

An example for  $\text{sup PCN}_L(K)$ :

①

$$L = \text{pr}(aa + baa + aba + bba)$$

$$K = \text{pr}(aba + ba + bba)$$

$$M(a) = M(b) \neq \varepsilon$$

Answer:  $\text{sup PCN}(K) = \text{pr}(a+b)$  if  $\Sigma_u = \emptyset$   
 $\text{sup PCN}(K) = \emptyset$  if  $\Sigma_u = \{b\}$

If the alg for computing  $\text{sup PCN}_L(K)$  is as following:

1.  $K_0 := K$ ,  $i=0$ .

2.  $K'_i = \text{sup PC}(K_i)$

$$K_{i+1} = K'_i - [M^T M (L - K'_i)] \Sigma^*$$

3. if  $K_{i+1} \neq K_i$ , let  $i=i+1$  and go back to 2. otherwise, stop.

Then 1st iteration:  $K'_0 = \text{sup PC}(K_0) = K$

$$K_1 = K - [M^T M (L - K)] \Sigma^* = \text{pr}(a+b)$$

$$K_1 \neq K_0$$

2nd iteration:  $K'_1 = \text{sup PC}(K_1) = \begin{cases} K_1 & \text{if } \Sigma_u = \emptyset \\ \emptyset & \text{if } \Sigma_u = \{b\} \end{cases}$

$$K_2 = \begin{cases} K_1 - [M^T M (L - K_1)] \Sigma^* = K_1 & \text{if } \Sigma_u = \emptyset \\ \emptyset & \text{if } \Sigma_u = \{b\} \end{cases}$$

$$\text{So, } \text{sup PCN}_L(K) = \begin{cases} \text{pr}(a+b) & \text{if } \Sigma_u = \emptyset \\ \emptyset & \text{if } \Sigma_u = \{b\} \end{cases}$$

Another Example:

$$L = \text{pr}(aa + ab)$$

$$K = \text{pr}(a + b)$$

$$M(a) = \varepsilon \quad M(b) = b$$

Answer:  $\sup PCN_L(K) = \emptyset$  for either  $\Sigma_u = \emptyset$  or  $\Sigma_u \neq \emptyset$