

## CprE 281: Digital Logic

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# Analysis of Synchronous Sequential Circuits 

CprE 281: Digital Logic
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## Goal for Today's Lecture

- Given a circuit diagram for a synchronous sequential circuit, the goal is to figure out the FSM.
- Figure out the present state variables, the next state variables, the state-assigned table, the state table, and finally the state diagram.
- In other words, the goal is to reverse engineer the circuit.


## What does this circuit do?


[ Figure 6.75 from the textbook ]

## Approach

- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.


## Where are the inputs?


[ Figure 6.75 from the textbook ]

## Where are the inputs?


[ Figure 6.75 from the textbook ]

## Where are the outputs?


[ Figure 6.75 from the textbook ]

## Where are the outputs?


[ Figure 6.75 from the textbook ]

Where kind of machine is this? Moore or Mealy?


## Moore: because the output does not depend directly on the primary input



## Where are the memory elements?



## Where are the memory elements?



## Where are the outputs of the flip-flops?



## Where are the outputs of the flip-flops?



## These are the present-state variables



Where are the inputs of the flip-flops?


Where are the inputs of the flip-flops?


These are the next-state variables


## What are their logic expressions?



## What are their logic expressions?



Where is the output, again?


Where is the output, again?


## What is its logic expression?



## What is its logic expression?



## This is what we have to work with now (we don't need the circuit anymore)

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

## Let's derive the state-assigned table

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | z |
| 0 |  |  |  |
| 00 |  |  |  |
| 01 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |

## Let's derive the state-assigned table

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | Z |  |  |
| 00 |  |  |  |
| 01 |  |  |  |
| 10 |  |  |  |
| 11 |  |  |  |

## Let's derive the state-assigned table

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\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | z |
| 00 |  |  | 0 |
| 00 |  | 0 |  |
| 01 |  |  | 0 |
| 10 |  |  | 1 |
| 11 |  |  |  |

## Let's derive the state-assigned table

$$
Y_{1}=w \bar{y}_{1}+w y_{2}
$$

$$
Y_{2}=w y_{1}+w y_{2}
$$

$$
z=y_{1} y_{2}
$$

| $\begin{array}{c}\text { Present } \\ \text { state } \\ \mathrm{y}_{2} \mathrm{y}_{1}\end{array}$ | Next State |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | Y | Y |  |$)$

## Let's derive the state-assigned table

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | z |
| 00 | 0 | 1 | 0 |
| 01 | 0 | 0 | 0 |
| 10 | 0 | 1 | 0 |
| 11 | 0 | 1 | 1 |

## Let's derive the state-assigned table

$$
Y_{1}=w \bar{y}_{1}+w y_{2}
$$

$$
Y_{2}=w y_{1}+w y_{2}
$$

$z=y_{1} y_{2}$

| Present <br> state <br> $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | Y2) $r_{1}$ | Y2 $r_{1}$ |  |
| 00 | 0 | 1 | 0 |
| 01 | 0 | 0 | 0 |
| 10 | 0 | 1 | 0 |
| 11 | 0 | 1 | 1 |

## Let's derive the state-assigned table

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | z |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

## We don't need the logic expressions anymore

$$
\begin{aligned}
& Y_{1}=w \bar{y}_{1}+w y_{2} \\
& Y_{2}=w y_{1}+w y_{2} \\
& z=y_{1} y_{2}
\end{aligned}
$$

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ | Output |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

## We don't need the logic expressions anymore

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ | Output |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

## Let's derive the state table



State table

| Present state $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | w = 1 |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $Y_{2} Y_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

## Let's derive the state table



State table

| Present state $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | w = 1 |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $Y_{2} Y_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

## Let's derive the state table

| Present state | Next state | Output <br> Z | $\begin{gathered} \text { Present } \\ \text { state } \\ \mathrm{y}_{2} \mathrm{y}_{1} \end{gathered}$ | Next | State | Output <br> Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |  | $\mathrm{w}=0$ | w = 1 |  |
| $\mathrm{A} \leftarrow$ |  |  |  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| B |  |  | -00 | 00 | 01 | 0 |
| $\mathrm{C} \leftarrow$ |  |  | -0 1 | 00 | 10 | 0 |
| D |  |  | -10 | 00 | 11 | 0 |
|  |  |  | -11 | 00 | 11 | 1 |

State table
State-assigned table

## Let's derive the state table



State table

| Present <br> state $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $w=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

## Let's derive the state table

| Present state | Next state | Outputz | Present state $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ |  |  | w = 0 | w = 1 |  |
| A | $\left(\begin{array}{l}\text { A } \\ A \\ A \\ A\end{array}\right)$ |  |  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| B |  |  | 00 | 00 | 01 | 0 |
| C |  |  | 0 | -00 | 10 | 0 |
| D |  |  | 10 | 00 | 11 | 0 |
|  |  |  | 11 | 00 | 11 | 1 |

State table
State-assigned table

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A |  |  |
| B | A |  |  |
| C | A |  |  |
| D | A |  |  |


| Present <br> state | Next State |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State table
State-assigned table

## Let's derive the state table

| Present state | Next state |  | Outputz | Present state $\mathrm{y}_{2} \mathrm{y}_{1}$ | Next State |  | Output <br> Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | w = 0 | $w=1$ |  |  | w = 0 | w = 1 |  |
| A | A | $\left(\begin{array}{l} B \\ C \\ D \\ D \end{array}\right)$ |  |  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| B | A |  |  | 00 |  |  | 0 |
| C |  |  |  | 01 | 0 | 10 | 0 |
| D |  |  |  | 10 | 00 | $\left(\begin{array}{ll}1 \\ 1\end{array}\right.$ | 0 |
|  |  |  |  | 11 | 00 |  | 1 |

State table
State-assigned table

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=1$ |  |  |
| A | A | B |  |
| B | A | C |  |
| C | A | D |  |
| D | A | D |  |


| Present <br> state | Next State |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State table
State-assigned table

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B |  |
| B | A | C |  |
| C | A | D |  |
| D | A | D |  |

State table

| Present <br> state | Next State |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

The output is the same in both tables

## The two tables for the initial circuit

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
|  | Output |  |  |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | z |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

## We don't need the state-assigned table anymore

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table

| Present <br> state | Next State |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ | Output |
|  | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ | $\mathrm{Y}_{2} \mathrm{Y}_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 00 | 11 | 0 |
| 11 | 00 | 11 | 1 |

State-assigned table

## We don't need the state-assigned table anymore

| Present <br> state | Next state |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table

## Let's Draw the State Diagram

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

## Let's Draw the State Diagram

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |



Because this is a Moore machine the output is tied to the state

## Let's Draw the State Diagram

| Present state | Next state |  | Output <br> Z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | (D) | 1 |



All transitions when the input $w$ is equal to 1

## Let's Draw the State Diagram

| Present <br> state | Next state |  | Output <br>  |
| :---: | :---: | :---: | :---: |
|  | w $=1$ |  |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |



## Let's Draw the State Diagram

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |



## Let's Draw the State Diagram



## We are done!



State diagram

## Almost done. What does this FSM do?

| Present <br> state | Next state |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table


State diagram

## Almost done. What does this FSM do?

It sets the output $z$ to 1 when three consecutive 1's occur on the input w. In other words, it is a sequence detector for the input pattern 111.

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table


State diagram

## Another Example (with JK flip-flops)

## What does this circuit do?


[ Figure 6.77 from the textbook]

## Approach

- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.


## Where are the inputs and outputs?


[ Figure 6.77 from the textbook ]

## Where are the inputs and outputs?



## What kind of machine is this?



## Where are the flip-flops?



## Where are the flip-flops?



## Where are the outputs of the flip-flops?



## Where are the outputs of the flip-flops?



## These are the present-state variables



Where are the inputs of the flip-flops?


Where are the inputs of the flip-flops?


## What are their logic expressions?



## What are their logic expressions?



## What is the logic expression of the output?



## What is the logic expression of the output?



## This is what we have to work with now (we don't need the circuit anymore)

$$
\begin{aligned}
\mathrm{J}_{1} & =\mathrm{w} \\
\mathrm{~K}_{1} & =\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
\mathrm{~J}_{2} & =\mathrm{w} \mathrm{y}_{1} \\
\mathrm{~K}_{2} & =\overline{\mathrm{w}} \\
\mathrm{z} & =\mathrm{y}_{1} \mathrm{y}_{2}
\end{aligned}
$$

## Let's derive the excitation table

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
& \mathrm{~J}_{2}=\mathrm{w} \mathrm{y}_{1} \\
& \mathrm{~K}_{2}=\overline{\mathrm{w}} \\
& z=y_{1} y_{2}
\end{aligned}
$$

## Let's derive the excitation table

$$
\begin{aligned}
\mathrm{J}_{1} & =\mathrm{w} \\
\mathrm{~K}_{1} & =\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
\mathrm{~J}_{2} & =\mathrm{w} \mathrm{y}_{1} \\
\mathrm{~K}_{2} & =\overline{\mathrm{w}}
\end{aligned}
$$

| $\begin{array}{c}\text { Present } \\ \text { state }\end{array}$ | Flip-flop inputs |  |  | Output |
| :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | $z$ |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ |  |$]$

$$
z=y_{1} y_{2}
$$

## Let's derive the excitation table

$$
\begin{aligned}
\mathrm{J}_{1} & =\mathrm{w} \\
\mathrm{~K}_{1} & =\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
\mathrm{~J}_{2} & =\mathrm{w} \mathrm{y}_{1} \\
\mathrm{~K}_{2} & =\overline{\mathrm{w}}
\end{aligned}
$$

$$
z=y_{1} y_{2}
$$

## Let's derive the excitation table

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2}
\end{aligned}
$$

$$
\mathrm{J}_{2}=\mathrm{w} \mathrm{y}_{1}
$$

$$
\mathrm{K}_{2}=\overline{\mathrm{w}}
$$

| Present <br> state | Flip-flop inputs |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Output |  |  |  |
| $y_{2} y_{1}$ | $w=0$ | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |
| $z$ |  |  |  |  |
| 00 |  |  | 0 |  |
| 01 |  |  | 0 |  |
| 10 |  |  | 0 |  |
| 11 |  |  |  |  |

$$
z=y_{1} y_{2}
$$

## Let's derive the excitation table

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2}
\end{aligned}
$$

$$
\mathrm{J}_{2}=\mathrm{w} \mathrm{y}_{1}
$$

$$
\mathrm{K}_{2}=\overline{\mathrm{w}}
$$

| Present state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 |  | 01 |  | 11 | 0 |
| 01 |  | 01 |  | 11 | 0 |
| 10 |  | 01 |  | 10 | 0 |
| 11 |  | 01 |  | 10 | 1 |

$$
z=y_{1} y_{2}
$$

## Let's derive the excitation table

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
& \mathrm{~J}_{2}=\mathrm{w} \mathrm{y}_{1} \\
& K_{2}=\bar{w} \\
& z=y_{1} y_{2}
\end{aligned}
$$

## The excitation table

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
& \mathrm{~J}_{2}=\mathrm{w} \mathrm{y}_{1} \\
& \mathrm{~K}_{2}=\overline{\mathrm{w}} \\
& z=y_{1} y_{2}
\end{aligned}
$$

## We don't need the logic expressions anymore

$$
\begin{aligned}
& \mathrm{J}_{1}=\mathrm{w} \\
& \mathrm{~K}_{1}=\overline{\mathrm{w}}+\overline{\mathrm{y}}_{2} \\
& \mathrm{~J}_{2}=\mathrm{w} \mathrm{y}_{1} \\
& \mathrm{~K}_{2}=\overline{\mathrm{w}} \\
& z=y_{1} y_{2}
\end{aligned}
$$

## We don't need the logic expressions anymore

| Present <br> state | Flip-flop inputs |  |  |  | Output |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ | $z$ |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |

[ Figure 6.78 from the textbook ]

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
|  |  |  |  |
|  |  |  |  |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |

State table

Excitation table

## Let's derive the state table

| Present state | Next state | $\begin{aligned} & \text { Output } \\ & \text { z } \end{aligned}$ | Presen state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |  |  | 0 | w |  |  |
| $\mathrm{A} \leftarrow$ |  |  |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| $\mathrm{B} \leftarrow$ |  |  | - 00 | 01 | 01 | 00 | 11 | 0 |
| $\mathrm{C} \leftarrow$ |  |  | - 01 | 01 | 01 | 10 | 11 | 0 |
| D $\leftarrow$ |  |  | - 10 | 01 | 01 | 00 | 10 | 0 |
|  |  |  | - 11 | 01 | 01 | 10 | 10 | 1 |

State table
Excitation table

This step is easy
(map 2-bit numbers to 4 letters)

## Let's derive the state table

| Present state | Next state | Output <br> Z | Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |  | $w=0$ |  | $w=1$ |  |  |
| A |  | 0 |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| B |  | 0 | 00 | 01 | 01 | 00 | 14 | - 0 |
| C |  | 0 | 01 | 01 | 01 | 10 | 11 | -0 |
| D |  | 1 | 10 | 01 | 01 | 00 | 10 | -0 |
|  |  |  | 11 | 01 | 01 | 10 | 10 | -1 |

State table
Excitation table

This step is easy too
(the outputs are the same in both tables)

## Let's derive the state table

| Present state | Next state | Output <br> Z | Present state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | w = 0 w = 1 |  |  |  | 0 | w |  |  |
| A | ? | 0 |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| B |  | 0 | 00 | 01 | 01 | 00 | 11 | 0 |
| C |  | 0 | 01 | 01 | 01 | 10 | 11 | 0 |
| D |  | 1 | 10 | 01 | 01 | 00 | 10 | 0 |
|  |  |  | 11 | 01 | 01 | 10 | 10 | 1 |

State table
Excitation table

How should we do this?

## JK Flip-Flop Refresher


[ Figure 5.16a from the textbook]

## JK Flip-Flop Refresher


(a) Circuit

| $J$ | $K$ | $Q(t+1)$ |
| :---: | :---: | :---: |
| 0 | 0 | $Q(t)$ |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | $\bar{Q}(t)$ |

(b) Truth table

(c) Graphical symbol
[ Figure 5.16 from the textbook ]

## Let's derive the state table

| Present state | Next state | Output <br> Z | Present state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | w = 0 w = 1 |  |  |  | 0 | w |  |  |
| A | ? | 0 |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| B |  | 0 | 00 | 01 | 01 | 00 | 11 | 0 |
| C |  | 0 | 01 | 01 | 01 | 10 | 11 | 0 |
| D |  | 1 | 10 | 01 | 01 | 00 | 10 | 0 |
|  |  |  | 11 | 01 | 01 | 10 | 10 | 1 |

State table
Excitation table

How should we do this?

## Let's derive the state table

| Present <br> state | Next state |  |
| :---: | :---: | :---: |
|  | Output |  |
| $\mathrm{z}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |


| $J$ | $K$ | $Q(t+1)$ |
| :---: | :---: | :---: |
| 0 | 0 | $Q(t)$ |
| 0 | 1 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | $\bar{Q}(t)$ |

## Let's derive the state table

| Present <br> state | Next state |  |
| :---: | :---: | :---: |
|  | Output |  |
| $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |


| $J$ | $K$ | $Q(t+1)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $J K$ | $Q(t+1)$ |  |  |
| 0 | 0 | $Q(t)$ |  | 0 | 0 |
| 0 | 1 | 0 |  | 0 | 0 |
| 0 | $(t)$ | 0 |  |  |  |
| 1 | 0 | 1 |  | 1 | 0 |
| 1 | 1 | $\bar{Q}(t)$ |  | 1 | 1 |
| 1 | 1 | $\bar{Q}(t)$ |  |  |  |

## Let's derive the state table

| Present state | Next state | Output <br> Z | Presen state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |  | $w=0$ |  | $w=1$ |  |  |
| A |  |  |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| B |  |  | 00 | 01 | 01 | 00 | 11 | 0 |
| C |  |  | 01 | 01 | 01 | 10 | 11 | 0 |
| D |  |  | 10 | 01 | 01 | 00 | 10 | 0 |
|  |  |  | 11 | 01 | 01 | 10 | 10 | 1 |
|  | Note that $\mathrm{A}=00$ |  | K | Q(t |  | J K | $Q(t+1)$ |  |
|  |  |  | $\begin{array}{ll} \hline & 0 \\ 0 & 1 \\ 0 & 1 \end{array}$ | $\frac{\mathrm{Q}}{0} \mathrm{t}$ |  | $\begin{array}{ll} \hline Q & 0 \\ 0 & 1 \end{array}$ | $\begin{aligned} & Q(t) \\ & 0 \end{aligned}$ |  |
|  |  |  | 10 | 1 |  | 10 | 1 |  |
|  |  |  | 11 | $\overline{\mathrm{Q}}(\mathrm{t})$ |  | 11 | $\overline{\mathrm{Q}}$ (t) |  |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A |  | 0 |
| B |  | $?$ | 0 |
| C |  |  | 0 |
| D |  |  | 1 |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |


| $J$ | $K$ | $Q(t+1)$ |  | $J K$ | $Q(t+1)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | $Q(t)$ |  | 0 | 0 |
| 0 | 1 | 0 |  | $Q(t)$ |  |
| 1 | 0 | 1 |  | 1 | 0 |
| 1 | 1 | $\bar{Q}(t)$ |  | 1 | 1 | $\bar{Q}(t)$

## Let's derive the state table

| Present <br> state | Next state |  |
| :---: | :---: | :---: |
|  | Output |  |
| $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A | A | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |


| $\begin{gathered} \text { Present } \\ \text { state } \\ y_{2} y_{1} \end{gathered}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |

$\left.\begin{array}{cc|ccc|c}J & K & Q(t+1) \\ & & J K & Q(t+1) \\ \hline 0 & 0 & Q(t) & & 0 & 0 \\ 0 & 1 & 0 & & Q(t) \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & & 1 & 0 \\ 1 & 1 & \bar{Q}(t) & & 1 & 1\end{array}\right] \quad \bar{Q}(t)$

## Let's derive the state table

| Present <br> state | Next state |  |
| :---: | :---: | :---: |
|  | Output |  |
| $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A | A | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |


| $\begin{gathered} \text { Present } \\ \text { state } \\ y_{2} y_{1} \end{gathered}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |


| J K | $Q(t+1)$ | J K | $Q(t+1)$ |
| :---: | :---: | :---: | :---: |
| 00 | Q (t) | 00 | Q (t) |
| 01 | 0 | 01 | 0 |
| 10 | 1 | 10 | 1 |
| 11 | $\overline{\mathrm{Q}}(\mathrm{t})$ | 11 | $\bar{Q}(\mathrm{t})$ |

## Let's derive the state table

| Present <br> state | Next state |  |
| :---: | :---: | :---: |
|  | Output |  |
| $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A | A | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |



## Let's derive the state table

| Present state | Next state | Output z |
| :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |
| A | A | 0 |
| B |  | 0 |
| C |  | 0 |
| D |  | 1 |



## Let's derive the state table

| Present state | Next state | Output <br> z | Presen state $y_{2} y_{1}$ | Flip-flop inputs |  |  |  | Outpu <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0 \quad w=1$ |  |  | $w=0$ |  | $w=1$ |  |  |
| A | A | 0 |  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| B |  |  | 00 | 01 | 01 | 00 | 11 | 0 |
| C |  |  | 01 | 01 | 01 | 10 | 11 | 0 |
| D |  |  | 10 | 01 | 01 | 00 | 10 | 0 |
|  |  |  | 11 | 01 | 01 | 10 | 10 | 1 |
|  | Note that C $=10$ |  | J K | Q(t) |  | J K | $Q(t+1)$ |  |
|  |  |  | Q 0 | Q (t) |  | 0 | Q (t) |  |
|  |  |  |  | 0 |  | 01 |  |  |
|  |  |  | 10 | 1 |  | 10 |  |  |
|  |  |  | 11 | $\overline{\mathrm{Q}}(\mathrm{t})$ |  | 11 | $\bar{Q}(\mathrm{t})$ |  |

## The two tables for the initial circuit

| Present <br> state | Next state |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |


| $\begin{aligned} & \text { Present } \\ & \text { state } \\ & y_{2} y_{1} \end{aligned}$ | Flip-flop inputs |  |  |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $w=0$ |  | $w=1$ |  |  |
|  | $J_{2} K_{2}$ | $J_{1} K_{1}$ | $J_{2} K_{2}$ | $J_{1} K_{1}$ |  |
| 00 | 01 | 01 | 00 | 11 | 0 |
| 01 | 01 | 01 | 10 | 11 | 0 |
| 10 | 01 | 01 | 00 | 10 | 0 |
| 11 | 01 | 01 | 10 | 10 | 1 |

State table

Excitation table

## The state diagram



State diagram

## The state diagram

Thus, this FSM is identical to the one in the previous example, even though the circuit uses JK flip-flops.

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table


State diagram

## Yet Another Example (with mixed flip-flops)

## What does this circuit do?


[ Figure 6.79 from the textbook ]

## Approach

- Find the flip-flops
- Outputs of the flip-flops = present state variables
- Inputs of the flip-flops determine the next state variables
- Determine the logical expressions for the outputs
- Given this info it is easy to do the state-assigned table
- Next do the state table
- Finally, draw the state diagram.


## What are the logic expressions?


[ Figure 6.79 from the textbook ]

## What are the logic expressions?



## What are the logic expressions?



## The Excitation Table

$$
\begin{aligned}
\mathrm{D}_{1} & =\mathrm{w}\left(\overline{\mathrm{y}}_{1}+\mathrm{y}_{2}\right) \\
\mathrm{T}_{2} & =\overline{\mathrm{w}} \mathrm{y}_{2}+\mathrm{w} \mathrm{y}_{1} \overline{\mathrm{y}}_{2} \\
\mathrm{z} & =\mathrm{y}_{1} \mathrm{y}_{2}
\end{aligned}
$$

| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | O |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

Excitation table

## Let's derive the state table



| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | Output |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

## Let's derive the state table

| Present state |  | Output <br> z | Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Next state |  |  | $w=0$ | $w=1$ |  |
|  | $w=0 \quad w=1$ |  |  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| A |  |  | 00 | 00 | 01 | 0 |
| C |  |  | 01 | 00 | 10 | 0 |
| D |  |  | 10 | 10 | 01 | 0 |
|  |  |  | 11 | 10 | 01 | 1 |

This step is easy
(map 2-bit numbers to 4 letters)

## Let's derive the state table

| Present state |  | $\begin{gathered} \text { Output } \\ z \end{gathered}$ | Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Next state |  |  | $w=0$ | $w=1$ |  |
|  | $w=0 \quad w=1$ |  |  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| A |  | 0 | 00 | 00 | 04 | - 0 |
| C |  | 0 | 01 | 00 | 10 | 0 |
| D |  | 1 | 10 | 10 | 01 | 0 |
|  |  |  | 11 | 10 | 01 |  |

This step is easy too
(the outputs are the same in both tables)

## Let's derive the state table

| Present state |  | Output <br> Z | Present state $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Next state |  |  | $w=0$ | $w=1$ |  |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ |  |  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| A | ? | 0 |  |  |  |  |
| B |  | 0 | 00 | 00 | 01 | 0 |
| C |  | 0 | 01 | 00 | 10 | 0 |
| D |  | 1 | 10 | 10 | 01 | 0 |
|  |  |  | 11 | 10 | 01 | 1 |

What should we do here?

## Let's derive the state table

| Present state | Next state | $\begin{gathered} \text { Output } \\ \mathrm{z} \\ \hline \end{gathered}$ | Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $w=0$ | $w=1$ |  |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ |  |  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| A | ? | 0 |  |  |  |  |
| B |  | 0 | 00 | 00 | 01 | 0 |
| C |  | 0 | 01 | 00 | 10 | 0 |
| D |  | 1 | 10 | 10 | 01 | 0 |
|  |  |  | 11 | 10 | 01 | 1 |

What should we do here?

| T | $\mathrm{Q}(t+1)$ | D | $\mathrm{Q}(t+1)$ |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t)$ | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | O |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

$\left.\begin{array}{c|cc|c}\mathrm{T} & \mathrm{Q}(t+1) & \mathrm{D} & \mathrm{Q}(t+1) \\ \hline 0 & \mathrm{Q}(t) & & 0 \\ 1 & \mathrm{Q}(t) & & 1\end{array}\right] 1$

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  |  | 0 |
| D |  | 1 |  |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ |  | D |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t+1)$ |  |  |
| 0 | $\mathrm{Q}(t)$ |  |  |
|  | $\overline{\mathrm{Q}}(t)$ |  | 0 |
| 1 | 1 | 1 |  |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Presen <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| d1 | 00 | 10 | 0 |
| 19 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |
| T ${ }^{1}$ | $\mathrm{Q}(2+1)$ | D | $\mathrm{Q}(t+1)$ |
|  |  |  |  |
| 0 | $\mathrm{Q}(t)$ | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ | 1 | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  |  | 0 |
| D |  | 1 |  |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ | D | $\mathrm{Q}(t+1)$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  |  | 0 |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  <br>  <br> $T_{2} D_{1}$ |
| $z$ | $T_{2} D_{1}$ |  |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ |  | D |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t+1)$ |  |  |
| 0 | 0 |  | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 0 |
|  | 1 | 1 |  |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | z |  |
| A |  | 0 |  |
| B |  | 0 |  |
| C |  |  | 0 |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  <br>  <br> $T_{2} D_{1}$ |
| $z$ | $T_{2} D_{1}$ |  |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ |  | D |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t+1)$ |  |  |
| 0 | 0 |  | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 0 |
|  | 1 | 1 |  |

## Let's derive the state table



## Let's derive the state table

| Present state |  |  | Present <br> state <br> $y_{2} y_{1}$ | Flip-flop in | Output <br> z |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Next state | $\begin{gathered} \text { Output } \\ \text { z } \end{gathered}$ |  | $w=0 \quad w=1$ |  |
|  | $w=0 \quad w=1$ |  |  | $\begin{array}{ll}T_{2} D_{1} & T_{2} D_{1}\end{array}$ |  |
| A | A | 0 | 00 | 0001 | 0 |
| C | ? | 0 | 01 | 0010 | 0 |
| D |  | 1 | 10 | $10-01$ | 0 |
|  |  |  | 11 | 1001 | 1 |

What should we do here?
$\left.\begin{array}{c|cc|c}\mathrm{T} & \mathrm{Q}(t+1) & \mathrm{D} & \mathrm{Q}(t+1) \\ \hline 0 & \mathrm{Q}(t) & 0 & 0 \\ 1 & \overline{\mathrm{Q}}(t) & & 1\end{array}\right] 1$

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | O |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

$\left.\begin{array}{c|cc|c}\mathrm{T} & \mathrm{Q}(t+1) & \mathrm{D} & \mathrm{Q}(t+1) \\ \hline 0 & \mathrm{Q}(t) & 0 & 0 \\ 1 & \overline{\mathrm{Q}}(t) & & 1\end{array}\right] 1$

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | O |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ |  | D | $\mathrm{Q}(t+1)$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t)$ |  | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 1 | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present state $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 1 | 10 | 01 | 1 |
|  | $Q(t+1)$ | D | ( $t$ |
|  |  |  |  |
| 0 | $\mathrm{Q}(t)$ | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ | 1 | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Presen state $y_{2} y_{1}$ | Flip-flop inputs |  | $\begin{gathered} \text { Output } \\ z \end{gathered}$ |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 17 | 10 | 01 | 1 |
|  | $Q(x+1)$ | D | $\mathrm{Q}(t+1)$ |
|  |  |  |  |
| 0 | 1 | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ | 1 | 1 |

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present <br> state <br> $y_{2} y_{1}$ | Flip-flop inputs |  | Output <br> z |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ |  |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

$\begin{array}{c|cc|c}\mathrm{T} & \mathrm{Q}(t+1) & & \mathrm{D} \\$\cline { 1 - 2 } \& $\left.\mathrm{Q}(t+1) \\ \hline 0 & 1 & & 0 \\ 1 & \overline{\mathrm{Q}}(t) & & 1\end{array}\right] 1$

## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0 \quad \mathrm{w}=1$ | Z |  |
| A | A | 0 |  |
| B |  | 0 |  |
| C |  | 0 |  |
| D |  | 1 |  |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | Output |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | $0 \dagger$ | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ |  | D |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{Q}(t+1)$ |  |  |
| 0 | 1 |  | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 1 |

## Let's derive the state table



## Let's derive the state table

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | O |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |


| T | $\mathrm{Q}(t+1)$ | D | $\mathrm{Q}(t+1)$ |
| :---: | :---: | :---: | :---: |
| 0 | $\mathrm{Q}(t)$ | 0 | 0 |
| 1 | $\overline{\mathrm{Q}}(t)$ |  | 1 |

## The two tables for the initial circuit

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |


| Present <br> state | Flip-flop inputs |  |  |
| :---: | :---: | :---: | :---: |
|  | $w=0$ | $w=1$ | Output |
|  | $T_{2} D_{1}$ | $T_{2} D_{1}$ |  |
| 00 | 00 | 01 | 0 |
| 01 | 00 | 10 | 0 |
| 10 | 10 | 01 | 0 |
| 11 | 10 | 01 | 1 |

State table
Excitation table

## The state diagram



State diagram

## The state diagram

Thus, this FSM is identical to the ones in the previous examples, even though the circuit uses one D and one T flip-flop.

| Present <br> state | Next state |  | Output |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{w}=0$ | $\mathrm{w}=1$ |  |
| A | A | B | 0 |
| B | A | C | 0 |
| C | A | D | 0 |
| D | A | D | 1 |

State table


State diagram

## Questions?

## THE END

