Piano key frequencies

This is a list of the fundamental frequencies in hertz (cycles per second) of the keys of a modern 88-key standard or 108-key extended piano in twelve-tone equal temperament, with the 49th key, the fifth A (called A_4), tuned to 440 Hz (referred to as A440).[1][2] Every octave is made of twelve steps called semitones. A jump from the lowest semitone to the highest semitone in one octave doubles the frequency (for example, the fifth A is 440 Hz and the sixth A is 880 Hz). The frequency of a pitch is derived by multiplying (ascending) or dividing (descending) the frequency of the previous pitch by the twelfth root of two (approximately 1.059463).[1][2] For example, to get the frequency one semitone up from A_4 ($A\sharp_4$), multiply 440 Hz by the twelfth root of two. To go from A_4 up two semitones (one whole tone) to B_4, multiply 440 twice by the twelfth root of two (or once by the sixth root of two, approximately 1.122462). To go from A_4 up three semitones to C_5 (a minor third), multiply 440 Hz three times by the twelfth root of two (or once by the fourth root of two, approximately 1.189207). For other tuning schemes, refer to musical tuning.

This list of frequencies is for a theoretically ideal piano. On an actual piano, the ratio between semitones is slightly larger, especially at the high and low ends, where string stiffness causes inharmonicity, i.e., the tendency for the harmonic makeup of each note to run sharp. To compensate for this, octaves are tuned slightly wide, stretched according to the inharmonic characteristics of each instrument.[3] This deviation from equal temperament is called the Railsback curve.

The following equation gives the frequency f (Hz) of the n^{th} key on the idealized standard piano with the 49th key tuned to A_4 at 440 Hz:

$$f(n) = \left(\sqrt[12]{2}\right)^{n-49} \times 440\,\text{Hz} = 2^{\frac{n-49}{12}} \times 440\,\text{Hz}$$

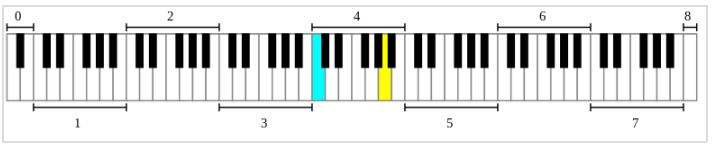
where n is shown in the table below.[1]

Conversely, the key number of a pitch with a frequency f (Hz) on the idealized standard piano is:

$$n = 12\,\log_2\left(\frac{f}{440\,\text{Hz}}\right) + 49$$

List

A printable version of the standard key frequencies (only including the 88 keys on a standard piano)



An 88-key piano, with the octaves numbered and Middle C (cyan) and A440 (yellow) highlighted

Values in **bold** are exact on an idealized standard piano. Keys shaded gray are rare and only appear on extended pianos. The normal 88 keys were numbered 1–88, with the extra low keys numbered 89–97 and the extra high keys numbered 98–108. A 108-key piano that extends from C_0 to B_8 was first built in 2018 by Stuart & Sons.[4] (Note: these piano key numbers 1-108 are not the n keys in the equations or the table.)

Piano key number	MIDI note number	Helmholtz name[5]	Scientific pitch name[5]	*n*	Frequency *f*(*n*) (Hz) (Equal temperament) [6]	Corresponding open strings on other instruments					
						Violin	Viola	Cello	Bass	Guitar	Ukulele
108	119	b′′′′′	B_8	99	7902.133						
107	118	a♯′′′′′/b♭′′′′′	$A♯_8/B♭_8$	98	7458.620						
106	117	a′′′′′	A_8	97	**7040.000**						
105	116	g♯′′′′′/a♭′′′′′	$G♯_8/A♭_8$	96	6644.875						
104	115	g′′′′′	G_8	95	6271.927						
103	114	f♯′′′′′/g♭′′′′′	$F♯_8/G♭_8$	94	5919.911						
102	113	f′′′′′	F_8	93	5587.652						
101	112	e′′′′′	E_8	92	5274.041						
100	111	d♯′′′′′/e♭′′′′′	$D♯_8/E♭_8$	91	4978.032						
99	110	d′′′′′	D_8	90	4698.636						
98	109	c♯′′′′′/d♭′′′′′	$C♯_8/D♭_8$	89	4434.922						
88	108	c′′′′′ 5-line octave	C_8 Eighth octave	88	4186.009						
87	107	b′′′′	B_7	87	3951.066						
86	106	a♯′′′′/b♭′′′′	$A♯_7/B♭_7$	86	3729.310						
85	105	a′′′′	A_7	85	**3520.000**						
84	104	g♯′′′′/a♭′′′′	$G♯_7/A♭_7$	84	3322.438						
83	103	g′′′′	G_7	83	3135.963						
82	102	f♯′′′′/g♭′′′′	$F♯_7/G♭_7$	82	2959.955						
81	101	f′′′′	F_7	81	2793.826						
80	100	e′′′′	E_7	80	2637.020						
79	99	d♯′′′′/e♭′′′′	$D♯_7/E♭_7$	79	2489.016						
78	98	d′′′′	D_7	78	2349.318						
77	97	c♯′′′′/d♭′′′′	$C♯_7/D♭_7$	77	2217.461						
76	96	c′′′′ 4-line octave	C_7 Double high C	76	2093.005						
75	95	b′′′	B_6	75	1975.533						
74	94	a♯′′′/b♭′′′	$A♯_6/B♭_6$	74	1864.655						
73	93	a′′′	A_6	73	**1760.000**						
72	92	g♯′′′/a♭′′′	$G♯_6/A♭_6$	72	1661.219						
71	91	g′′′	G_6	71	1567.982						
70	90	f♯′′′/g♭′′′	$F♯_6/G♭_6$	70	1479.978						
69	89	f′′′	F_6	69	1396.913						
68	88	e′′′	E_6	68	1318.510						
67	87	d♯′′′/e♭′′′	$D♯_6/E♭_6$	67	1244.508						
66	86	d′′′	D_6	66	1174.659						
65	85	c♯′′′/d♭′′′	$C♯_6/D♭_6$	65	1108.731						
64	84	c′′′ 3-line octave	C_6 Soprano C (High C)	64	1046.502						
63	83	b′′	B_5	63	987.7666						
62	82	a♯′′/b♭′′	$A♯_5/B♭_5$	62	932.3275						
61	81	a′′	A_5	61	**880.0000**						
60	80	g♯′′/a♭′′	$G♯_5/A♭_5$	60	830.6094						
59	79	g′′	G_5	59	783.9909						
58	78	f♯′′/g♭′′	$F♯_5/G♭_5$	58	739.9888						
57	77	f′′	F_5	57	698.4565						
56	76	e′′	E_5	56	659.2551	E	E (5 String Viola)				
55	75	d♯′′/e♭′′	$D♯_5/E♭_5$	55	622.2540						

Piano key number	MIDI note number	Helmholtz name[5]	Scientific pitch name[5]	*n*	Frequency *f*(*n*) (Hz) (Equal temperament) [6]	Corresponding open strings on other instruments					
						Violin	Viola	Cello	Bass	Guitar	Ukulele
54	74	d′′	D_5	54	587.3295						
53	73	c♯′′/d♭′′	$C♯_5/D♭_5$	53	554.3653						
52	72	c′′ 2-line octave	C_5 Tenor C	52	523.2511						
51	71	b′	B_4	51	493.8833					High B (Optional for 12 String Guitar)	
50	70	a♯′/b♭′	$A♯_4/B♭_4$	50	466.1638						
49	69	a′	A_4 A440	49	**440.0000**	A	A			High A (Optional)	A
48	68	g♯′/a♭′	$G♯_4/A♭_4$	48	415.3047				High Ab (12 Single String Bass)		
47	67	g′	G_4	47	391.9954						High G
46	66	f♯′/g♭′	$F♯_4/G♭_4$	46	369.9944						
45	65	f′	F_4	45	349.2282						
44	64	e′	E_4	44	329.6276			High E (5 String Cello)		High E	E
43	63	d♯′/e♭′	$D♯_4/E♭_4$	43	311.1270				High Eb (12 String Single String Bass)		
42	62	d′	D_4	42	293.6648	D	D				
41	61	c♯′/d♭′	$C♯_4/D♭_4$	41	277.1826						
40	60	c′ 1-line octave	C_4 Middle C	40	261.6256						C
39	59	b	B_3	39	246.9417					B	
38	58	a♯/b♭	$A♯_3/B♭_3$	38	233.0819						
37	57	a	A_3	37	**220.0000**			A			
36	56	g♯/a♭	$G♯_3/A♭_3$	36	207.6523						
35	55	g	G_3	35	195.9977	G	G			G	Low G
34	54	f♯/g♭	$F♯_3/G♭_3$	34	184.9972				High F (7 String)		
33	53	f	F_3	33	174.6141						
32	52	e	E_3	32	164.8138						
31	51	d♯/e♭	$D♯_3/E♭_3$	31	155.5635						
30	50	d	D_3	30	146.8324			D		D	
29	49	c♯/d♭	$C♯_3/D♭_3$	29	138.5913						
28	48	c small octave	C_3	28	130.8128	C (5 String)	C		C (6 string)		
27	47	B	B_2	27	123.4708						
26	46	A♯/B♭	$A♯_2/B♭_2$	26	116.5409						
25	45	A	A_2	25	**110.0000**					A	
24	44	G♯/A♭	$G♯_2/A♭_2$	24	103.8262						
23	43	G	G_2	23	97.99886			G	G		
22	42	F♯/G♭	$F♯_2/G♭_2$	22	92.49861						
21	41	F	F_2	21	87.30706	F (6 String)	F (6 String)				
20	40	E	E_2	20	82.40689					Low E	
19	39	D♯/E♭	$D♯_2/E♭_2$	19	77.78175						
18	38	D	D_2	18	73.41619				D		
17	37	C♯/D♭	$C♯_2/D♭_2$	17	69.29566						
16	36	C great octave	C_2 Deep C	16	65.40639			C			

Piano key number	MIDI note number	Helmholtz name[5]	Scientific pitch name[5]	*n*	Frequency *f*(*n*) (Hz) (Equal temperament) [6]	Corresponding open strings on other instruments					
						Violin	Viola	Cello	Bass	Guitar	Ukulele
15	35	$B_{,}$	B_1	15	61.73541					Low B (7 string)	
14	34	$A♯_{,}/B♭_{,}$	$A♯_1/B♭_1$	14	58.27047						
13	33	$A_{,}$	A_1	13	**55.00000**				A		
12	32	$G♯_{,}/A♭_{,}$	$G♯_1/A♭_1$	12	51.91309						
11	31	$G_{,}$	G_1	11	48.99943						
10	30	$F♯_{,}/G♭_{,}$	$F♯_1/G♭_1$	10	46.24930					Low F♯ (8 string)	
9	29	$F_{,}$	F_1	9	43.65353						
8	28	$E_{,}$	E_1	8	41.20344				E		
7	27	$D♯_{,}/E♭_{,}$	$D♯_1/E♭_1$	7	38.89087						
6	26	$D_{,}$	D_1	6	36.70810						
5	25	$C♯_{,}/D♭_{,}$	$C♯_1/D♭_1$	5	34.64783					Low C#(9 String)	
4	24	$C_{,}$ contra-octave	C_1 Pedal C	4	32.70320				C (Upright Extension)		
3	23	$B_{,,}$	B_0	3	30.86771				B (5 string)		
2	22	$A♯_{,,}/B♭_{,,}$	$A♯_0/B♭_0$	2	29.13524						
1	21	$A_{,,}$	A_0	1	**27.50000**						
97	20	$G♯_{,,}/A♭_{,,}$	$G♯_0/A♭_0$	0	25.95654					Low G# (10 String)	
96	19	$G_{,,}$	G_0	-1	24.49971						
95	18	$F♯_{,,}/G♭_{,,}$	$F♯_0/G♭_0$	-2	23.12465						
94	17	$F_{,,}$	F_0	-3	21.82676						
93	16	$E_{,,}$	E_0	-4	20.60172						
92	15	$D♯_{,,}/E♭_{,,}$	$D♯_0/E♭_0$	-5	19.44544						
91	14	$D_{,,}$	D_0	-6	18.35405						
90	13	$C♯_{,,}/D♭_{,,}$	$C♯_0/D♭_0$	-7	17.32391						
89	12	$C_{,,}$ sub-contra-octave	C_0 Double Pedal C	-8	16.35160						

See also

- Piano tuning
- Scientific pitch notation
- Music and mathematics

References

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External links

- interactive piano frequency table (http://shakahara.com/pianopitch2.php) – A PHP script allowing the reference pitch of A4 to be altered from 440 Hz.
- PySynth (https://web.archive.org/web/20190510200224/https://mdoege.github.io/PySynth/) – A simple Python-based software synthesizer that prints the key frequencies table and then creates a few demo songs based on that table.
- "Keyboard and frequencies (http://www.sengpielaudio.com/calculator-notenames.htm)", *SengpielAudio.com*.
- Notefreqs (http://www.deimos.ca/notefreqs) – A complete table of note frequencies and ratios for midi, piano, guitar, bass, and violin. Includes fret measurements (in cm and inches) for building instruments.

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