HCI/CprE/ComS 575: Computational Perception
Homework 1 (part 1)
Out: Friday Jan 20, 2023
Due: Thursday Feb 2, 2023

For extra credit submit part 1 (only part 1) by Sunday Jan 22 before midnight.
(upload a scanned copy (as only 1 PDF) on Canvas)

These problems are designed to test your background knowledge in linear algebra and math in general. Please show the intermediary steps of your calculations (not just the final answers).

1. What is 1+1?
2. What is 1+1 in the binary system?
3. What is 2+2?
4. What is 2+2 in the ternary number system (i.e., one with base 3)?
5. Convert the decimal number 145 to binary.
6. In the hexadecimal system, what is the value of B + C?
7. Convert the hexadecimal number C0FFEE to binary.
8. Solve the following equations for x.

   a) $4(2x-3) = 8(2x+5)$ b) $9(2x-1) - 3x = 3(12+x)$ c) $\frac{2}{7x+3} = \frac{9}{2x-5}$
9. Solve the following quadratic equations:

   a) $x^2 - 11x + 30 = 0$ b) $x^2 + 6x - 27 = 0$ c) $2x^2 + 3x - 20 = 0$
10. Solve the following systems of equations:

    a)
    $$\begin{aligned} 4x - 3y &= -3 \\ 2x - 5y &= 9 \end{aligned}$$
    b)
    $$\begin{aligned} x + 2y + 3z &= 5 \\ 2x + 5y + 3z &= 3 \\ x + 8z &= 17 \end{aligned}$$
    c)
    $$\begin{aligned} x + 3y + z &= 4 \\ 2x + 2y + z &= -1 \\ 2x + 3y + z &= 3 \end{aligned}$$
    d)
    $$\begin{aligned} -x - 2y - 3z &= 0 \\ w + x + 4y + 4z &= 7 \\ w + 3x + 7y + 9z &= 4 \\ -w - 2x - 4y - 6z &= 6 \end{aligned}$$
11. Two standard dice are rolled. One is red and one is green. What is the probability that the sum of the two numbers on top is equal to 7?

12. Compute the value of "n choose k", i.e., $\begin{pmatrix} n \\ k \end{pmatrix}$, for $n = 7$ and $k = 4$.

13. Find a formula for the sum $\sum_{k=0}^{n} 2^k$.

14. Derive a formula for the following sum $1 + 2 + 3 + \ldots + n$.

15. Given that $1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ derive a similar formula for $1^2 + 2^2 + 3^2 + \ldots + (n-1)^2$.

16. What is the value of $\log_2 1024$?

17. Compute $\int_0^1 x \, \mathrm{d}x$.

18. Compute $\int_0^1 x^2 \, \mathrm{d}x$.

19. Compute $\int_0^1 (\cos^2 x + \sin^2 x) \, \mathrm{d}x$.

20. Prove the Pythagorean theorem in two different ways.

21. Let A be an $m \times n$ matrix and let x be a column vector of size n. How many additions and multiplications are required to compute the matrix-vector product Ax?

22. Let A be an $m \times n$ matrix and let B be an $n \times p$ matrix. How many additions and multiplications are required to compute the matrix product $A\,B$?

23. What is the computational complexity, using big-O notation, for the following algorithms: Bubble sort, Insertion sort, Quicksort, Linear search, Binary search, Matrix-Vector product, Matrix-Matrix product, Matrix inversion, FFT, IFFT? Assume that all arrays and vectors are of size N and all matrices are $N \times N$.

24. Calculate the following determinants.

$$(a) \begin{vmatrix} 1 & 3 \\ -6 & 5 \end{vmatrix} \quad (b) \begin{vmatrix} \alpha - 3 & 7 \\ -4 & \alpha - 2 \end{vmatrix} \quad (c) \begin{vmatrix} 1 & 2 & 3 \\ 2 & 9 & 3 \\ 1 & 0 & 4 \end{vmatrix} \quad (d) \begin{vmatrix} -2 & 4 & 2 \\ 2 & 1 & -1 \\ 4 & 7 & 3 \end{vmatrix}$$

25. Show that the value of the determinant does not depend on θ.

$$\begin{vmatrix} sin\theta & cos\theta & 0 \\ -cos\theta & sin\theta & 0 \\ sin\theta - cos\theta & sin\theta + cos\theta & 1 \end{vmatrix}$$

26. Plot the functions $\sin(x)$, $\sin(2x)$, and $2\sin(x)$ on the same plot.

27. Draw a table with the values of $\sin(x)$, $\cos(x)$, and $\tan(x)$ for $x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$.

28. Calculate the inverse of the given matrix.

$$(a)\ \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \quad (b)\ \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix} \quad (c)\ \begin{bmatrix} 3 & 4 & -1 \\ 1 & 0 & 3 \\ 2 & 5 & -4 \end{bmatrix} \quad (d)\ \begin{bmatrix} 0 & 0 & 0 & k_1 \\ 0 & 0 & k_2 & 0 \\ 0 & k_3 & 0 & 0 \\ k_4 & 0 & 0 & 0 \end{bmatrix}$$

29. Let $u = (3, 2, -1)$, $v = (0, 2, -3)$, and $w = (2, 6, 7)$. Compute:

$$(a)\ u \cdot v \qquad (b) \|u\| \qquad (c)\ v \times w \qquad (d)\ u \times (v \times w) \qquad (e)\ (u \times v) \times w$$

30. Find the equation for the plane that passes through the points: $P(-4, -1, -1), Q(-2, 0, 1), R(-1, -2, -3)$.

31. Find all values of λ for which the determinant of the matrix is equal to 0.

$$(a)\ \begin{vmatrix} \lambda - 3 & 0 \\ 8 & \lambda + 1 \end{vmatrix} \quad (b)\ \begin{vmatrix} \lambda & 3 \\ 4 & \lambda \end{vmatrix} \quad (c)\ \begin{vmatrix} \lambda - 5 & 0 & 1 \\ 1 & \lambda - 1 & 0 \\ -7 & 1 & \lambda \end{vmatrix}$$

32. Find the eigenvalues of the following matrices.

$$(a)\ \begin{bmatrix} -1 & 6 \\ 0 & 5 \end{bmatrix} \quad (a)\ \begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & -2 \end{bmatrix} \quad (b)\ \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$$

33. Given the vector $v = [x, y]^T$ find a rotation matrix R that rotates the vector by 60° counterclockwise. Give the values for the vector $v' = [x', y']^T$ in terms of x and y, where $v' = Rv$.

34. By example, show that matrix multiplication is not commutative.

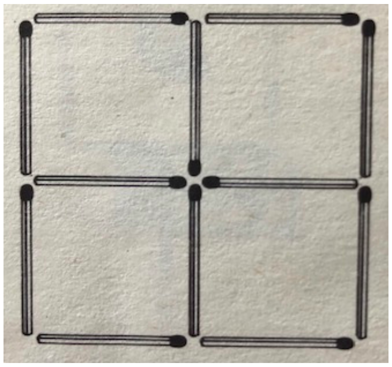
35. By example, show that vector cross-product is not commutative.

36. Prove that any pair of 2×2 upper-triangular matrices commute under multiplication.

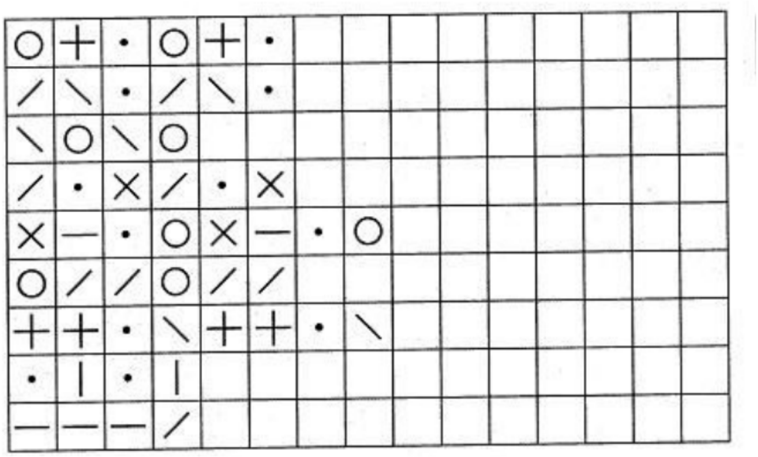
37. Give the formulas for the 1D and 2D probability density function of the normal distribution.

38. Give the formulas for the discrete Fourier transform and its inverse. Explain their meaning with one paragraph for each.

39. The figure below is constructed with safety matches. Move only four of them such that the new figure contains exactly 3 squares. Draw a picture of your solution. If it is not possible, say that it can't be done.



40. Find the pattern and complete the sequence in each row.



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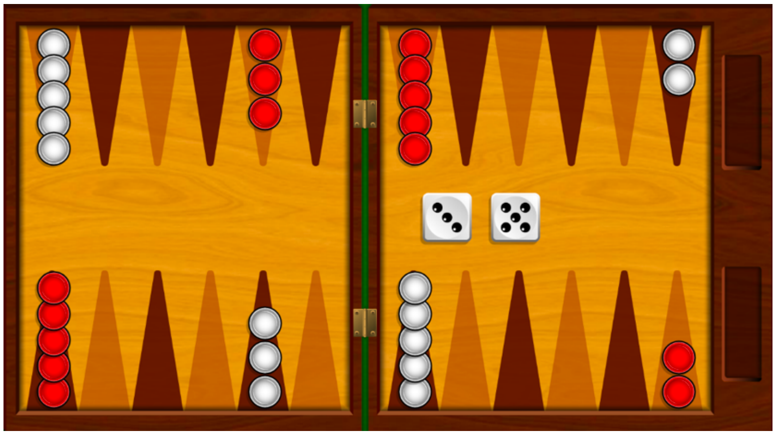
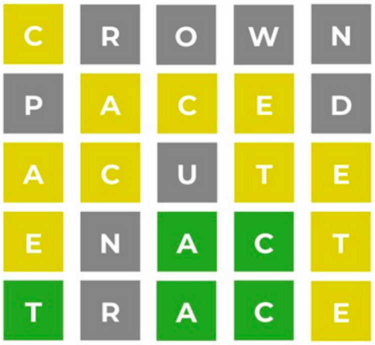
41. Find the sum of the numbers on the two dice shown in this picture.



42. Solve the Wordle puzzle shown in this picture.



C	R	O	W	N
P	A	C	E	D
A	C	U	T	E
E	N	A	C	T
T	R	A	C	E

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Download matlab on your computer. Then, take the “Matlab Onboarding” tutorial. As you progress through the tutorial, take several screenshots (start, middle, and end) to document that you did this task. Attach these images to your solutions for Part2.

The Matlab book, which is a required textbook for this class, shows several plot types on pages 170-174 (old edition of the book) or pages 184-188 (new edition).

Pick one plot type from each page and try to draw something similar, but it should not be the exact same plot as in the book. For each problem submit your matlab file and an output image. Name your files: myplot1.m, myplot2.m, ... , myplot5.m. Similarly, name your output images myoutput1.jpg, myoutput2.jpg, ... , myoutput5.jpg. (PNG or GIF file format is also acceptable).

Upload a ZIP file with all of your files to Canvas.

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Homework 1 (part 3)
Out: Friday Jan 20, 2023
Due: Thursday Feb 2, 2023

Solve 10 of the following 19 problems (see the following pages). You can pick any 10. Your grade will not depend on your problem selection. Submit a separate matlab file (and any supporting files if needed) for each problem. Name your files using the problem and page number. For example, your solution file for exercise 3 on page 84 should be named e3_page84.m

Upload a ZIP file with all of your files to Canvas.