



CprE 281: Digital Logic

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<http://www.ece.iastate.edu/~alexs/classes/>

Examples of Solved Problems

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Iowa State University, Ames, IA
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Administrative Stuff

- **HW4 is due today**

Administrative Stuff

- **HW5 is out**
- **It is due on Monday Sep 25 @ 10pm.**
- **It is posted on the class web page**
- **I also sent you an e-mail with the link.**
- **You can use this as a preparation for the exam.**

Administrative Stuff

- **Midterm Exam #1**
- **When: Friday Sep 22.**
- **What: Chapter 1 and Chapter 2 plus number systems**
- **The exam will be closed book but open notes (you can bring up to 3 pages of handwritten notes).**
- **It will be in this room.**
- **Review session: This Wednesday during lecture**

Topics for the Midterm Exam

- **Binary Numbers**
- **Octal Numbers**
- **Hexadecimal Numbers**
- **Conversion between the different number systems**
- **Truth Tables**
- **Boolean Algebra**
- **Logic Gates**
- **Circuit Synthesis with AND, OR, NOT**
- **Circuit Synthesis with NAND, NOR**
- **Converting an AND/OR/NOT circuit to NAND circuit**
- **Converting an AND/OR/NOT circuit to NOR circuit**
- **SOP and POS expressions**

Topics for the Midterm Exam

- **Mapping a Circuit to Verilog code**
- **Mapping Verilog code to a circuit**
- **Multiplexers**
- **Venn Diagrams**
- **K-maps for 2, 3, and 4 variables**
- **Minimization of Boolean expressions using theorems**
- **Minimization of Boolean expressions with K-maps**
- **Incompletely specified functions (with don't cares)**
- **Functions with multiple outputs**
- **Something from Star Wars**

**All possible Boolean functions
with two input variables**

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



constant 0




constant 1

There are 16 possible Boolean functions with two input variables


x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



NOR(x, y)


OR(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


$$m_1 = \bar{x} y$$


$$M_1 = x + \bar{y}$$

There are 16 possible Boolean functions with two input variables


x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


↑
NOT(x)

↑
x

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1


 $m_2 = x \bar{y}$


 $M_2 = \bar{x} + y$

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

↑
NOT(y)

↑
y

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



XOR(x, y)



XNOR(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1



NAND(x, y) AND(x, y)

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0
 NOR
 m_1
 \bar{x}
 m_2
 \bar{y}
 XOR
 NAND
 AND
 XNOR
 y
 M_2
 x
 M_1
 OR
 1

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0 m₀ m₁ \bar{x} m₂ \bar{y} XOR M₃ m₃ XNOR y M₂ x M₁ M₀ 1

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0 m₀ m₁ \bar{x} m₂ \bar{y} m₁ + m₂ M₃ m₃ M₁ • M₂ y M₂ x M₁ M₀ 1

There are 16 possible Boolean functions with two input variables

x	y	F ₀	F ₁	F ₂	F ₃	F ₄	F ₅	F ₆	F ₇	F ₈	F ₉	F ₁₀	F ₁₁	F ₁₂	F ₁₃	F ₁₄	F ₁₅
0	0	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
0	1	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1

0 m_0 m_1 $m_0 + m_1$ m_2 $m_0 + m_2$ $m_1 + m_2$ M_3 m_3 $M_1 \cdot M_2$ $M_0 \cdot M_2$ M_2 $M_0 \cdot M_1$ M_1 M_0 1

K-Maps for all 16 possible functions with two input variables

K-Maps for the 16 functions

		x	
	y	0	1
0		1	0
1		0	0

$$\bar{x} \bar{y}$$

F_1

		x	
	y	0	1
0		0	0
1		1	0

$$\bar{x} y$$

F_2

		x	
	y	0	1
0		0	1
1		0	0

$$x \bar{y}$$

F_4

		x	
	y	0	1
0		0	0
1		0	1

$$x y$$

F_8

K-Maps for the 16 functions

		x	
	y	0	1
0		1	0
1		1	0

\bar{x}

F_3

		x	
	y	0	1
0		0	1
1		0	1

x

F_{12}

		x	
	y	0	1
0		1	1
1		0	0

\bar{y}

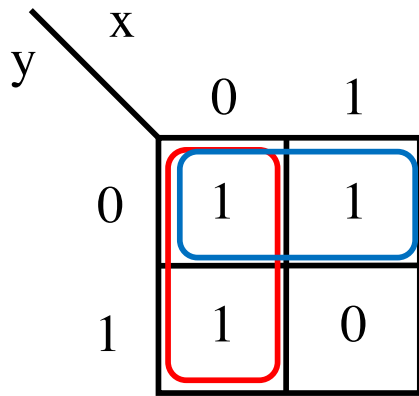
F_5

		x	
	y	0	1
0		0	0
1		1	1

y

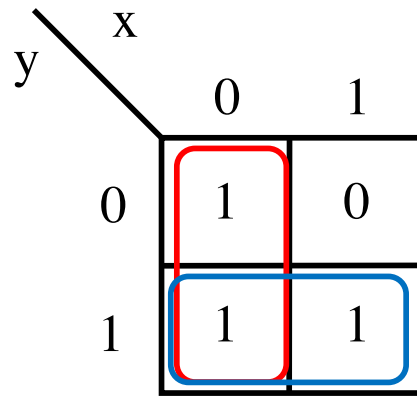
F_{10}

K-Maps for the 16 functions



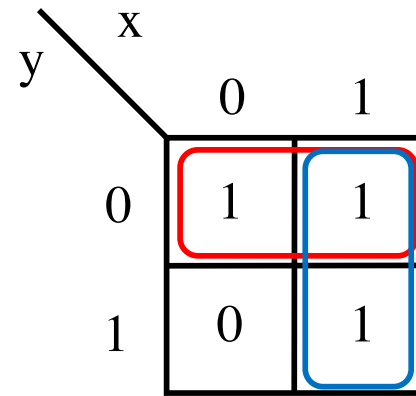
$$\bar{x} + \bar{y}$$

F_7



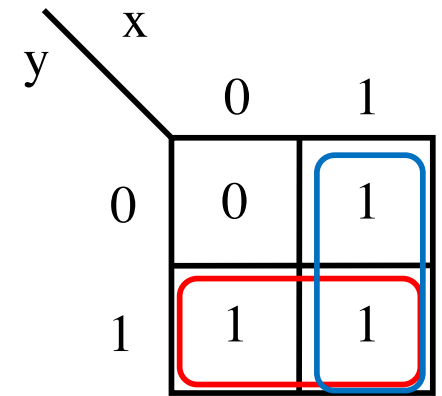
$$\bar{x} + y$$

F_{11}



$$x + \bar{y}$$

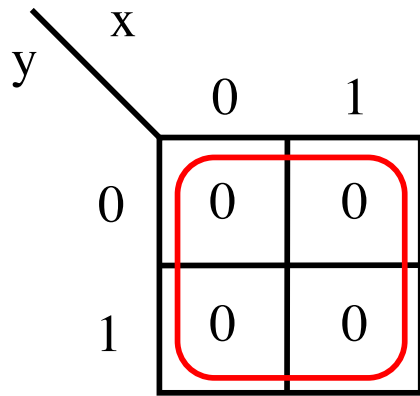
F_{13}



$$x + y$$

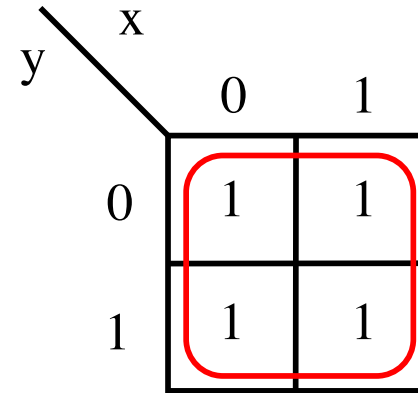
F_{14}

K-Maps for the 16 functions



constant 0

F_0



constant 1

F_{15}

**These are not valid groupings,
but they correspond to XOR and XNOR**

		x	
	y		
		0	1
0		0	1
1		1	0

A 2x2 truth table for XOR. The columns are labeled 'x' (0, 1) and the rows are labeled 'y' (0, 1). The values in the cells are: (0,0)=0, (0,1)=1, (1,0)=1, (1,1)=0. A red diamond shape is drawn around the cells (0,1) and (1,0), which both contain the value 1.

		x	
	y		
		0	1
0		1	0
1		0	1

A 2x2 truth table for XNOR. The columns are labeled 'x' (0, 1) and the rows are labeled 'y' (0, 1). The values in the cells are: (0,0)=0, (0,1)=1, (1,0)=0, (1,1)=1. A red diamond shape is drawn around the cells (0,0) and (1,1), which both contain the value 0.

**These are not valid groupings,
but they correspond to XOR and XNOR**

		x	
			y
		0	1
0		0	1
1		1	0

XOR(x, y)

F_6

		x	
			y
		0	1
0		1	0
1		0	1

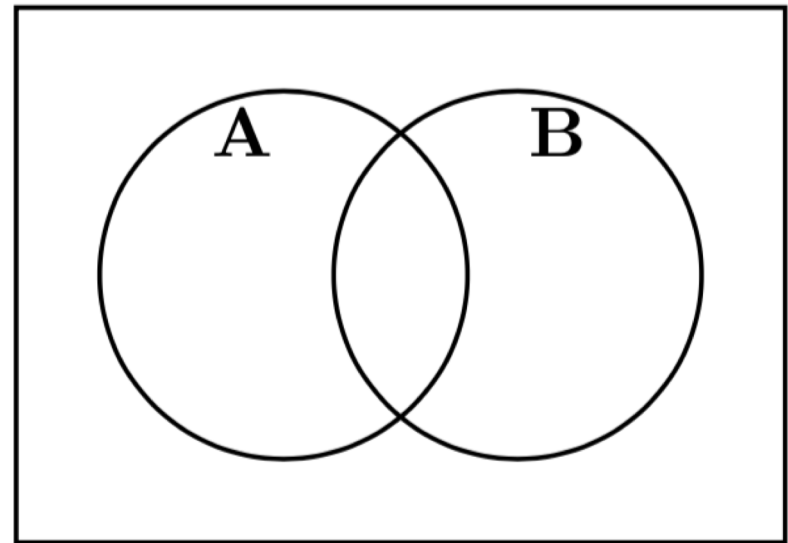
XNOR(x, y)

F_9

The Link Between Truth Tables and Venn Diagrams

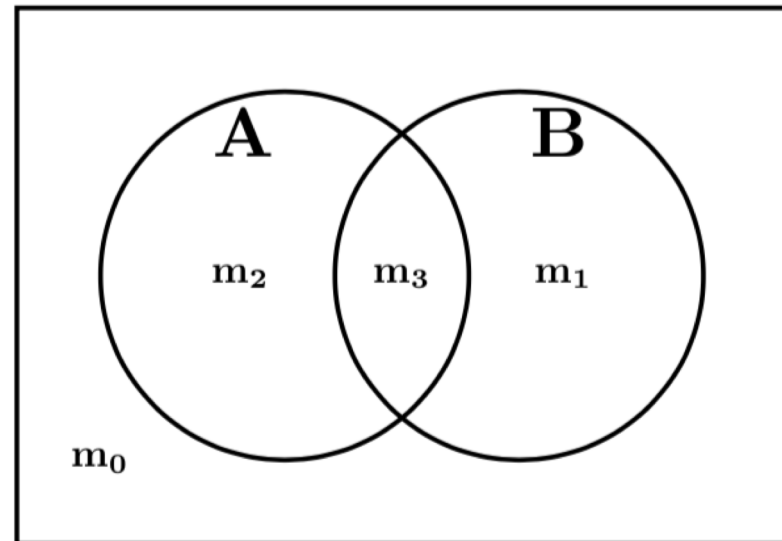
Place the minterms on the Venn diagram

A	B	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3



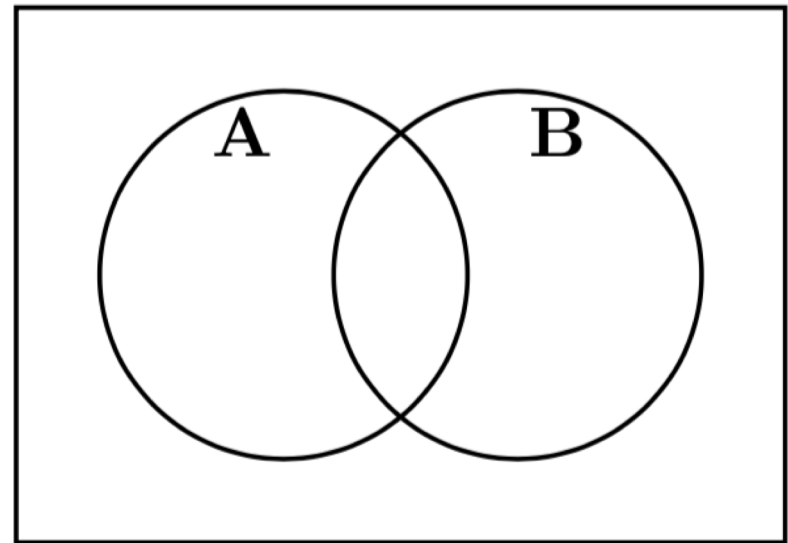
Place the minterms on the Venn diagram

A	B	
0	0	m_0
0	1	m_1
1	0	m_2
1	1	m_3



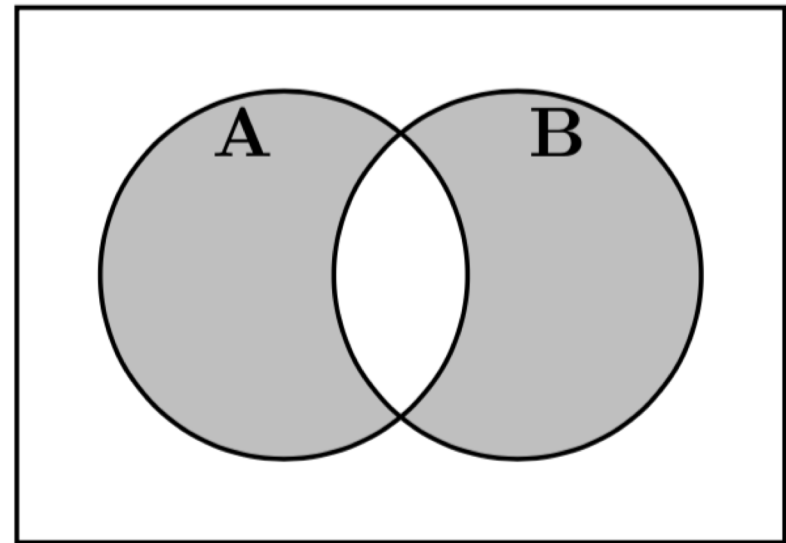
Color the Venn diagram for XOR

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



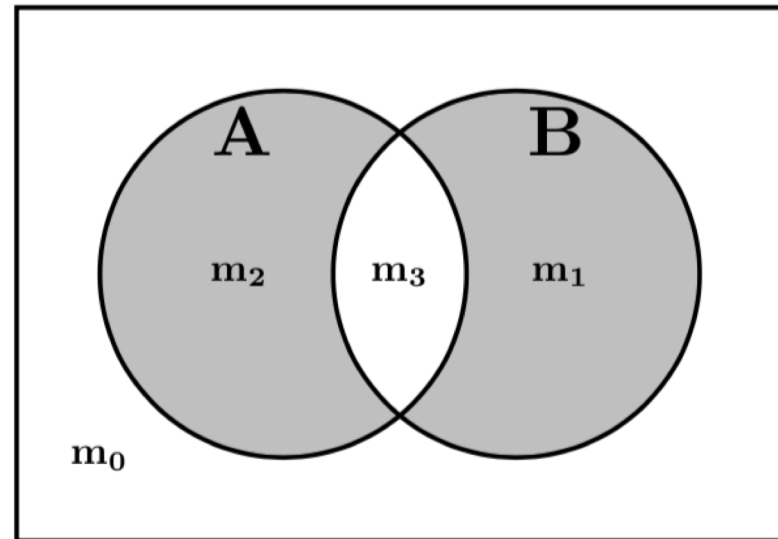
Color the Venn diagram for XOR

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0



Color the Venn diagram for XOR

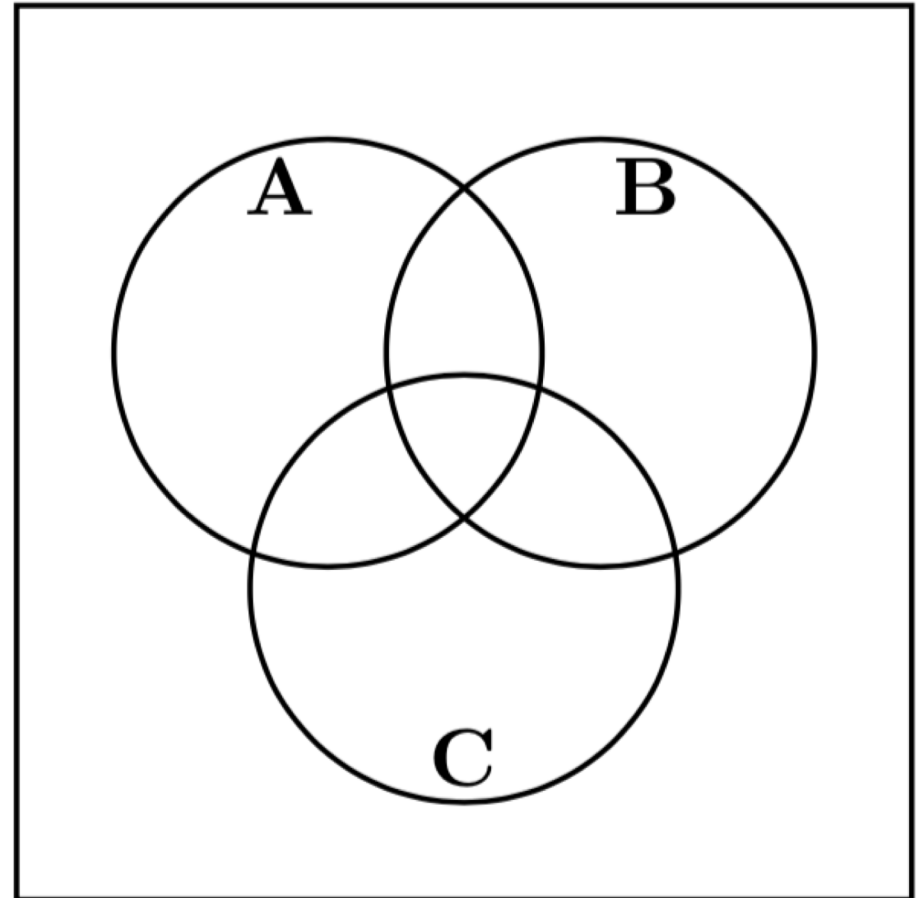
	A	B	F
m_0	0	0	0
m_1	0	1	1
m_2	1	0	1
m_3	1	1	0



$$F = \bar{A}B + A\bar{B}$$

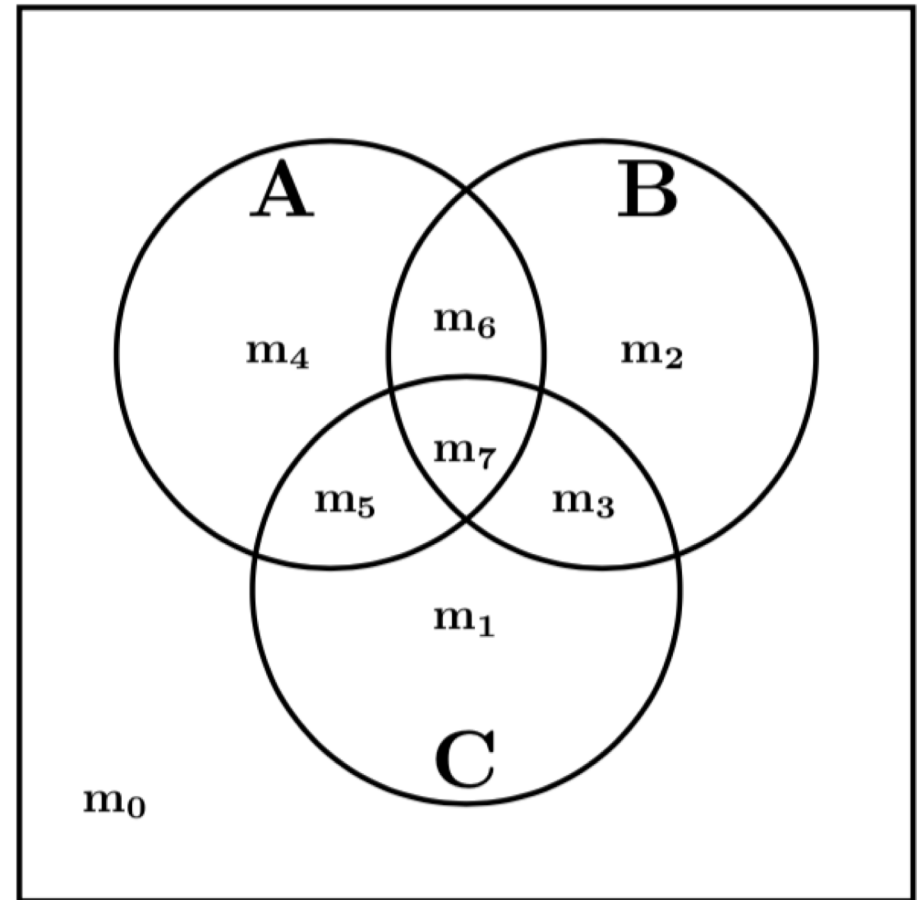
Place the minterms on the Venn diagram

A	B	C	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



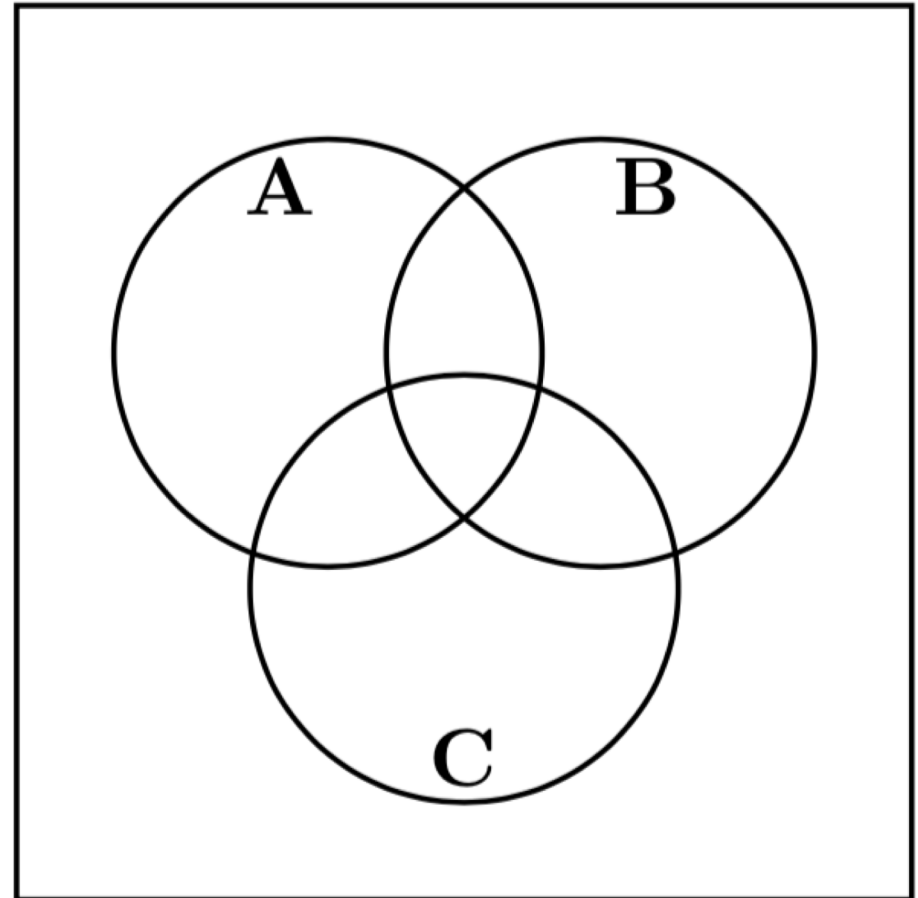
Place the minterms on the Venn diagram

A	B	C	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



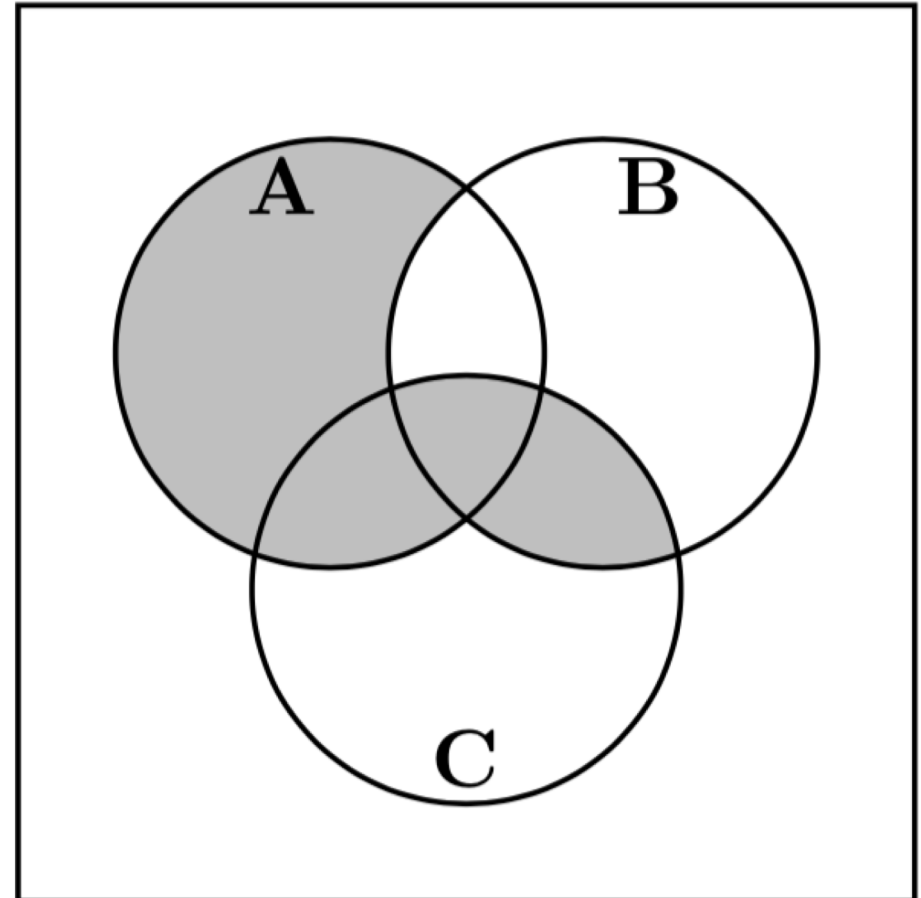
Color the Venn diagram for this function

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



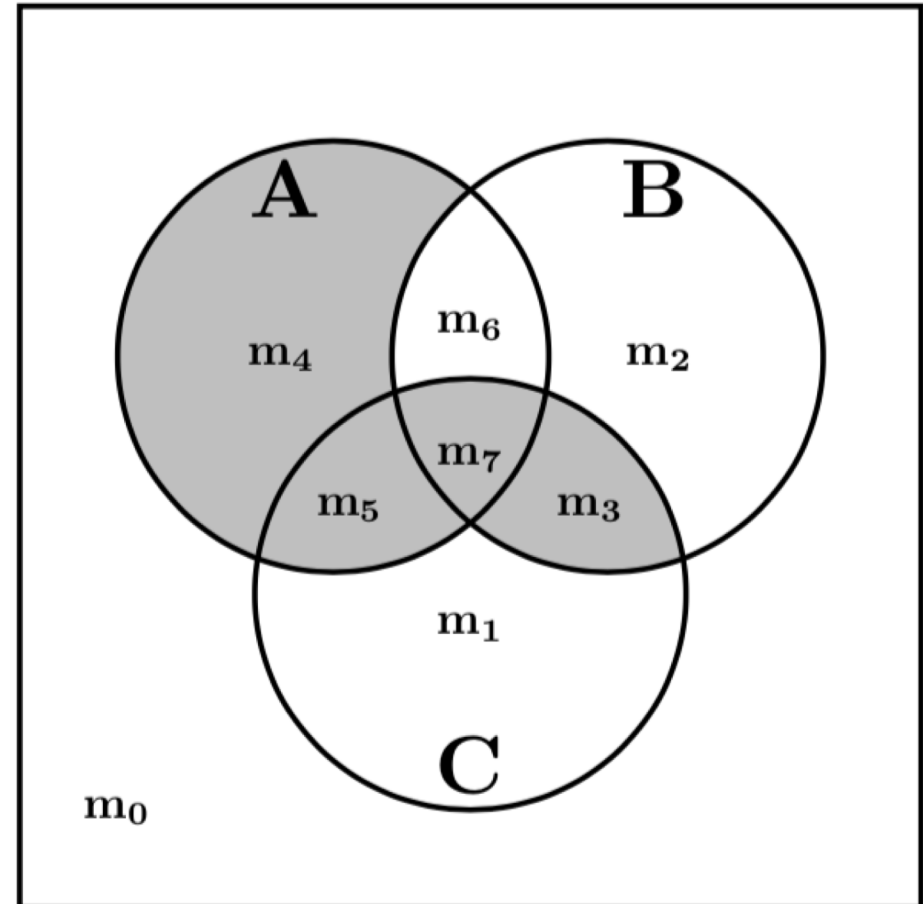
Color the Venn diagram for this function

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1



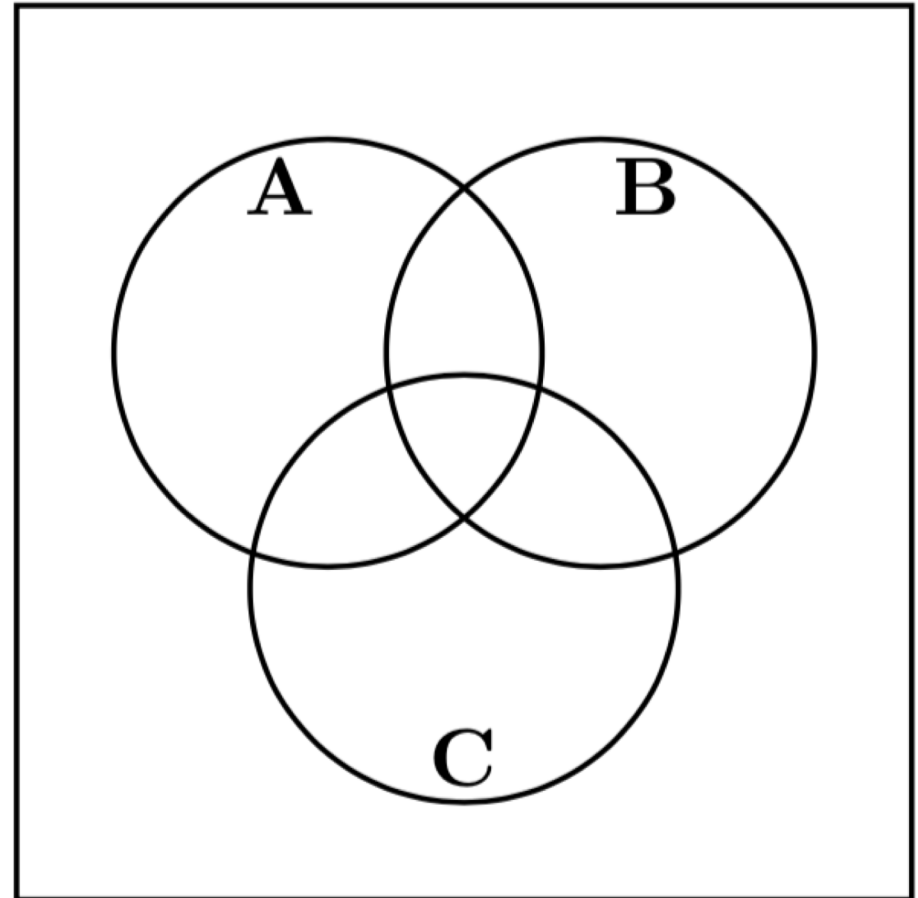
Place the minterms on the Venn diagram

	A	B	C	F
m_0	0	0	0	0
m_1	0	0	1	0
m_2	0	1	0	0
m_3	0	1	1	1
m_4	1	0	0	1
m_5	1	0	1	1
m_6	1	1	0	0
m_7	1	1	1	1



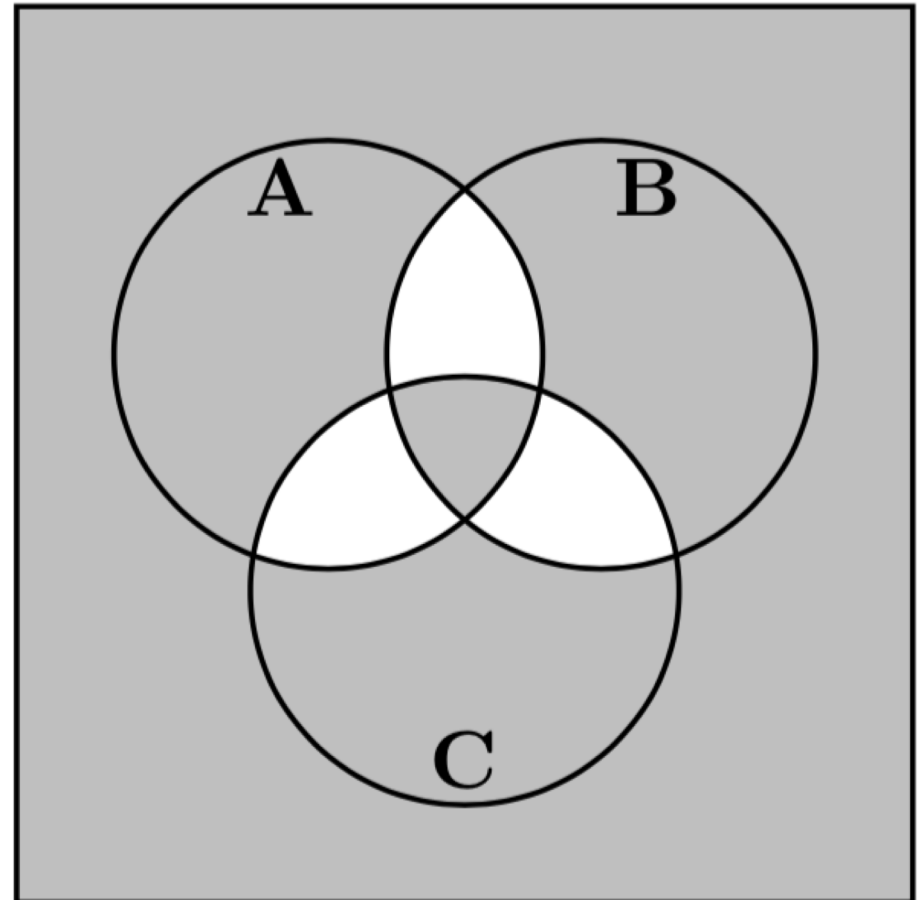
Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



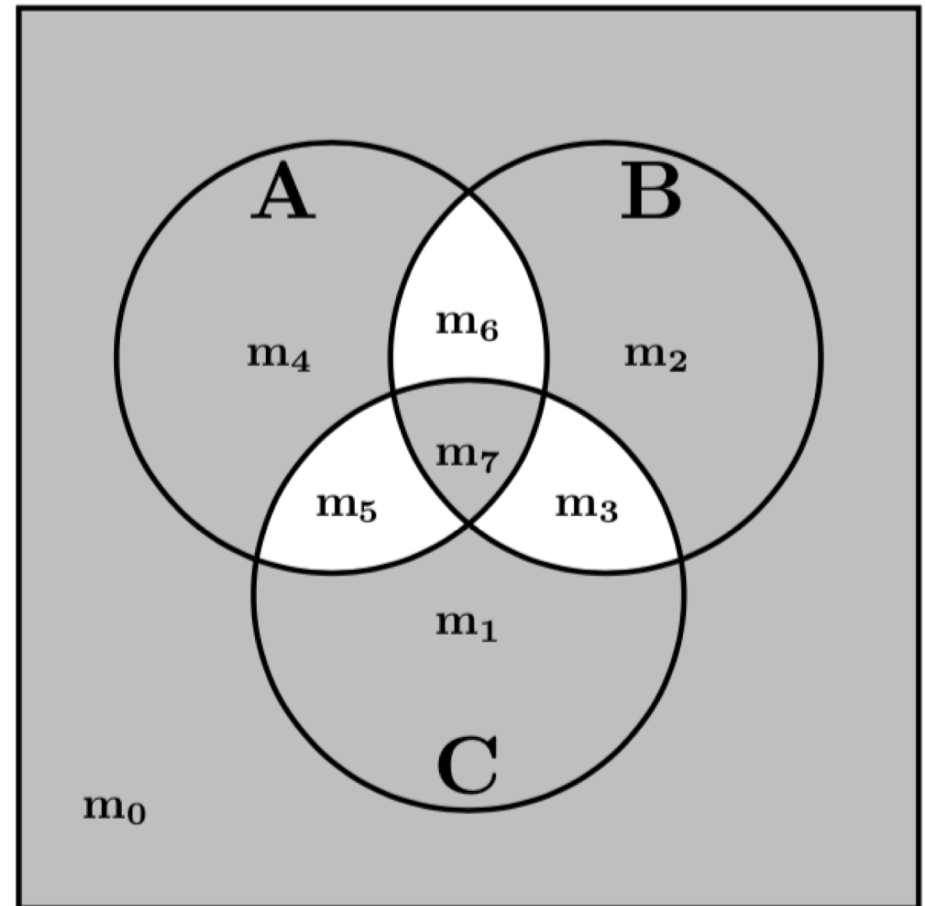
Color the Venn diagram for this function

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

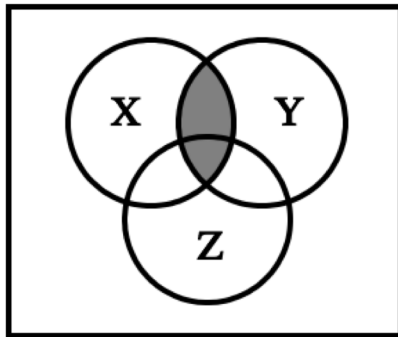


Place the minterms on the Venn diagram

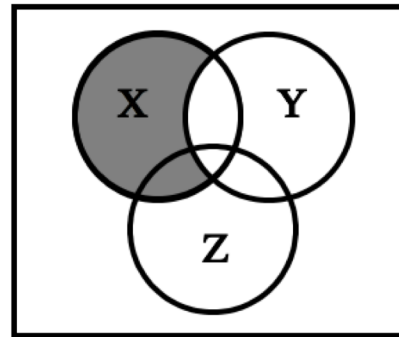
	A	B	C	F
m_0	0	0	0	1
m_1	0	0	1	1
m_2	0	1	0	1
m_3	0	1	1	0
m_4	1	0	0	1
m_5	1	0	1	0
m_6	1	1	0	0
m_7	1	1	1	1



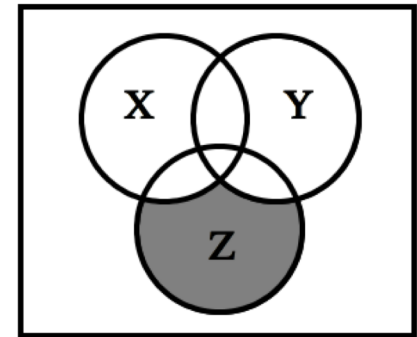
Write the expression that is represented by each of the three Venn diagrams:



(A)

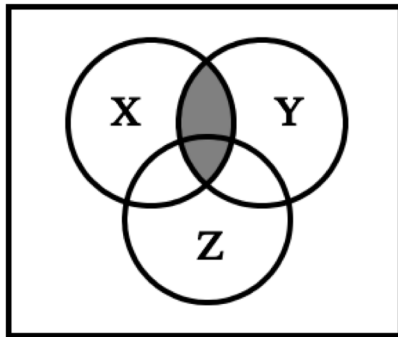


(B)



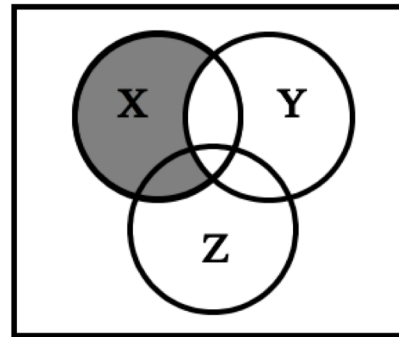
(C)

Write the expression that is represented by each of the three Venn diagrams:



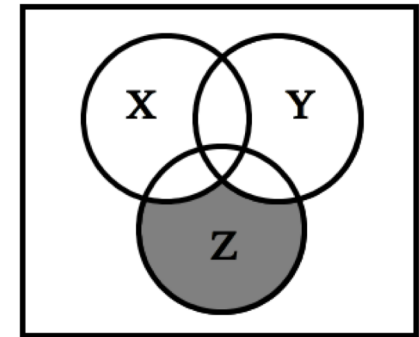
(A)

$$XY$$



(B)

$$X\bar{Y}$$



(C)

$$\bar{X}\bar{Y}Z$$

Example 1

Determine if the following equation is valid

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 = \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

$$\underbrace{\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2}_{\text{LHS}} \stackrel{?}{=} \underbrace{\bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3}_{\text{RHS}}$$

LHS

RHS

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	f
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	f
0	0	0	0	1	0	0	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	0	1	0	
4	1	0	0	0	0	1	
5	1	0	1	0	0	1	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_3}$	$x_2 x_3$	$x_1 \overline{x_2}$	f
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2}$ $\overline{x_3}$	f
0	0	0	0				
1	0	0	1				
2	0	1	0				
3	0	1	1				
4	1	0	0				
5	1	0	1				
6	1	1	0				
7	1	1	1				

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	f
0	0	0	0	0	0	1	
1	0	0	1	0	0	0	
2	0	1	0	1	0	0	
3	0	1	1	1	0	0	
4	1	0	0	0	0	1	
5	1	0	1	0	1	0	
6	1	1	0	0	0	0	
7	1	1	1	0	1	0	

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\overline{x_1 x_2}$	$x_1 x_3$	$\overline{x_2} \overline{x_3}$	f
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \stackrel{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

LHS
RHS

f

1
0
1
1
1
1
1
0
1

f

1
0
1
1
1
1
1
0
1

They are equal.

Example 2

Design the minimum-cost product-of-sums expression for the function

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is
1 for these rows**

Minterms and Maxterms (with three variables)

Row number	x_1	x_2	x_3	Minterm	Maxterm
0	0	0	0	$m_0 = \bar{x}_1\bar{x}_2\bar{x}_3$	$M_0 = x_1 + x_2 + x_3$
1	0	0	1	$m_1 = \bar{x}_1\bar{x}_2x_3$	$M_1 = x_1 + x_2 + \bar{x}_3$
2	0	1	0	$m_2 = \bar{x}_1x_2\bar{x}_3$	$M_2 = x_1 + \bar{x}_2 + x_3$
3	0	1	1	$m_3 = \bar{x}_1x_2x_3$	$M_3 = x_1 + \bar{x}_2 + \bar{x}_3$
4	1	0	0	$m_4 = x_1\bar{x}_2\bar{x}_3$	$M_4 = \bar{x}_1 + x_2 + x_3$
5	1	0	1	$m_5 = x_1\bar{x}_2x_3$	$M_5 = \bar{x}_1 + x_2 + \bar{x}_3$
6	1	1	0	$m_6 = x_1x_2\bar{x}_3$	$M_6 = \bar{x}_1 + \bar{x}_2 + x_3$
7	1	1	1	$m_7 = x_1x_2x_3$	$M_7 = \bar{x}_1 + \bar{x}_2 + \bar{x}_3$

**The function is
1 for these rows**

**The function is
0 for these rows**

Two different ways to specify the same function f of three variables

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \prod M(1, 3)$$

Two different ways to specify the same function f of three variables

$$f(x_1, x_2, x_3) = \sum m(0, 2, 4, 5, 6, 7)$$

$$f(x_1, x_2, x_3) = \prod M(1, 3)$$

$$= M_1 \cdot M_3$$

$$= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3)$$

The Minimum POS Expression

$$\begin{aligned}f(x_1, x_2, x_3) &= (x_1 + x_2 + \bar{x}_3) \cdot (x_1 + \bar{x}_2 + \bar{x}_3) \\ &= (x_1 + \bar{x}_3 + x_2) \cdot (x_1 + \bar{x}_3 + \bar{x}_2) \\ &= (x_1 + \bar{x}_3)\end{aligned}$$

Hint: Use the following Boolean Algebra theorem

$$**14b. \quad (x + y) \cdot (x + \bar{y}) = x**$$

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Alternative Solution Using K-Maps

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$x_1 x_2$			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

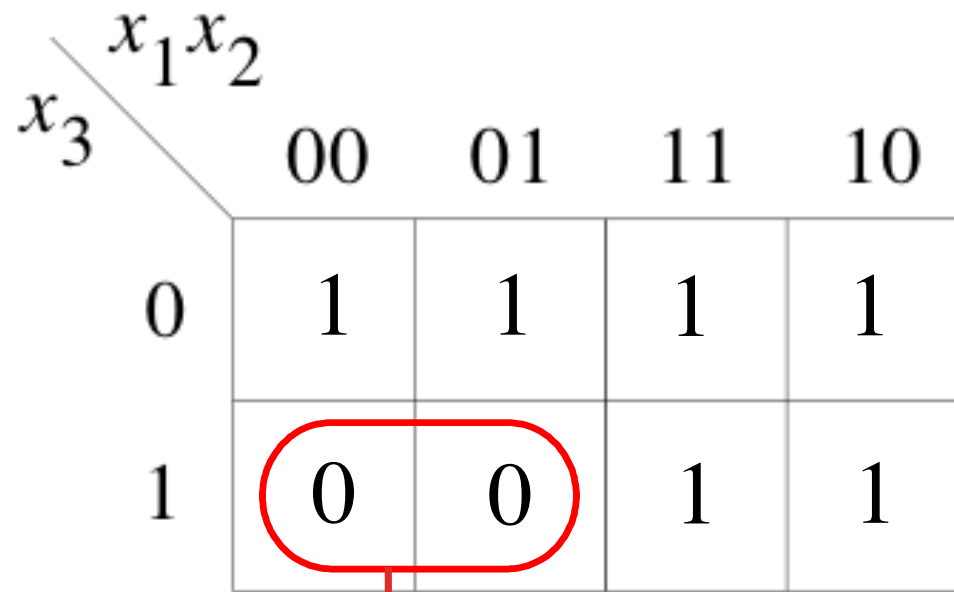
Alternative Solution Using K-Maps

		x_1x_2			
		00	01	11	10
x_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

Alternative Solution Using K-Maps

		x_1x_2			
		00	01	11	10
x_3	0	1	1	1	1
	1	0	0	1	1

Alternative Solution Using K-Maps



$x_3 \backslash x_1 x_2$	00	01	11	10
0	1	1	1	1
1	0	0	1	1

$$(x_1 + \bar{x}_3)$$

Example 3

Problem: A circuit that controls a given digital system has three inputs: x_1 , x_2 , and x_3 . It has to recognize three different conditions:

- Condition A is true if x_3 is true and either x_1 is true or x_2 is false
- Condition B is true if x_1 is true and either x_2 or x_3 is false
- Condition C is true if x_2 is true and either x_1 is true or x_3 is false

The control circuit must produce an output of 1 if at least two of the conditions A , B , and C are true. Design the simplest circuit that can be used for this purpose.

Condition A

Condition A is true if x_3 is true and either x_1 is true or x_2 is false

Condition A

Condition A is true if x_3 is true and either x_1 is true or x_2 is false

$$A = x_3(x_1 + \bar{x}_2) = x_3x_1 + x_3\bar{x}_2$$

Condition B

Condition B is true if x_1 is true and either x_2 or x_3 is false

Condition B

Condition B is true if x_1 is true and either x_2 or x_3 is false

$$B = x_1(\bar{x}_2 + \bar{x}_3) = x_1\bar{x}_2 + x_1\bar{x}_3$$

Condition C

Condition C is true if x_2 is true and either x_1 is true or x_3 is false

Condition C

Condition C is true if x_2 is true and either x_1 is true or x_3 is false

$$C = x_2(x_1 + \bar{x}_3) = x_2x_1 + x_2\bar{x}_3$$

The output of the circuit can be expressed as

$$f = \mathbf{AB} + AC + BC$$

$$AB = (x_3x_1 + x_3\bar{x}_2)(x_1\bar{x}_2 + x_1\bar{x}_3)$$

$$= x_3x_1x_1\bar{x}_2 + x_3x_1x_1\bar{x}_3 + x_3\bar{x}_2x_1\bar{x}_2 + x_3\bar{x}_2x_1\bar{x}_3$$

$$= x_3x_1\bar{x}_2 + 0 + x_3\bar{x}_2x_1 + 0$$

$$= x_1\bar{x}_2x_3$$

The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$AC = (x_3x_1 + x_3\bar{x}_2)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_3x_1x_2x_1 + x_3x_1x_2\bar{x}_3 + x_3\bar{x}_2x_2x_1 + x_3\bar{x}_2x_2\bar{x}_3$$

$$= x_3x_1x_2 + 0 + 0 + 0$$

$$= x_1x_2x_3$$

The output of the circuit can be expressed as

$$f = AB + AC + BC$$

$$BC = (x_1\bar{x}_2 + x_1\bar{x}_3)(x_2x_1 + x_2\bar{x}_3)$$

$$= x_1\bar{x}_2x_2x_1 + x_1\bar{x}_2x_2\bar{x}_3 + x_1\bar{x}_3x_2x_1 + x_1\bar{x}_3x_2\bar{x}_3$$

$$= 0 + 0 + x_1\bar{x}_3x_2 + x_1\bar{x}_3x_2$$

$$= x_1x_2\bar{x}_3$$

Finally, we get

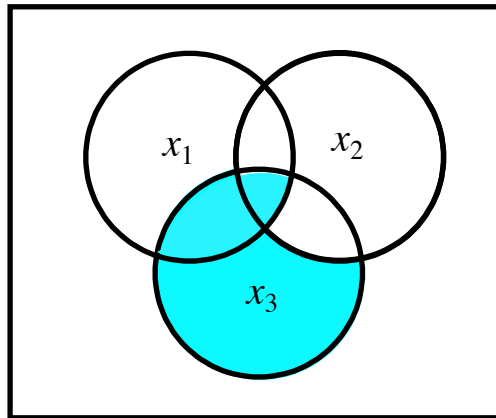
$$\begin{aligned} f &= x_1 \bar{x}_2 x_3 + x_1 x_2 x_3 + x_1 x_2 \bar{x}_3 \\ &= x_1 (\bar{x}_2 + x_2) x_3 + x_1 x_2 (x_3 + \bar{x}_3) \\ &= x_1 x_3 + x_1 x_2 \\ &= x_1 (x_3 + x_2) \end{aligned}$$

Example 4

Solve the previous problem using Venn diagrams.

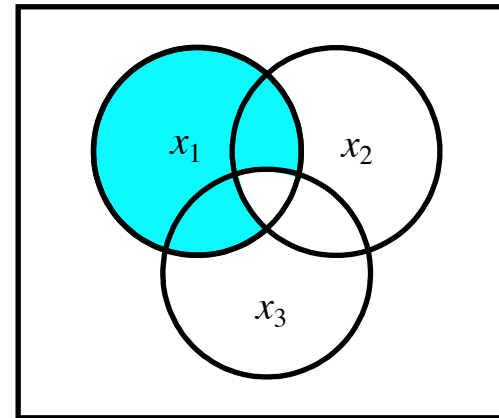
Venn Diagrams

(find the areas that are shaded at least two times)



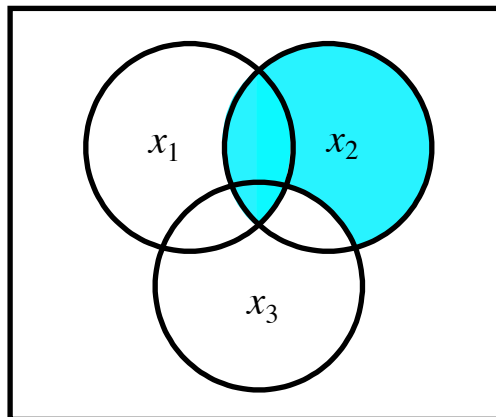
(a) Function A:

$$x_3x_1 + x_3\bar{x}_2$$



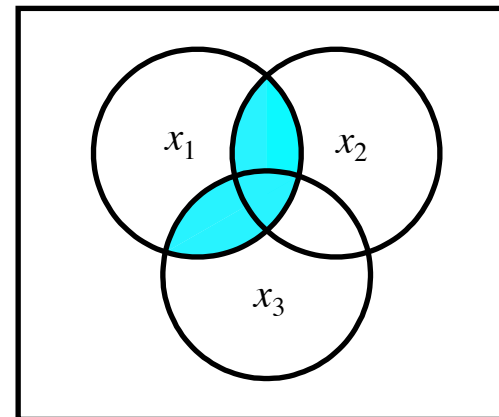
(b) Function B

$$x_1\bar{x}_2 + x_1\bar{x}_3$$



(c) Function C

$$x_2x_1 + x_2\bar{x}_3$$



(d) Function f

$$x_1(x_3 + x_2)$$

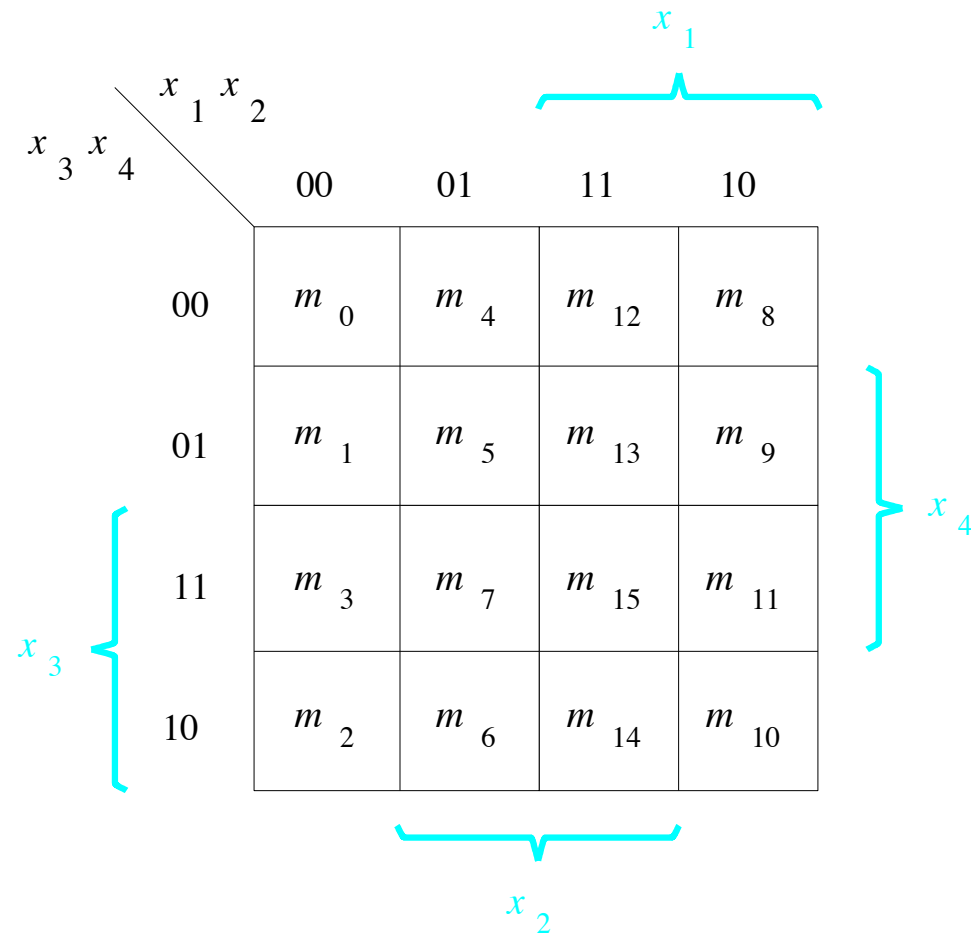
Example 5

Design the minimum-cost SOP and POS expression for the function

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

The Karnaugh map is annotated with cyan brackets indicating groupings for variables x_1 , x_2 , and x_3 . A horizontal bracket above the top two columns (11 and 10) is labeled x_1 . A horizontal bracket below the bottom two columns (11 and 10) is labeled x_2 . A vertical bracket to the left of the bottom two rows (11 and 10) is labeled x_3 . A vertical bracket to the right of the middle two rows (01 and 11) is labeled x_4 .

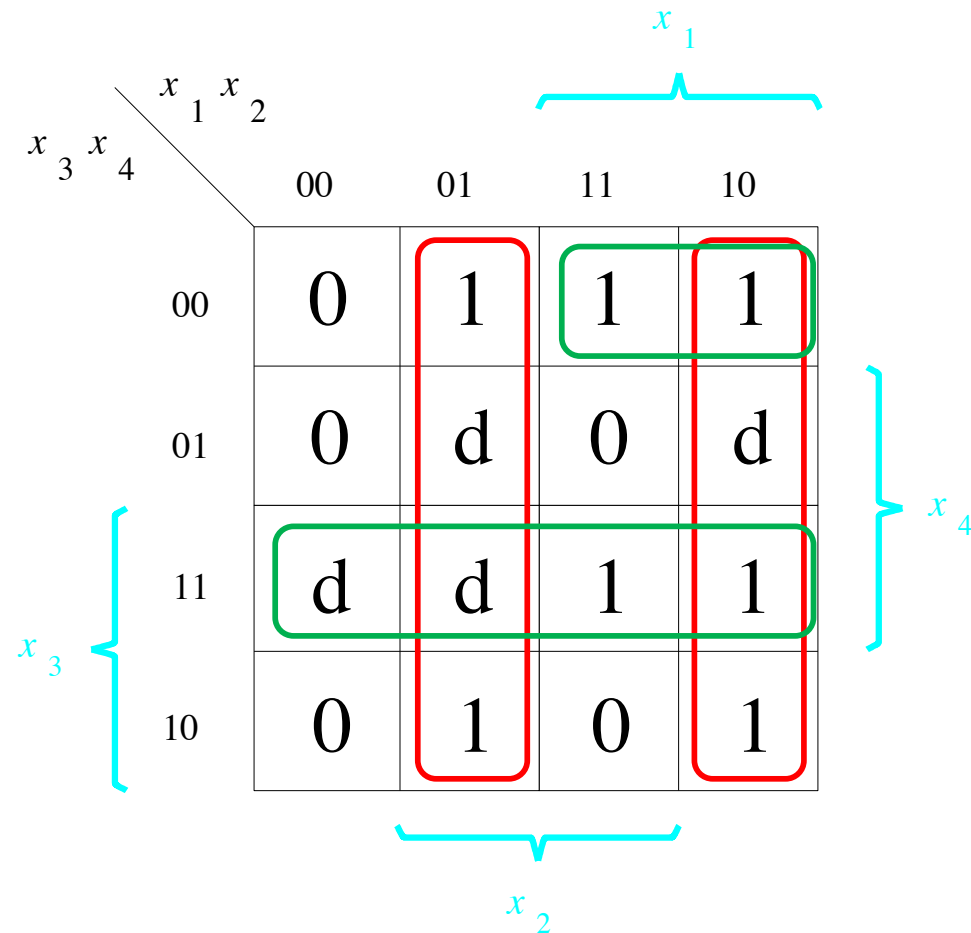
Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	0	1	1	1
01	0	d	0	d
11	d	d	1	1
10	0	1	0	1

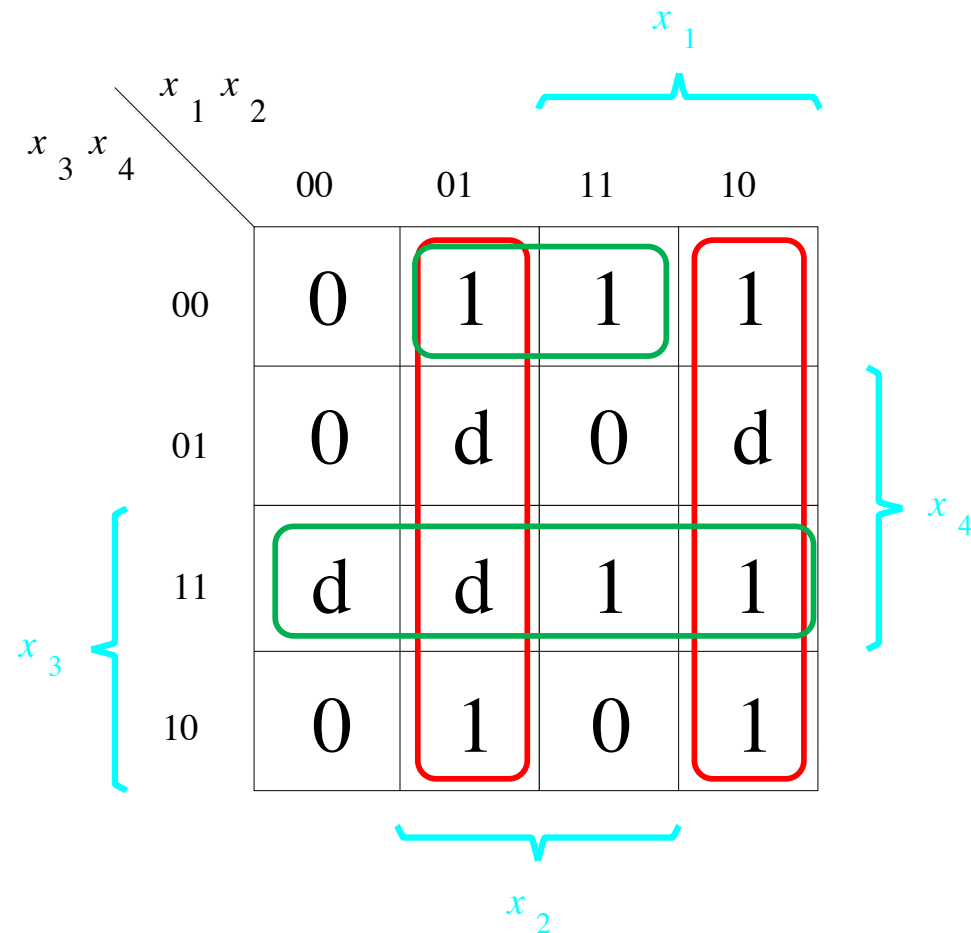
Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



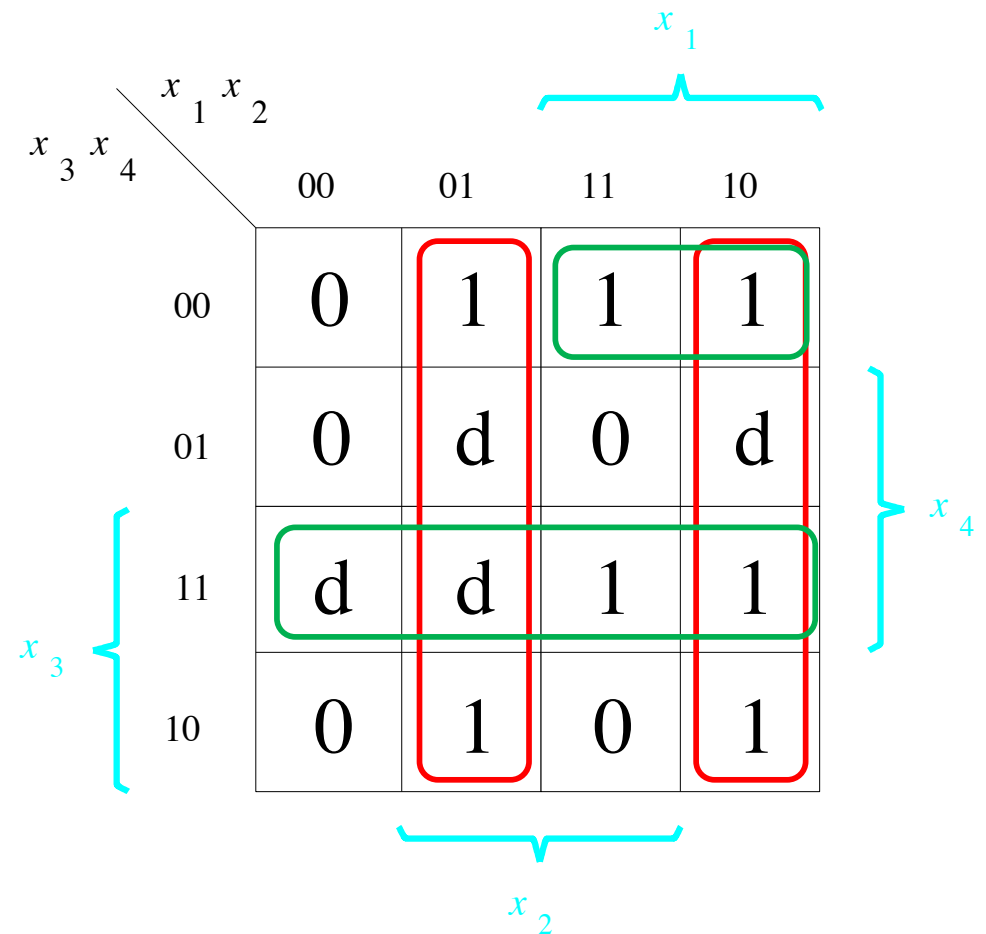
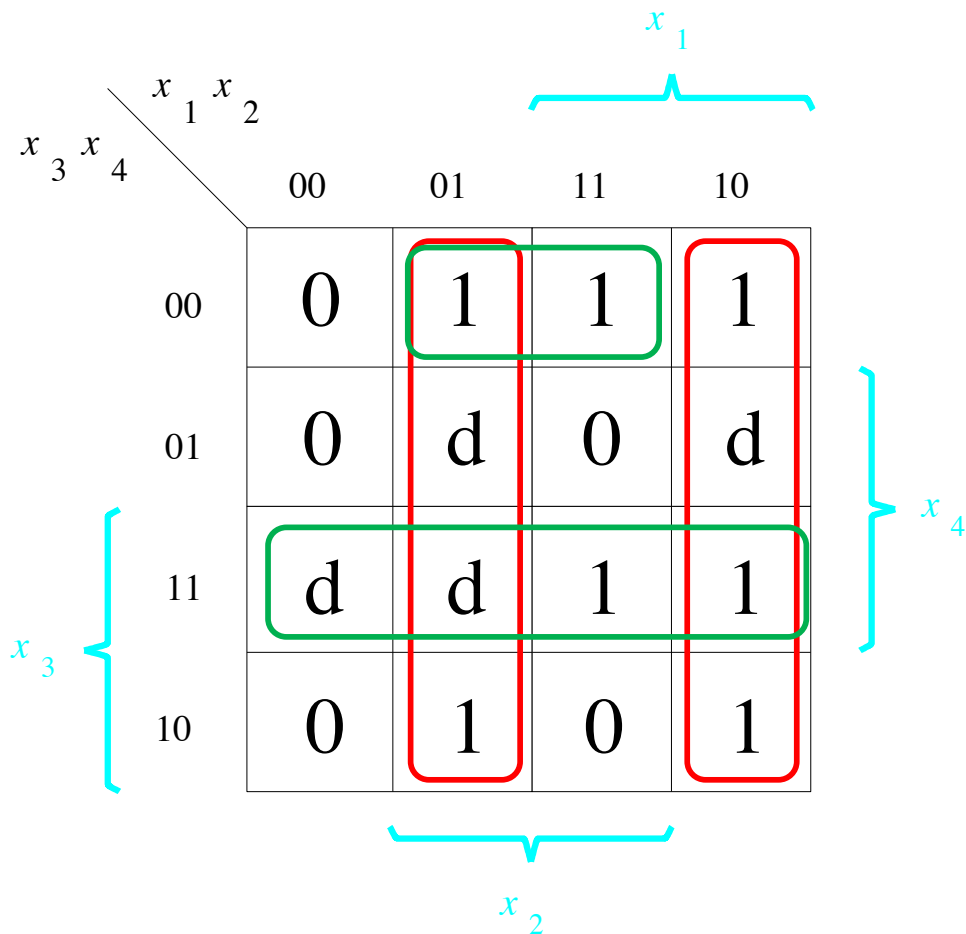
Let's Use a K-Map

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

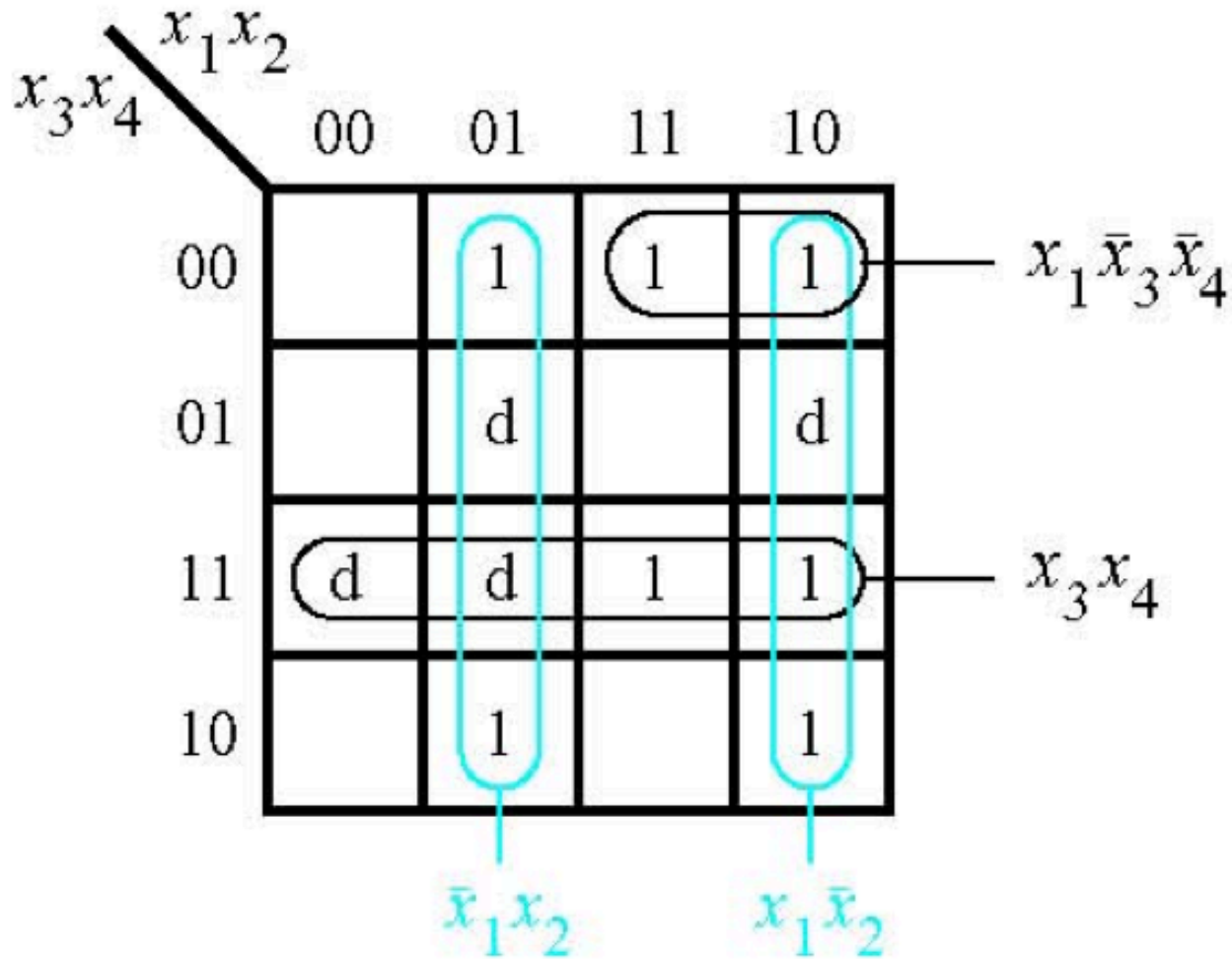


Two Alternative Solutions

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$



The SOP Expression



$$f = \bar{x}_1 x_2 + x_1 \bar{x}_2 + x_1 \bar{x}_3 \bar{x}_4 + x_3 x_4$$

What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

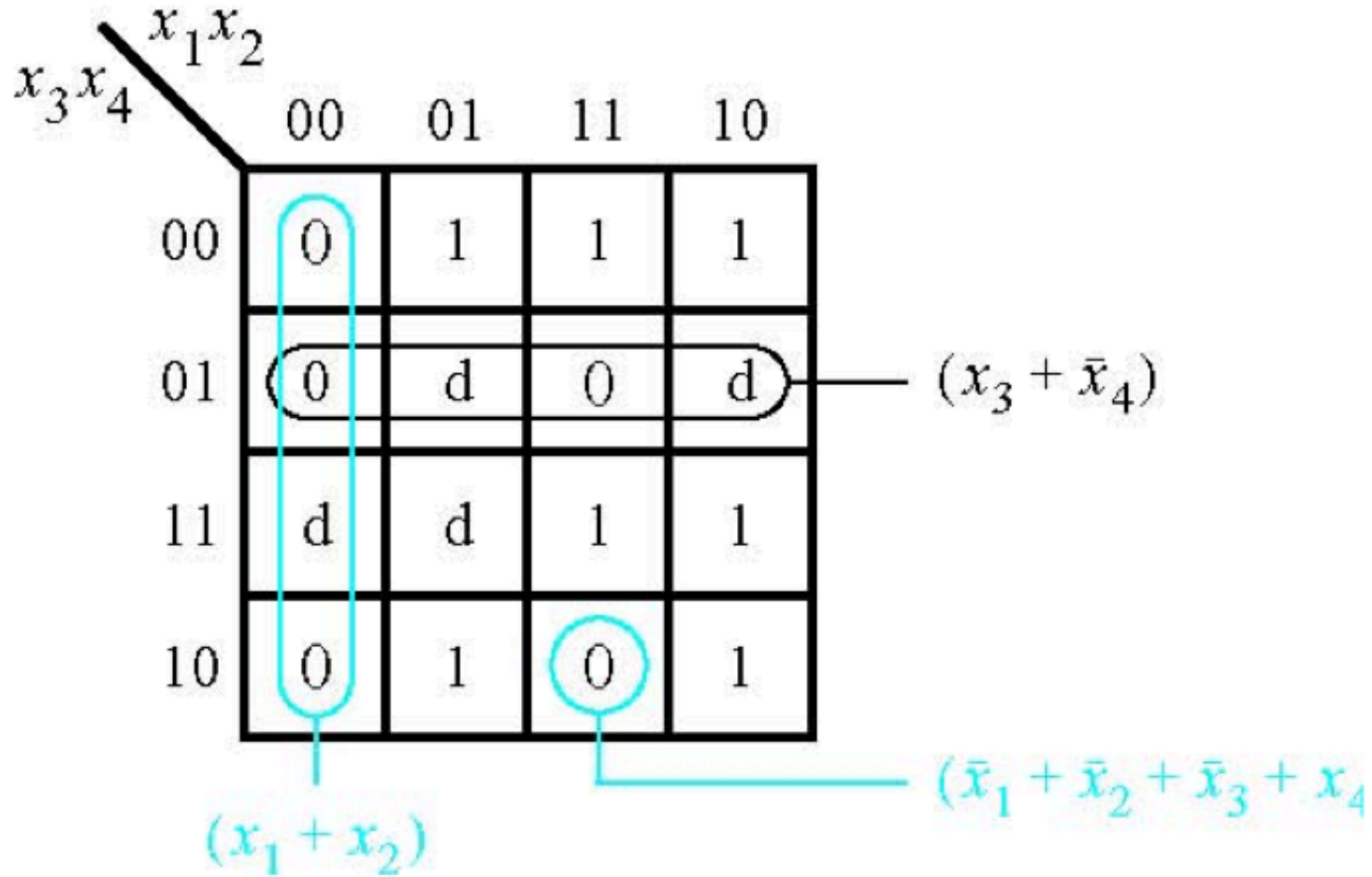
		$x_1 x_2$			
		00	01	11	10
$x_3 x_4$	00	0	1	1	1
	01	0	d	0	d
	11	d	d	1	1
	10	0	1	0	1

What about the POS Expression?

$$f(x_1, x_2, x_3, x_4) = \sum m(4, 6, 8, 10, 11, 12, 15) + D(3, 5, 7, 9)$$

		x_1		x_2	
		00	01	11	10
x_3	00	0	1	1	1
	01	0	d	0	d
x_4	11	d	d	1	1
	10	0	1	0	1

The POS Expression



$$f = (x_1 + x_2) \cdot (x_3 + \bar{x}_4) \cdot (\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + x_4)$$

Example 6

Use K-maps to find the minimum-cost SOP and POS expression for the function

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

assuming that there are also don't-cares defined as $D = \sum(9, 12, 14)$.

Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

$x_3 \ x_4$	$x_1 \ x_2$	00	01	11	10
00	m_0	m_4	m_{12}	m_8	
01	m_1	m_5	m_{13}	m_9	
11	m_3	m_7	m_{15}	m_{11}	
10	m_2	m_6	m_{14}	m_{10}	

Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$$D = \sum(9, 12, 14).$$

$x_3 \ x_4$		$x_1 \ x_2$					
		00	01	11	10	x_1	
x_3	00	m_0	m_4	d	m_8	x_4	
	01	m_1	m_5	m_{13}	d		
	11	m_3	m_7	m_{15}	m_{11}		
	10	m_2	m_6	d	m_{10}		
		x_2					

Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

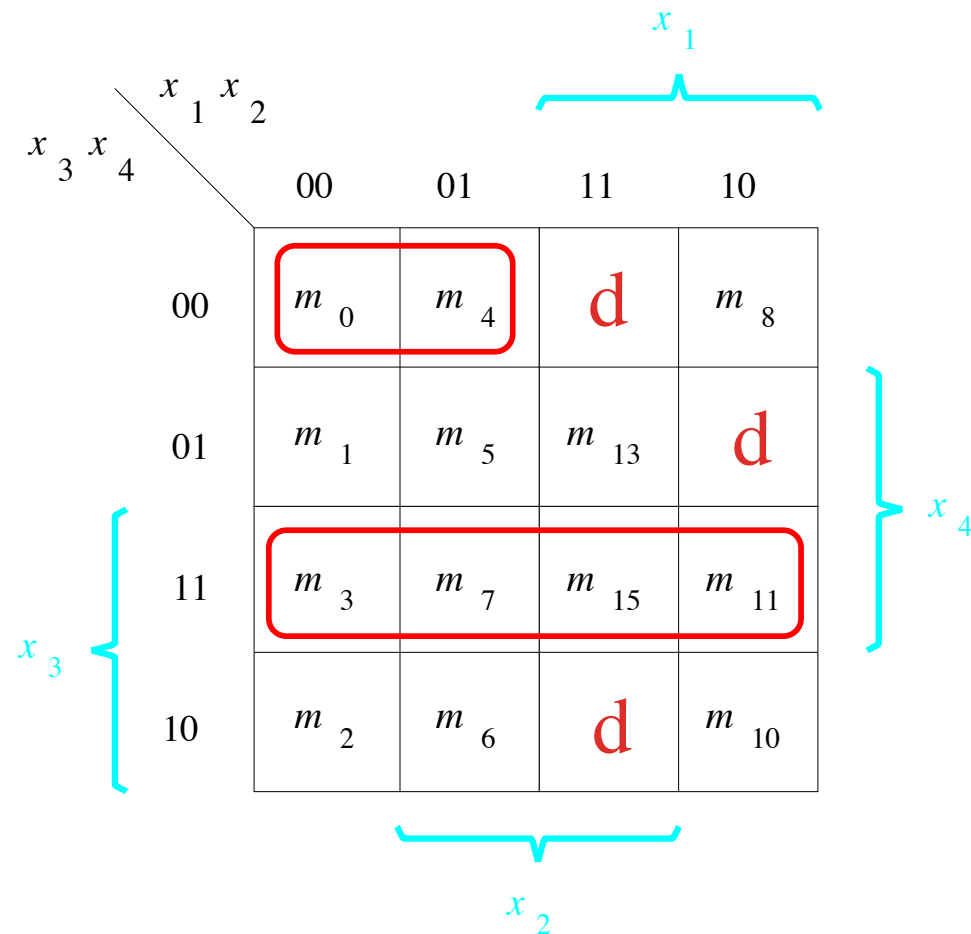
Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4 \backslash x_1 x_2$	00	01	11	10
00	m_0	m_4	d	m_8
01	m_1	m_5	m_{13}	d
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	d	m_{10}

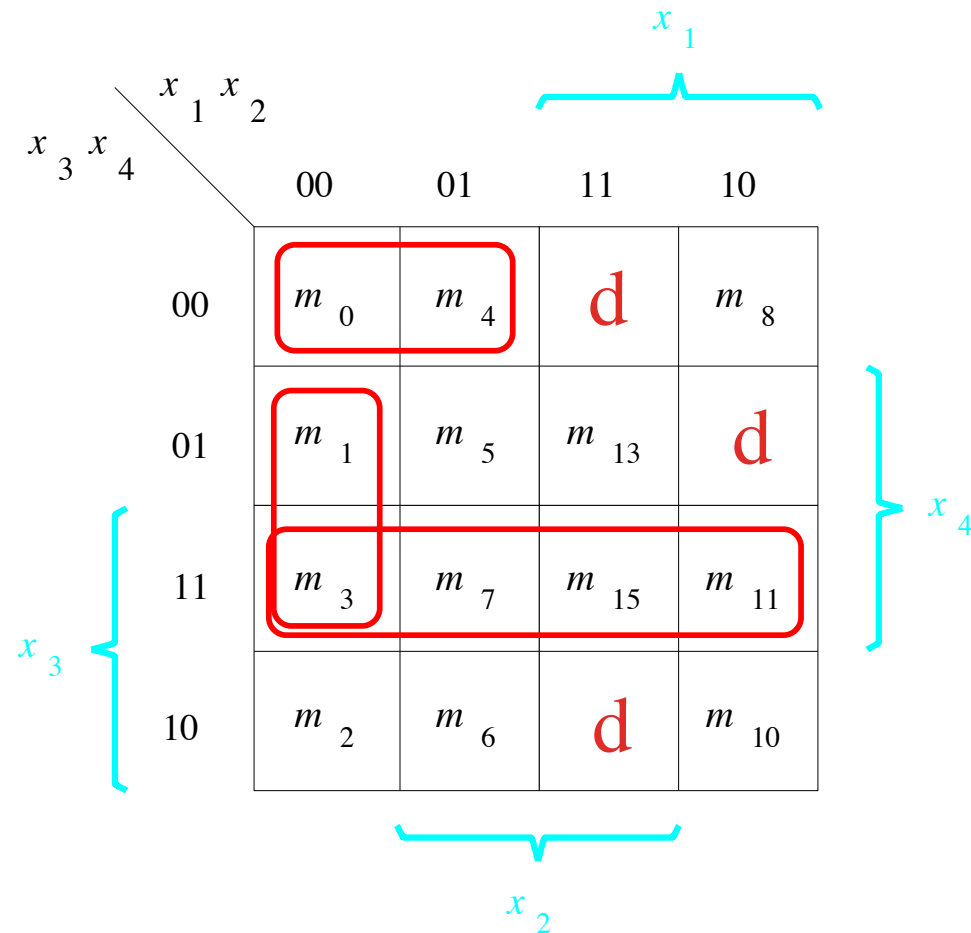
Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + \boxed{x_3x_4} + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



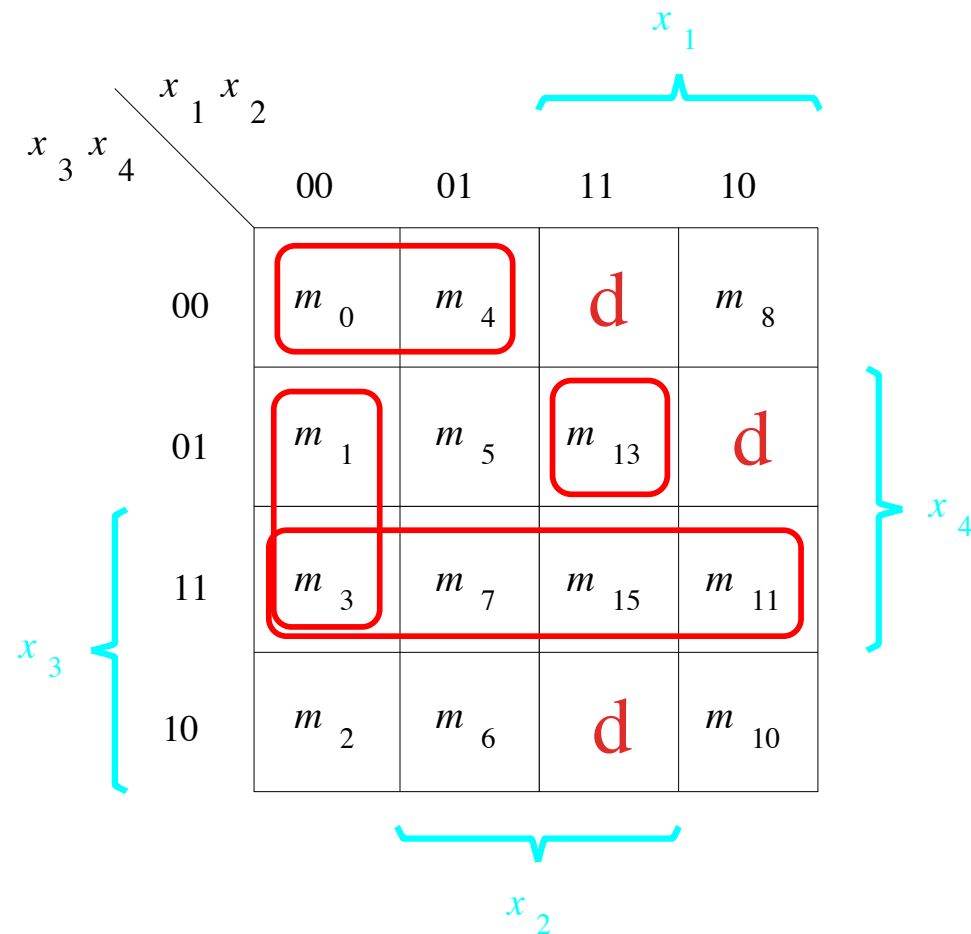
Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \boxed{\bar{x}_1\bar{x}_2x_4} + x_1x_2\bar{x}_3x_4$$



Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

$x_3 x_4$ \ $x_1 x_2$	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

Let's map the expression to the K-Map

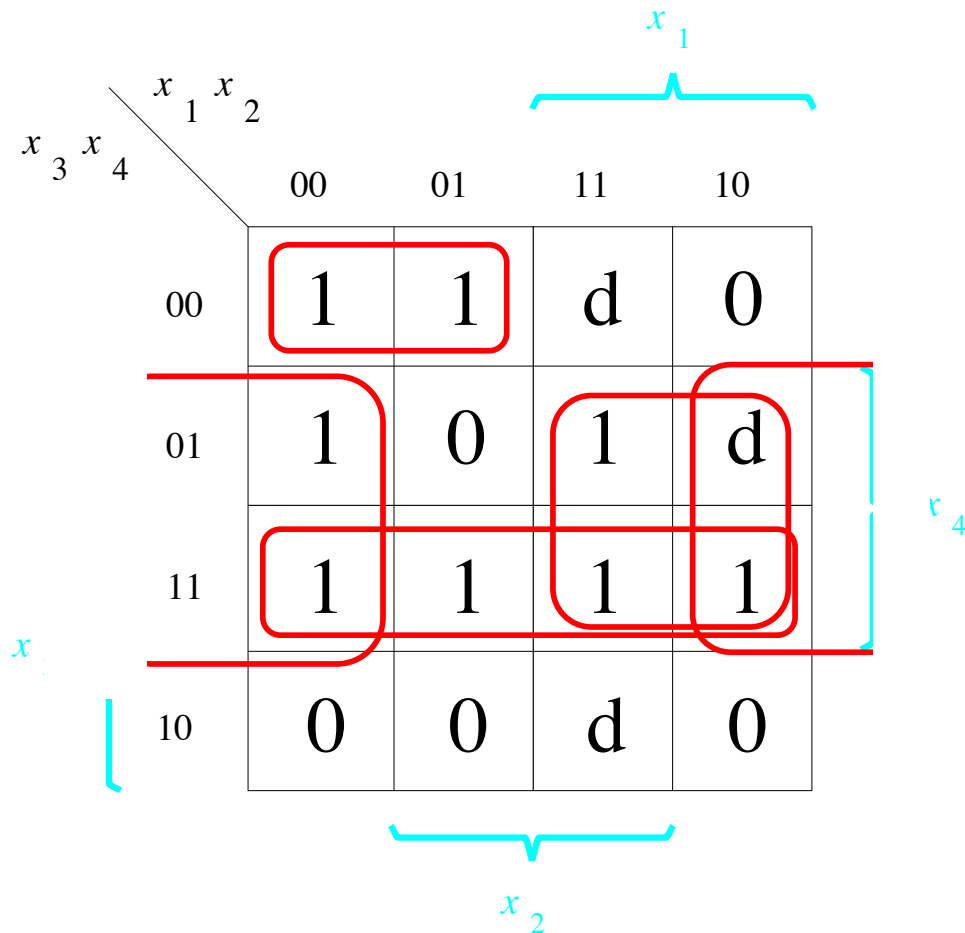
$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$

A 4x4 Karnaugh map for the function $f(x_1, x_2, x_3, x_4)$. The map is a 4x4 grid with columns labeled x_1x_2 (00, 01, 11, 10) and rows labeled x_3x_4 (00, 01, 11, 10). The cells contain the values 1, 0, or d (don't care). Cyan brackets indicate the variables x_1 , x_2 , x_3 , and x_4 associated with the map's structure.

$x_3x_4 \backslash x_1x_2$	00	01	11	10
00	1	1	d	0
01	1	0	1	d
11	1	1	1	1
10	0	0	d	0

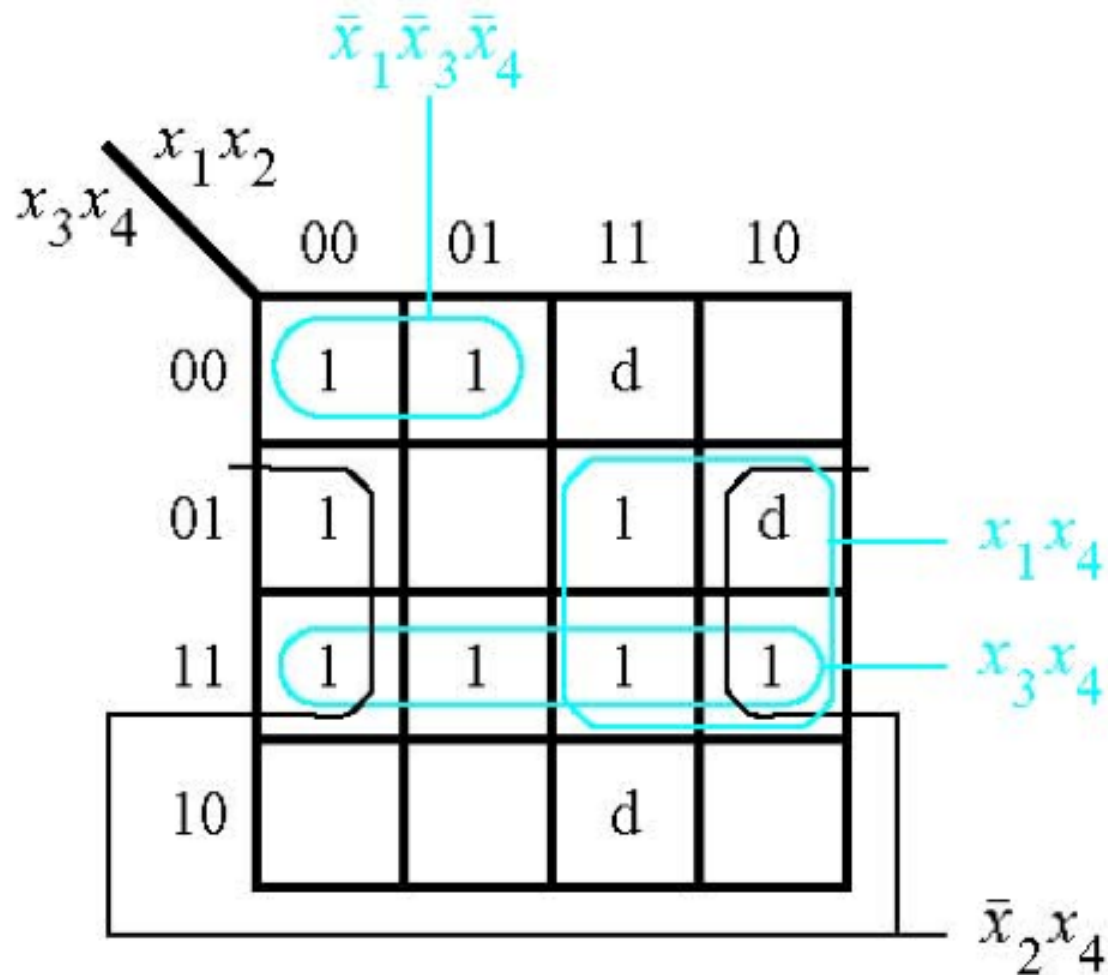
Let's map the expression to the K-Map

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



The SOP Expression

$$f(x_1, \dots, x_4) = \bar{x}_1\bar{x}_3\bar{x}_4 + x_3x_4 + \bar{x}_1\bar{x}_2x_4 + x_1x_2\bar{x}_3x_4$$



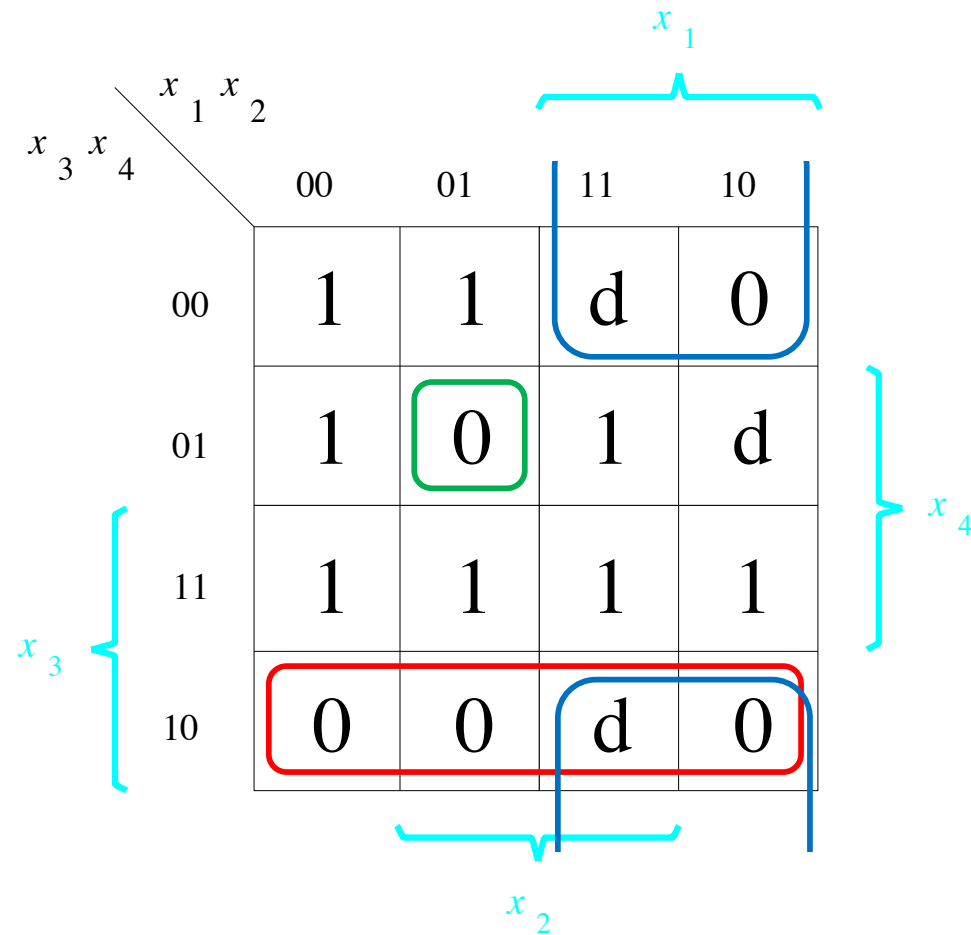
$$f = x_3x_4 + \bar{x}_1\bar{x}_3\bar{x}_4 + \bar{x}_2x_4 + x_1x_4$$

[Figure 2.68a from the textbook]

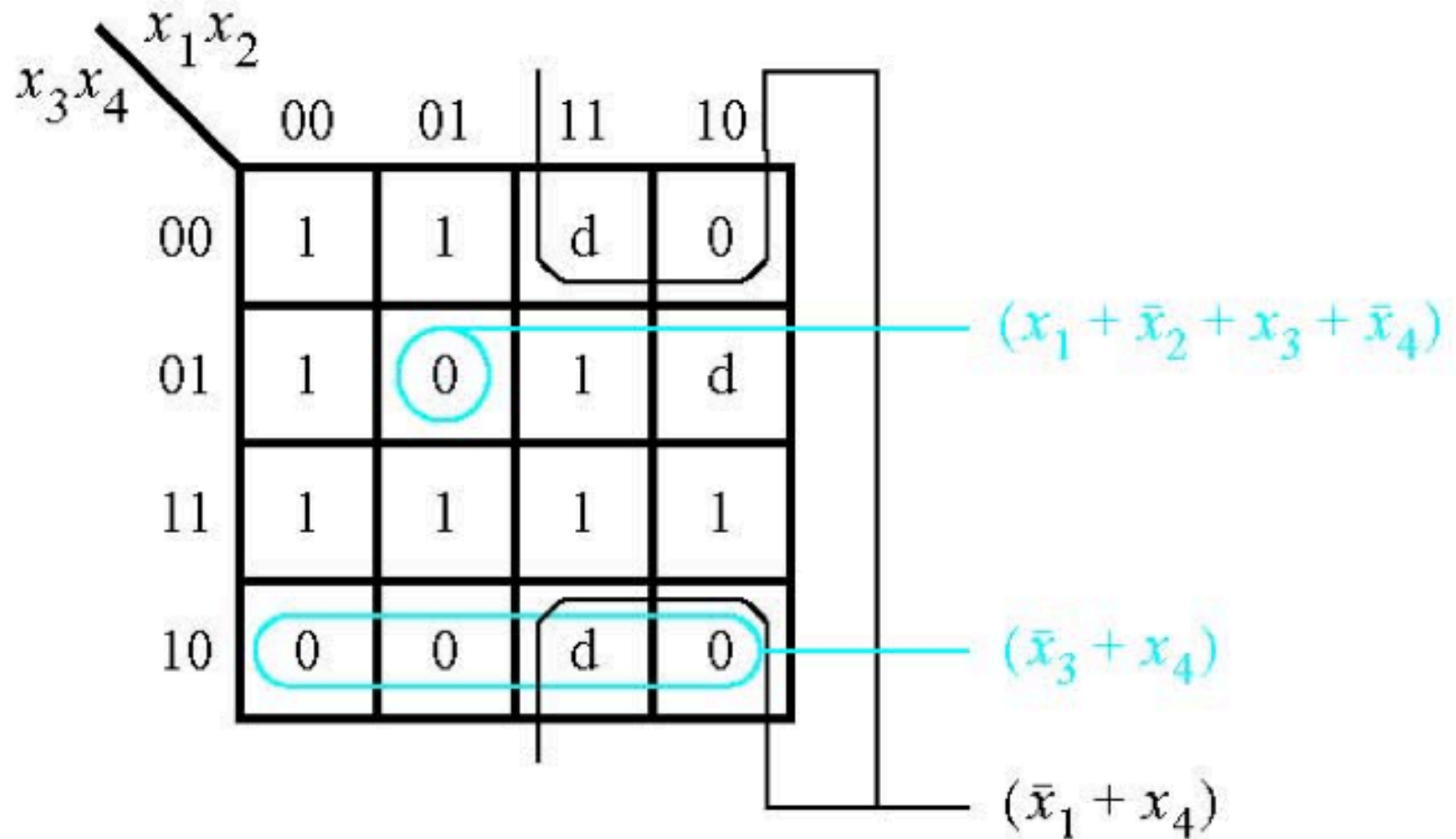
What about the POS Expression?

		x_1		x_2	
		x_3	x_4	x_3	x_4
x_3	x_4	00	01	11	10
00	1	1	d	0	
01	1	0	1	d	
11	1	1	1	1	
10	0	0	d	0	

What about the POS Expression?



The POS Expression



$$f = (\bar{x}_3 + x_4)(\bar{x}_1 + x_4)(x_1 + \bar{x}_2 + x_3 + \bar{x}_4)$$

Example 7

Derive the minimum-cost SOP expression for

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$

**First, expand the expression
using property 12a**

$$f = s_3(\bar{s}_1 + \bar{s}_2) + s_1s_2$$

$$f = \bar{s}_1s_3 + \bar{s}_2s_3 + s_1s_2$$

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
s_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
s_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
s_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
s_3	0	m_0	m_2	m_6	m_4
	1	m_1	m_3	m_7	m_5

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

		$s_1 s_2$			
		00	01	11	10
s_3	0	0	0	1	0
	1	1	1	1	1

(b) Karnaugh map

Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

s_1	s_2	s_3	
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table

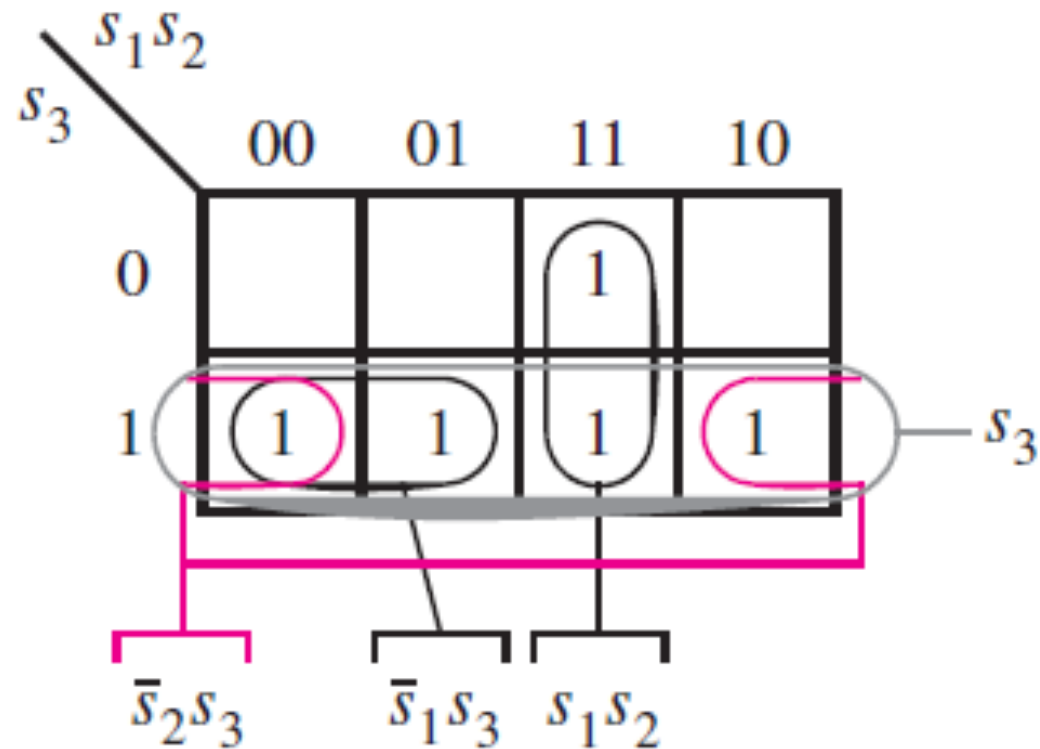
		$s_1 s_2$			
		00	01	11	10
s_3	0	0	0	1	0
	1	1	1	1	1

(b) Karnaugh map

Simplified Expression: $f = s_3 + s_1 s_2$

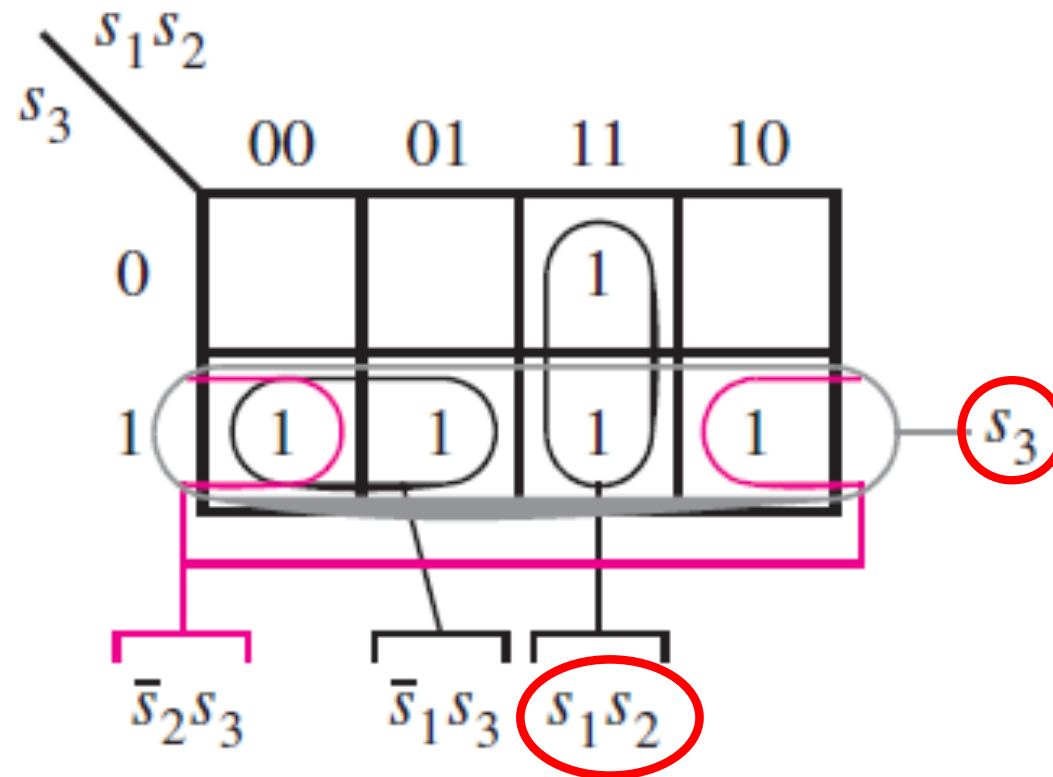
Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$



Construct the K-Map for this expression

$$f = \bar{s}_1 s_3 + \bar{s}_2 s_3 + s_1 s_2$$

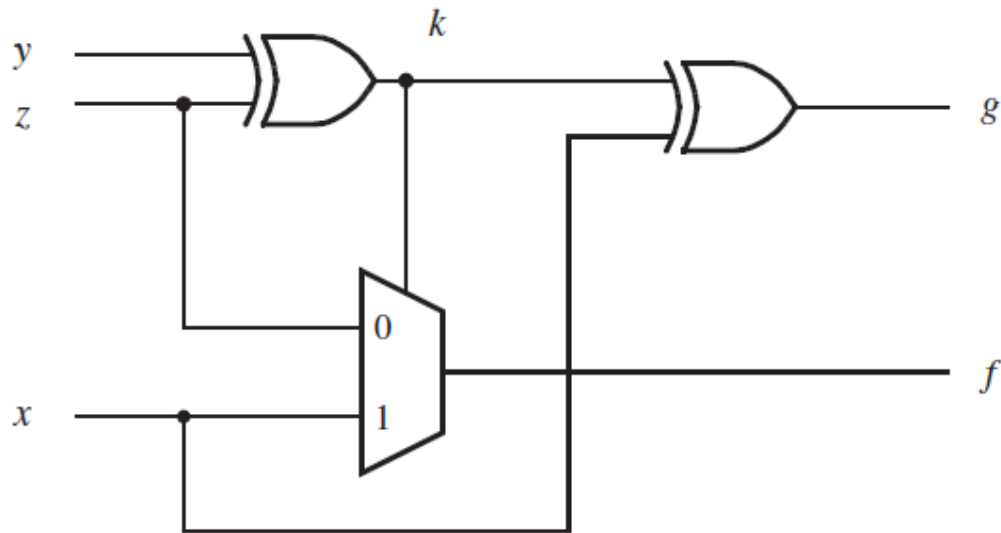


Simplified Expression: $f = s_3 + s_1 s_2$

Example 8

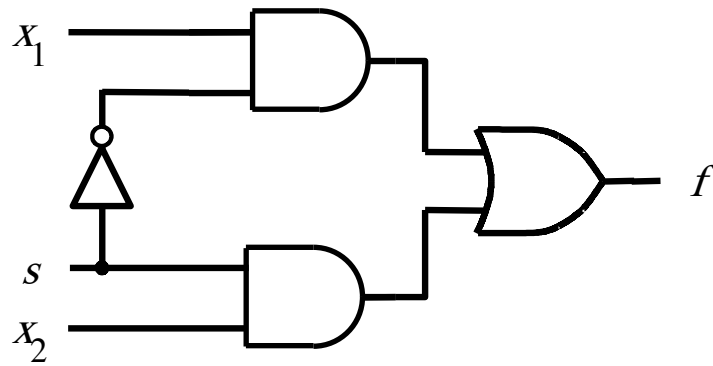
Write the Verilog code for the following circuit ...

Logic Circuit

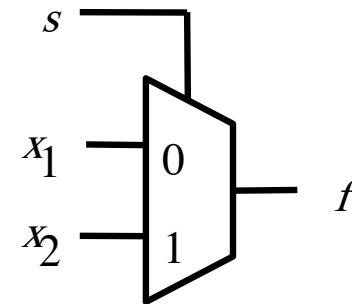


[Figure 2.70 from the textbook]

Circuit for 2-1 Multiplexer



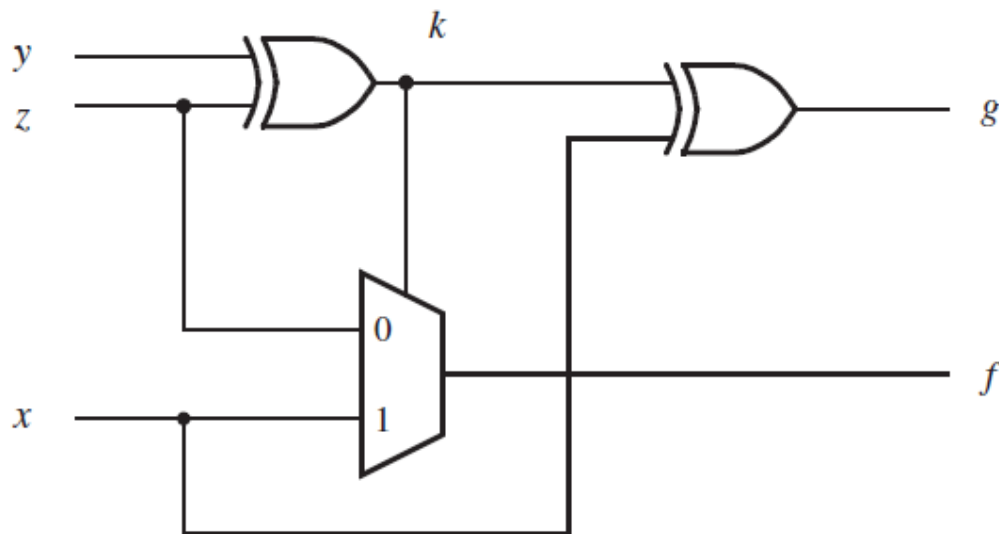
(b) Circuit



(c) Graphical symbol

$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

Logic Circuit vs Verilog Code

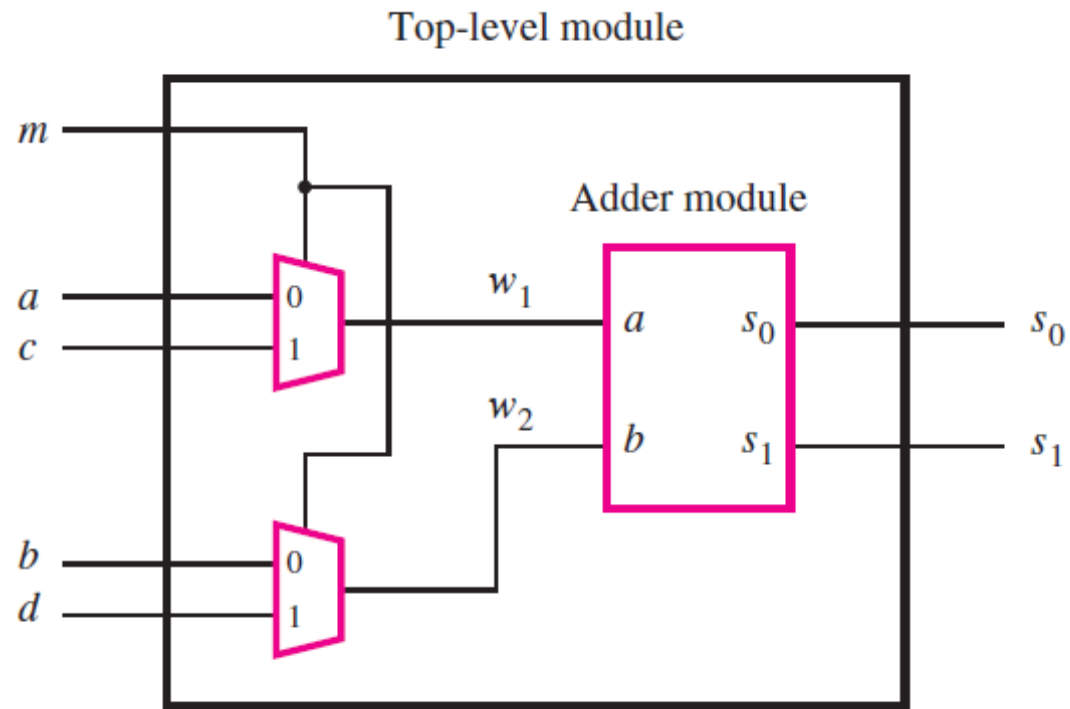


```
module f_g (x, y, z, f, g);  
  input x, y, z;  
  output f, g;  
  wire k;  
  
  assign k = y ^ z;  
  assign g = k ^ x;  
  assign f = (~k & z) | (k & x);  
  
endmodule
```

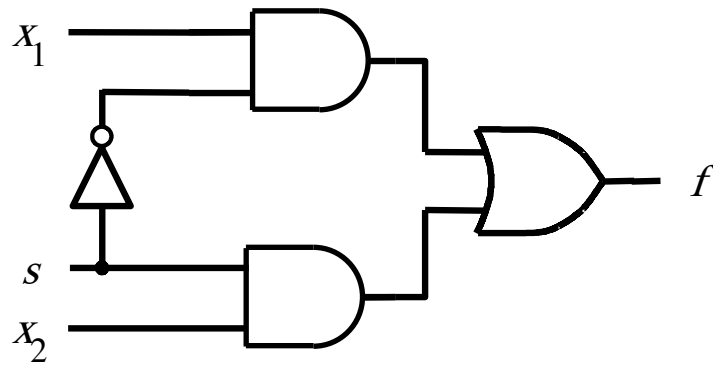
Example 9

Write the Verilog code for the following circuit ...

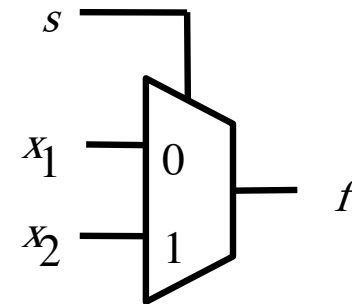
The Logic Circuit for this Example



Circuit for 2-1 Multiplexer



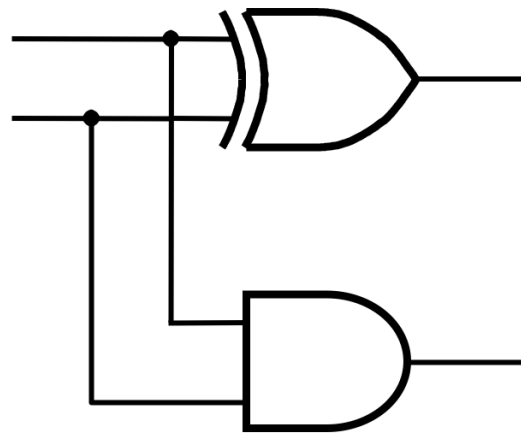
(b) Circuit



(c) Graphical symbol

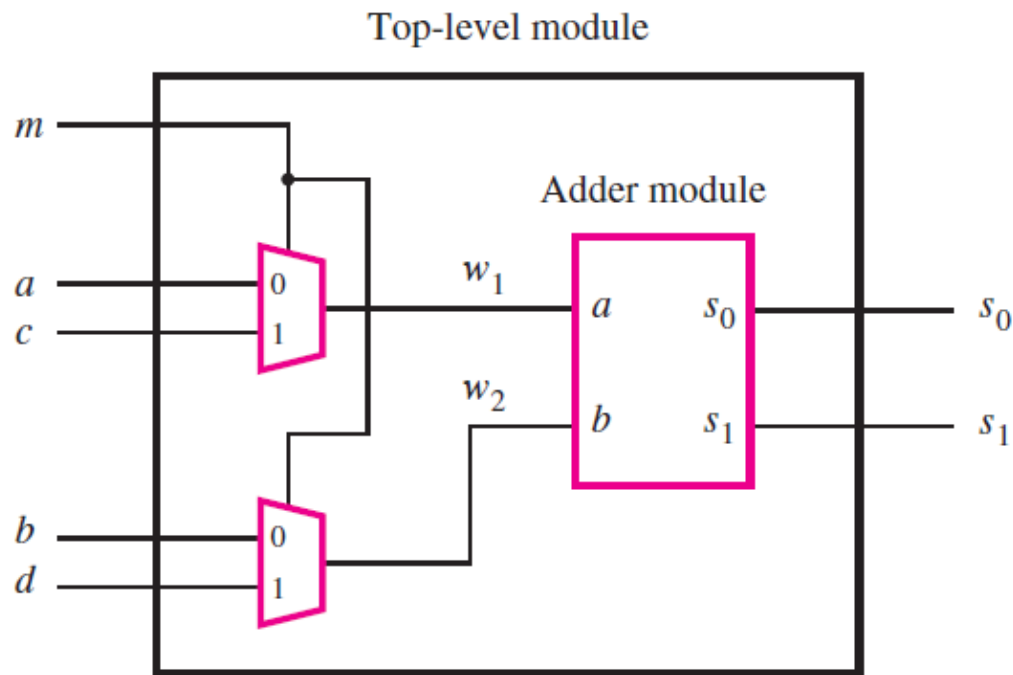
$$f(s, x_1, x_2) = \bar{s} x_1 + s x_2$$

Addition of Binary Numbers



a	b	s_1	s_0
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Logic Circuit vs Verilog Code



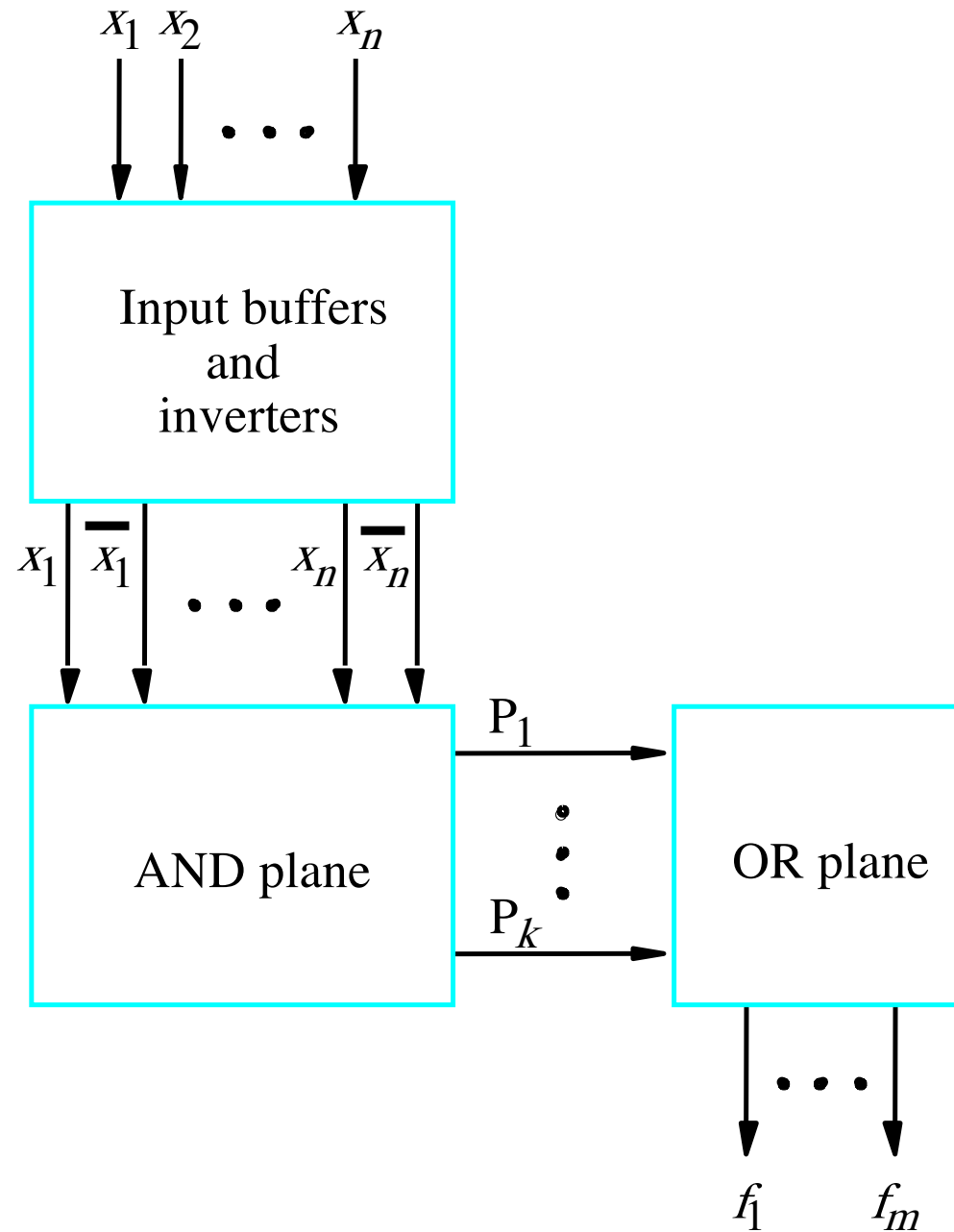
```
module shared (a, b, c, d, m, s1, s0);  
  input a, b, c, d, m;  
  output s1, s0;  
  wire w1, w2;  
  mux2to1 U1 (a, c, m, w1);  
  mux2to1 U2 (b, d, m, w2);  
  adder U3 (w1, w2, s1, s0);  
endmodule
```

```
module mux2to1 (x1, x2, s, f);  
  input x1, x2, s;  
  output f;  
  assign f = (~s & x1) | (s & x2);  
endmodule
```

```
module adder (a, b, s1, s0);  
  input a, b;  
  output s1, s0;  
  assign s1 = a & b;  
  assign s0 = a ^ b;  
endmodule
```

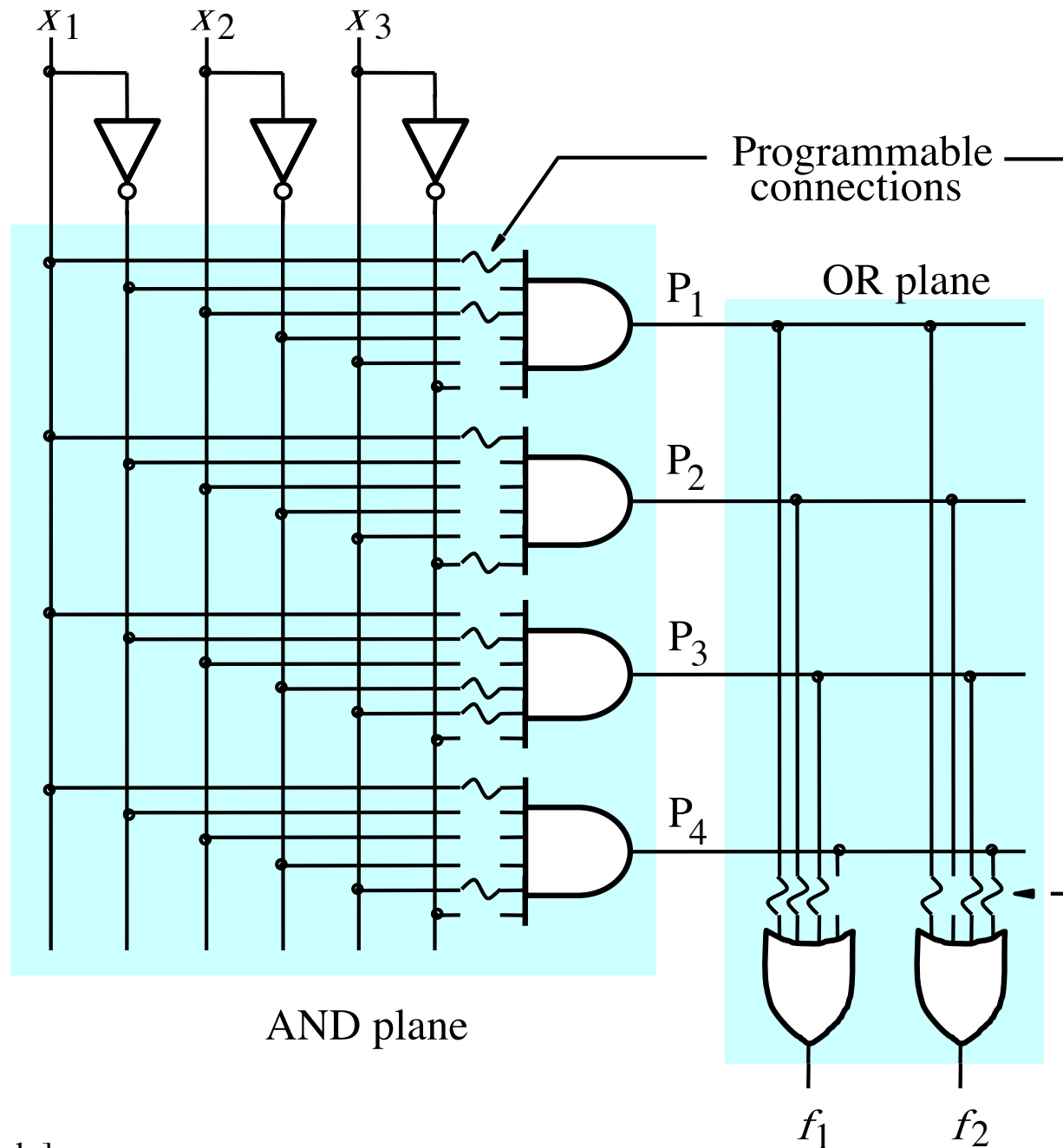
Some material from Appendix B

Programmable Logic Array (PLA)



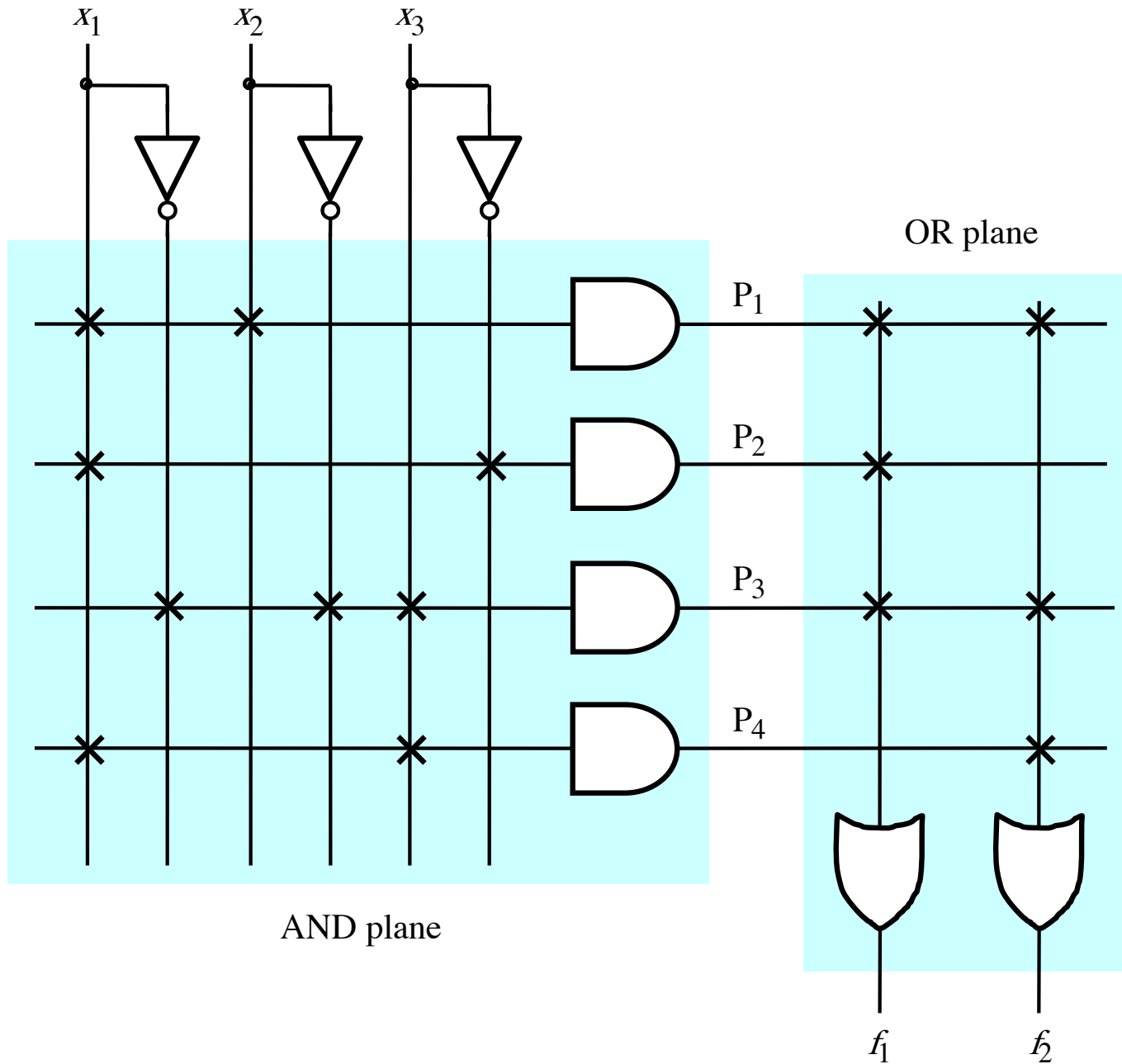
[Figure B.25 from textbook]

Gate-Level Diagram of a PLA



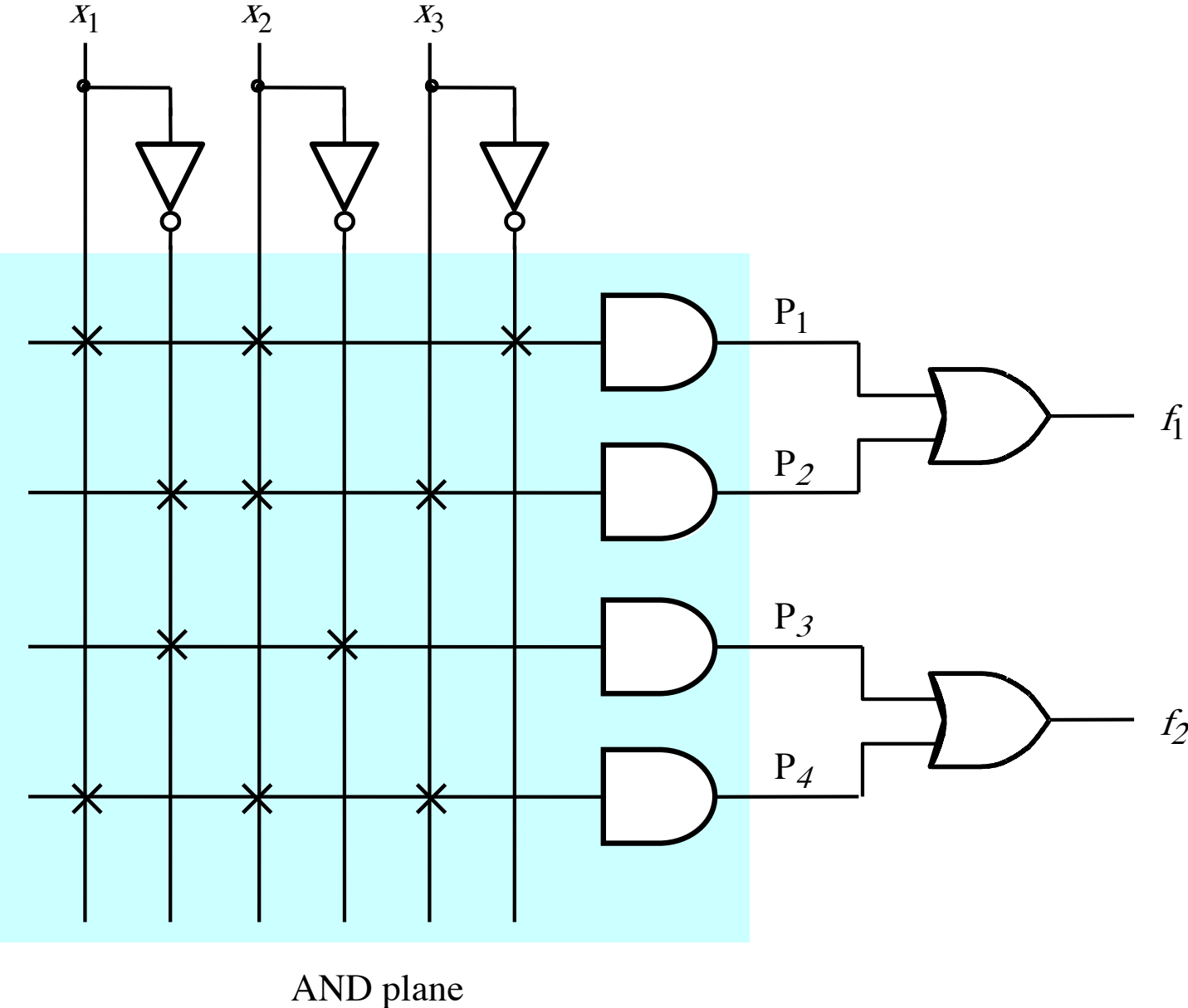
[Figure B.26 from textbook]

Customary Schematic for PLA



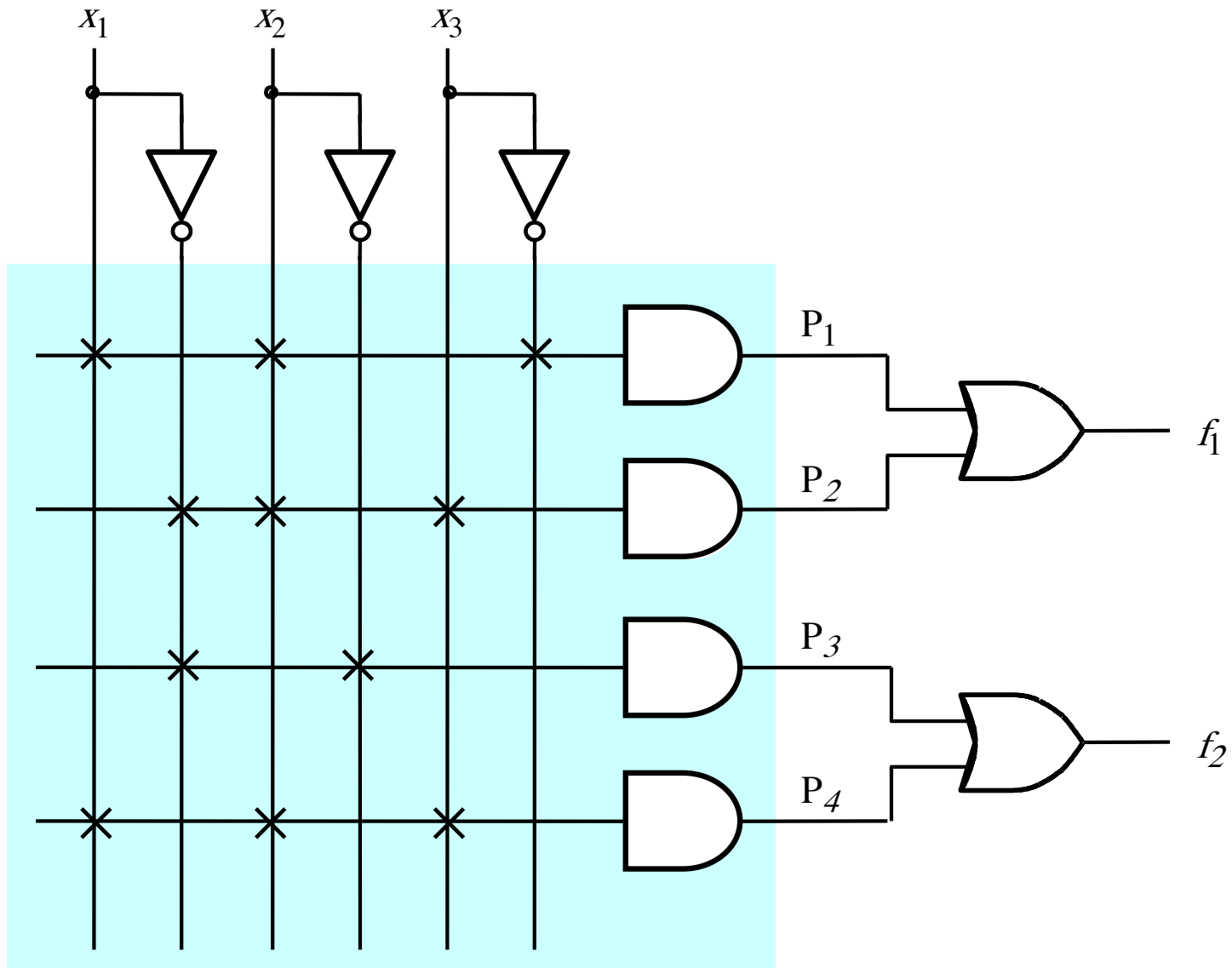
[Figure B.27 from textbook]

Programmable Array Logic (PAL)



[Figure B.28 from textbook]

Programmable Array Logic (PAL)



AND plane

Only the AND plane is programmable.
The OR plane is fixed.

Questions?

THE END