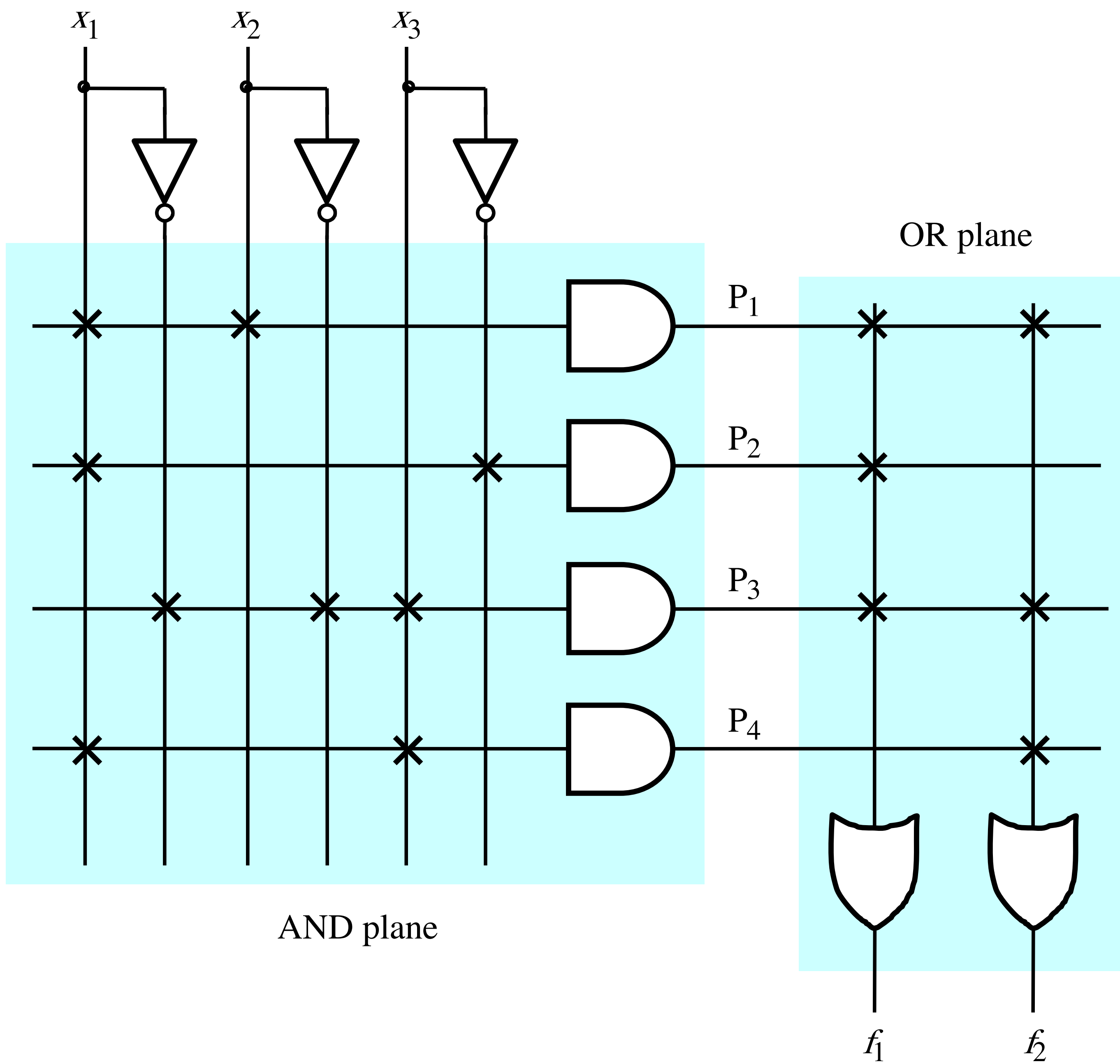
[Figure B.25 from textbook]

Gate-Level Diagram of a PLA



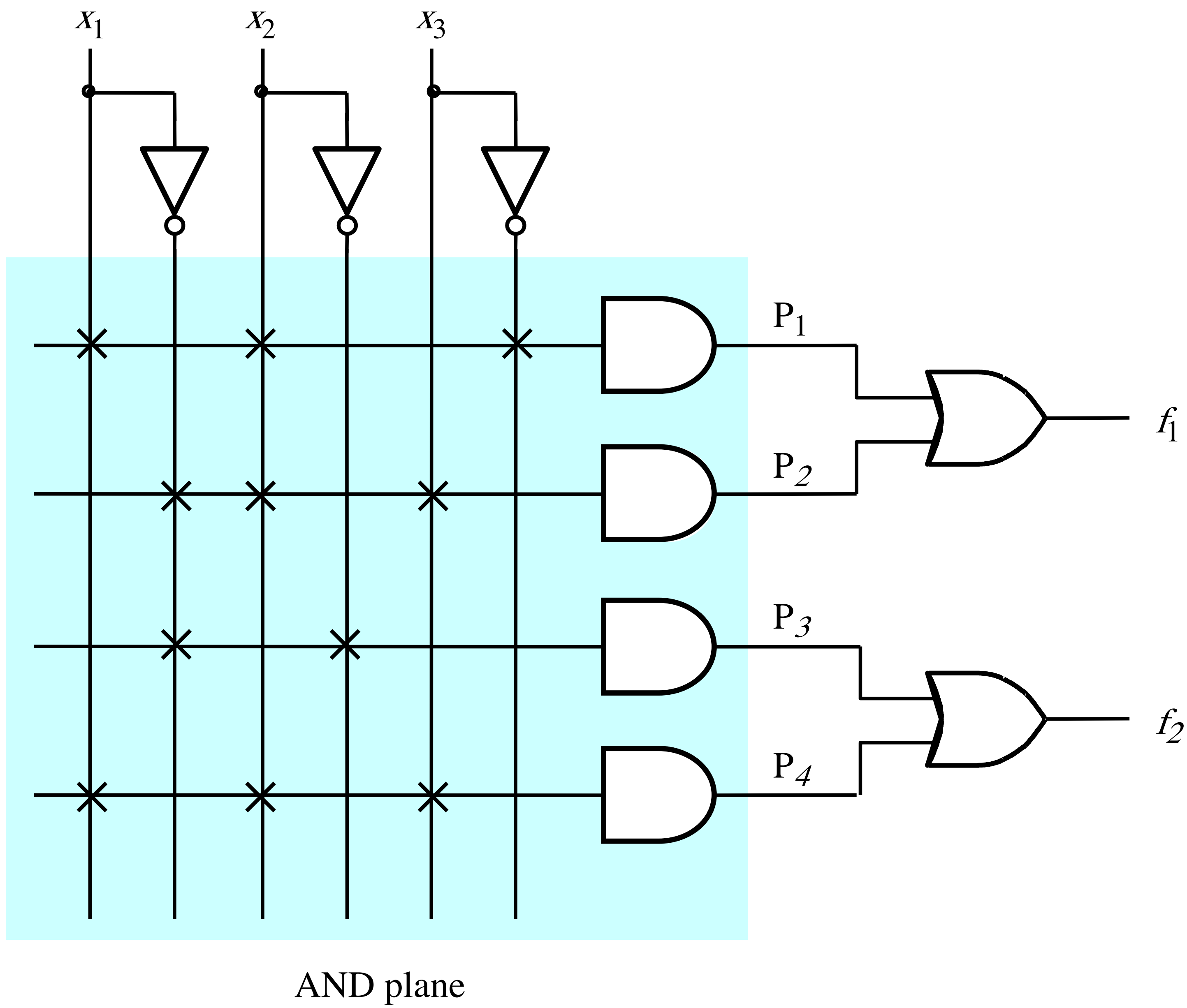
[Figure B.26 from textbook]

Customary Schematic for PLA



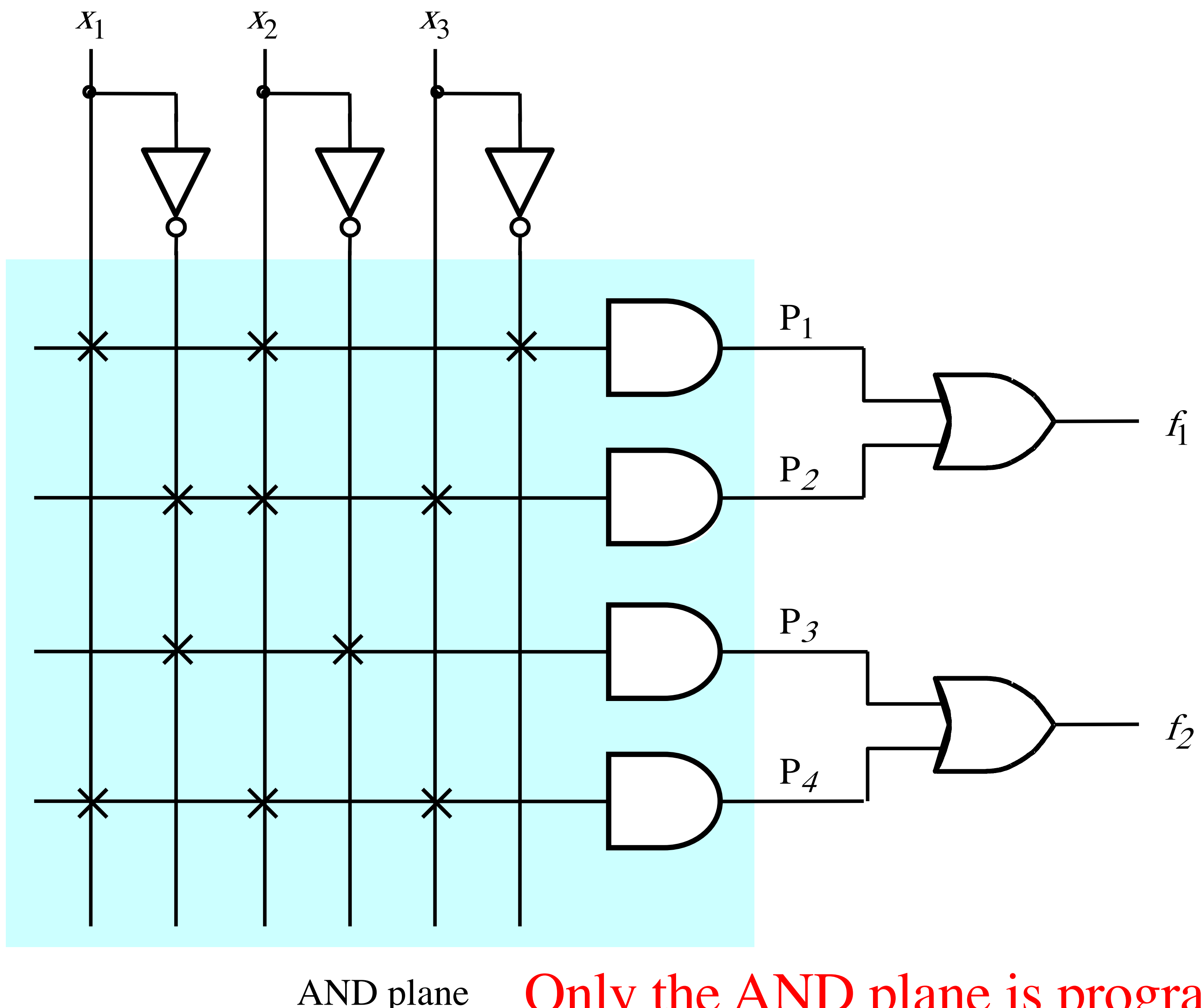
[Figure B.27 from textbook]

Programmable Array Logic (PAL)



[Figure B.28 from textbook]

Programmable Array Logic (PAL)



Only the AND plane is programmable.
The OR plane is fixed.

[Figure B.28 from textbook]

Questions?

THE END